Second-Order Anisotropic Hydrodynamics

Michael Strickland

Primary References and Collaborators

D. Bazow, U. Heinz, and MS, forthcoming (this week?)
F. Florkowski, R. Ryblewski, and MS, 1305.7234
M. Martinez, R. Ryblewski, and MS, 1204.1473
M. Martinez and MS, arXiv:1007.0889

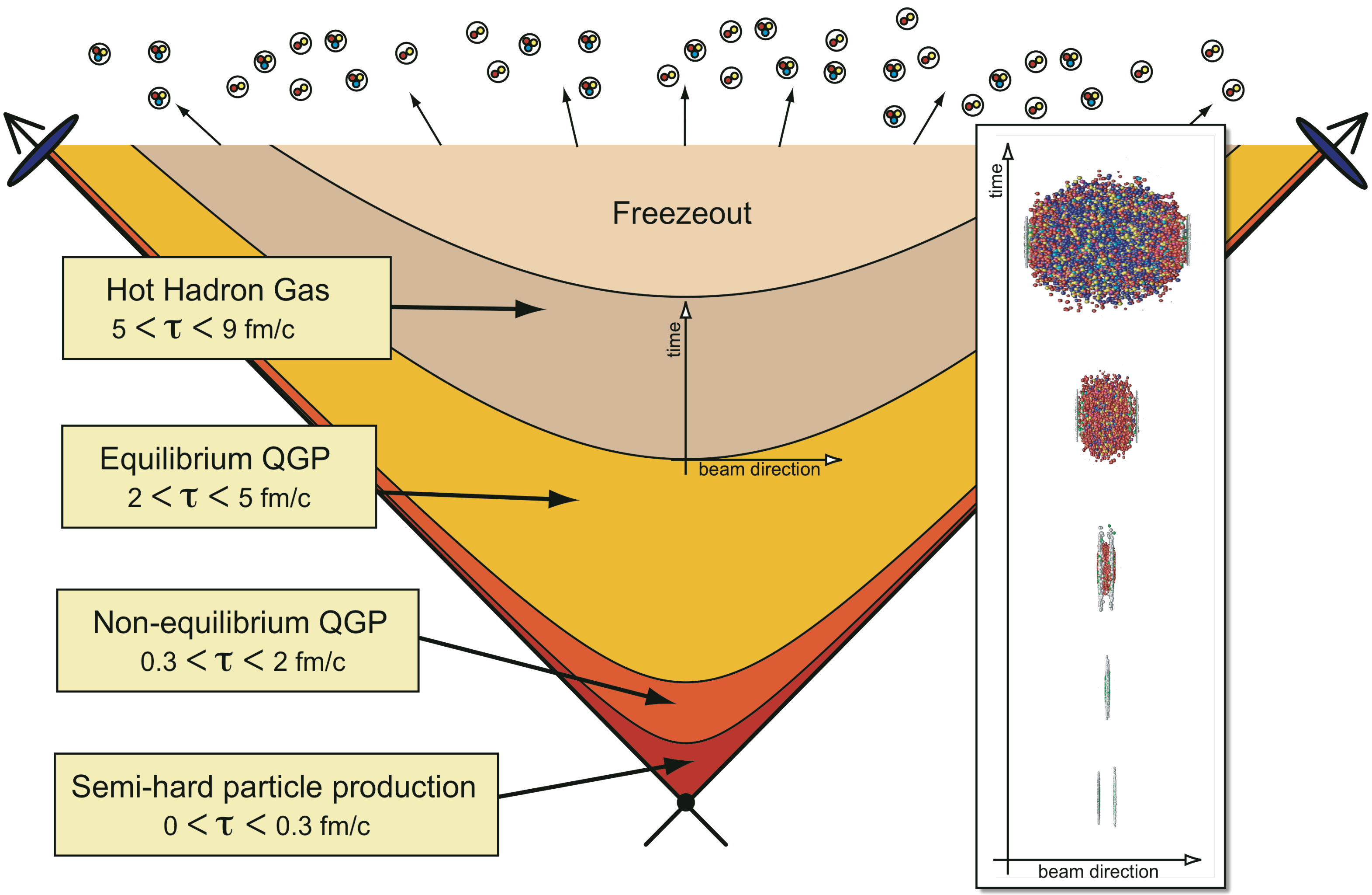
New Frontiers in QCD 2013

--- *Insight into QCD matter from heavy-ion collisions* ---

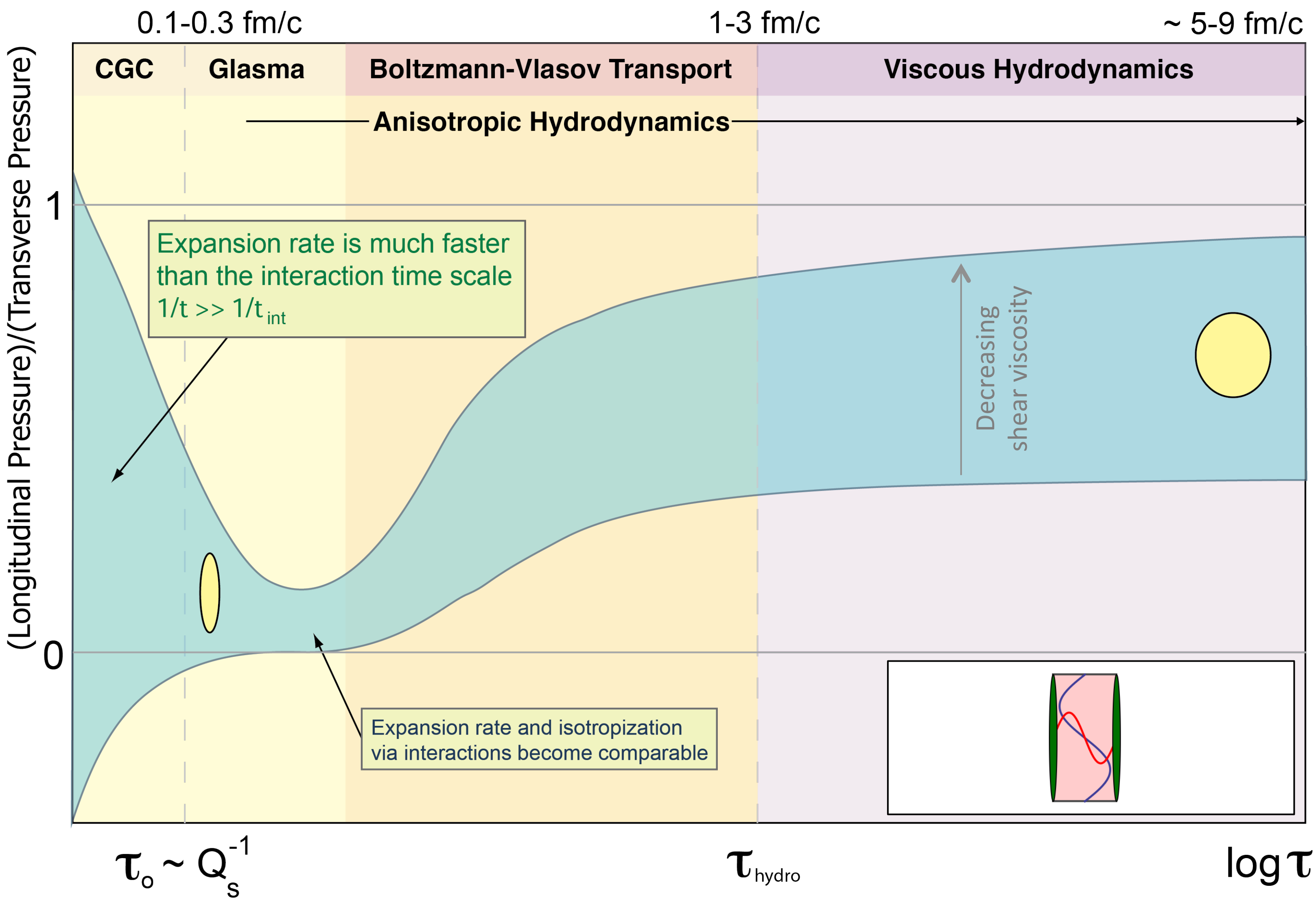
Motivation

- Relativistic viscous hydrodynamical modeling for heavy ion collisions at RHIC and LHC is ubiquitous
- Application is justified a priori by the (relative) smallness of the shear viscosity of the plasma
- The canonical way to derive viscous hydrodynamics relies on a linearization around an isotropic equilibrium state
- However, the **QGP is not isotropic** → there are large corrections to ideal hydrodynamics due to strong longitudinal expansion
- Alternative approach: Anisotropic hydrodynamics builds in momentum-space anisotropies in the local rest frame from the beginning
- The goal is to create a quantitatively reliable viscous-hydro-like code that more accurately describes:
    - Early time dynamics
    - Dynamics near the transverse edges of the overlap region
    - Temperature-dependent (and potentially large) η/S

LHC Heavy Ion Collision Timescales



QGP momentum anisotropy cartoon



Estimating Early-time Pressure Anisotropy

- CGC @ leading order predicts negative → approximately zero longitudinal pressure
- QGP scattering + plasma instabilities work to drive the system <u>towards</u> isotropy on the fm/c timescale, but don't seem to fully restore it [see e.g. talk by M. Attems, F. Gelis, and perhaps others in this program]
- Viscous hydrodynamics predicts early-time anisotropies ≤ 0.35 → 0.5 (see next slide)
- AdS-CFT dynamical calculations in the strong coupling limit predict anisotropies of ≤ 0.3 (discussion in three slides from now)

Estimating Anisotropy – Viscous hydro

- To get a feel for the magnitude of pressure anisotropies to expect let's consider the Navier-Stokes limit

$$\left(\frac{\mathcal{P}_L}{\mathcal{P}_T}\right)_{\rm NS} = \frac{P_{\rm eq} + \pi^{zz}_{\rm NS}}{P_{\rm eq} + \pi^{xx}_{\rm NS}} = \frac{3\tau T - 16\bar\eta}{3\tau T + 8\bar\eta}$$

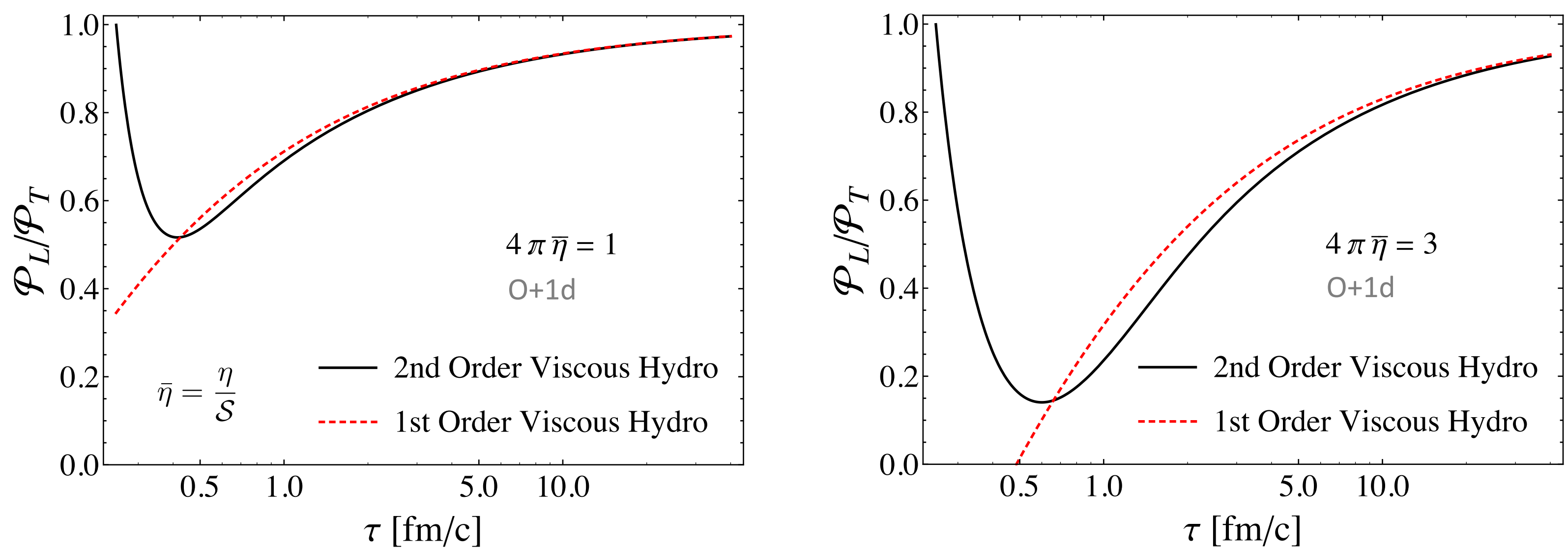
$$\bar\eta = \frac{\eta}{\mathcal{S}}$$

$$\pi^{zz}_{\rm NS} = -2\pi^{xx}_{\rm NS} = -2\pi^{yy}_{\rm NS} = -4\eta/3\tau$$

- Anisotropy increases with increasing η/S. Assume $\eta/S = 1/4\pi$ in order to get an upper bound on the anisotropy
- Using RHIC initial conditions (T_0 = 400 MeV @ τ_0 = 0.5 fm/c) we obtain $P_L/P_T \leq 0.5$
- Using LHC initial conditions (T_0 = 600 MeV @ τ_0 = 0.25 fm/c) we obtain $P_L/P_T \leq 0.35$
- Negative P_L at large η/S or low temperatures!

Estimating Anisotropy – Viscous hydro

- Navier-Stokes solution is "attractor" for 2nd order solution
- τ_π sets timescale to approach Navier-Stokes evolution
- $\tau_\pi \sim 5\eta/(TS) \sim 0.1$ fm/c at LHC temperatures
- Assume isotropic LHC initial conditions T_0= 600 MeV @ τ_0 = 0.25 fm/c and solve for the 0+1d viscous hydro dynamics



Estimating Anisotropy – AdS/CFT

- In 0+1d case there are now numerical solutions of Einstein's equations to compare with.
  [Heller, Janik, and Witaszczyk, 1103.3452]

- They studied a wide variety of initial conditions and found a kind of universal lower bound for the thermalization time

RHIC 200 GeV/nucleon:
T_0 = 350 MeV, τ_0 > 0.35 fm/c

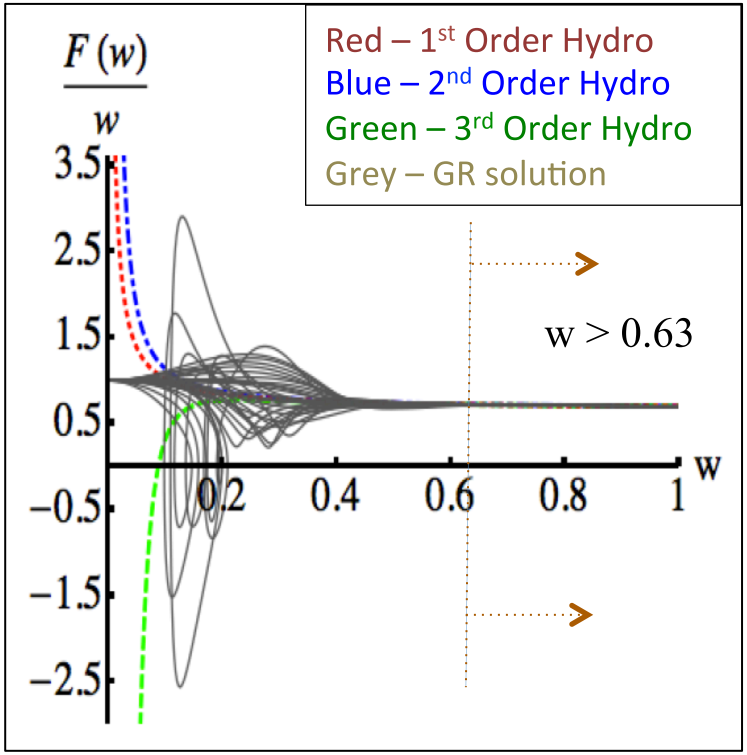
LHC 2.76 TeV/nucleon:
T_0 = 600 MeV, τ_0 > 0.2 fm/c

$$\langle T_{\tau\tau}\rangle \equiv \varepsilon(\tau) \equiv N_c^2 \cdot \frac{3}{8}\pi^2 \cdot T_{eff}^4 \cdot$$

$$w = T_{eff} \cdot \tau$$

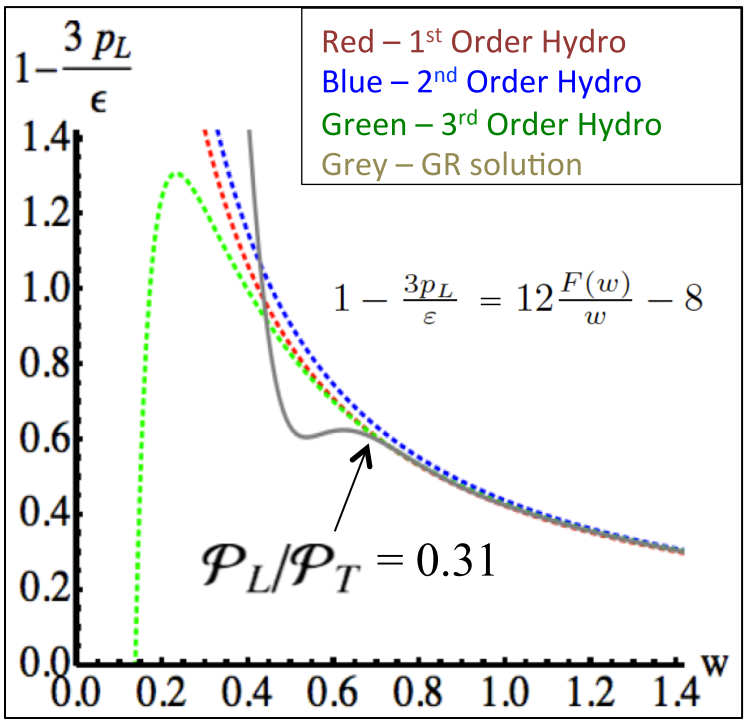
$$\frac{\tau}{w}\frac{d}{d\tau}w = \frac{F_{hydro}(w)}{w},$$

F_{hydro} known up to 3rd order hydro analytically

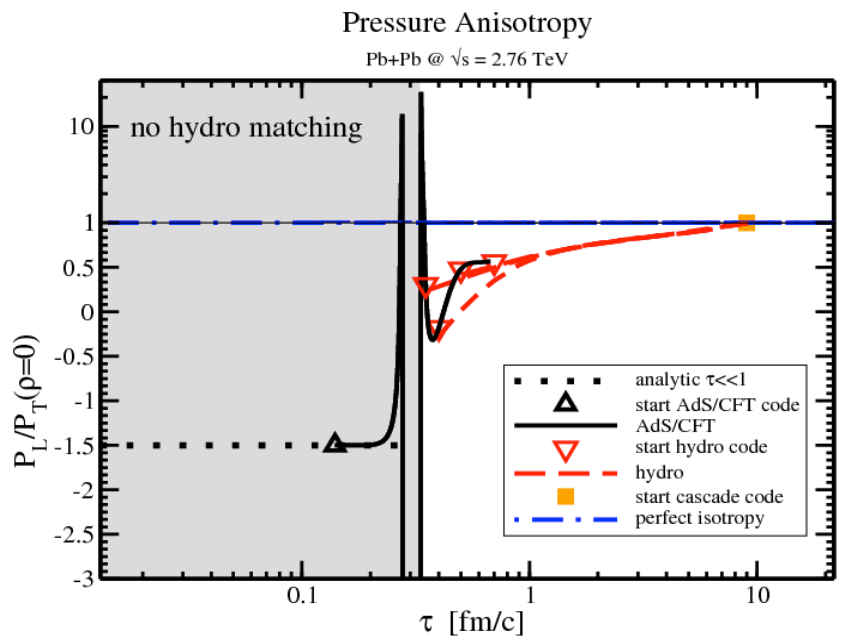


N=4 SUSY using AdS/CFT

However, at that time the system is not isotropic and remains anisotropic for the entirety of the evolution



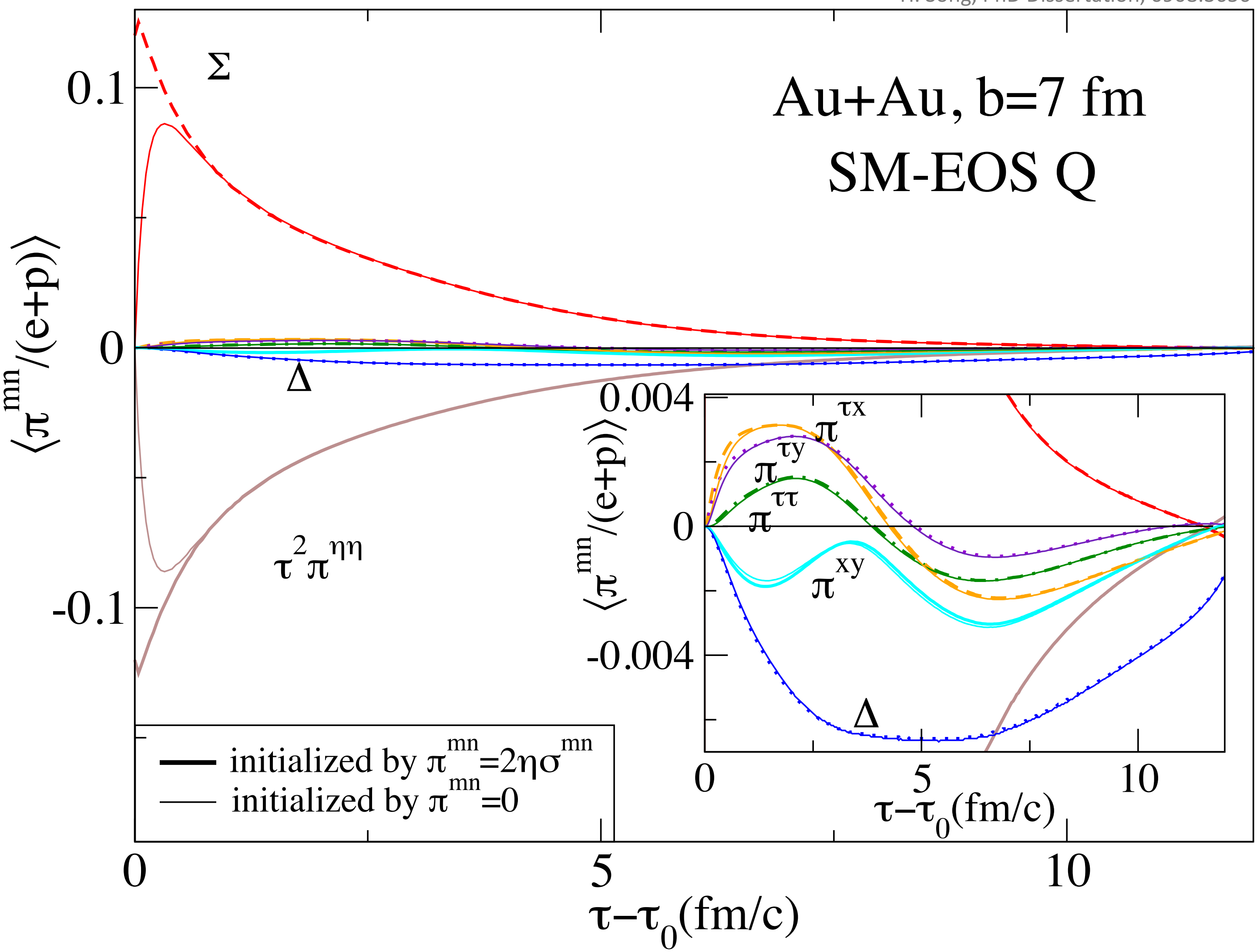
Another AdS/CFT numerical GR paper which includes transverse expansion reaches a similar conclusion
[van der Schee et al. 1307.2539]



See also J. Casalderrey-Solana et al. arXiv: 1305.4919

Hints from Viscous Hydro

H. Song, PhD Dissertation, 0908.3656




Adding another piece to the tower…

Anisotropic Hydrodynamics Basics

M. Martinez and MS, 1007.0889

Viscous Hydrodynamics Expansion

$$f(\tau,\mathbf{x},\mathbf{p}) = \underbrace{f_{\rm eq}(\mathbf{p},T(\tau,\mathbf{x}))}_{\text{Isotropic in momentum space}} + \delta f$$

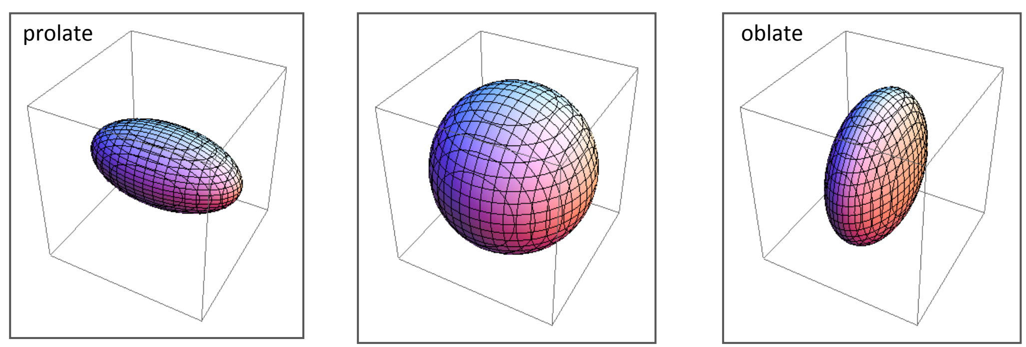
Anisotropic Hydrodynamics Expansion

$$f(\tau,\mathbf{x},\mathbf{p}) = f_{\rm aniso}(\mathbf{p},\underbrace{\Lambda(\tau,\mathbf{x})}_{T_\perp},\underbrace{\xi(\tau,\mathbf{x})}_{\text{anisotropy}}) + \delta\tilde{f}$$

→ "Romatschke-Strickland" form in LRF

$$\boxed{f^{LRF}_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2+\xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)}$$

$$\boxed{\xi = \frac{\langle p_T^2\rangle}{2\langle p_L^2\rangle} - 1}$$



$-1 < \xi < 0$ $\xi = 0$ $\xi > 0$

Anisotropic Hydrodynamics Basics

M. Martinez and MS, 1007.0889

Viscous Hydrodynamics Expansion

$$f(\tau, \mathbf{x}, \mathbf{p}) = \underbrace{f_{\rm eq}(\mathbf{p}, T(\tau, \mathbf{x}))} + \delta f$$

Isotropic in momentum space

Anisotropic Hydrodynamics Expansion

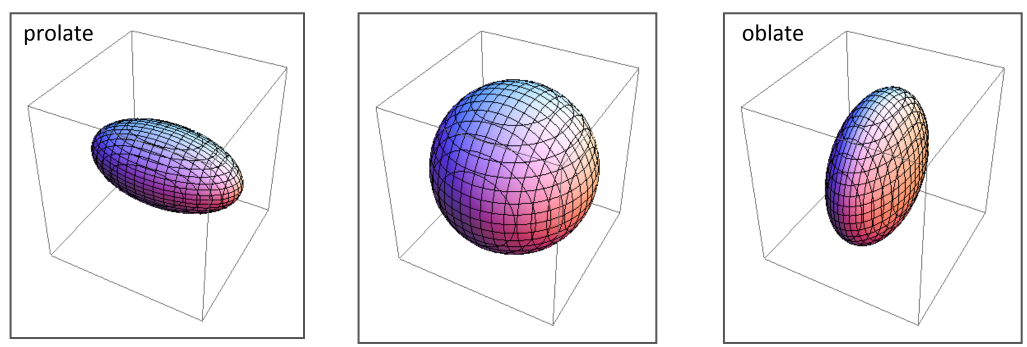
$$f(\tau, \mathbf{x}, \mathbf{p}) = f_{\rm aniso}(\mathbf{p}, \underbrace{\Lambda(\tau, \mathbf{x})}_{T_\perp}, \underbrace{\xi(\tau, \mathbf{x})}_{\text{anisotropy}}\ ) + \cancel{\delta \tilde{f}}$$

First, let's consider what happens when we ignore this term…

→ "Romatschke-Strickland" form in LRF

$$\boxed{f_{\rm aniso}^{LRF} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2 + \xi(\mathbf{x},\tau) p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)}$$

$$\boxed{\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1}$$



$-1 < \xi < 0$ $\qquad$ $\xi = 0$ $\qquad$ $\xi > 0$

First-order Anisotropic Hydrodynamics

LO (Spheroidal) Distribution

- Consider conformal system to start with
- In the conformal (massless) limit all bulk observables factorize into a product of two functions
- Note that, in the general case, it is also possible to define an anisotropic EOS

$$n(\Lambda,\xi) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_{\text{aniso}} = \frac{n_{\text{iso}}(\Lambda)}{\sqrt{1+\xi}}$$

$$\mathcal{E}(\Lambda,\xi) = T^{\tau\tau} = \mathcal{R}(\xi)\,\mathcal{E}_{\text{iso}}(\Lambda)$$

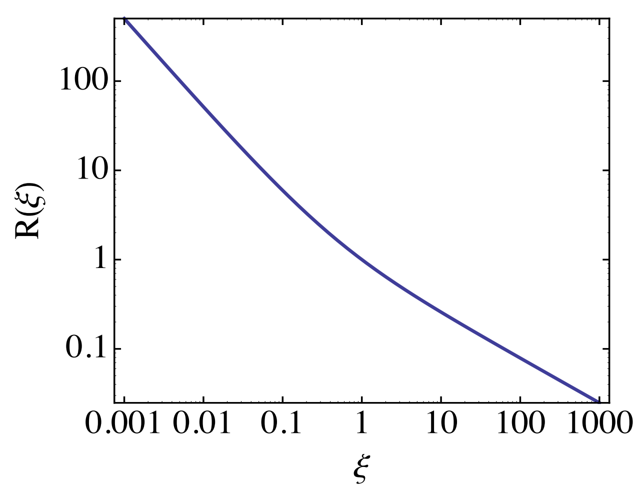
$$\mathcal{P}_\perp(\Lambda,\xi) = \frac{1}{2}\left(T^{xx}+T^{yy}\right) = \mathcal{R}_\perp(\xi)\mathcal{P}_{\text{iso}}(\Lambda)$$

$$\mathcal{P}_L(\Lambda,\xi) = -T^{\varsigma}_{\varsigma} = \mathcal{R}_{\rm L}(\xi)\mathcal{P}_{\text{iso}}(\Lambda)$$

$$\mathcal{R}(\xi) \equiv \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)$$

$$\mathcal{R}_\perp(\xi) \equiv \frac{3}{2\xi}\left(\frac{1+(\xi^2-1)\mathcal{R}(\xi)}{\xi+1}\right)$$

$$\mathcal{R}_L(\xi) \equiv \frac{3}{\xi}\left(\frac{(\xi+1)\mathcal{R}(\xi)-1}{\xi+1}\right)$$

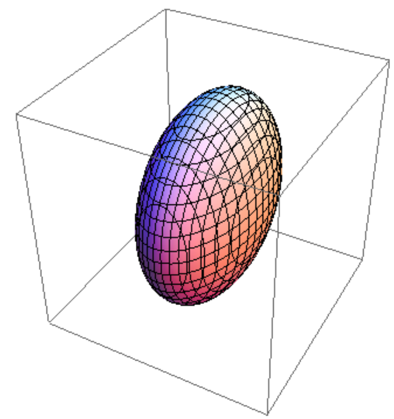


Azimuthally symmetric $T^{\mu\nu}$

$$T^{\mu\nu}(t,\mathbf{x}) = t_{00} g^{\mu\nu} + \sum_{i=1}^{3} t_{ii} X_i^\mu X_i^\nu + \sum_{\substack{\alpha,\beta=0 \\ \alpha>\beta}}^{3} t_{\alpha\beta}(X_\alpha^\mu X_\beta^\nu + X_\beta^\mu X_\alpha^\nu) \, ,$$

$$T_{\rm LRF}^{00} = \mathcal{E} = t_{00} \, ,$$
$$T_{\rm LRF}^{xx} = \mathcal{P}_\perp = -t_{00} + t_{11} \, ,$$
$$T_{\rm LRF}^{yy} = \mathcal{P}_\perp = -t_{00} + t_{22} \, ,$$
$$T_{\rm LRF}^{zz} = \mathcal{P}_L = -t_{00} + t_{33} \, ,$$

Assume, at leading order, rotational symmetry around p_z-axis in LRF



$$T^{\mu\nu} = (\mathcal{E} + \mathcal{P}_\perp) u^\mu u^\nu - \mathcal{P}_\perp g^{\mu\nu} + (\mathcal{P}_L - \mathcal{P}_\perp) z^\mu z^\nu \, ,$$

L. Satarov et al, hep-ph/0611099 R. Ryblewski and W. Florkowski, 1103.1260 M. Martinez, R. Ryblewski, and MS, 1204.1473

0+1d case – new Bjorken eqs

0th Moment of Boltzmann EQ

$$\partial_\alpha N^\alpha \neq 0$$

$$\frac{1}{1+\xi}\partial_\tau \xi - \frac{2}{\tau} - 6\,\partial_\tau \log\Lambda = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

1st Moment of Boltzmann EQ

$$\partial_\alpha T^{\alpha\beta} = 0$$

$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau \xi + 4\,\partial_\tau \log\Lambda = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$

Where (original MS prescription)

$$\Gamma = \frac{2T(\tau)}{5\bar{\eta}} = \frac{2\mathcal{R}^{1/4}(\xi)\Lambda}{5\bar{\eta}}$$

M. Martinez and MS, 1007.0889

$$\mathcal{R}(\xi) \equiv \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)$$

$$\mathcal{E}(\Lambda,\xi) = \mathcal{R}(\xi)\mathcal{E}_{\rm iso}(\Lambda)$$

Linearized Equations

If we expand the energy-momentum tensor to linear order in the anisotropy parameter and match to 2nd-order viscous hydro, we find

$$\frac{\Pi}{\mathcal{E}_{\rm eq}} = \frac{8}{45}\xi + \mathcal{O}(\xi^2)$$

If we similarly expand the coupled nonlinear differential equations to lowest order in the anisotropy parameter and rewrite in terms of the shear using the relation above, we obtain

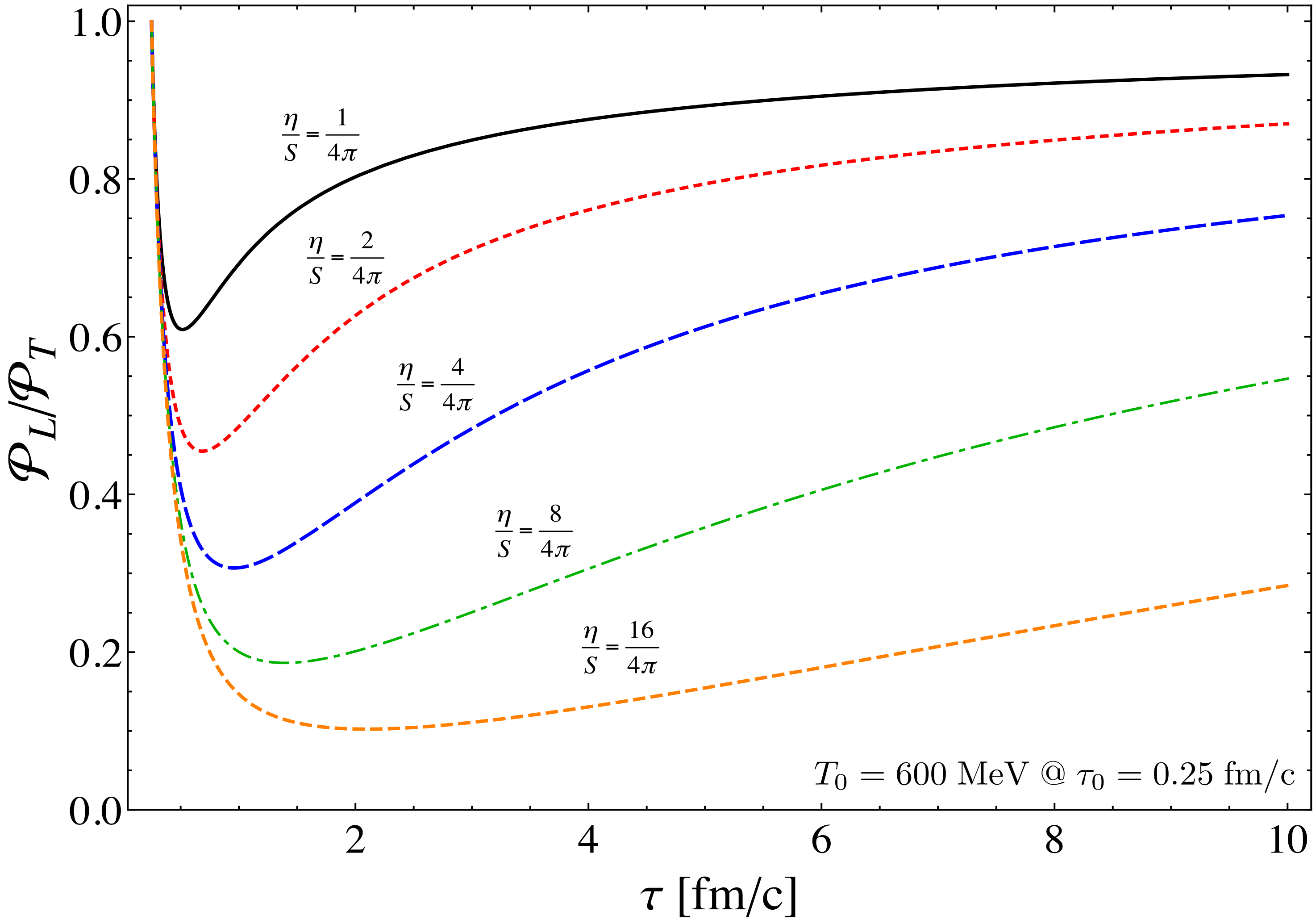
$$\boxed{\begin{aligned} \partial_\tau \mathcal{E} &= -\frac{\mathcal{E}+\mathcal{P}}{\tau} + \frac{\Pi}{\tau} \\ \partial_\tau \Pi &= -\frac{\Pi}{\tau_\pi} + \frac{4}{3}\frac{\eta}{\tau_\pi \tau} - \frac{4}{3}\frac{\Pi}{\tau} \end{aligned}} \qquad \boxed{\begin{aligned} \Gamma &= \frac{2}{\tau_\pi} \\ \tau_\pi &= \frac{5}{4}\frac{\eta}{\mathcal{P}} \end{aligned}}$$

- Reproduces 2nd-order viscous hydro in the small anisotropy limit!
- Also correctly describes the free streaming limit! (not shown here)

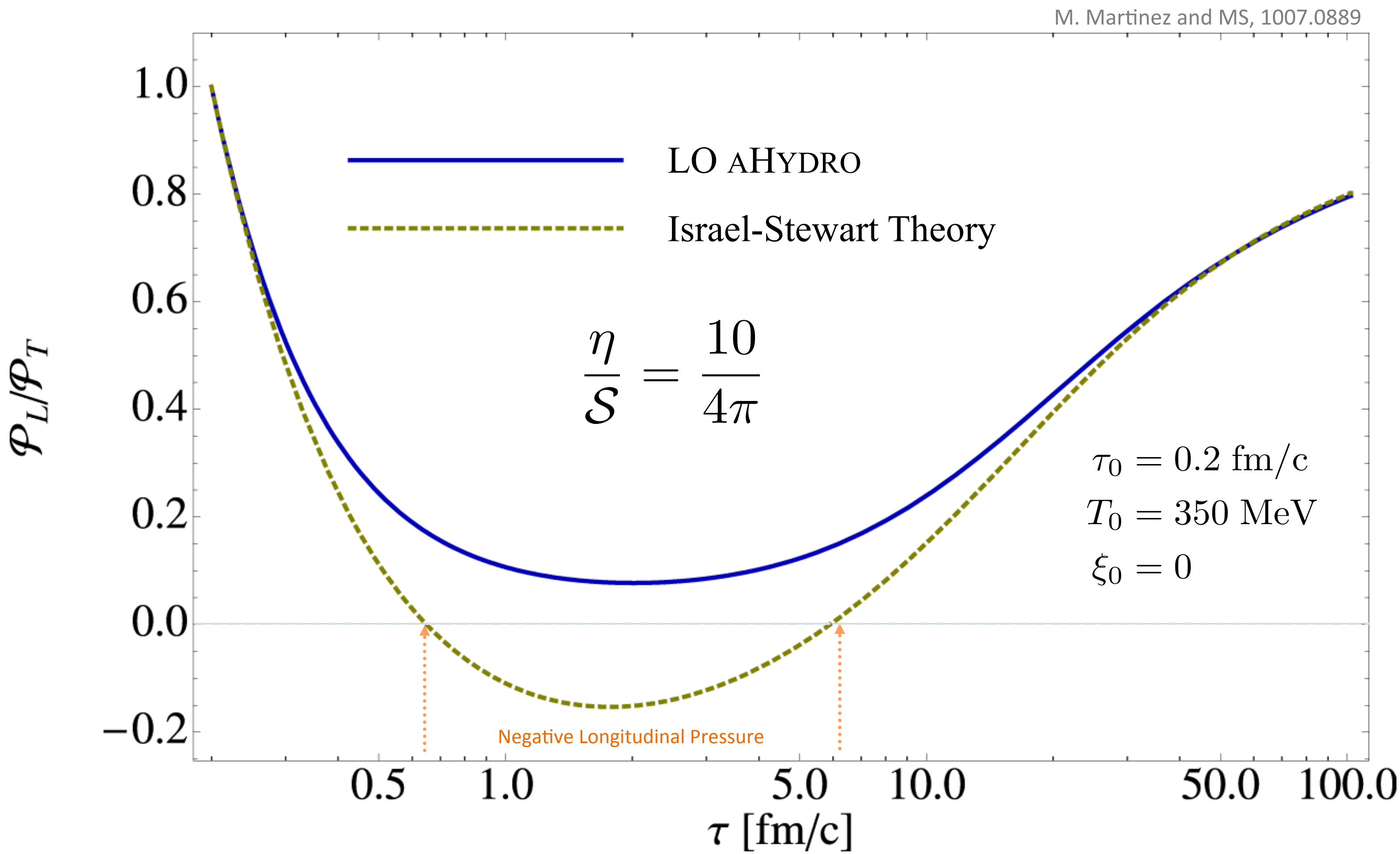
Pressure Anisotropy

$$\frac{1}{1+\xi}\partial_\tau \xi - \frac{2}{\tau} - 6\,\partial_\tau \log\Lambda = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau \xi + 4\,\partial_\tau \log\Lambda = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$



Viscous Hydro vs LO aHydro



Including Transverse Dynamics

M. Martinez, R. Ryblewski, and MS, 1204.1473

- Allowing variables to depend on x and y while still assuming boost-invariance, we obtain the "2+1d" dimensional AHYDRO equations
- Conformal system → four equations for four variables u_x, u_y, ξ, and Λ.

0th moment

$$Dn + n\theta = J_0 \,.$$

$$D \equiv u^\mu \partial_\mu \,,$$
$$\theta \equiv \partial_\mu u^\mu \,,$$

$$u_0 = \sqrt{1 + u_x^2 + u_y^2}$$

1st moment

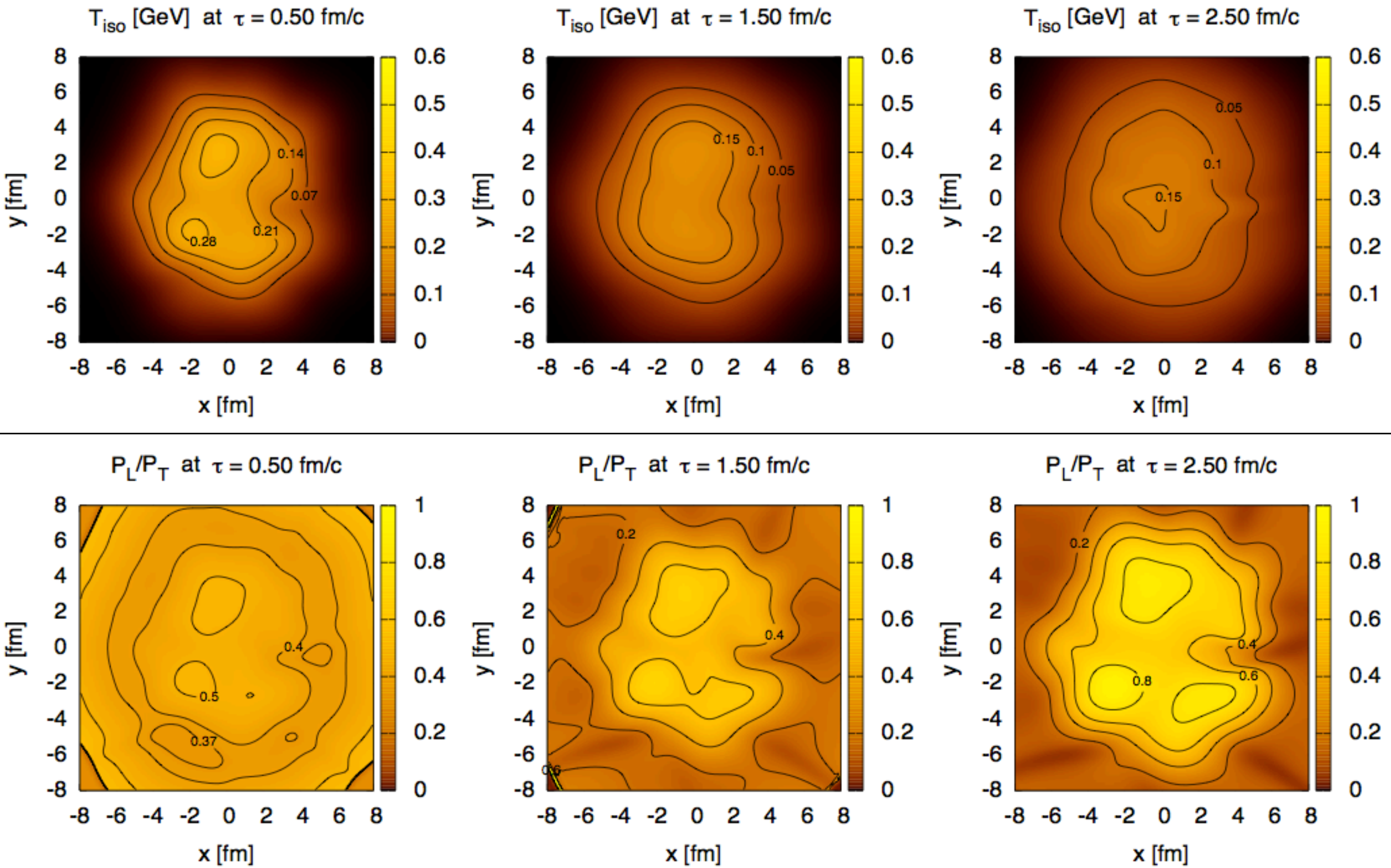
$$D\mathcal{E} + (\mathcal{E} + \mathcal{P}_\perp)\theta + (\mathcal{P}_L - \mathcal{P}_\perp)\frac{u_0}{\tau} = 0 \,,$$
$$(\mathcal{E} + \mathcal{P}_\perp)Du_x + \partial_x \mathcal{P}_\perp + u_x D\mathcal{P}_\perp + (\mathcal{P}_\perp - \mathcal{P}_L)\frac{u_0 u_x}{\tau} = 0 \,,$$
$$(\mathcal{E} + \mathcal{P}_\perp)Du_y + \partial_y \mathcal{P}_\perp + u_y D\mathcal{P}_\perp + (\mathcal{P}_\perp - \mathcal{P}_L)\frac{u_0 u_y}{\tau} = 0 \,.$$

Transverse Dynamics

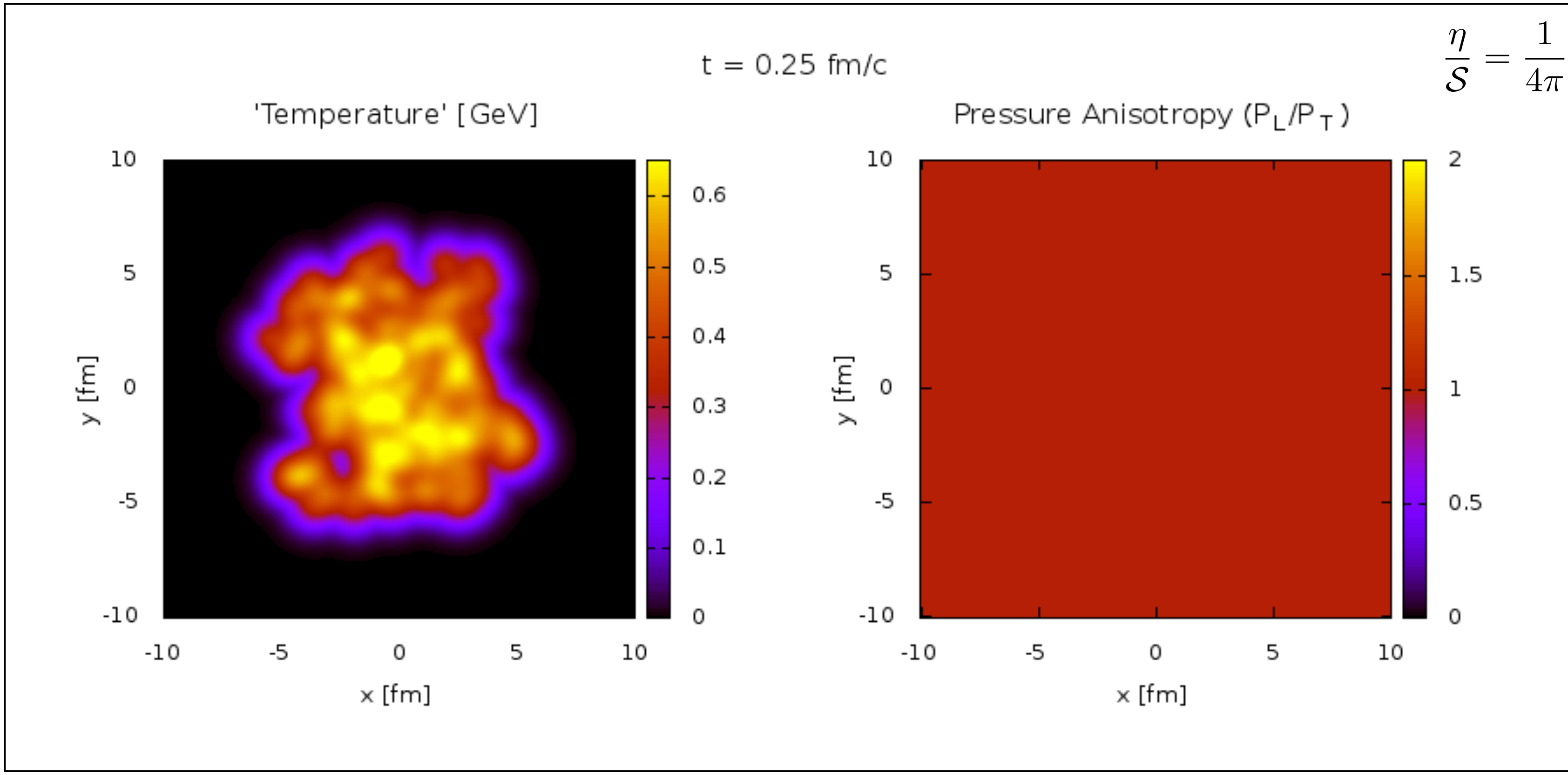
M. Martinez, R. Ryblewski, and MS, 1204.1473.

Pb-Pb @ 2.76 TeV
T_0 = 600 MeV
τ_0 = 0.25 fm/c
b = 7 fm

$$\frac{\eta}{\mathcal{S}} = \frac{1}{4\pi}$$



Spatiotemporal Evolution



- Pb-Pb, b = 7 fm collision with Monte-Carlo Glauber initial conditions
  T_0 = 600 MeV @ τ_0 = 0.25 fm/c
- Left panel shows temperature and right shows pressure anisotropy

Second-order Anisotropic Hydrodynamics

Anisotropic Hydrodynamics Basics

M. Martinez and MS, 1007.0889

Viscous Hydrodynamics Expansion

$$f(\tau,\mathbf{x},\mathbf{p}) = \underbrace{f_{\rm eq}(\mathbf{p},T(\tau,\mathbf{x}))}_{\text{Isotropic in momentum space}} + \delta f$$

Anisotropic Hydrodynamics Expansion

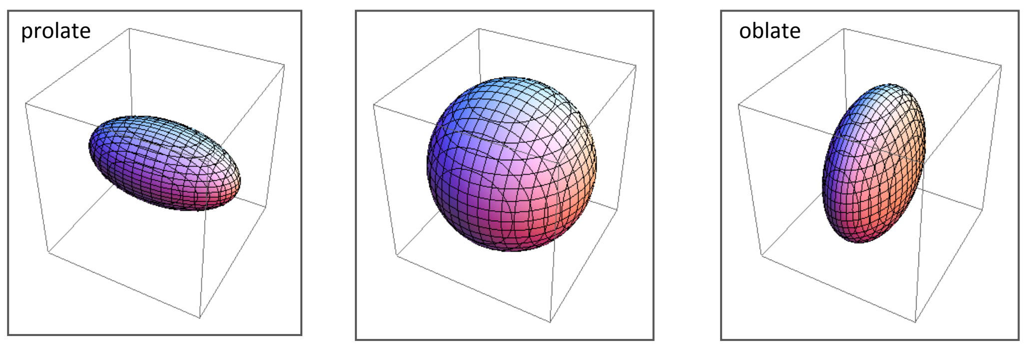
$$f(\tau,\mathbf{x},\mathbf{p}) = f_{\rm aniso}(\mathbf{p},\underbrace{\Lambda(\tau,\mathbf{x})}_{T_\perp},\underbrace{\xi(\tau,\mathbf{x})}_{\text{anisotropy}}\,) + \delta\tilde{f}$$

Now let's treat this term "perturbatively"
[D. Bazow, U. Heinz, and MS, forthcoming]

→ "Romatschke-Strickland" form in LRF

$$\boxed{f^{LRF}_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2+\xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)}$$

$$\boxed{\xi = \frac{\langle p_T^2\rangle}{2\langle p_L^2\rangle} - 1}$$



$-1 < \xi < 0$ $\quad\quad$ $\xi = 0$ $\quad\quad$ $\xi > 0$

2^{nd}-order Anisotropic Hydrodynamics

- Treat LO term "non-perturbatively" assuming spheroidal "RS" form but couple it to the dissipative currents
- Treat corrections $\delta\tilde{f}$ "perturbatively" → viscous aHydro (vaHydro)
- Use the very impressive method of Denicol et al [1202.4551] adapted to an anisotropic background
- Complete and orthogonal relativistic polynomial basis + systematic expansion in Knudsen number and (modified) inverse Reynolds number
- For the results shown today, we used the 14-moment approximation, however, our forthcoming paper, like Denicol et al, contains exact equations of motion for the moments and can be straightforwardly extended to higher moment approximations

Resulting Equations

Skipping over the gory details the final 14-moment approximation result is

$$\dot{\mathcal{N}} = -\mathcal{N}\theta - \boxed{\partial_\mu \tilde{V}^\mu} + \mathcal{C}.$$

[D. Bazow, U. Heinz, and MS, forthcoming]

$$\dot{\mathcal{E}} + (\mathcal{E}+\mathcal{P}_\perp+\boxed{\tilde{\Pi}})\theta + (\mathcal{P}_\mathrm{L}-\mathcal{P}_\perp)\frac{u_0}{\tau} + \boxed{u_\nu \partial_\mu \tilde{\pi}^{\mu\nu}} = 0,$$

$$(\mathcal{E}+\mathcal{P}_\perp+\boxed{\tilde{\Pi}})\dot{u}_x + \partial_x(\mathcal{P}_\perp+\boxed{\tilde{\Pi}}) + u_x(\dot{\mathcal{P}}_\perp+\boxed{\dot{\tilde{\Pi}}}) + (\mathcal{P}_\perp-\mathcal{P}_\mathrm{L})\frac{u_0 u_x}{\tau} - \boxed{\Delta^{1\nu}\partial^\mu \tilde{\pi}_{\mu\nu}} = 0,$$

$$(\mathcal{E}+\mathcal{P}_\perp+\boxed{\tilde{\Pi}})\dot{u}_y + \partial_y(\mathcal{P}_\perp+\boxed{\tilde{\Pi}}) + u_y(\dot{\mathcal{P}}_\perp+\boxed{\dot{\tilde{\Pi}}}) + (\mathcal{P}_\perp-\mathcal{P}_\mathrm{L})\frac{u_0 u_y}{\tau} - \boxed{\Delta^{2\nu}\partial^\mu \tilde{\pi}_{\mu\nu}} = 0,$$

$$\begin{aligned}
\dot{\tilde{\Pi}} &= -\frac{\dot{\tilde{\gamma}}_r^{\Pi}}{\tilde{\gamma}_r^{\Pi}}\tilde{\Pi} + \frac{1}{\tilde{\gamma}_r^{\Pi}}\mathcal{C}_{r-1} + \mathcal{W}_r + \mathcal{U}_r^{\mu\nu}\nabla_\mu u_\nu \\
&\quad + \lambda_{\Pi\pi}^r \tilde{\pi}^{\mu\nu}\sigma_{\mu\nu} + \tau_{\Pi V}^r \tilde{V}^\mu \dot{u}_\mu - \frac{1}{\tilde{\gamma}_r^{\Pi}}\nabla_\mu\left(\tilde{\gamma}_{r-1}^V \tilde{V}^\mu\right) - \delta_{\Pi\Pi}^r \tilde{\Pi}\theta \\
\dot{\tilde{V}}^{\langle\mu\rangle} &= -\frac{\dot{\tilde{\gamma}}_r^{V}}{\tilde{\gamma}_r^{V}}\tilde{V}^\mu + \frac{1}{\tilde{\gamma}_r^{V}}\mathcal{C}_{r-1}^{\langle\mu\rangle} + \mathcal{Z}_r^\mu - \tilde{V}^\nu \omega_\nu{}^\mu + \delta_{VV}^r \tilde{V}^\mu\theta - \Delta^\mu{}_\lambda \frac{1}{\tilde{\gamma}_r^{V}}\nabla_\nu\left(\tilde{\gamma}_{r-1}^\pi \tilde{\pi}^{\nu\lambda}\right) \\
&\quad + \tau_{q\pi}^r \tilde{\pi}^{\mu\nu}\dot{u}_\nu + \lambda_{VV}^r \tilde{V}_\nu \sigma^{\nu\mu} + \tau_{q\Pi}^r \tilde{\Pi}\dot{u}^\mu + \ell_{q\Pi}^r \nabla^\mu \tilde{\Pi} + \tilde{\Pi}\mathcal{O}^\mu\,, \\
\dot{\tilde{\pi}}^{\langle\mu\nu\rangle} &= -\frac{\dot{\tilde{\gamma}}_r^{\pi}}{\tilde{\gamma}_r^{\pi}}\tilde{\pi}^{\mu\nu} + \mathcal{T}^{\langle\mu}V^{\nu\rangle} + \frac{1}{\tilde{\gamma}_r^{\pi}}\mathcal{C}_{r-1}^{\langle\mu\nu\rangle} + \mathcal{K}_r^{\mu\nu} + \mathcal{L}_r^{\mu\nu} + \mathcal{H}_r^{\mu\nu\lambda}\dot{z}_\lambda + \mathcal{Q}_r^{\mu\nu\lambda\alpha}\nabla_\lambda u_\alpha + \mathcal{X}_r^{\mu\nu\lambda}u^\alpha \nabla_\lambda z_\alpha \\
&\quad - 2\lambda_{\pi\pi}^r \tilde{\pi}_\alpha{}^{\langle\mu}\sigma^{\nu\rangle\alpha} + 2\tilde{\pi}^{\lambda\langle\mu}\omega^{\nu\rangle}{}_\lambda + 2\lambda_{\pi\Pi}^r \tilde{\Pi}\sigma^{\mu\nu} + 2\lambda_{\pi V}^r \nabla^{\langle\mu}\tilde{V}^{\nu\rangle} + 2\tau_{\pi V}^r \tilde{V}^{\langle\mu}\dot{u}^{\nu\rangle} - 2\delta_{\pi\pi}^r \tilde{\pi}^{\mu\nu}\theta\,.
\end{aligned}$$

- Orange-boxed terms are new
- Dot indicates a convective derivative
- Complicated bits in last two equations correspond to dissipative "forces" and anisotropic transport coefficients

(2+1)-dimensional Equations

For conformal boost-invariant systems assuming no gradients in the chemical potential and using an RTA collisional kernel, the equations reduce to

[D. Bazow, U. Heinz, and MS, forthcoming]

$$\frac{\dot{\xi}}{1+\xi} - 6\frac{\dot{\Lambda}}{\Lambda} - 2\theta = 2\Gamma\left(1 - \sqrt{1+\xi}\,\mathcal{R}^{3/4}(\xi)\right)$$

$$\mathcal{R}'\dot{\xi} + 4\mathcal{R}\frac{\dot{\Lambda}}{\Lambda} = -\left(\mathcal{R}+\frac{1}{3}\mathcal{R}_\perp\right)\theta_\perp - \left(\mathcal{R}+\frac{1}{3}\mathcal{R}_L\right)\frac{u_0}{\tau} + \frac{\tilde{\pi}^{\mu\nu}\sigma_{\mu\nu}}{\mathcal{E}_0(\Lambda)},$$

$$\begin{aligned}[3\mathcal{R}+\mathcal{R}_\perp]\,\dot{u}_\perp = {} & -\mathcal{R}'_\perp\partial_\perp\xi - 4\mathcal{R}_\perp\frac{\partial_\perp\Lambda}{\Lambda} - u_\perp\left(\mathcal{R}'_\perp\dot{\xi}+4\mathcal{R}_\perp\frac{\dot{\Lambda}}{\Lambda}\right) \\ & -u_\perp(\mathcal{R}_\perp-\mathcal{R}_L)\frac{u_0}{\tau} + \frac{3}{\mathcal{E}_0(\Lambda)}\left(\frac{u_x\Delta^1{}_\nu + u_y\Delta^2{}_\nu}{u_\perp}\right)\partial_\mu\tilde{\pi}^{\mu\nu},\end{aligned}$$

$$[3\mathcal{R}+\mathcal{R}_\perp]\,u_\perp\dot{\phi}_u = -\mathcal{R}'_\perp D_\perp\xi - 4\mathcal{R}_\perp\frac{D_\perp\Lambda}{\Lambda} - \frac{3}{\mathcal{E}_0(\Lambda)}\left(\frac{u_y\partial_\mu\tilde{\pi}^{\mu 1} - u_x\partial_\mu\tilde{\pi}^{\mu 2}}{u_\perp}\right).$$

$$\begin{aligned}\dot{\tilde{\pi}}^{\mu\nu} = {} & -2\dot{u}_\alpha\tilde{\pi}^{\alpha(\mu}u^{\nu)} - \Gamma\Big[\big(\mathcal{P}(\Lambda,\xi)-\mathcal{P}_\perp(\Lambda,\xi)\big)\Delta^{\mu\nu} + \big(\mathcal{P}_\mathrm{L}(\Lambda,\xi)-\mathcal{P}_\perp(\Lambda,\xi)\big)z^\mu z^\nu + \tilde{\pi}^{\mu\nu}\Big] \\ & +\mathcal{K}_0^{\mu\nu} + \mathcal{L}_0^{\mu\nu} + \mathcal{H}_0^{\mu\nu\lambda}\dot{z}_\lambda + \mathcal{Q}_0^{\mu\nu\lambda\alpha}\nabla_\lambda u_\alpha + \mathcal{X}_0^{\mu\nu\lambda}u^\alpha\nabla_\lambda z_\alpha - 2\lambda^0_{\pi\pi}\tilde{\pi}^{\lambda\langle\mu}\sigma^{\nu\rangle}_\lambda + 2\tilde{\pi}^{\lambda\langle\mu}\omega^{\nu\rangle}_\lambda - 2\delta^0_{\pi\pi}\tilde{\pi}^{\mu\nu}\theta,\end{aligned}$$

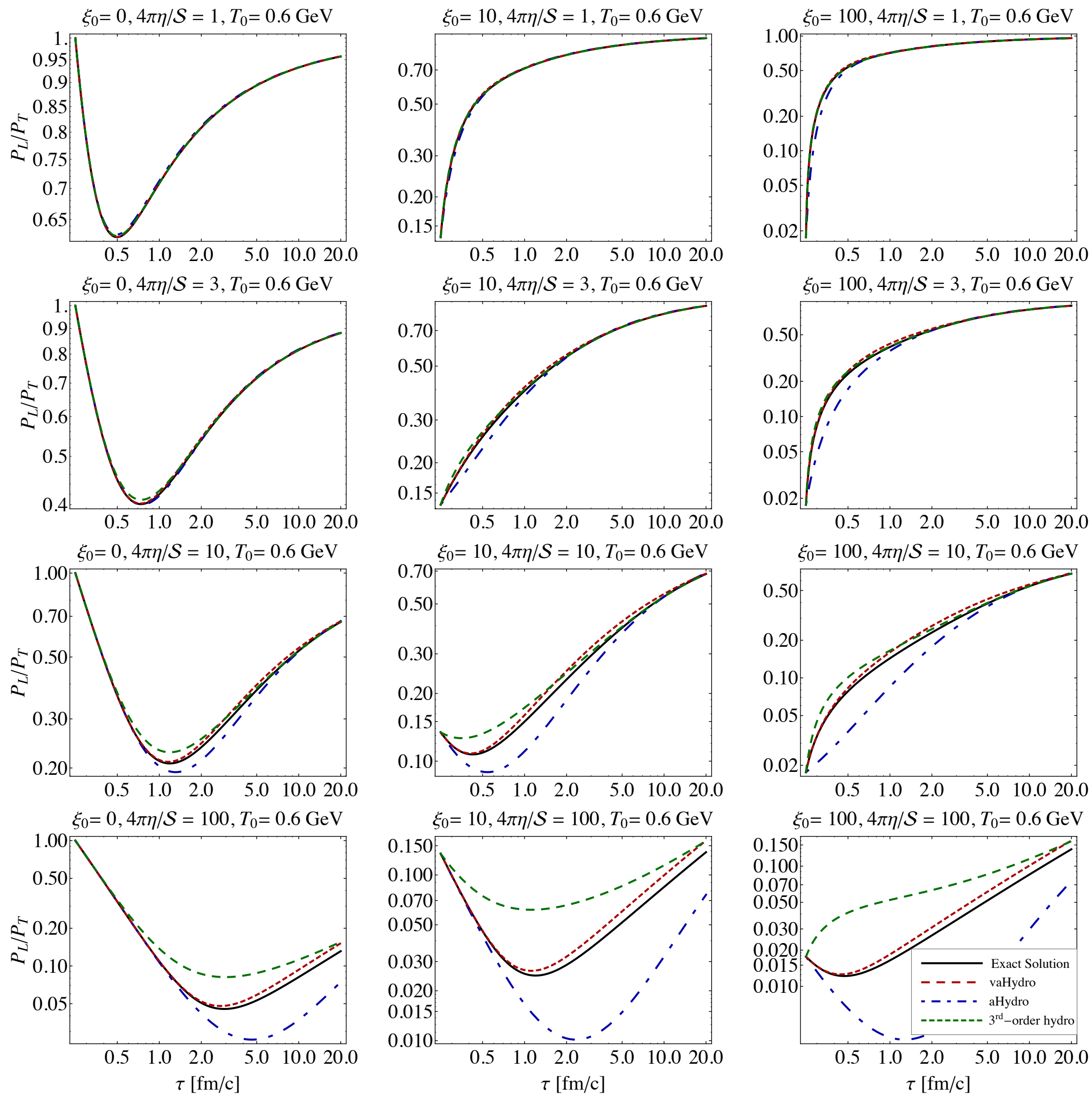
A prime indicates a derivative with respect to ξ.

Beauty vs the Beast

- You might say at this point, “But Mike, aren’t complicated things inherently bad?”
- Perhaps, but sometimes they are necessary
- The real question is how can we test this beast to see if it is, in fact, better in some sense
- For this purpose we can compare to exact solutions of the RTA Boltzmann equation for transversely homogenous boost-invariant systems [W. Florkowski, R. Ryblewski, MS 1304.0665, 1305.7234]

Pressure Ratio Comparisons

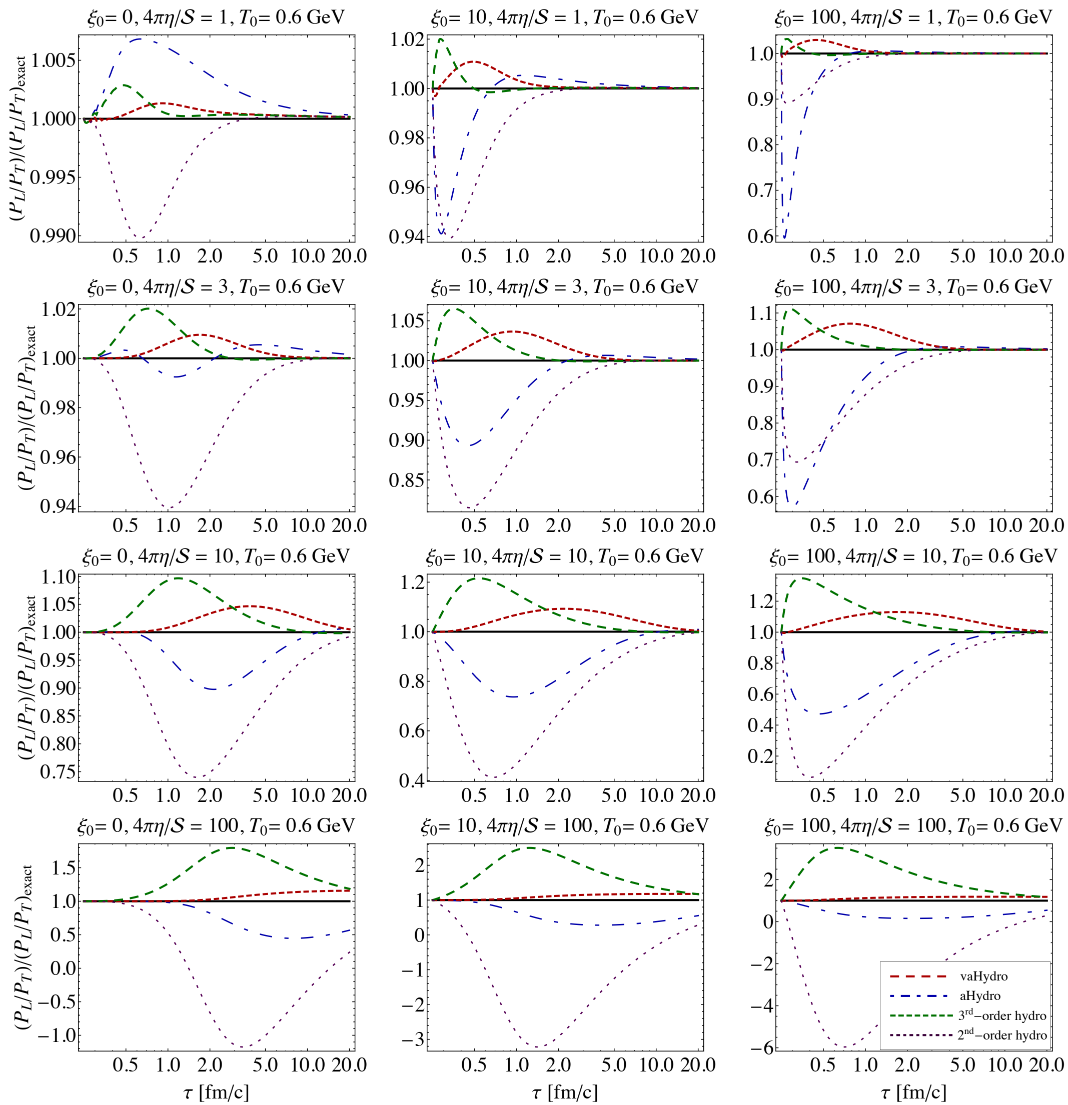
[D. Bazow, U. Heinz, and MS, forthcoming]



- Panels show ratio of longitudinal to transverse pressure
- T_0 = 600 MeV @ τ_0 = 0.25 fm/c
- Left to right is increasing initial momentum-space anisotropy
- Top to bottom is increasing η/S
- Black line is the exact solution
- Red dashed line is the aHydro approximation
- Blue dot-dashed line is the vaHydro approximation
- Green dashed line is a third-order Chapman-Enskog-like viscous hydrodynamics approximation
  [A. Jaiswal, 1305.3480]
- As we can see from these plots vaHydro does quite well indeed!

Pressure Ratio Error Comparisons

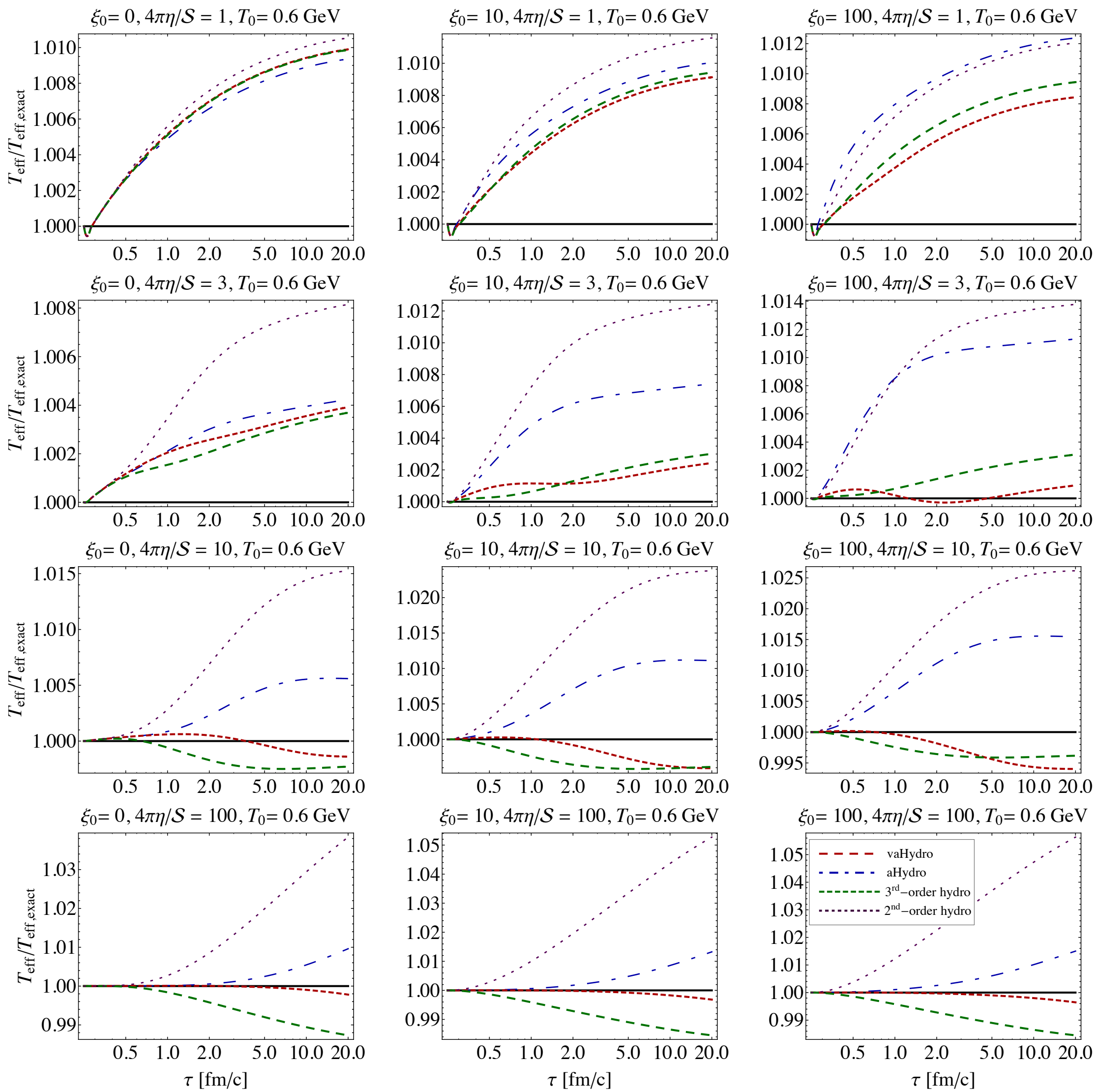
[D. Bazow, U. Heinz, and MS, forthcoming]



- Panels now show the relative error in the ratio of longitudinal to transverse pressure
- Same params as the previous slide, but now we have included comparison to the second-order viscous hydrodynamic approximation of Denicol et al [1202.4551]
- We do not show the Israel-Stewart equations, because they are <u>vastly inferior to all approximations shown</u>
- It seems that vaHydro "outperforms" all competitors, so … perhaps the beast is beautiful after all!

Effective Temperature Comparisons

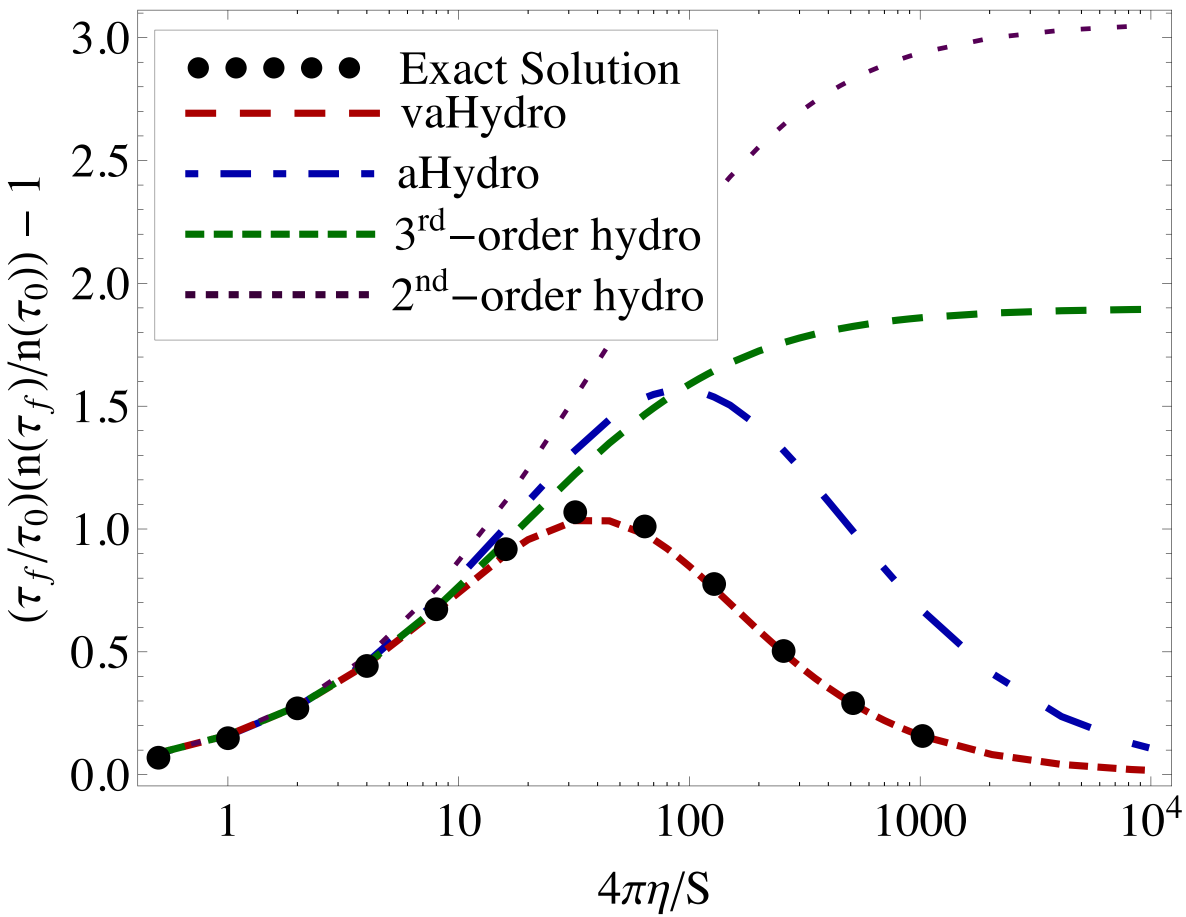
[D. Bazow, U. Heinz, and MS, forthcoming]



- But maybe I'm cheating and only showing you one measure? Let's check the temperature to make sure all is good…
- Panels show relative error in the effective temperature
- Same params as the previous slide etc.
- Once again, vaHydro "outperforms" all competitors
- That being said, one should note the scale on the axes here. All approximations considered are quite accurate for the effective temperature evolution.

Entropy Generation

[D. Bazow, U. Heinz, and MS, forthcoming]

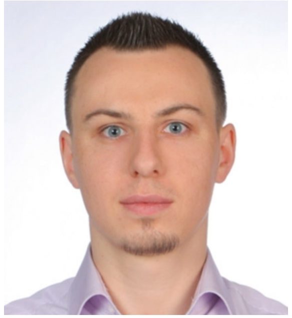


- Number (entropy) production vanishes in two limits: ideal hydrodynamic and free streaming limits
- In the conformal model which we are testing with, number density is proportional to entropy density

Thanks to my collaborators

Dennis Bazow

Mauricio Martinez



Radoslaw Ryblewski

Wojciech Florkowski

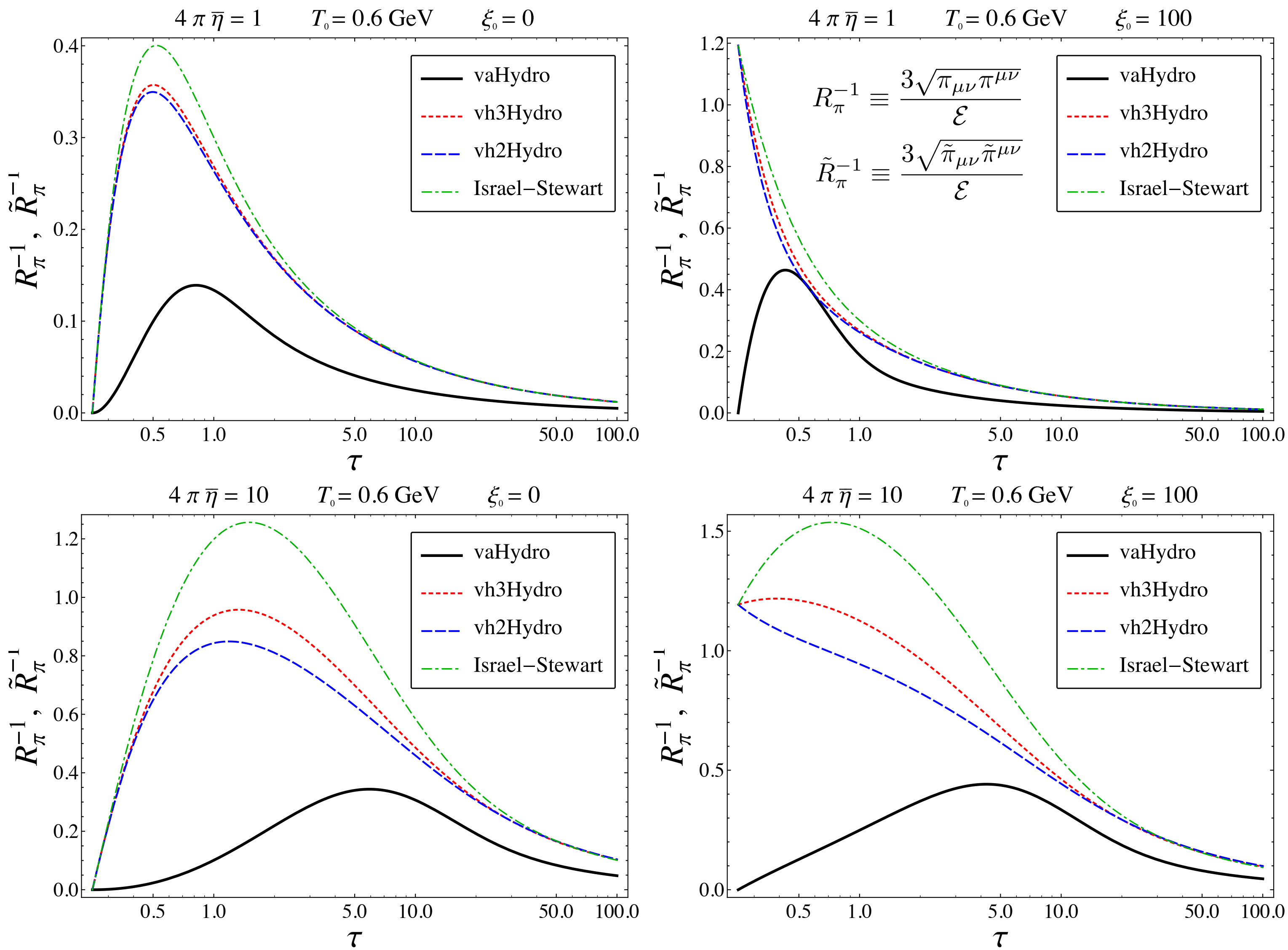
Ulrich Heinz

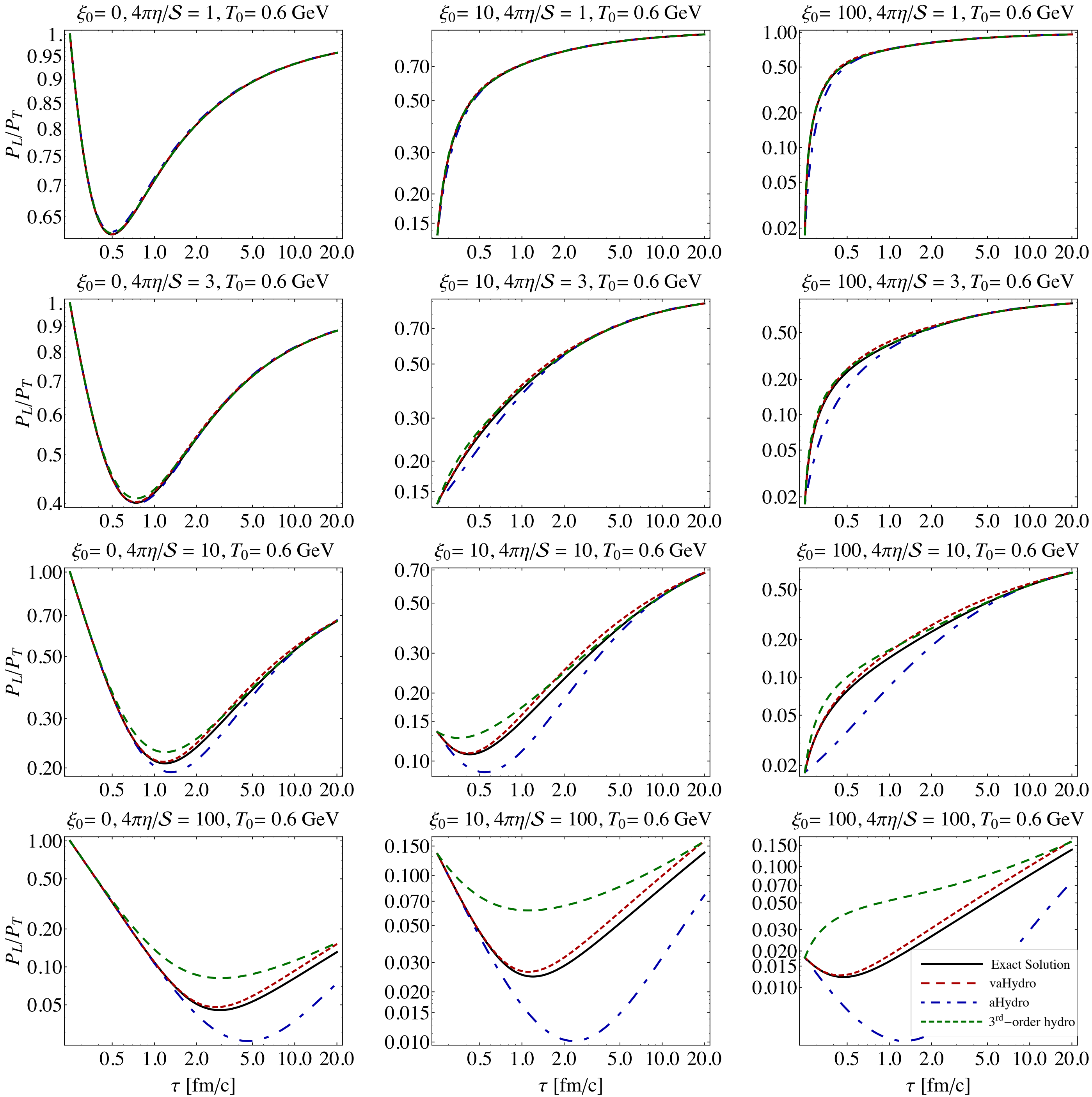
Conclusions and Outlook

- Anisotropic hydrodynamics builds upon prior advances in relativistic hydrodynamics in an attempt to create an even more quantitatively reliable tool
- It incorporates some "facts of life" specific to the conditions generated in relativistic heavy ion collisions and, in doing so, optimizes the non-ideal hydrodynamics approach
- Having second-order anisotropic hydrodynamics (VAHYDRO) allows us to proceed to numerical modeling of heavy ion collisions
- The evolution of the matter (particularly at early times, near the transverse edges, or with large temperature-dependent η/S) should now be more reliably described
- Extensions coming: (3+1)d VAHYDRO, anisotropic transport coefficients (!), realistic collisional kernels, bulk viscosity, higher moment approximations, ellipsoidal anisotropic hydrodynamics, … MUCH TO DO!

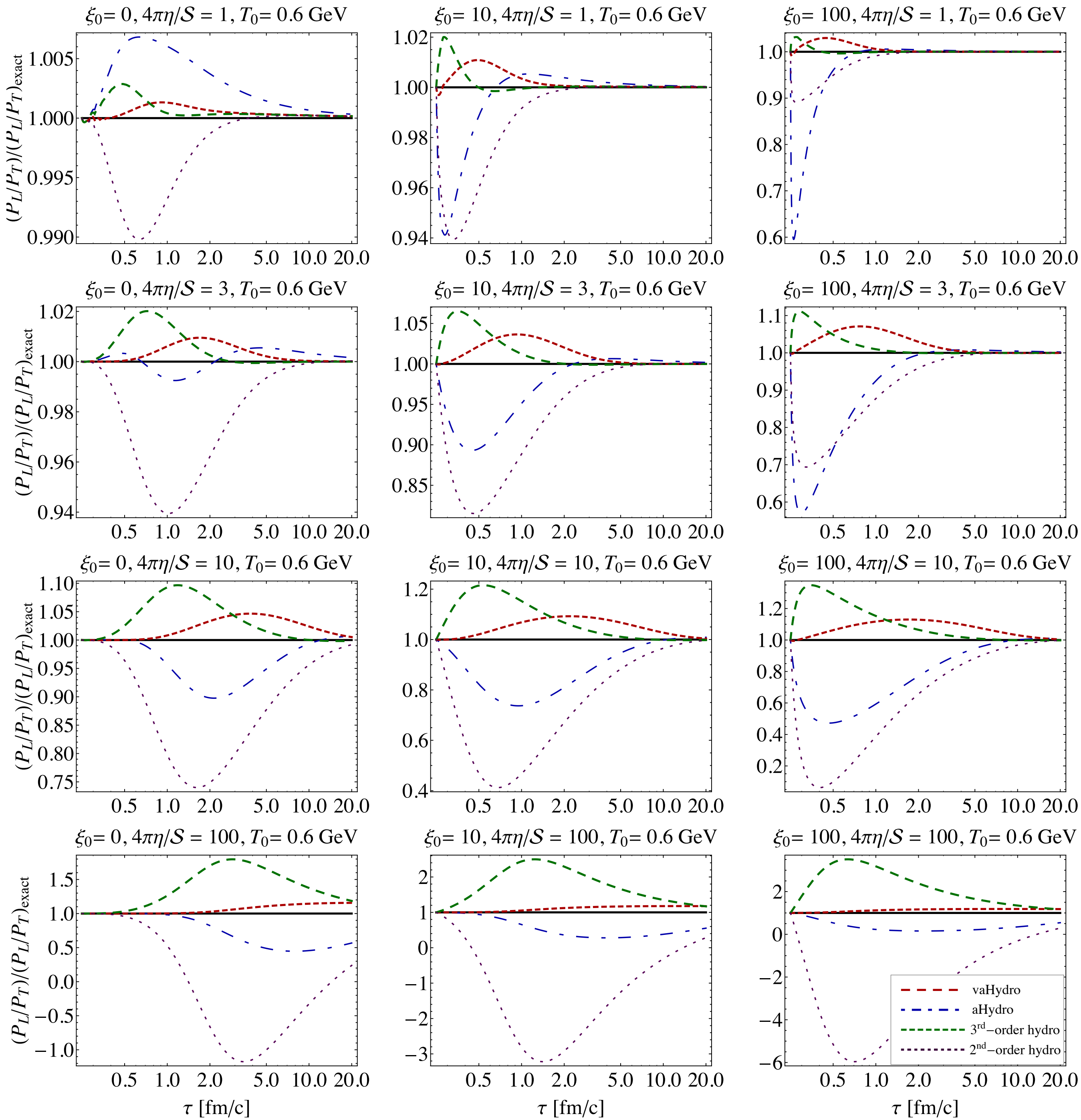
Backup Slides

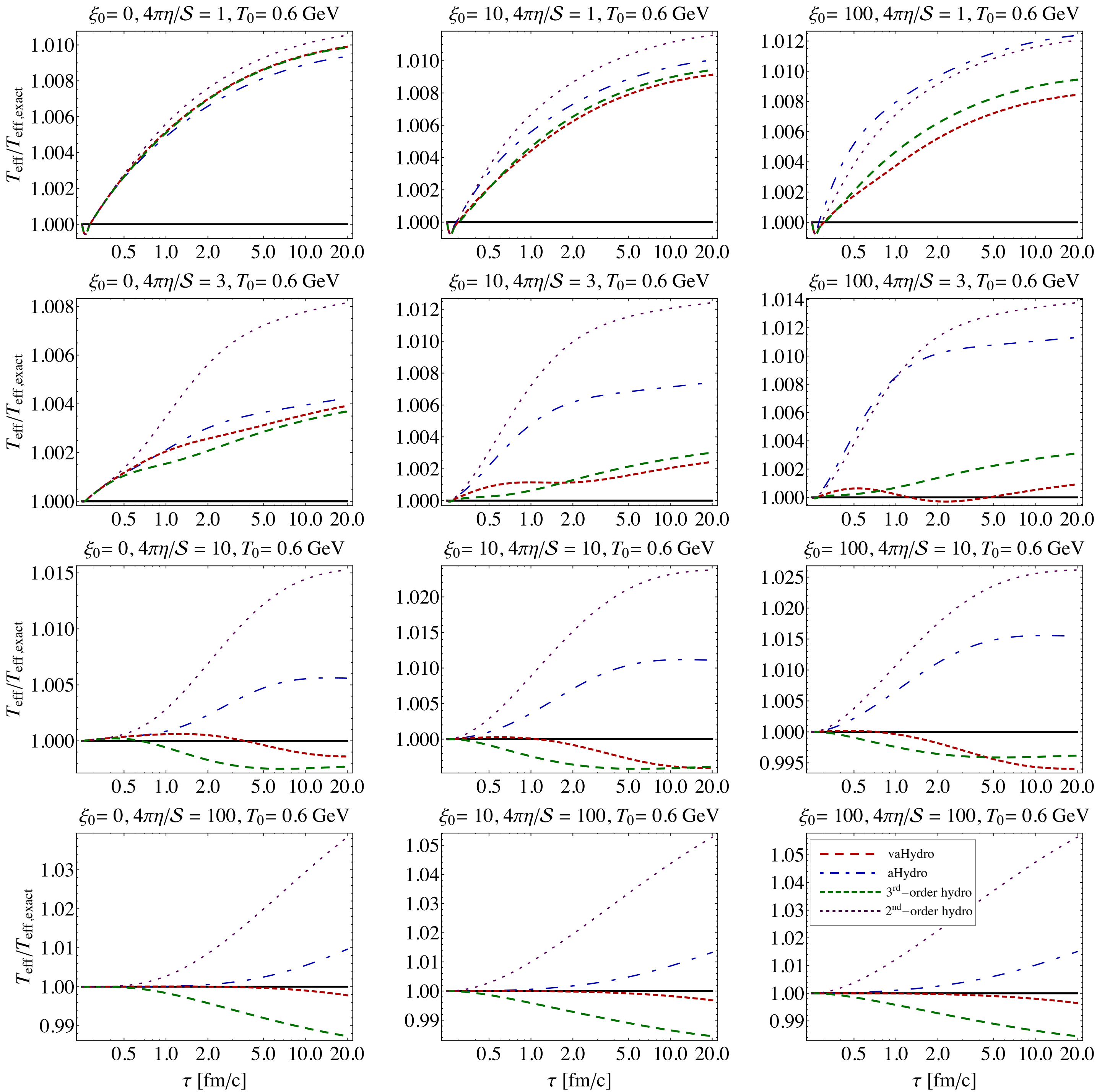
Reynolds Number Comparison





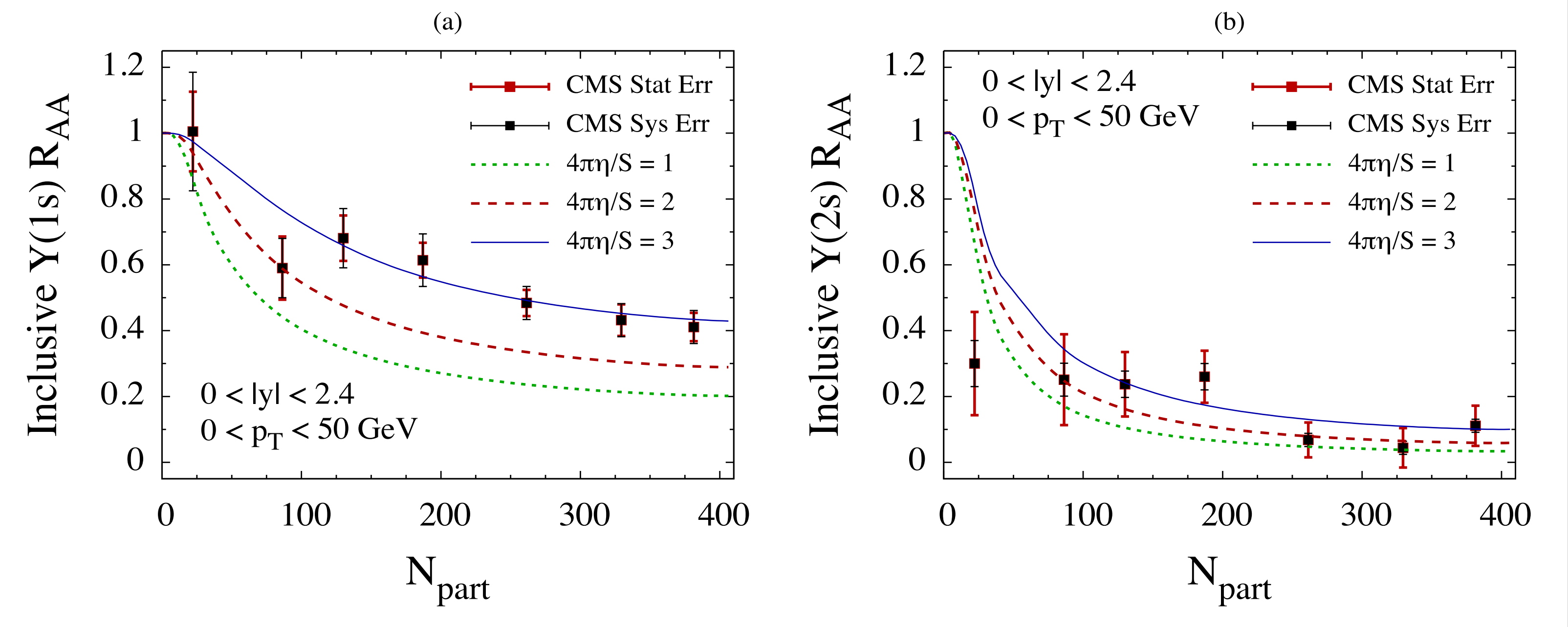
ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 1, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 1, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 1, T_0= 0.6 GeV
ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 3, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 3, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 3, T_0= 0.6 GeV
ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 10, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 10, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 10, T_0= 0.6 GeV
ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 100, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 100, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 100, T_0= 0.6 GeV
P_L/P_T
τ [fm/c]
Exact Solution
vaHydro
aHydro
3rd–order hydro


ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 1, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 1, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 1, T_0= 0.6 GeV
ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 3, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 3, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 3, T_0= 0.6 GeV
ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 10, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 10, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 10, T_0= 0.6 GeV
ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 100, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 100, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 100, T_0= 0.6 GeV
$(P_L/P_T)/(P_L/P_T)_{\rm exact}$
τ [fm/c]
vaHydro
aHydro
3rd−order hydro
2nd−order hydro

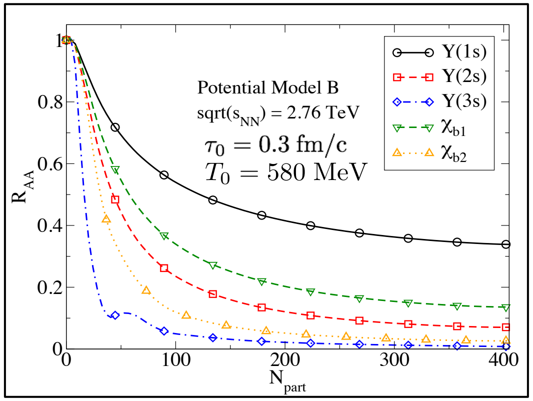

ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 1, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 1, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 1, T_0= 0.6 GeV
ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 3, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 3, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 3, T_0= 0.6 GeV
ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 10, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 10, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 10, T_0= 0.6 GeV
ξ_0= 0, $4\pi\eta/\mathcal{S}$ = 100, T_0= 0.6 GeV
ξ_0= 10, $4\pi\eta/\mathcal{S}$ = 100, T_0= 0.6 GeV
ξ_0= 100, $4\pi\eta/\mathcal{S}$ = 100, T_0= 0.6 GeV
$T_{\rm eff}/T_{\rm eff,exact}$
τ [fm/c]
vaHydro
aHydro
3rd–order hydro
2nd–order hydro

Example: Bottomonium Suppression

MS, arXiv:1207.5327; MS and D. Bazow, arXiv:1112.2761; MS arXiv:1106.2571



Compute ϒ(1s) suppression including effects of feed-down, formation time, and aHydro evolution with anisotropic complex-valued quarkonium potential.



Collective Flow