QCD effective potential with strong magnetic field

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arXiv: 1311.3137

NFQCD 2013, Nov. 27, Kyoto

Introduction

- Recently, QCD under strong magnetic fields attracts much attention.

- In particular, it is interesting question how the QCD vacuum and hadron properties are affected when the strength of the fields approach or exceed the QCD scale.

    - Chiral magnetic effect
    - (Inverse) Magnetic catalysis
    - Charged rho meson in strong magnetic field

- Such strong magnetic fields are generated in the relativistic heavy ion collisions.

- Lattice QCD can simulate strongly interacting quark and gluon systems in the presence of the strong magnetic fields.

In QCD under the strong magnetic field...

two kinds of strong dynamics coexist.

Strongly interacting quark and gluon dynamics

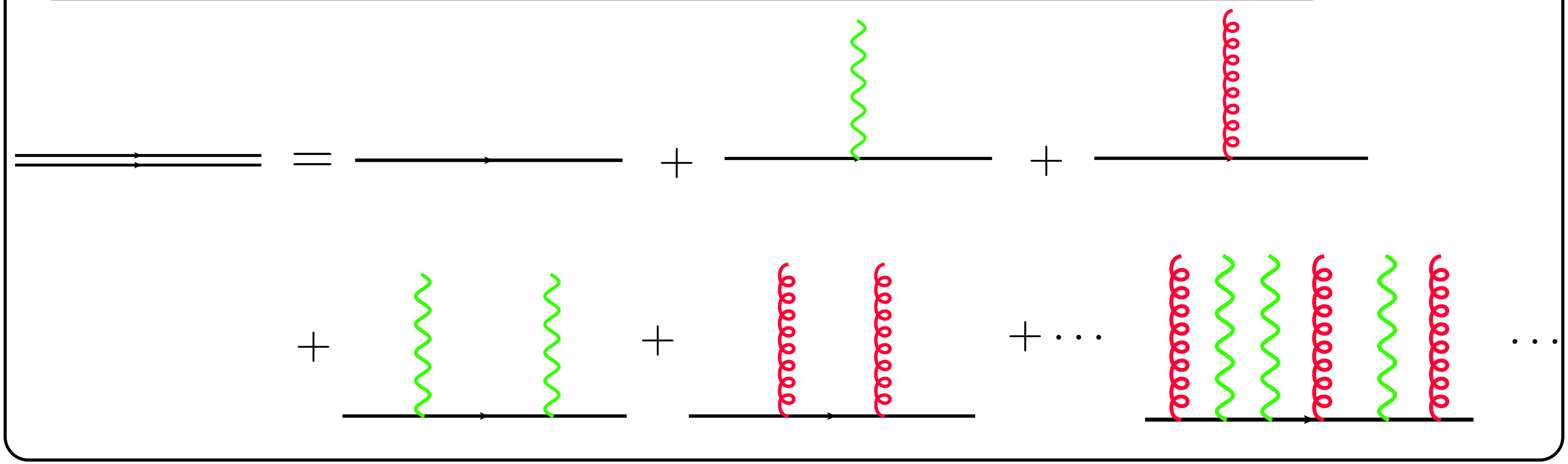
Non-linear QED dynamics with strong B field

Gluon couples to quark and gluon itself.

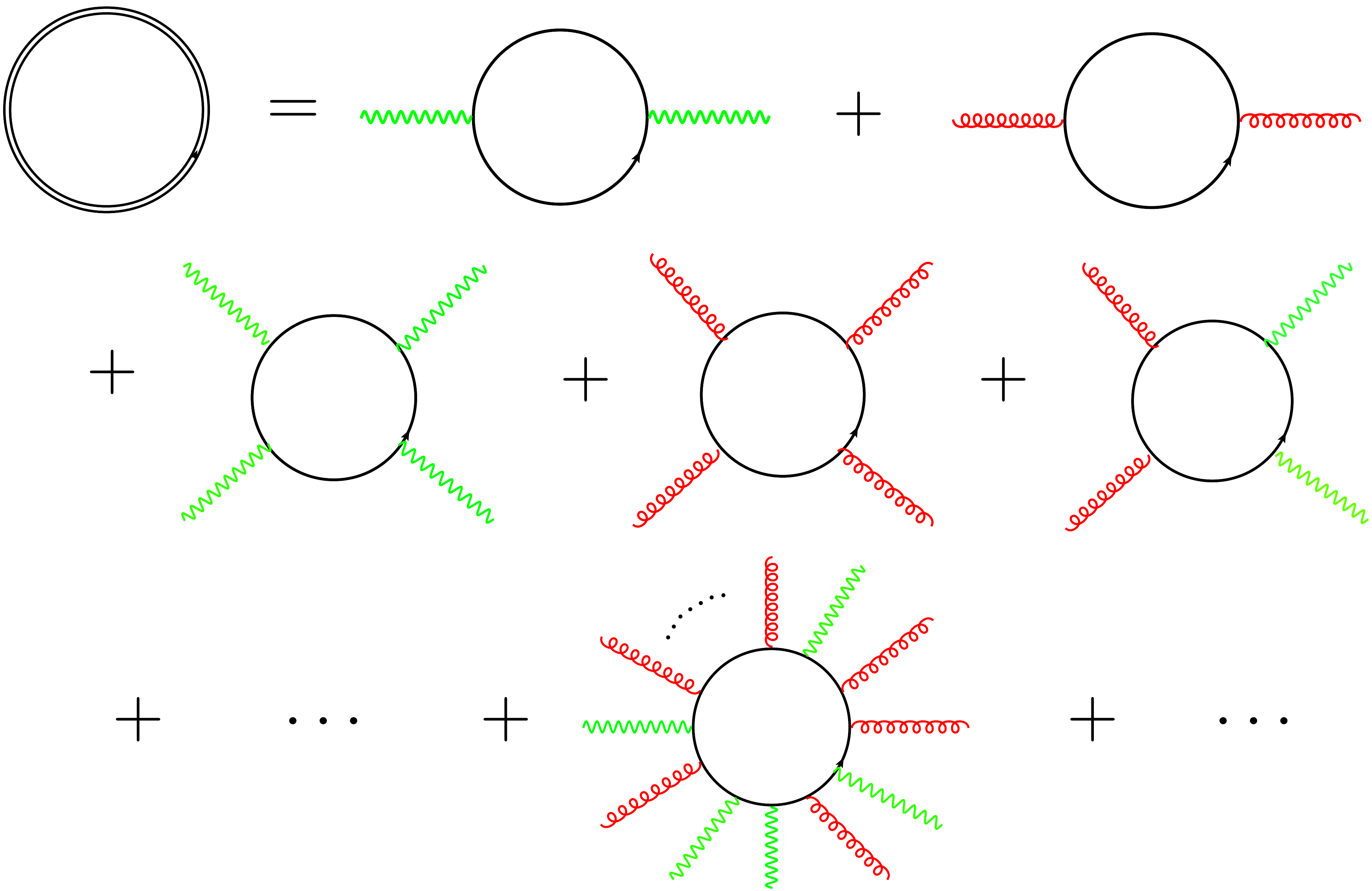
Magnetic field (photon) does not couple to gluon directly but interacts with quarks.

The effect of magnetic field must be reflected on QCD through the quark.

Quark propagator non-linearly interacting with photons and gluons

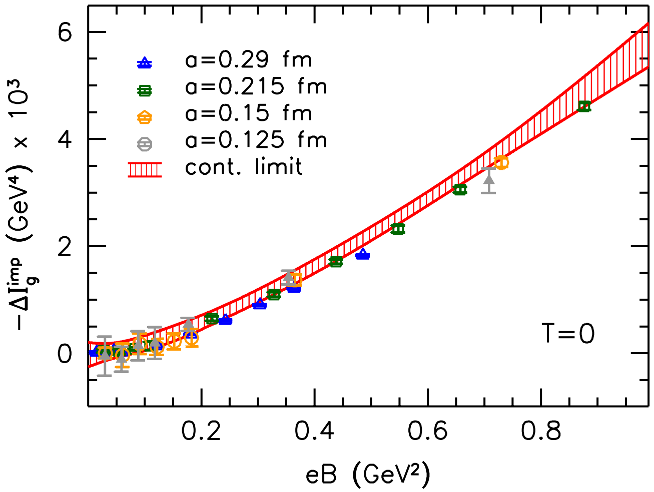


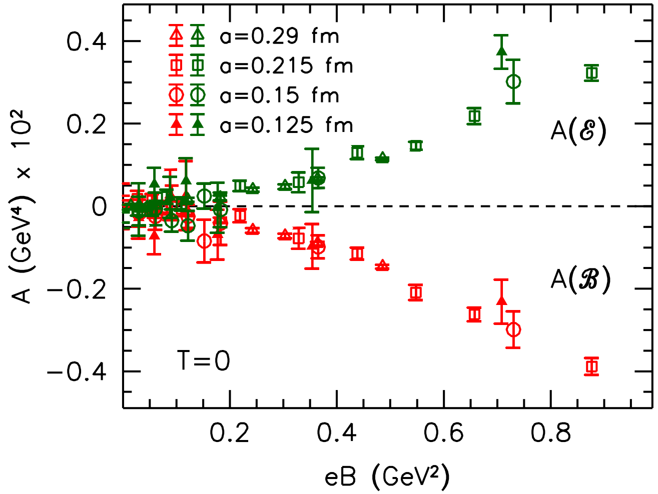
Using quark loop non-linearly interacting with gluon and photon, one can calculate effective Lagrangian for QCD+QED.



Recent Lattice simulation

G. S. Bali et al, JHEP 1304(2013) 130





$\langle -S_g \rangle = \langle +\frac{1}{4} F^2 \rangle$ increases with eB

$$A(\mathcal{B}) = \frac{T}{V} \left\langle \frac{\beta}{6} \sum_n \left(\mathrm{tr}\, \mathcal{B}_\perp^2(n) - \mathrm{tr}\, \mathcal{B}_\parallel^2(n) \right) \right\rangle$$

$\longrightarrow$ $\langle \mathrm{tr}\, \mathcal{B}_\parallel^2 \rangle > \langle \mathrm{tr}\, \mathcal{B}_\perp^2 \rangle$

Euler-Heisenberg effective action

$$S_{\mathrm{eff}}^{(2,2)}(\mathcal{F}_{\mu\nu}; B) = -\frac{V_4}{180\pi^2} \frac{(qB)^2}{m^4} \left[3\, \mathrm{tr}\, \mathcal{B}_\parallel^2 + \mathrm{tr}\, \mathcal{B}_\perp^2 + \mathrm{tr}\, \mathcal{E}_\perp^2 - \frac{5}{2} \mathrm{tr}\, \mathcal{E}_\parallel^2 \right]$$

Caution : Euler-Heisenberg effective action is basically the expansion of e*field/m, and in the current case this expansion obviously breaks down.

$\longrightarrow$ Full order calculation with respect to the fields is needed!

QCD Lagrangian with electromagnetic fields

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \underline{- \frac{1}{4}f_{\mu\nu}f^{\mu\nu}} + \bar{q}(i\gamma_\mu D^\mu - M_q)q$$

Covariant derivative

$$D_\mu = \partial_\mu - igA^a_\mu T^a - \underline{ieQa_\mu}$$

Field strengths

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

$$f^{\mu\nu} = \partial^\mu a^\nu - \partial^\nu a^\mu$$

$\boxed{\partial f = 0 : \text{constant fields}}$

Charge and mass matrices

$$Q = diag(Q_{q_1}, Q_{q_2}, \cdots, Q_{q_f}) \ , \qquad M_q = diag(m_{q_1}, m_{q_2}, \cdots, m_{q_f})$$

Background field method

I. Batalin et al, Sov. J. Nucl.Phys. 26 (1977)214

M. Gyulassy and A. Iwazaki, Phys. Lett. B 165(1985) 157

N. Tanji and K. Itakura, Phys. Lett. B 713(2012) 117

$$A^a = \underline{\hat{A}^a} + \underline{\mathcal{A}^a}$$

$\hat{A}^a$: Slowly varying classical background field

$\mathcal{A}^a$: Quantum fluctuation

We apply the Covariantly-constant field as the background field.

$$\hat{D}^{ab}_\rho \hat{F}^b_{\mu\nu} = 0 \qquad \hat{D}^{ab}_\rho = \partial\delta^{ab} + gf^{acb}\hat{A}^c$$

$\hat{F}$ is varying very slowly $(\partial\hat{F} = 0)$

$$\hat{F}^a_{\mu\nu} = F_{\mu\nu}\hat{n}^a \qquad \hat{n}^2 = 1$$

$$\hat{A}^a_\mu = A_\mu\hat{n}^a \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Gauge fixing (background gauge)

$$\hat{D}^{ab}_\mu \mathcal{A}^{b\mu} = 0$$

Effective action for $\hat{A}$

$$\exp\left[iS_{eff}(\hat{A}_\mu)\right] = \int \mathcal{D}\mathcal{A}_\mu \mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}q\mathcal{D}\bar{q}\, \exp\left\{i\int d^4x \left[-\frac{1}{4}\left(\hat{F}^a_{\mu\nu} + (\hat{D}^{ab}_\mu \mathcal{A}^b_\nu - \hat{D}^{ab}_\nu \mathcal{A}^b_\mu) + gf^{abc}\mathcal{A}^b_\mu \mathcal{A}^c_\nu\right)^2 \right.\right.$$
$$\left.\left. -\frac{1}{2\xi}(\hat{D}^{ab}_\mu \mathcal{A}^{b\mu})^2 - \bar{c}^a(\hat{D}_\mu D^\mu)^{ac}c^c + \bar{q}(i\gamma_\mu \hat{D}^\mu - M_q)q + \bar{q}(ig\gamma_\mu \mathcal{A}^{a\mu}\cdot T^a)q - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}\right]\right\}$$

Functional integral for second order fluctuations with $\xi = 1$

Gluon

$$\int \mathcal{D}\mathcal{A} e^{i\int d^4x\ -\frac{1}{2}\mathcal{A}^{a\mu}\{-(\hat{D}^2)^{ac}g_{\mu\nu} - 2gf^{abc}\hat{F}^b_\mu\}\mathcal{A}^{c\nu}} = \det\left[-(\hat{D}^2)^{ac}g_{\mu\nu} - 2gf^{abc}\hat{F}^b_\mu\right]^{-1/2}$$

Ghost

$$\int \mathcal{D}c\mathcal{D}\bar{c} e^{i\int d^4x\ \bar{c}[-(\hat{D}^2)^{ac}]c} = \det\left[-(\hat{D}^2)^{ac}\right]^{+1}$$

The results are well known.

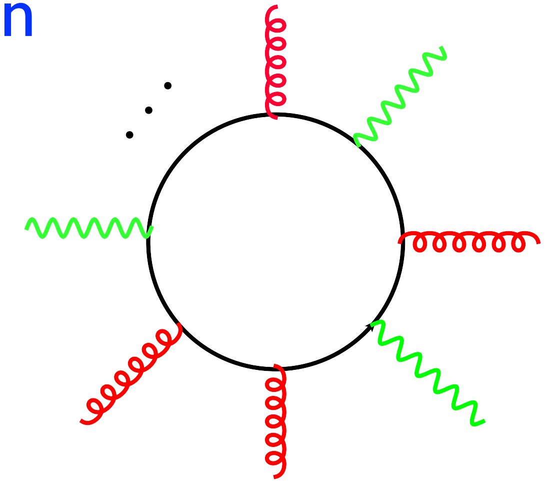
Quark

$$\int \mathcal{D}q\mathcal{D}\bar{q} e^{i\int d^4x \bar{q}(i\gamma_\mu \hat{D}^\mu - M_q)q} = \det\left[i\gamma_\mu \hat{D}^\mu - M_q\right]$$

This work

The quark contribution to the effective action

$$i\Delta S_q = \log\det[i\gamma_\mu \hat{D}^\mu - M_q]$$



$$\hat{D}^\mu = \partial^\mu - \underline{igA^\mu \hat{n}^a T^a} - \underline{ieQa^\mu}$$

Diagonalization in color space

$$U\hat{n}^a T^a U^\dagger \xrightarrow{SU(2)} \begin{pmatrix} w_1 & 0 \\ 0 & w_2 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

$$U\hat{n}^a T^a U^\dagger \xrightarrow{SU(3)} \begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix}$$

$$w_1 = \frac{1}{\sqrt{3}}\cos\left(\Theta + \frac{\pi}{6}\right), \quad w_2 = -\frac{1}{\sqrt{3}}\cos\left(\Theta - \frac{\pi}{6}\right), \quad w_3 = \frac{1}{\sqrt{3}}\sin(\Theta)$$

$$\sin^2 3\Theta = 3C_2, \quad C_2 = [d^{abc}\hat{n}^a\hat{n}^b\hat{n}^c]^2$$

$$\boxed{\sum_{a=1}^{N_c} w_a^2 = \frac{1}{2}, \quad \sum_{a=1}^{N_c} w_a = 0}$$

G. C. Nayak and P. Nieuwenhuizen, PRD71 (2005)

G. C. Nayak PRD72 (2005)

N. Tanji, Ann. Phys. 324(2009)

- We apply the Schwinger's proper time method to evaluate the effective potential.

- Performing the proper time integral, we derive the analytic expression of the effective potential of quark part.

- In order to focus on the chromo-magnetic field, we employ the pure chromo-magnetic background for the gluon field.

Effective potential for quark part

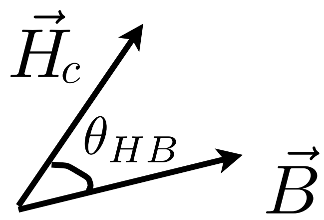
$$V_q = V_q^{fin} + V_q^{div}$$

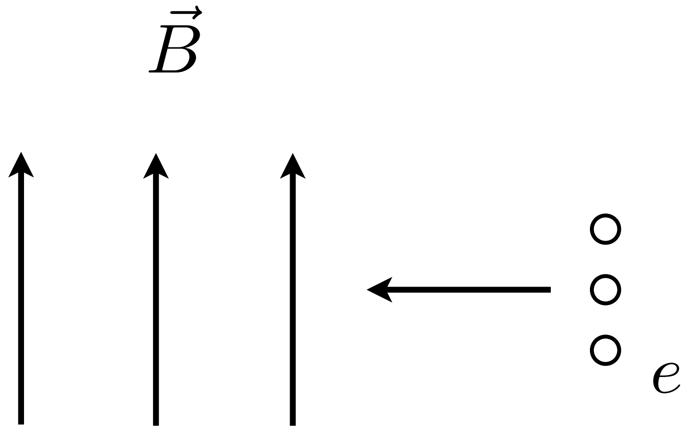
$$H_c = \sqrt{\vec{H}_c^2}\,, \quad B = \sqrt{\vec{B}^2}$$

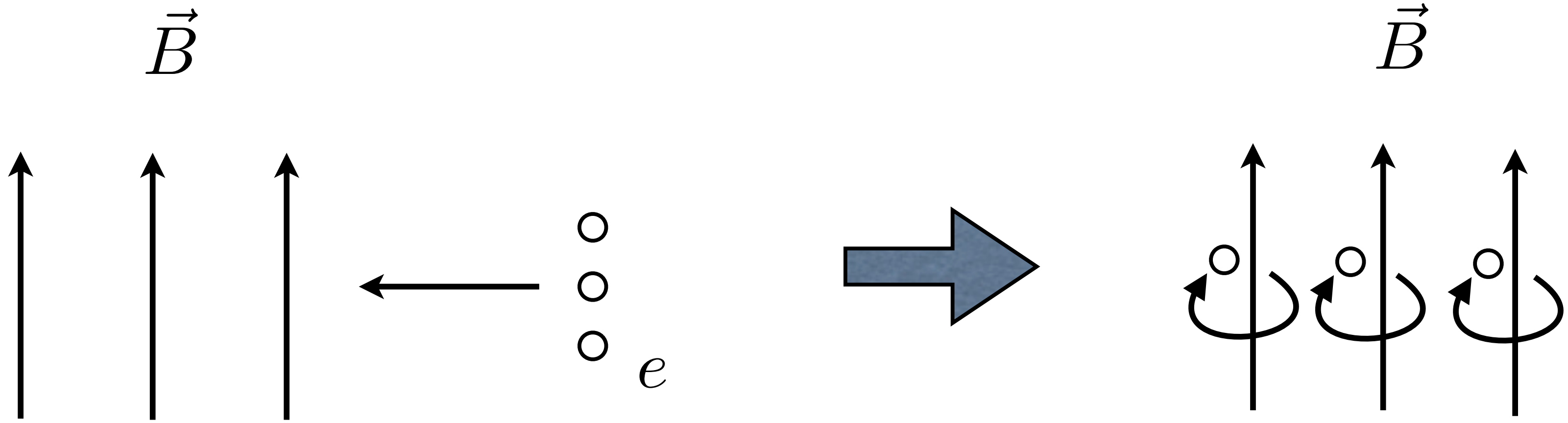
$$V_q^{fin} = \sum_{a=1}^{N_c}\sum_{i=1}^{N_f}\left\{ -\frac{a_{a,i}^2}{24\pi^2}\left(\log(2a_{a,i}) + 12\zeta'(-1, \frac{m_{q_i}^2}{2a_{a,i}}) - 1 \right) + \frac{m_{q_i}^2 a_{a,i}}{8\pi^2}\log\left(\frac{2a_{a,i}}{m_{q_i}^2}\right) - \frac{m_{q_i}^4}{16\pi^4}\left(\log\left(\frac{2a_{a,i}}{m_{q_i}^2}\right) + \frac{1}{2}\right)\right\}$$

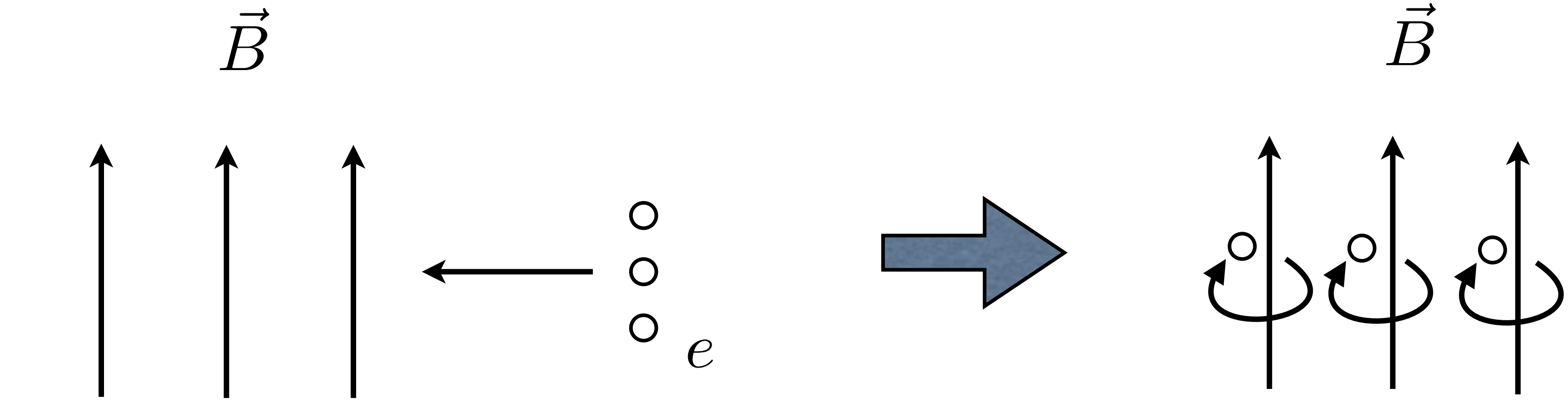
$$V_q^{div} = \frac{N_f}{48\pi^2}(gH_c)^2\log\Lambda^2 + \frac{N_c}{24\pi^2}\left(\sum_{i=1}^{N_f} Q_{q_i}^2\right)^2 (eB)^2\log\Lambda^2$$

$$a_{a,i} = \sqrt{\left(gw_a\vec{H}_c + eQ_{q_i}\vec{B}\right)^2} = \sqrt{g^2w_a^2H_c^2 + e^2Q_{q_i}^2B^2 + 2gw_aeQ_{q_i}H_cB\cos\theta_{HB}}$$

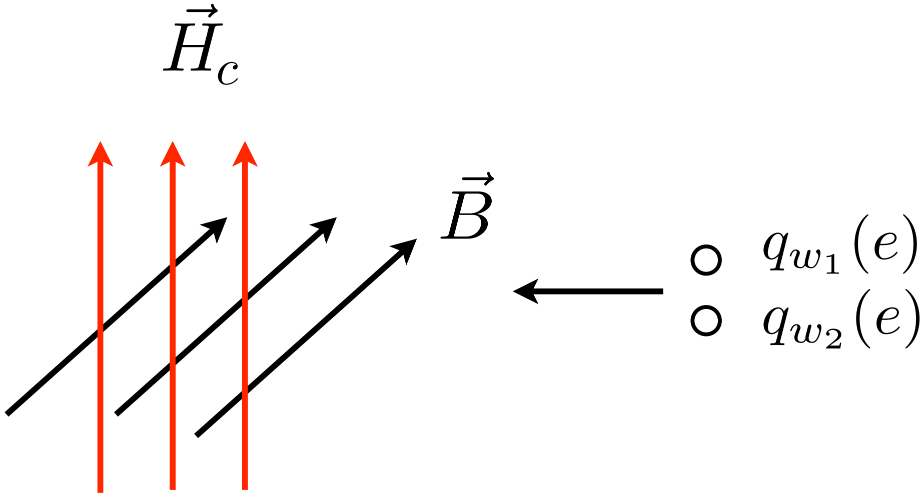


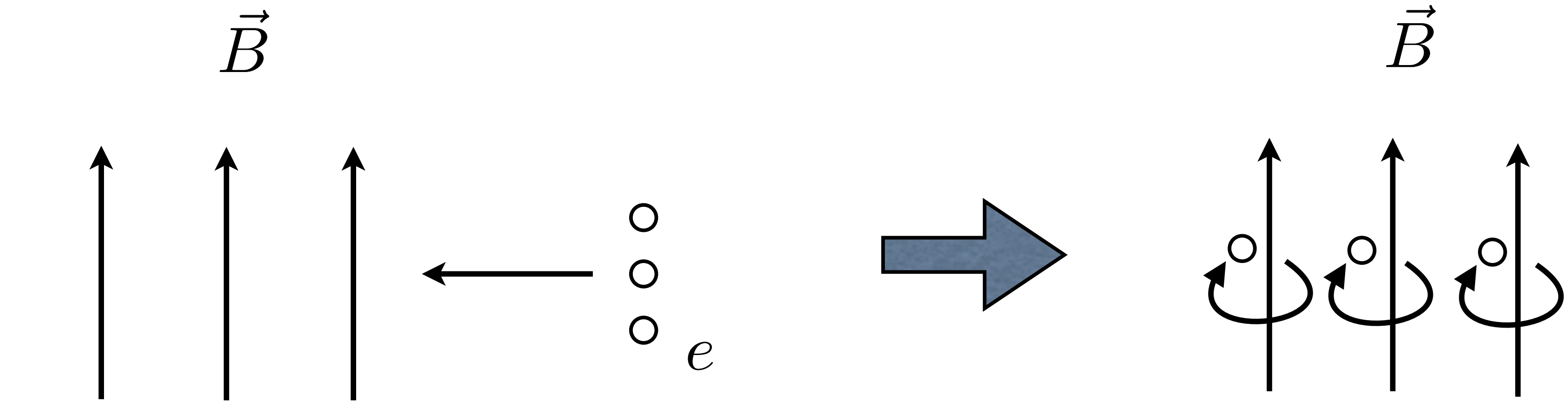

$\vec{B}$
e


$\vec{B}$
e
$\vec{B}$



Color $SU(2)$ case with $Q_q = 1$





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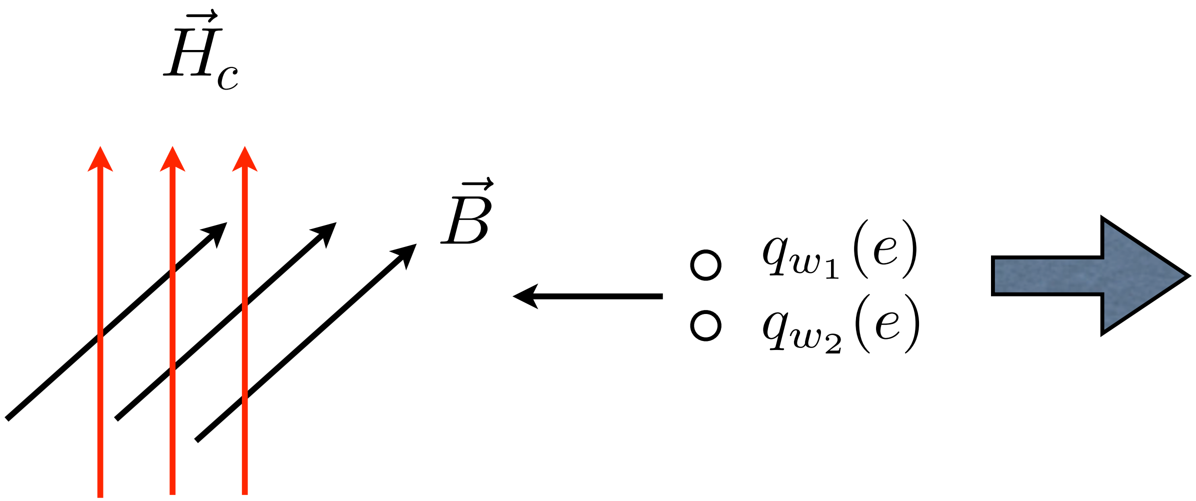
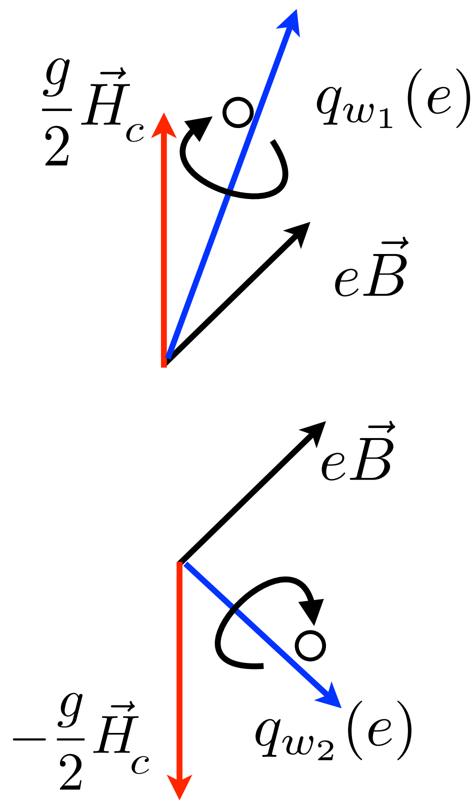
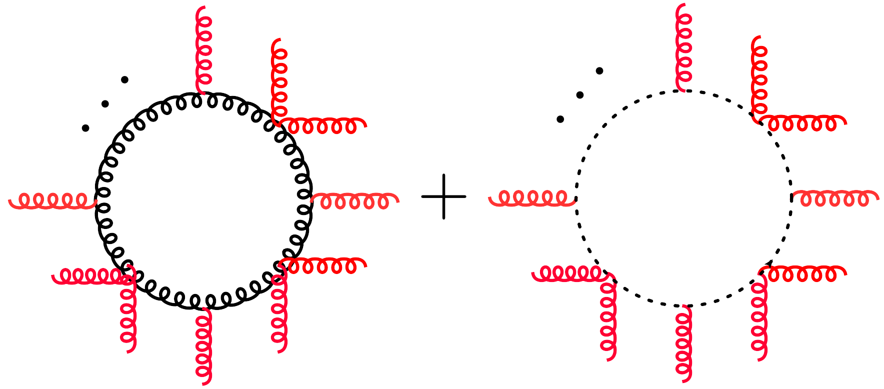




Gluon+ghost part effective potential



Real part

$$ReV_g = V_g^{fin} + V_g^{div}$$

$$V_g^{fin} = \frac{11N_c}{96\pi^2}(gH_c)^2 \left\{ \log(gH_c) - c_g + \frac{1}{N_c}\sum_{a=1}^{N_c} \lambda_a^2 \log\lambda_a^2 \right\}$$

$$V_g^{div} = -\frac{11N_c}{96\pi^2}(gH_c)^2 \log\Lambda^2$$

G. K. Savvidy, Phys. Lett. B71(1977)

N. Nielsen and Olesen, Nucl. Phys. B144(1978)

Color charges

$SU(2)$

$$\lambda_1 = +1 \ , \ \lambda_2 = -1$$

$SU(3)$

$$\lambda_1^2 = \frac{1}{2}\left[1 - \cos\left(2\Theta - \frac{\pi}{3}\right)\right], \lambda_2^2 = \frac{1}{2}\left[1 - \cos\left(2\Theta + \frac{\pi}{3}\right)\right], \lambda_3^2 = \frac{1}{2}\left[1 + \cos(2\Theta)\right]$$

G. C. Nayak and P. Nieuwenhuizen, PRD71 (2005)

Imaginary part

$$ImV_g = -\frac{N_c}{16\pi^2}(gH_c)^2$$: Nilesen-Olesen instability

Logarithmic divergences and renormalization

$$V^{div} = \frac{1}{2}\left\{-\frac{1}{(4\pi)^2}\left[\frac{11}{3}N_c - \frac{2}{3}N_f\right]\right\}(gH_c)^2 \log\Lambda^2 + \frac{1}{2}\frac{N_c}{12\pi^2}\left(\sum_{i=1}^{N_f} Q_{q_i}^2\right)(eB)^2 \log\Lambda^2$$

replacing the couplings and field in the potential by bare couplings g_0, e_0 and fields H_0, B_0

$$V_{eff} = \frac{H_0^2}{2} + \frac{B_0^2}{2} + V_0^{div} + V_0^{fin}$$

and rescale the couplings and fields as

$$H_0 = Z_H^{1/2} H_c \quad , \qquad B_0 = Z_B^{1/2} B$$

$$g = Z_H^{1/2} g_0 \quad , \qquad e = Z_B^{1/2} e_0$$

The rescale factors are given by

$$Z_H = 1 + \delta_H \ , \qquad Z_B = 1 + \delta_B$$

counterterms

Using renormalized couplings g, e and fields H_c, B we can write the effective potential as

$$V_{eff} = \frac{1}{2} Z_H H_c^2 + \frac{1}{2} Z_B B^2 + V^{div} + V^{fin}$$

Introducing the counterterms so that log divergences cancel

$$\delta_H = \frac{g^2}{(4\pi)^2} \left[\frac{11}{3} N_c - \frac{2}{3} N_f \right] \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\delta_B = -\frac{N_c e^2}{12\pi^2} \left(\sum_{i=1}^{N_f} Q_{q_i}^2 \right) \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

we finally get renormalized effective potential

$$V_{eff} = \frac{H_c^2}{2} + \frac{B^2}{2} + V^{fin}$$

Furthermore, we can calculate the beta functions of QCD and QED

$$\beta_{QCD} = \mu \frac{\partial g}{\partial \mu} = \frac{1}{2} g \mu \frac{\partial Z_H}{\partial \mu} = \underline{-\frac{g^3}{(4\pi)^2} \left[\frac{11}{3} N_c - \frac{2}{3} N_f \right]}$$

$$\beta_{QED} = \mu \frac{\partial e}{\partial \mu} = \frac{1}{2} e \mu \frac{\partial Z_B}{\partial \mu} = \underline{+\frac{N_c e^3}{12\pi^2} \left(\sum_{i=1}^{N_f} Q_{q_i}^2 \right)}$$

correct one-loop β functions

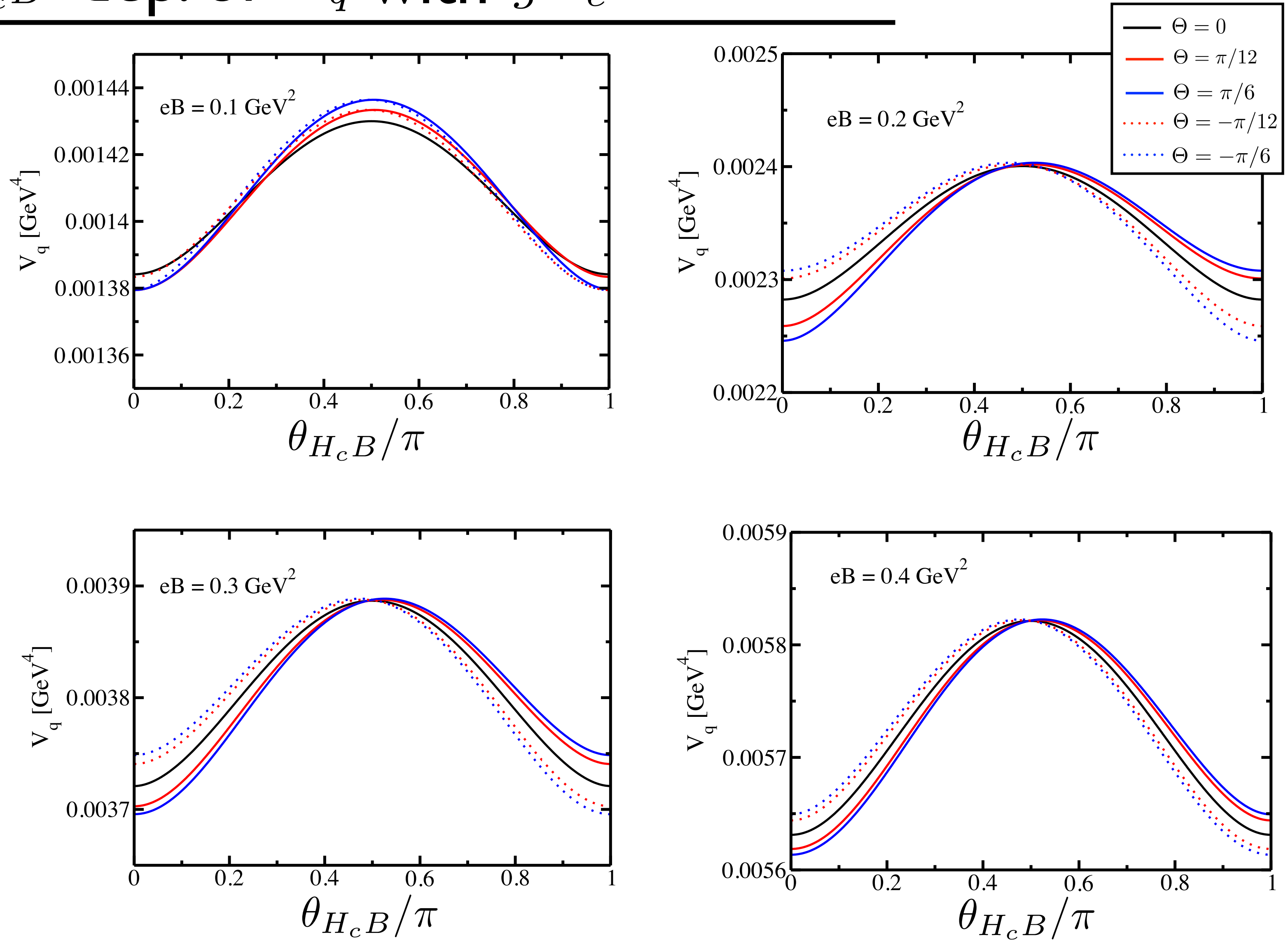
We investigate the magnetic field dependence of the QCD effective potential.

In this study, we consider the color SU(3) case with the three flavor (u,d,s).

We use the following parameters

- $$Q_u = +\frac{2}{3}\,,\;\; Q_d = Q_s = -\frac{1}{3}$$
- $$m_u = m_d = 5\ \mathrm{MeV}\,,\;\; m_s = 140\ \mathrm{MeV}$$
- $$\alpha_s = 1\,,\;\; \alpha_{EM} = \frac{1}{137}\,,\quad \mu = 1\ \mathrm{GeV}$$

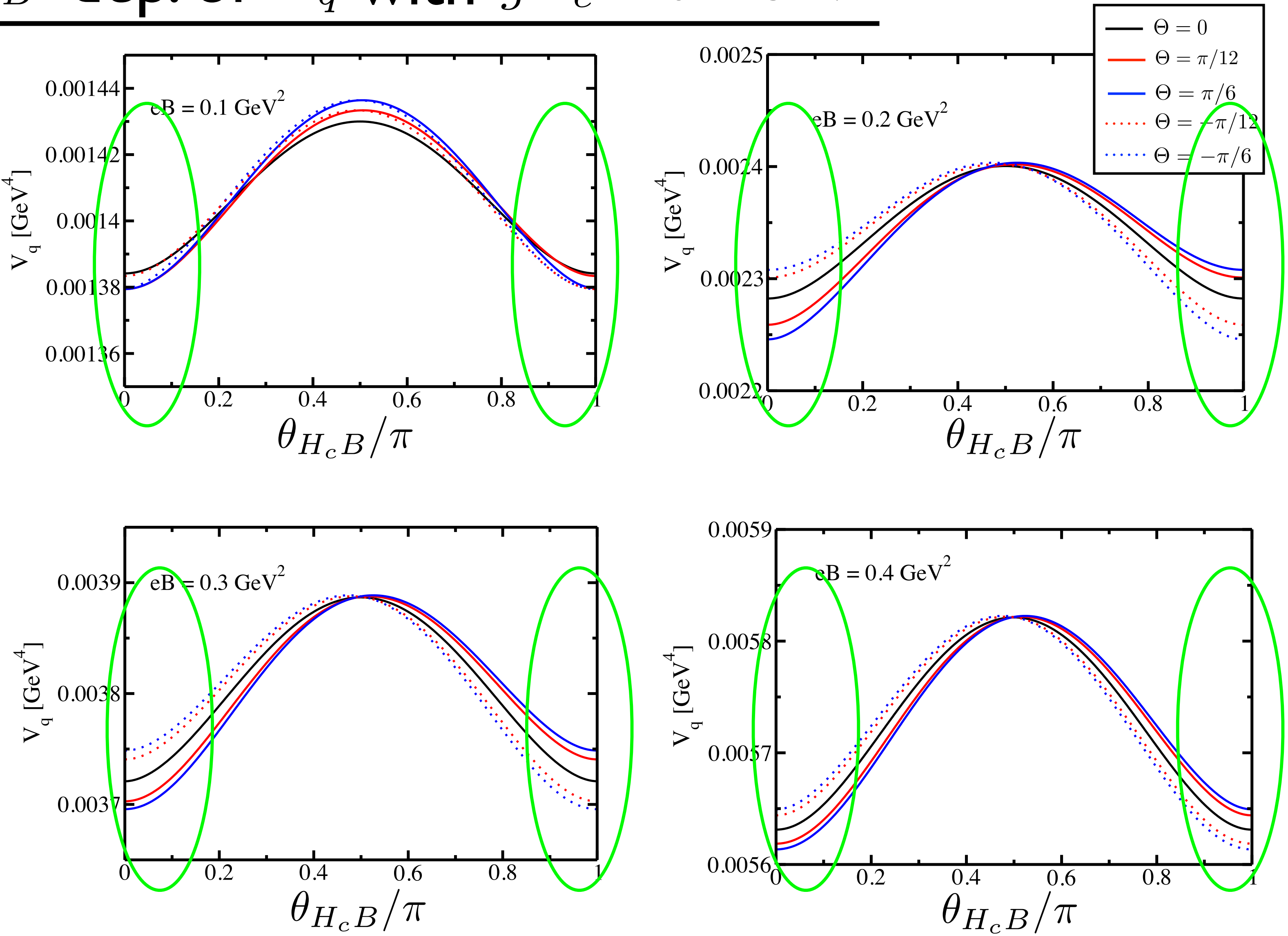
θ_{H_cB} -dep. of V_q with $gH_c = 0.2\ \mathrm{GeV}^2$



Chromo-magnetic fields prefer to be parallel (or anti-paralell) to the external magnetic field, which is consistent with recent lattice results.

$$\longrightarrow \boxed{H_{c\parallel} > H_{c\perp}}$$

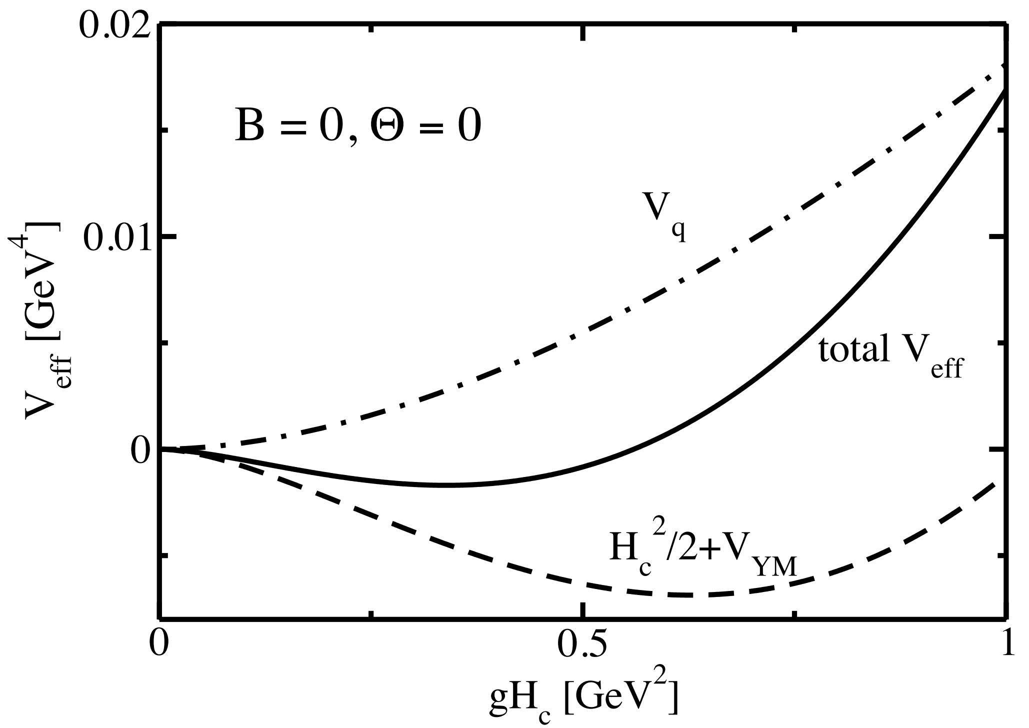
θ_{H_cB} -dep. of V_q with $gH_c = 0.2\ \mathrm{GeV}^2$



Chromo-magnetic fields prefer to be parallel (or anti-paralell) to the external magnetic field, which is consistent with recent lattice results.

$\longrightarrow \boxed{H_{c\parallel} > H_{c\perp}}$

QCD effective potential at $\underline{B = 0}$



- The one-loop YM effective potential $H_c^2/2 + V_{YM}$ has a minimum away from the origin, which corresponds to the dynamical generation of the chromo-magnetic condensate.

$\longrightarrow$ This result is qualitatively in agreement with LQCD and FRG analyses.

J. Amebjorn, V. K. Mitrjushkin and A. M. Zadorozhnyi, PLB 245 (1990) 575

A. Eichhorn, H. Gies and J. M. Pawlowski, PRD83 (2011) 045014

- Quark loop contributions attenuate the gluonic contributions.

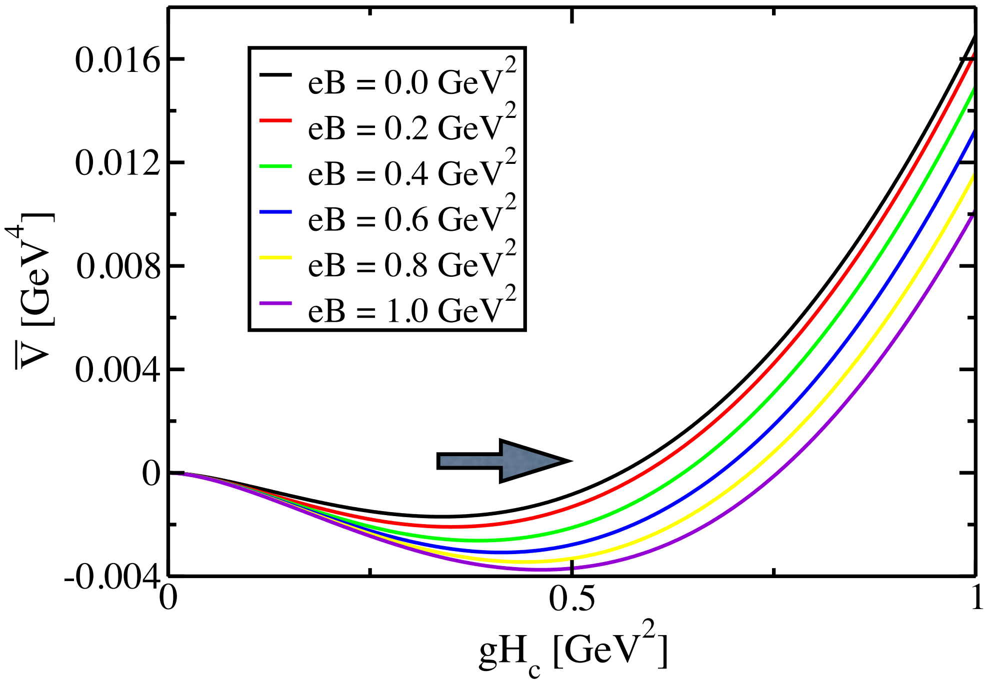
$\longrightarrow$ How the condensate behaves in the presence of the magnetic field?

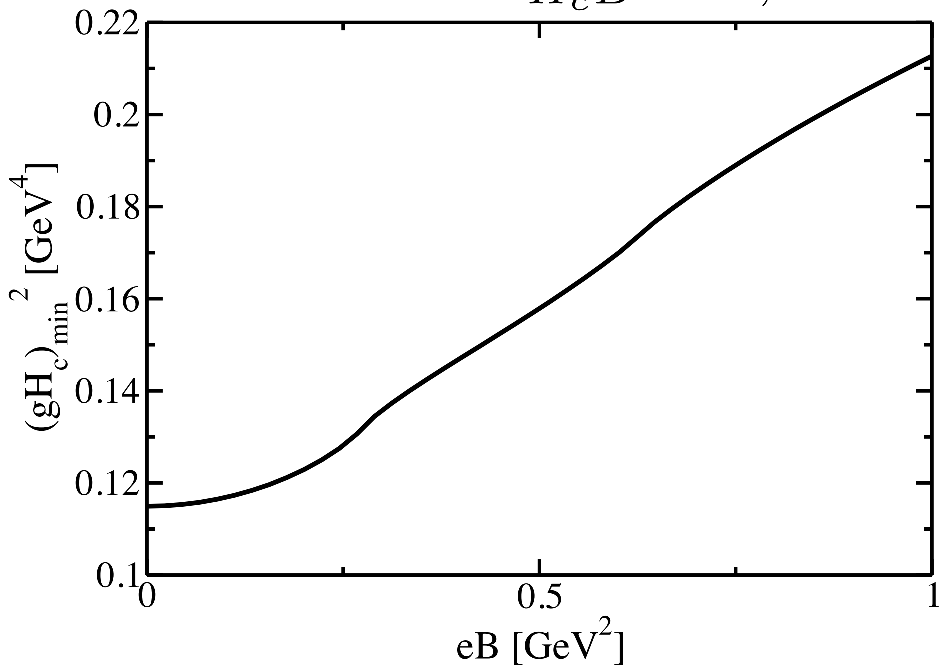
QCD effective potential with finite magnetic fields

We defined the normalized potential:

$$\bar{V}(H_c, B) = V(H_c, B) - V(0, B)$$

$$\theta_{H_c B} = 0,\ \Theta = 0$$





As the magnetic field increases, the minimum shift to the right hand side.

⟶ The chromo-magnetic condensate increases with an increasing magnetic field.

This behavior is quite similar to the recent observed gluonic magnetic catalysis in lattice QCD.

- In the mass less limit of the quark $m_q \to 0$, one can obtain the analytic expression of $(gH_c)^2_{min}$ with $eB = 0$:

$$(gH_c)^2_{min,0} = \mu^4 \exp\left\{-\frac{8\pi}{b_0\alpha_s} - 1 + \frac{2}{b_0}\left(\frac{11N_c}{3}c_g - \frac{2N_f}{3}c_q\right)\right\}, \quad b_0 = \frac{11N_c}{3} - \frac{2N_f}{3}$$

where c_g and c_q are some constants.

- In the small eB region, $(gH_c)_{min,0} >> eB$, we find

$$(gH_c)^2_{min} = (gH_c)^2_{min,0} + \frac{(4\pi)^2}{b_0}\frac{N_c}{12\pi^2}\left(\sum_{i=1}^{N_f} Q^2_{q_i}\right)(eB)^2$$

Note that the coefficient of the second term is the ratio of the coefficients of β_{QCD} and β_{QED} .

- In the large eB region, $eB > (gH_c)_{min}$, $(gH_c)^2_{min}$ still monotonically increases as the magnetic field increases.

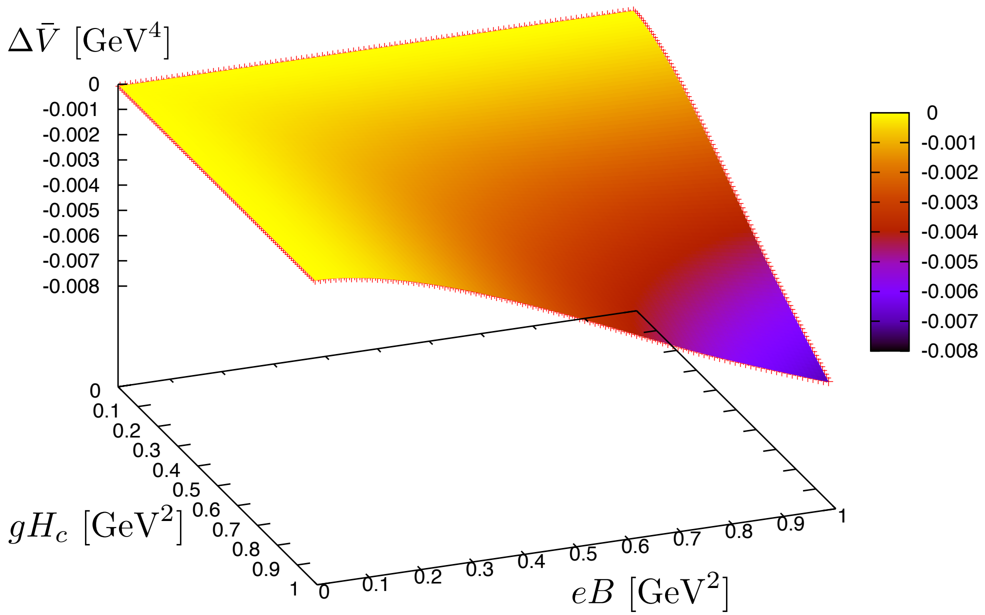
In our results, quark loop contributions should be important, since only V_q has B-dependence.

To see the importance, we define the following quantity

$$\Delta\bar{V}(H_c, B) = \bar{V}(H_c, B) - \bar{V}(H_c, 0)$$

$$= V_q(H_c, B) - V_q(0, B) - V_q(H_c, 0)$$

$$= \frac{i}{\int d^4x} \log\left[\frac{\det(i\hat{\not{D}}(H_c, B) - M_q)}{\det(i\hat{\not{D}}(H_c, 0) - M_q)\ \det(i\hat{\not{D}}(0, B) - M_q)}\right].$$



- $\Delta\bar{V}$ is negative in the whole region of gH_c-eB plane.
- $\Delta\bar{V}$ is monotonically decreasing as either gH_c or eB increases.

Using the definition of $\Delta\bar{V}$, we can rewrite the normalized effective potential

$$\begin{aligned}\bar{V}(H_c, B) &= \bar{V}(H_c, 0) + \Delta\bar{V}(H_c, B) \\ &= \frac{H_c^2}{2} + V_{YM} + \left[\underline{V_q(H_c, 0)} + \underline{\Delta\bar{V}(H_c, B)}\right]\end{aligned}$$

$\underline{V_q(H_c, 0)}$: B-independent part of quark loop which attenuates the gluonic contributions.

Completely opposite roles

$\underline{\Delta\bar{V}(H_c, B)}$: B-dependent part of quark loop which enhances the gluonic contributions.

- Thanks to the property of the B-dep. part of the quark loop, $(gH_c)^2_{min}$ monotonically increases with an increasing magnetic field.
- This property of the quark loop supports the gluonic magnetic catalysis at zero temperature, observed in current lattice data.

Summary

- We derive the analytic expression of the one-lop QCD effective potential with the magnetic field.

- After the renormalization of couplings and fields, we obtain the correct β-functions of both QCD and QED.

- Our result shows that the chromo-magnetic field prefers to be parallel (or anti-parallel) to the external magnetic field, which is consistent with recent lattice results.

- Quark loop contributions with magnetic fields enhance the gluonic contributions and thus the chromo-magnetic condensate monotonically increase with an increasing magnetic field.

  $\longrightarrow$ This result supports the gluonic magnetic catalysis at zero temperature, observed in current lattice data.