Nature of QCD phase transition at finite temperature and density

Endpoint of first order transition by a histogram method

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Heavy quark region: WHOT-QCD(H. Saito, S. Ejiri, S. Aoki, K. Kanaya, Y. Nakagawa, H. Ohno, K. Okuno, and T. Umeda), arXiv:1309.2445 [hep-lat]

Many-flavor QCD: S. Ejiri and N. Yamada, Phys. Rev. Lett. 110, 172001 (2013) [arXiv:1212.5899]

Histogram method mini-review: S. Ejiri, Euro. Phys. J. A 49, 86 (2013) [arXiv:1306.0295]

NFQCD 2013, YITP, Kyoto, Japan, Nov. 27, 2013

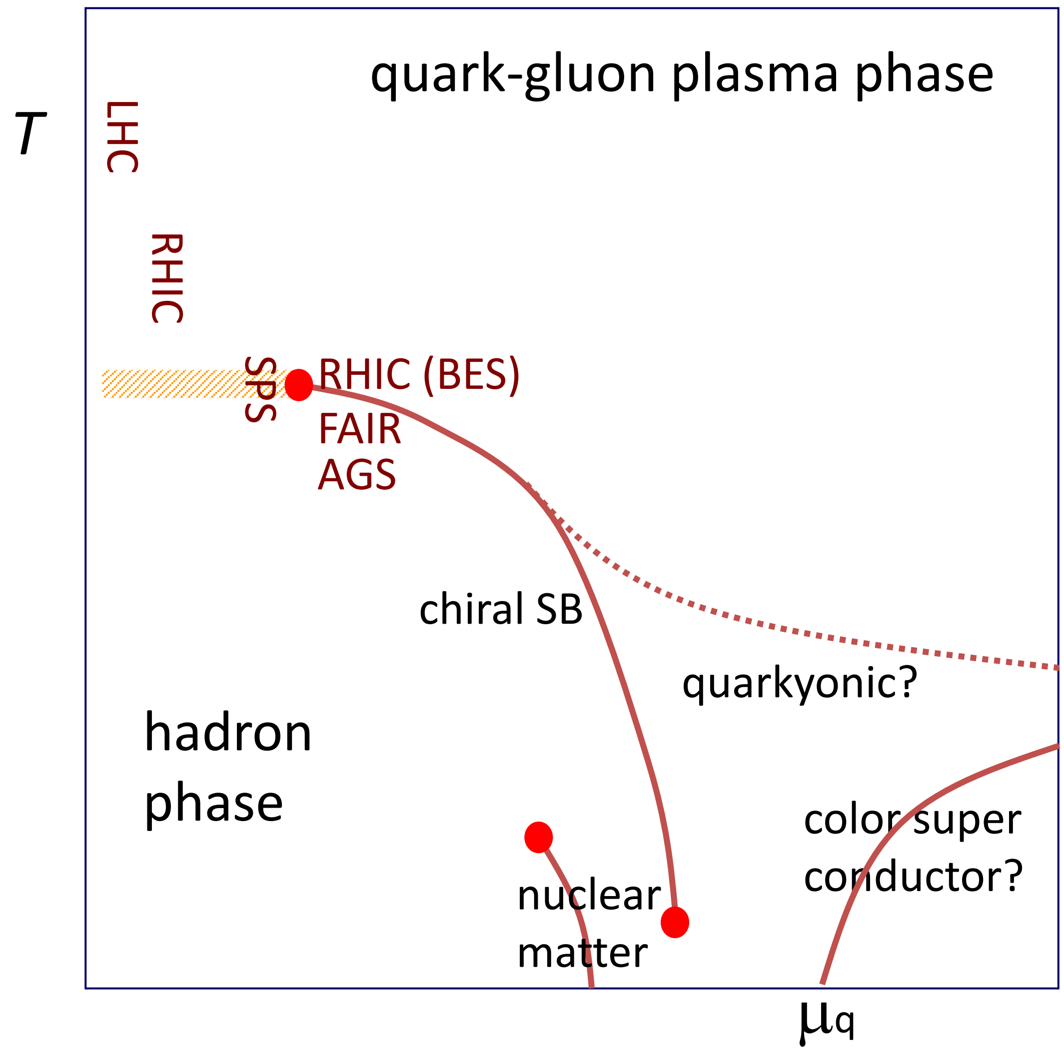
Phase structure of QCD at high temperature and density

- Phase transition lines
- Critical point

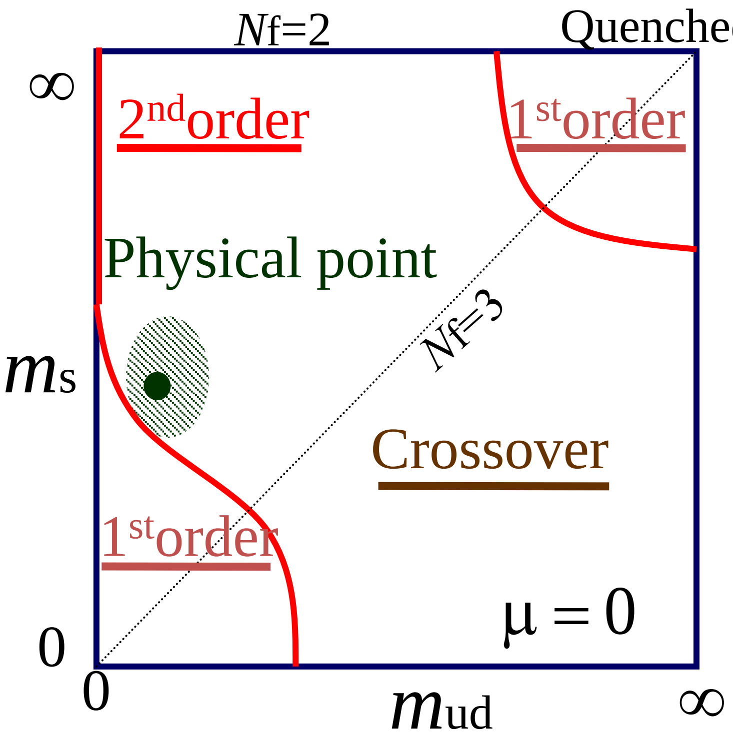
Lattice QCD Simulations

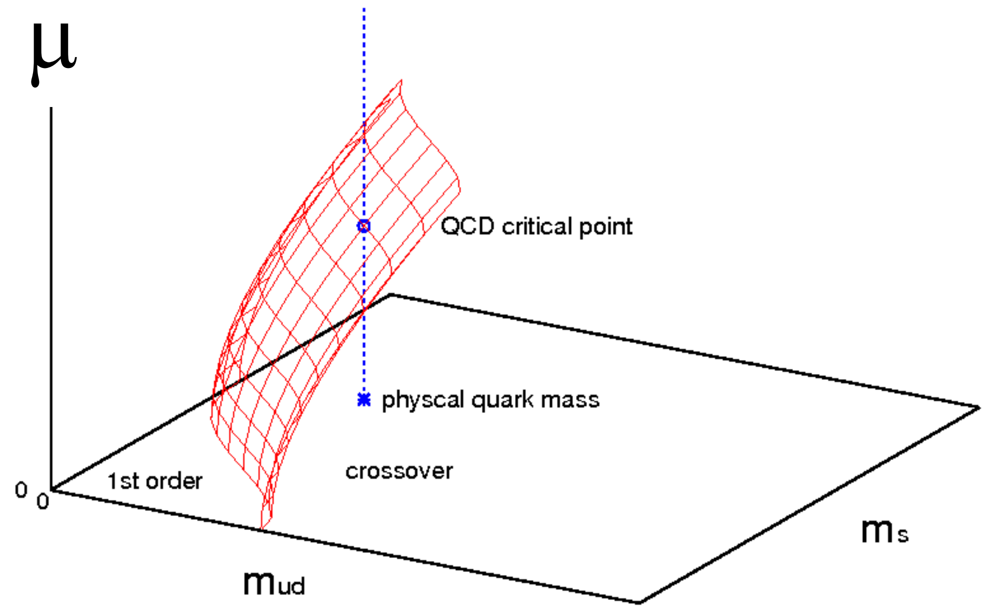
- Direct simulation:
  Impossible at $\mu \neq 0$.

- Reweighting method



Quark Mass dependence of QCD phase trantion





- The determination of the boundary of 1^{st} order region: important.
- On the line of physical mass, the crossover at low density ➡ 1^{st} order transition at high density.
- However, the 1^{st} order region is very small, and simulations with very small quark mass are required. ➡ Difficult to study.

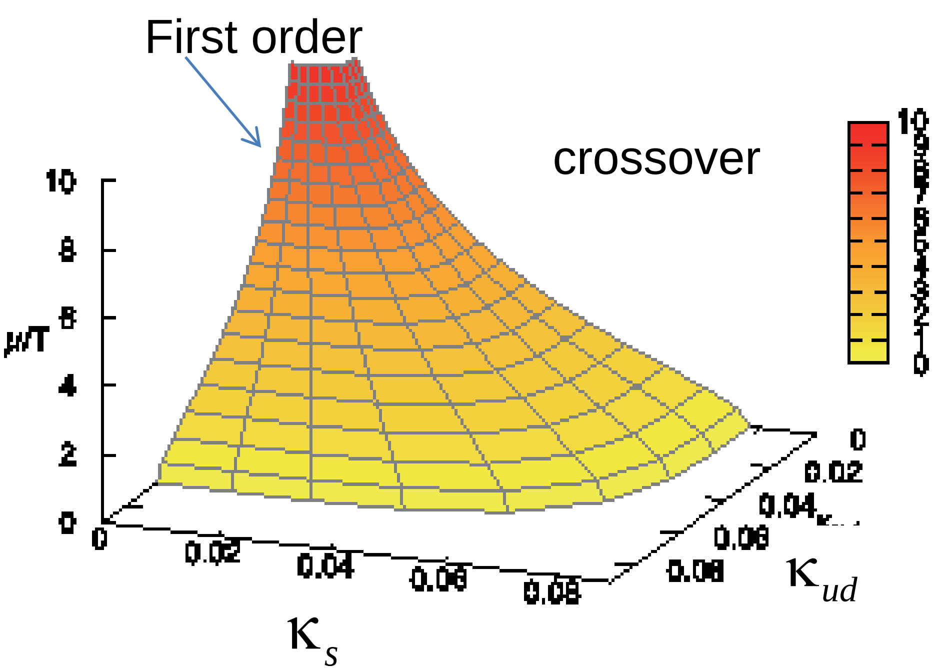
➡ Heavy quark region, many-flavor QCD

Critical surface at finite density
simple examples

Heavy quark region

All quarks are heavy.

$\kappa \sim 1/(\text{quark mass})$

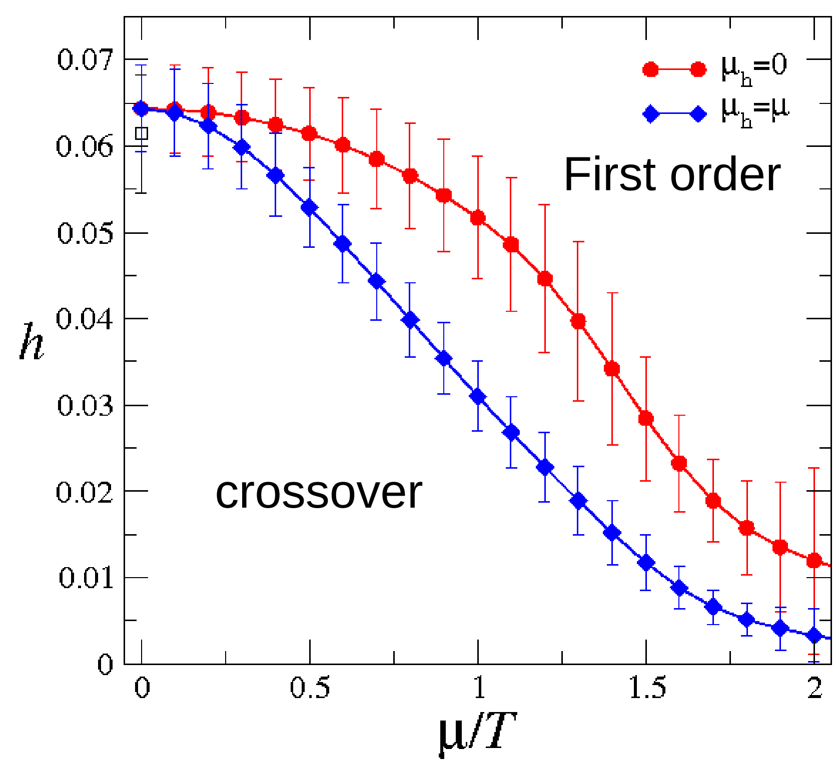


arXiv:1309.2445 [hep-lat]

(2+N_f)-flavor QCD (N$_f$: many)

Two light quarks and many massive quarks

$$h \propto \frac{N_f}{(\text{quark mass})^{N_t}}$$



Phys. Rev. Lett. 110, 172001 (2013)

Histogram method

- Monte-Carlo method (*S*g: gauge action, *M*: qaurk matrix)
  - Generate configurations with the probability of the Boltzmann weight.

$$\langle O \rangle_{(m,T,\mu)} = \frac{1}{Z} \int DU\, O \left(\det M(m,\mu)\right)^{N_f} e^{-S_g} \approx \frac{1}{N_{\text{conf.}}} \sum_{\{\text{conf.}\}} O$$

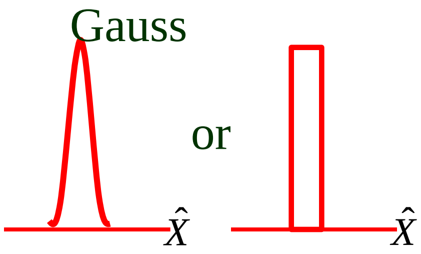
- Distribution function in Density of state method (Histogram method)

  X: order parameters, total quark number, average plaquette etc.

$$W(X; m, T, \mu) \equiv \int DU\, \delta(X - \hat{X}) \left(\det M(m,\mu)\right)^{N_f} e^{-S_g}$$

$$\frac{W(X)}{Z} \approx \frac{1}{N_{\text{conf.}}} \sum_{\{\text{conf.}\}} \delta(X - \hat{X})$$

$$\delta(\hat{X}) \approx$$



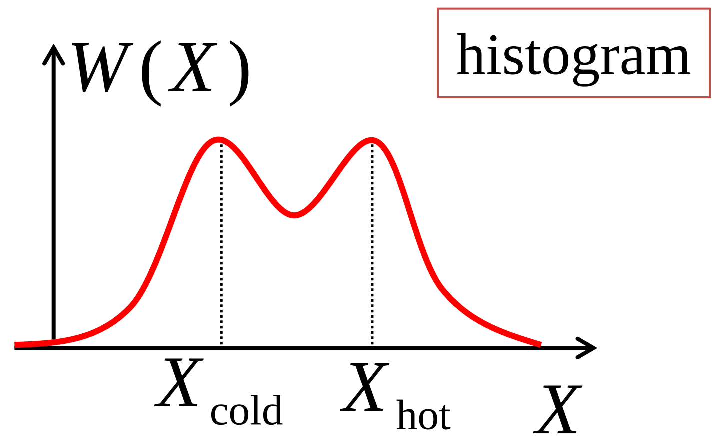
  - Expectation values

$$\langle O[X] \rangle_{(m,T,\mu)} = \frac{1}{Z} \int dX\, O[X]\, W(X, m, T, \mu), \quad Z(m,T,\mu) = \int dX\, W(X, m, T, \mu)$$

Effective potential $V_{\mathrm{eff}}(X)$
Probability distribution function (histogram)

- First order phase transition

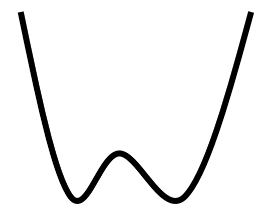
  Two phases coexists at T_c



- If $W(X)$ have two peaks,

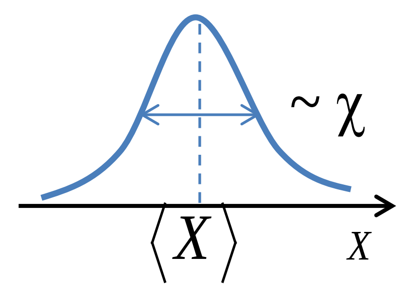
  ➡ first order transition

- Effective potential: $V_{\mathrm{eff}}(X) \equiv -\ln\left(W(X)\right)$



- If W(X) is a Gaussian distribution,
  - The peak position of $W(X)$ ➡ ($\langle X \rangle$)
  - The width of $W(X)$ ➡ susceptibilities

$$\chi = V\left\langle \left(X - \langle X \rangle\right)^2 \right\rangle$$



$$W(X) \approx \sqrt{\frac{A}{\pi}}\, e^{-A\left(X - \langle X \rangle\right)^2}$$

$$A \propto V/\chi$$

Reweighting method for plaquette distribution function

$$W(P,\beta,m,\mu) \equiv \int DU \delta(\hat{P}-P) \prod_{f=1}^{N_f} \det M(m_f,\mu_f) e^{6N_{site}\beta\hat{P}}$$

$$S_g = -6N_{site}\beta\hat{P}$$

$$(\beta = 6/g^2)$$

plaquette P (1x1 Wilson loop for the standard action)

$$R(P,\beta,\beta_0 m,m_0,\mu) \equiv W(P,\beta,m,\mu)/W(P,\beta_0,m_0,0)$$ (Reweight factor)

$$R(P) = \frac{\left\langle \delta(\hat{P}-P) e^{6N_{site}(\beta-\beta_0)\hat{P}} \prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)} \right\rangle_{(\beta_0,\mu=0)}}{\left\langle \delta(\hat{P}-P) \right\rangle_{(\beta_0,\mu=0)}} \equiv \left\langle e^{6N_{site}(\beta-\beta_0)\hat{P}} \prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)} \right\rangle_{P:\text{fixed}}$$

Effective potential:

$$V_{\text{eff}}(P,\beta,m,\mu) = -\ln[W(P,\beta,m,\mu)] = V_{\text{eff}}(P,\beta_0,m_0,0) - \ln R(P,\beta,\beta_0 m,m_0,\mu)$$

$$\ln R(P) = \underline{6N_{site}(\beta-\beta_0)P} + \ln \left\langle \underline{\prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)}} \right\rangle_{P:\text{fixed}}$$

Sign problem

$$\left\langle \left(\frac{\det M(m,\mu)}{\det M(m_0,0)} \right)^{N_f} \right\rangle_{X \text{ fixed}} = \left\langle e^{i\theta} \left| \frac{\det M(m,\mu)}{\det M(m_0,0)} \right|^{N_f} \right\rangle_{X \text{ fixed}}$$

θ: complex phase of $(\det M)^{N_f}$

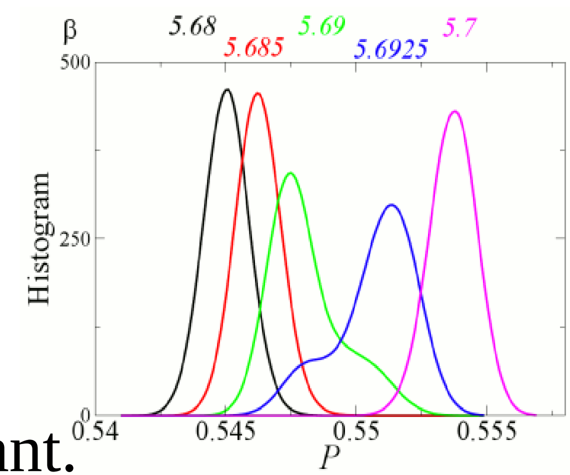
- Sign problem: if $e^{i\theta}$ changes the sign frequently,

$$W(X) \sim \left\langle e^{i\theta} \left| \frac{\det M(m,\mu)}{\det M(m_0,0)} \right|^{N_f} \right\rangle_{X \text{ fixed}} \ll \text{(statistical error)}$$

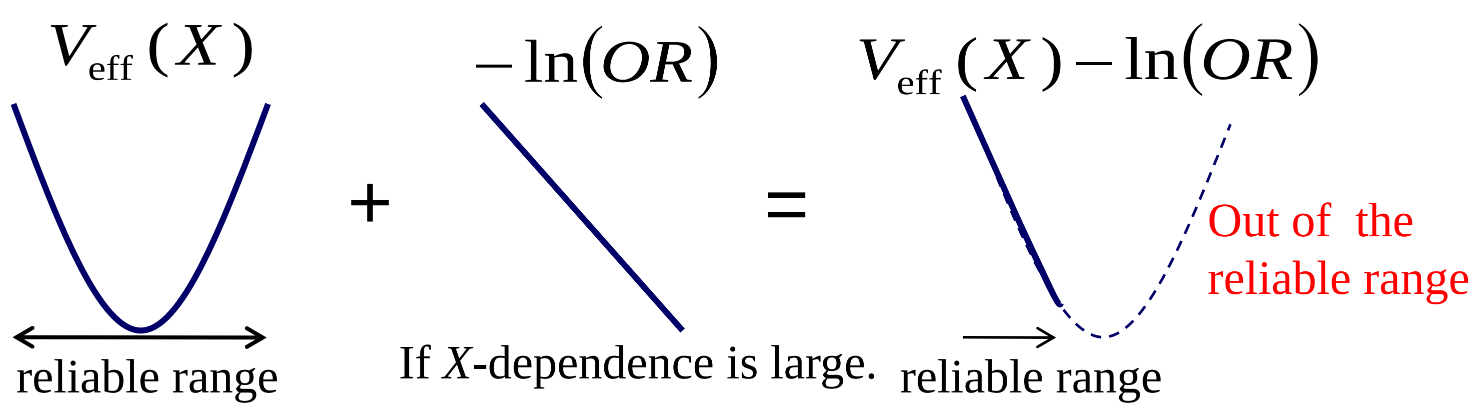
Overlap problem

$$\langle OR \rangle = \frac{1}{Z}\int ORW(X)\,dX \;\; = \frac{1}{Z}\int \exp\left(-V_{\rm eff}(X) + \ln(OR)\right) dX$$

$$V_{\rm eff}(X) = -\ln W(X)$$

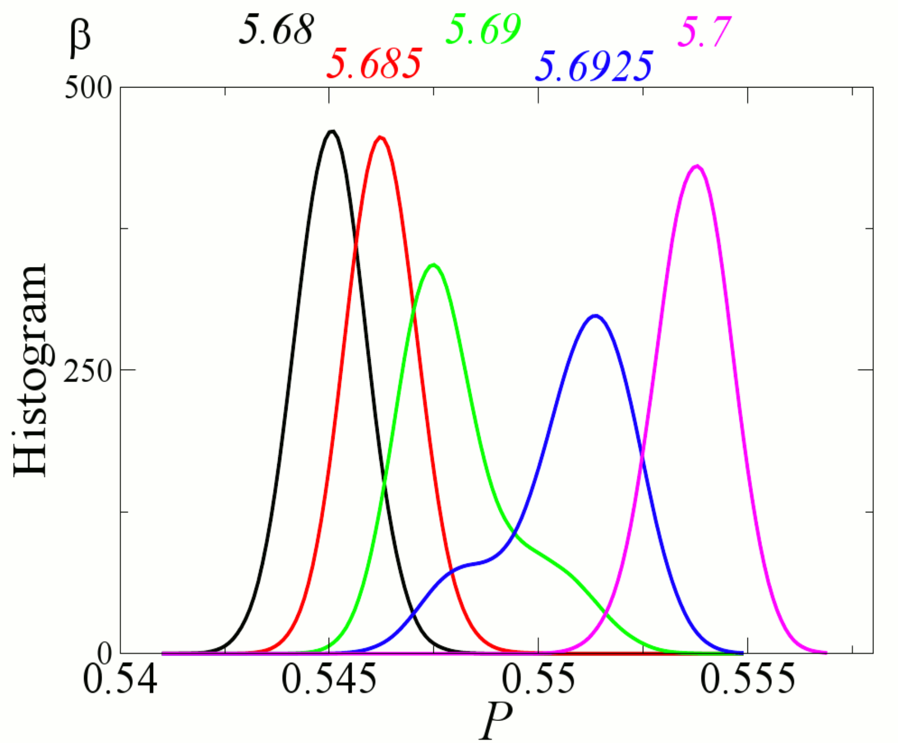


- W is computed from the histogram.
- Distribution function around X where $V_{\rm eff}(X) - \ln(OR)$ is minimized: important.
- $V_{\rm eff}$ must be computed in a wide range.



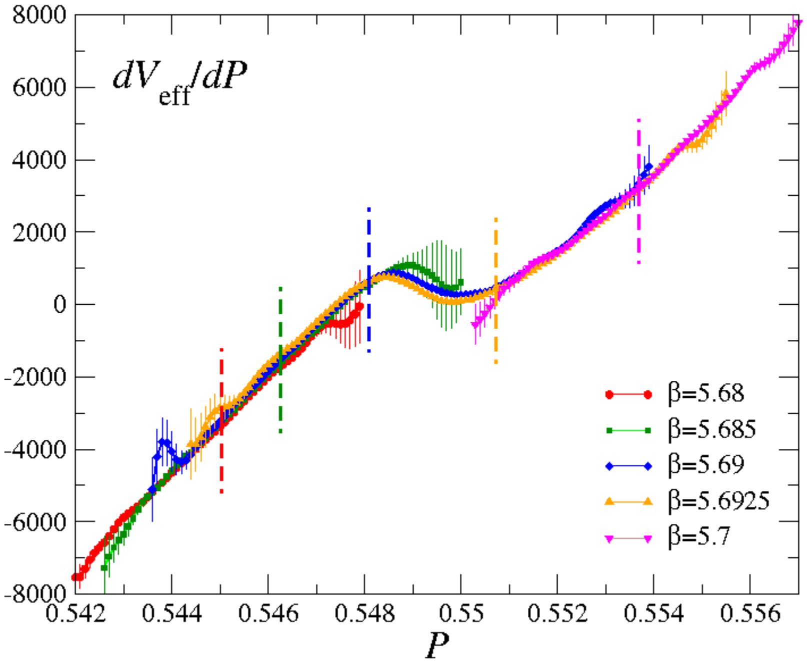
Distribution function in quenched simulations

Plaquette histogram at $K=1/m_q=0$.



$$V_{\rm eff}(\beta_2) = V_{\rm eff}(\beta_1) - 6N_{\rm site}(\beta_2 - \beta_1)P$$

Derivative of $V_{\rm eff}$ at $\beta=5.69$



$$\frac{dV_{\rm eff}}{dP}(\beta_2) = \frac{dV_{\rm eff}}{dP}(\beta_1) - 6N_{\rm site}(\beta_2 - \beta_1)$$

First order phase transition

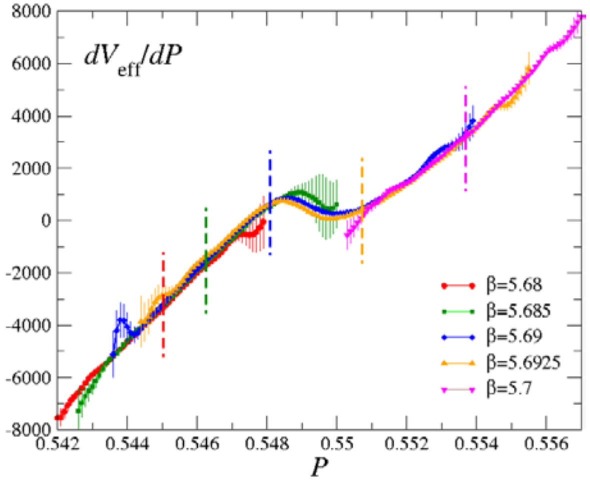
$dV_{\rm eff}/dP=0$ at the peak position of $V_{\rm eff}(P)$.

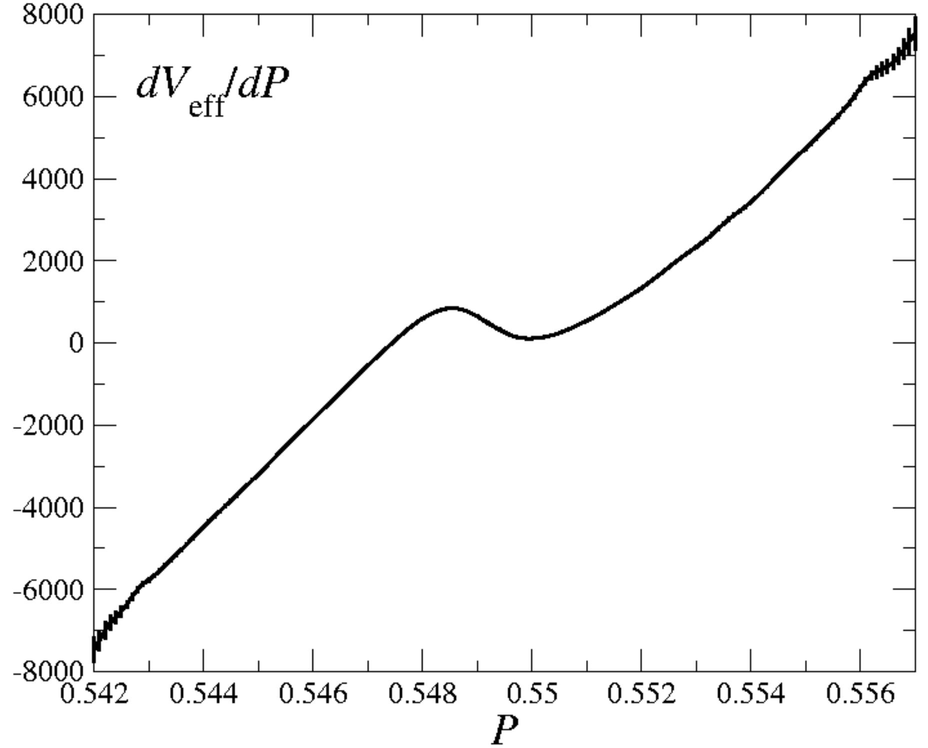
In this case, the curvature of $V_{\rm eff}$ is independent of β.

$$N_{\rm site} = 24^3 \times 4$$

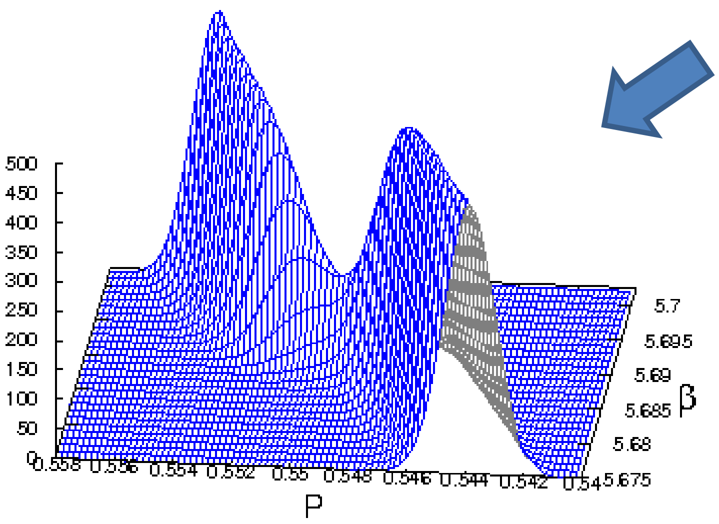
Distribution function in a quenched simulation

Derivative of the plaquette effective potential





Plaquette distribution function



multi-point reweighting method

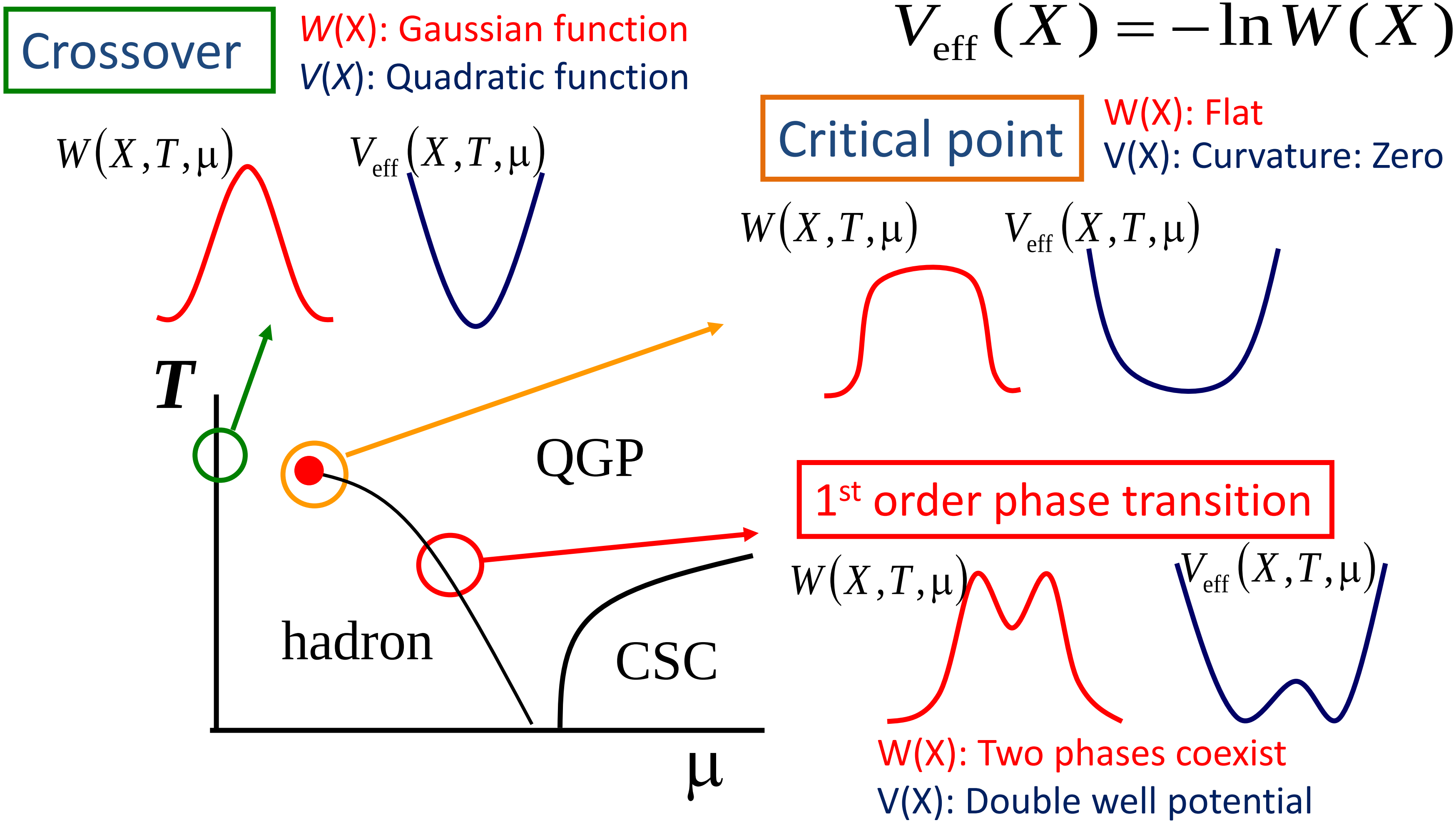
- Adopting β, average with the weight of N_{conf}
- Ferrenberg-Swendsen, Phys.Rev.Lett. 63, 1195 (1989); S.E., Phys. Rev. D78, 074507 (2008); WHOT-QCD, arXiv:1309.2445.

Distribution function & the effective potential

$$W(X;m,T,\mu)\equiv\int DU\delta\left(X-\hat{X}\right)\left(\det M(m,\mu)\right)^{N_f}e^{-S_g}$$ (Histogram)

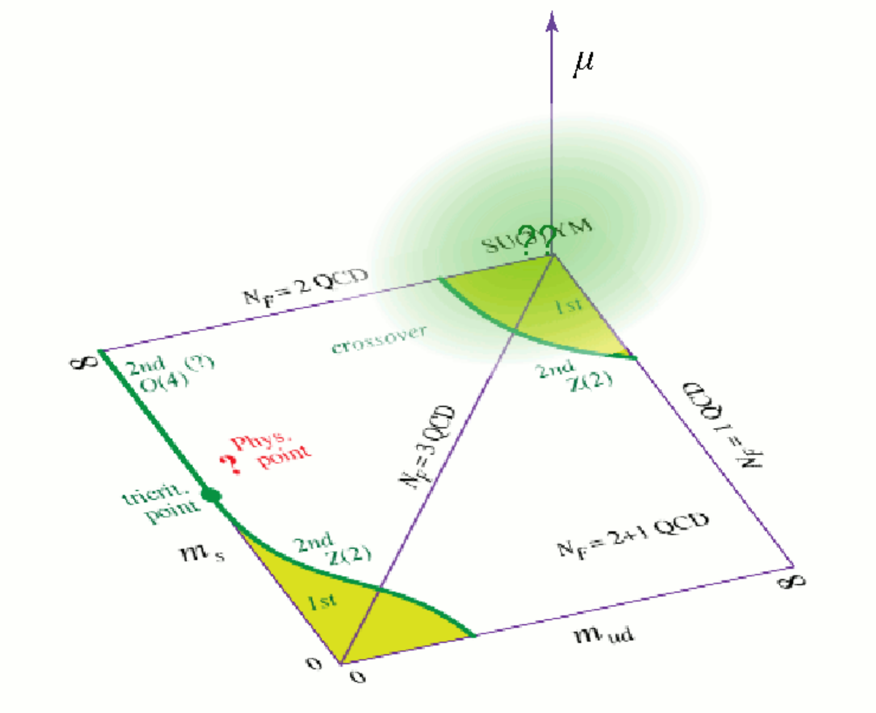
X: order parameters, total quark number, average plaquette, etc.

$$V_{\text{eff}}(X)=-\ln W(X)$$



Distribution function in the heavy quark region

(WHOT-QCD Collab., Phys.Rev.D84, 054502(2011); arXiv:1309.2445)



- We study the properties of $W(X)$ in the heavy quark region.
- Performing quenched simulations + Reweighting.
- We find the critical surface.
- Standard Wilson quark action + plaquette gauge action, $S_g = -6 N_{\mathrm{site}} \beta P$
- 24^3x4 lattice

Hopping parameter expansion

κ～1/(quark mass)

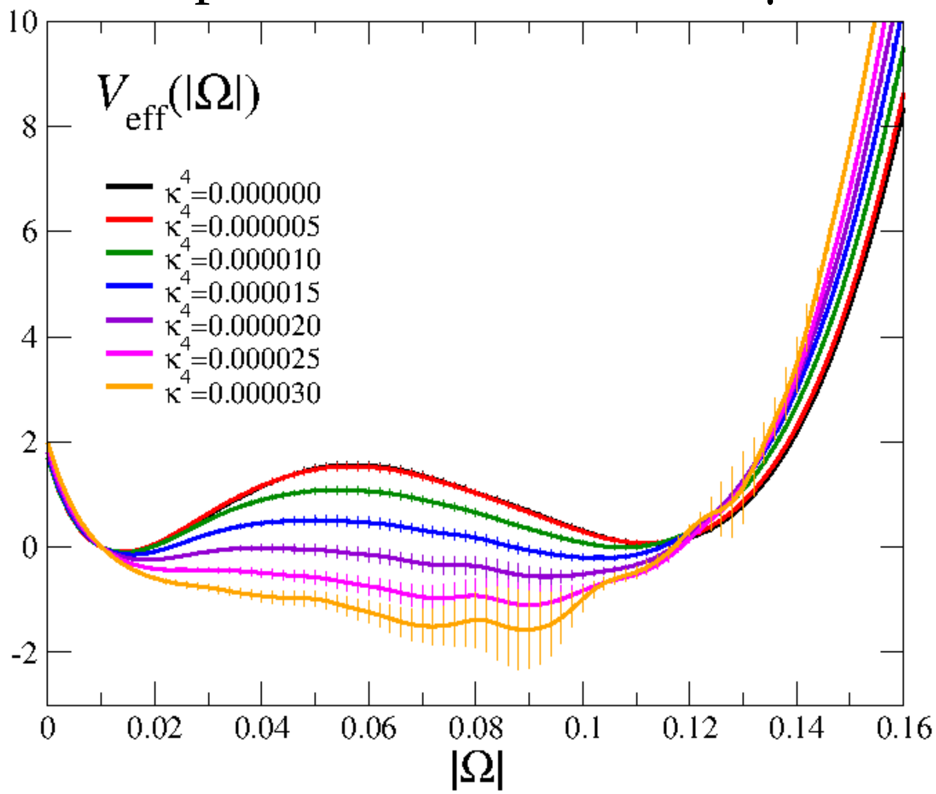
$$N_{\mathrm{f}} \ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)} \right) = N_{\mathrm{f}} \left(288 N_{\mathrm{site}} \kappa^4 P + 12 \cdot 2^{N_t} N_s^3 \kappa^{N_t} \left(\cosh(\mu/T) \Omega_R + \underline{\underline{i \sinh(\mu/T) \Omega_I}} \right) + \cdots \right)$$

phase

P: plaquette, $\Omega = \Omega_R + i\Omega_I$: Polyakov loop $\qquad \det M(0,0) = 1$

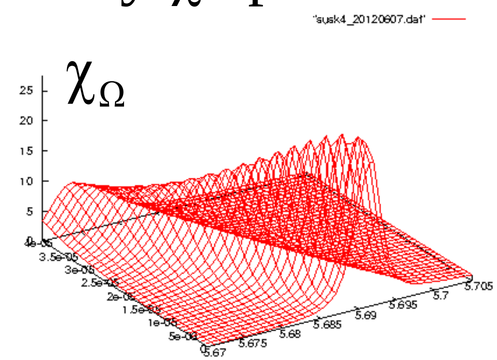
Order of the phase transition
Polyakov loop distribution (2-flavor)

Effective potential of $|\Omega|$
on the pseudo-critical line at $\mu=0$



$$\text{Critical point}: \kappa^4 \approx 2.0\times10^{-5}$$

- The pseudo-critical line is determined by χ_Ω peak.



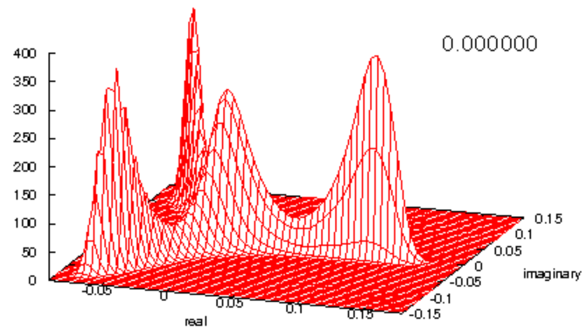
- Double-well at small κ
  - First order transition
- Single-well at large κ
  - Crossover

$\kappa \sim 1/(\text{quark mass})$

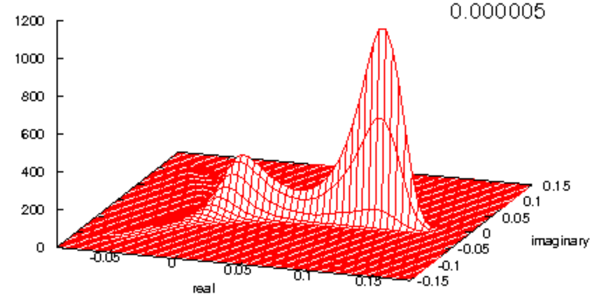
Polyakov loop distribution in the complex plane

(2-flavor, μ=0)

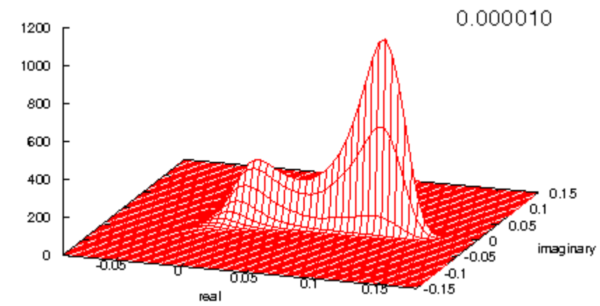
$\kappa^4 = 0.0$ Z(3) symmetric



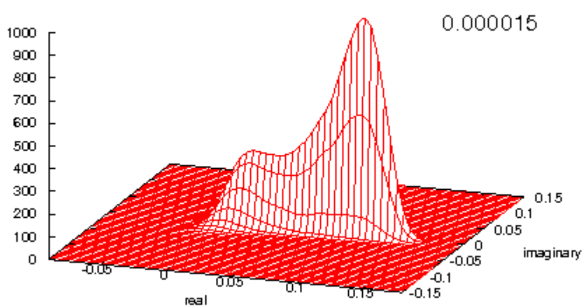
$\kappa^4 = 5.0\times10^{-6}$



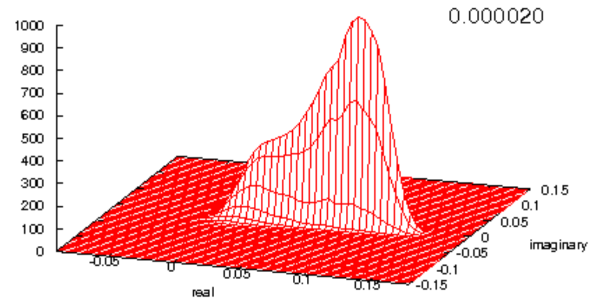
$\kappa^4 = 1.0\times10^{-5}$



$\kappa^4 = 1.5\times10^{-5}$

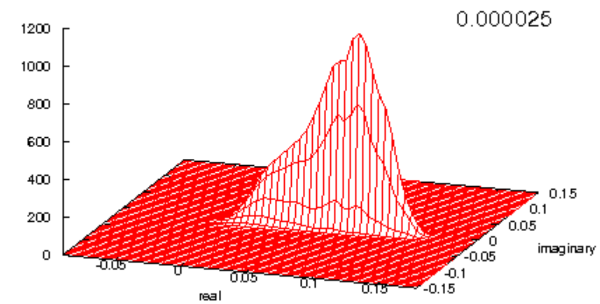


$\kappa^4 = 2.0\times10^{-5}$



critical point

$\kappa^4 = 2.5\times10^{-5}$

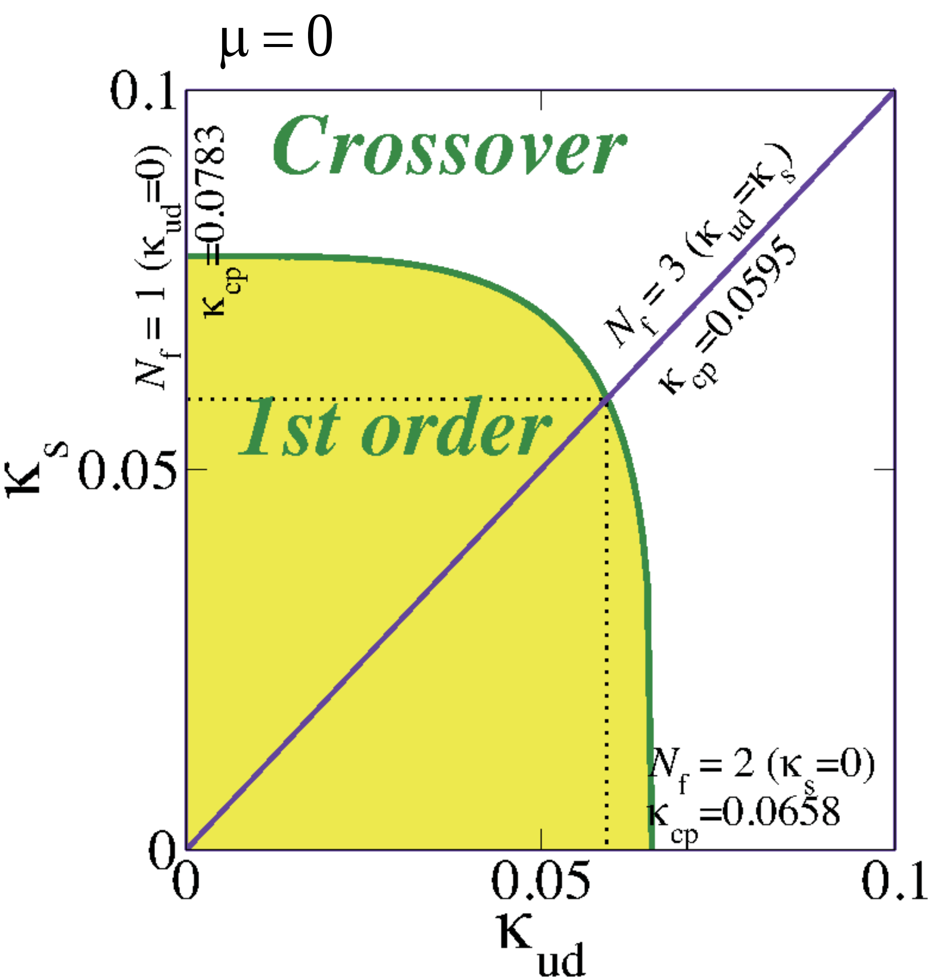


- on β_{pc} measured by the Polyakov loop susceptibility.

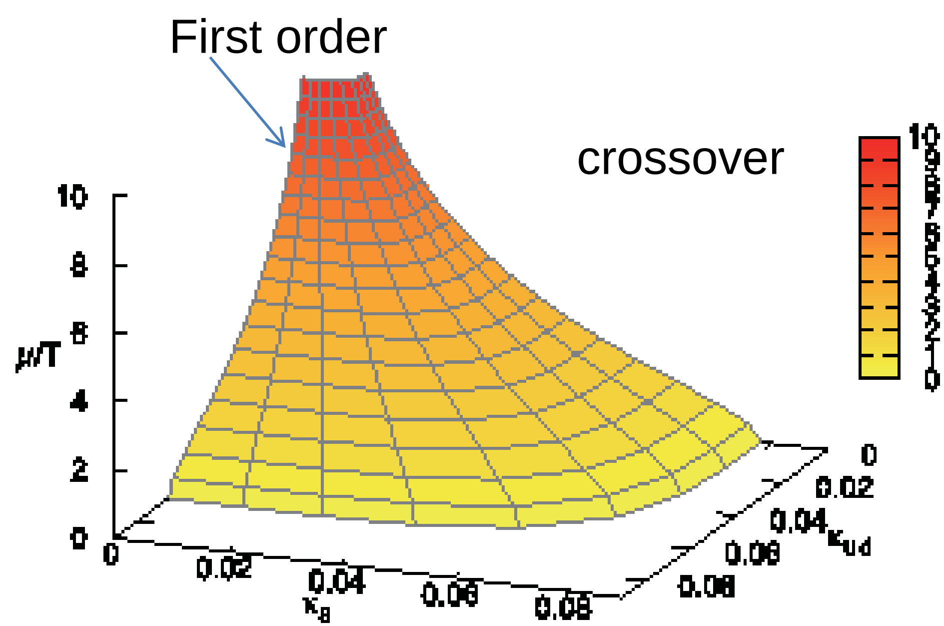
Critical surface in the heavy quark region of (2+1)-flavor QCD

$\left(24^3 \times 4 \text{ lattice}\right)$

$\mu = 0$



Critical surface at finite density



$$\frac{T_c}{m_\pi} \approx 0.02$$ at κ_{cp} for 2-flavor

Control Parameters in $W(X)$

- Distribution function

$$W(X;\kappa,\beta,\mu)\equiv\int DU\delta\left(X-\hat{X}\right)\prod_{f=1}^{N_{\mathrm{f}}}\det M\left(\kappa_f,\mu_f\right)e^{-S_g}$$

$$S_g=-6N_{\mathrm{site}}\beta\hat{P}$$

- Hopping parameter expansion

$$\ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)}\right)=288N_{\mathrm{site}}\kappa^4\hat{P}+12\cdot 2^{N_t}N_s^3\kappa^{N_t}\left(\cosh(\mu/T)\hat{\Omega}_R+i\sinh(\mu/T)\hat{\Omega}_I\right)+\cdots$$

- Three quantities in W: P, Ω_R, Ω_I
- Three parameters

$$\left(0\le\left|\tanh(\mu_f/T)\right|<1\right)$$

$$\beta^*\equiv\beta+\sum_{f=1}^{N_{\mathrm{f}}}48\kappa_f^4,\qquad \sum_{f=1}^{N_{\mathrm{f}}}\kappa_f^{N_t}\cosh(\mu_f/T),\qquad \sum_{f=1}^{N_{\mathrm{f}}}\kappa_f^{N_t}\sinh(\mu_f/T)$$

$$=\sum_{f=1}^{N_{\mathrm{f}}}\kappa_f^{N_t}\cosh(\mu_f/T)\tanh(\mu_f/T)$$

Distribution function of Ω_R at finite density

$$W(\Omega_R,\beta,\kappa,\mu)=\int DU\,\delta\left(\Omega_R-\hat{\Omega}_R\right)\left(\det M(\kappa)\right)^{N_f}\ e^{-6N_{site}\hat{P}}$$

- Hopping parameter expansion

$$\frac{W(\beta,\kappa,\mu)}{W(\beta_0,0,0)}=\left\langle \exp\left[\left(6(\beta-\beta_0)+288N_f\kappa^4\right)N_{site}\hat{P}-12\times 2^{N_t}N_fN_s^3\kappa^{N_t}\cosh(\mu/T)\hat{\Omega}_R+i\theta\right]\right\rangle_{\Omega_R;\beta_0,\kappa=\mu=0}$$

$$\left(\theta=12\cdot 2^{N_t}N_s^3N_f\kappa^{N_t}\sinh(\mu/T)\hat{\Omega}_I\right)$$

- Adopting $\beta_0\equiv\beta+48N_f\kappa^4$,
- Effective potential: $V_{eff}(\Omega_R;\beta,\kappa,\mu)=-\ln W(\Omega_R;\beta,\kappa,\mu)$

$$V_{eff}(\beta,\kappa,\mu)=V_{eff}(\beta_0,0,0)-12\times 2^{N_t}N_fN_s^3\kappa^{N_t}\cosh(\mu/T)\Omega_R-\ln\left\langle e^{i\theta}\right\rangle_{\Omega_R;\beta_0,\kappa=\mu=0}$$

$$\equiv V_0(\beta,\kappa,\mu)\qquad -\ln\left\langle e^{i\theta}\right\rangle_{\Omega_R;\beta_0,\kappa=\mu=0}$$

Phase-quenched part Phase average

- V_0 is V_{eff} (μ=0) when we replace $\kappa^{N_t}\Rightarrow\kappa^{N_t}\cosh(\mu/T)$

(at μ=0, $V_{eff}(\beta,\kappa,0)=V_{eff}(\beta_0,0,0)-12\times 2^{N_t}N_fN_s^3\kappa^{N_t}\Omega_R$)

Avoiding the sign problem

(SE, Phys.Rev.D77,014508(2008), WHOT-QCD, Phys.Rev.D82, 014508(2010))

θ: complex phase $\theta \equiv \mathrm{Im} \ln \det M \approx 12 \cdot 2^{N_t} N_s^3 N_f \kappa^{N_t} \sinh(\mu/T) \Omega_I$

- Sign problem: If $e^{i\theta}$ changes its sign,

$$\left\langle e^{i\theta} \right\rangle_{\Omega_R \text{ fixed}} << \text{(statistical error)}$$

- Cumulant expansion

$$\left\langle e^{i\theta} \right\rangle_{\Omega_R} = \exp\left[i\langle\theta\rangle_C - \frac{1}{2}\left\langle\theta^2\right\rangle_C - \frac{i}{3!}\left\langle\theta^3\right\rangle_C + \frac{1}{4!}\left\langle\theta^4\right\rangle_C + \cdots \right]$$

$\rightarrow 0$ $\rightarrow 0$

cumulants

$$\langle\theta\rangle_C = \langle\theta\rangle_{\Omega_R}, \quad \left\langle\theta^2\right\rangle_C = \left\langle\theta^2\right\rangle_{\Omega_R} - \langle\theta\rangle_{\Omega_R}^2, \quad \left\langle\theta^3\right\rangle_C = \left\langle\theta^3\right\rangle_{\Omega_R} - 3\left\langle\theta^2\right\rangle_{\Omega_R}\langle\theta\rangle_{\Omega_R} + 2\langle\theta\rangle_{\Omega_R}^3, \quad \left\langle\theta^4\right\rangle_C = \cdots$$

– Odd terms vanish from a symmetry under $\mu \leftrightarrow -\mu$ ($\theta \leftrightarrow -\theta$)
  Source of the complex phase

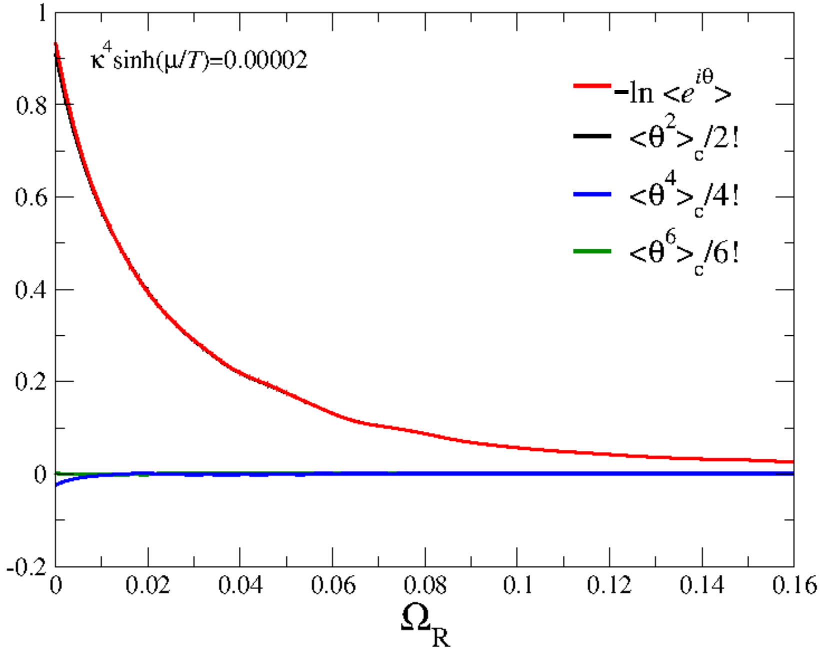
If the cumulant expansion converges, No sign problem.

Cumulant expansion

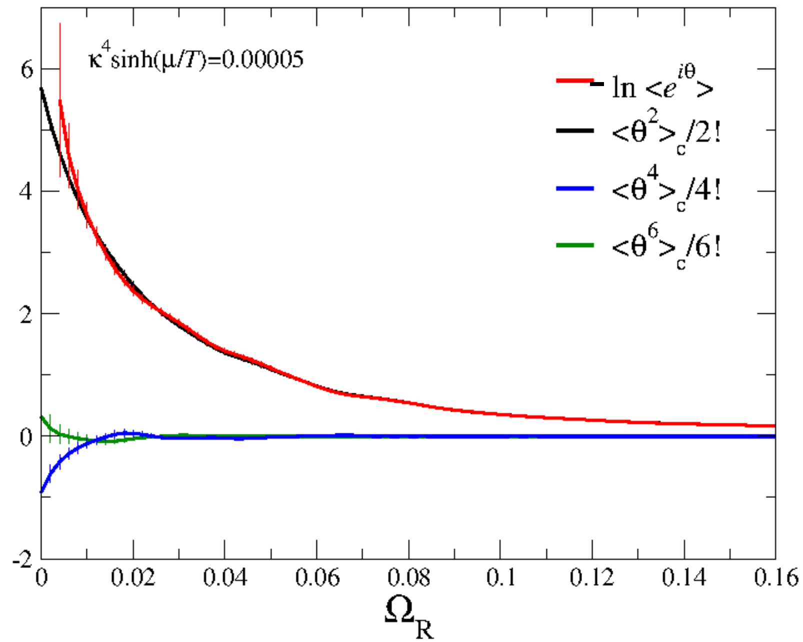
$\beta_0 = 5.69$

$$\ln\left\langle e^{i\theta}\right\rangle_{\Omega_R} = -\frac{1}{2}\left\langle\theta^2\right\rangle_C + \frac{1}{4!}\left\langle\theta^4\right\rangle_C - \frac{1}{6!}\left\langle\theta^6\right\rangle_C + \cdots$$

$K^4(\mu)\sinh(\mu/T) = 0.00002$



$K^4(\mu)\sinh(\mu/T) = 0.00005$



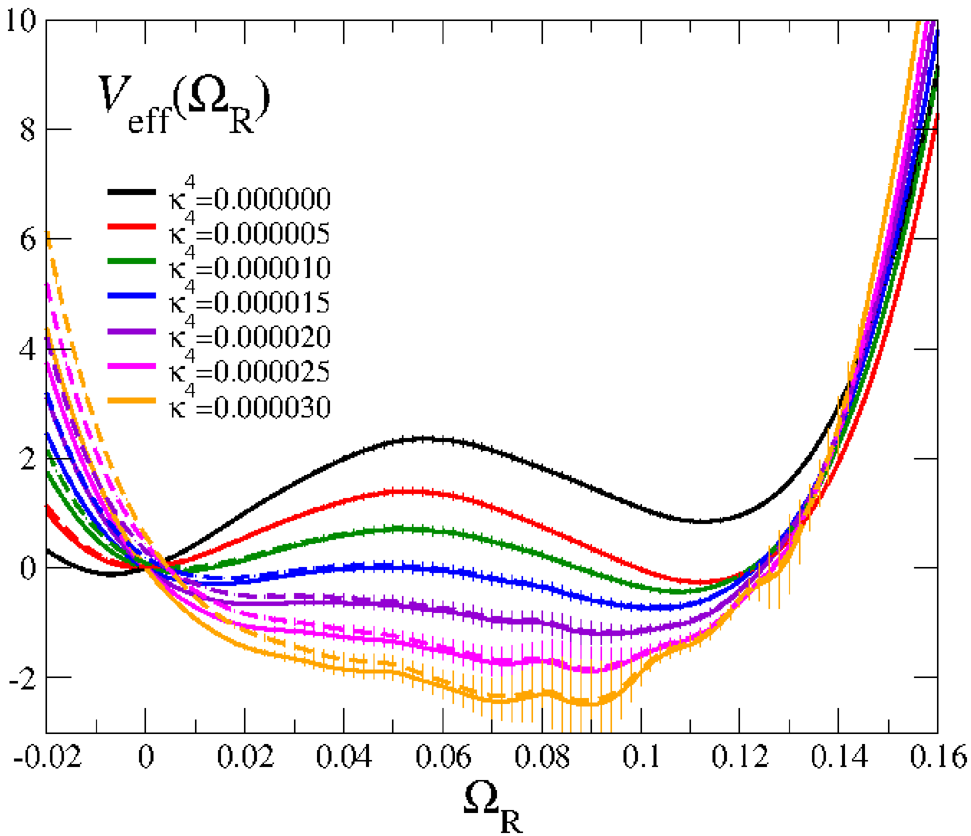
- At the critical point of phase-quenched part, the effect of higher order terms: small.

$$\kappa_{cp}^{N_t}(0) = \kappa_{cp}^{N_t}(\mu)\cosh(\mu/T) > \kappa_{cp}^{N_t}(\mu)\sinh(\mu/T)$$

~0.00002

Effect from the complex phase factor (2-flavor)

- Polyakov loop effective potential at various $\kappa^{N_t} \cosh(\mu/T)$ at the transition point. (β^* is adjusted at the transition point.)
  - Solid lines: μ=0 , i.e., $\cosh(\mu/T)=1, \quad \tanh(\mu/T)=0$
  - Dashed lines: $\tanh(\mu/T)=1$



The effect from the complex phase factor is very small except near Ω_R=0.

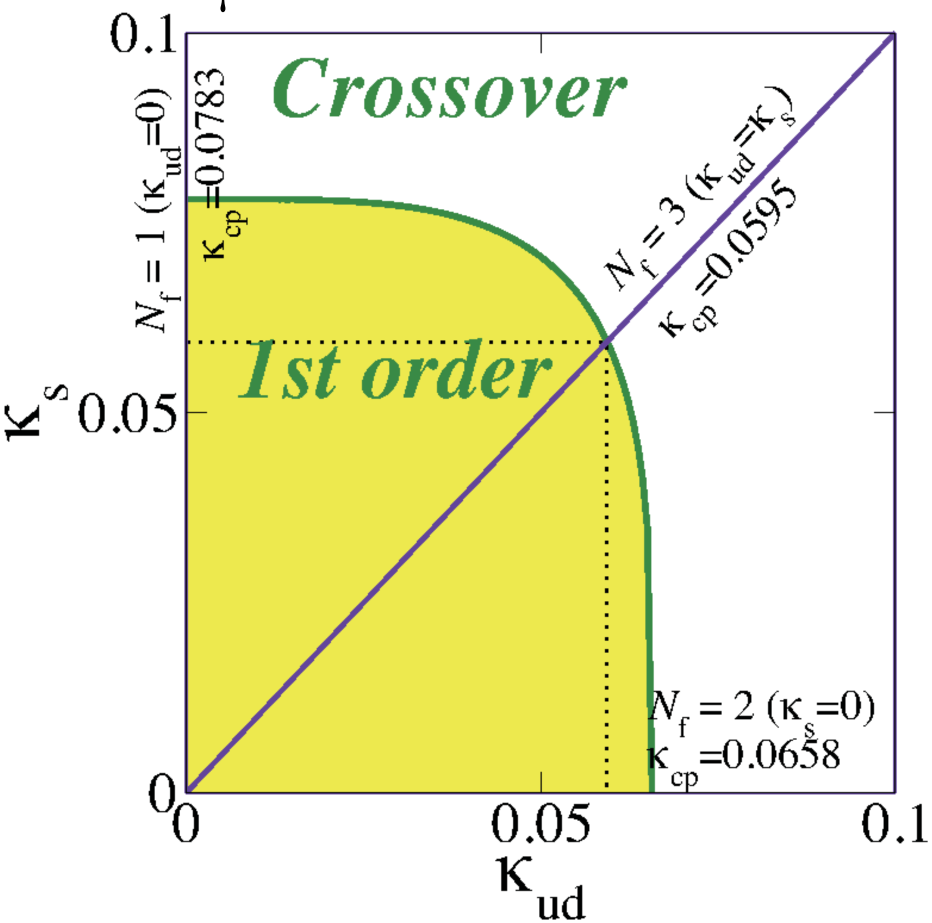
The nature of the phase transition is controlled only by

$$\sum_{f=1}^{N_f} \kappa_f^{N_t} \cosh(\mu_f/T)$$

Critical surface in the heavy quark region of (2+1)-flavor QCD

$\left(24^3 \times 4 \text{ lattice}\right)$

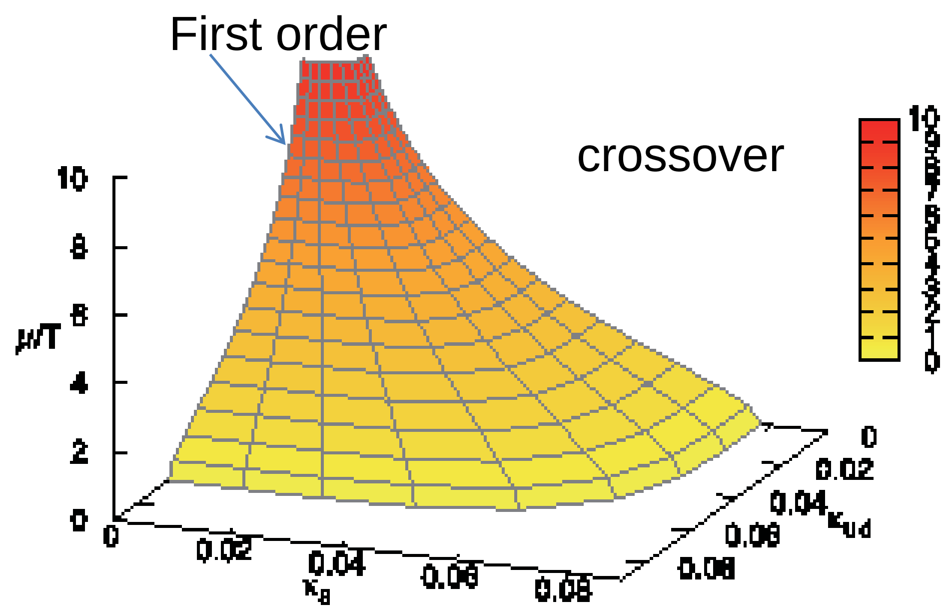
$\mu = 0$



$$\sum_{f=1}^{N_f} \kappa_{cp\,f}^{N_t} \cosh\left(\mu_f / T\right) = 4 \times 10^{-5}$$

$$\left(N_t = 4\right)$$

Critical surface at finite density



arXiv:1309.2445 [hep-lat]

$$\frac{T_c}{m_\pi} \approx 0.02$$ at κ_{cp} for 2-flavor

Phase transitions in many-flavor QCD

Phys. Rev. Lett. 110, 172001 (2013)

- Technicolor model
- First order transition

  ⇨ Electro-weak baryogenesis

- Good test for (2+1)-flavor QCD

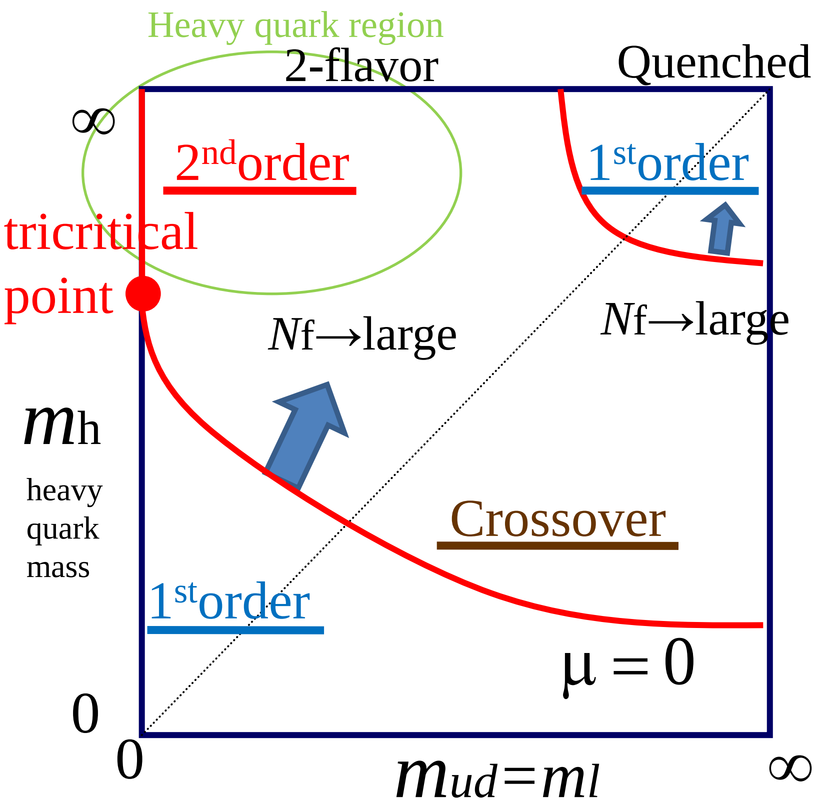
Finite T and μ phase transition in (2+many)-flavor QCD

(Cf. Kikukawa, Kohda and Yasuda,Phys.Rev.D77, 015014(2008))

- Technicolor model constructed by many-flavor QCD
- Chiral phase transition of QCD

  → Electroweak phase transition at finite temperature
- Nambu-Goldstone bosons
  - 3 bosons are absorbed into the gauge bosons. (3 massless bosons)
  - The other bosons have not observed yet. (The other bosons: heavy)
  - 2 techni-felmions are massless, and the others are heavy.
- Electro-weak baryogenesis
  - Strong first order transition: required.
  - From the analogy of 2+1-flavor QCD, 1st order at small mass; 2nd order or crossover at large mass.
- <u>It is important to determine the endpoint of the first order region in (2+many)-flavor QCD.</u>

Nature of phase transition of 2+N_f-flavor QCD



- Assumption: N_f-flavors are heavy.
  - Hopping parameter κ expansion
- Parameter: $N_f \kappa^{N_t}$ ➡ $1/m_{h,ct} \sim \kappa_{ct} \propto 1/N_f^{1/N_t}$
- As increasing N_f, critical mass becomes larger.
- Tricritical scaling: the same as (2+1)-flavor QCD

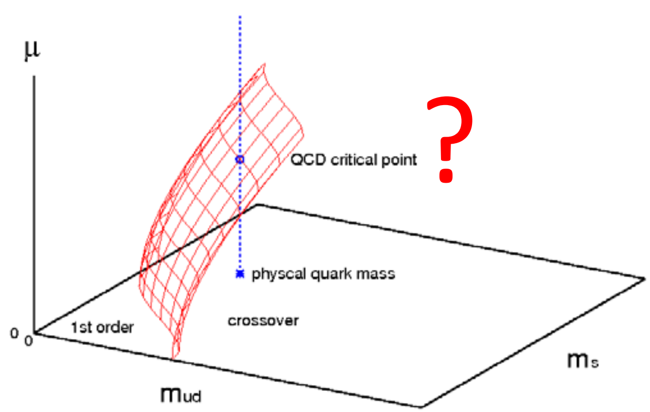
Tricritical point m_E:

$$m_{ud}^c \sim (m_E - m_h)^{5/2}$$

$$m_{ud}^c \sim \mu^5$$

Good test ground

At finite density?



Reweighting method for plaquette distribution function

$$W(P,\beta,m,\mu) \equiv \int DU\,\delta(\hat{P}-P)\prod_{f=1}^{N_f}\det M(m_f,\mu_f)\,e^{6N_{\rm site}\beta\hat{P}}$$

$$S_g = -6N_{\rm site}\beta\hat{P}$$

$$(\beta = 6/g^2)$$

plaquette P (1x1 Wilson loop for the standard action)

$$R(P,\beta,\beta_0 m,m_0,\mu) \equiv W(P,\beta,m,\mu)/W(P,\beta_0,m_0,0)$$ (Reweight factor)

$$R(P) = \frac{\left\langle \delta(\hat{P}-P)\,e^{6N_{\rm site}(\beta-\beta_0)\hat{P}}\prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)} \right\rangle_{(\beta_0,\mu=0)}}{\left\langle \delta(\hat{P}-P) \right\rangle_{(\beta_0,\mu=0)}} \equiv \left\langle e^{6N_{\rm site}(\beta-\beta_0)\hat{P}}\prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)} \right\rangle_{P:\text{fixed}}$$

Effective potential:

$$V_{\rm eff}(P,\beta,m,\mu) = -\ln[W(P,\beta,m,\mu)] = V_{\rm eff}(P,\beta_0,m_0,0) - \ln R(P,\beta,\beta_0 m,m_0,\mu)$$

$$\ln R(P) = \underline{6N_{\rm site}(\beta-\beta_0)P} + \ln\left\langle \underline{\prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)}} \right\rangle_{P:\text{fixed}}$$

First order transition point: two phases coexist

Plaquette distribution function

- Performing simulations of 2-flavor QCD,
- Dynamical effect of N_f-flavors are included by the reweighting.
- We assume N_f-flavors are heavy.
- Hopping parameter (κ) expansion（Wilson quark）

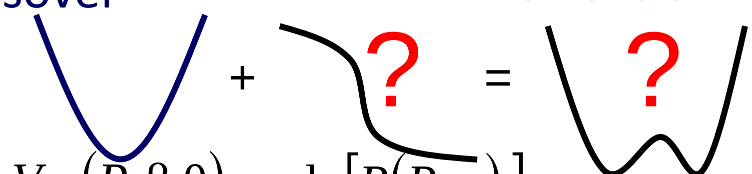
$$N_{\rm f} \ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)}\right) = N_{\rm f}\left(288 N_{\rm site}\kappa^4 P + 12\cdot 2^{N_t} N_s^3 \kappa^{N_t}\left(\cosh(\mu/T)\Omega_R + i\sinh(\mu/T)\Omega_I\right)+\cdots\right)$$

- Effective potential

2−flavor
crossover

2+Nf-flavor
1st order transition

$$V_{\rm eff}(P,\beta,\kappa) = -\ln\left[R(P,\kappa)W(P,\beta,0)\right] =$$



$V_{\rm eff}(P,\beta,0)$ $\quad -\ln[R(P,\kappa)]$

Negative curvature

$$\ln R(P) = \ln\left\langle e^{6N_{\rm site}(\beta-\beta_0)\hat{P}} \prod_f \frac{\det M(\kappa_f,\mu_f)}{\det M(\kappa_0,0)}\right\rangle_{P:{\rm fixed}} \approx \ln\left\langle \exp\left(6hN_s^3\hat{\Omega}_R\right)\right\rangle_{P:{\rm fixed}}$$

+(linear term of P)

(degenerate mass case at μ=0)

Curvature of the effective potential

$$V_{\rm eff}(P,\beta,h,\mu) = V_{\rm eff}(P,\beta_0,0,0) - \ln \overline{R}(P,h,\mu) + \text{ (linear term of } P\text{)}$$

$$\overline{R}(P) = \left\langle \exp\left(6N_s^3 h \Omega_R\right)\right\rangle_{P:\text{fixed}} \qquad \text{(for the case of } \mu=0\text{)}$$

Wilson quark

$$h = 2N_{\rm f}\left(2\kappa_{\rm h}\right)^{N_t}$$

Staggered quark

$$h = N_{\rm f} \Big/ \left(4\left(2m_{\rm h}\right)^{N_t}\right)$$

- Linear term of P is irrelevant to the curvature
- β-dependence is only in the linear term.
- The curvature is independent of β.

χ_P: plaquette susceptibility

$$\frac{d^2 V_{\rm eff}(0)}{dP^2} \approx \frac{6N_{\rm site}}{\chi_P}$$

$$\frac{d^2 V_{\rm eff}}{dP^2}(P,h,\mu) = \frac{d^2 V_{\rm eff}}{dP^2}(P,0,0) - \frac{d^2 \ln \overline{R}}{dP^2}(P,h,\mu)$$

2-flavor

- If there exists the negative curvature region,

$\Rightarrow$ First order transition (double-well potential)

Effective potential at $h \neq 0$

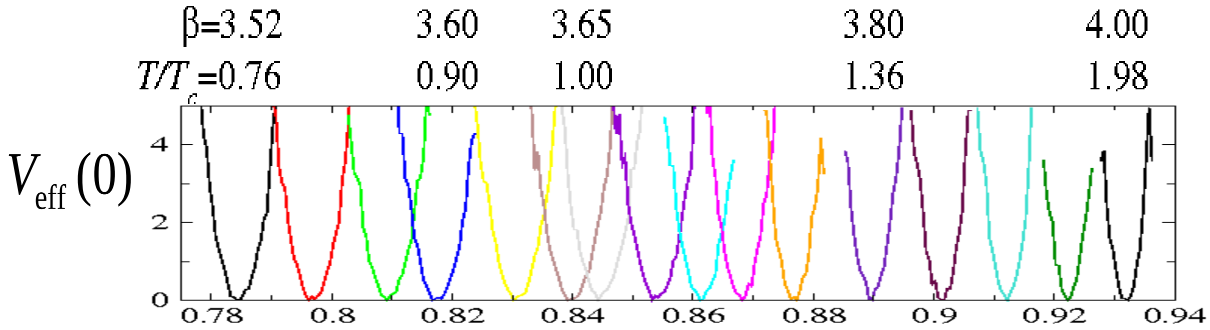
$$V_{\rm eff}(P,\beta,h) = V_{\rm eff}(P,\beta,0) - \ln R(P,h)$$

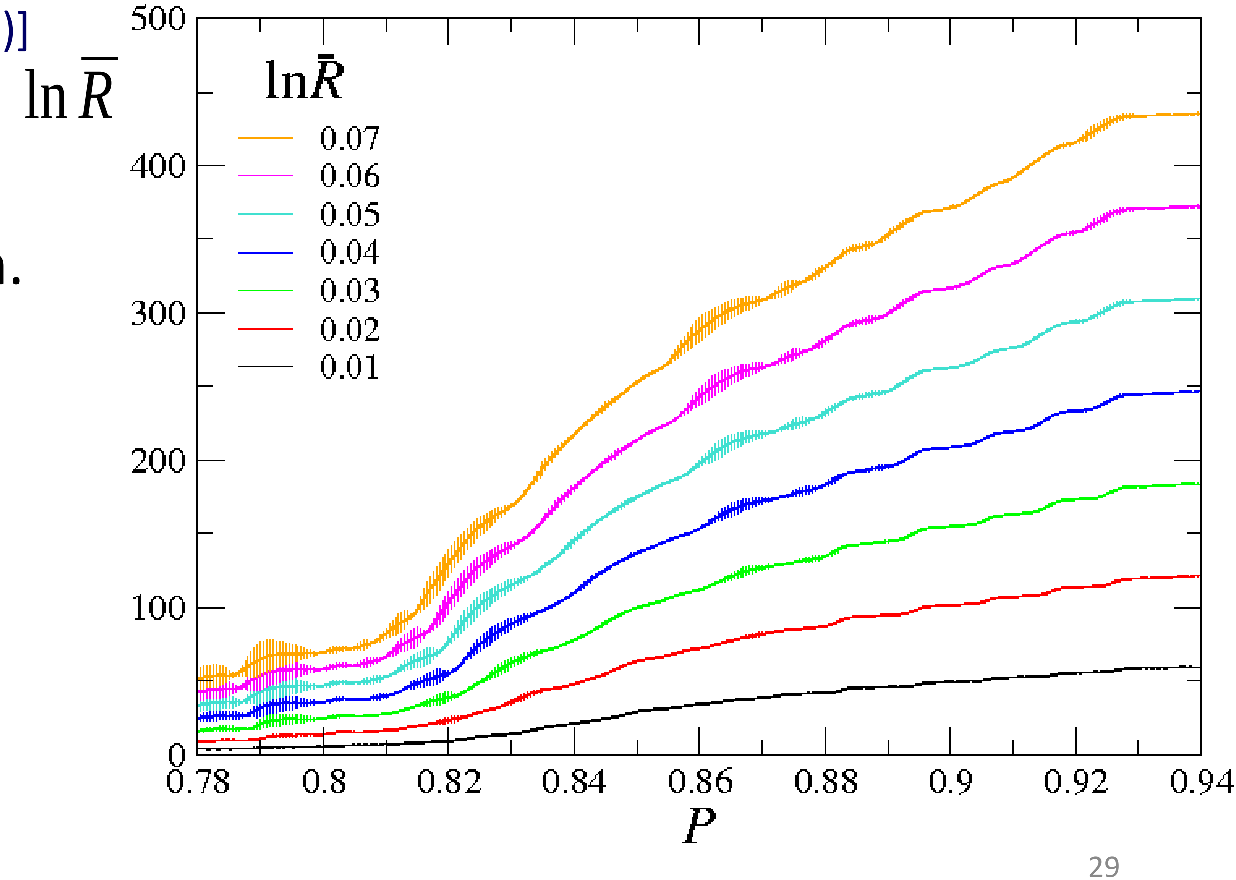
Nf=2 p4-staggared,
$m_\pi/m_\rho \approx 0.7$

[data: Beilefeld-Swansea Collab.,PRD71,054508(2005)]

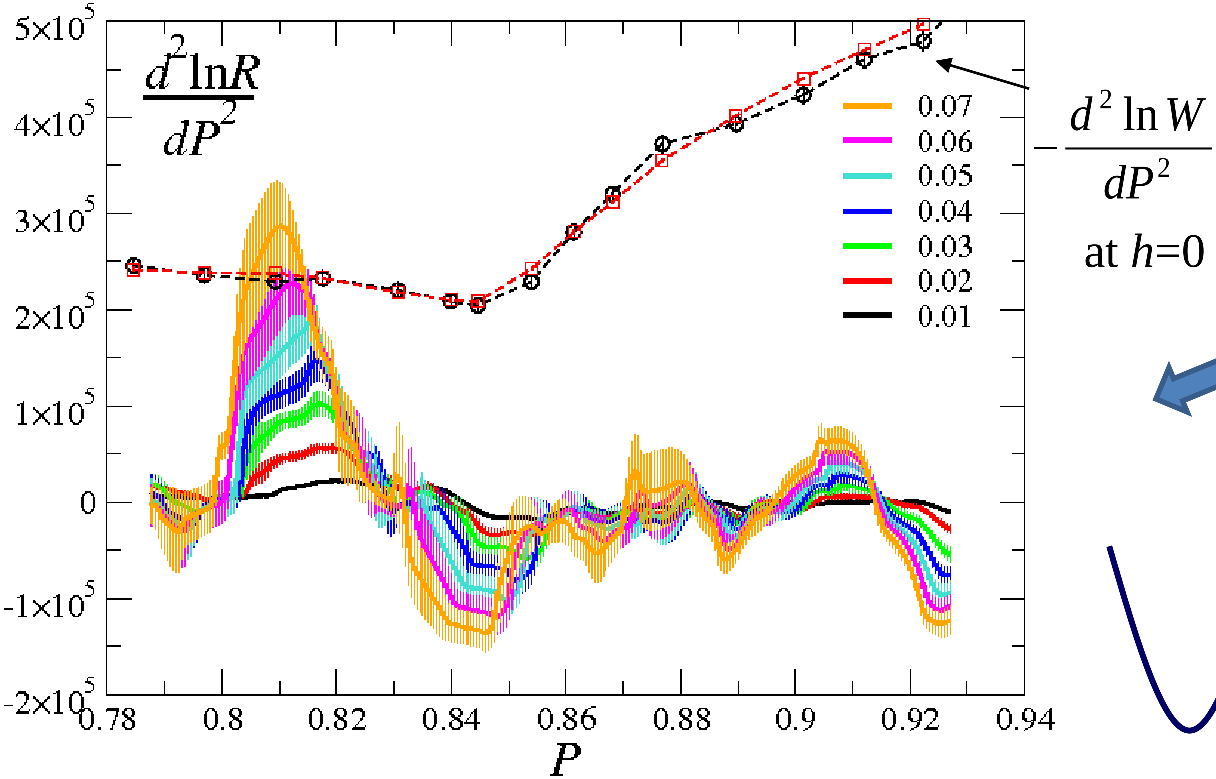
- det$M$: hopping parameter expansion.

- lnR increases as increasing h.
- The curvature increases with h.

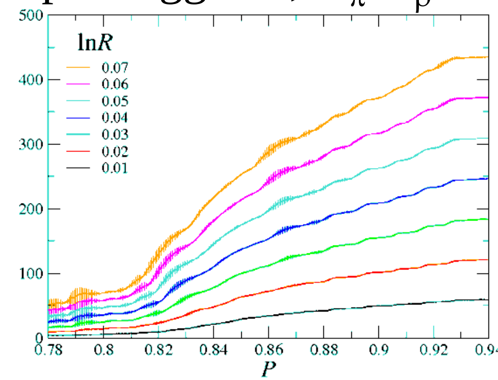


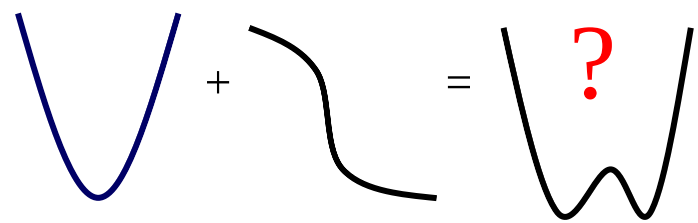


Curvature of the effective potential



Nf=2 p4-staggared, $m_\pi/m_\rho \approx 0.7$





First order transition: $\frac{d^2 V_{\text{eff}}(P,\beta,h)}{dP^2} = \frac{d^2 V_{\text{eff}}(P,\beta,0)}{dP^2} - \frac{d^2 \ln \overline{R}(P,h)}{dP^2} < 0$

$$h = 2N_f (2\kappa_h)^{N_t}$$

(Wilson quarks)

- First order transition for h > 0.06

Critical value: $h_c = 0.0614(69)$

N_f–dependence of the critical mass

$$h_c = 0.0614(69)$$

- Critical mass increases as N_f increases.

$$h = 2N_\mathrm{f}\left(2\kappa_\mathrm{h}\right)^{N_t} \Rightarrow \kappa_h^c = \frac{1}{2}\left(\frac{h_c}{2N_\mathrm{f}}\right)^{1/N_t}$$

  - When N_f is large, κ is small. Then, the hopping parameter (κ) expansion is good.
  - On the hand, when N_f is small, the κ-expansion is bad.

- In a quenched simulation with N_t=4, the first and second terms becomes comparable around κ=0.18.
- For N_f=10, N_t=4, $h_c = 0.0614(69)$ $\Rightarrow$ $\kappa_h^c \approx 0.118$
  - It may be applicable for N_f~10.

Curvature of the effective potential at finite μ

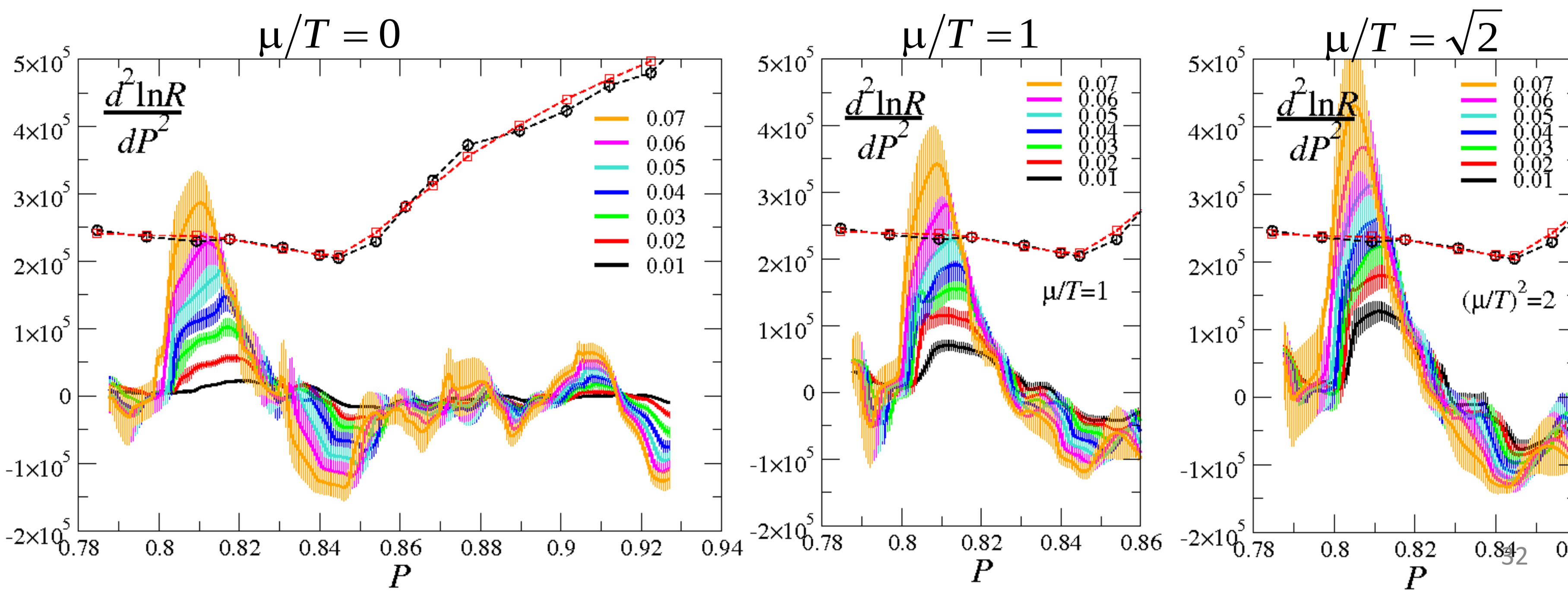
$$\frac{d^2 V_{\mathrm{eff}}}{dP^2}(P,h,\mu) = \frac{d^2 V_{\mathrm{eff}}}{dP^2}(P,0,0) - \frac{d^2 \ln \bar{R}}{dP^2}(P,h,\mu)$$

$$h = 2N_{\mathrm{f}}(2\kappa_{\mathrm{h}})^{N_t}$$ for Wilson quarks

$$\ln R(P) = \ln \left\langle \left(\frac{\det M(m,\mu)}{\det M(m,0)} \right)^2 \left(\frac{\det M(h,\mu_h)}{\det M(0,0)} \right)^{N_{\mathrm{f}}} \right\rangle_{P:\mathrm{fixed}}$$

- Calculations of detM: Taylor expansion up to O(μ^6)
- Distribution function of the complex phase of detM: approximated by a Gaussian function

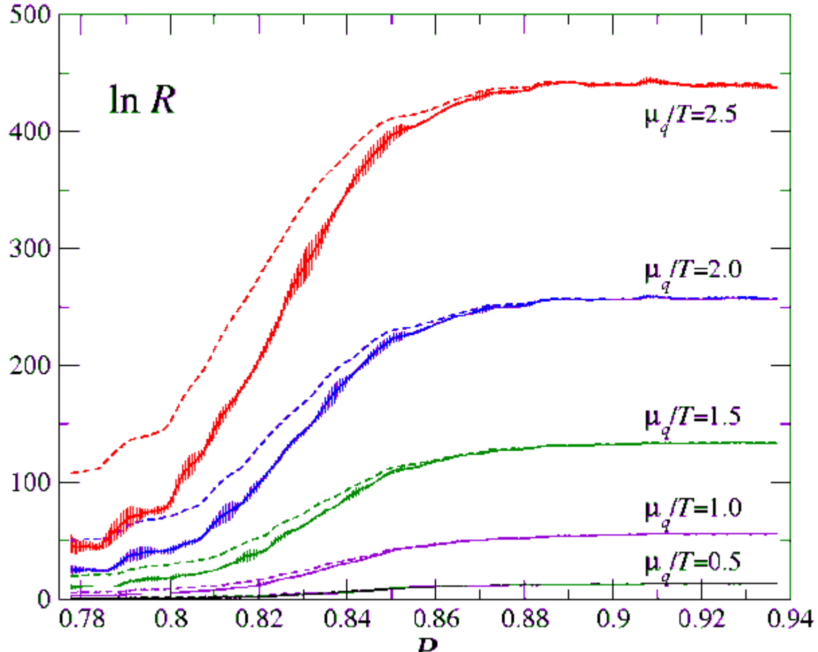
$$\mu_h / T = 0$$



Reweighting factors at $h\neq 0$ $\mu\neq 0$

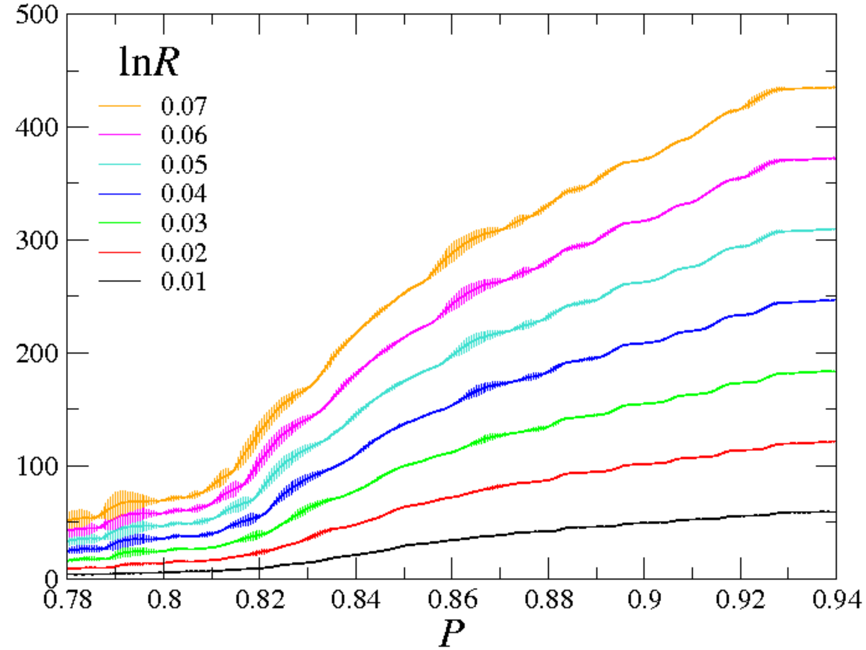
$$\ln R(P;h,\mu)=\ln\left\langle \left(\frac{\det M(m,\mu)}{\det M(m,0)}\right)^2 \left(\frac{\det M(h,0)}{\det M(0,0)}\right)^{N_f} \right\rangle_{P:\text{fixed}} \approx \ln R(P;0,\mu)+\ln R(P;h,0)$$

$\ln R(P;0,\mu)$



(S. Ejiri, Phys. Rev. D 77 (2008) 014508)

$\ln R(P;h,0)$



N_f=2 p4-staggared, $m_\pi/m_\rho\approx 0.7$ [data in PRD71,054508(2005)]

- The curvatures of lnR(P;μ,0) and lnR(P;0,h) are large at the same P.

  ⇨ The curvature of lnR(P;μ,h) is enhanced.

Critical line at finite density

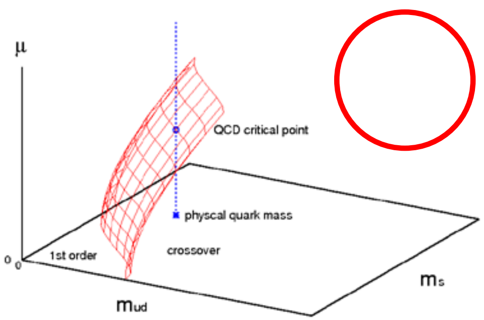
$$h = 2N_{\mathrm{f}}\left(2\kappa_{\mathrm{h}}\right)^{N_t}$$

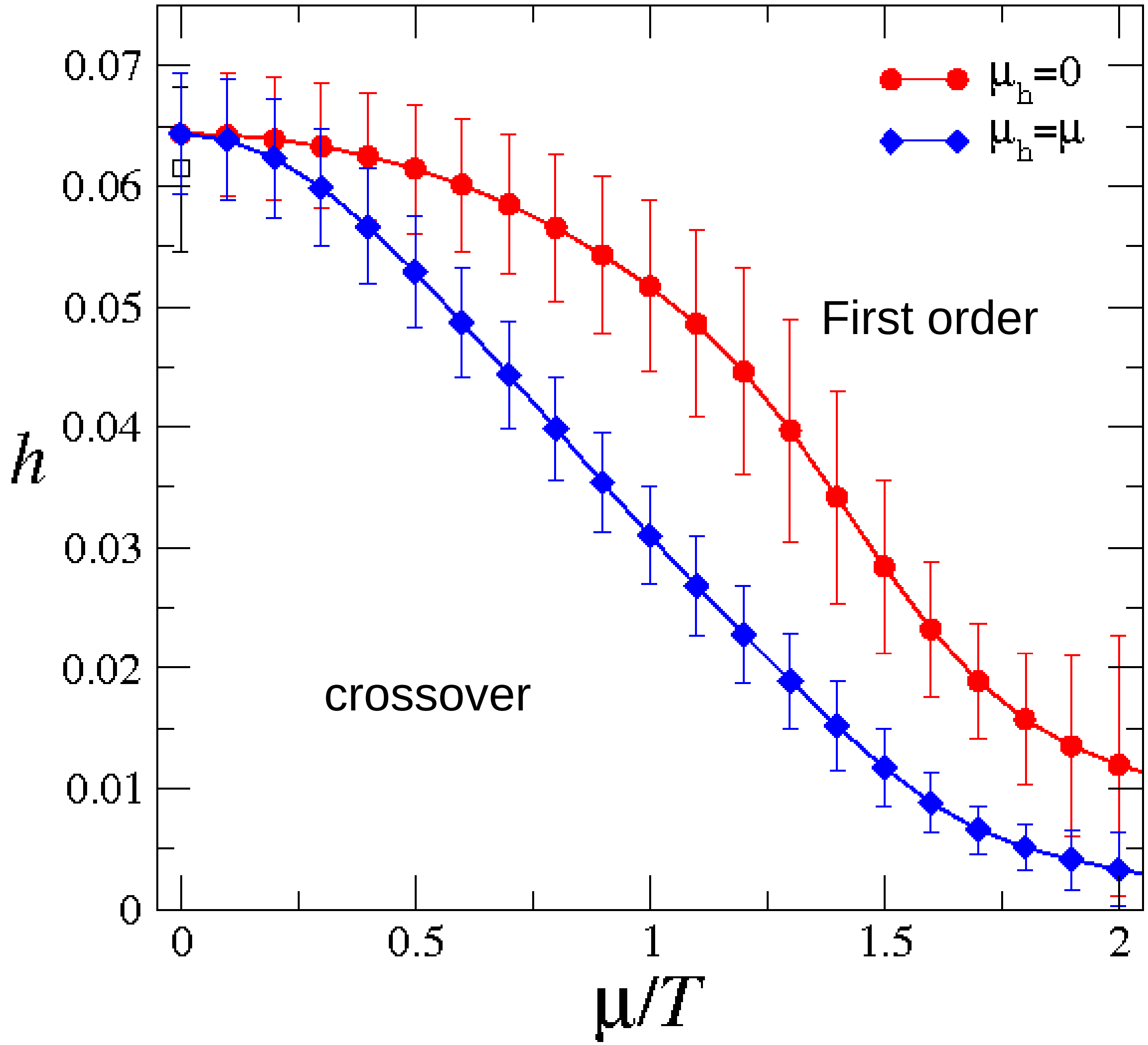
for Wilson quarks

$$h = N_{\mathrm{f}} \Big/ \left(4\left(2m_{\mathrm{h}}\right)^{N_t}\right)$$

for staggered quarks

- Calculations of detM: Taylor expansion up to O(μ^6)
- Distribution function of the complex phase of detM: approximated by a Gaussian function





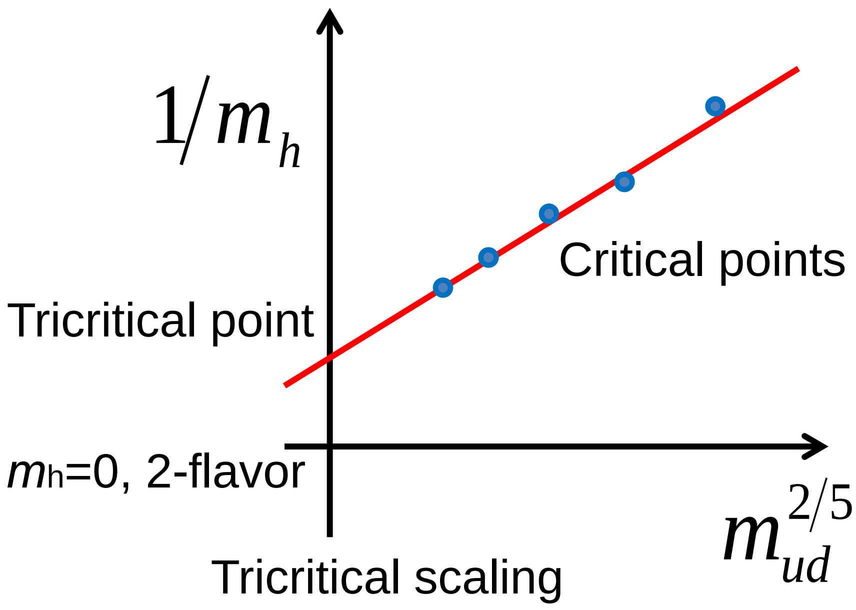
Phys. Rev. Lett. 110, 172001 (2013)

Phase structure of (2+many)-flavor QCD using Wilson quark action

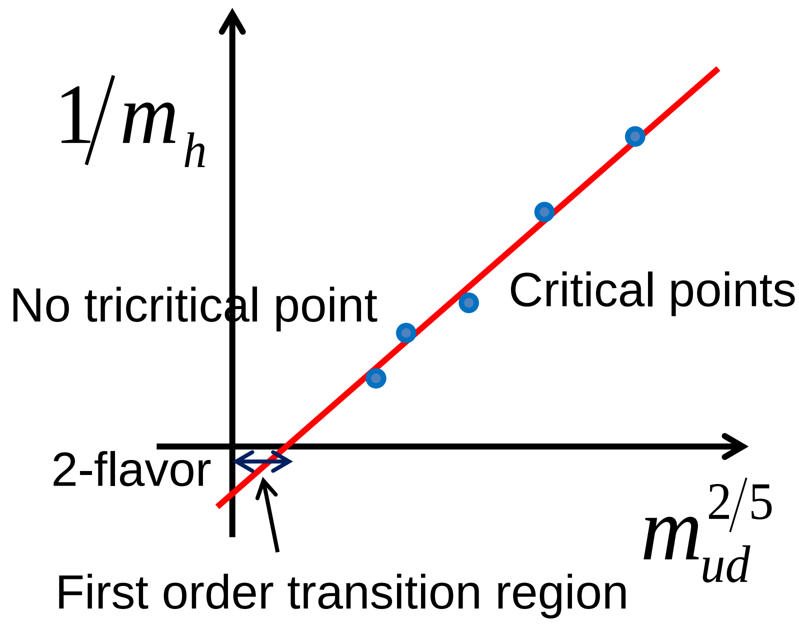
2-flavor QCD simulations + reweighting

Light quark mass dependence of the critical line

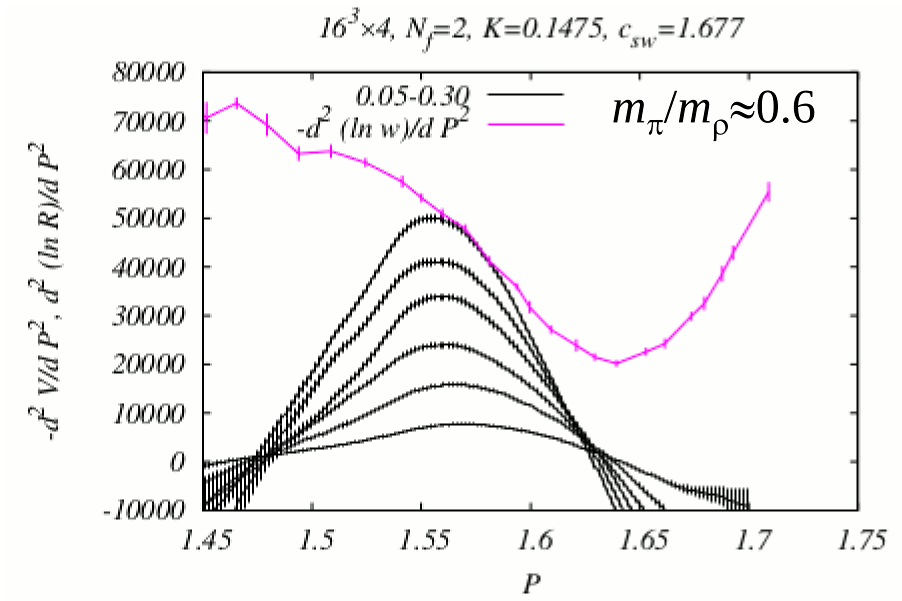
- Trictitical scaling behavior?
- Is there a first order transition region in 2-flavor QCD?

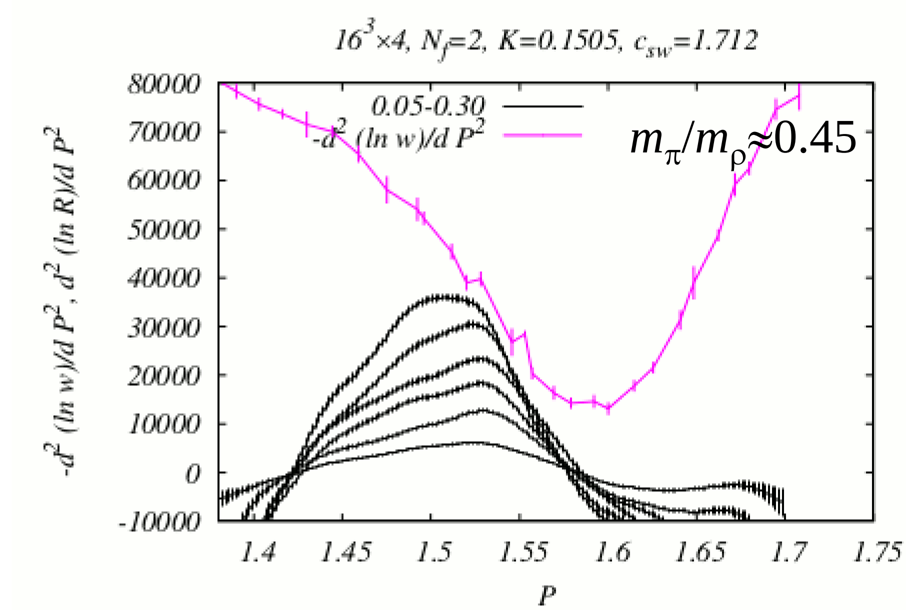


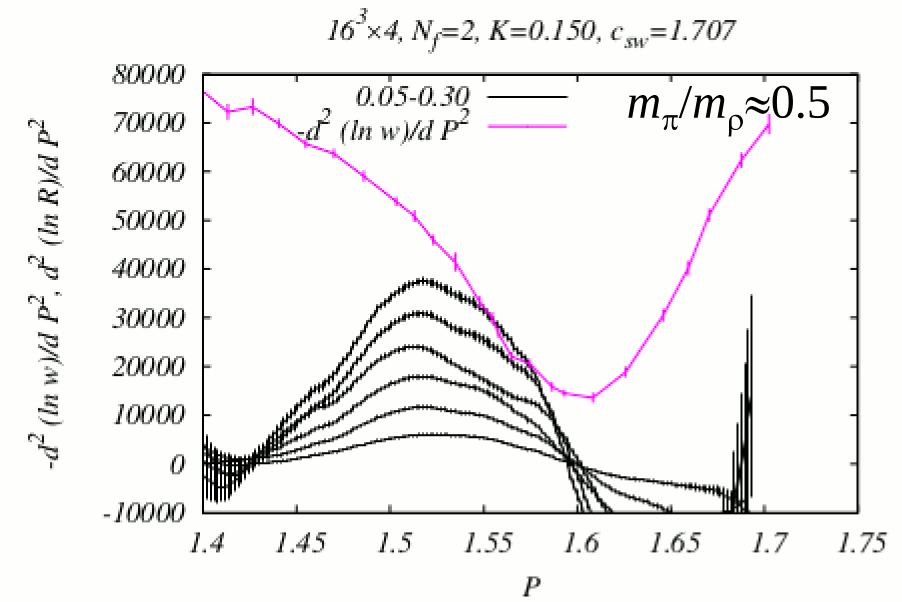
or



Light quark mass dependence (preliminary)







h=0.05, 0.10, 0.15, 0.20, 0.25, 0.30

- Critical point: light quark mass dependence is small in this mass region.
- In progress

Summary

- We discussed the QCD phase transition in the heavy quark region.

  WHOT-QCD(H. Saito, S. Ejiri, S. Aoki, K. Kanaya, Y. Nakagawa, H. Ohno, K. Okuno, and T. Umeda), arXiv:1309.2445

  S. Ejiri, Euro. Phys. J. A 49, 86 (2013) [arXiv:1306.0295]

  - The critical surface in the heavy quark region of (2+1)-flavor QCD is computed.
- We investigated the phase structure of (2+Nf)-flavor QCD.

  S. Ejiri & N. Yamada, Phys. Rev. Lett. 110, 172001 (2013) [arXiv:1212.5899]

  - This model is interesting for the feasibility study of the electroweak baryogenesis in the technicolor scenario.
  - An appearance of a first order phase transition at finite temperature is required for the baryogenesis.
  - Applying the reweighting method, we determine the critical mass of heavy flavors terminating the first order region.
    - The critical mass becomes larger with N_f.
    - The first order region becomes wider as increasing μ.
- This may be a good test for the determination of boundary of the first order region in (2+1)-flavor QCD at finite density.