Microscopic identification of dissipative modes in relativistic field theories

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goal

understand long-time evolution of nonequilibrium state
from microscopic interactions between particles

- dynamics at the critical point

  $\longrightarrow$ naïve perturbation breaks down

dynamics at the QCD critical point
from dynamics of quarks and gluons

contents

1. identify dissipative modes from microscopic theories
   using resummation by two-particle irreducible (2PI) formalism

   YS, H. Fujii, K. Itakura and O. Morimatsu, arXiv:1309.4892

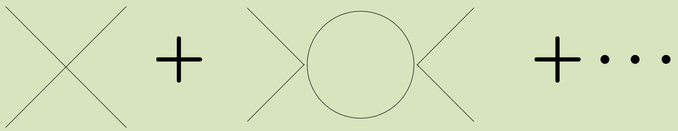
2. evaluate the dynamic exponent z
   (the divergence of the relaxation time at the critical point)

1. Background

Hydrodynamic time scales and secularities

microscopic perturbation

- perturbation
  ~ expansion by a number of collisions



e.g. cross section

hydrodynamics

- effective theories of long distance and time $(\boldsymbol{x} \gg \xi,\ t \gg \tau)$

(ξ : correlation length, τ : correlation time)

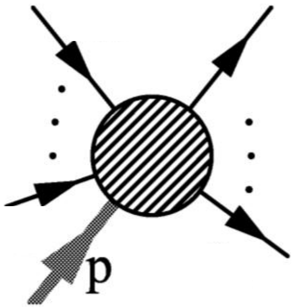
e.g. heat diffusion eq. $\partial_t Q = \kappa \nabla^2 Q$

hydrodynamic time scales
($t \gg \tau \gg$ collision time)

multi-particle scatterings (higher order terms) becomes important.

breakdown of naïve perturbation (**secularity**)

Resummation framework



multi-particle scattering processes

infinite series of self-energies

resummation is useful

two-particle-irreducible (2PI) effective action

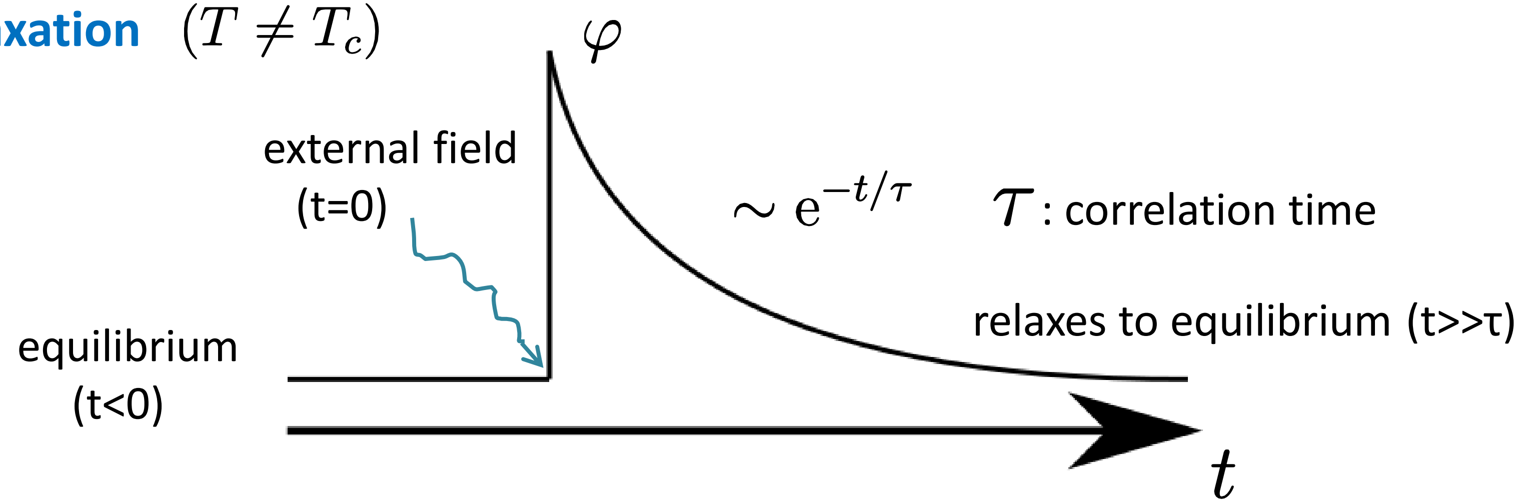
systematically resums infinite series of self-energies

through solving Schwinger-Dyson eq. self-consistently

Apply this framework to dynamics in long-distance and -time scales such as critical dynamics

Relaxation in near-equilibrium state

- **relaxation** $(T \neq T_c)$



- **relaxation at the critical point** $(T = T_c)$

  (dynamic critical phenomena)

  - large fluctuations, correlation length ($\xi \to \infty$)
  - **divergence of correlation time** $\tau \sim \xi^{z}$
  - divergences of transport coefficients
    (diffusion constant, heat conductivity, …)

z: dynamic critical exponent

Critical phenomena
and dissipative modes

critical dynamics the longest time-scale in the system. $(\xi \sim |T - T_c|^{-\nu}, \quad \tau \sim \xi^z)$

- microscopic details are integrated out (white noises)
- critical dynamics can be approximately described by **hydrodynamic modes and order parameter**

	solution of	time evolution	
propagating modes e.g. sound wave	**wave eq.**	$\sim \mathrm{e}^{\pm i c_s \|\boldsymbol{p}\| t}$	(move)
dissipative modes e.g. heat diffusion	**diffusion eq.**	$\sim \mathrm{e}^{-\Gamma \boldsymbol{p}^2 t}$	(damp)

Dissipative modes are relevant in critical regions from scale transformation

identifying dissipative modes is important to deal with critical dynamics

2. Identification of dissipative modes

$$(T \neq T_c)$$

YS, H. Fujii, K. Itakura and O. Morimatsu, arXiv:1309.4892

Setup

- model

relativistic $O(N)\ \varphi^4$ theory φ : order parameter $(a = 1, \cdots, N)$

$$S[\varphi] = \int dt d^3x \left[\frac{1}{2}\partial_\mu \varphi_a(x) \partial^\mu \varphi_a(x) - \frac{m_0^2}{2}\varphi_a(x)\varphi_a(x) - \frac{\lambda_0}{4!N}(\varphi_a(x)\varphi_a(x))^2 \right]$$

verify how φ relaxes in long-time scale (low frequency)

- linear response theory

relaxation of φ in near-equilibrium state corresponds to

retarded Green function G_{R} in the **equilibrium state**

$$G_{\mathrm{R}}(t, \boldsymbol{x}) = \theta(t)\ \mathrm{Tr}\{\mathrm{e}^{-\beta H}[\varphi(t, \boldsymbol{x}),\ \varphi(0, \mathbf{0})]\}$$

- 1/N expansion : typical non-perturbative expansion

Dissipative condition

$$(T \neq T_c)$$

Phenomenology

time evolution of φ
in infrared regions $\longrightarrow$ **diffusion eq.** $(\Gamma = \text{const.})$

$$G_{\text{pheno.}}(p) \sim \frac{1}{\boldsymbol{p}^2 + m^2 - i\Gamma^{-1}p_0}$$

Relativistic field theory

retarded Green function : $G_{\mathrm{R}}(p) = \dfrac{1}{-p_0^2 + \boldsymbol{p}^2 + m_0^2 - \mathrm{Re}\Sigma_{\mathrm{R}}(p) - i\mathrm{Im}\Sigma_{\mathrm{R}}(p)}$

dissipative condition $(p_0 \ll |\boldsymbol{p}|)$

$$\Gamma^{-1} = \lim_{\boldsymbol{p}\to\boldsymbol{0}} \lim_{p_0\to 0} \left(\frac{\partial}{\partial p_0} \mathrm{Im}\,\Sigma_{\mathrm{R}}(p_0, \boldsymbol{p}) \right) \neq 0, \ \infty$$

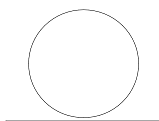
Order of Self-energies

dissipation is a result of **scatterings** between particles in heat bath

self-energies must have imaginary parts.

- 1/N expansion at LO

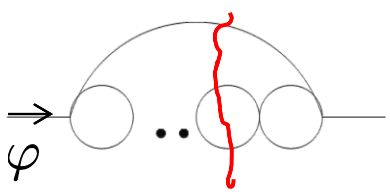
$$\Sigma_{\mathrm{R}}^{\mathrm{LO}} =$$



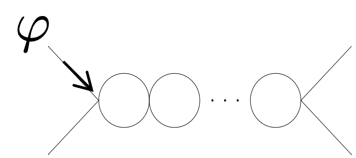
no imaginary parts (only mass shift)

- 1/N expansion at NLO

$$\Sigma_{\mathrm{R}}^{\mathrm{NLO}} =$$



imaginary part



scattering processes exist

evaluate the self-energy at NLO

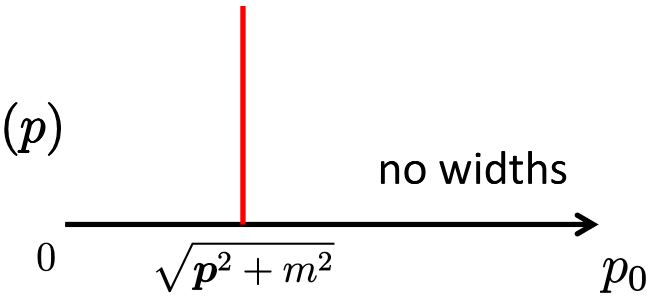
Self-energy of 1PI-NLO

YS, H. Fujii, K. Itakura and O. Morimatsu, arXiv:1309.4892

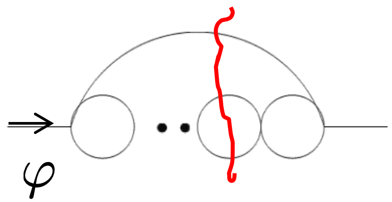
In 1PI-framework,
all internal lines of $\Sigma_{\mathrm{R}}^{\mathrm{1PI}}(p)$
are free propagators.

spectral function of $G_0(p)$

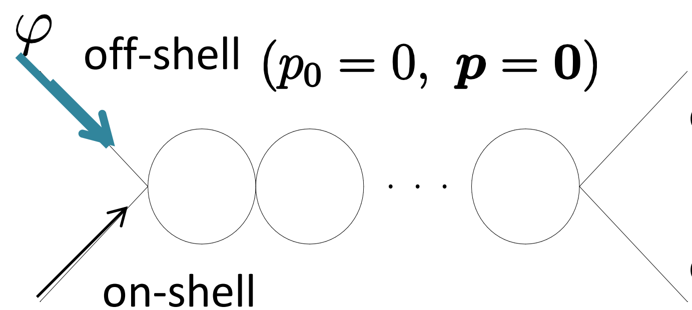
$$\rho(p) = 2\mathrm{Im}\,\langle \varphi\varphi \rangle$$



$$\Sigma_{\mathrm{R}}^{\mathrm{1PI}}(p) =$$



$$\Gamma^{-1} = \frac{\partial}{\partial p_0}\mathrm{Im}\Sigma_{\mathrm{R}}(p_0, \mathbf{0})\Big|_{p_0=0} =$$



scattering process
(off + on → on + on)

kinematically forbidden

$$\Gamma^{-1} = 0$$

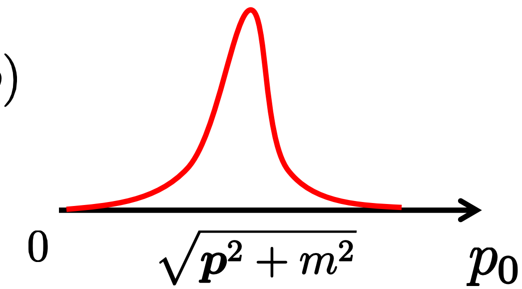
No dissipative modes in 1PI-NLO,
although the self-energy contains scattering processes.

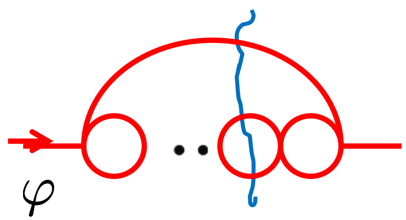
Self-energy of 2PI-NLO

In 2PI-framework,
all internal lines of $\Sigma_{\mathrm{R}}^{\mathrm{2PI}}(p)$
are **full** propagators.

full two-point function $G(p)$

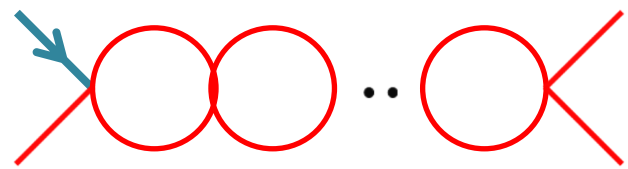
$\rho(p)$ has finite widths



$\Sigma_{\mathrm{R}}^{\mathrm{2PI}}(p) =$ 

$$\Gamma^{-1} = \frac{\partial}{\partial p_0} \mathrm{Im}\Sigma_{\mathrm{R}}(p_0, \mathbf{0}) \bigg|_{p_0=0} =$$

$\varphi \quad (p_0 = 0, \ \boldsymbol{p} = \mathbf{0})$



scattering processes
between off-shell particles

kinematically allowed

$\Gamma^{-1} \neq 0$ **dissipative modes exist in 2PI-NLO**

- dissipative mode is a result of multi-particle scatterings
- 1PI-1/N expansion at NLO fails to extract dissipative processes
- dissipative modes appear in 2PI-1/N expansion at NLO

- dissipative modes play the important role in critical dynamics

3. Dynamic critical exponent

$$(T = T_c)$$

Dynamic scale invariance

Correlation-length and -time diverge at $T = T_c$

$$\boxed{\xi \sim |T - T_c|^{-\nu}\,, \quad \tau \sim \xi^z}$$

→ **The system becomes invariant under dynamic scale transformation**

$$\boldsymbol{p} \to b\boldsymbol{p}, \quad p_0 \to b^z p_0 \qquad (b > 1)$$

The retarded two-point function changes :

$$G_{\mathrm{R}}(p_0, \boldsymbol{p}) = b^{2-\eta}\, G_{\mathrm{R}}(b^z p_0, b\boldsymbol{p}) \sim \frac{1}{\boldsymbol{p}^{2-\eta}}\, F\left(\frac{p_0}{\boldsymbol{p}^z}\right)$$

η : static ($p_0 = 0$) exponent, z : ratio between powers of p_0 and $\boldsymbol{p}$

$F(x)$: shape function

Scaling form

useful forms to evaluate critical exponents

$$G_{\mathrm{R}}(p_0, \boldsymbol{p}) \sim \frac{1}{\boldsymbol{p}^{2-\eta}} F\left(\frac{p_0}{\boldsymbol{p}^z}\right) \qquad (T = T_c)$$

Both $G_{\mathrm{R}}(0, \boldsymbol{p})$ and $G_{\mathrm{R}}(p_0, \mathbf{0})$ are finite

$$\Rightarrow F(0) = \mathrm{const.}, \quad \lim_{x\to\infty} F(x) = x^{-(2-\eta)/z}$$

- scaling form at $p_0 = 0$ $\Rightarrow$ $G_{\mathrm{R}}(0, \boldsymbol{p}) \sim \dfrac{1}{\boldsymbol{p}^{2-\eta}}$

  evaluation of static critical exponent η

- scaling form at $\boldsymbol{p} = \mathbf{0}$ $\Rightarrow$ $G_{\mathrm{R}}(p_0, \mathbf{0}) \sim \dfrac{1}{p_0^{(2-\eta)/z}}$

  evaluation of dynamic critical exponent z

The exponent z at O(1)

two-point function has the dissipative term in the infrared region

$$G_{\mathrm{R}}(p) \sim \frac{1}{-i\Gamma^{-1}p_0 + \boldsymbol{p}^2} = \frac{1}{\boldsymbol{p}^2}\,\frac{1}{1 - i\Gamma^{-1}\frac{p_0}{\boldsymbol{p}^2}}$$

$$G_{\mathrm{R}}(p) \sim \frac{1}{\boldsymbol{p}^{2-\eta}}\,F\left(\frac{p_0}{\boldsymbol{p}^z}\right)$$

$\boxed{z = 2}$ **Dissipative term determines z at LO**

If one evaluates the exponent z in the framework without dissipative terms, one obtains a wrong result even at LO.

1PI-1/N at NLO : **no dissipative terms**

$$G_{\mathrm{R}}(p) \sim \frac{1}{-a_1 p_0^2 + a_2 \boldsymbol{p}^2} \quad \Rightarrow \quad z_{(\text{no dis.})} = 1?$$

(wrong even at LO)

outline of our evaluation

2PI formalism (self-consistent eq.)

$$G_{\mathrm{R}}^{-1}(p_0,\boldsymbol{p}) = G_{\mathrm{R},0}^{-1}(p_0,\boldsymbol{p}) - \Sigma_{\mathrm{R}}[G]$$

$$G_{\mathrm{R}}(p) \sim \frac{1}{\boldsymbol{p}^{2-\eta}}\, F\left(\frac{p_0}{\boldsymbol{p}^z}\right)$$: “Full” dissipative propagator

It is difficult to determine the shape function $F(p_0/\boldsymbol{p}^z)$

We use $G_{\mathrm{R}}(p) \sim \dfrac{1}{-i\Gamma^{-1}p_0 + \boldsymbol{p}^{2-\eta}}$

and evaluate the exponent z.

outline of evaluation

- **static part** $(p_0 = 0) \longrightarrow \eta$

$$G_{\mathrm{R}}^{-1}(0,\boldsymbol{p}) = G_{\mathrm{R},0}^{-1}(0,\boldsymbol{p}) - \Sigma_{\mathrm{R}}(0,\boldsymbol{p})$$

⇒ evaluate self-consistently

Alford, Berges, Cheyne, PRD 70, 125002 (2004)

$$1 = \frac{4\eta(1-2\eta)\cos(\pi\eta)}{N(3-\eta)(2-\eta)\sin^2(\pi\eta/2)}$$

- **dynamic part** $(\boldsymbol{p} = \boldsymbol{0}) \longrightarrow \kappa$

$$G_{\mathrm{R}}^{-1}(p_0,\boldsymbol{0}) = G_{\mathrm{R},0}^{-1}(p_0,\boldsymbol{0}) - \Sigma_{\mathrm{R}}(p_0,\boldsymbol{0})$$

⇒ evaluate at leading-log-order (LLO)

$$G_{\mathrm{R}}^{-1}(p_0,\boldsymbol{0}) \sim -i\Gamma^{-1}p_0\ (1-\kappa\ln p_0) \sim -i\Gamma^{-1}p_0^{1-\kappa} \sim -i\Gamma^{-1}p_0^{(2-\eta)/z}$$

$$z = \frac{2-\eta}{1-\kappa}$$

Result (preliminary)

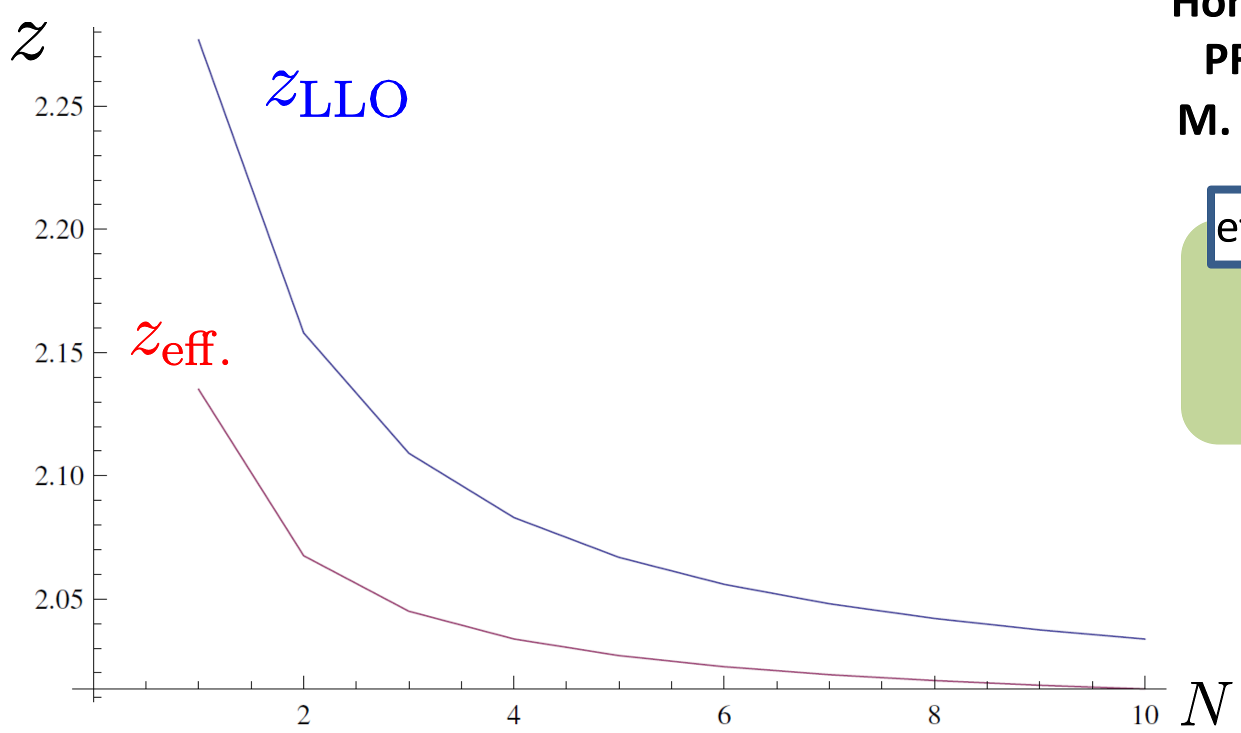
the result of our calculation

$$z_{\mathrm{LLO}} = \frac{2-\eta_{\mathrm{2PI}}}{1-\kappa}$$

$$1 = \frac{4\eta(1-2\eta)\cos(\pi\eta)}{N(3-\eta)(2-\eta)\sin^2(\pi\eta/2)}$$

$$\left\{\begin{aligned}
\kappa &= \frac{1}{2\pi^2 N}\left[\mathcal{B}^{-1}(\eta_{\mathrm{2PI}}) - \mathcal{A}^{-1}(\eta_{\mathrm{2PI}})\right] \\
\mathcal{A}(\eta_{\mathrm{2PI}}) &= \frac{1}{8\pi^{3/2}}\,\frac{\Gamma\left(\frac{1}{2}-\eta\right)\ \left[\Gamma\left(\frac{1+\eta}{2}\right)\right]^2}{\left[\Gamma\left(\frac{1-\eta}{2}\right)\right]^2\ \Gamma(1+\eta)} \\
\mathcal{B}(\eta_{\mathrm{2PI}}) &= \int \frac{d\hat{\boldsymbol{k}}}{(2\pi)^3}\,\frac{\hat{\boldsymbol{k}}^{\eta-2} + |\boldsymbol{e}-\hat{\boldsymbol{k}}|^{\eta-2}}{1+\hat{\boldsymbol{k}}^2 + |\boldsymbol{e}-\hat{\boldsymbol{k}}|^2}
\end{aligned}\right.$$

z (preliminary)



Hohenberg, Halperin and Ma, PRL 29, 1548 (1972)
M. Suzuki, PTP 53, 97 (1975)

effective theory (model A)

$$z_{\rm eff.} = 2 + \frac{4}{3\pi^2 N}$$

our result

$$z_{\rm LLO} = \frac{2-\eta_{\rm 2PI}}{1-\kappa}$$

$z_{\rm LLO}$ agreeds with $z_{\rm eff.}$ in $N \to \infty$.

$N \leq 1$ out of validity region of $1/N$ expansion

spatial dimensions d=3

summary

dissipative modes

- dominate critical dynamics
- identify dissipative modes microscopically
  by 2PI-1/N expansion at NLO

The exponent z

- characterize time scales of critical dynamics $\tau \sim \xi^z$
- microscopic calculation using resummation framework

static part : self-consistent calculation

dynamic part : leading-log-order calculation

future prospects

- evaluate z self-consistently
- find correspondence between 2PI formalism and
  effective theories of critical dynamics (mode coupling theories)