



UNIVERSITÀ DEGLI STUDI DI CATANIA
INFN-LNS



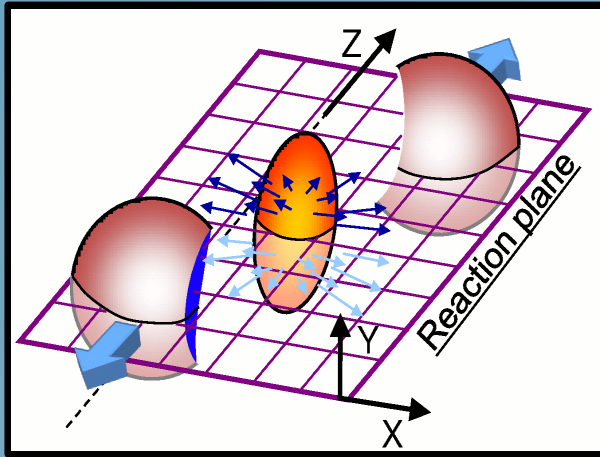
**Anisotropic flows and the shear viscosity of the QGP
within a kinetic approach**

**S. Plumari, A. Puglisi, L. Guardo,
M. Ruggieri, F. Scardina, V. Greco**

Outline

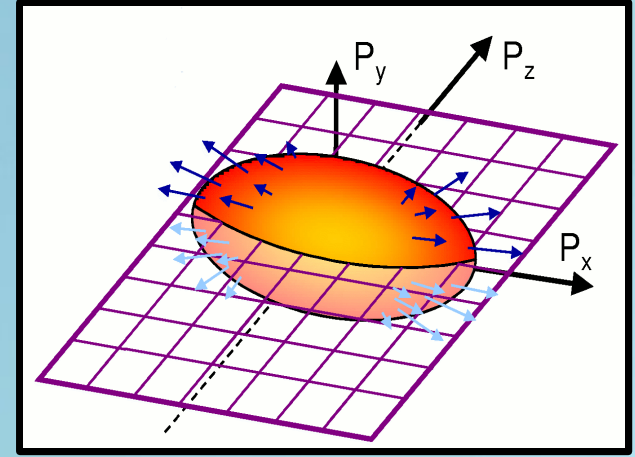
- **Transport approach at fixed η/s :**
 - **Motivation**
 - **How to fix locally $\eta/s \leftrightarrow \sigma(\theta), M, T$ -
Chapman-Enskog approach.**
- **η/s and generation of v_2 : from RHIC to LHC**
- **V_n from initial state fluctuations (preliminary)**
- **Conclusions**

Information from non-equilibrium: elliptic flow



$\lambda = (\sigma\rho)^{-1}$ or η/s viscosity

$c_s^2 = dP/d\varepsilon$, EoS-IQCD



$$\varepsilon_x = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle$$

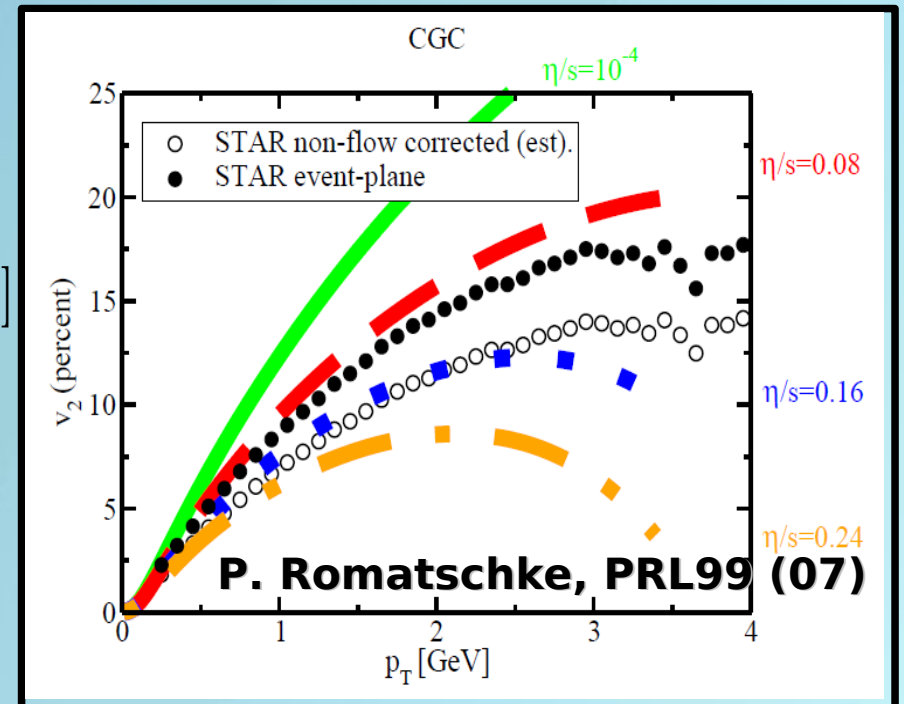
The v_2/ε measures efficiency in converting the eccentricity from Coordinate to Momentum space
 J.Y. Ollitrault, PRD 46 (1992).

$$v_2 = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

Can be seen also as Fourier expansion

$$\frac{dN}{dp_T d\phi} = \frac{dN}{dp_T} [1 + 2v_2 \cos(2\phi) + 2v_4 \cos(4\phi) + \dots]$$

by symmetry v_n with n odd expected to be zero ... (but event by event fluctuations)



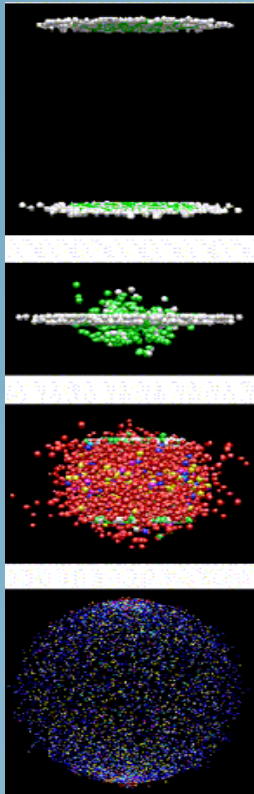
Motivation for a kinetic approach:

$$\left\{ p^\mu \partial_\mu + \left[p_\nu F^{\mu\nu} + M \partial^\mu M \right] \partial_\mu^p \right\} f(x, p) = C_{22} + C_{23} + \dots$$

Free
streaming

Field Interaction $\rightarrow \epsilon \neq 0$

Collisions $\rightarrow \eta \neq 0$



- Starting from 1-body distribution function and not from $T^{\mu\nu}$: possible to include $f(x, p)$ out of equilibrium.
- It is not a gradient expansion in η/s .
- Valid at intermediate p_T out of equilibrium.
- Valid at high η/s (cross over region): + self consistent kinetic freeze-out
- Include hadronization by coalescence + fragmentation.

Parton Cascade model

$$p^\mu \partial_\mu f(X, p) = C = C_{22} + C_{23} + \dots$$

Collisions



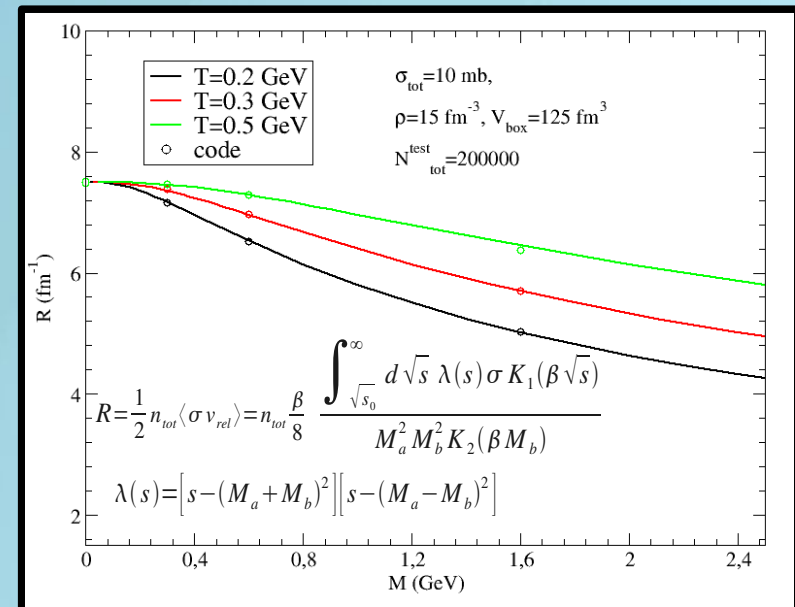
$$\left. \begin{array}{l} \varepsilon - 3p = 0, \\ \eta \neq 0 \end{array} \right\}$$

$$C_{22} = \frac{1}{2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{1}{v} \int \frac{d^3 p'_1}{(2\pi)^3 2E'_1} \frac{d^3 p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |M_{1'2' \rightarrow 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

For the numerical implementation of the collision integral we use the stochastic algorithm. (Z. Xu and C. Greiner, PRC 71 064901 (2005))

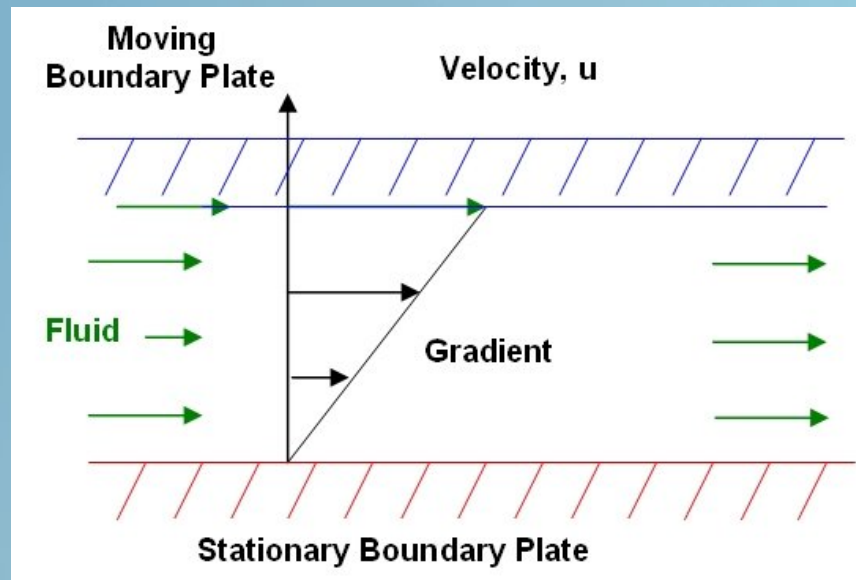
$$P_{22} = \frac{\Delta N_{coll}^{2 \rightarrow 2}}{\Delta N_1 \Delta N_2} = v_{rel} \sigma_{22} \frac{\Delta t}{\Delta^3 x}$$

$\Delta t \rightarrow 0$
 $\Delta^3 x \rightarrow 0$ \Rightarrow **right solution**



Do we really have the wanted shear viscosity η with the relax. time approx.?

- Check η with the Green-Kubo correlator



$$\frac{F}{A} = \eta \frac{\partial u}{\partial y}$$

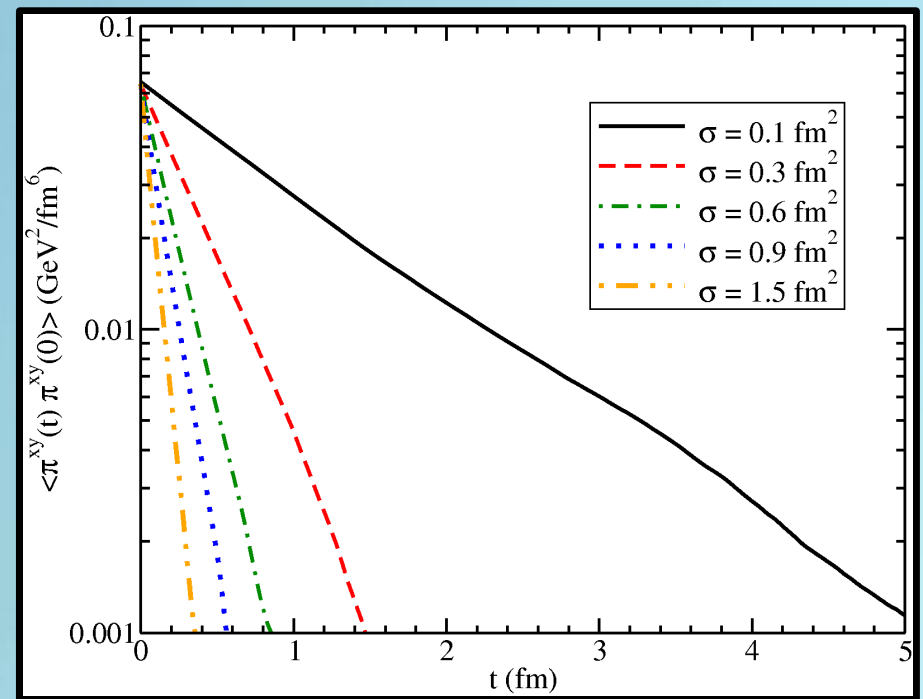
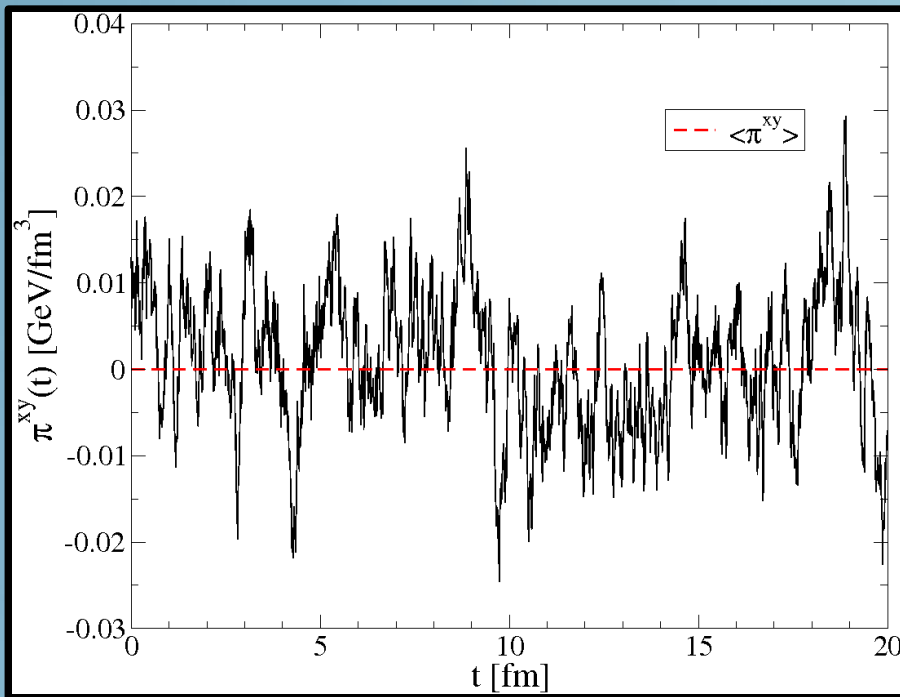
Extraction of the Shear Viscosity: Box calculation

Green – Kubo relation

$$\eta = \frac{1}{T} \int_0^{\infty} dt \int_V d^3x \langle \pi^{xy}(x,t) \pi^{xy}(0,t) \rangle$$

$$\langle \pi^{xy}(\vec{x},t) \pi^{xy}(\vec{0},t) \rangle = \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \cdot e^{-t/\tau}$$

$$\eta = \frac{V}{T} \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \cdot \tau$$



- S. Plumari et al., Phys. Rev. C86 (2012) 054902.
 C. Wesp et al., Phys. Rev. C 84, 054911 (2011).
 J. Fuini III et al. J. Phys. G38, 015004 (2011).

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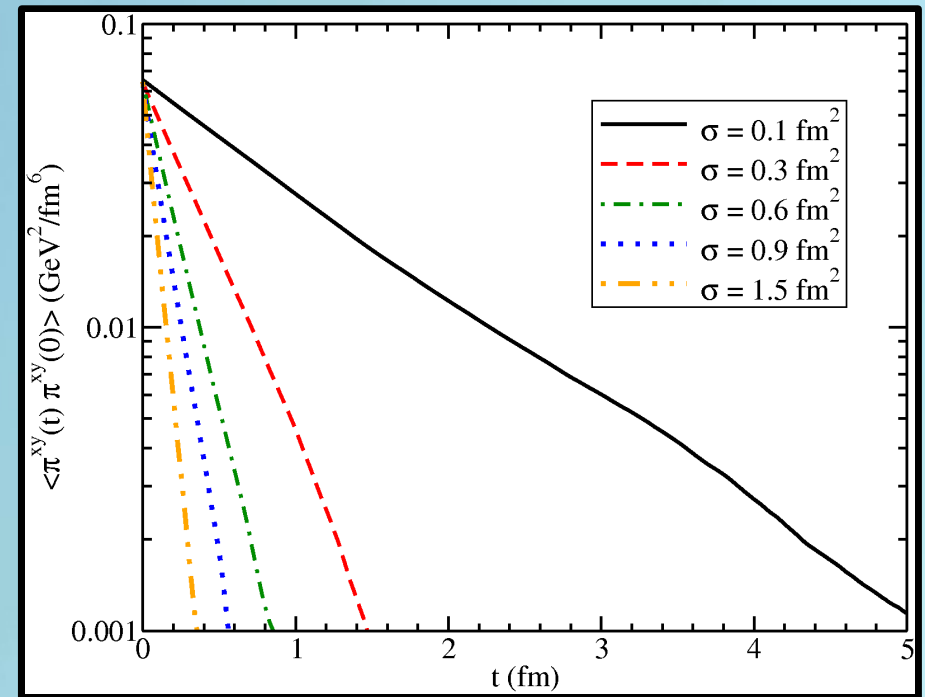
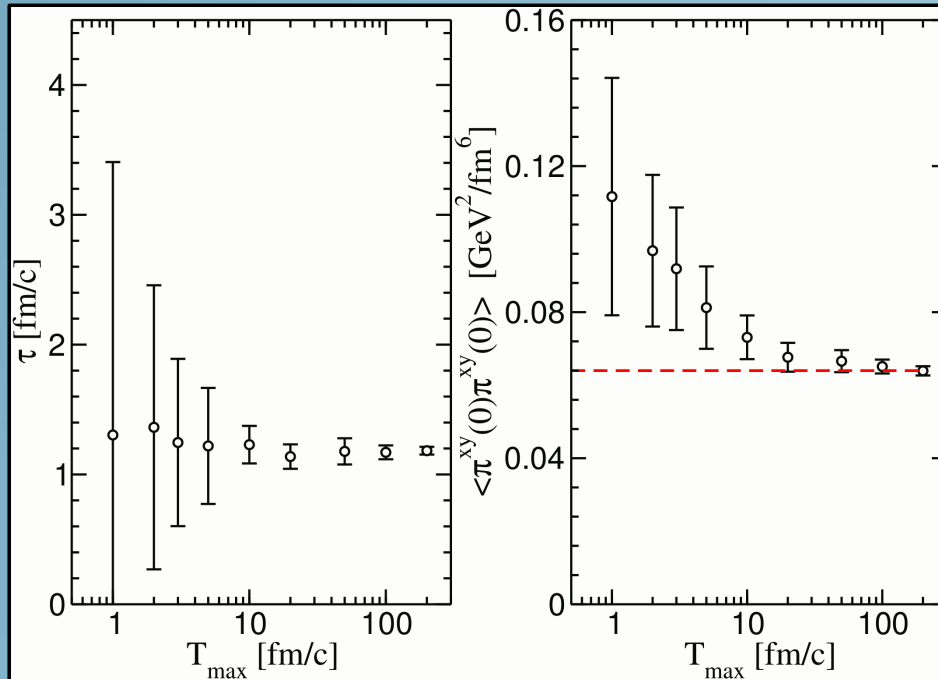
$$\langle \pi^{xy}(\vec{x},t) \pi^{xy}(\vec{0},t) \rangle = \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \cdot e^{-t/\tau}$$

Depends on microscopical details: $\tau(\sigma)$

$$\eta = \frac{V}{T} \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \cdot \tau$$

Depends on macroscopical details:

$$= \frac{4}{15} \frac{eT}{V}$$



- S. Plumari et al., Phys. Rev. C86 (2012) 054902.
 C. Wesp et al., Phys. Rev. C 84, 054911 (2011).
 J. Fuini III et al. J. Phys. G38, 015004 (2011).

Extraction of the Shear Viscosity: Box calculation

Relaxation Time Approximation

Kapusta, PRC(2010); Gavin NPA(1985);

$$\eta = \frac{1}{15T} \int_0^\infty \frac{d^3 p}{(2\pi)^3} \frac{p^4}{E^2} \tau(E) f^{eq}(E) \quad \tau^{-1}(E) = \rho \langle \sigma_{tot} v_{rel} \rangle$$

$$\eta_{relax} = 0.8 \frac{T}{\sigma_{tot}} \quad \longrightarrow \quad \eta \sim \frac{1}{\sigma_{tot}}$$

Usual as Relax. Time Approx. - Israel Stewart $\sigma_{tot} \rightarrow \sigma_{tr} = (2/3) \sigma_{tot}$

$$\eta_{relax}^{IS} = 0.8 \frac{T}{\sigma_{tr}} = 1.2 \frac{T}{\sigma_{tot}}$$

Molnar-Huovinen PRC(2009),
G. Ferini PLB(2009),
Khvorostukhin PRC (2010)
....

Isotropic cross section: massless case

- At 1st order of approx. in the Chapman-Enskog:

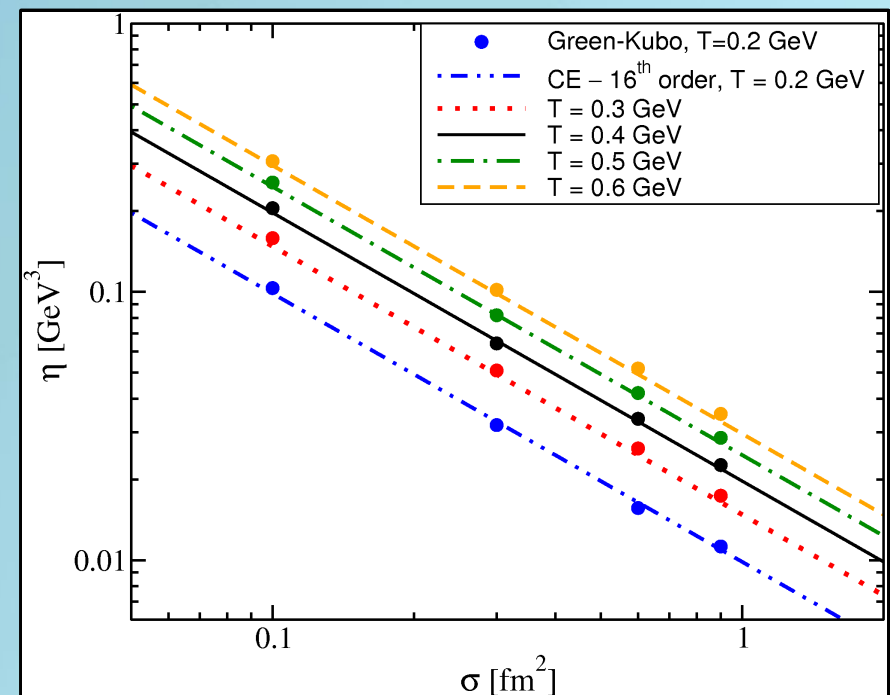
$$[\eta]_{1st}^{CE} = 1.2 \frac{T}{\sigma_{tot}}$$

- successive approx. up to 16 order:

$$[\eta]_{CE}^{16th} = 1.267 \frac{T}{\sigma_{tot}}$$

A. Wiranata, M. Prakash, PRC85 (2012) 054908.
O. N. Moroz, arXiv:1112.0277 [hep-ph].

S. Plumari et al., PRC86 (2012) 054902.



Extraction of the Shear Viscosity: Box calculation

$$\eta_{relax}^{IS}/s = \frac{1}{15} \langle p \rangle \tau_r = \frac{1}{15} \frac{\langle p \rangle}{\sigma_{tot} \langle f(a) v_{rel} \rangle \rho}$$

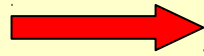
Employed also for non-isotropic cross section:

G.Ferini, PLB(2009); D. Molnar, JPG35(2008);
V.Greco, PPNP(2009);

$$\sigma_{tr} = \int d\Omega \sin^2(\theta_{cm}) \frac{d\sigma}{d\Omega_{cm}} = \sigma_{tot} f(a) \leq \frac{2}{3} \sigma_{tot}$$

For the standard pQCD-like cross section

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{9\pi\alpha_s^2}{2} \frac{1}{(q^2 + m_D^2)^2} \left(1 + \frac{m_D^2}{s}\right)$$



m_D regulates the anisotropy of collision
 $m_D \rightarrow \infty$ we recover the isotropic limit

$$f(a) = 4a(1+a)[(2a+1)\ln(1+a^{-1}) - 2], \quad a = m_D^2/s$$

1st Chapman-Enskog approximation

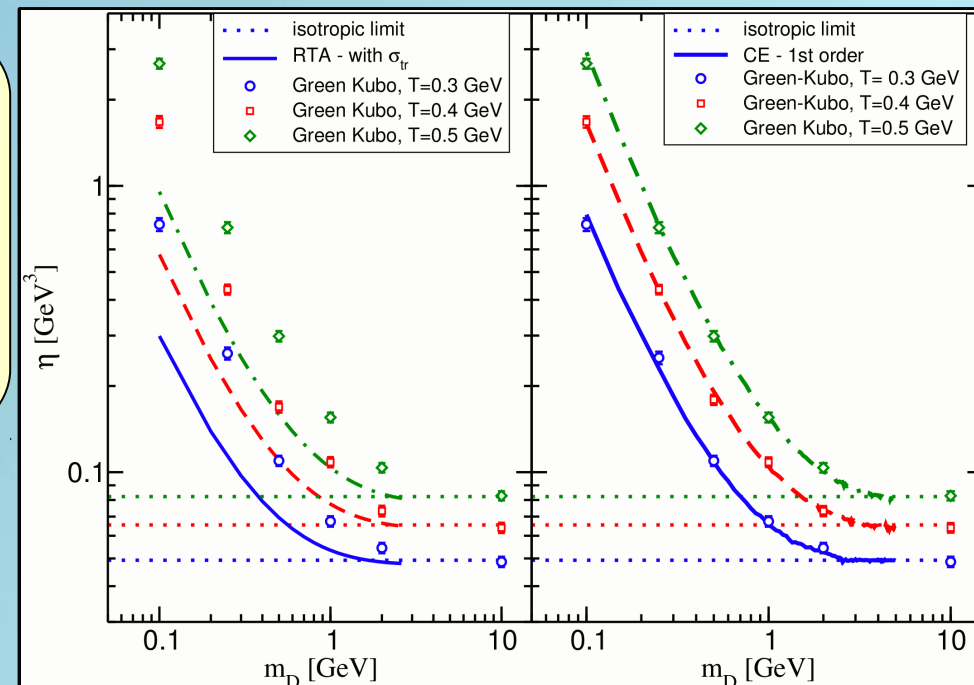
$$[\eta]_{1st}/s = \frac{1}{15} \langle p \rangle \tau_\eta = \frac{1}{15} \frac{\langle p \rangle}{\sigma_{tot} g(a) \rho}$$

$$g(a) = \frac{1}{50} \int_0^\infty dy y^6 \left[\left(y^2 + \frac{1}{3}\right) K_3(2y) - y K_2(2y) \right] f(a), \quad a = \frac{m_D}{2T}$$

- CE and RTA can differ by a factor of 2
- Green-Kubo agree with CE (< 5%)

A. Wiranata, M. Prakash, PRC85 (2012) 054908.
O. N. Moroz, arXiv:1112.0277 [hep-ph].

S. Plumari et al., PRC86 (2012) 054902.



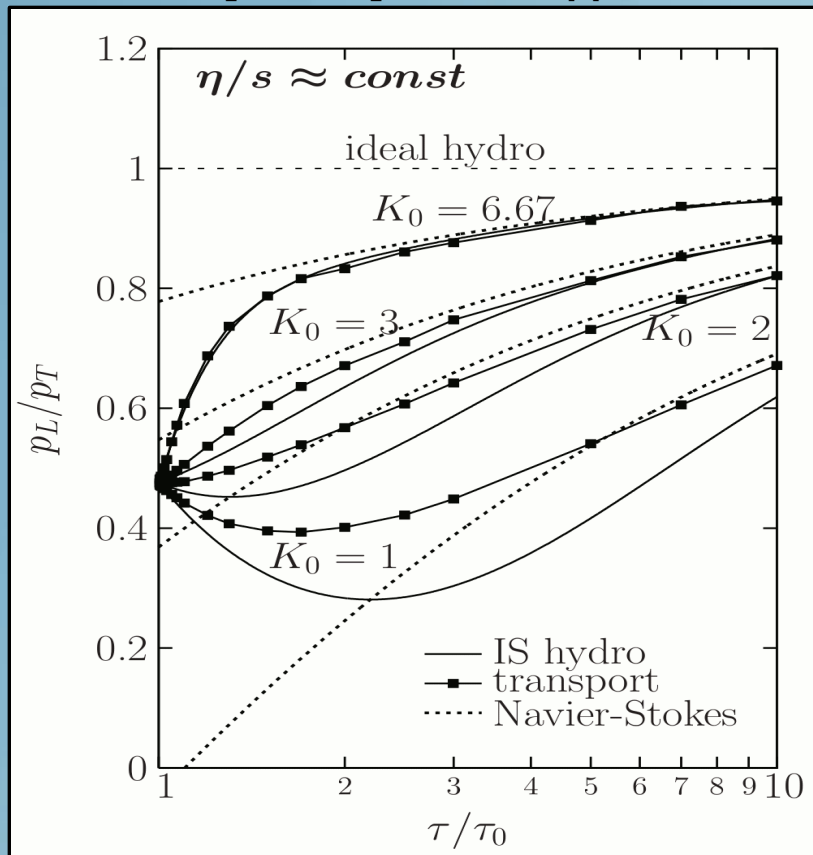
- **We know how to fix locally $\eta/s(T)$**
- **We have checked the Chapmann-Enskog:**
 - *CE good already at I° order $\approx 5\%$ ($\approx 3\%$ at II° order)*
 - *RTA even with σ_{tr} severely underestimates η*

Simulating a constant η/s

For the general case of anisotropic cross section and massless particles:

$$\eta(\vec{x}, t)/s = \frac{1}{15} \langle p \rangle \tau_\eta \quad \longrightarrow \quad \sigma_{tot}^{\eta/s} = \frac{1}{15} \frac{\langle p \rangle}{g(m_D/2T)n} \frac{1}{\eta/s}$$

σ is evaluated in such way to keep fixed the η/s during the dynamics according the Chapman-Enskog equation. (similar to D. Molnar, arXiv:0806.0026[nucl-th] but our approach is local.)



Huovinen and Molnar, PRC79(2009)

Knudsen number

$$K = \frac{L}{\lambda} \rightarrow \frac{\tau}{\lambda}$$

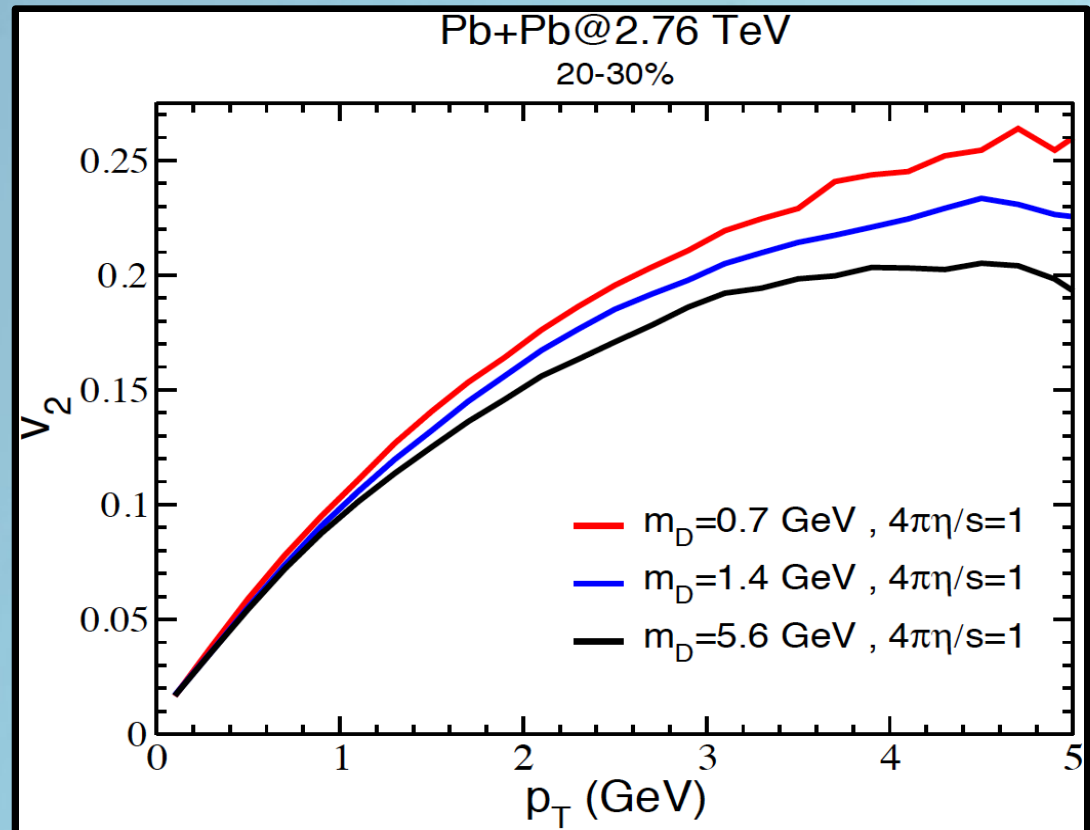
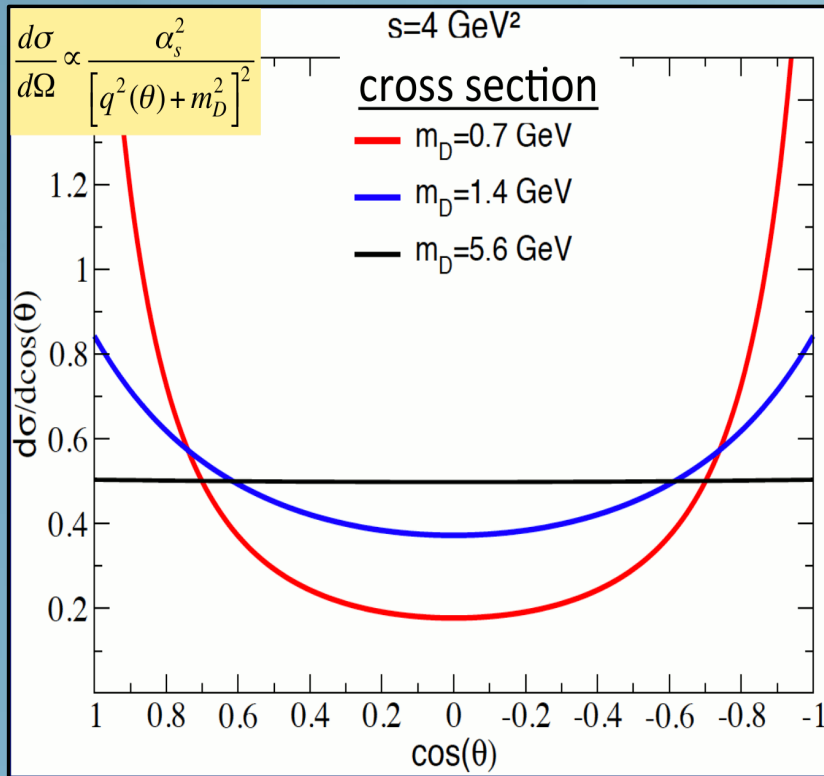
Large K small η/s

$$K = \frac{1}{5} \frac{T_0 \tau_0}{\eta/s}$$

$$\frac{\eta}{s} = \frac{1}{5} T \cdot \lambda$$

In the limit of small η/s (<0.16) and for small pT equivalent viscous hydro

η/s or detail of the cross section



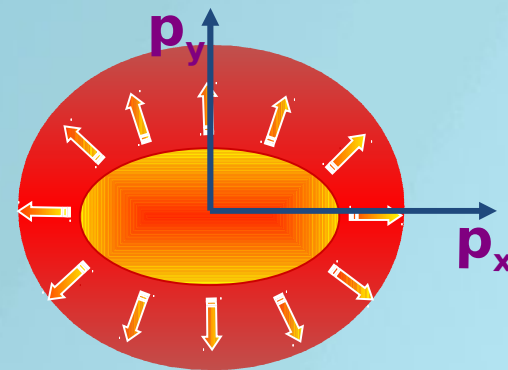
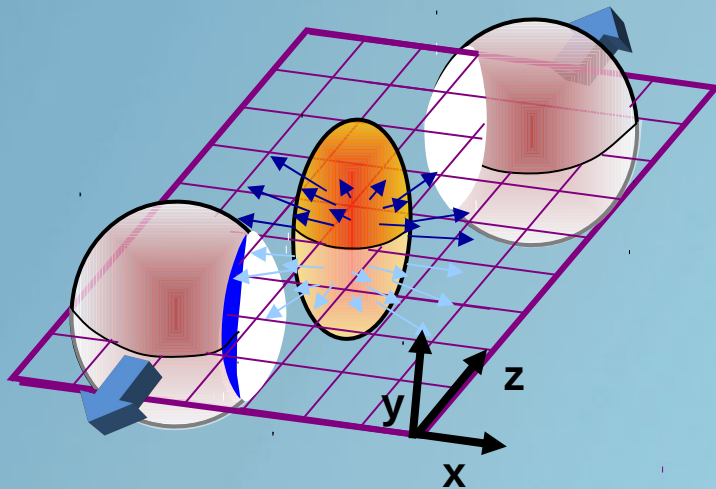
$$\frac{\eta}{s} = \frac{1}{15} \langle p \rangle \tau_\eta$$

$$\tau_\eta = \frac{1}{\sigma_{tot} g(a) \rho}$$

- η/s is the physical parameter determining the v_2 at least up to p_T 1.5 -2 GeV.
- microscopic details becomes important at higher p_T .

Applying kinetic theory to A+A Collisions....

- Impact of $\eta/s(T)$ on the build-up of $v_2(p_T)$ vs. beam energy



Initial condition of our simulation

- ◇ **r-space: standard Glauber model**
- ◇ **p-space: Boltzmann-Juttner $T_{\max}=1.7-3.5 T_c$**
- ◇ **[$p_T < 2 \text{ GeV}$] + minijet [$p_T > 2-3 \text{ GeV}$]**
Discarded in viscous hydro

We fix maximum initial T at RHIC 200 AGeV

$$T_{\max 0} = 340 \text{ MeV}$$

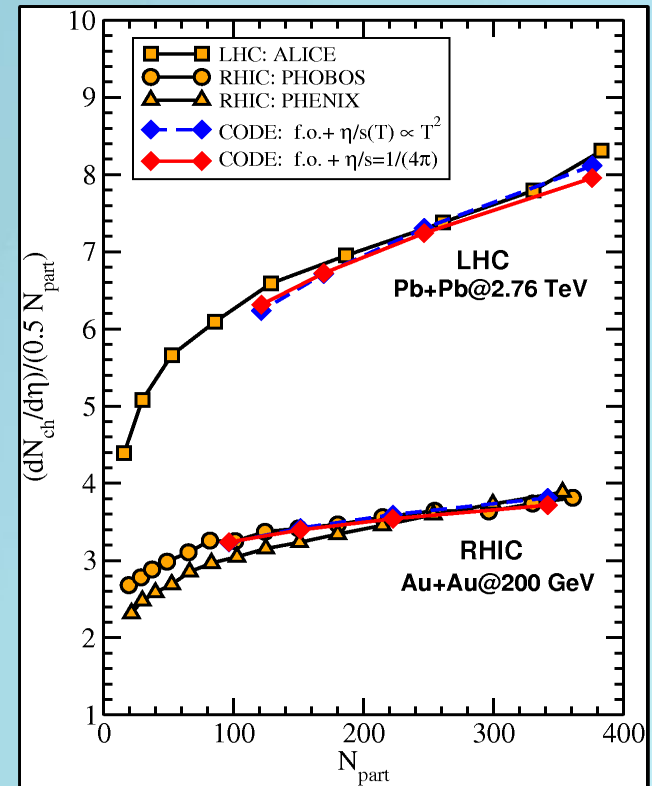
$$T_0 \tau_0 = 1 \rightarrow \tau_0 = 0.6 \text{ fm/c}$$

Typical
hydro
condition

**Then we scale it
according to**

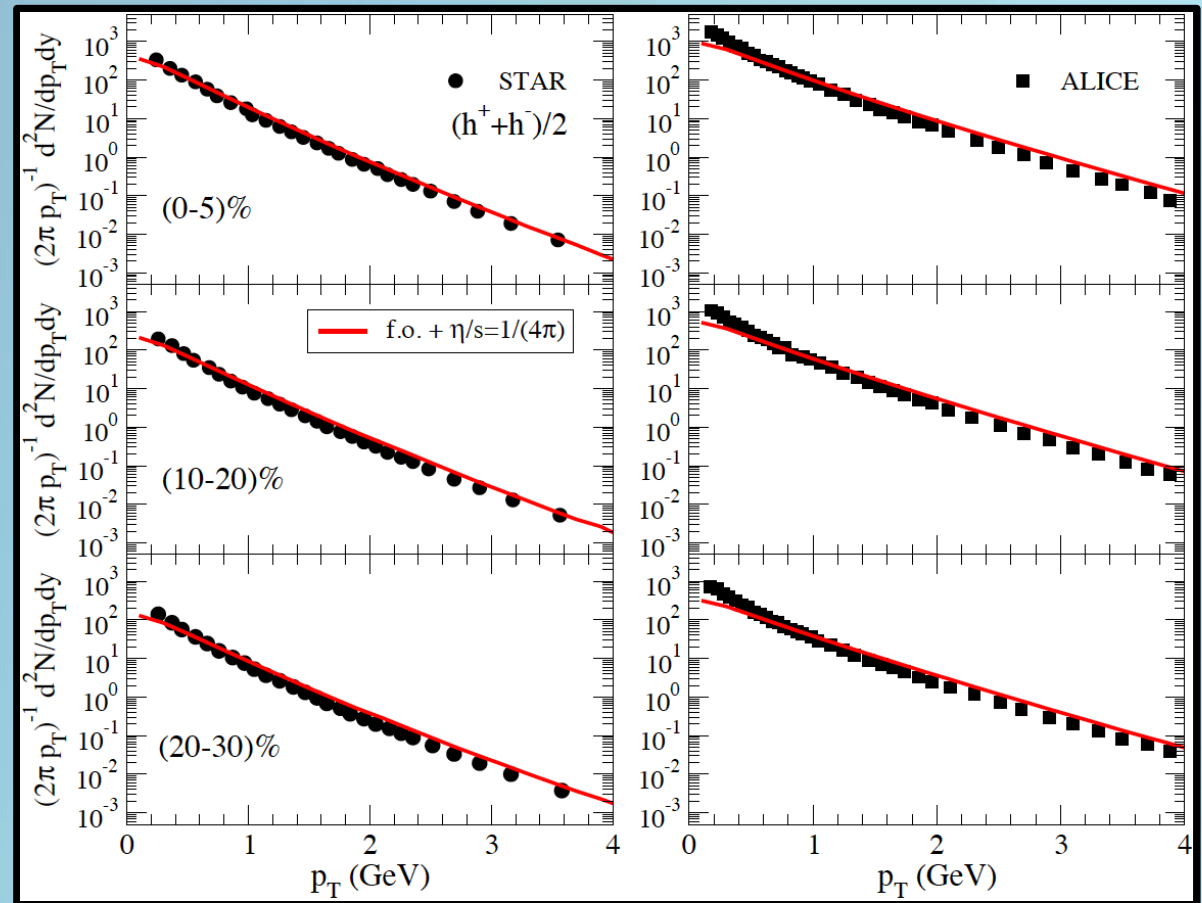
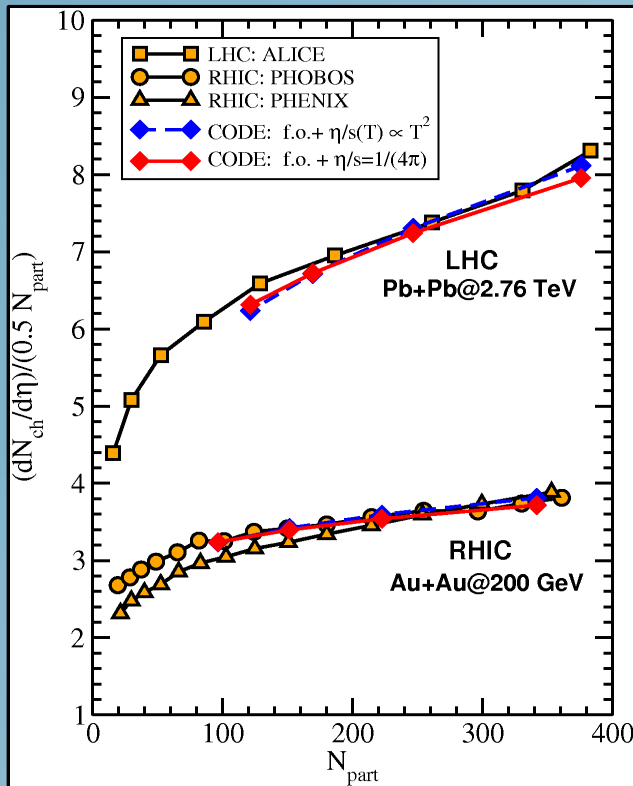
$$\frac{1}{\tau A_T} \frac{dN_{ch}}{d\eta} \propto T^3$$

	62 GeV	200 GeV	2.76 TeV
\sqrt{s}			
T_0	290 MeV	340 MeV	590 MeV
τ_0			
	0.7 fm/c	0.6 fm/c	0.3 fm/c



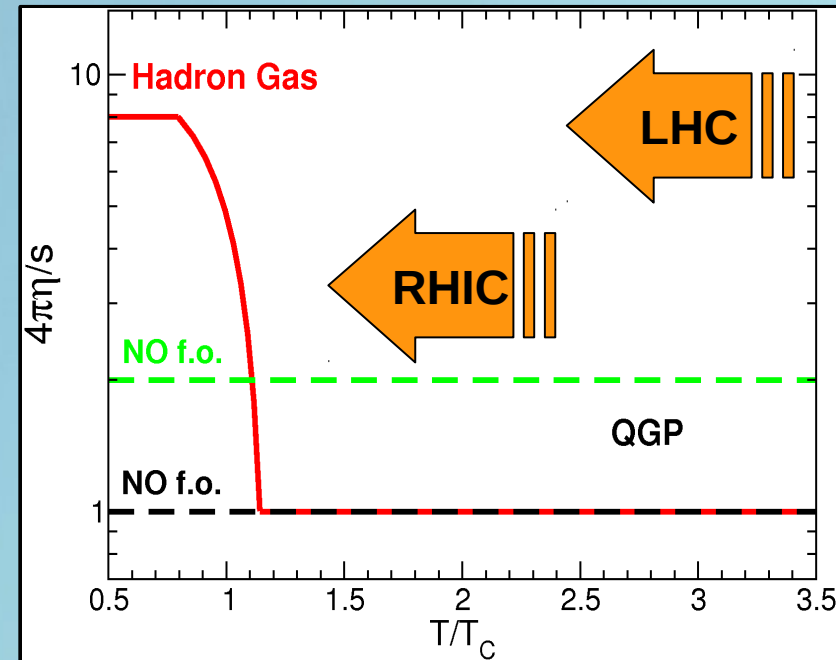
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Discarded in viscous hydro**



kinetic freeze-out scheme

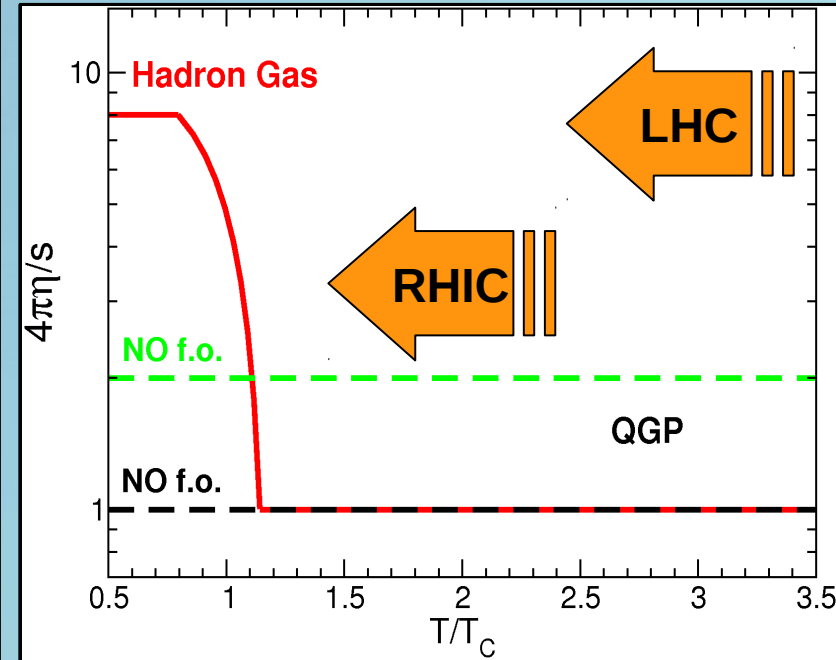
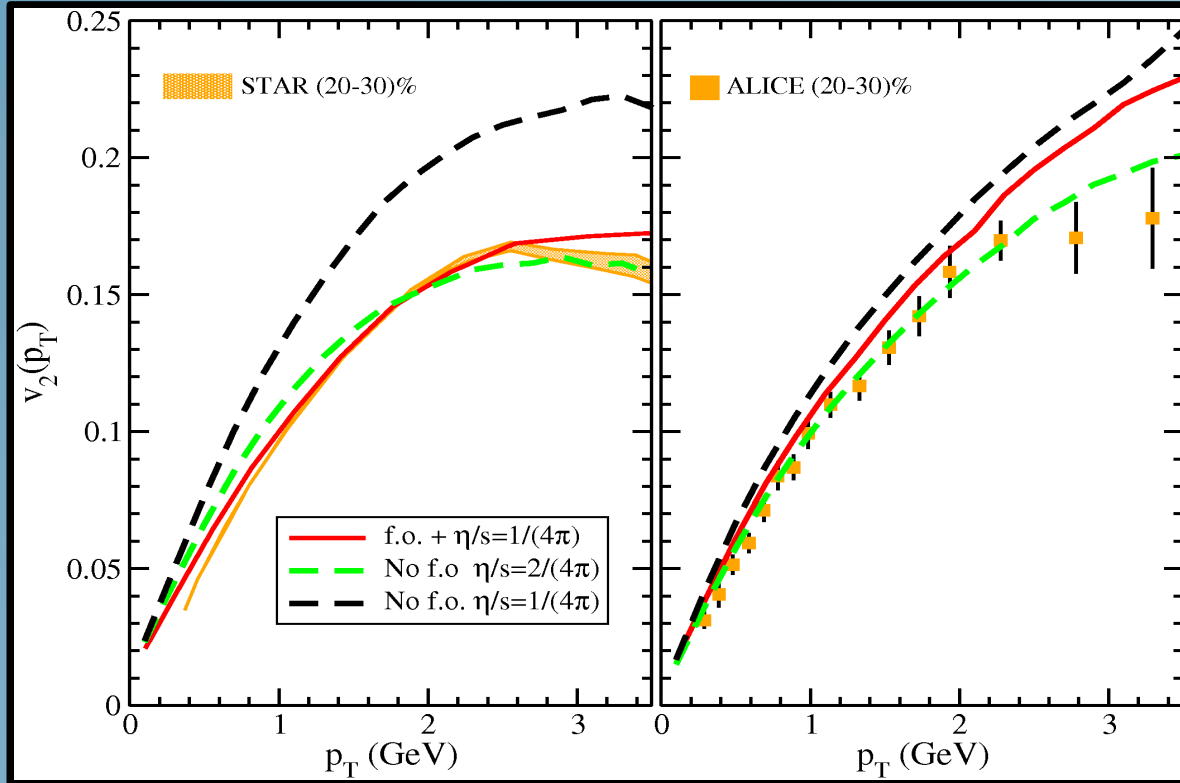
- The f.o. is the increase of η/s in the cross-over region, with a smooth transition between the QGP and the hadronic phase, the collisions are switched off.



For the v_2 similar to cut-off at $\varepsilon_0 = 0.7 \text{ GeV}/\text{fm}^3$

kinetic freeze-out scheme

K. Aamodt et al. [ALICE Collaboration], Phys. Rev.Lett. 105, 252302 (2010).



S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013). arXiv:1209.0601.

RHIC:

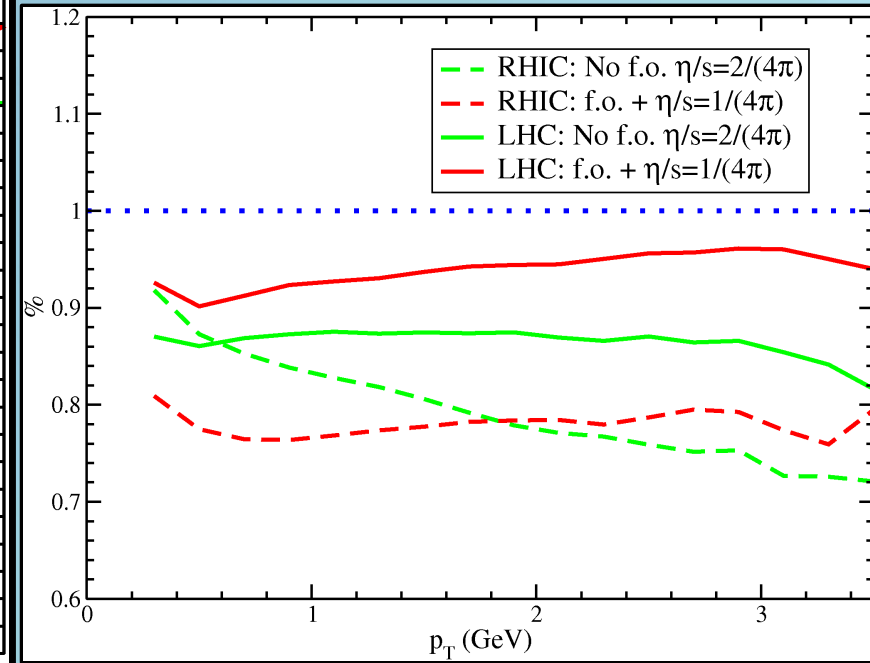
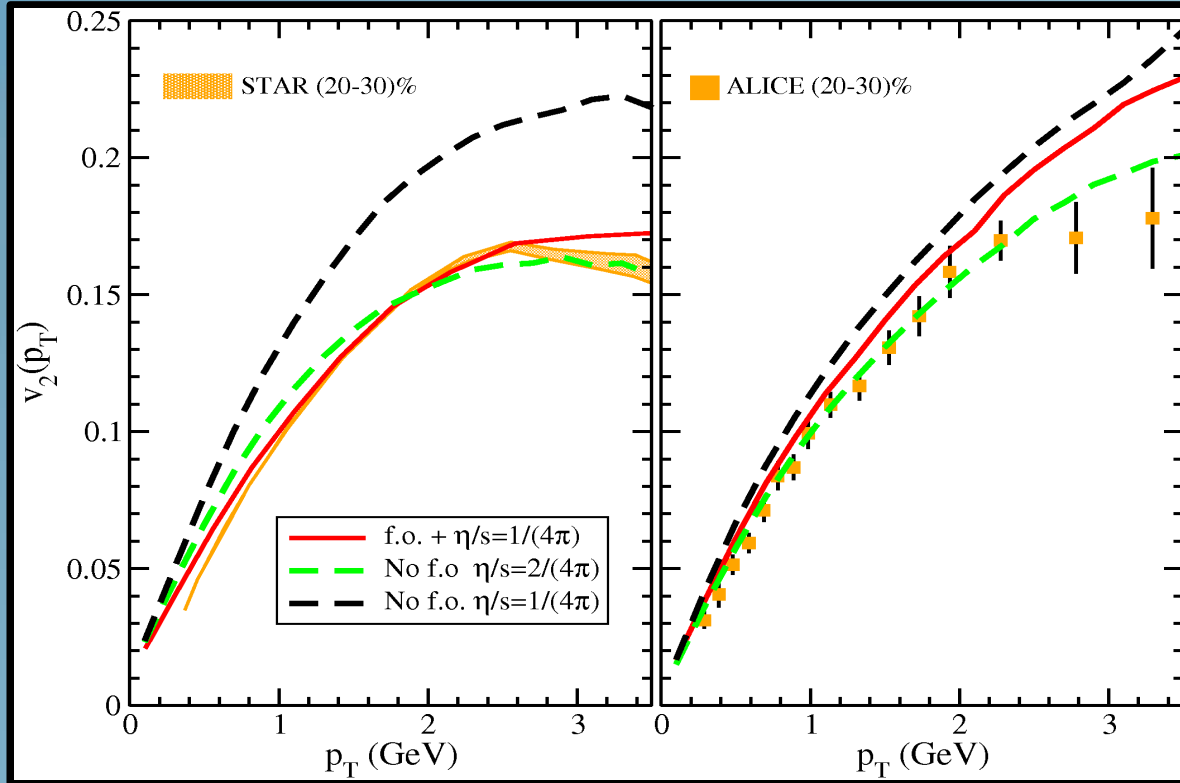
- Like viscous hydro the data are close to $\eta/s=1/(4\pi) + \text{f.o.}$
- Sensitive reduction of the v_2 when the f.o. is included the effect is about of 20%.
- $p_T < 2.5$ GeV good agreement with the experimental data.

LHC:

- $p_T < 2$ GeV like hydro data described with $\eta/s=1/(4\pi) + \text{f.o.}$
- Smaller effect on the reduction of the v_2 when the f.o. is included an effect of about 5%.
- Without the kinetic freezeout the effect of a constant $\eta/s=2(4\pi)^{-1}$ is to reduce the v_2 of 15%.

kinetic freeze-out scheme

K. Aamodt et al. [ALICE Collaboration], Phys. Rev.Lett. 105, 252302 (2010).



S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013). arXiv:1209.0601.

At LHC the contamination of mixed and hadronic phase becomes negligible

Longer life time of QGP $\rightarrow v_2$ completely developed in the QGP phase
(at least up to 3 GeV)

$\eta/s(T)$ around to a phase transition

- Quantum mechanism

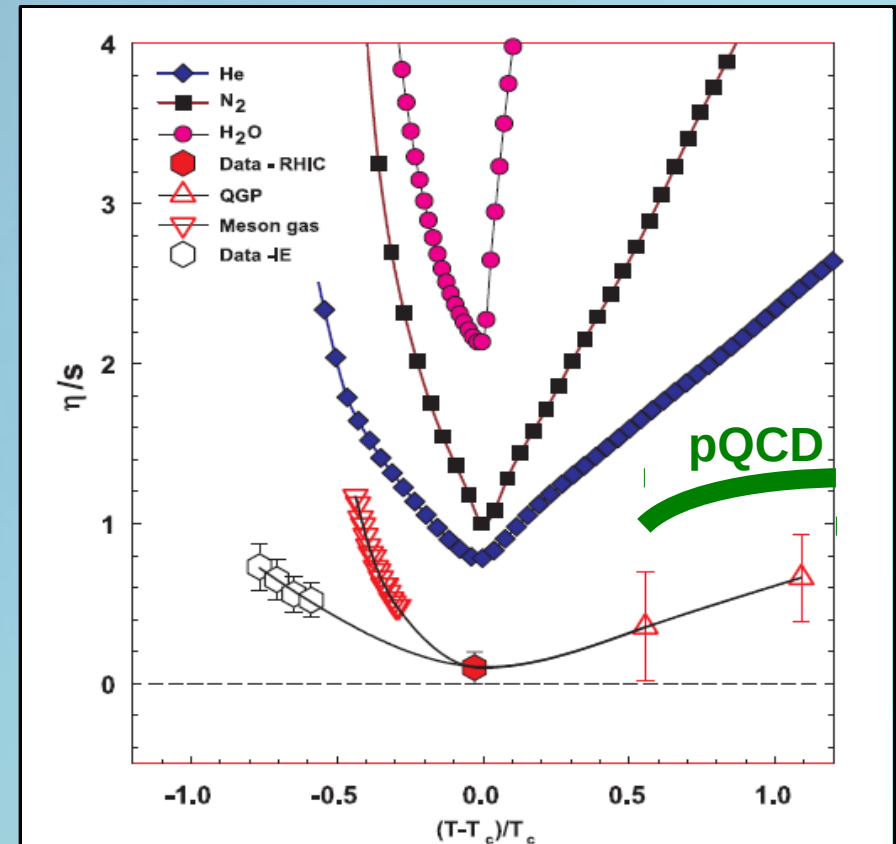
$$\Delta E \cdot \Delta t \geq 1 \rightarrow \eta/s \approx \frac{1}{15} \langle p \rangle \cdot \tau > \frac{1}{15}$$

- AdS/CFT suggest a lower bound $\eta/s = 1/(4\pi) \sim 0.08$

The QGP viscosity is close to this bound!

Do we have signature of a 'U' shape of $\eta/s(T)$ for the QCD matter ?

P. Kovtun et al., Phys.Rev.Lett. 94 (2005) 111601.
 L. P. Csernai et al., Phys.Rev.Lett. 97 (2006) 152303.
 R. A. Lacey et al., Phys.Rev.Lett. 98 (2007) 092301.



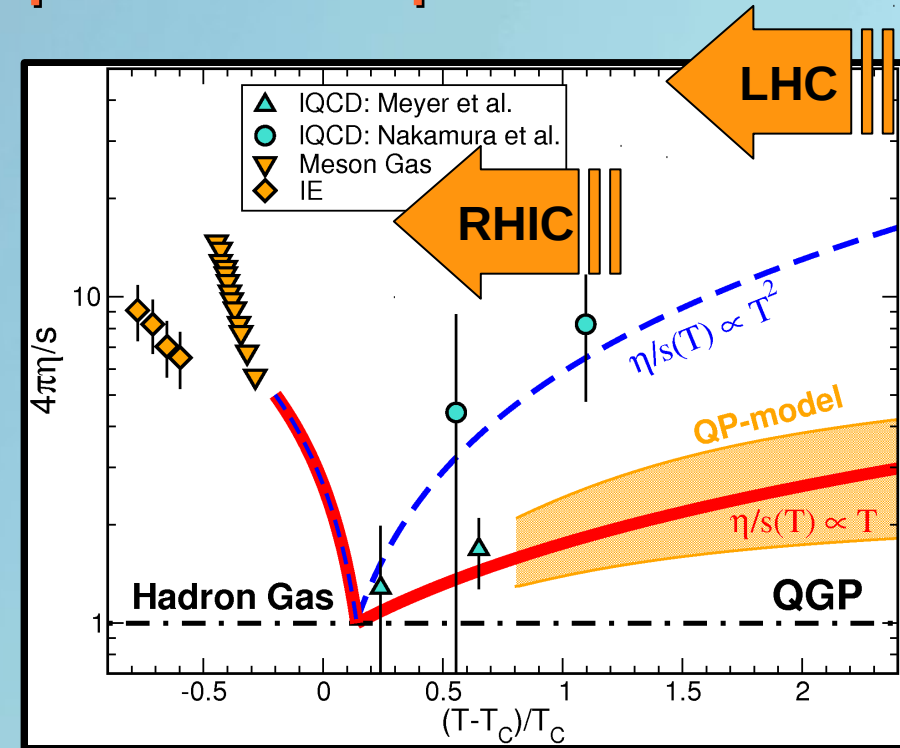
From pQCD: $\eta/s \sim \frac{1}{g^4 \ln(1/g)} \Rightarrow \eta/s \sim 1$

P. Arnold et al., JHEP 0305 (2003) 051.

Temperature dependent $\eta/s(T)$

Phase transition physics suggest a T dependence of η/s also in the QGP phase

- LQCD some results for quenched approx. large error bars
- Quasi-Particle models seem to suggest a $\eta/s \sim T^\alpha$ $\alpha \sim 1 - 1.5$.
- Chiral perturbation theory \rightarrow Meson Gas
- Intermediate Energies – IE ($\mu_B > T$)

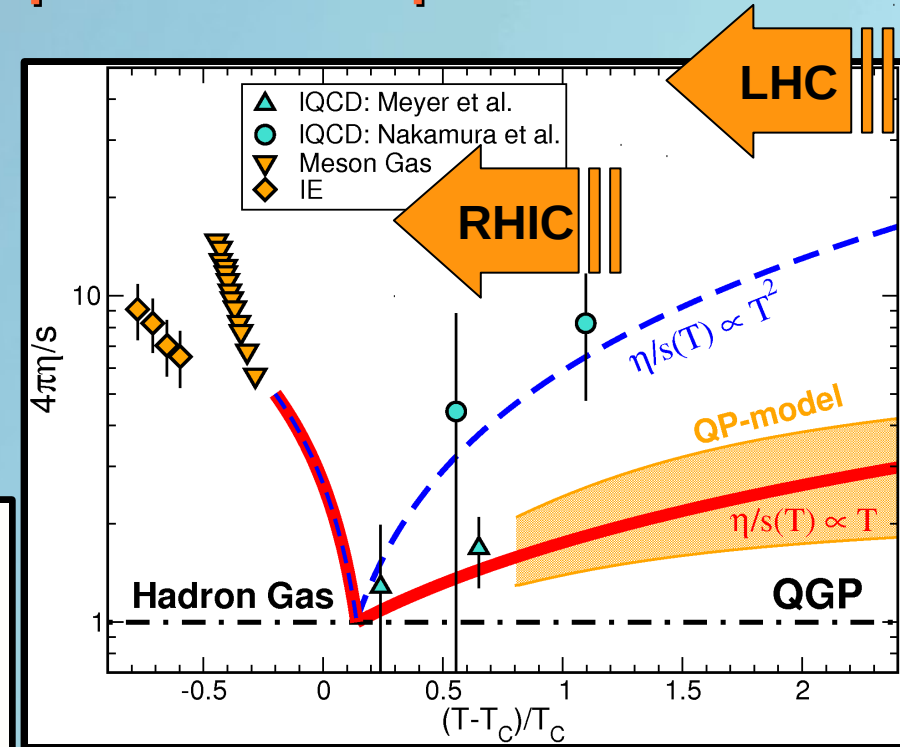
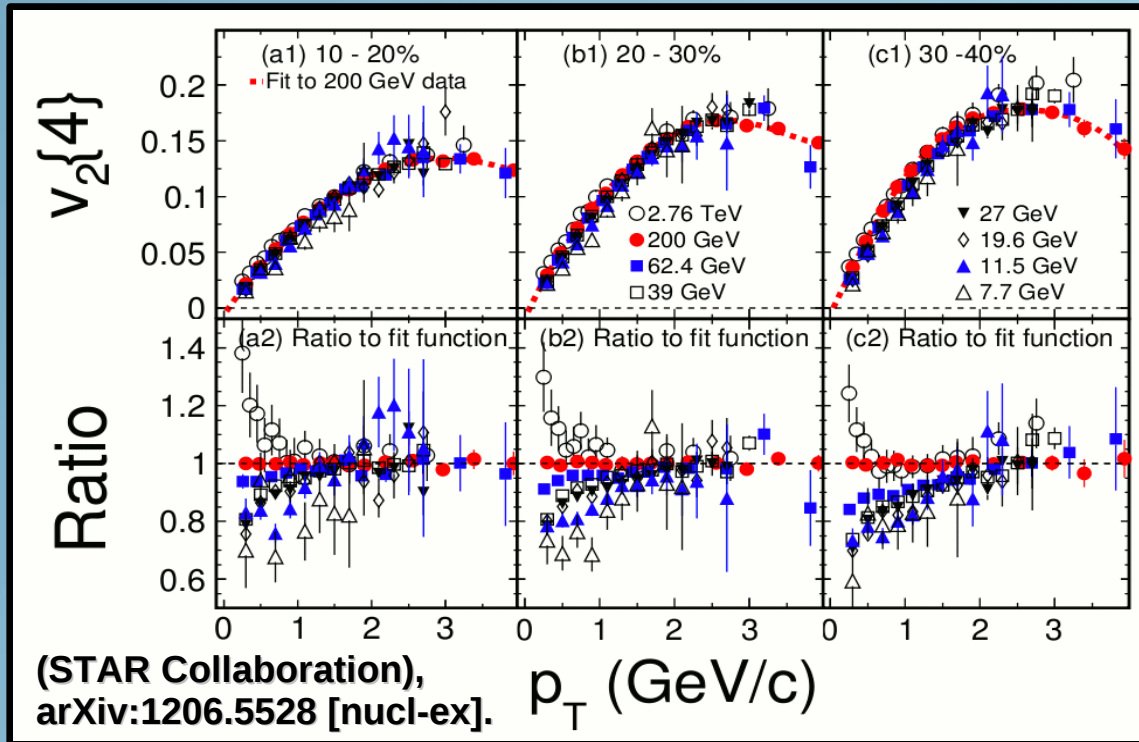


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Temperature dependent $\eta/s(T)$

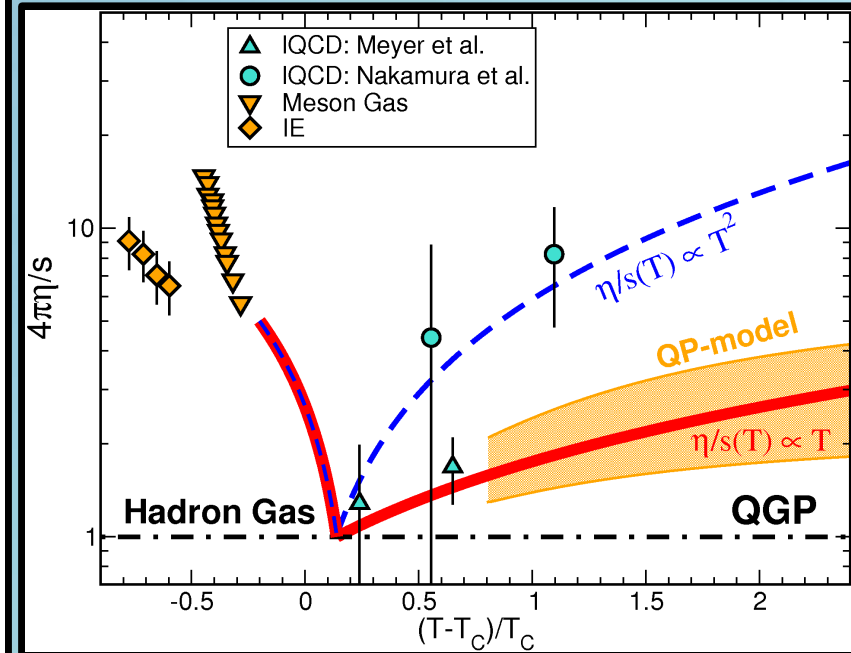
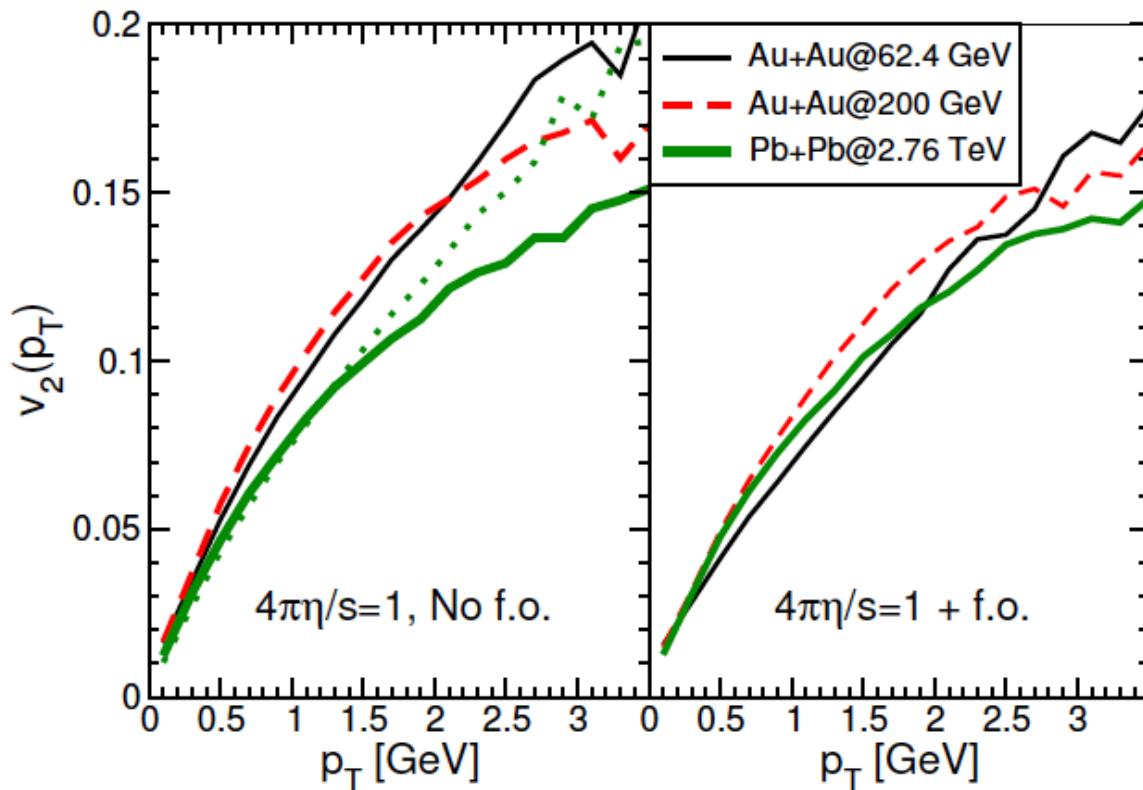
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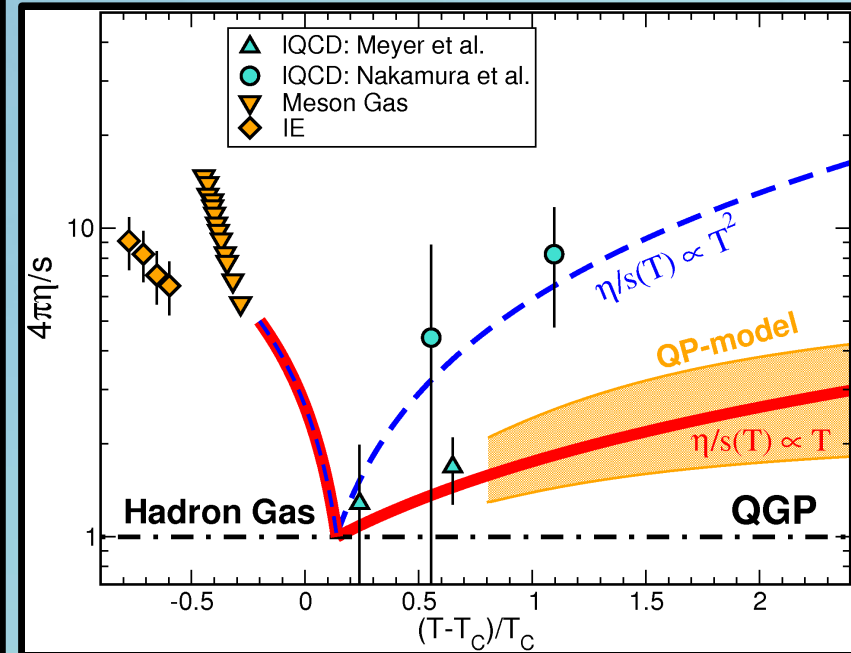
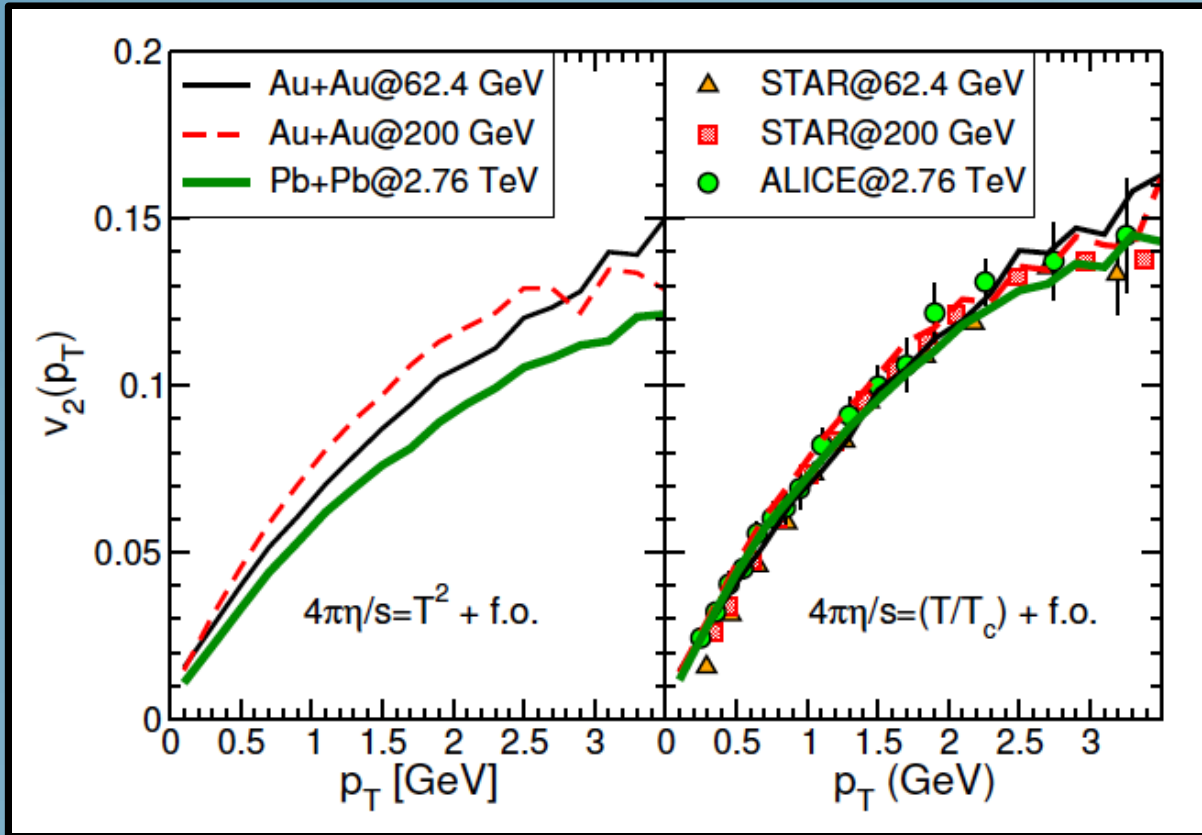
Temperature dependent $\eta/s(T)$



Plumari, Greco, Csernai,
arXiv:1304.6566

- For $4\pi\eta/s=1$ during all the evolution of the fireball we get a discrepancy for the $v_2(p_T)$, in particular we observe a smaller $v_2(p_T)$ at LHC.
- Similar results for $\eta/s \propto T^2 \rightarrow$ a discrepancy about 20%.
- Notice only with RHIC \rightarrow scaling for $4\pi\eta/s=1$ LHC data play a key role

Temperature dependent $\eta/s(T)$

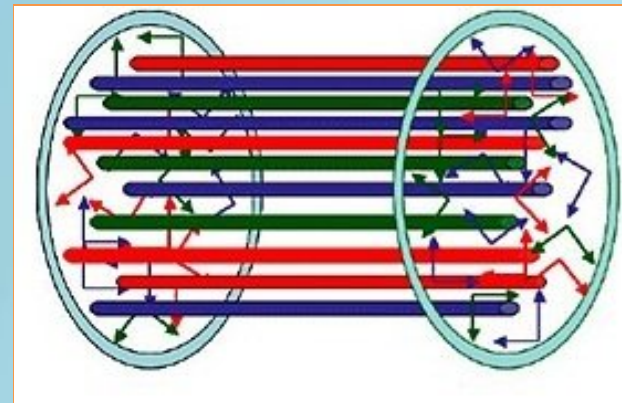
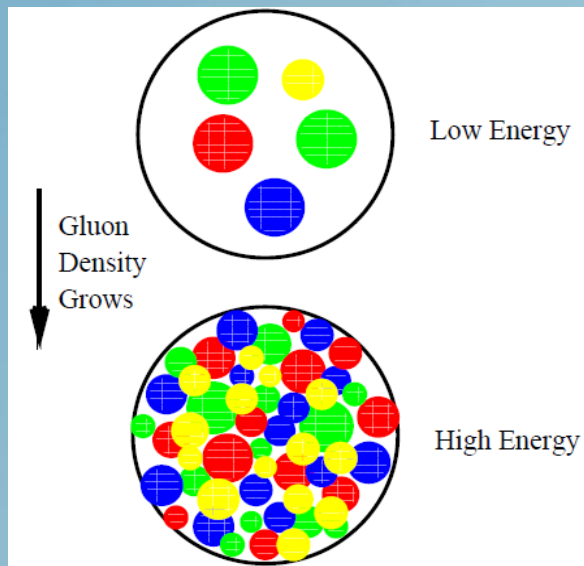


Plumari, Greco, Csernai,
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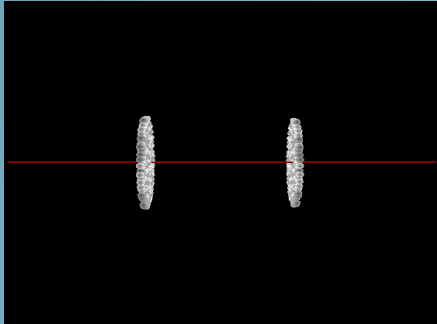
- Invariance of $v_2(p_T)$ in BES suggest that the system goes through a phase transition.
- Hope: v_n , $n > 3$ with an event-by-event analysis will put even stronger constraint
- Implementation of local fluctuation under development
- Similar results: R. A. Lacey et al., arXiv:1305.3341 [nucl-ex].

What about Color Glass condensate initial state

- Kinetic Theory with a Q_s saturation scale



Initial Conditions: Glasma



The two nuclei could be described as two tiny disks of Color Glass Condensate (CGC)

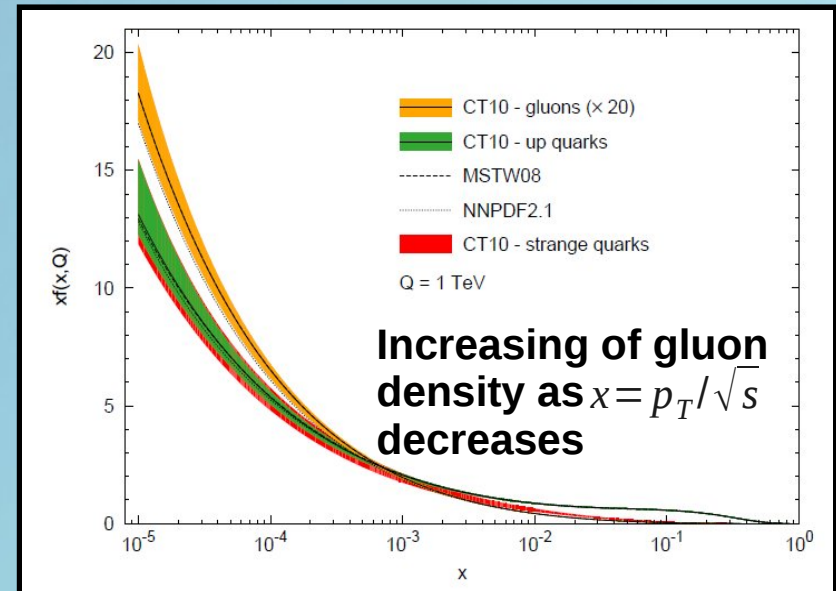
Saturation scale

$$Q_{sat}^2(s) \propto \alpha_s(Q^2) \frac{xg(x, Q^2)}{\pi R^2} \propto A^{1/3}$$

At RHIC $Q_s^2 \sim 1-2 \text{ GeV}^2$

At LHC $Q_s^2 \sim 2-5 \text{ GeV}^2$

The production of particle HIC is controlled by the Q_s



[Brandt and Klasen, arXiv:1305.5677]

Reviews

McLerran, 2011

Iancu, 2009

McLerran, 2009

Lappi, 2010

Gelis, 2010

Fukushima, 2011

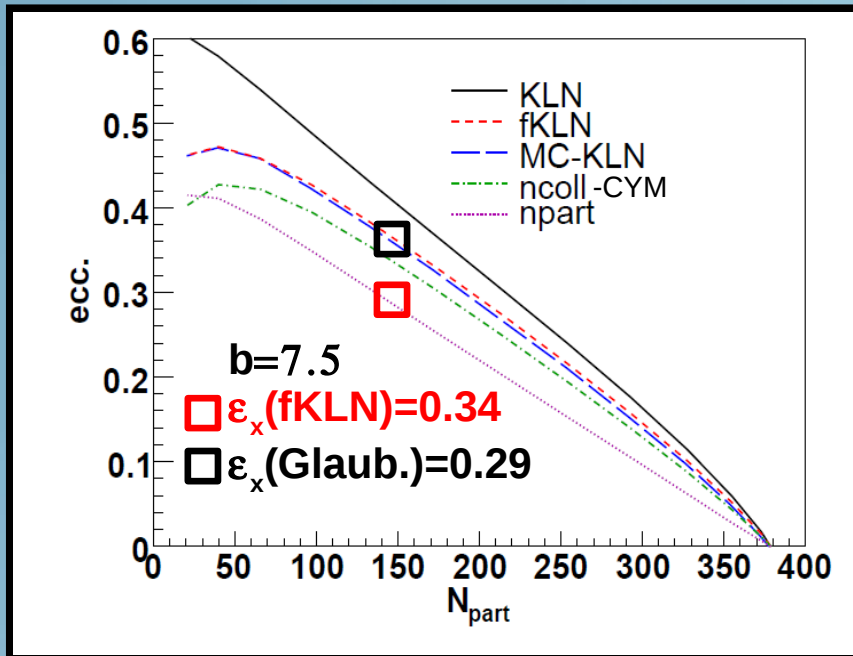
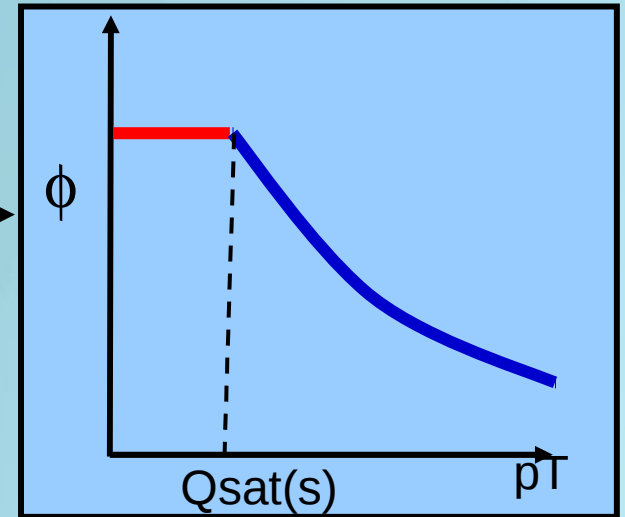
Initial Conditions: fKLN

Kharzeev *et al.*, Phys. Lett. B561, 93 (2003)
 Nardi *et al.*, Phys. Lett. B507, 121 (2001)
 Drescher and Nara, PRC75, 034905 (2007)
 Hirano and Nara, PRC79, 064904 (2009)
 Albacete and Dumitru, arXiv:1011.5161[hep-ph]

$$\begin{aligned}
 \frac{dN_g}{d^2x_\perp dy} &\propto \int \frac{d^2p_T}{p_T^2} \int_0^{p_T} d^2k_T \alpha_s(Q^2) \\
 &\times \phi_A \left(x_A, \frac{(p_T + k_T)^2}{4}; \mathbf{x}_\perp \right) \\
 &\times \phi_B \left(x_B, \frac{(p_T - k_T)^2}{4}; \mathbf{x}_\perp \right)
 \end{aligned}$$

Saturation effects built in the ϕ .

$$\phi_A(x_1, k_T^2; \mathbf{x}_\perp) = \frac{\kappa Q_s^2}{\alpha_s(Q_s^2)} \left[\frac{\theta(Q_s - k_T)}{Q_s^2 + \Lambda^2} + \frac{\theta(k_T - Q_s)}{k_T^2 + \Lambda^2} \right]$$



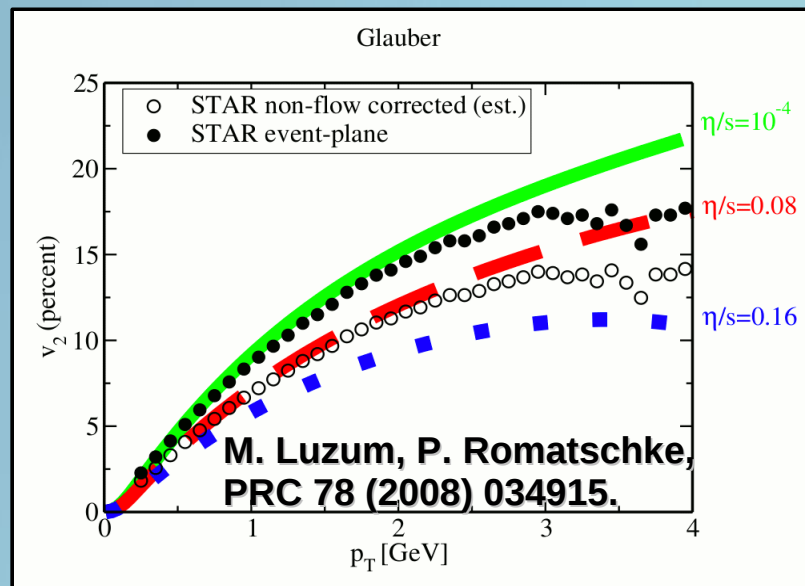
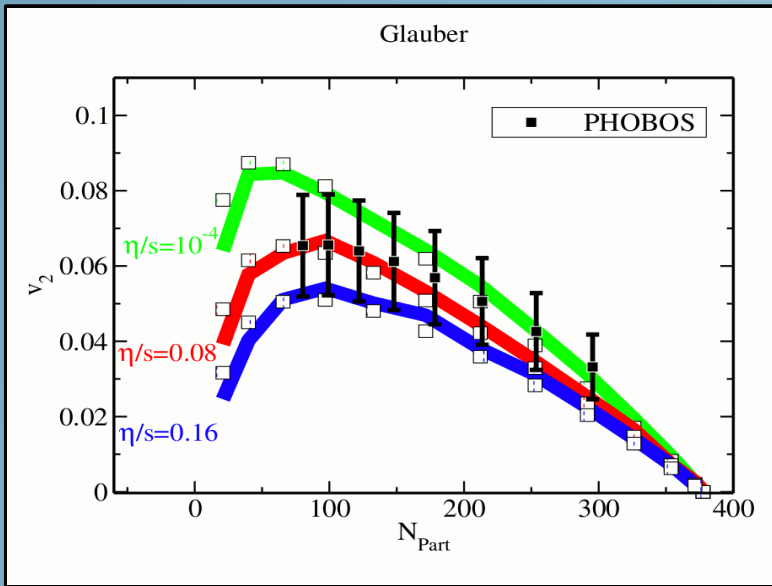
$$Q_{s,A}^2(x, \mathbf{x}_\perp) \propto Q_s^2 T_A(\mathbf{x}_\perp) x^{-\lambda}$$

Universal saturation scale, in agreement with:
 Lappi and Venugopalan, PRC 74 054905 (2006)

V2 from fKLN in viscous hydro

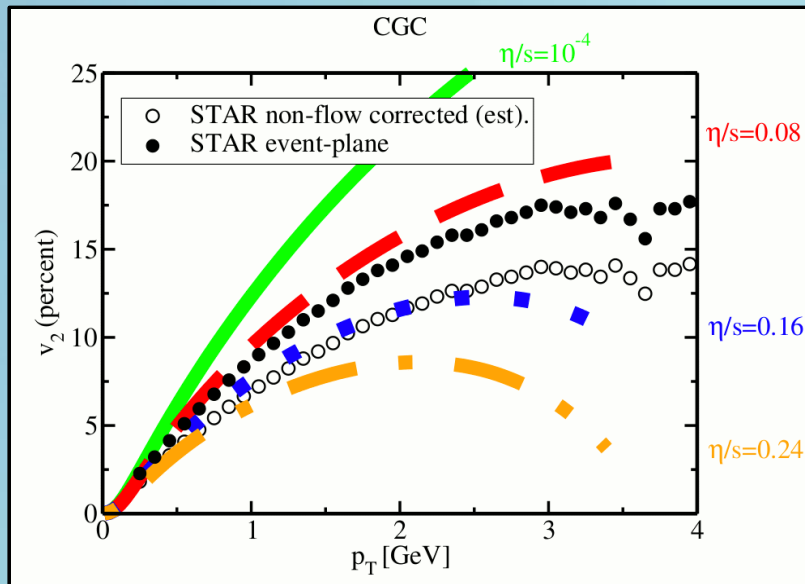
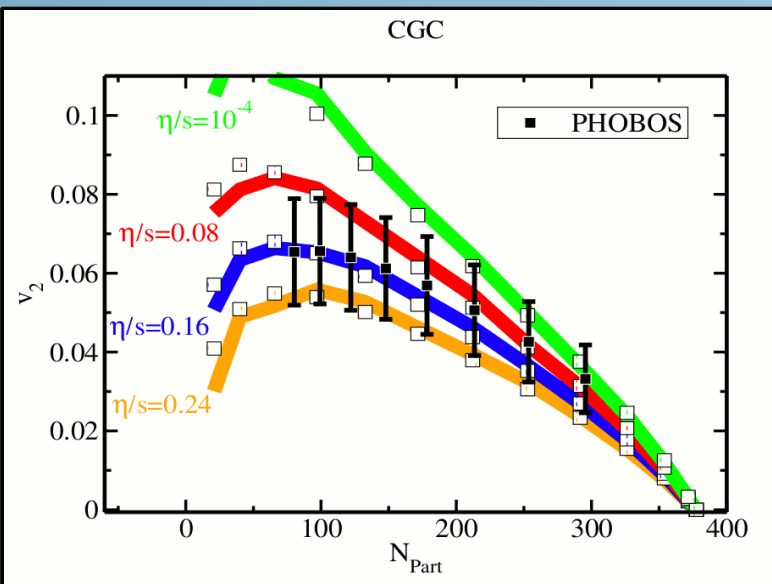
1) r-space from KLN (larger ϵ_x)

2) p-space thermal at $t_0 \approx 0.6$ fm/c - we call it fKLN-Th



Glauber

$$\eta/s \approx \frac{1}{4\pi}$$



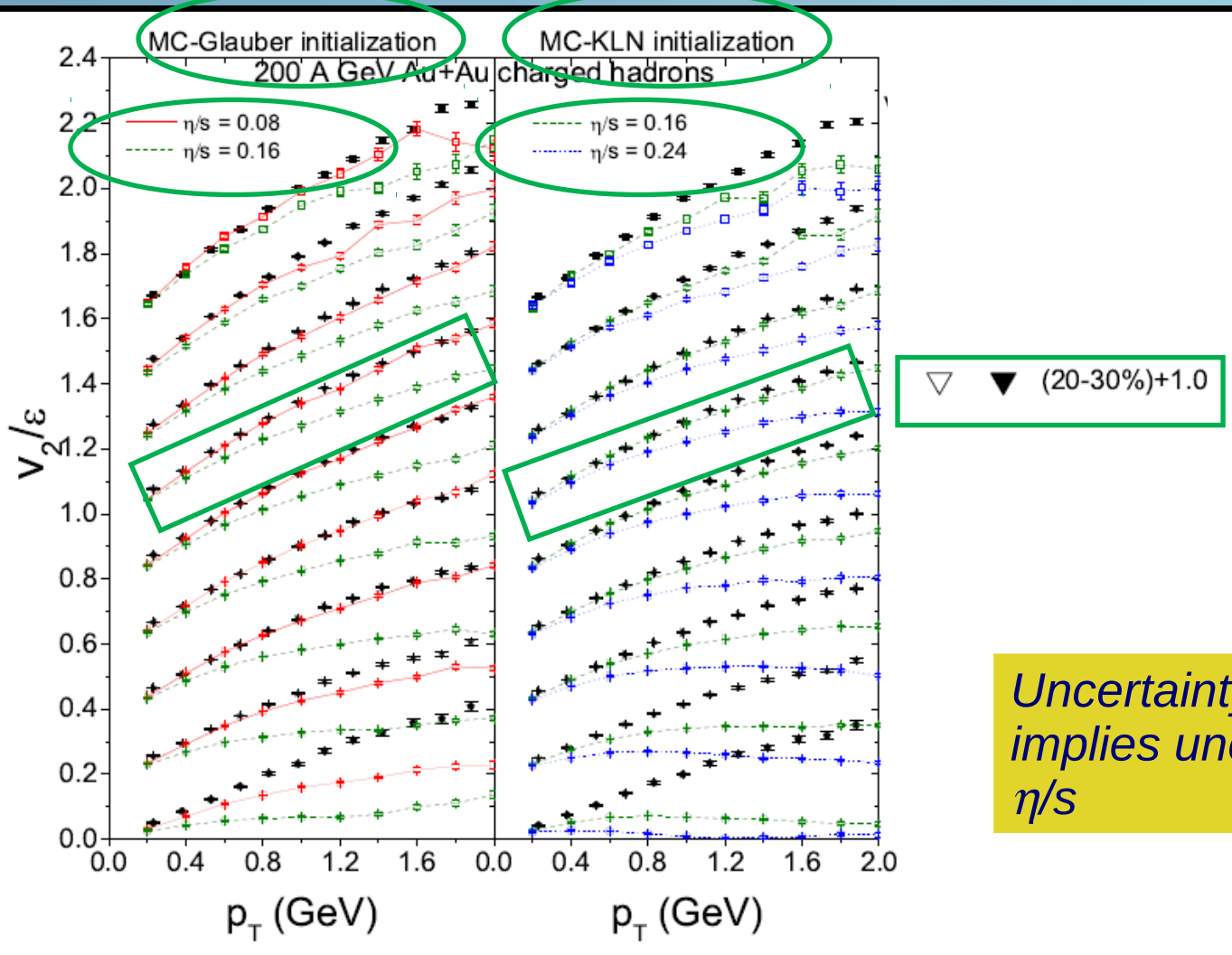
CGC

$$\eta/s \approx \frac{2}{4\pi}$$

V2 from fKLN in viscous hydro

1) r-space from KLN (larger ϵ_x)

2) p-space thermal at $t_0 \approx 0.6$ fm/c - we call it fKLN-Th



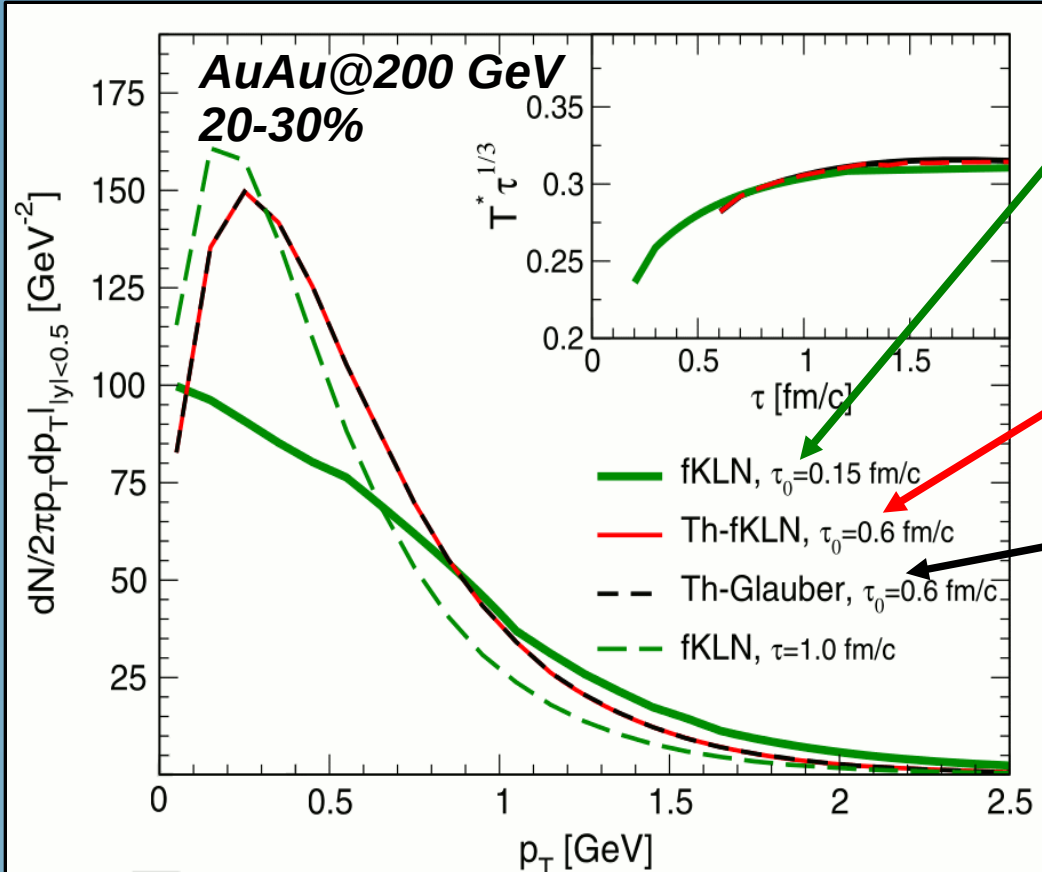
Larger ϵ_x - \rightarrow higher η/s
to get the same $v_2(p_T)$

Glauber: $\eta/s \approx \frac{1}{4\pi}$

CGC: $\eta/s \approx \frac{2}{4\pi}$

Uncertainty on initial conditions
implies uncertainty of a factor 2 on
 η/s

Implementing fKLN pT distribution



Using kinetic theory at finite η/s
we can implement full fKLN
(x & p space) - $\epsilon_x = 0.34$, $Q_s = 1.44$ GeV

fKLN only in x space (like in Hydro)
 $\epsilon_x = 0.34$, $Q_s = 0$

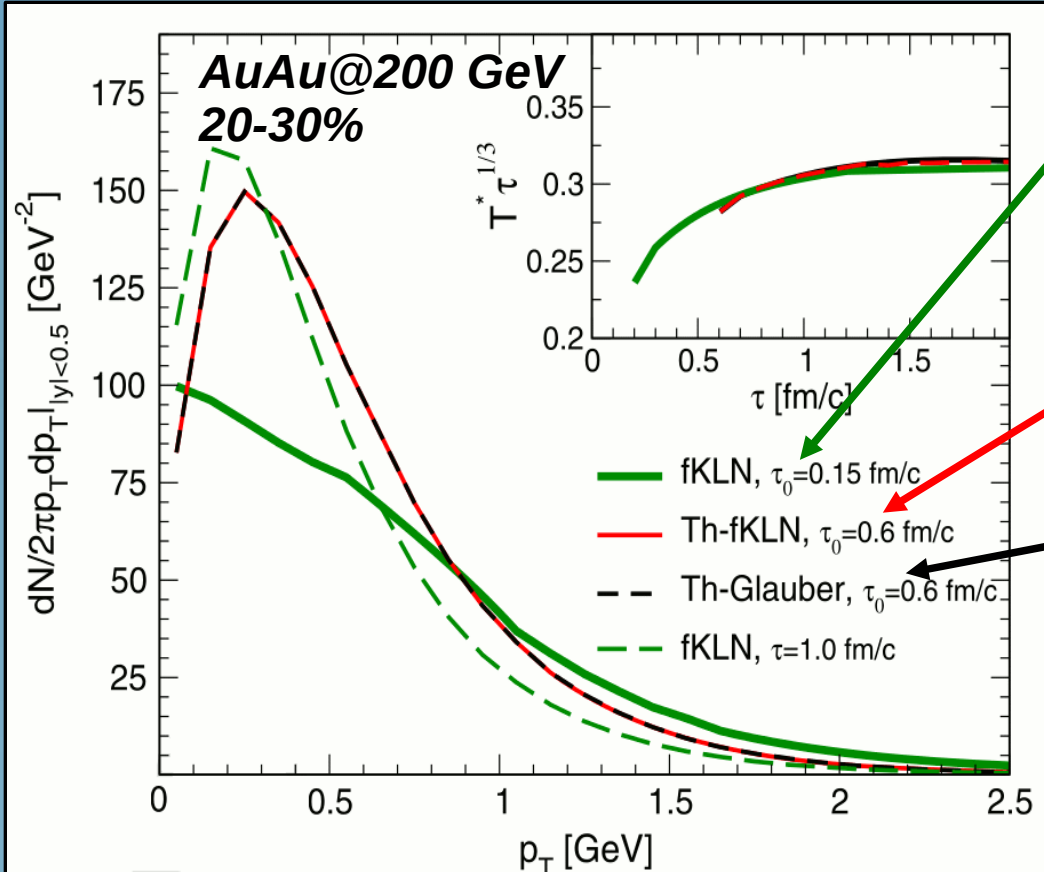
Glauber in x and thermal in p
 $\epsilon_x = 0.289$, $Q_s = 0$

M. Ruggieri *et al.*, 1303.3178 [nucl-th]

- Thermalization in less than 1 fm/c, in agreement with Greiner *et al.*, NPA806, 287 (2008).
- Not so surprising: η/s is small \rightarrow large effective scattering rate \rightarrow fast thermalization.

$$\sigma_{tot} = \frac{\langle p \rangle}{\rho g(a)} \frac{1}{\eta/s}$$

Implementing fKLN pT distribution



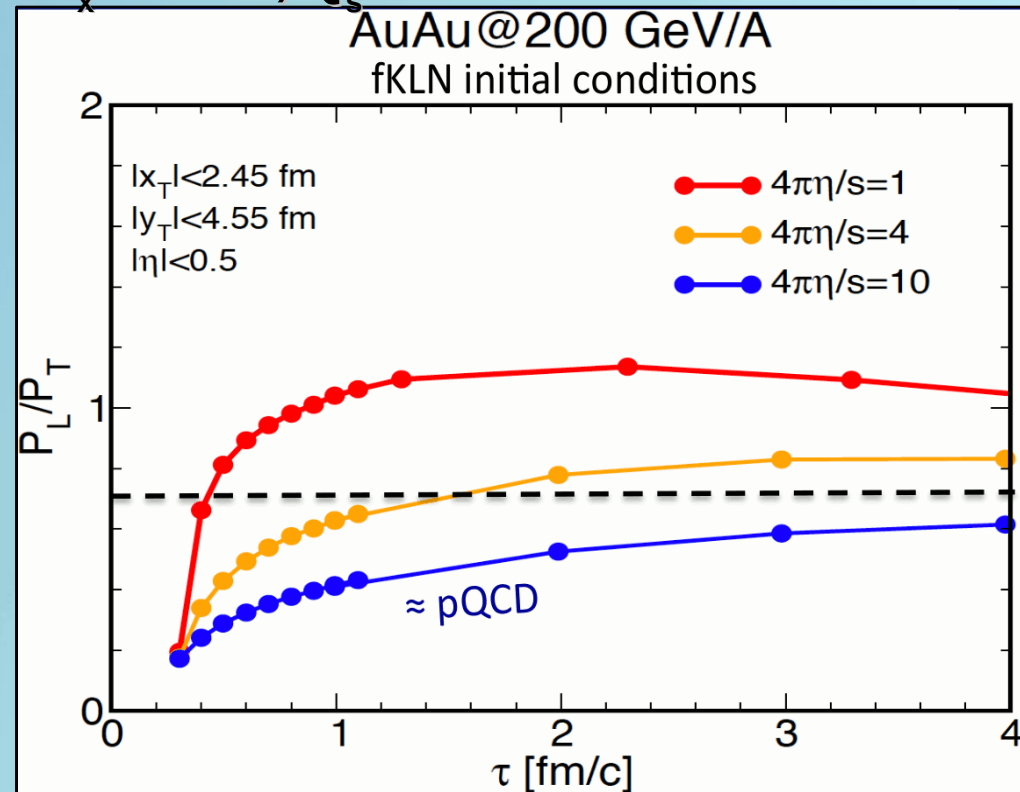
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 $\epsilon_x = 0.34$, $Q_s = 0$

Glauber in x and thermal in p
 $\epsilon_x = 0.289$, $Q_s = 0$

• Semi-quantitative agreement with
Florkowski PRD88 (2013) 034028.

our approach is 3+1D no relaxation time
approximation but no field

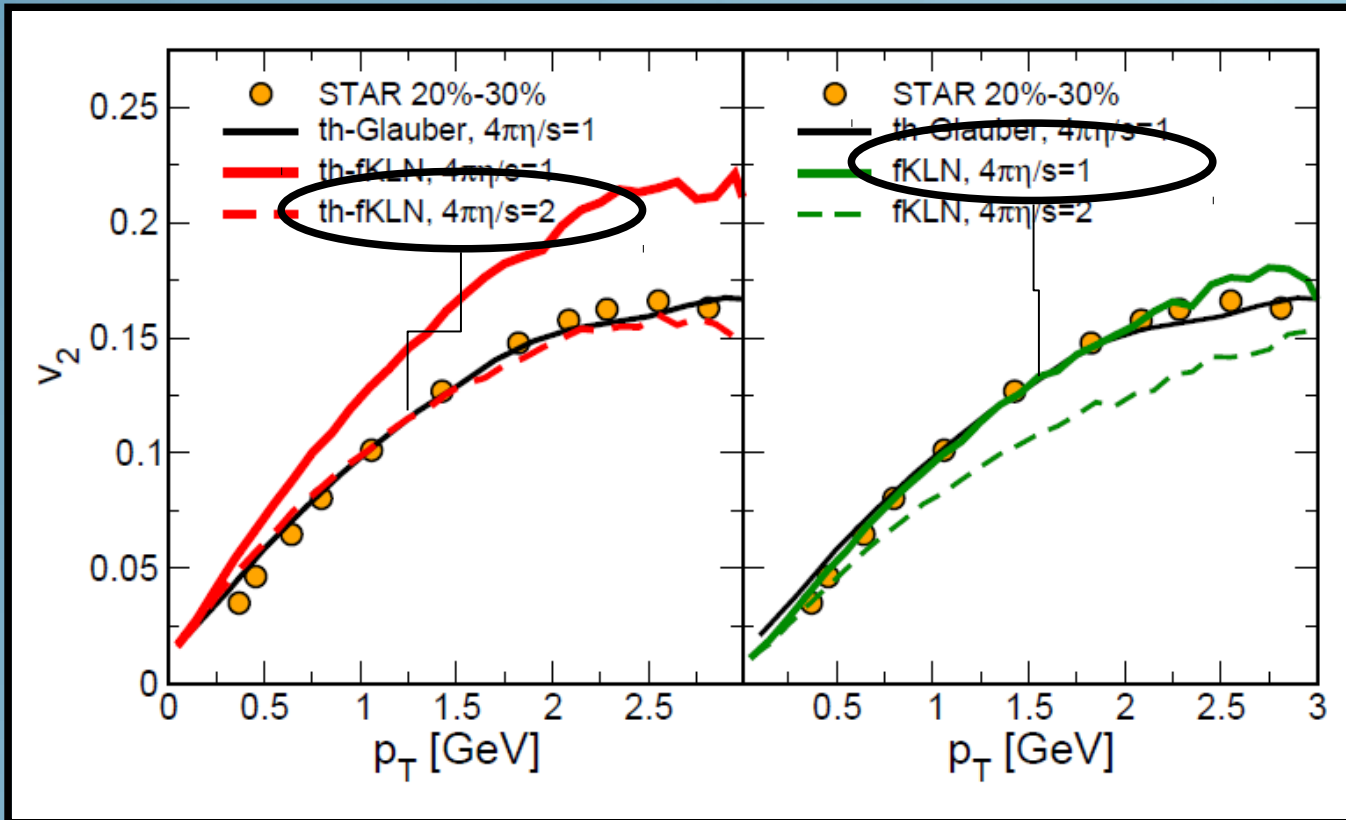


Elliptic flow at RHIC from: fKLN Glasma

In agreement with:

[Heinz *et al.*, PRC 83, 054910 (2011)]

Au+Au@200 GeV

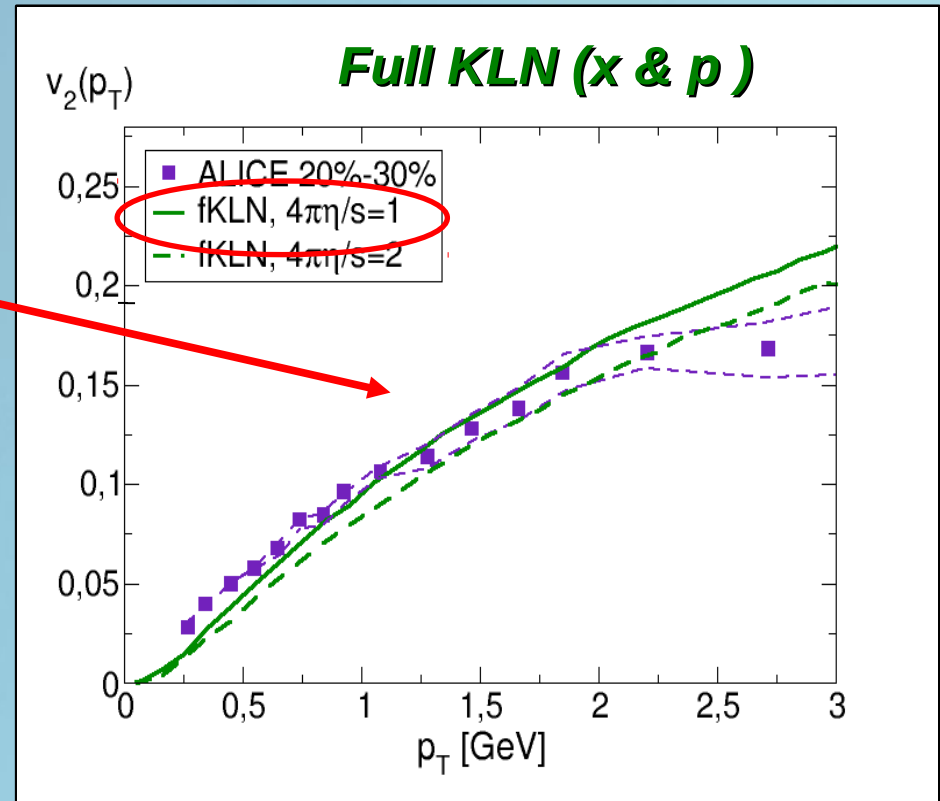
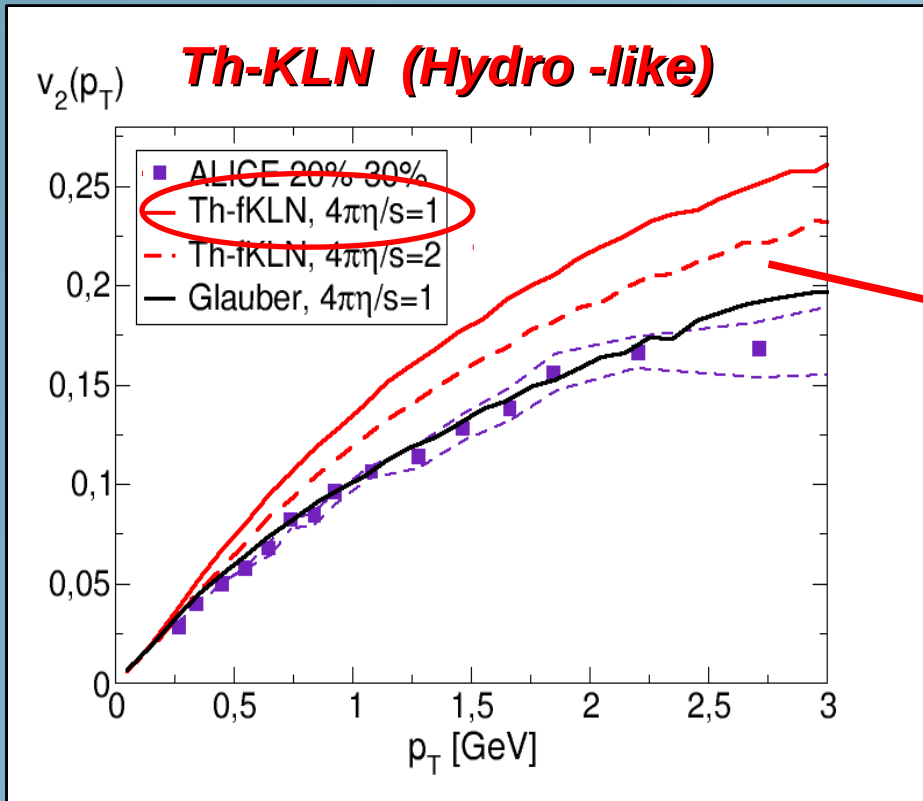


M. Ruggieri *et al.*, 1303.3178 [nucl-th]

- **When implementing KLN and Glauber like in Hydro we get the same of Hydro**
- **When implementing full KLN we get close to the data with $4\pi\eta/s = 1$: larger ε_x compensated by Q_s saturation scale (non-equilibrium distribution)**

Elliptic flow at LHC from: fKLN Glasma

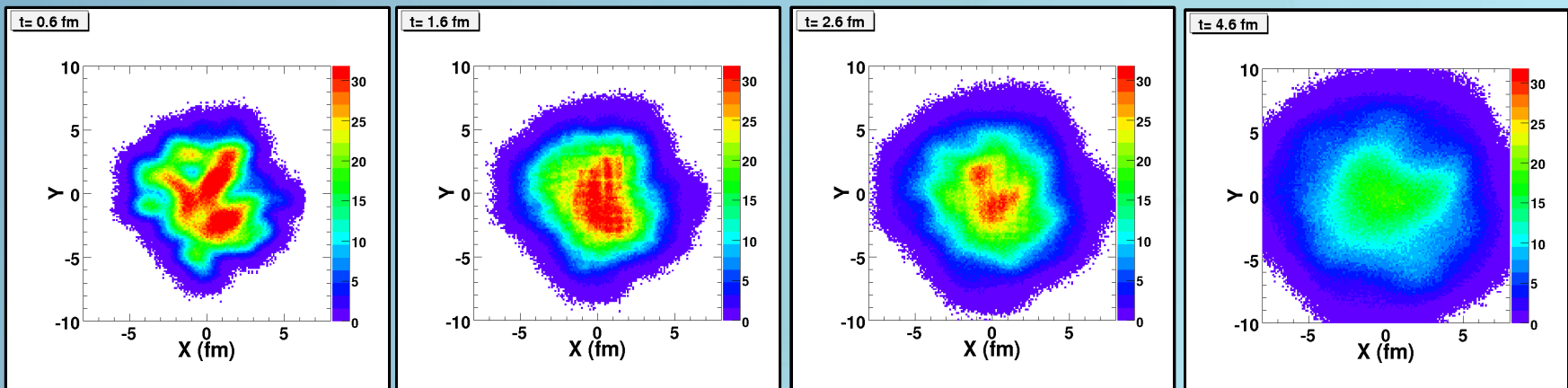
Pb+Pb@2.76 TeV



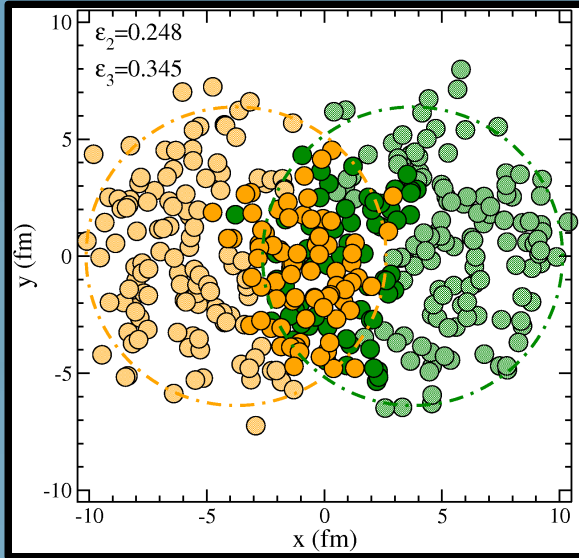
At LHC the larger saturation Q_s (≈ 2.4 GeV) scale makes the effect larger:

- $4\pi\eta/s=2$ not sufficient to get close to the data for Th-KLN
- $4\pi\eta/s=1$ it is enough if one implements both x & p

**Next step –
To include the Initial State Fluctuations
(Preliminary results)**



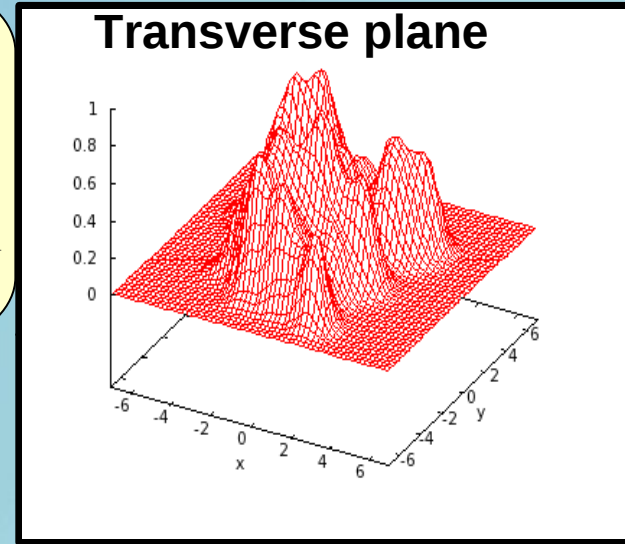
Initial State Fluctuations (Preliminary)



Monte Carlo Glauber



$$\rho_{\perp}(x, y) \propto \sum_{i=1}^{N_{part}} \exp\left\{-\left[\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma^2}\right]\right\}$$

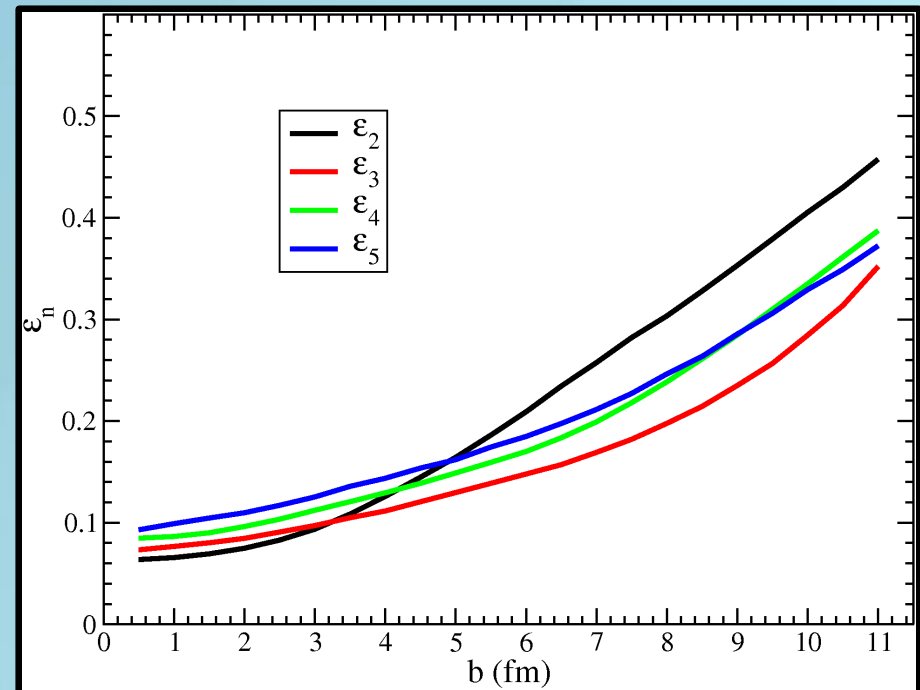


Characterization of the initial profile in terms of Fourier coefficients

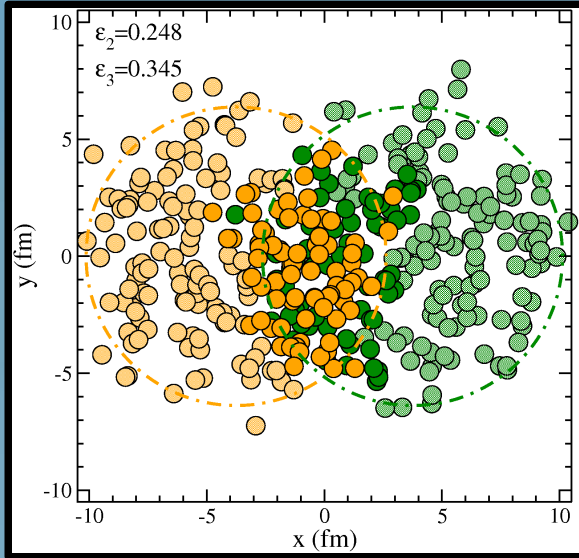
$$\epsilon_n = \frac{\langle r_{\perp}^n \cos[n(\phi - \Phi_n)] \rangle}{\langle r_{\perp}^n \rangle} \quad \Phi_n = \frac{1}{n} \arctan \frac{\langle r_{\perp}^n \sin(n\phi) \rangle}{\langle r_{\perp}^n \cos(n\phi) \rangle}$$

$$r_{\perp} = \sqrt{x^2 + y^2}, \quad \phi = \arct(y/x)$$

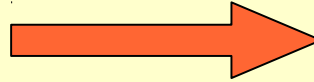
G-Y. Qin, H. Petersen, S.A. Bass and B. Muller,
 PRC82,064903 (2010).
 H.Holopainen, H. Niemi and K.J. Eskola,
 PRC83, 034901 (2011).



Initial State Fluctuations (Preliminary)

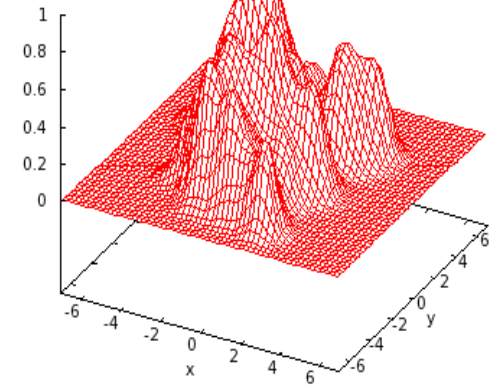


Monte Carlo Glauber



$$\rho_{\perp}(x, y) \propto \sum_{i=1}^{N_{part}} \exp\left\{-\left[\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma^2}\right]\right\}$$

Transverse plane

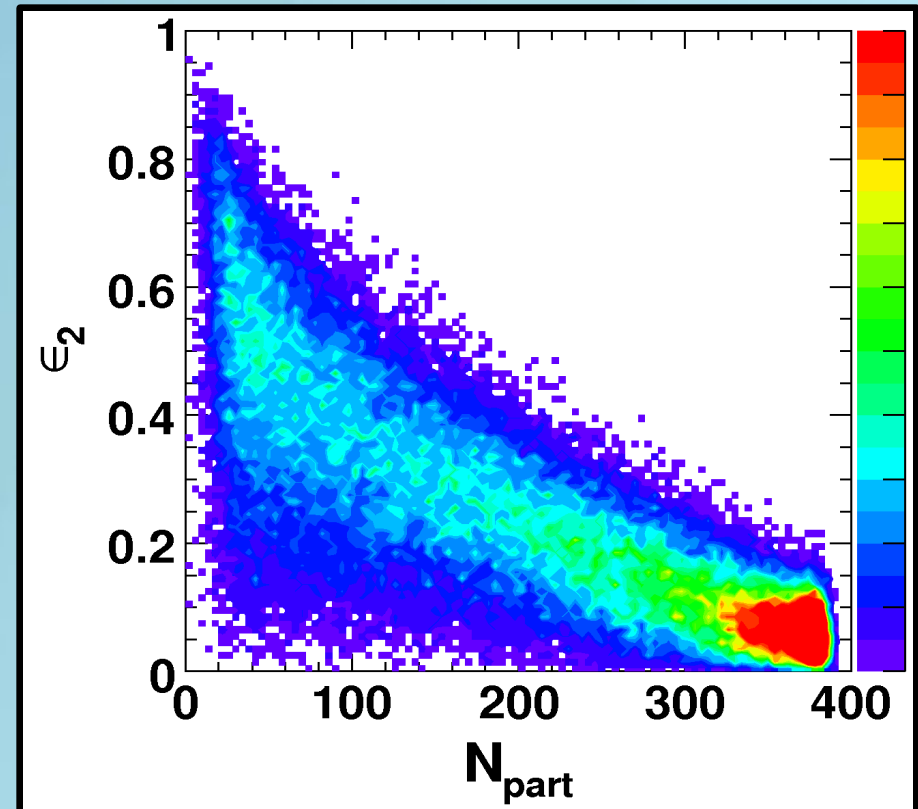


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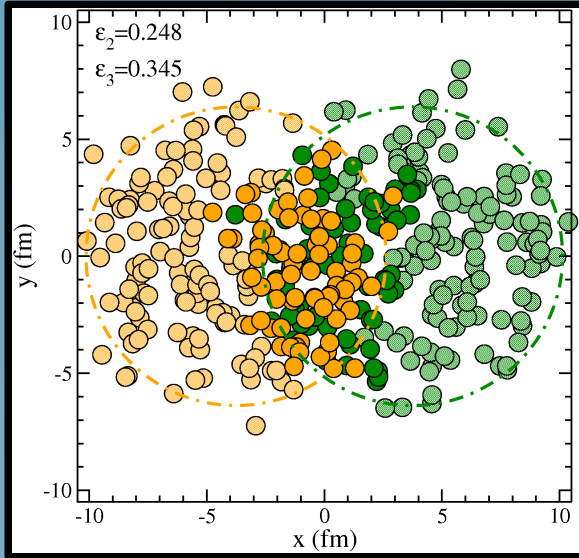
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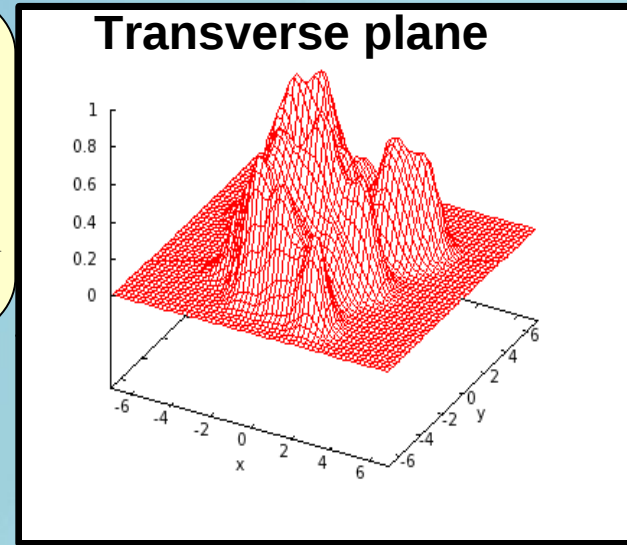
Initial State Fluctuations (Preliminary)



Monte Carlo Glauber



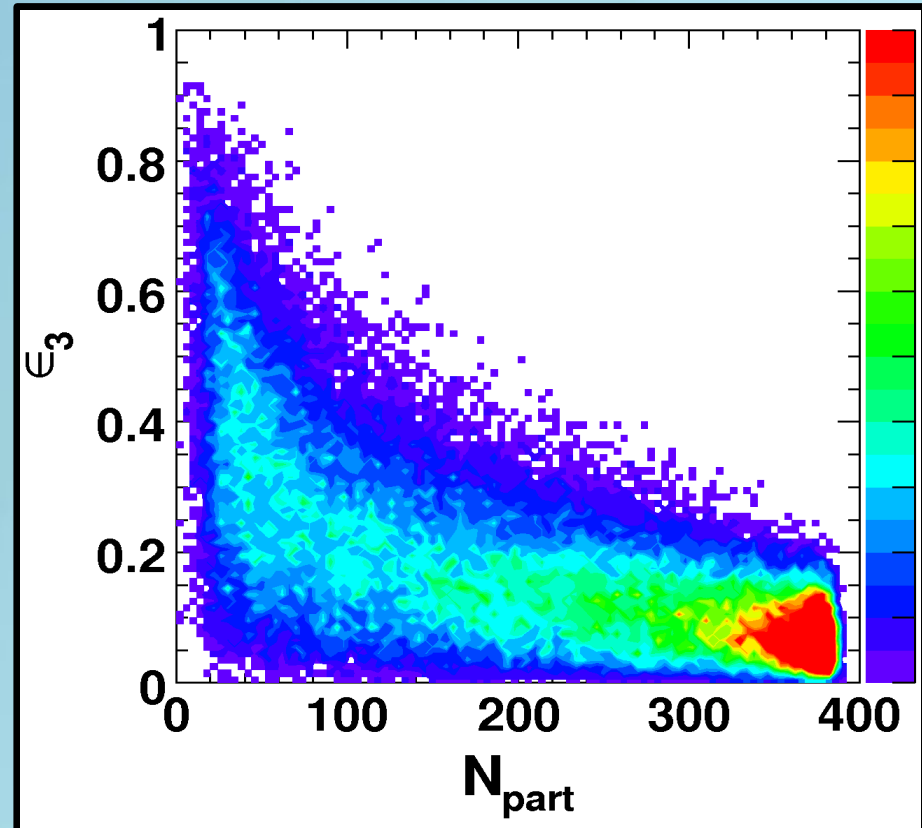
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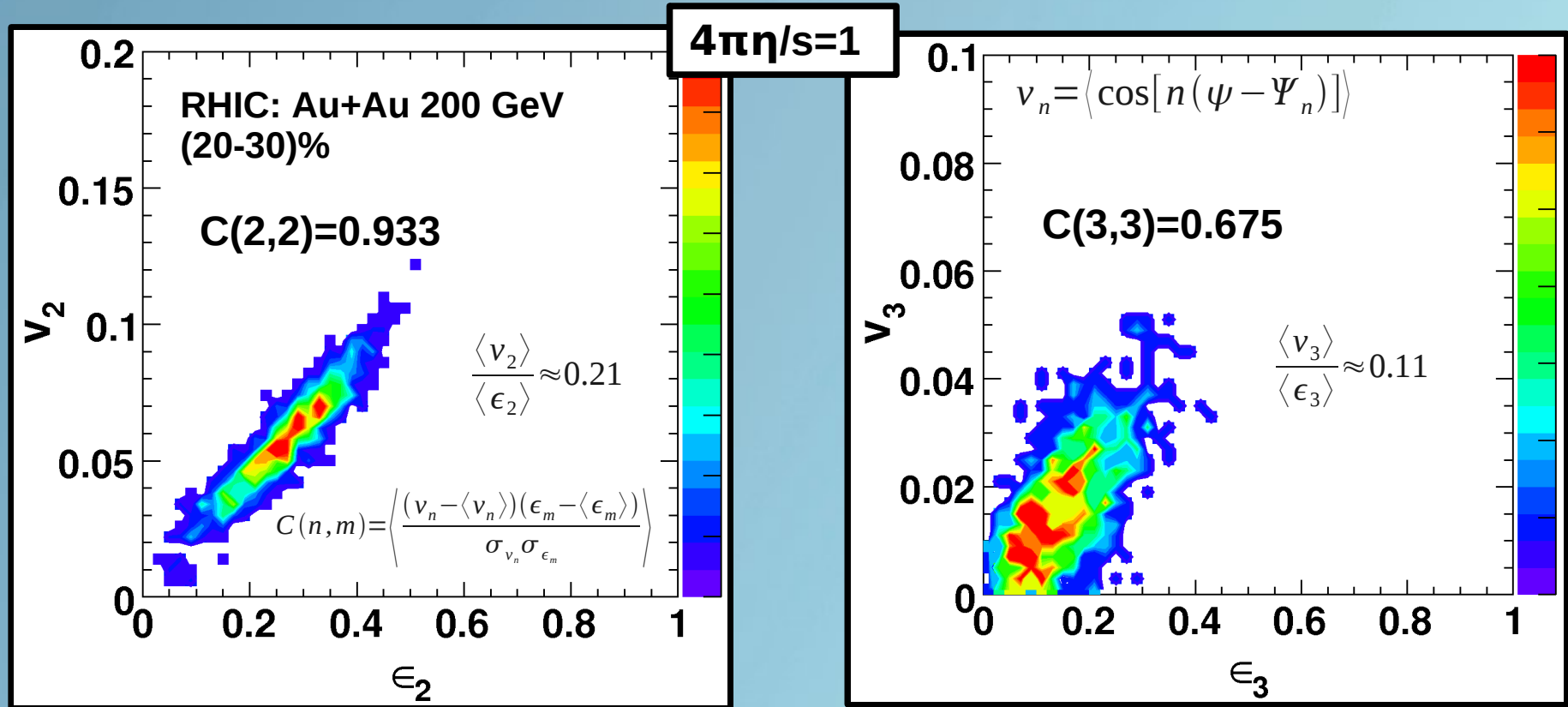
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 PRC82,064903 (2010).
 H.Holopainen, H. Niemi and K.J. Eskola,
 PRC83, 034901 (2011).

Initial State Fluctuations: v_n vs ϵ_n (Preliminary)

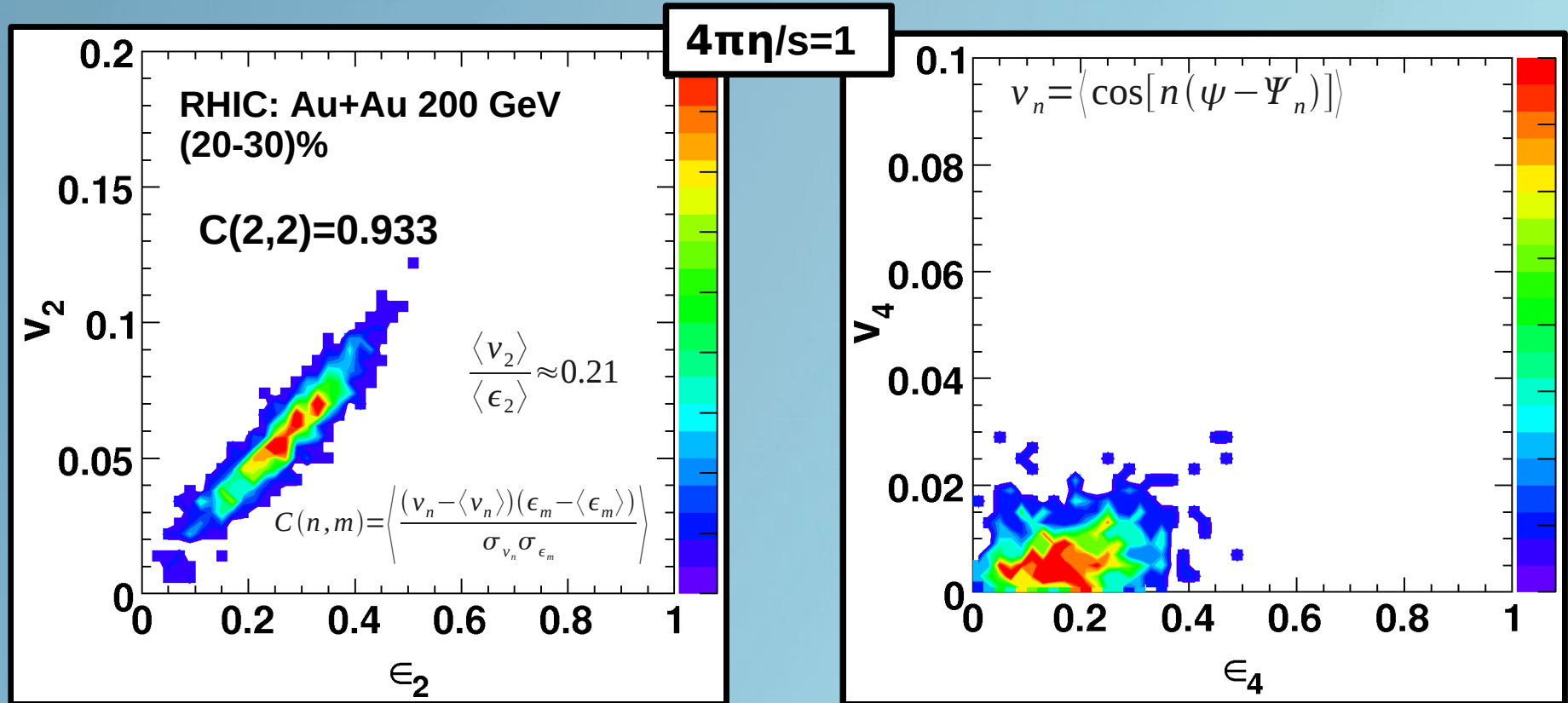


- v_2 and v_3 linearly correlated to the corresponding eccentricities ϵ_2 and ϵ_3 respectively.
- v_4 and ϵ_4 weak correlated similar to hydro calculations:

F.G.Gardim, F.Grassi, M.Luzum and J.Y.Ollitrault NPA904 (2013) 503.

Niemi, Denicol, Holopainen and Huovinen PRC87(2013) 054901.

Initial State Fluctuations: v_n vs ϵ_n (Preliminary)

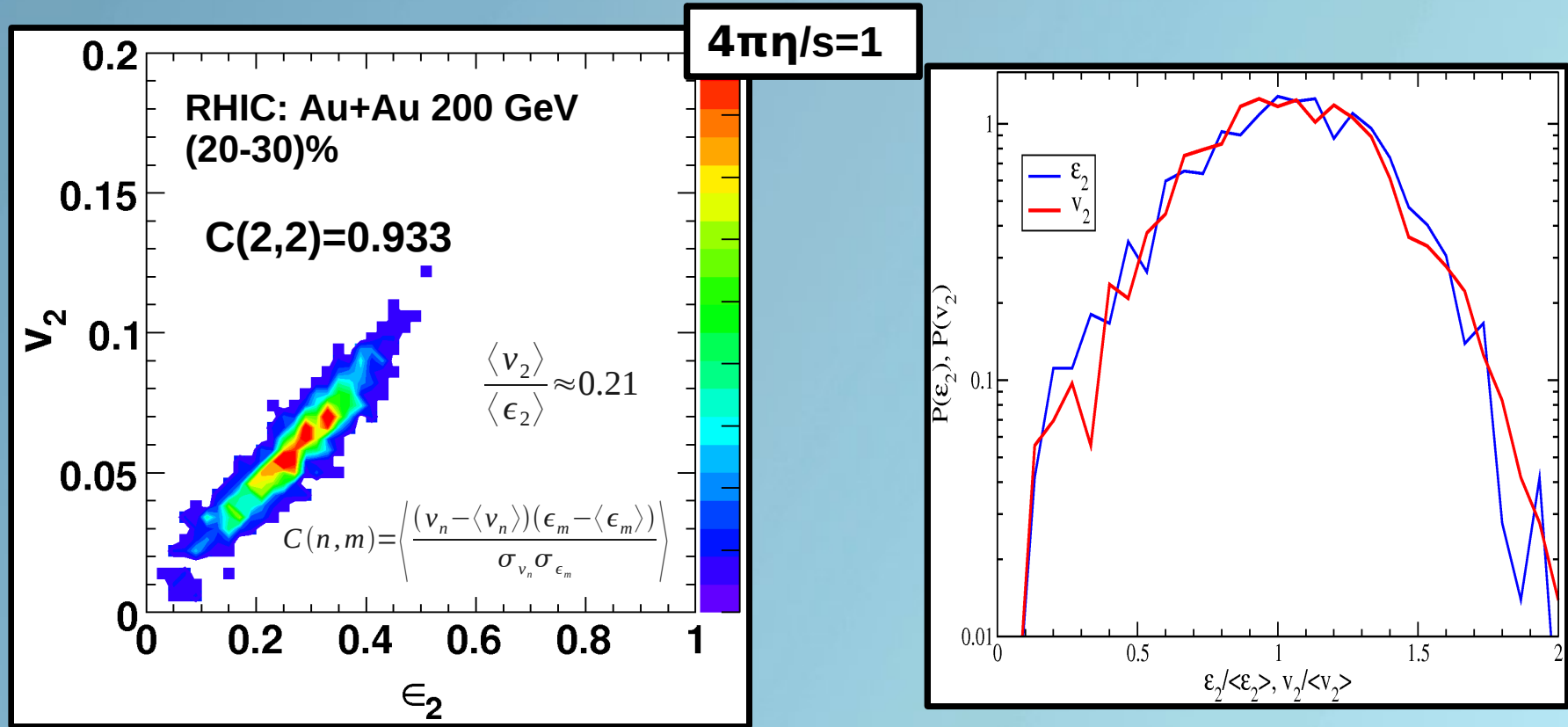


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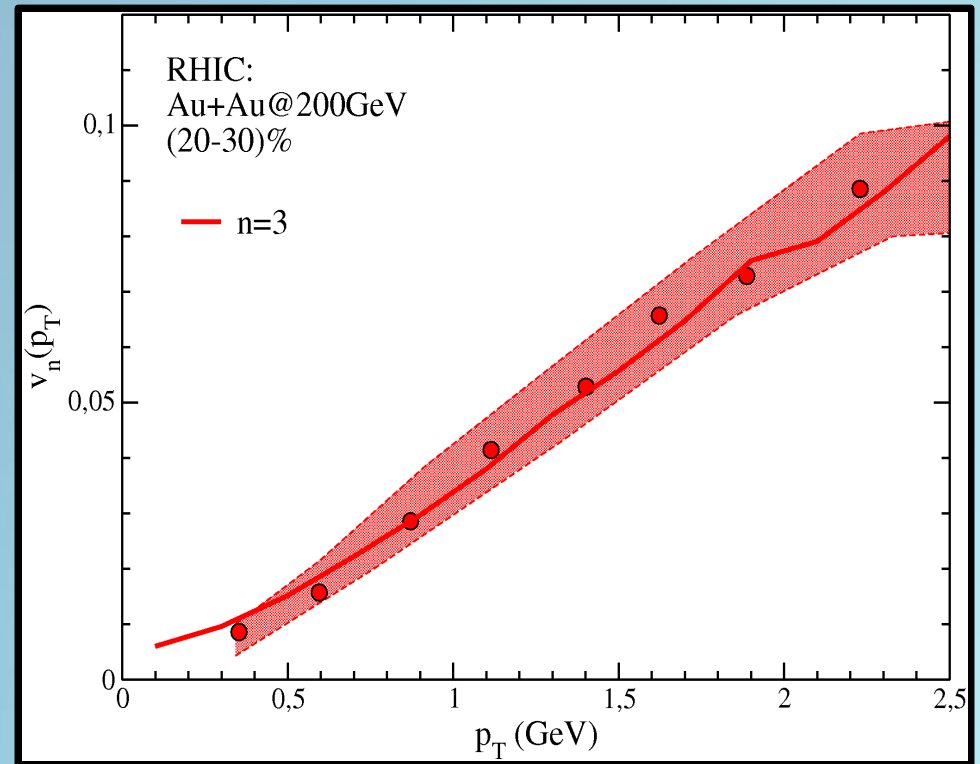
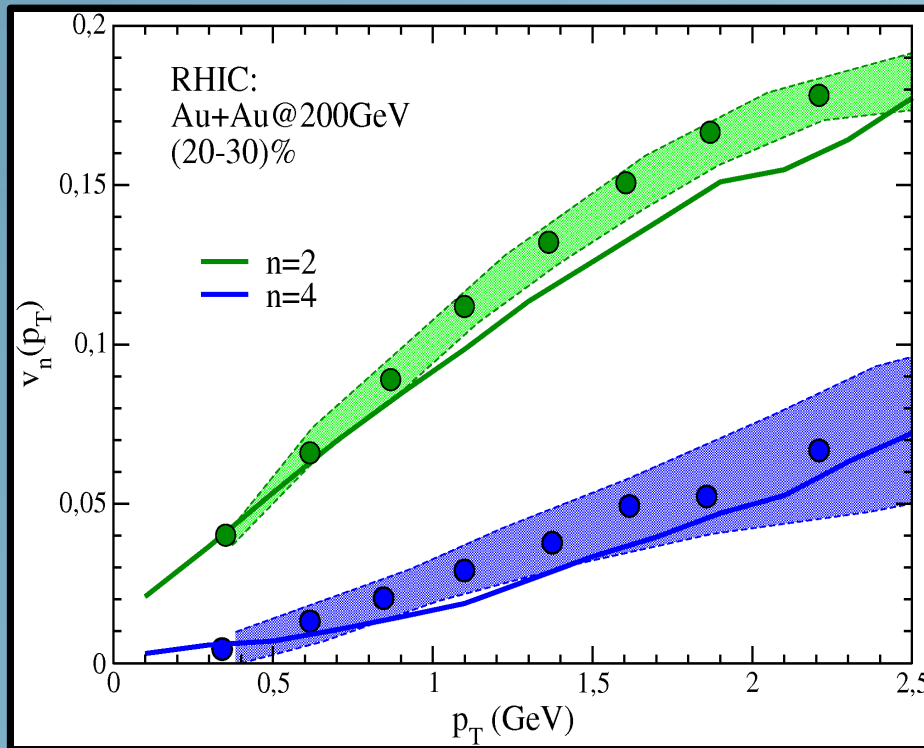
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Niemi, Denicol, Holopainen and Huovinen PRC87(2013) 054901.

Initial State Fluctuations: $v_n(p_T)$ (Preliminary)

Data taken from: A. Adare et al. [PHENIX collaboration], Phys.Rev. Lett. 107, 252301 (2011).



- Like in viscous hydro the data of $v_n(p_T)$ at RHIC energies are described with $4\pi\eta/s=1$.

Conclusions

Enhancement of $\eta/s(T)$ in the cross-over region affect differently the expanding QGP from RHIC to LHC.

At LHC nearly all the v_2 from the QGP phase.

The scaling of $v_2(p_T)$ from Beam Energy Scan indicate a 'U' shape of $\eta/s(T)$ this would be a signature of $\eta/s(T)$ behavior typical of a phase transition.

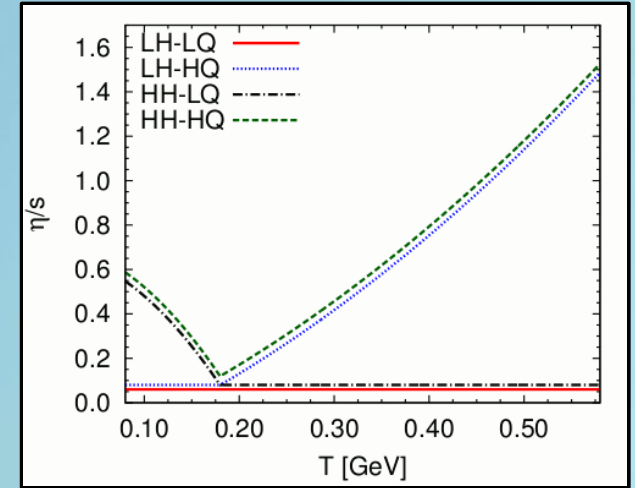
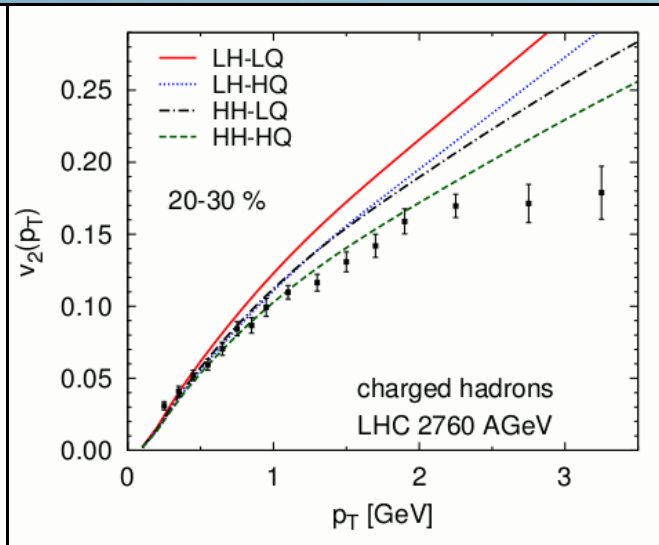
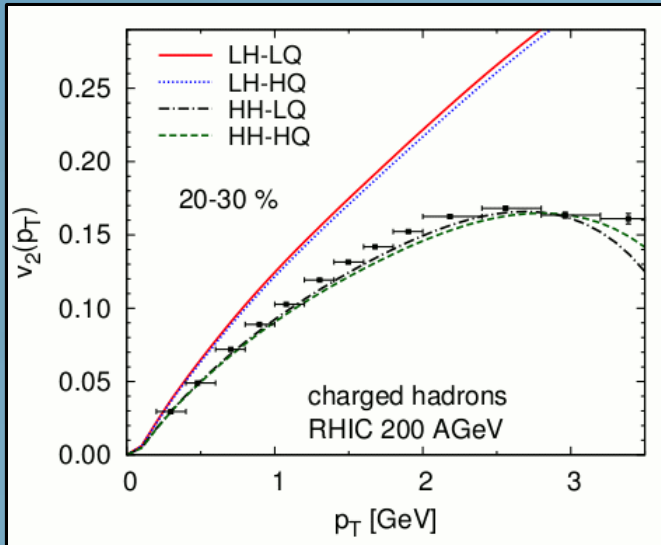
For $4\pi\eta/s=1$ v_2 and v_3 are linearly correlated to the corresponding eccentricities ϵ_2 and ϵ_3 . While v_4 and ϵ_4 are weakly correlated similar to hydro calculation. (More detailed study is going on)

Outlook

To study the role of $\eta/s(T)$ on the v_n and their correlation on the initial eccentricities ϵ_n .

To study the effect of different initial condition (glasma) on $v_n \leftrightarrow \epsilon_n$ correlation.

Effect of $\eta/s(T)$ in Hydro: Niemi et al.



Niemi et al., PRL 106 (2011).

$$T^{\mu\nu} = T_{eq}^{\mu\nu} + \delta T^{\mu\nu} \leftarrow f_{eq} + \delta f$$

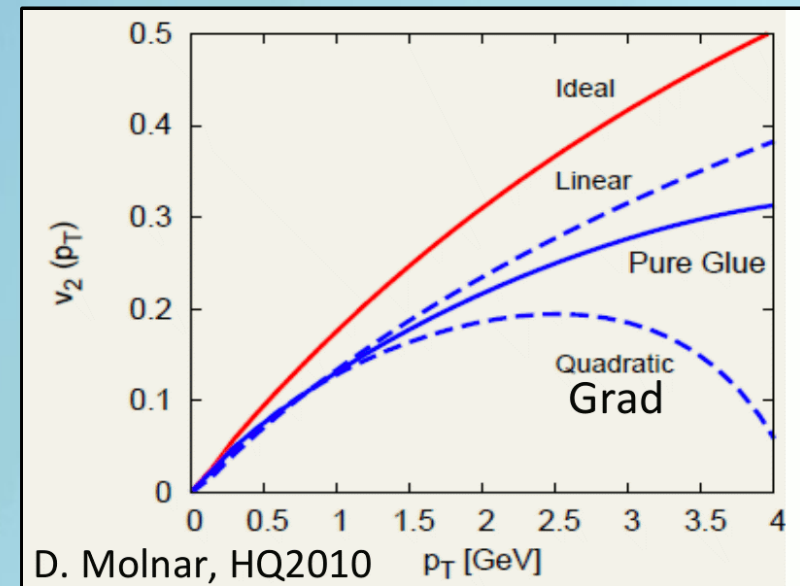
Grad ansatz

R. Lacey et al., PRC82

$$\delta f = \frac{\pi^{\mu\nu} p_\mu p_\nu}{(\epsilon + p) T^2} f_{eq} \approx \frac{\eta}{3s} \frac{p_T^2}{\tau T^2} f_{eq}$$

- This implies that the η is in Relaxation Time Approximation
D. Teaney, Phys.Rev. C68 (2003) 034913

- Hydro is valid up to $p_T \sim 3$ GeV



D. Molnar, HQ2010

Extraction of the Shear Viscosity: Box calculation

Isotropic cross section: massive case

Massive case is relevant in quasi-particle models where $M(T)$.
 Good agreement with CE 1st order for isotropic cross section
 and massive particles.

1st Chapman-Enskog approximation

$$[\eta]_{1st} = 10 T \left[\frac{K_3(z)}{K_2(z)} \right]^2 \frac{1}{c_{00}} = g(m_D, T) \frac{T}{\sigma_{tot}}$$

$$c_{00} = 16 \left[\omega_2^{(2)} - z^{-1} \omega_1^{(2)} + (3z^2)^{-1} \omega_0^{(2)} \right] \quad \text{for } s=2 \propto \sigma_{tr}$$

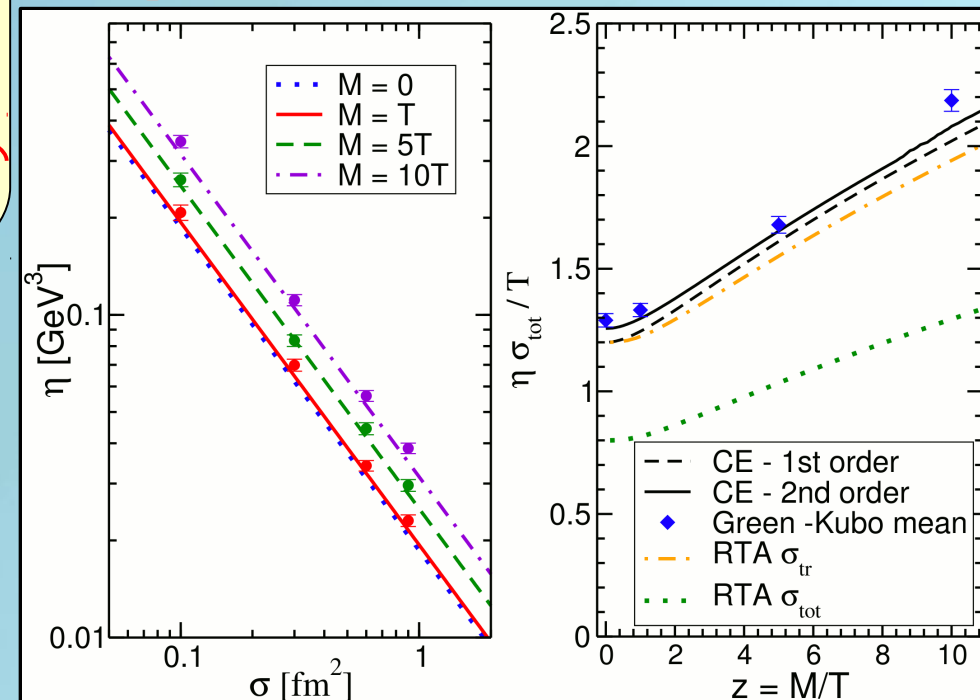
$$\omega_i^{(s)} = \frac{2\pi z^3}{[K_2(z)]^2} \int_1^\infty dy (y^2 - 1)^3 y^i K_j(2zy) \int_0^\pi d\Theta \sin \Theta \frac{d\sigma}{d\Omega} (1 - \cos^s \Theta)$$

$$[\eta]_{1st}^{CE} = f(z) \frac{T}{\sigma_{tot}}$$

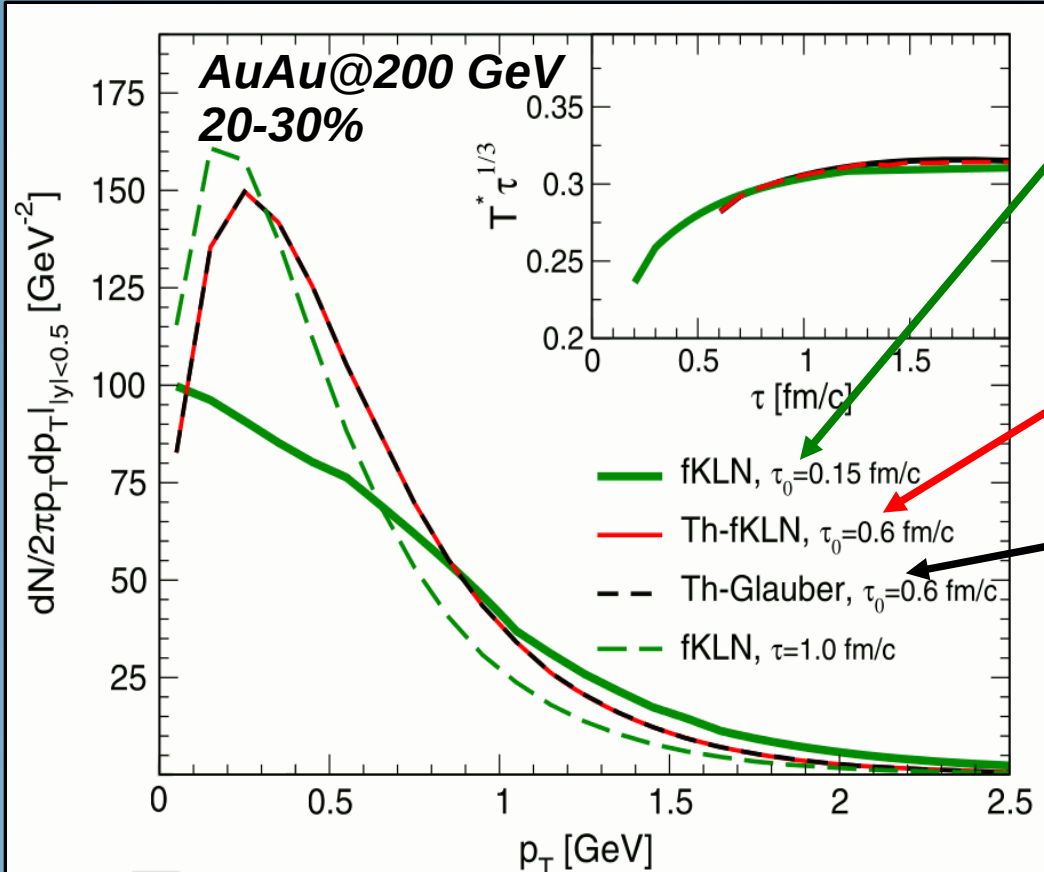
$$f(z) = \frac{15}{16} \frac{z^4 K_3^2(z)}{(15z^2 + 2)K_2(2z) + (3z^3 + 49z)K_3(2z)}$$

A. Wiranata, M. Prakash, arXiv:1203.0281 [nucl-th].
 O. N. Moroz, arXiv:1112.0277 [hep-ph].

S. Plumari et al., arXiv:1208.0481 [nucl-th].



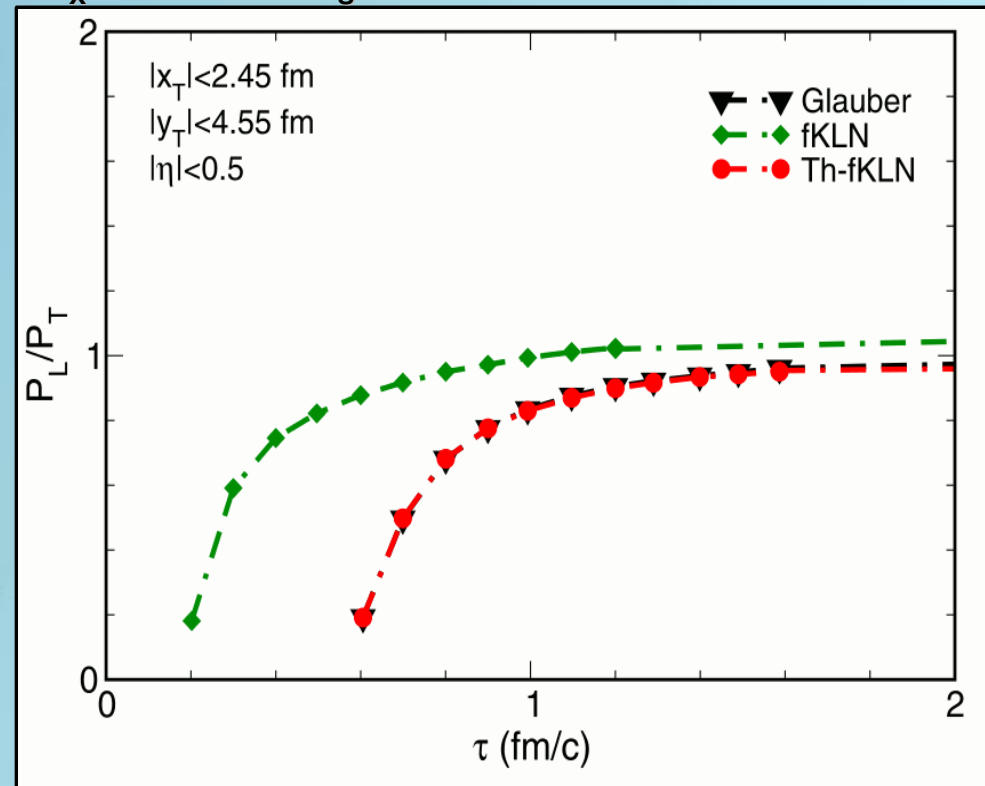
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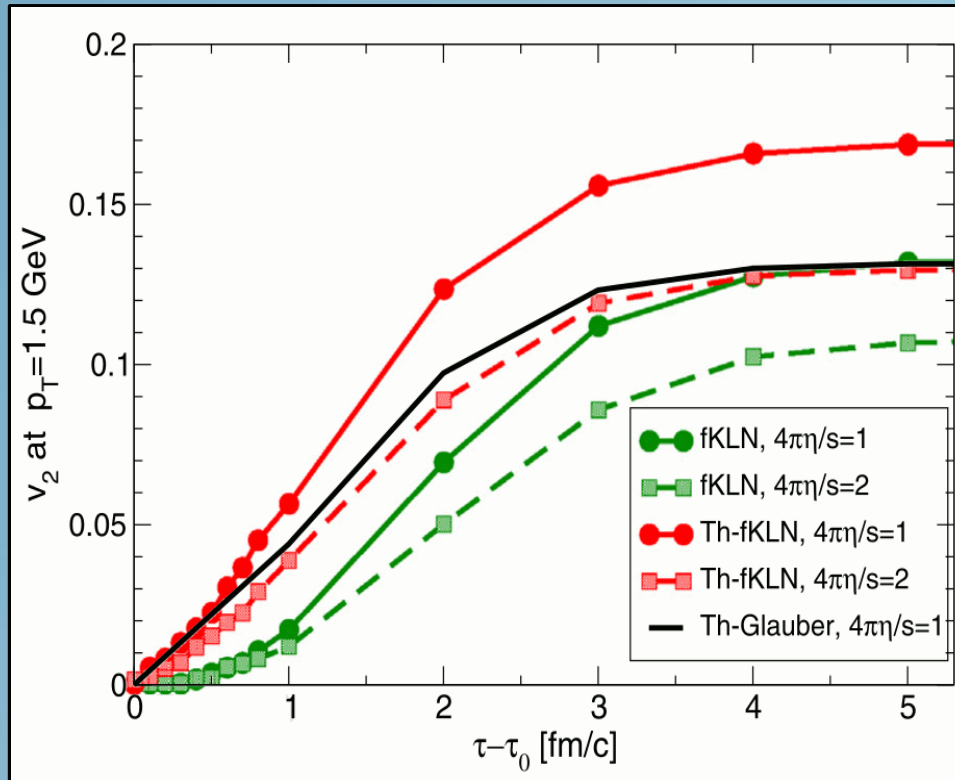
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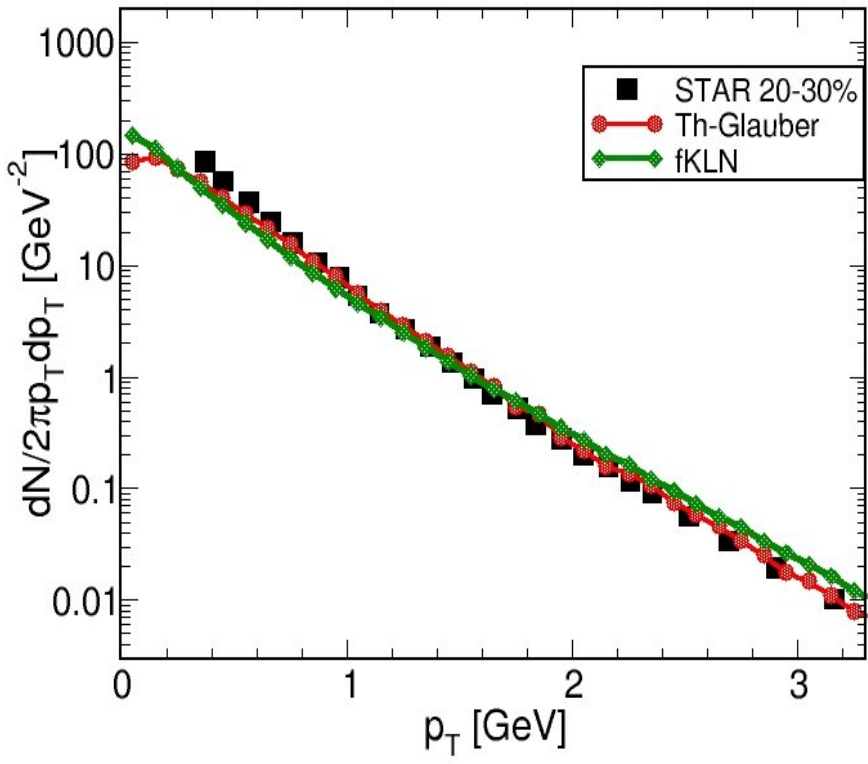
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- Not so surprising: η/s is small \rightarrow large effective scattering rate \rightarrow fast thermalization.

$$\sigma_{tot} = \frac{\langle p \rangle}{\rho g(a)} \frac{1}{\eta/s}$$

Time evolution of v_2



We see that when non-equilibrium distribution is implemented in the initial stage (≈ 1 fm/c) v_2 grows slowly respect to thermal one



Finite masses and EoS

$$p^\mu \partial_\mu f(x, p) = C_{22}$$

$$M \neq 0 \longrightarrow \left\{ \begin{array}{l} \epsilon - 3p \neq 0 \\ C_s^2 \leq \frac{1}{3} \end{array} \right.$$

