Dynamics of Non-Gaussianity in Heavy Ion Collisions

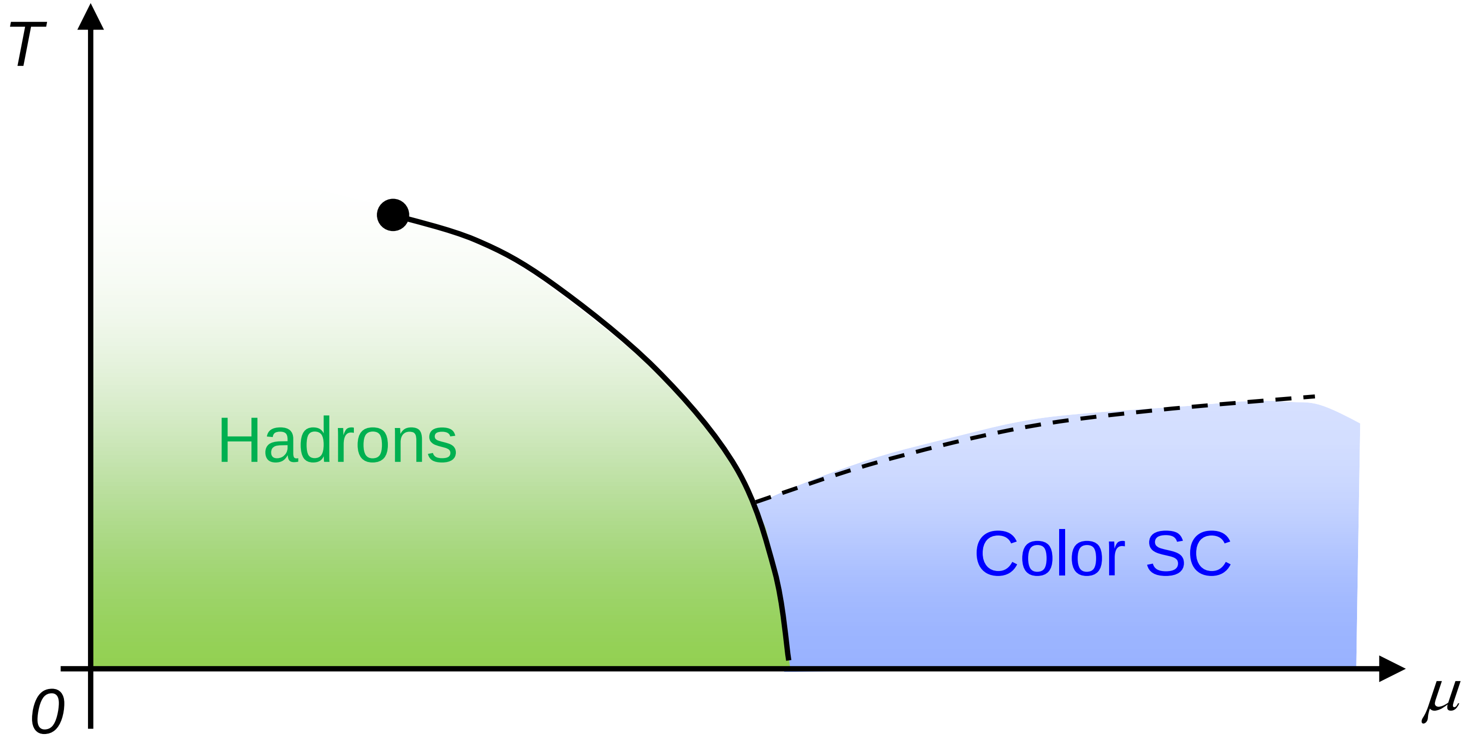
Masakiyo Kitazawa

(Osaka U.)

MK, Asakawa, PRC85,021901C(2012); PRC86, 024904(2012)
MK, Asakawa, Ono, arXiv:1307.2978

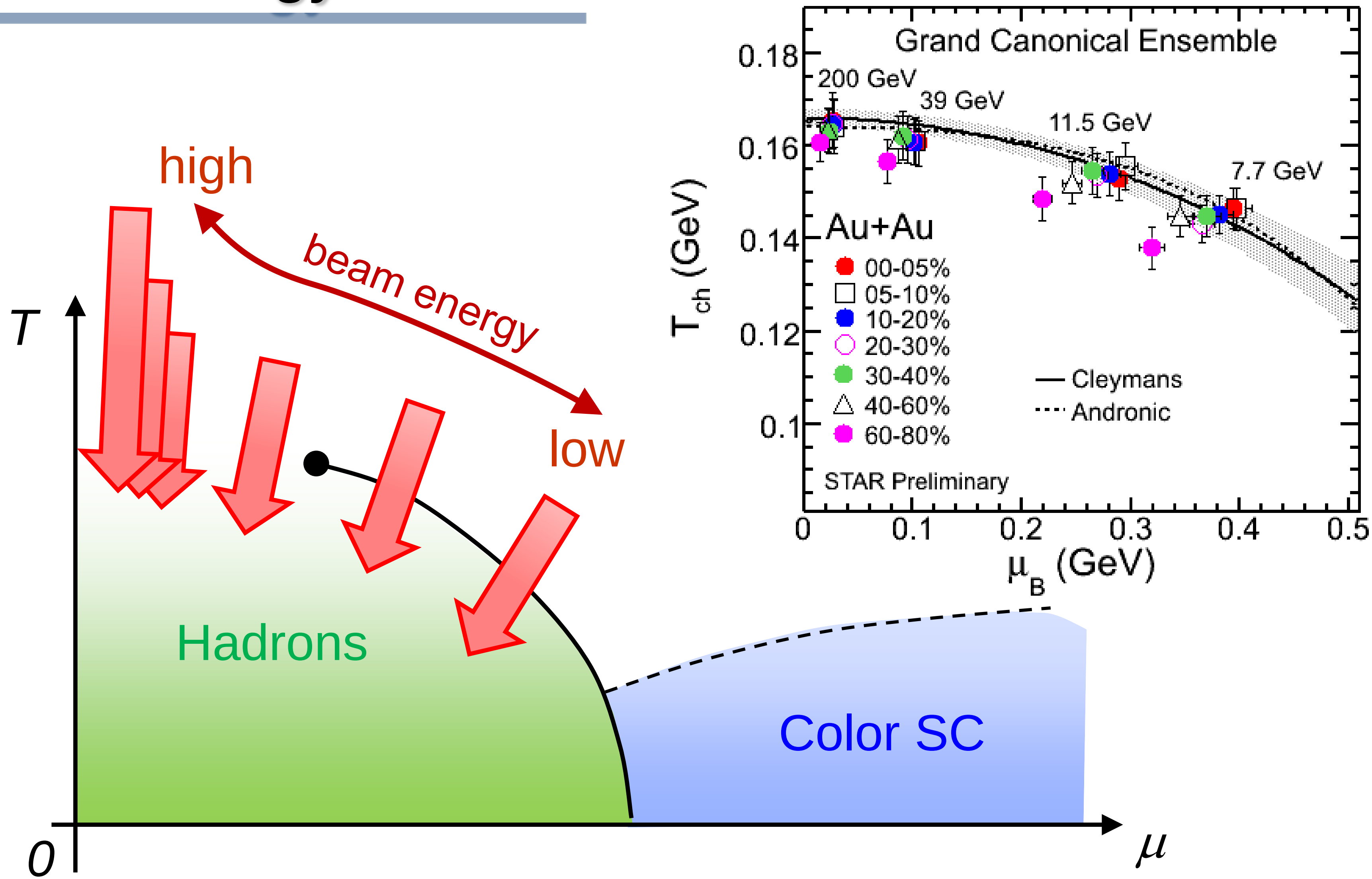
NFQCD, YITP, Kyoto, 27/Nov./2013

Beam-Energy Scan



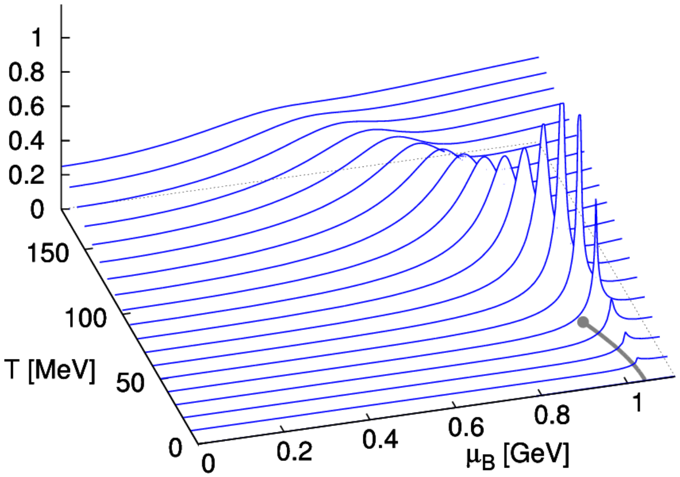
Beam-Energy Scan

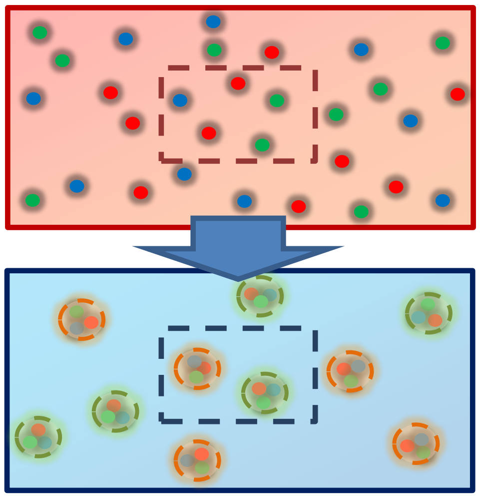
STAR 2012

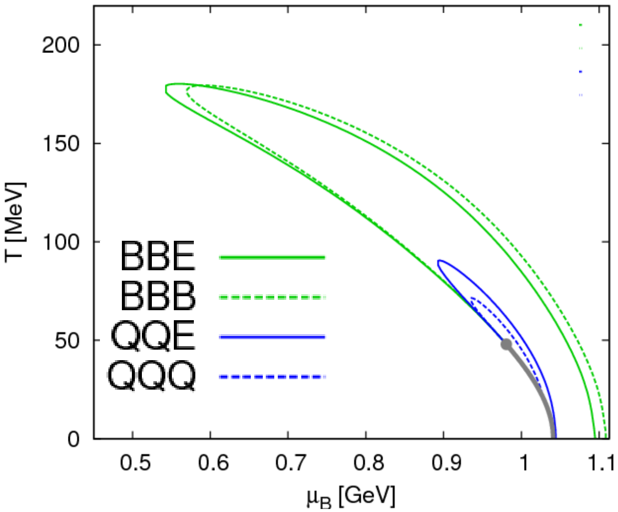


Fluctuations

- Fluctuations reflect properties of matter.
  - Enhancement near the critical point
    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges
    Asakawa,Heinz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants
    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)



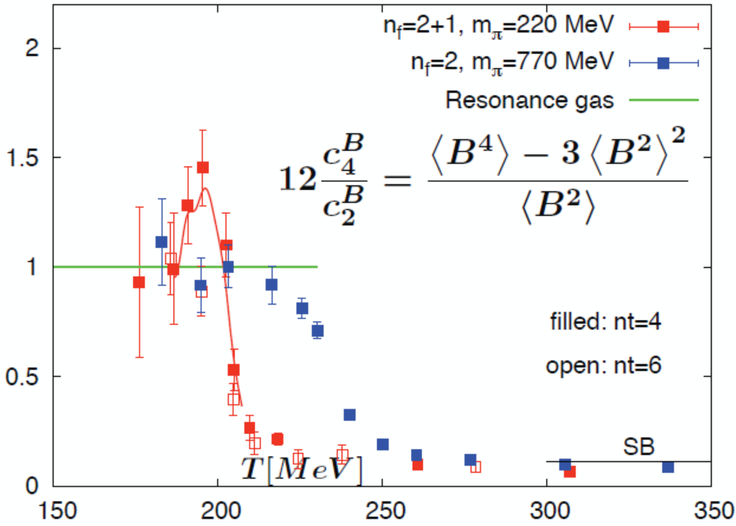




Conserved Charges : Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory

ex: on the lattice



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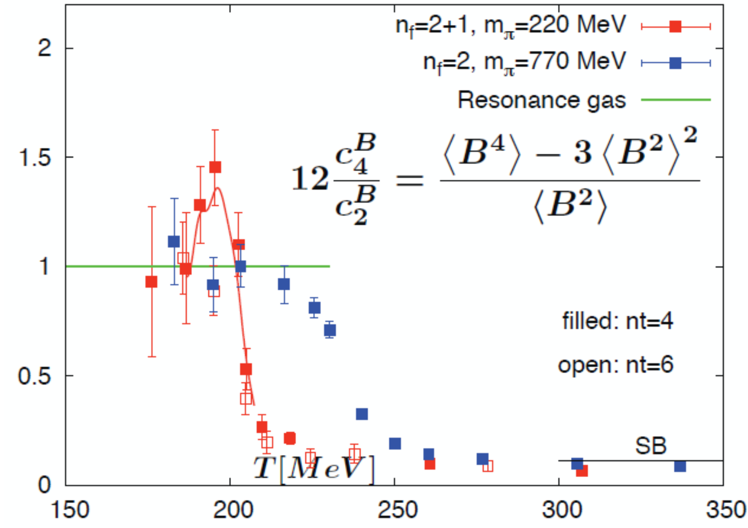


- Simple thermodynamic relations

$$\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

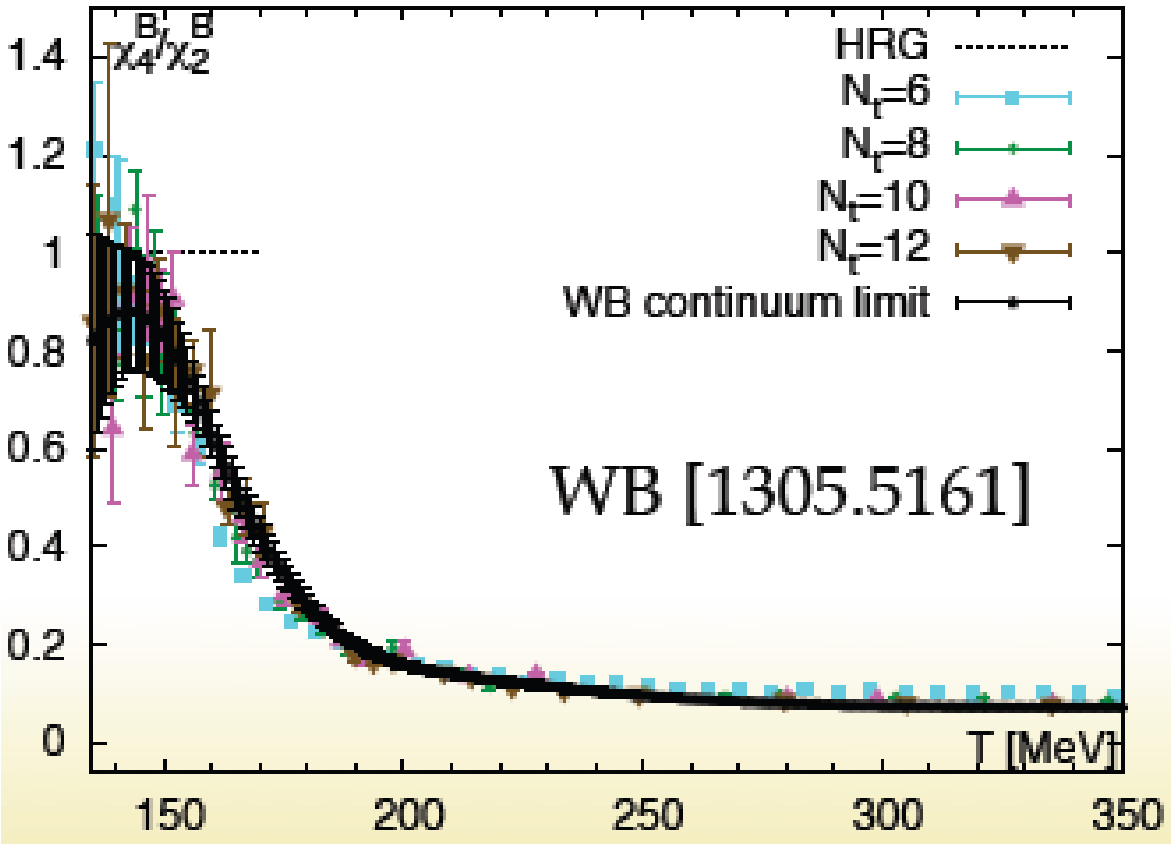
  - Intuitive interpretation for the behaviors of cumulants

ex: $\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$

Asakawa, Ejiri, MK, 2009

Conserved Charge Fluctuations

$$\frac{\langle \delta N_B^4 \rangle_c}{\langle \delta N_B^2 \rangle}$$

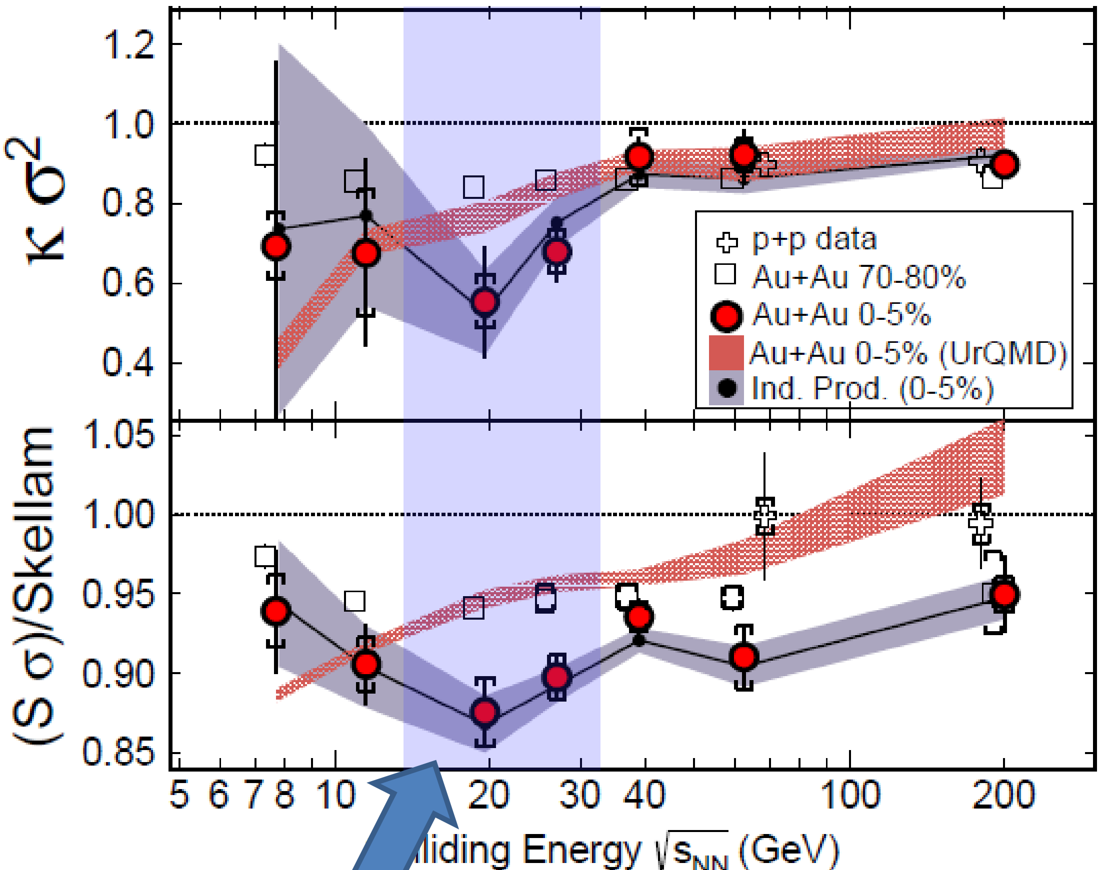


Cumulants of N_B and N_Q are **suppressed** at high T.

Asakawa, Heinz, Muller, 2000; Jeon, Koch, 2000;
Ejiri, Karsch, Redlich, 2006; Asakawa, Ejiri, MK, 2009;
Friman, et al., 2011; Stephanov, 2011

Proton # Cumulants @ STAR-BES

STAR,1309.5681



$$\frac{C_4}{C_2}$$

$$\frac{C_3}{C_1} = \frac{C_3/C_2}{\text{Poissonian}}$$

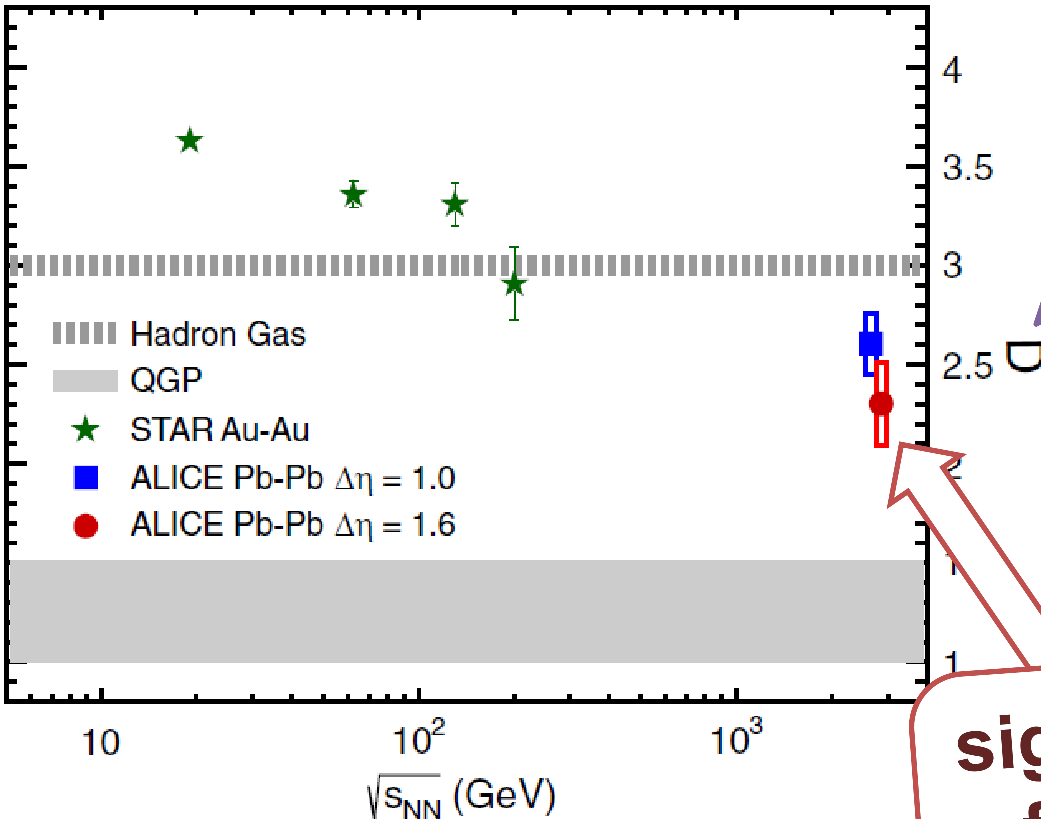
Something interesting??

CAUTION!

proton number $\neq$ baryon number

MK, Asakawa, 2011;2012

Charge Fluctuation @ LHC



ALICE, PRL110,152301(2013)

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

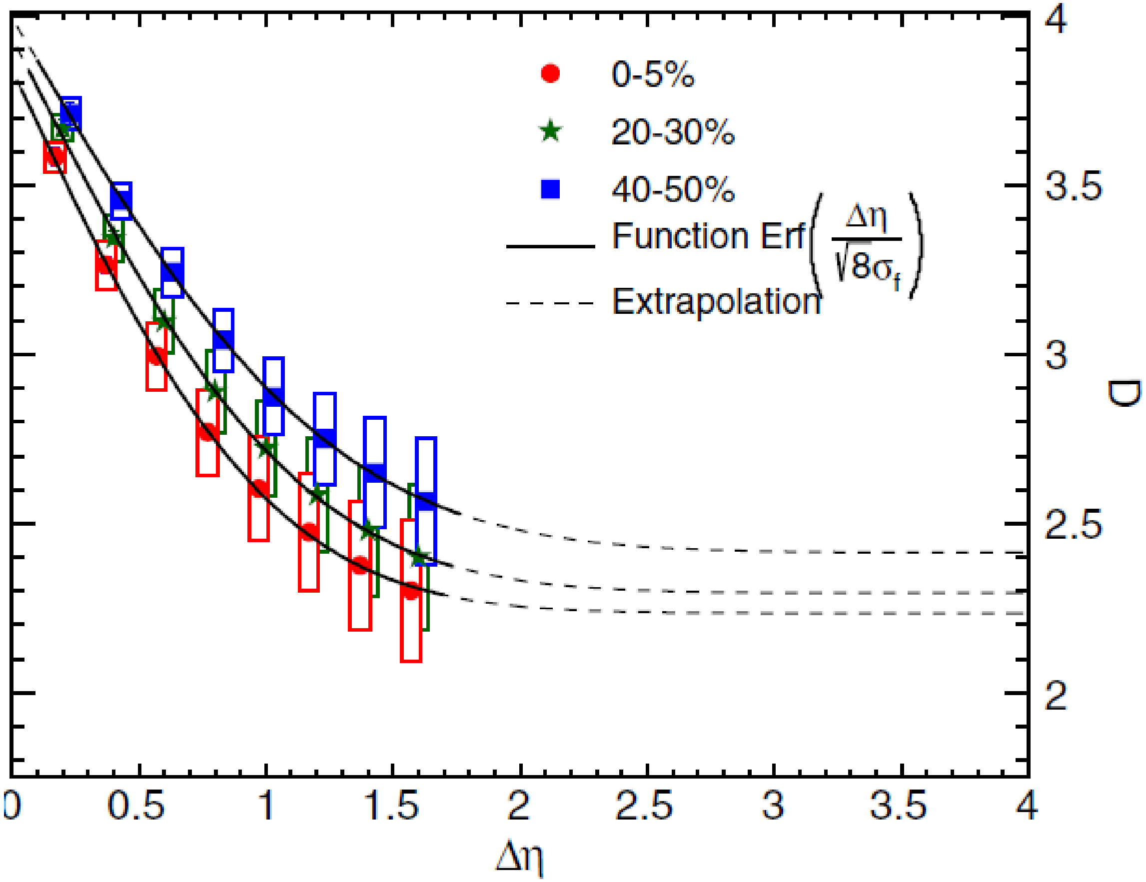
- D ~ 3-4 Hadronic
- D ~ 1-1.5 Quark

significant suppression from hadronic value at LHC energy!

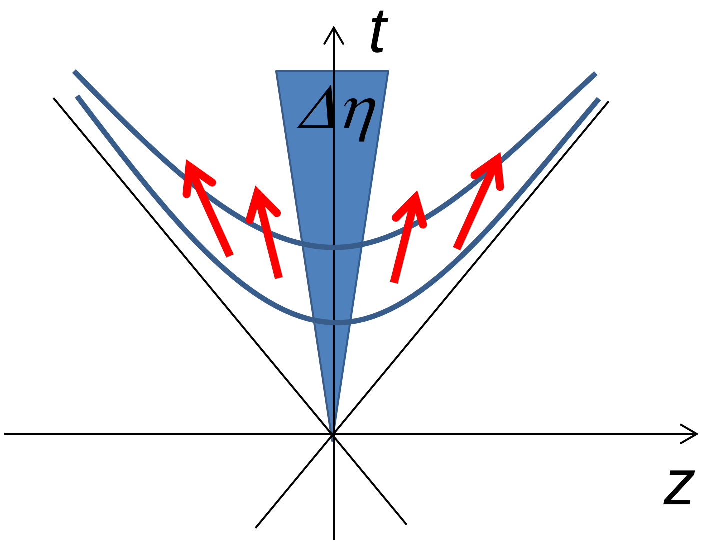
$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Δη Dependence @ ALICE

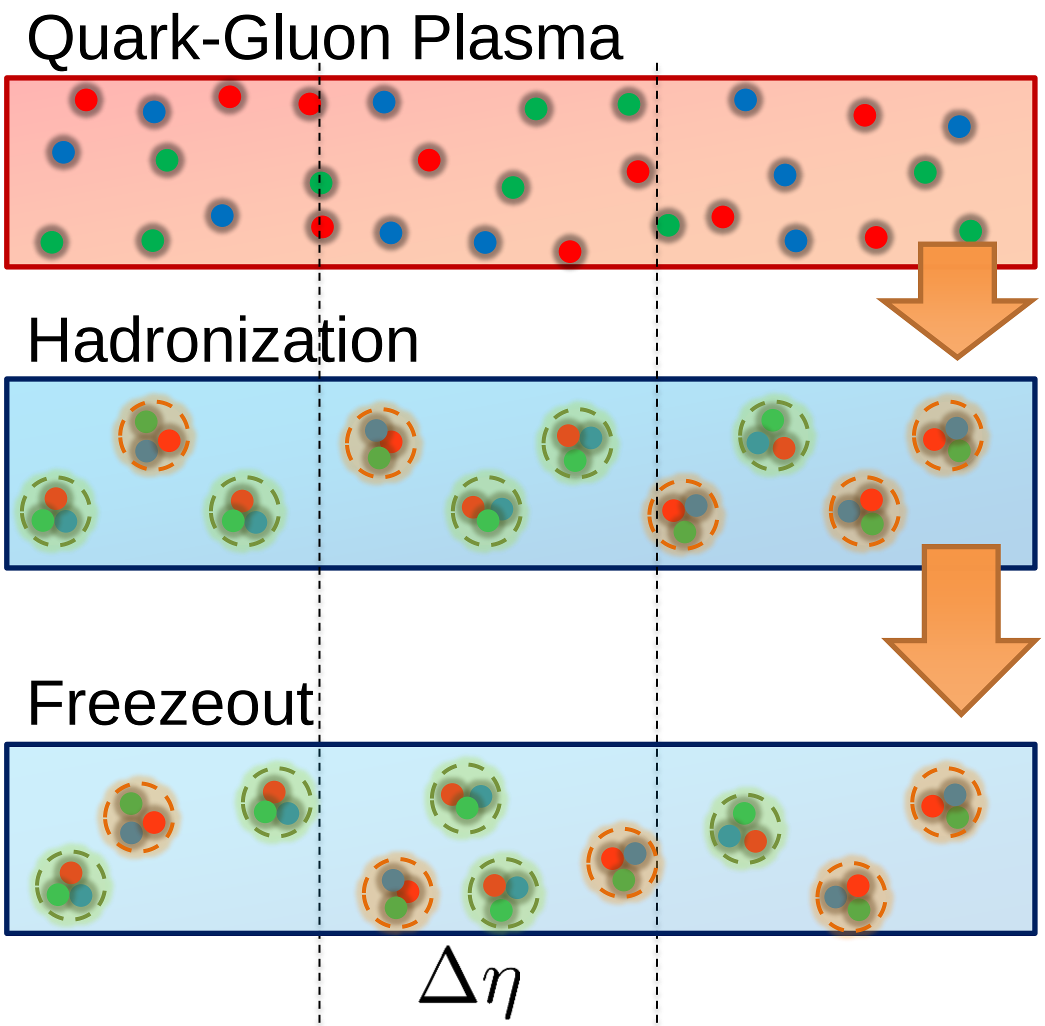
ALICE
PRL 2013

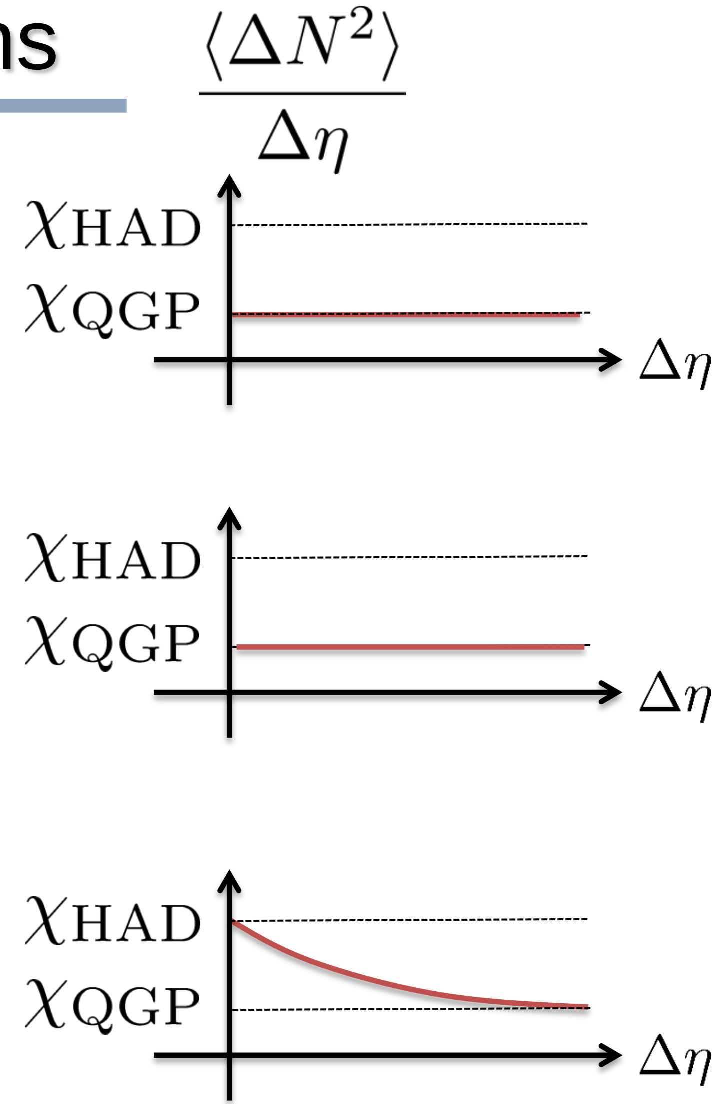


rapidity window



Time Evolution of Fluctuations

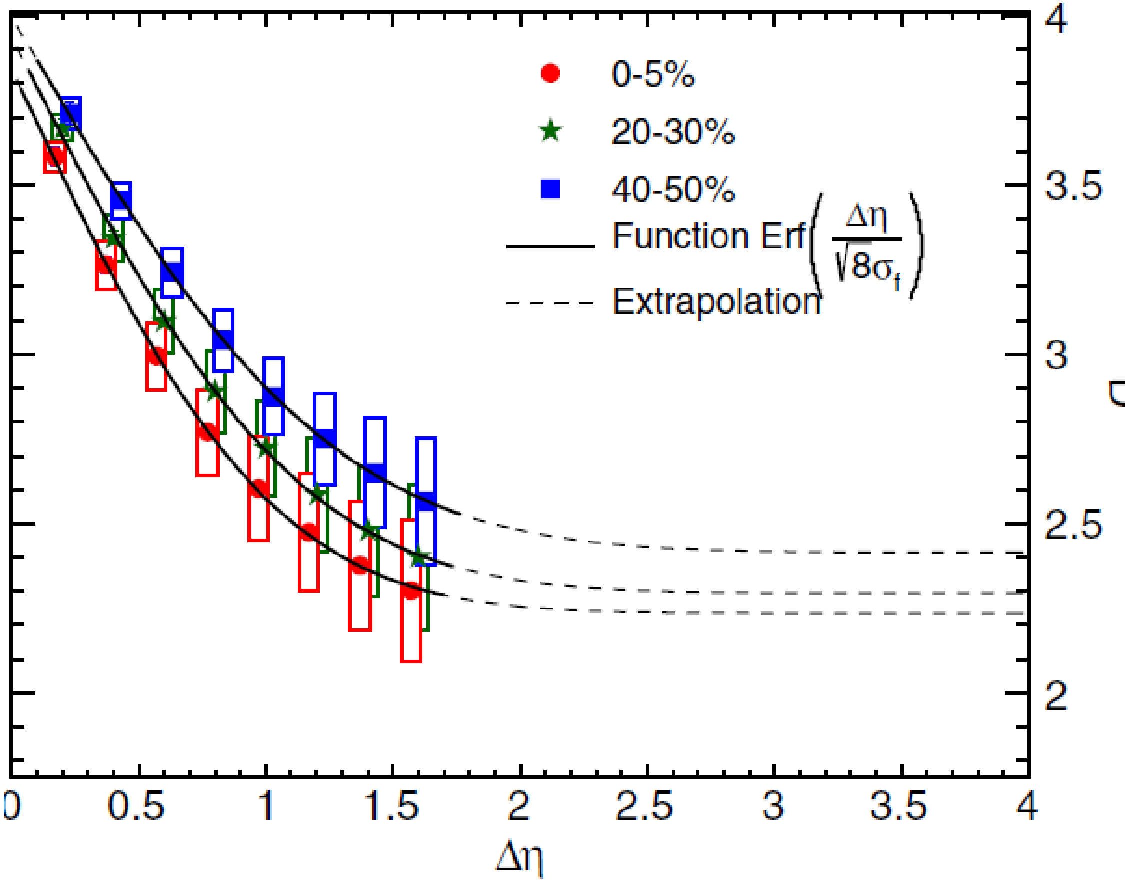


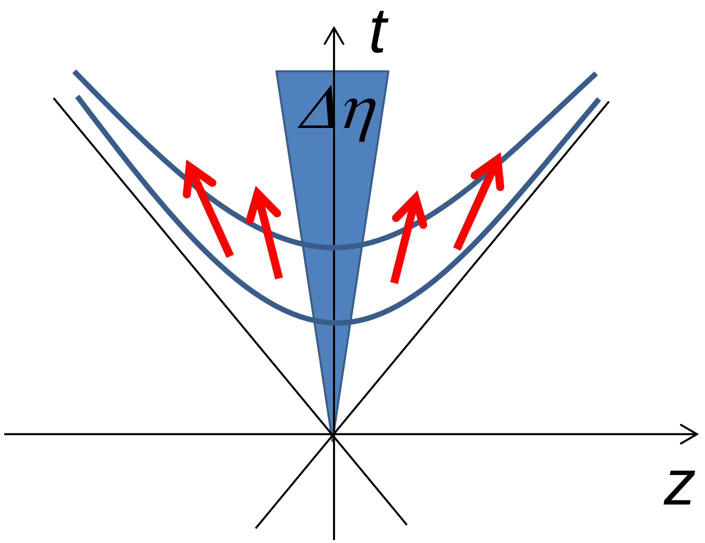


Variation of a conserved charge is achieved only through diffusion. ⇨ The larger $\Delta\eta$, the slower diffusion

Δη Dependence @ ALICE

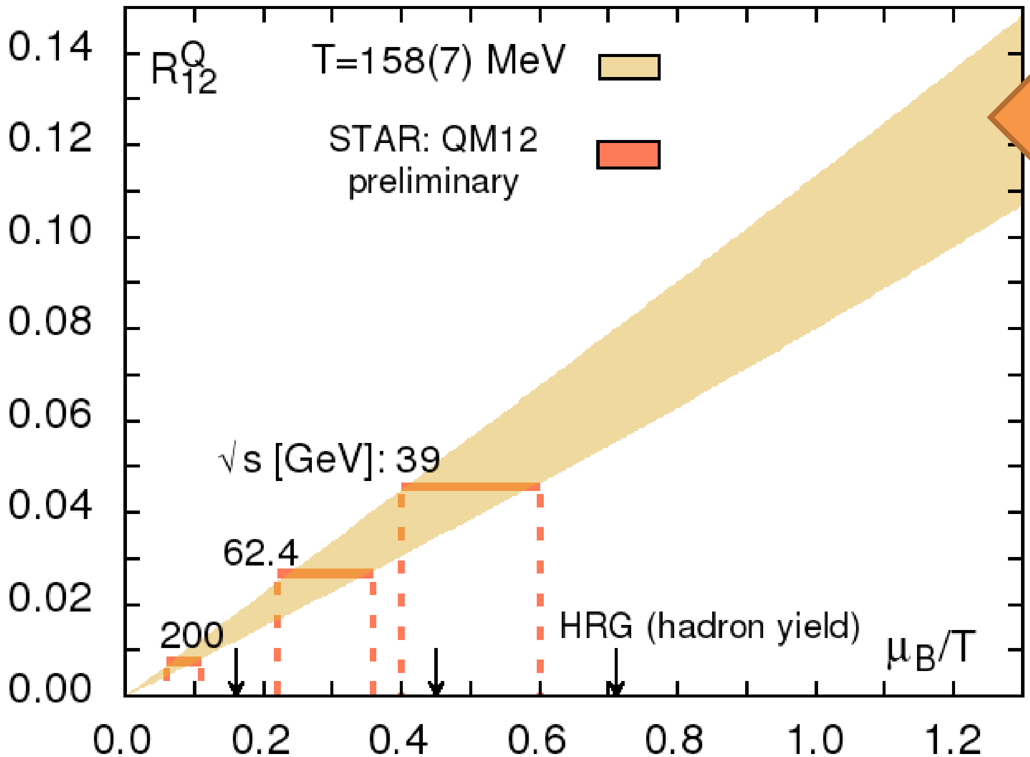
ALICE
PRL 2013





Δη dependences of fluctuation observables encode history of the hot medium!

Cumulants : HIC@RHIC vs Lattice



parameter window constrained by lattice

HotQCD, LATTICE2013

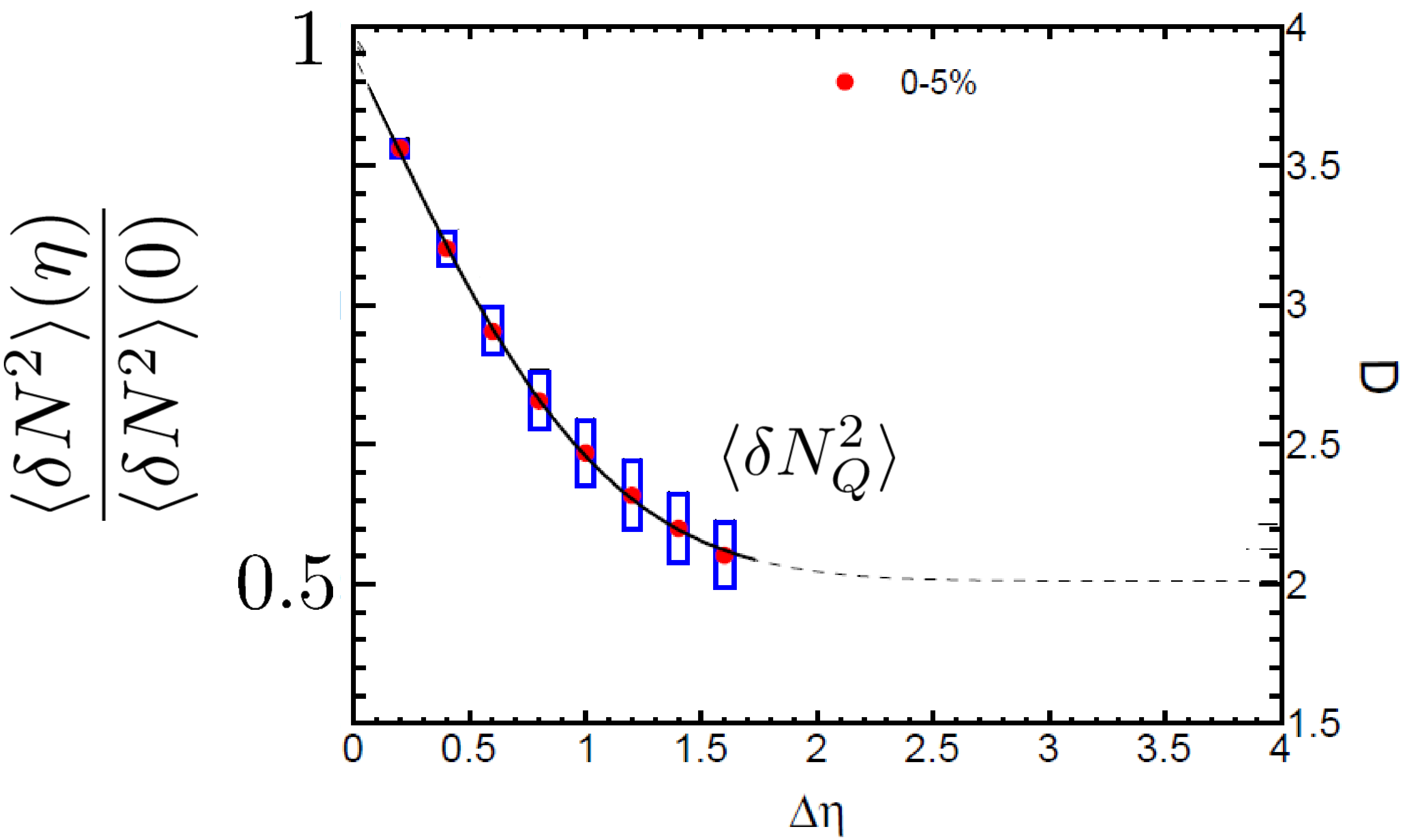
fluctuations "exp + lattice" ⟷ μ/T ⟷ particle abundance (chem. freezeout *T*)

discrepancy

$\langle\delta N_B^2\rangle$ and $\langle\delta N_p^2\rangle$ @ LHC ?

$\langle\delta N_Q^2\rangle, \langle\delta N_B^2\rangle, \langle\delta N_p^2\rangle$

should have different Δη dependence.



Baryon # cumulants are experimentally observable! MK, Asakawa, 2011;2012

$<\delta N_B^2>$ and $<\delta N_p^2>$ @ LHC ?

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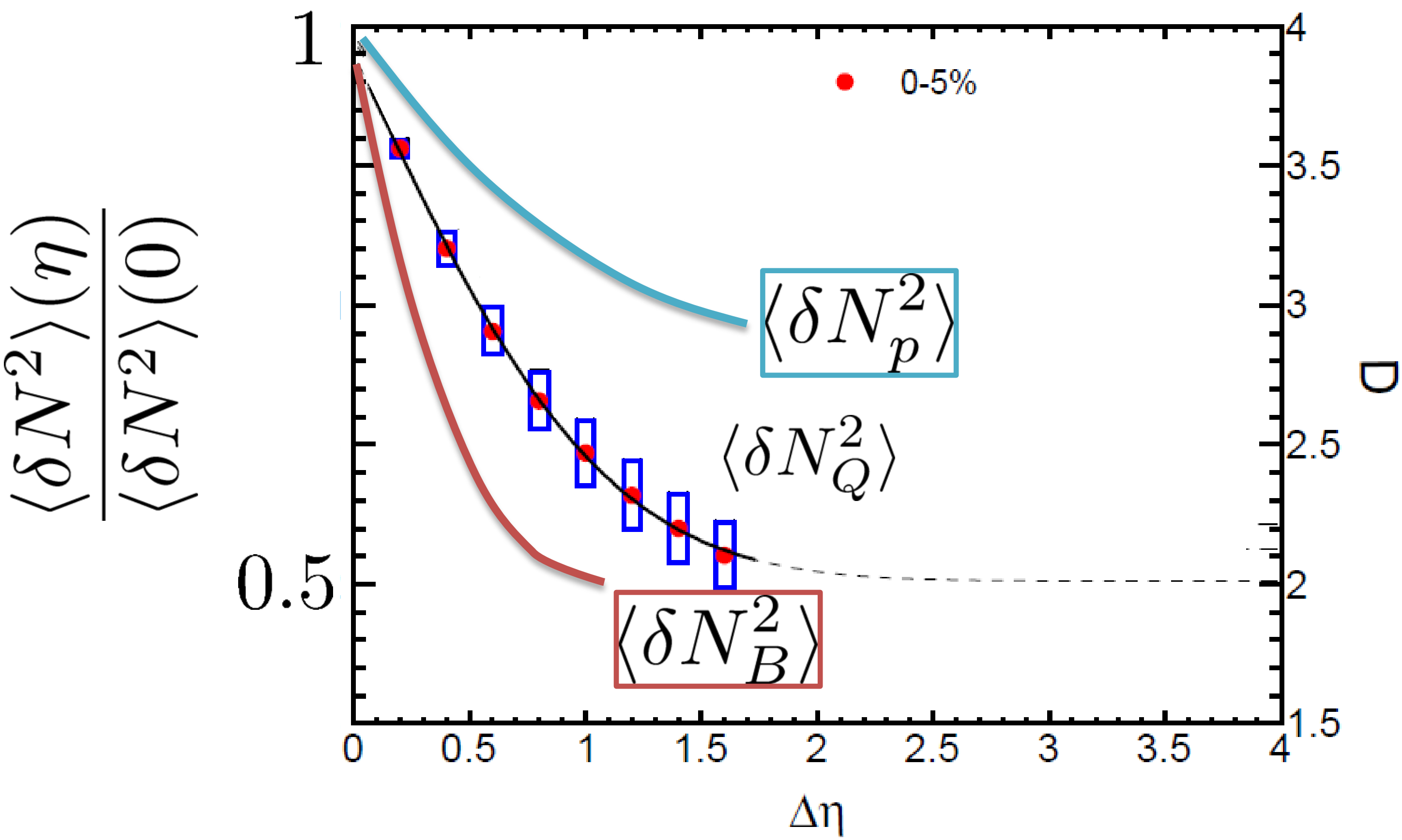

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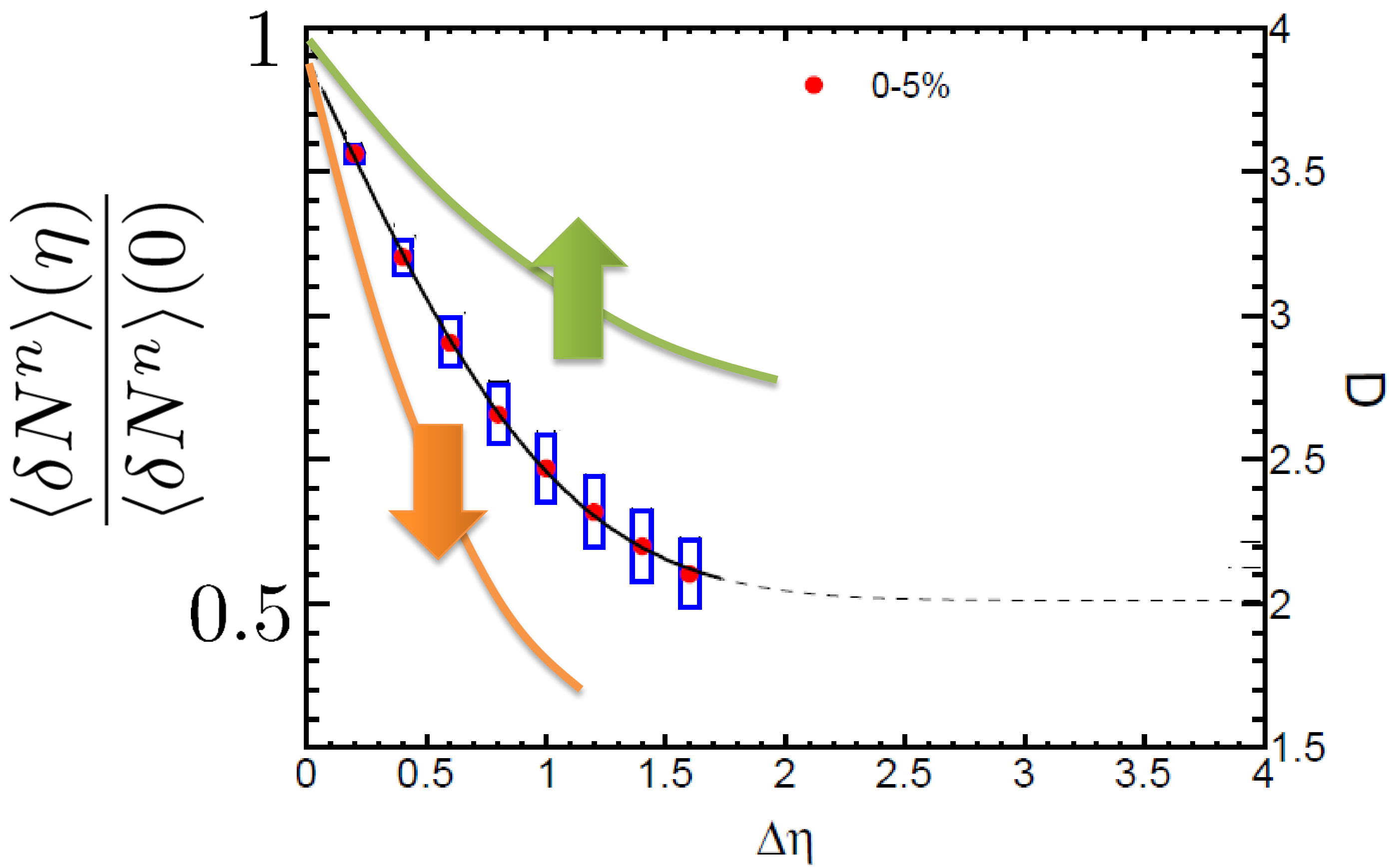
$\langle \delta N_Q^4 \rangle$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

suspression or enhancement

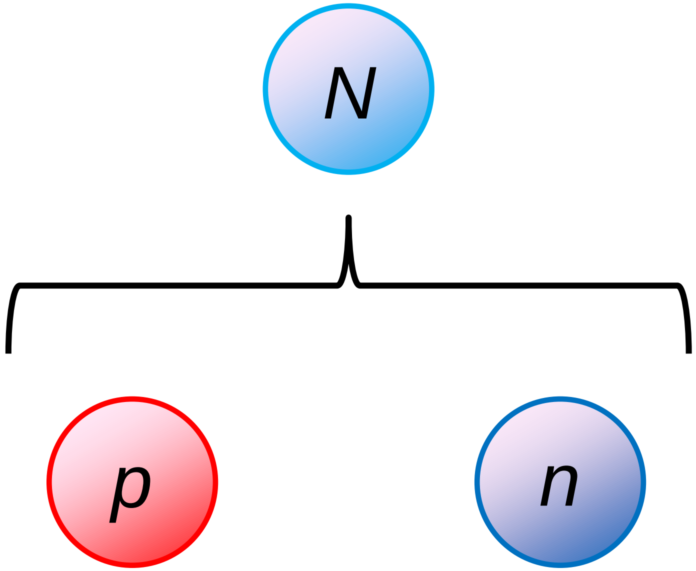


Baryon vs Proton Number Fluctuations

MK, Asakawa, PRC85,021901C(2012); PRC86, 024904(2012)

- $$\frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} \neq \frac{\langle \delta N_p^n \rangle_c}{\langle \delta N_p^m \rangle_c}$$
- $\langle \delta N_B^n \rangle_c$ are experimentally observable

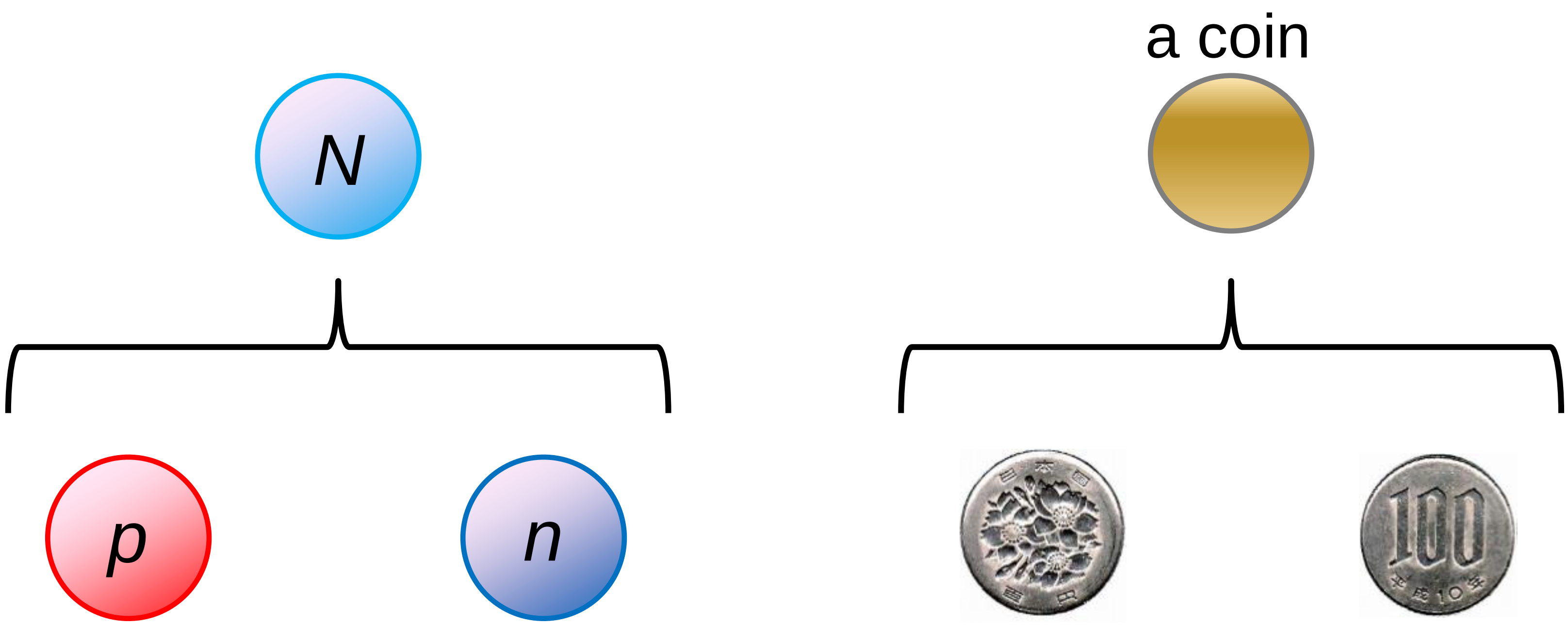
Nucleon Isospin as Two Sides of a Coin



Nucleons have
two isospin states.

MK, Asakawa,2012

Nucleon Isospin as Two Sides of a Coin

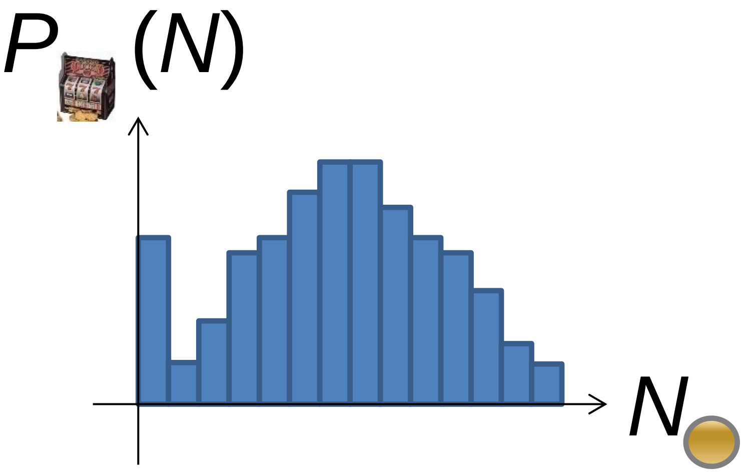


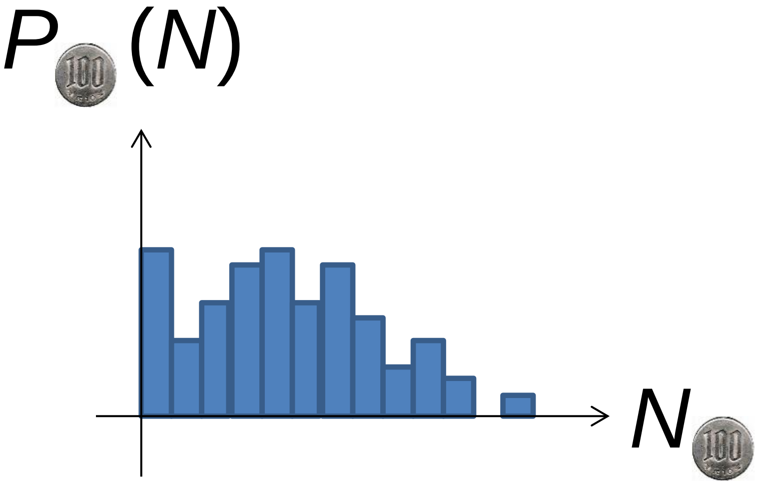
Nucleons have
two isospin states.

Coins have two sides.

MK, Asakawa,2012

Slot Machine Analogy

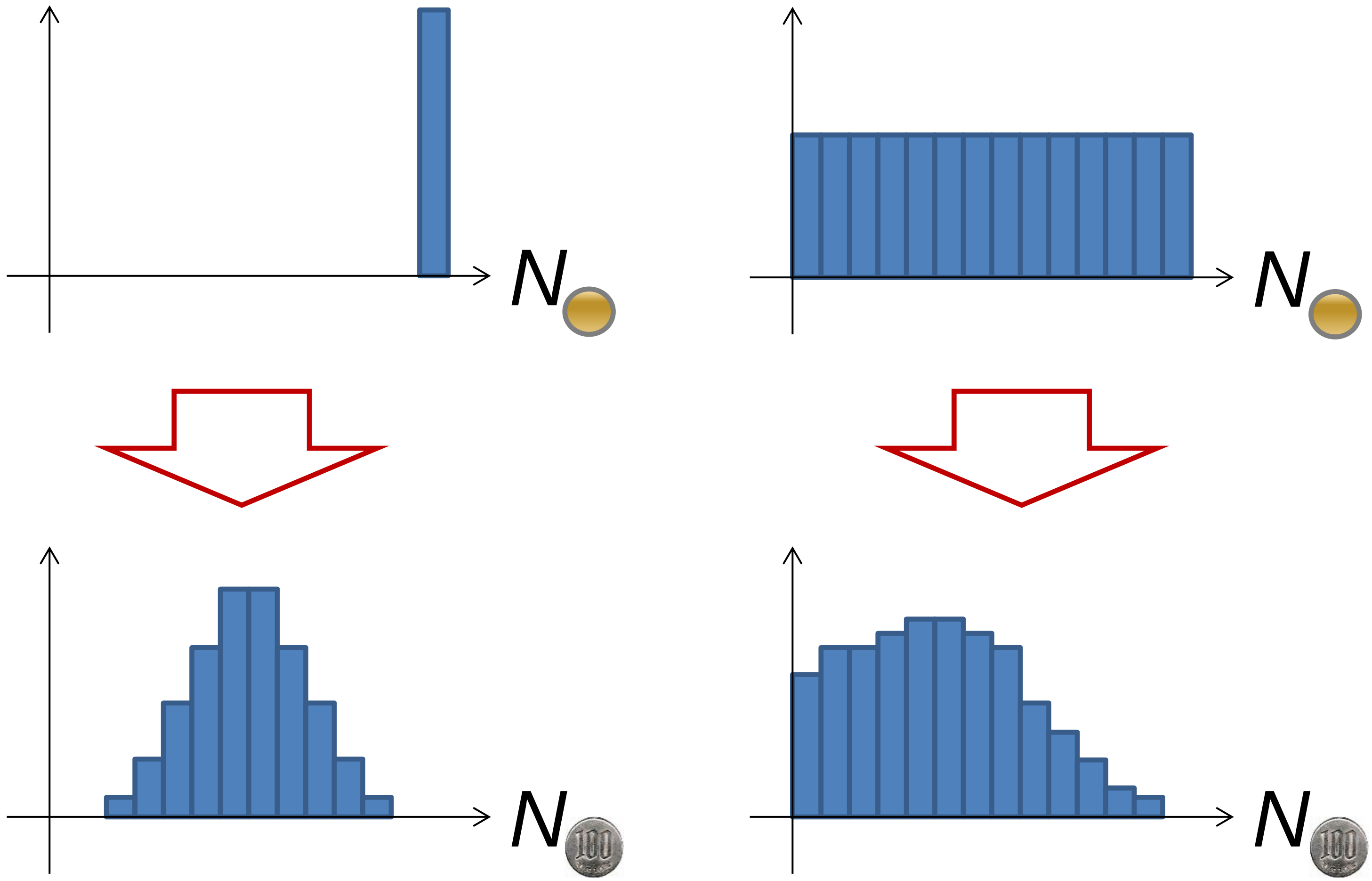




Extreme Examples

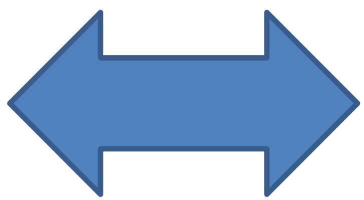
Fixed # of coins

Constant probabilities



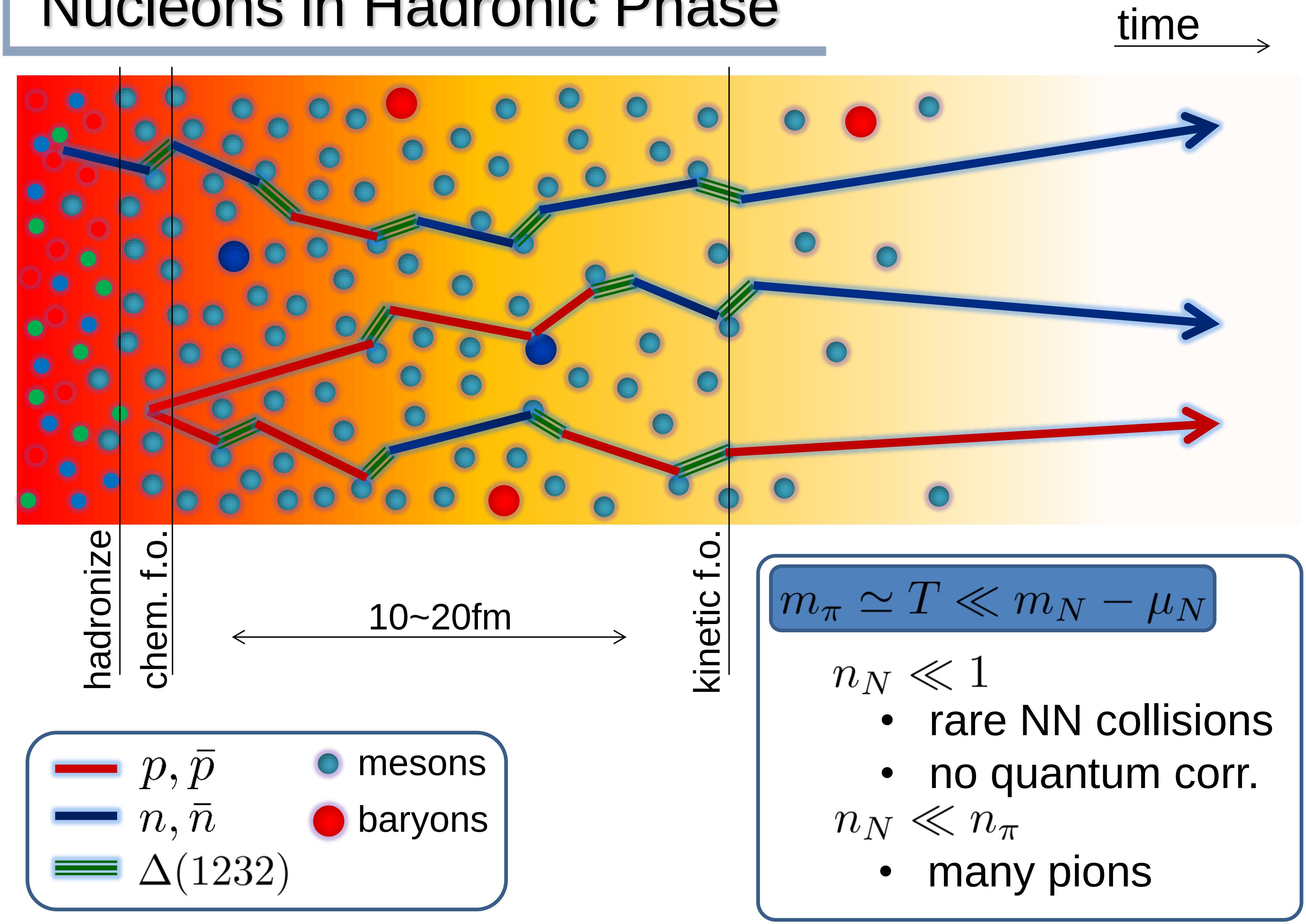
Reconstructing Total Coin Number

$$P_{100}(N_{100}) = \sum_{\bullet} P_{\text{slot}}(N_{\bullet}) B_{1/2}(N_{100}; N_{\bullet})$$



$$B_p(k;N) = p^k (1-p)^{N-k} \ {}_kC_N$$:binomial distr. func.

Nucleons in Hadronic Phase



$m_\pi \simeq T \ll m_N - \mu_N$

$n_N \ll 1$

- rare NN collisions
- no quantum corr.

$n_N \ll n_\pi$

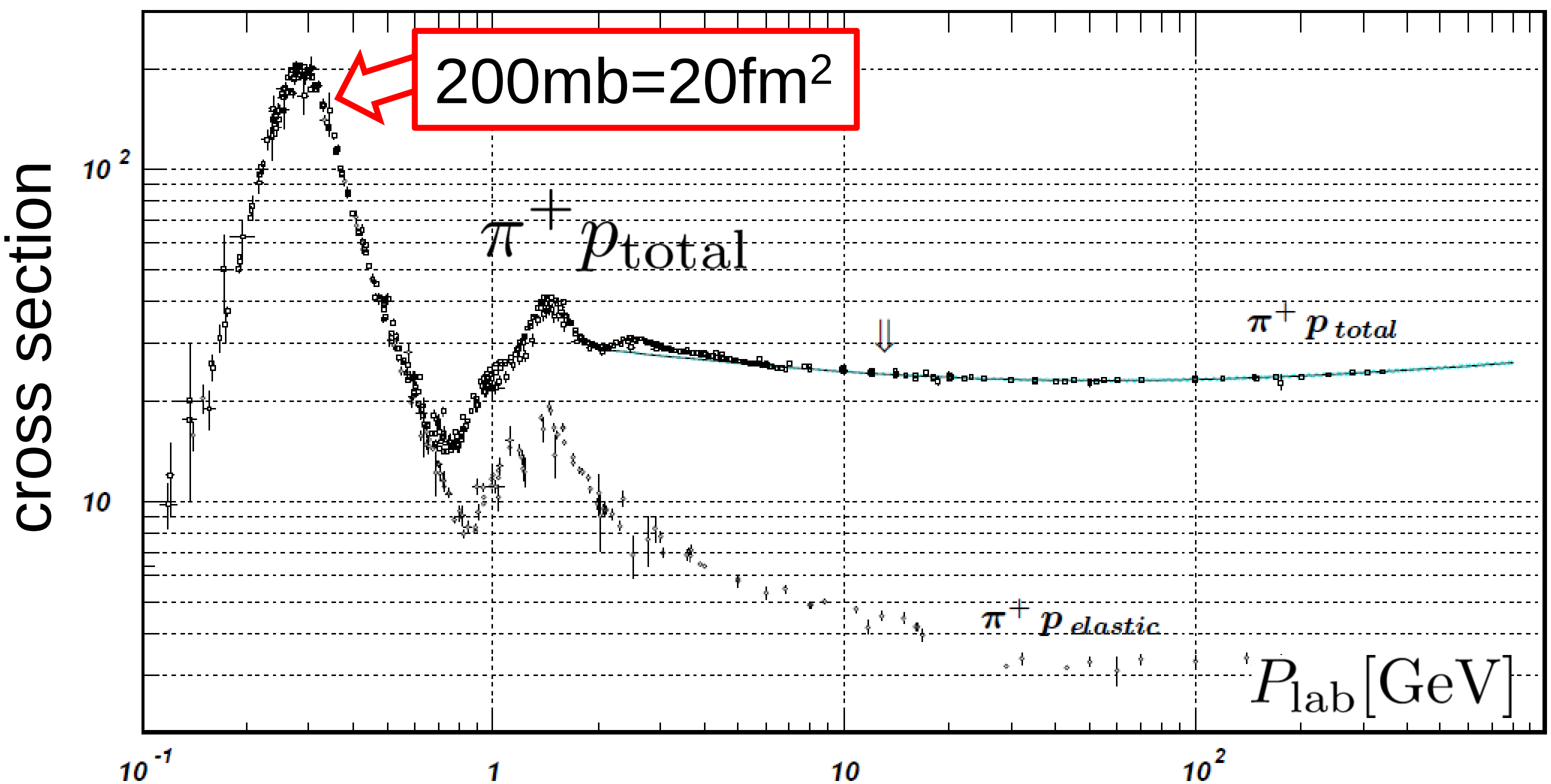
- many pions

Nucleon Isospin in Hadronic Medium

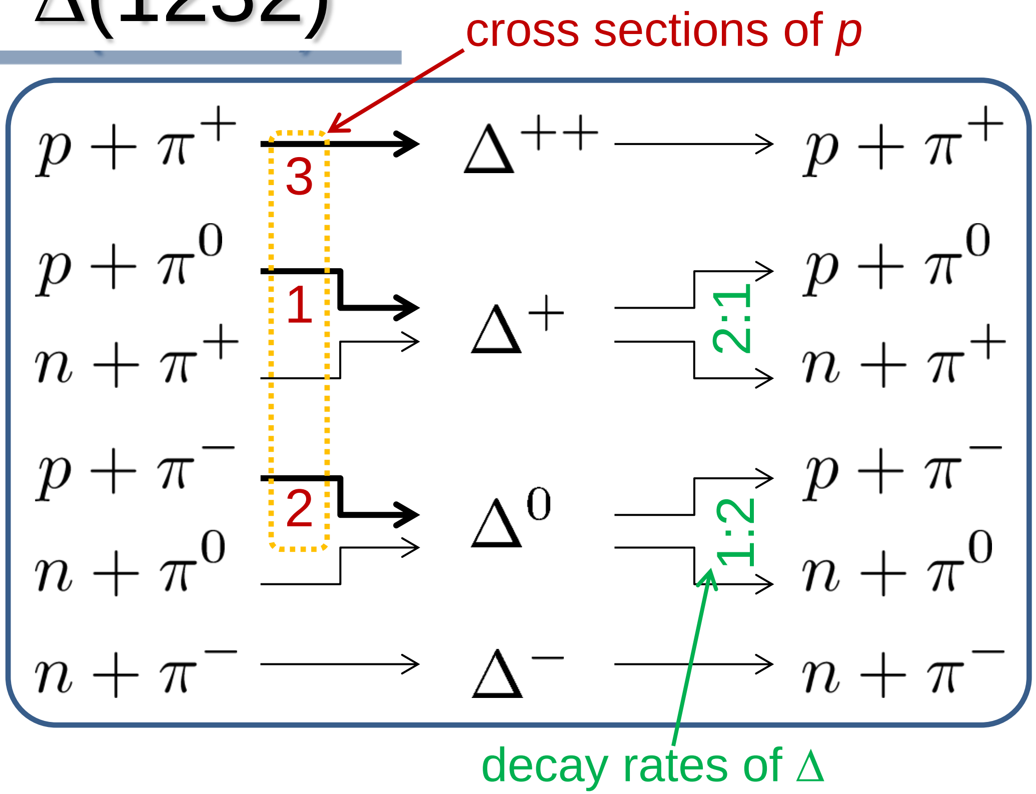
- Isospin of baryons can vary after chemical freezeout via charge exchange reactions mediated by Δ(1232):

$$p, n \rightarrow \Delta(1232) \rightarrow p, n$$
$$\pi \rightarrow \qquad\qquad \rightarrow \pi$$

$$I = 3/2$$
$$\Gamma \simeq 1.8\ [\mathrm{fm}]$$

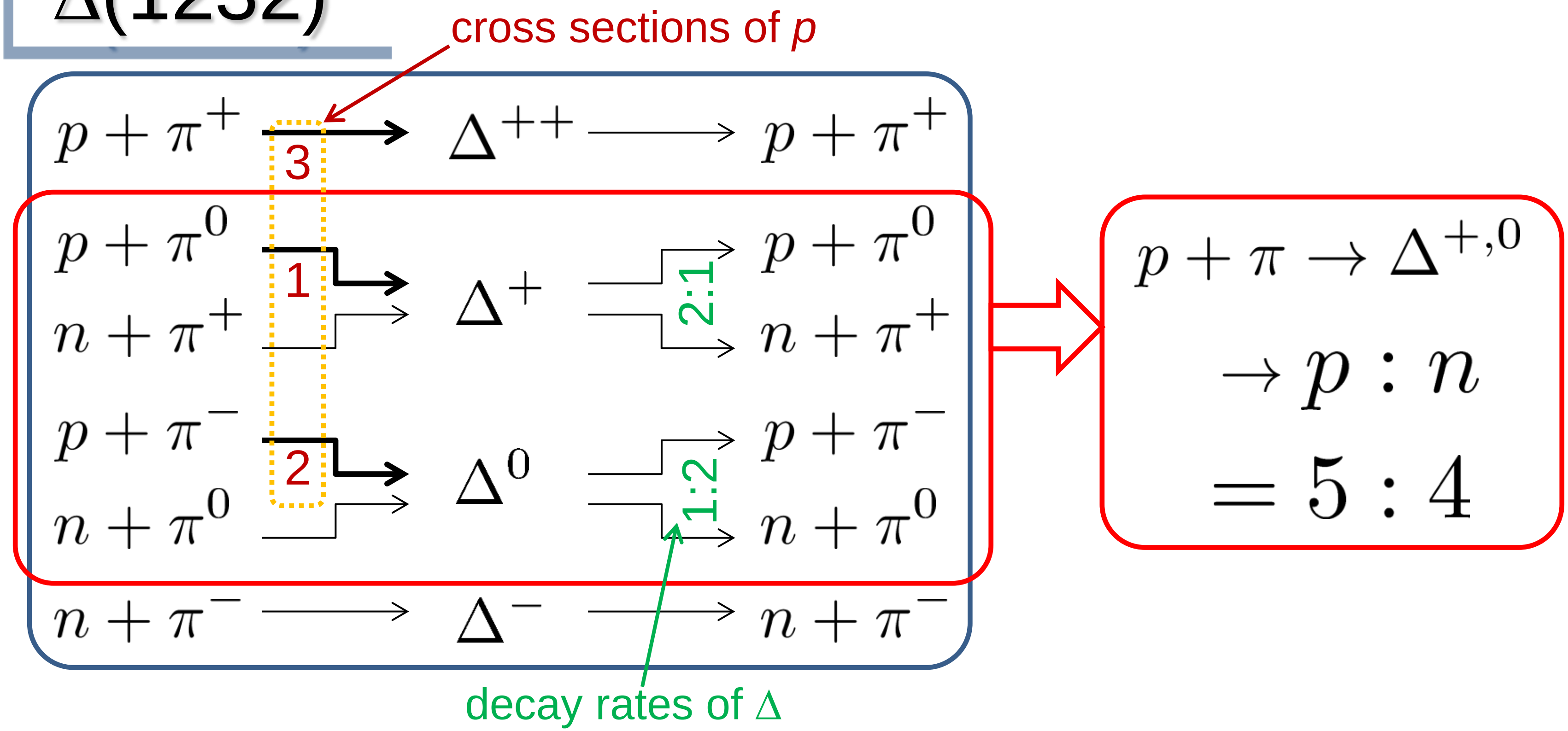


Δ(1232)

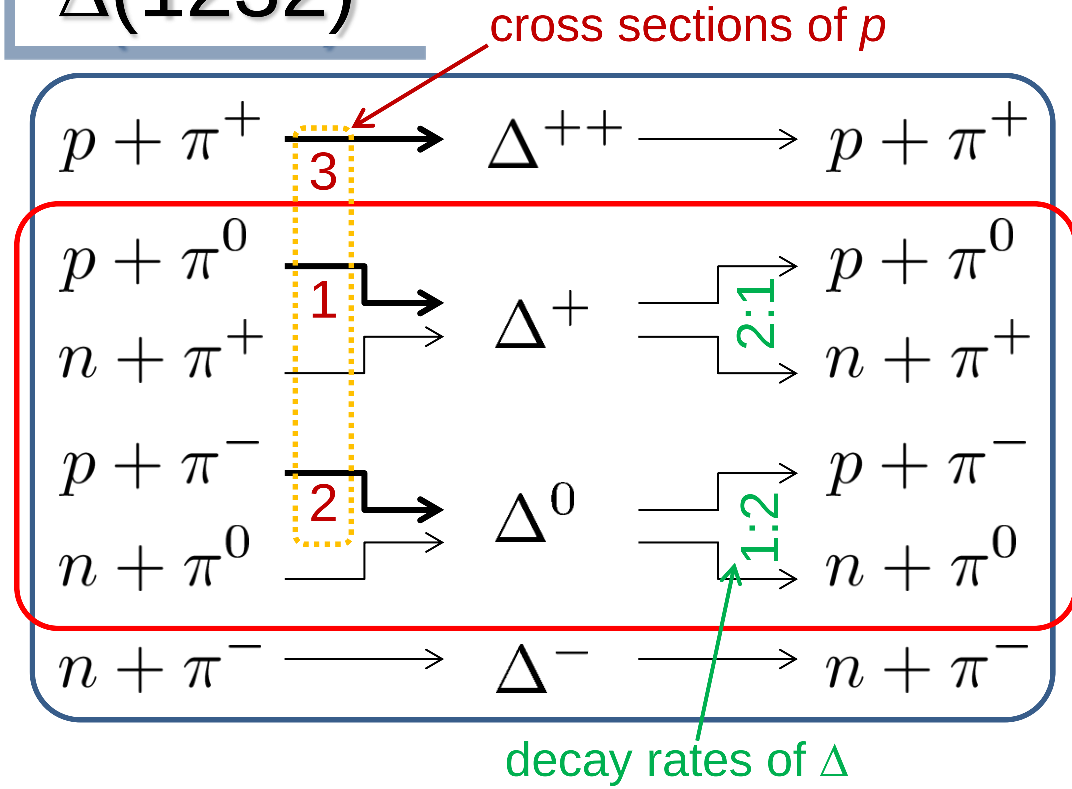


cross sections of p
$p+\pi^+$
Δ^{++}
$p+\pi^+$
3
$p+\pi^0$
1
Δ^+
$p+\pi^0$
$n+\pi^+$
2:1
$n+\pi^+$
$p+\pi^-$
2
Δ^0
$p+\pi^-$
$n+\pi^0$
1:2
$n+\pi^0$
$n+\pi^-$
Δ^-
$n+\pi^-$
decay rates of Δ

Δ(1232)


cross sections of p
$p+\pi^+$
3
Δ^{++}
$p+\pi^+$
$p+\pi^0$
1
$n+\pi^+$
Δ^+
2:1
$p+\pi^0$
$n+\pi^+$
$p+\pi^-$
2
$n+\pi^0$
Δ^0
1:2
$p+\pi^-$
$n+\pi^0$
$n+\pi^-$
Δ^-
$n+\pi^-$
decay rates of Δ
$p+\pi \to \Delta^{+,0}$
$\to p:n$
$= 5:4$

Δ(1232)

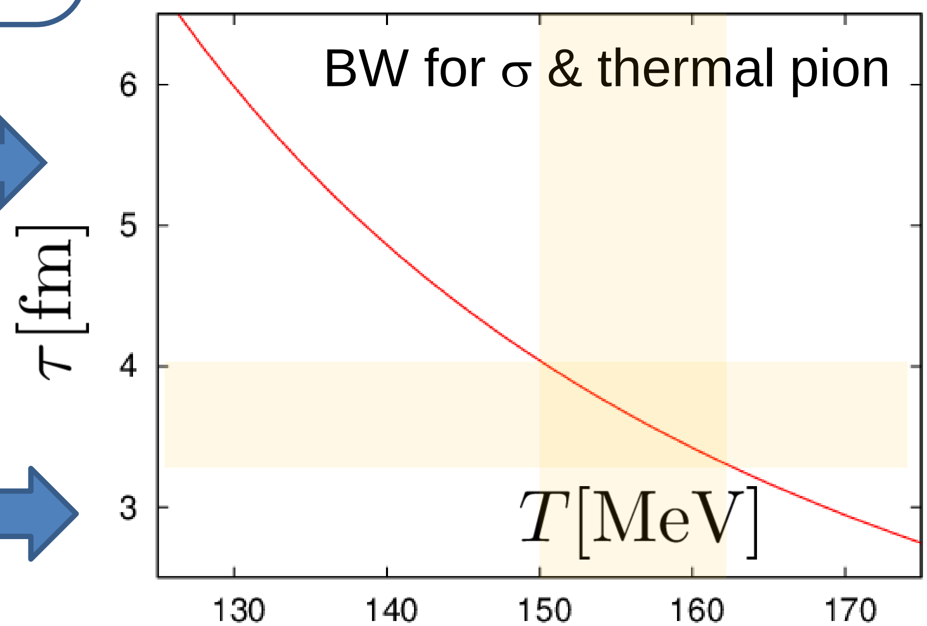


$$p + \pi \to \Delta^{+,0}$$
$$\to p : n$$
$$= 5 : 4$$

Lifetime to create Δ^+ or Δ^0

$$\tau^{-1} = \int \frac{d^3 k_\pi}{(2\pi)^3} \sigma(E_{\rm cm}) v_\pi n(E_\pi)$$

$$(\text{freezeout time}) \simeq 20[\text{fm}]$$



Difference btw Baryon and Proton Numbers

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

$$\begin{cases} 2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\ 2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\ 2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots \end{cases}$$

genuine info. Poissonian noise

Difference from Poisson (thermal) distribution
is suppressed in proton number fluctuations.

Difference btw Baryon and Proton Numbers

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$$2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}}$$

$$\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots$$

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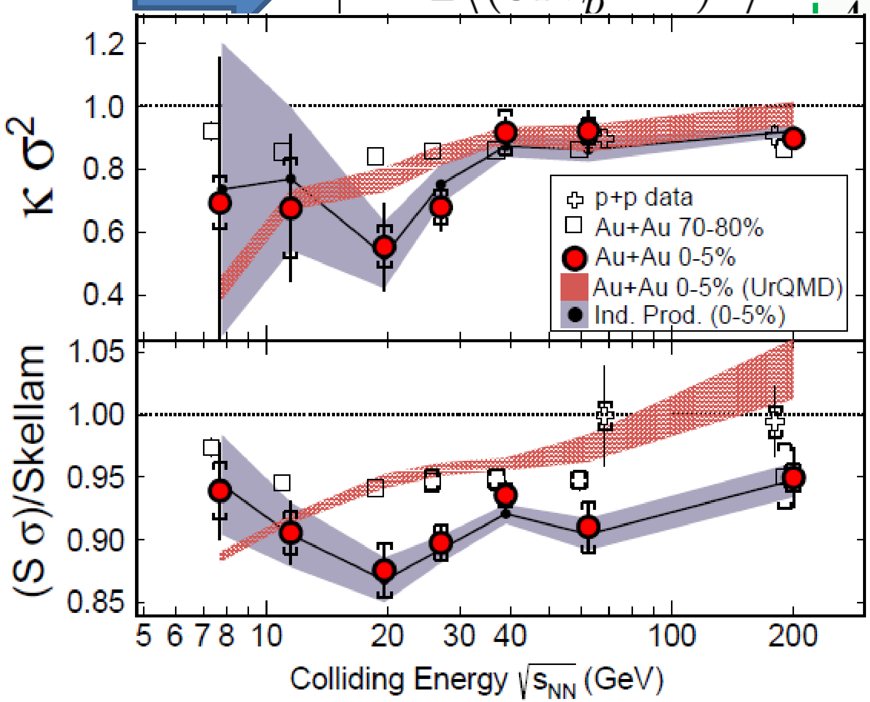


from Poisson (thermal) distribution

ssed in proton number fluctuations.

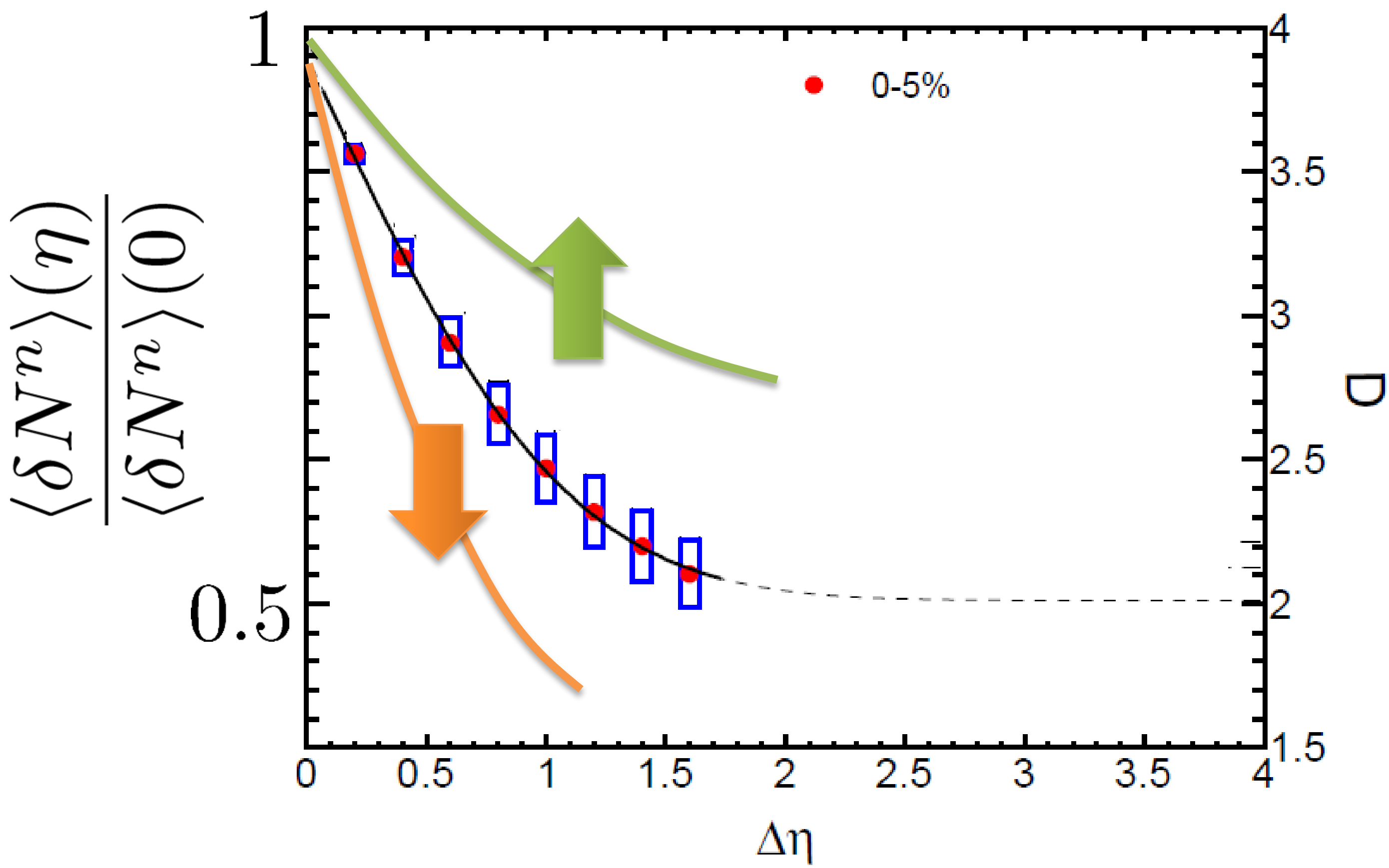
Time Evolution of Higher Order Cumulants

MK, Asakawa, Ono, arXiv:1307.2978

$<\delta N_Q{}^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

suppression or enhancement



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012
Stephanov, Shuryak, 2001

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Markov (white noise)
+
continuity

Gaussian noise

cf) Gardiner, "Stochastic Methods"

Fluctuation of *n* is
Gaussian in equilibrium

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n*

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

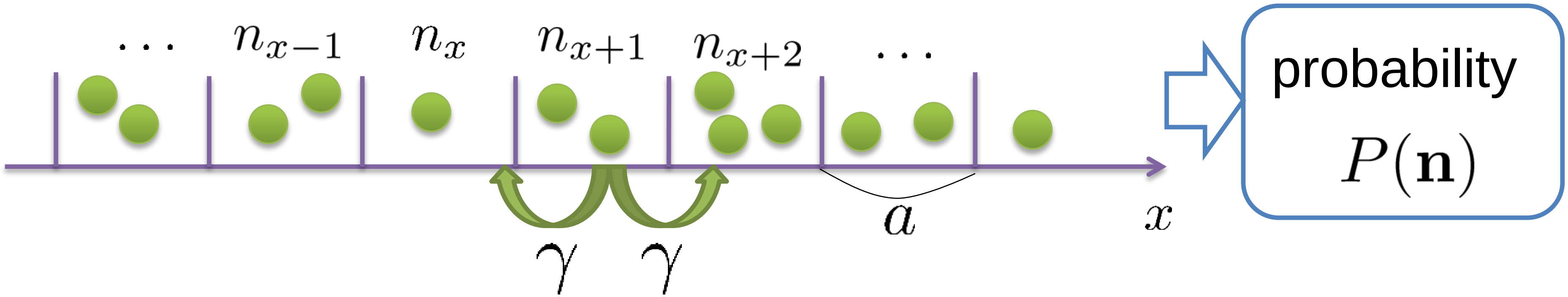
$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n* ← **our choice**

REMARK: Fluctuations measured in HIC are almost Poissonian.

Diffusion Master Equation

Divide spatial coordinate into discrete cells



Diffusion Master Equation

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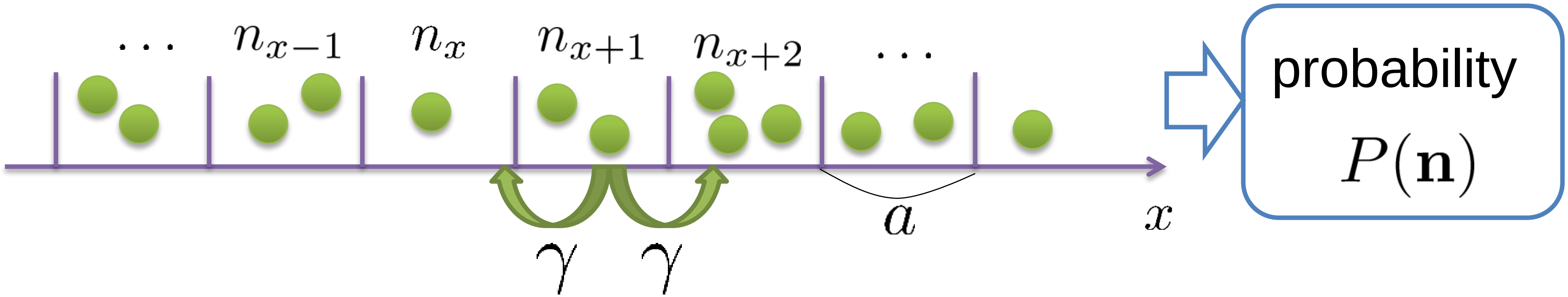


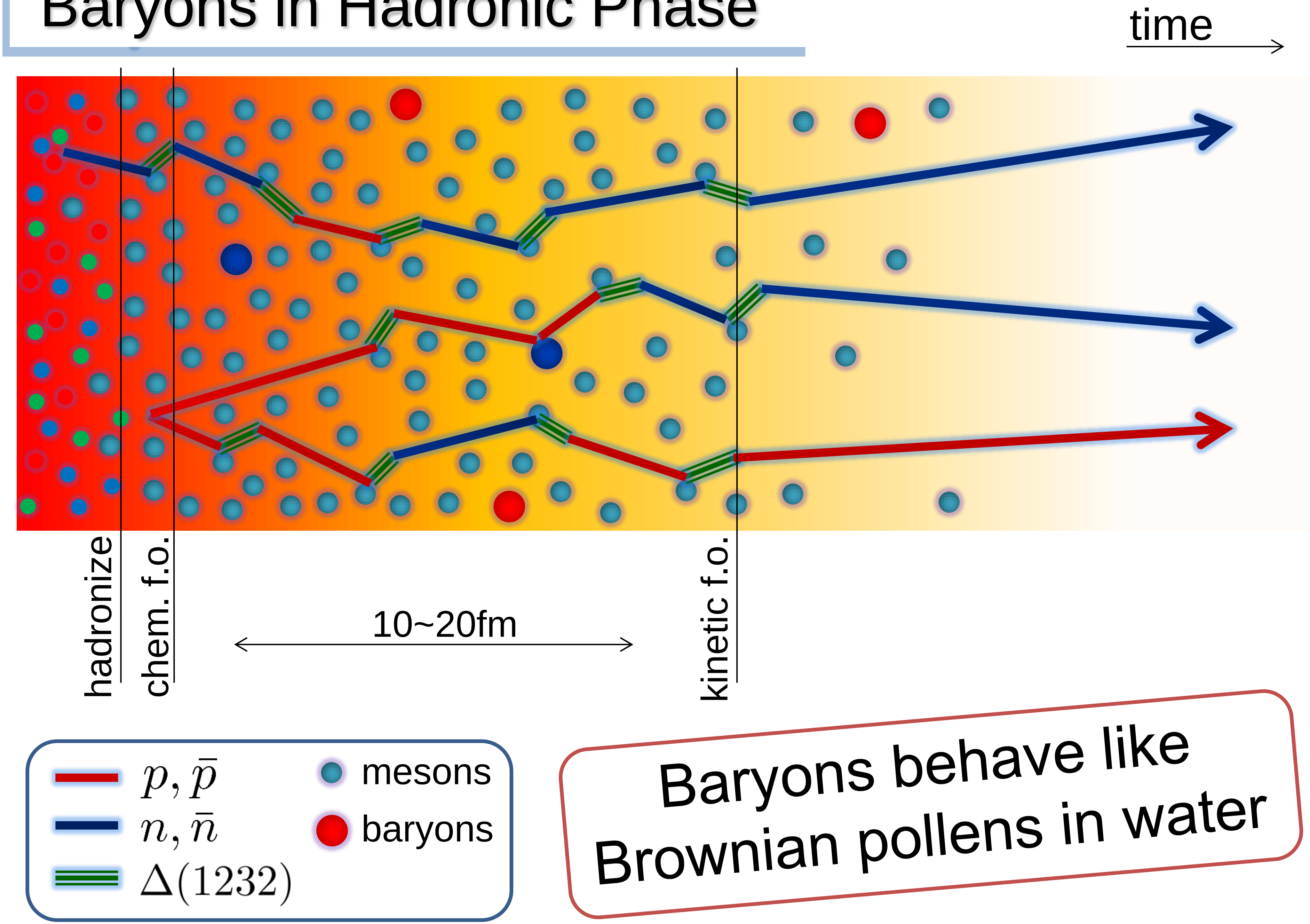
Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x + 1)\{P(\mathbf{n} + \mathbf{e}_x - \mathbf{e}_{x+1}) + P(\mathbf{n} + \mathbf{e}_x - \mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen’s system size expansion

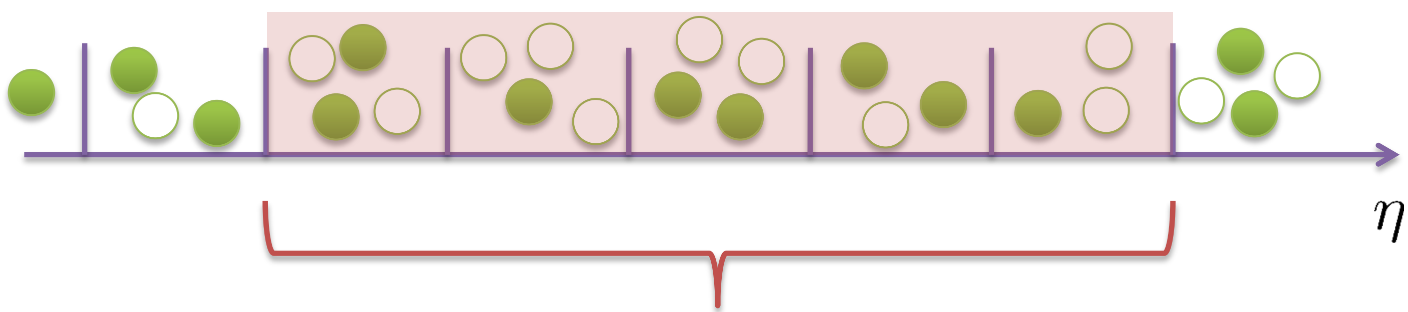
Baryons in Hadronic Phase



Baryons behave like Brownian pollens in water

Net Charge Number

Prepare 2 species of (non-interacting) particles



$$\bar{Q}(\tau) = \int_0^{\Delta\eta} d\eta \, (n_1(\eta,\tau) - n_2(\eta,\tau))$$

Let us investigate

$\langle \bar{Q}^2 \rangle_c$ $\langle \bar{Q}^4 \rangle_c$ at freezeout time t

Solution of DME in a→0 Limit

1st order (deterministic) $\langle n \rangle$

- consistent with diffusion equation with $D=\gamma a^2$

  ➡ Continuum limit with fixed $D=\gamma a^2$

2nd order $\langle \delta n^2 \rangle$

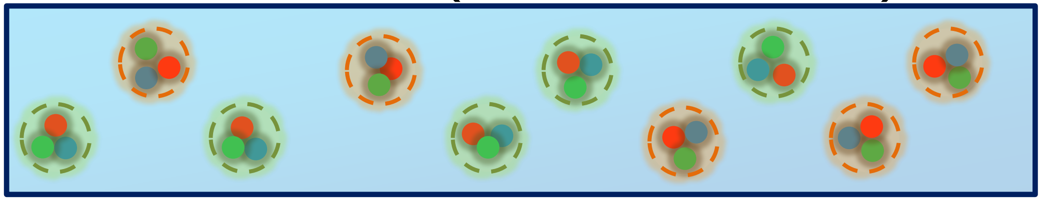
- consistent with stochastic diffusion eq.
  (for sufficiently smooth initial conditions)

Shuryak, Stephanov, 2001

Nontrivial results for non-Gaussian fluctuations

Time Evolution in Hadronic Phase

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ | $\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ | $\langle Q^2_{(\mathrm{tot})} \rangle_c$

suppression owing to local charge conservation

strongly dependent on hadronization mechanism

Time Evolution in Hadronic Phase

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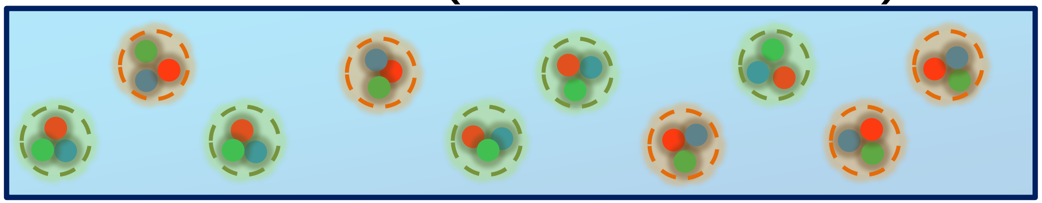


Time evolution via DME

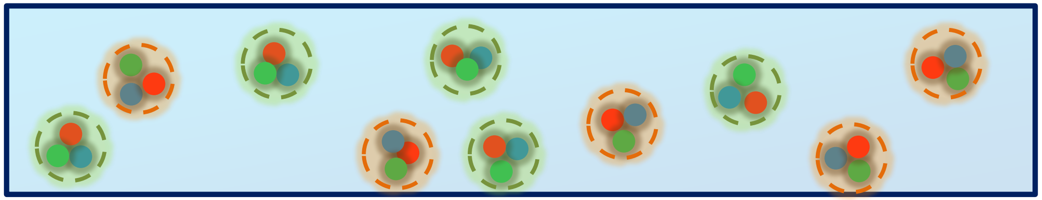
- Boost invariance / infinitely long system
- Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ — suppression owing to local charge conservation

$\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$

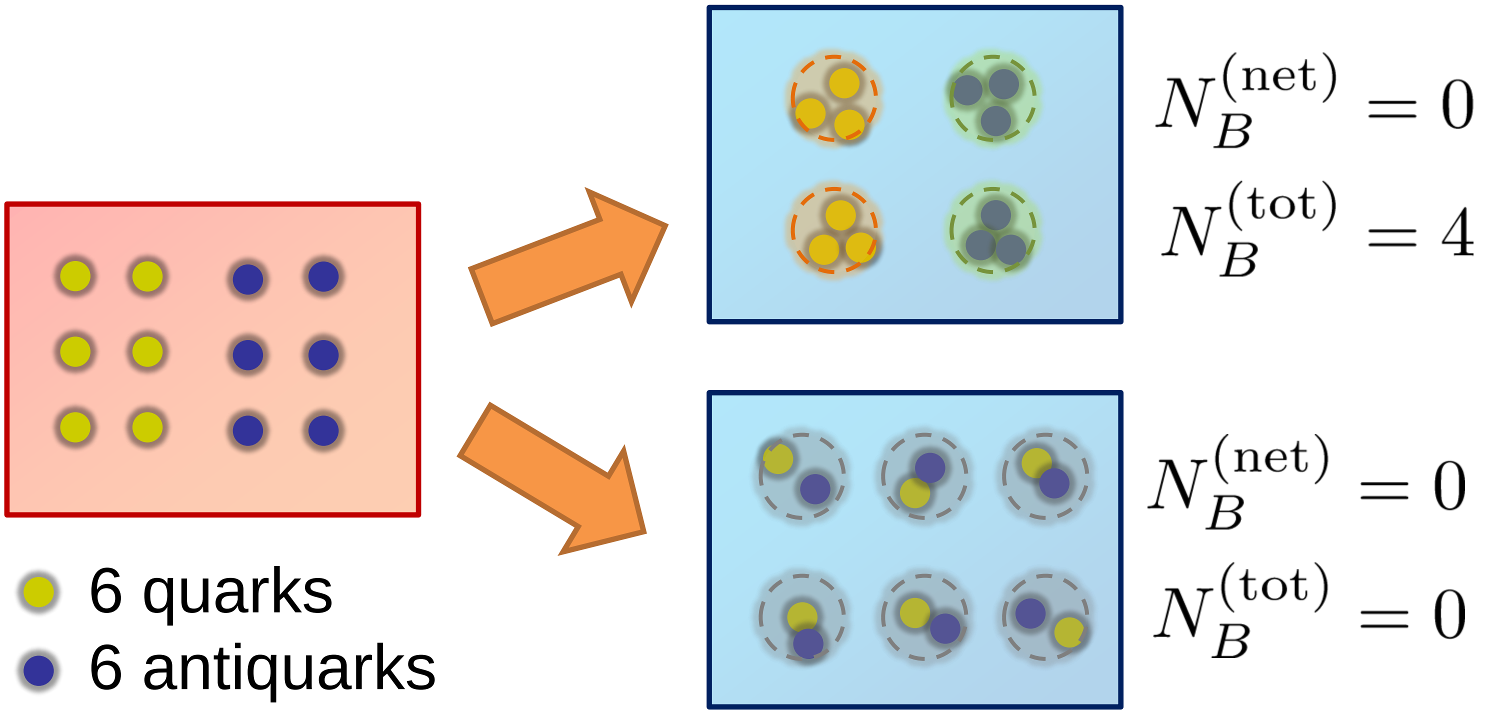
$\langle Q^2_{(\mathrm{tot})} \rangle_c$ — strongly dependent on hadronization mechanism

Freezeout



Total Charge Number

In recombination model,

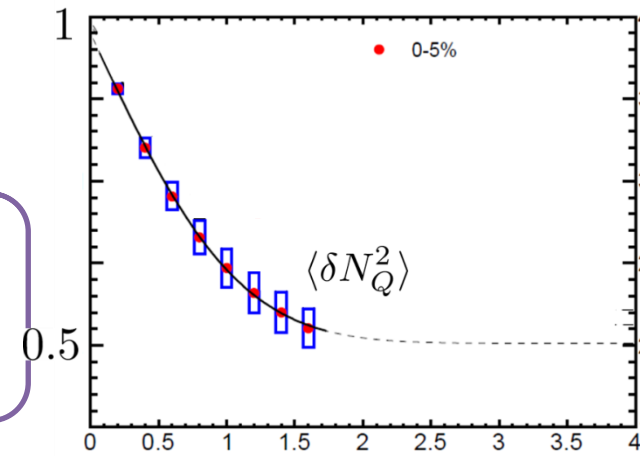


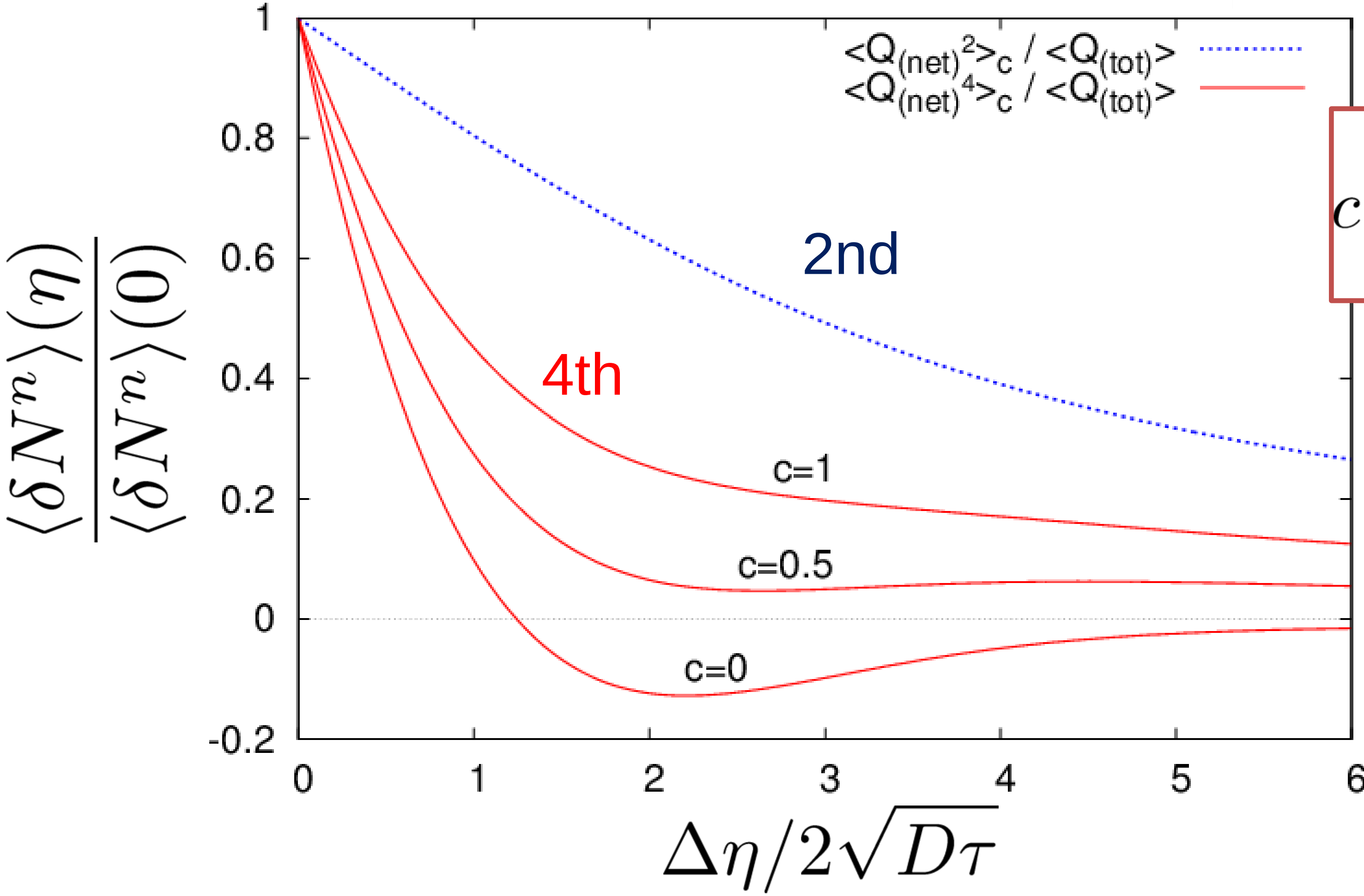
- $N_B^{(\mathrm{tot})}$ can fluctuate, while $N_B^{(\mathrm{net})}$ does not.

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$





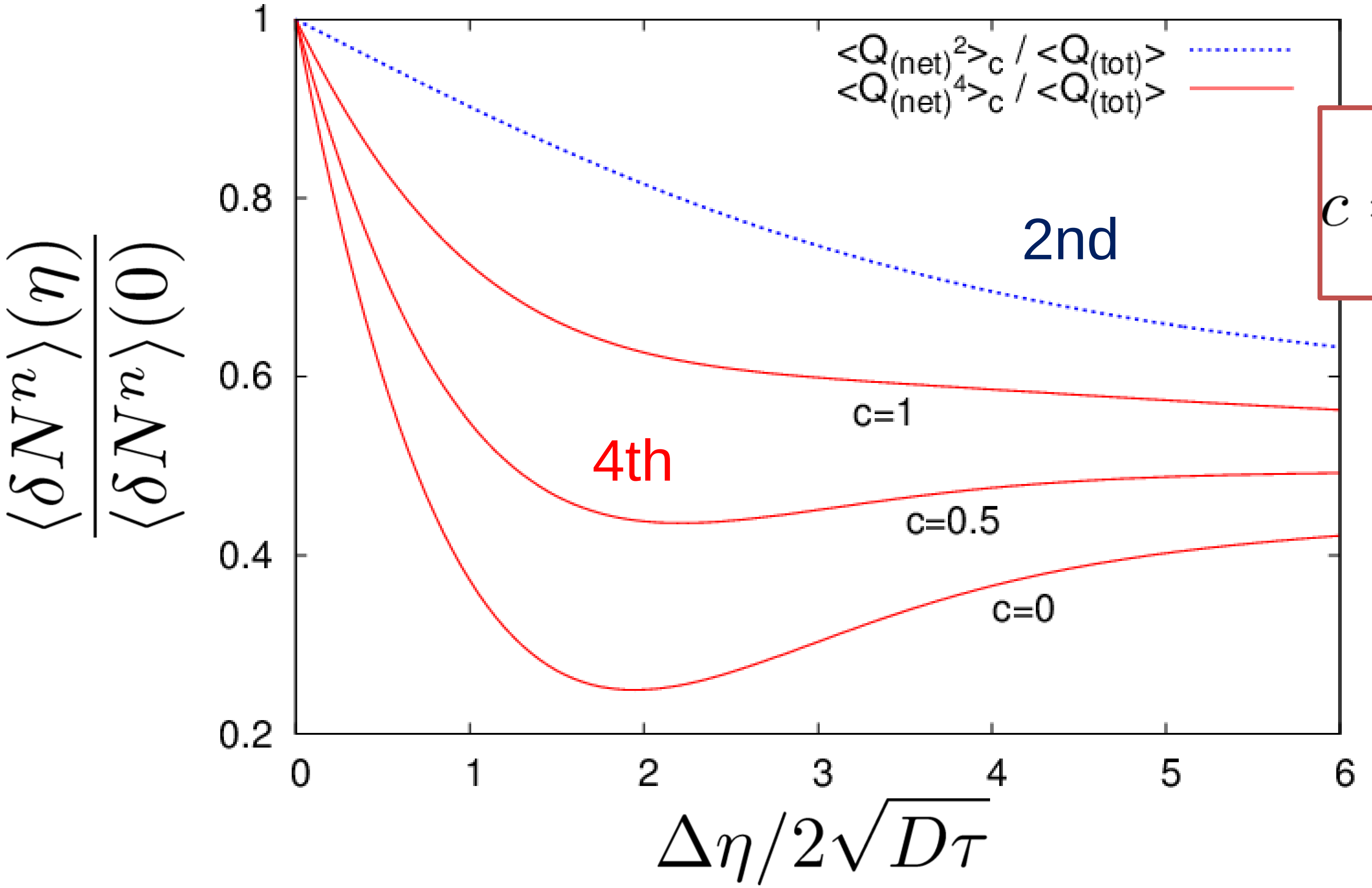
$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0.5 \langle Q_{(\mathrm{tot})} \rangle$$

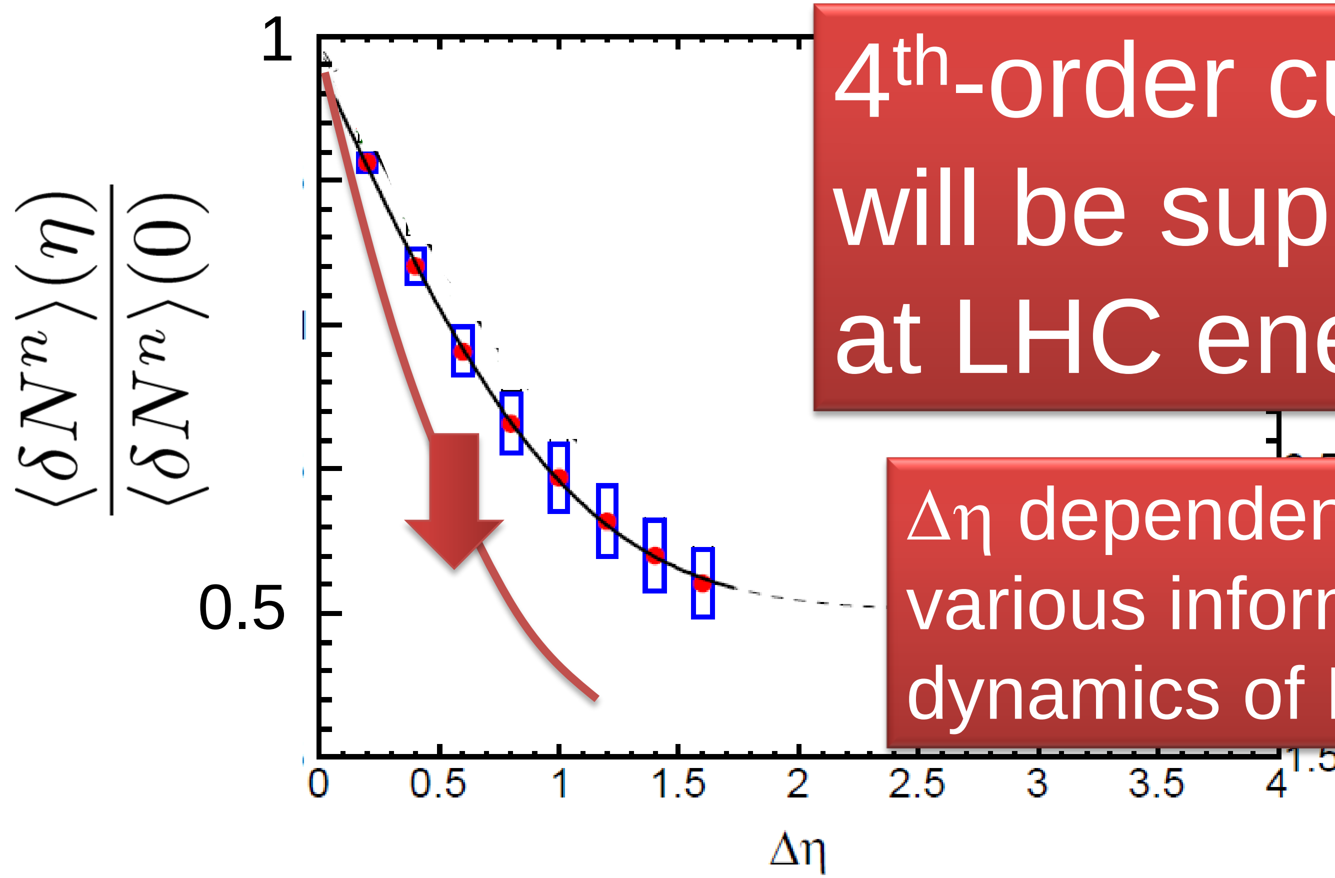


$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

$<\delta N_Q{}^4>$ @ LHC

Assumptions
- boost invariant system
- small fluctuations of CC at hadronization
- short correlation in hadronic stage



$\frac{\langle \delta N^n \rangle (\eta)}{\langle \delta N^n \rangle (0)}$

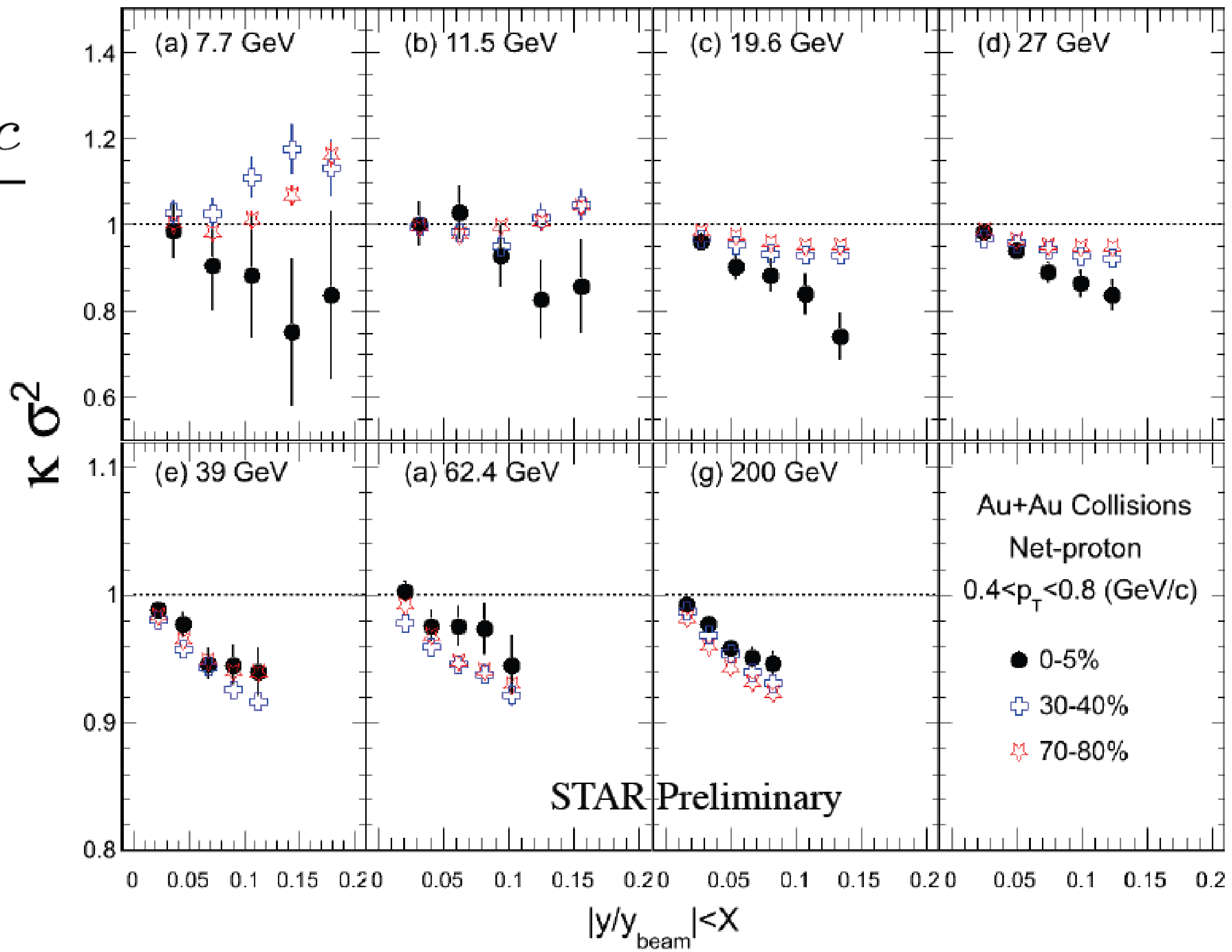
4^{th}-order cumulant will be suppressed at LHC energy!

Δη dependences encode various information on the dynamics of HIC!

Δη Dependence at STAR

STAR, QM2012

$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$

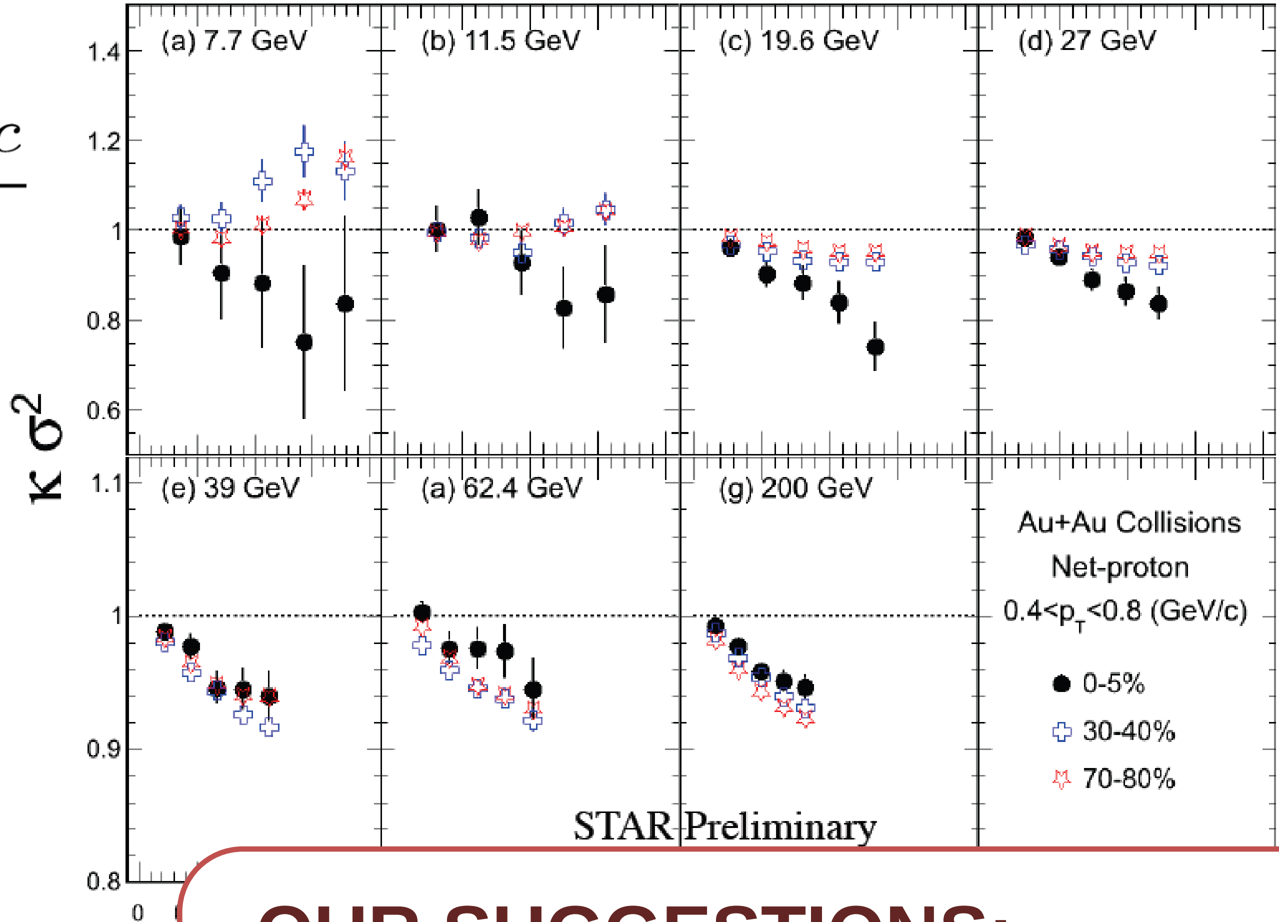


$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$ decreases as Δη becomes larger at RHIC energy.

Δη Dependence at STAR

STAR, QM2012

$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$



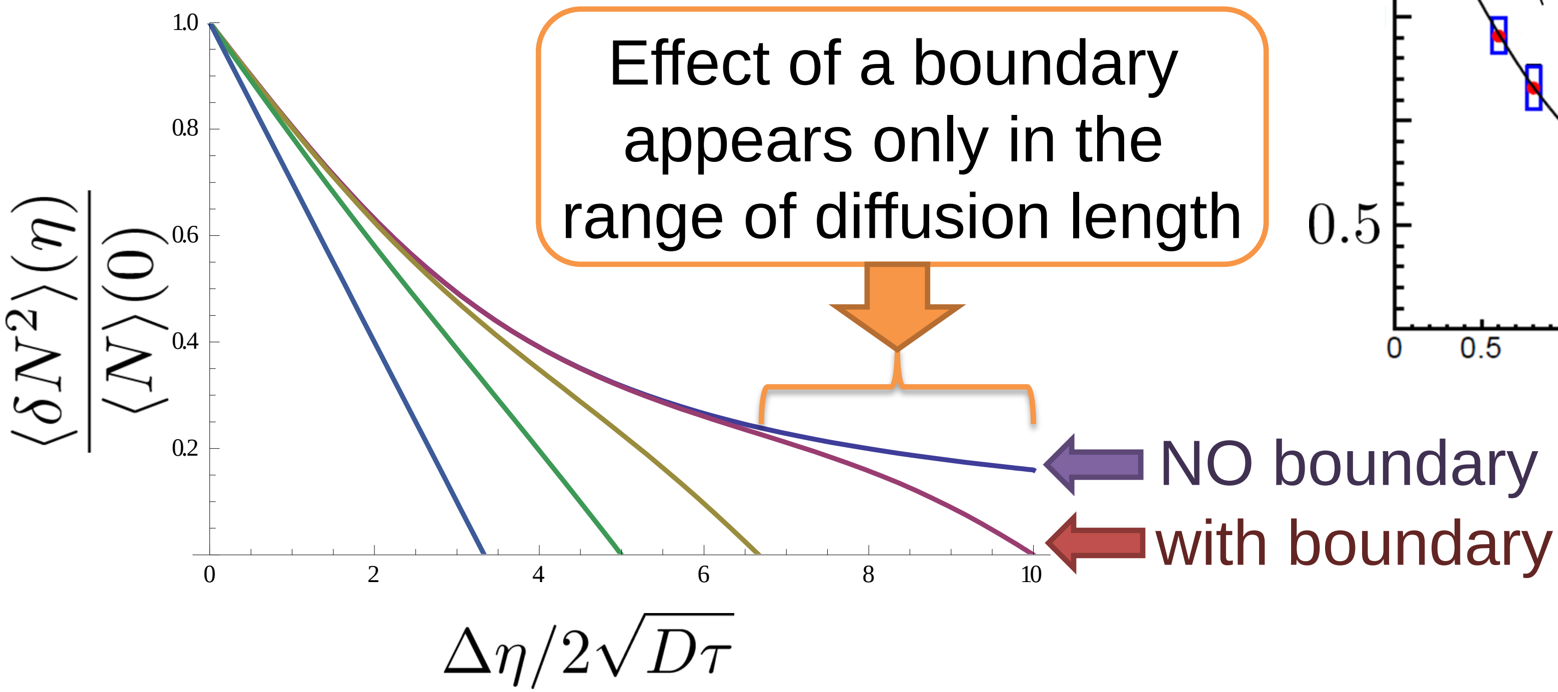
OUR SUGGESTIONS:

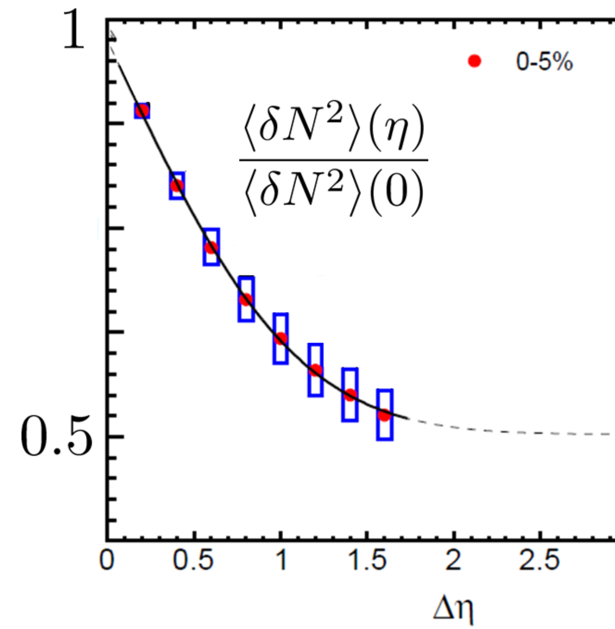
- Plot <δN^2> and <δN^4> separately
- Plot baryon number cumulants

Global Charge Conservation

Sakaida,
poster session (3rd week)

Solve SDE or DME in a finite volume





- Effect of GCC can be read off from $\Delta\eta$ dependence.
- No GCC effect in ALICE experiments!

Summary

Fluctuations in HIC are nonthermal!

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c, \ \langle N_B^2 \rangle_c, \ \langle N_Q^4 \rangle_c, \ \langle N_B^4 \rangle_c, \ \langle N_{ch}^2 \rangle_c, \cdots$$

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC

- history of hot medium
- mechanism of hadronization
- diffusion constant

LATTICE inputs

Summary

Fluctuations in HIC are nonthermal!

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c,\ \langle N_B^2 \rangle_c,\ \langle N_Q^4 \rangle_c,\ \langle N_B^4 \rangle_c,\ \langle N_{ch}^2 \rangle_c, \cdots$$

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC

- ☐ history of hot medium
- ☐ mechanism of hadronization
- ☐ diffusion constant

LATTICE inputs

Search of QCD Phase Structure in HIC

Open Questions & Future Work

- Why the primordial fluctuations are observed only at LHC, and not RHIC ?
- Extract more information on each stage of fireballs using fluctuations

- Model refinement
  - Including the effects of
    nonzero correlation length / relaxation time
    global charge conservation

  - Non Poissonian system ← interaction of particles

Chemical Reaction 1

$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

x: # of X
a: # of A (**fixed**)

Master eq.: $$\frac{\partial}{\partial t}P(x,t) = k_2 a P(x-1,t) + k_1(x+1)P(x+1,t) - (k_1 x + k_2 a)P(x,t)$$

Cumulants with fixed initial condition $P(x,0) = \delta_{x,N_0}$

$$\langle x(t)\rangle = N_0 e^{-k_1 t} + N_{eq}(1 - e^{-k_1 t})$$

$$\langle \delta x(t)^2\rangle = N_0(e^{-k_1 t} - e^{-2k_1 t}) + N_{eq}(1 - e^{-k_1 t})$$

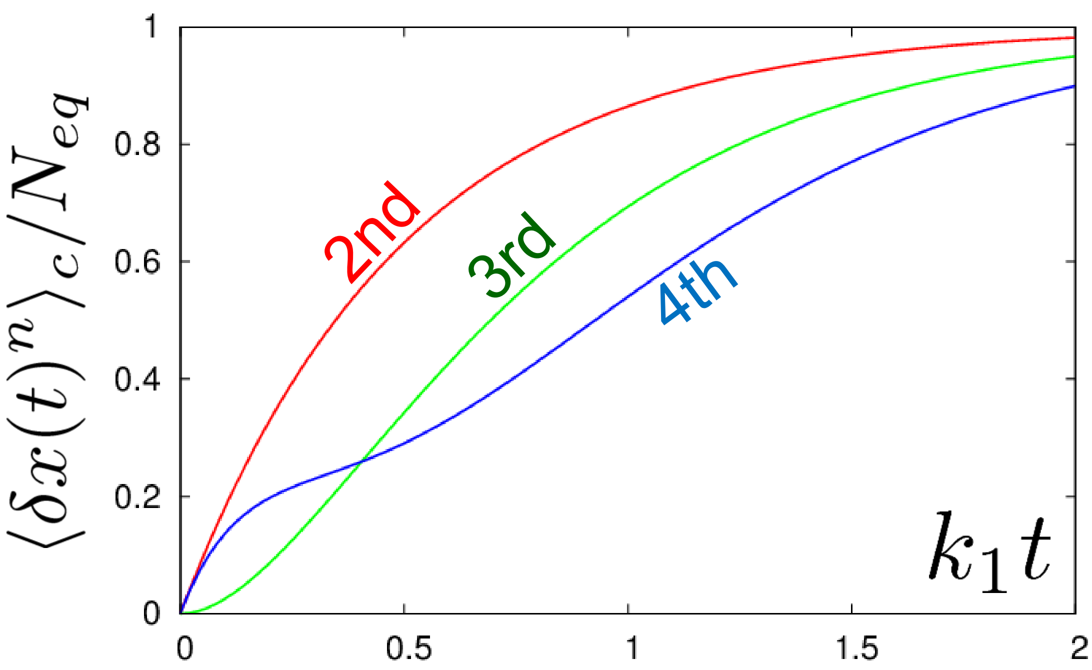
$$\langle \delta x(t)^3\rangle = N_0(e^{-k_1 t} - 3e^{-2k_1 t} + 2e^{-3k_1 t}) + N_{eq}(1 - e^{-k_1 t})$$

initial equilibrium

Chemical Reaction 2

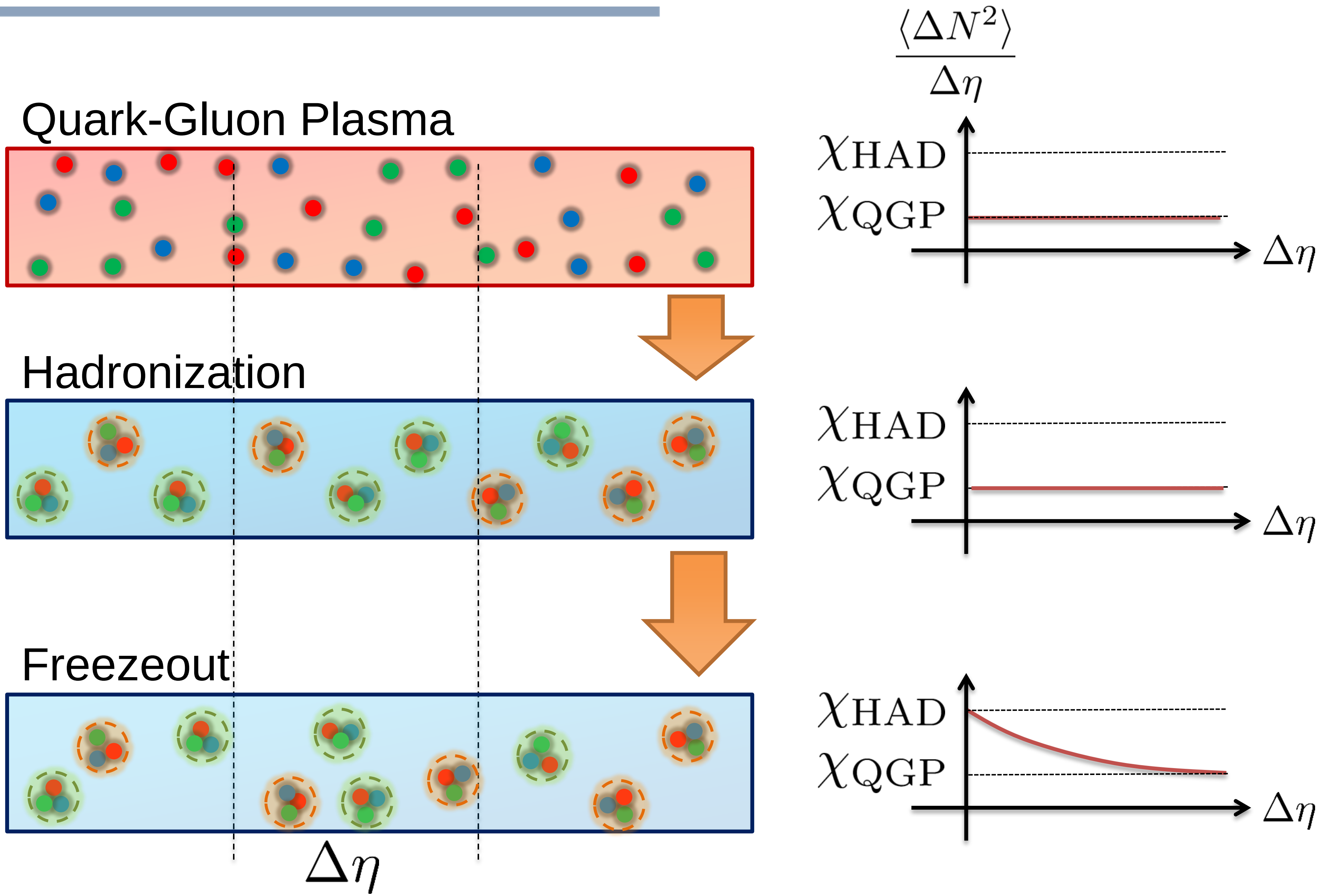
$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

$$N_0 = N_{eq} \Rightarrow \begin{cases} \langle x(t) \rangle = N_{eq} \\ \langle \delta x(t)^2 \rangle = N_{eq}(1 - e^{-2k_1 t}) \\ \langle \delta x(t)^3 \rangle = N_{eq}(1 - 3e^{-2k_1 t} + 2e^{-3k_1 t}) \end{cases}$$



Higher-order cumulants grow slower.

Time Evolution in HIC



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n$$

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

Stochastic Force

determined by fluctuation-dissipation relation

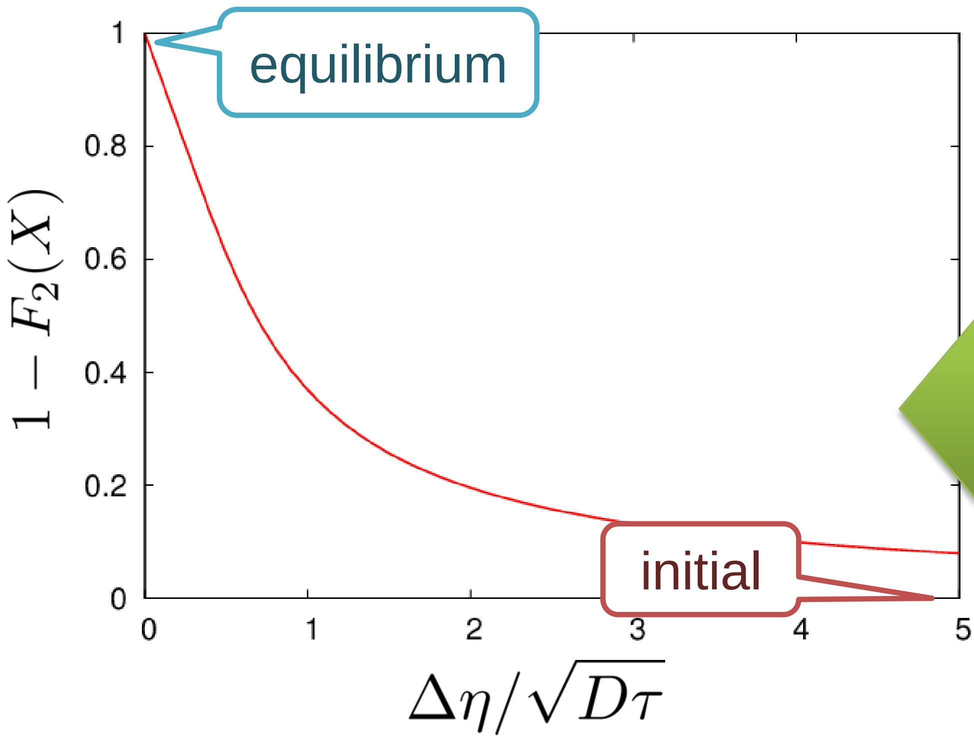
$\Delta\eta$ Dependence

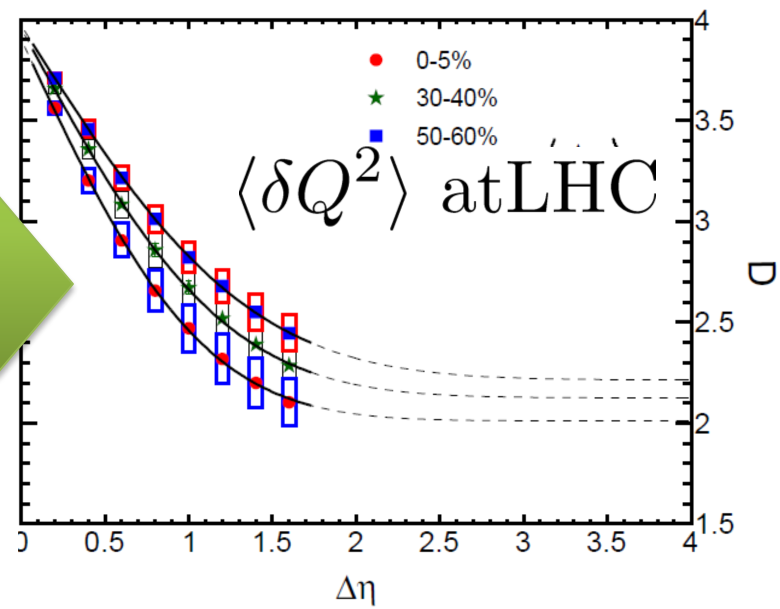
Shuryak, Stephanov, 2001

- Initial condition: $\langle \delta n(\eta_1, 0)\delta n(\eta_2, 0)\rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$

$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2 (1 - F_2(X))}_{\text{equilibrium}}$$





Non-Gaussianity in Fluctuating Hydro?

It is **impossible** to directly extend the theory of hydro fluctuations to treat higher orders.

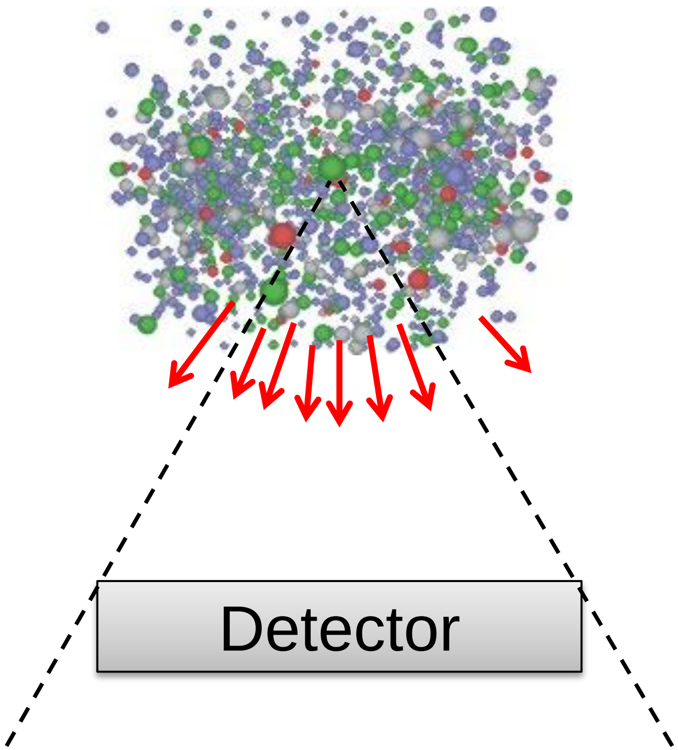
- No a priori extension of FD relations to higher orders
- **Theorem**

  Markov process + continuous variable
  → Gaussian random force

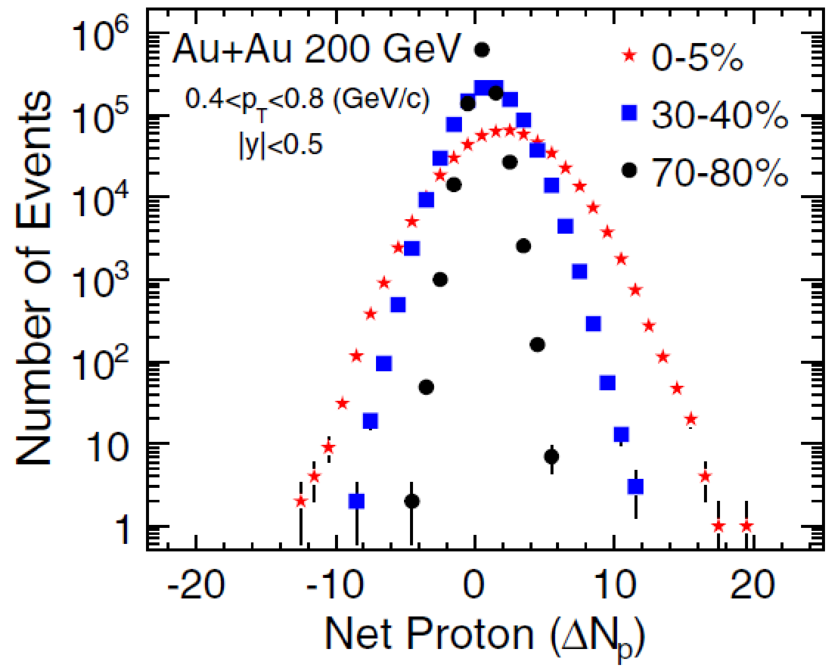
  cf) Gardiner, "Stochastic Methods"

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in HIC.



STAR, PRL**105** (2010)



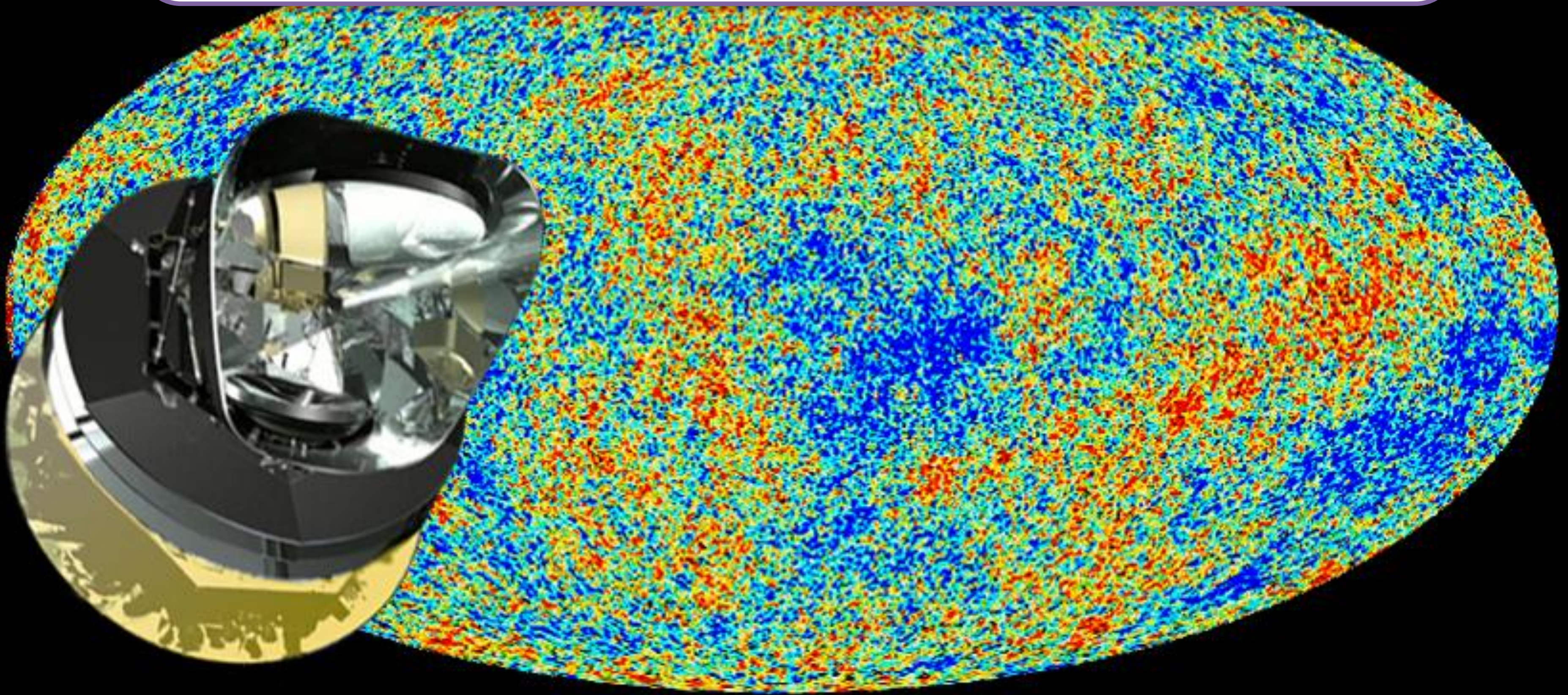
$$\langle \delta N^2 \rangle, \langle \delta N^3 \rangle, \langle \delta N^4 \rangle_c, \cdots$$

Non-Gaussianity

fluctuations (correlations)

$$\langle \delta n_1 \delta n_2 \rangle, \langle \delta n_1 \delta n_2 \delta n_3 \rangle, \langle \delta n_1 \delta n_2 \delta n_3 \delta n_4 \rangle_c, \cdots$$

→ Non-Gaussianity



PLANCK : statistics insufficient to see non-Gaussianity…(2013)