Fluctuations and the QCD Phase Structure from Effective Theories

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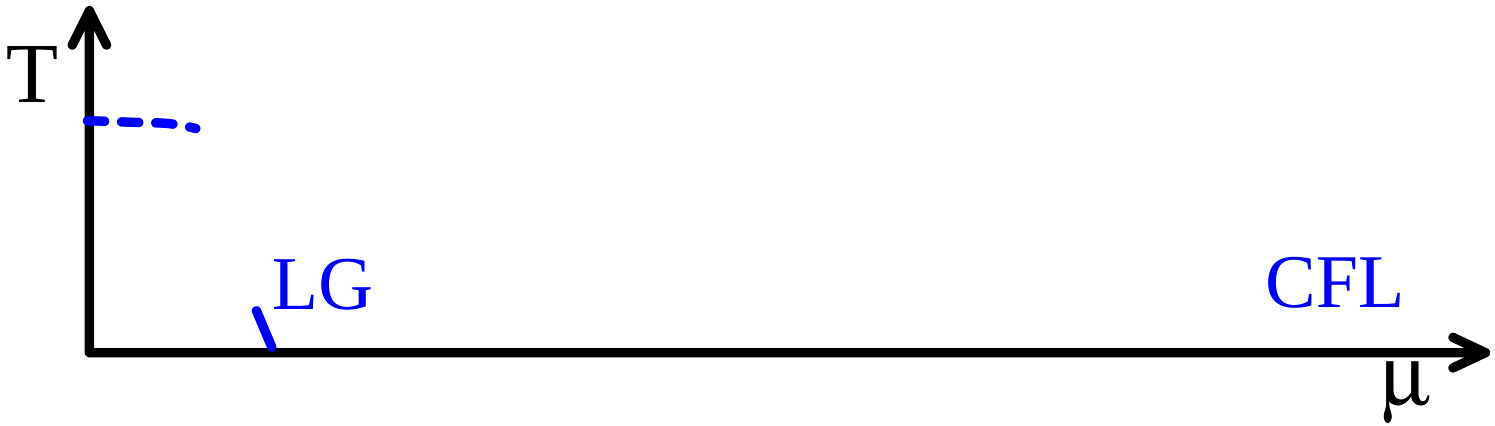
based on:

- C.S. and K. Redlich, Phys. Rev. D **86**, 014007 (2012).
- C.S., I. Mishustin and K. Redlich, arXiv:1308.3635 [hep-ph].
- P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C.S., Phys. Rev. D **88**, 014506 (2013); arXiv:1307.5958 [hep-lat], to appear in PRD.

- **QCD phase diagram? modeling QCD thermodynamics?**



  - good effective theories $\Leftarrow$ good symmetries; $p\,, N_c\,, N_f\,, T\,, \mu_B$
  - effective theories, toy models $\Leftarrow$ lattice QCD (pure YM, zero μ_B)

Outline

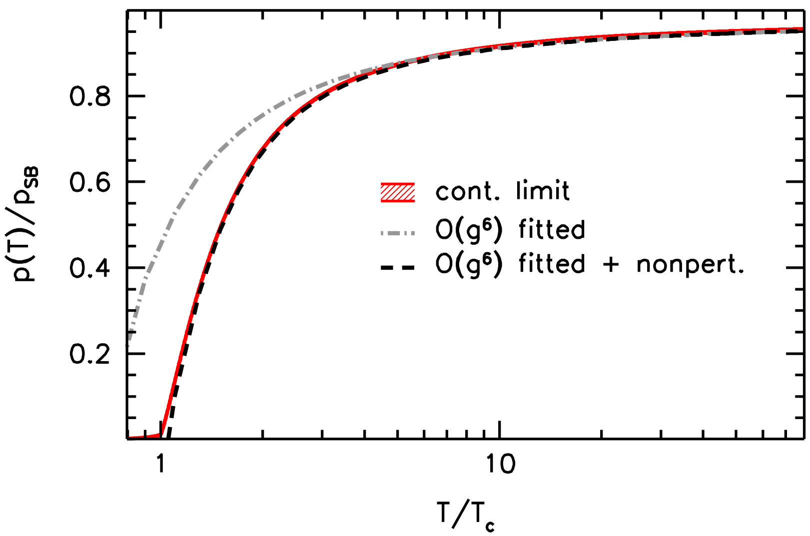
- **Yang-Mills thermodynamics**
- **interplay between color-electric and color-magnetic gluons**
- **how to characterize (de)confinement in QCD?**

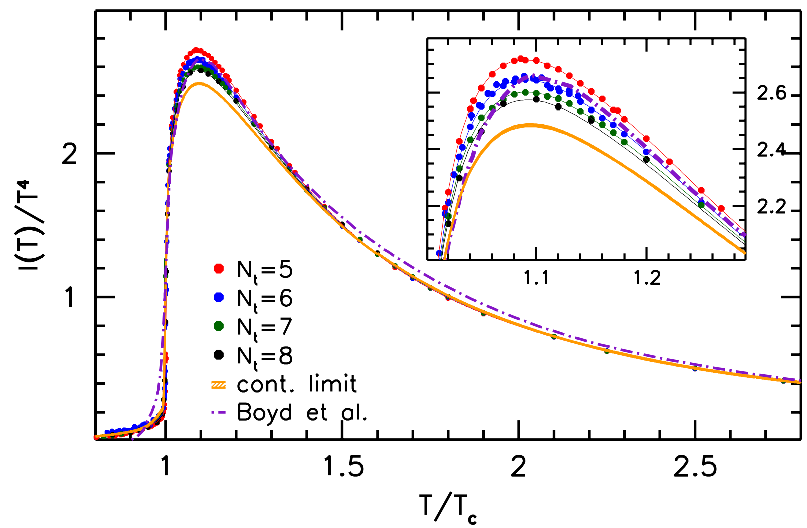
I. Modeling Yang-Mills Thermodynamics

Yang-Mills thermodynamics from Lattice QCD

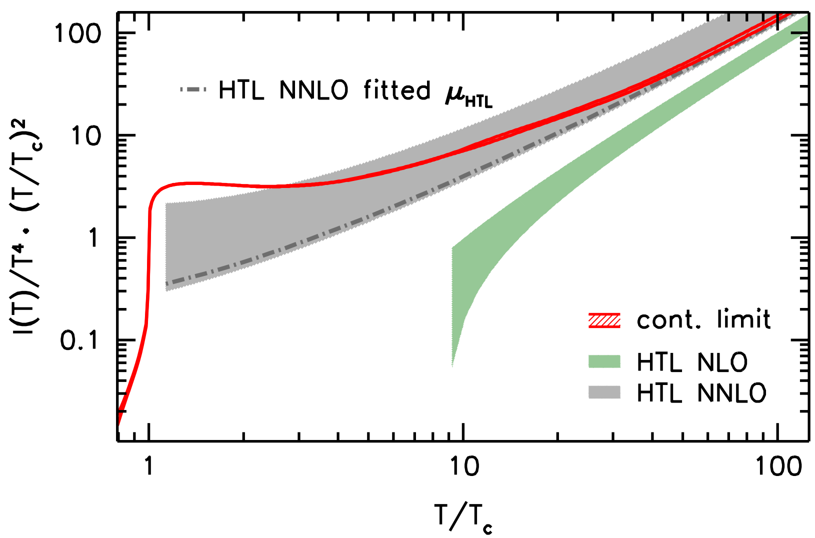
[Borsanyi et al. (12)]

- EoS *does not reach SB limit.* $I = \mathcal{E} - 3P$ *does not vanish.*





- $I/T^2 T_c^2 \sim$ *constant in intermediate temperatures.*



cf. $I^{\rm pert} \propto (gT)^4$

$\Rightarrow \quad I^{\rm pert}/T^2 \propto (gT)^2$

hot gluon matter in deconf. phase:

still nontrivial!

Residual interaction above $T_{\rm dec}$

- quasi-particle approach: $m_g(T) \sim g_{\rm eff}(T)T$ [Peshier et al., (96), Levai-Heinz (97)]
  interaction measure (if $m_g/T \ll 1$): constant m_g? what is $g_{\rm eff}$?

$$I \sim I_0 + C m_g^2 T^2 + \cdots \quad \Rightarrow \quad I/T^2 \sim C m_g^2 \,.$$

- how to model these aspects? $\Rightarrow$ effective theory approach
  $\cdots$ Z(3) center symmetry and Polyakov loops Φ

- **Polyakov loop model** [Pisarski (00)]
  made based on $Z(3)$ symmetry:

$$\mathcal{U} = a(T)\bar{\Phi}\Phi + b(T)\left(\bar{\Phi}^3 + \Phi^3\right) + c(T)\left(\bar{\Phi}\Phi\right)^2 + \cdots$$

  - T-dep. coefficients *unconstrained* by $Z(3)$
    $\Rightarrow$ putting T-dep. by hand & fitting Lattice EoS **not unique!**
  - drawback: insufficient $\mathcal{U}$ for fluctuations [CS et al. (06), Lo et al. (13)]
  - where T-dep. comes from? $\cdots$ thermal gluon excitations $\sim \mathcal{L}_{\rm YM}$
    $\Rightarrow$ closer contact with the underlying theory

Deriving partition function from YM Lagrangian

- background field method, a constant uniform background $\bar{A}_0$

$$A_\mu = \bar{A}_\mu + g\check{A}_\mu$$
$$\bar{A}^a_\mu = \bar{A}^a_0 \delta_{\mu 0}\,, \quad \bar{A}_0 = \bar{A}^3_0 T^3 + \bar{A}^8_0 T^8$$

$$\sum_n \ln\det\left(D^{-1}\right) = \ln\det\left(1 - \hat{L}_A e^{-|\vec{p}|/T}\right)$$

[Gross-Pisarski-Yaffe (81)]

- Polyakov loop matrix in adjoint representation (8x8 matrix)

$$\hat{L}_A = \text{diag}\left(1\,,1\,,e^{i(\phi_1-\phi_2)}\,,e^{-i(\phi_1-\phi_2)}\,,e^{i(2\phi_1+\phi_2)}\,,e^{-i(2\phi_1+\phi_2)}\,,e^{i(\phi_1+2\phi_2)}\,,e^{-i(\phi_1+2\phi_2)}\right)$$

rank of SU(3) group $= 2 \;\Rightarrow$ elements expressed in 2 variables

- express in terms of

$$\Phi = \text{tr}\hat{L}_F/3\,,\ \bar{\Phi} = \text{tr}\hat{L}^\dagger_F/3\,,\ \hat{L}_F = \text{diag}\left(e^{i\phi_1}\,,e^{i\phi_1}\,,e^{-i(\phi_1+\phi_2)}\right)\,.$$

- full thermodynamic potential:

$$\Omega = \underbrace{\Omega_g}_{\sim a(T)\bar{\Phi}\Phi?} + \underbrace{\Omega_{\text{Haar}}}_{\text{responsible for Z(3) breaking}}$$

- full thermodynamics potential: $\Omega = \Omega_g + \Omega_{\rm Haar}$ [CS-Redlich (2012)]

$$\Omega_g = 2T \int \frac{d^3p}{(2\pi)^3} \ln \left(1 + \sum_{n=1}^{8} C_n(\Phi, \bar{\Phi})\, e^{-n|\vec{p}|/T} \right) ,$$

$$\Omega_{\rm Haar} = -a_0 T \ln \left[1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2 \right] ,$$

$$C_1 = C_7 = 1 - 9\bar{\Phi}\Phi\,, \quad C_2 = C_6 = 1 - 27\bar{\Phi}\Phi + 27\left(\bar{\Phi}^3 + \Phi^3\right) ,$$

$$C_3 = C_5 = -2 + 27\bar{\Phi}\Phi - 81\left(\bar{\Phi}\Phi\right)^2 ,$$

$$C_4 = 2\left[-1 + 9\bar{\Phi}\Phi - 27\left(\bar{\Phi}^3 + \Phi^3\right) + 81\left(\bar{\Phi}\Phi\right)^2 \right] , \quad C_8 = 1$$

$\Rightarrow$ energy distributions solely determined by group characters of SU(3)

– no free parameter in Ω_g

– one parameter in $\Omega_{\rm Haar}$: $a_0 \Leftrightarrow T_c^{\rm lat} = 270$ MeV

all the phenomenological potentials deduced from Ω_g!

$\Omega \to \alpha(T)\bar{\Phi}\Phi + \Omega_{\rm Haar}\,, \quad \Omega \to$ polynomials

Character expansion of Ω_g

- effective gluon mass: $M_g(T) \sim g(T)T$ *nonperturbative!*

$$M_g/T \gg 1 \quad \Leftrightarrow \quad g(T) \gg 1$$

$$\Omega_g \sim \frac{T^2 M_g^2}{\pi^2} \sum_{n=1}^{8} \frac{C_n}{n} K_2\left(n M_g/T\right) .$$

- effective action in the strong coupling exp. [Wozar-Kaestner-Wipf-Heinzl-Pozsgay (06)]

$$\Omega_{\text{eff}}^{(\mathrm{SC})} = \lambda_{10} S_{10} + \lambda_{20} S_{20} + \lambda_{11} S_{11} + \lambda_{21} S_{21}$$

Ω_g vs. $\Omega^{(\mathrm{SC})}$: one-to-one correspondence!

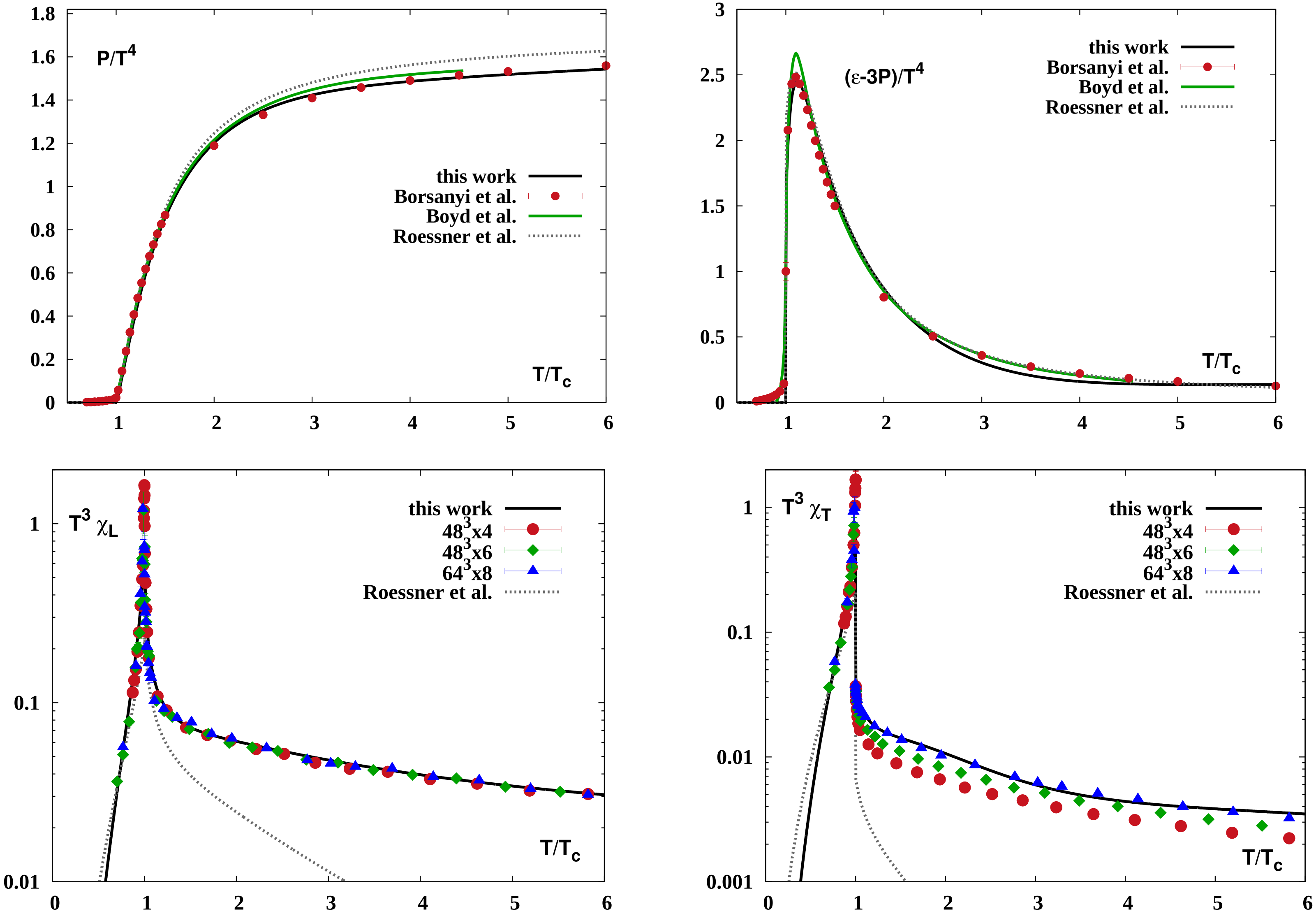
$$C_{1,7} = S_{10}\,, \quad C_{2,6} = S_{21}\,, \quad C_{3,5} = S_{11}\,, \quad C_4 = S_{20}$$

- T-dep. coefficients λ_{pq}
  - SC: derived from the underlying action
  - extracted from $\Omega_g \Rightarrow$ 1st-order phase transition with $\langle\Phi\rangle$

Truncation?

$$\frac{\Omega}{T^4} = -a(T)\bar{\Phi}\Phi + b(T)\ln M_{\text{Haar}}\left(\bar{\Phi},\Phi\right) + c(T)\left(\bar{\Phi}^3+\Phi^3\right) + d(T)\left(\bar{\Phi}\Phi\right)^2.$$

[Lo-Friman-Kaczmarek-Redlich-CS (2013)]



Thermodynamics

- any finite temperature in confined phase: $\Phi = 0$ thus $\Omega_{\rm Haar} \sim 0$

$$\Omega_g(\Phi = \bar{\Phi} = 0) \sim 2T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-|\vec{p}|/T}\right)$$

wrong sign! $\Rightarrow$ unphysical EoS $p, s, \epsilon < 0$

Gluons are NOT correct dynamical variables below T_c!

★ "conventional" potential $\Omega_{\rm polynomial} \Rightarrow$ gluon confinement NOT seen
★ correct physics from the terms obtained in character expansion.

- cf. PNJL/PQM: quarks are suppressed but exist at any T.

[Meisinger-Ogilvie (96), Fukushima (03), Ratti et al. (06)]

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ - A\!\!\!/\right) q + G\left(\bar{q}q\right)^2 - \mathcal{U}(\bar{\Phi}, \Phi)\,, \quad A_\mu = \delta_{\mu 0} A^0$$

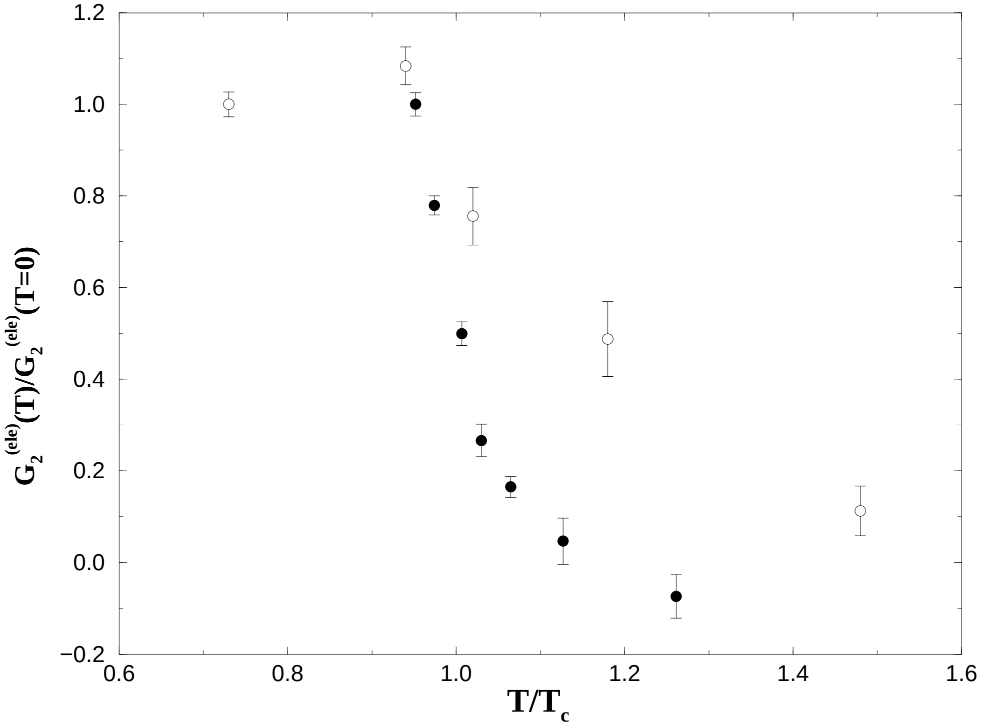
$$\Omega_q = -d_q\, T \int \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3\Phi e^{-E_+/T} + 3\bar{\Phi} e^{-2E_+/T} + e^{-3E_+/T}\right]$$

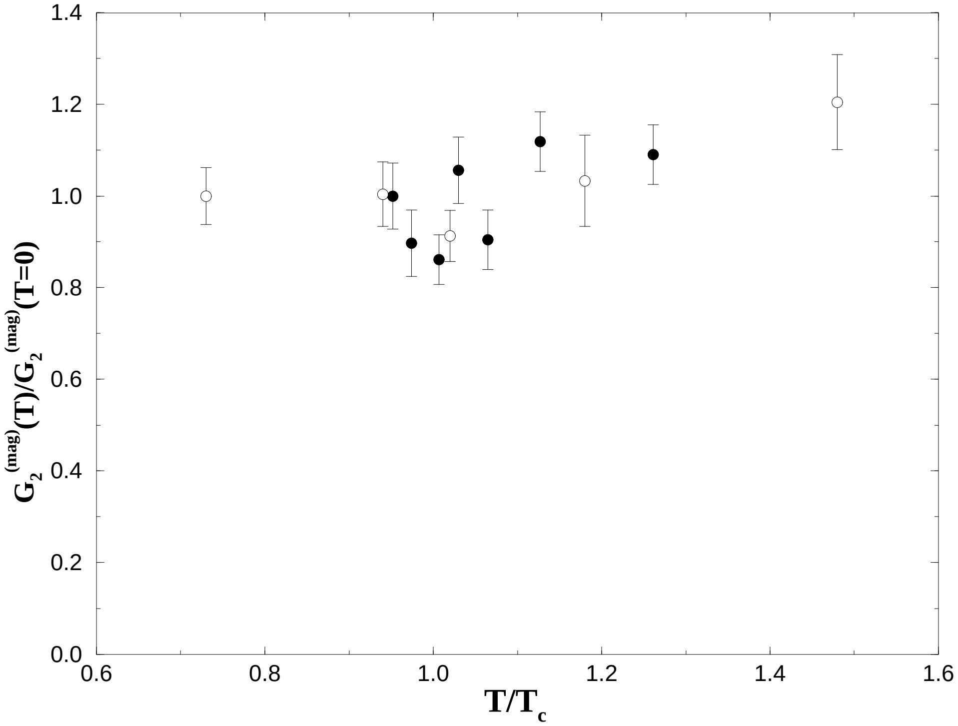
$\langle\Phi\rangle \simeq 0$ at low T: 1- and 2-quark states *thermodynamically* irrelevant
$\Rightarrow$ *mimicking confinement*

II. Chromoelectric and Chromomagnetic Dynamics

Gluon condensate from LQCD

[D'Elia-Di Giacomo-Meggiolaro (02)]





$$D_{\mu\nu\sigma\lambda}(x) = \langle g^2 \mathrm{tr} G_{\mu\nu}(x)\Phi(x,0)G_{\sigma\lambda}(0)\Phi(0,x)\rangle \,,$$

$$\Phi(x,y) = \mathcal{P}\exp\left[ig\int_x^y dx^\mu A_\mu(x)\right] \,.$$

Chromoelectric vs. chromomagnetic gluons

- $\langle \Phi \rangle \to 1$ limit: $P \to P_{\rm ideal}\,, I \to 0 \cdots$ QCD trace anomaly?
  $\Rightarrow$ scale symmetry breaking and a dilaton [Schechter (80)]

$$V_\chi = \frac{B}{4}\left(\frac{\chi}{\chi_0}\right)^4 \left[\ln\left(\frac{\chi}{\chi_0}\right)^4 - 1\right]$$

- *electric sector* $(G_{0i}^2 \sim E^2)$ changes qualitatively with T:
  deconfinement phase transition driven by electric gluons
- *magnetic sector* $(G_{ij}^2 \sim B^2)$ unaffected:
  string tension non-vanishing at any T [Borgs (85), Manousakis-Polonyi (87)]

dilatons can survive above T_c*!*

$\Phi \Leftrightarrow A_0$: electric $\chi \Leftrightarrow G_{\mu\nu}G^{\mu\nu}$: electric plus magnetic

cf. $\langle E^2 \rangle_T, \langle B^2 \rangle_T$ from D'Elia-Di Giacomo-Meggiolaro (02)

non-vanishing $\langle \chi \rangle \Leftrightarrow$ broken scale symmetry $\Leftrightarrow$ magnetic confinement

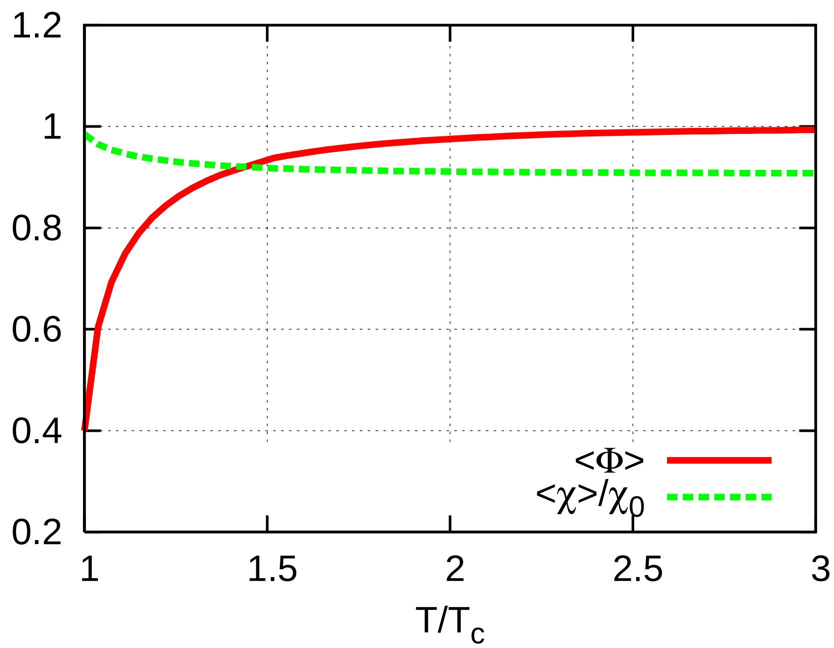
An effective theory [CS-Mishustin-Redlich (13)]

- effective potential

$$\Omega(\Phi, \chi; T) = \Omega_g(\Phi; T) + \Omega_{\rm Haar}(\Phi; T) + V_\chi(\chi) + V_{\rm mix}(\Phi, \chi)\,,$$

$$V_{\rm mix}(\Phi, \chi) = G_{\phi\chi}\left(\frac{\chi}{\chi_0}\right)^4 \bar{\Phi}\Phi$$: invariant under Z(3) and scale sym.

- two condensates: $\chi/\chi_0 = \exp\left[-G_{\phi\chi}\bar{\Phi}\Phi/B\right]$



- $\langle\chi\rangle$ at higher temperature? $\cdots$ magnetic scale $g^2(T)T$!

- residual interaction at high temperature: *magnetic confinement*
  - matching to 3d YM theory (high T effective theory)

$$\mathcal{D} \sim \langle B^2 \rangle e^{-|\vec{x}|/\xi}\,, \quad \sigma_s \sim \langle B^2 \rangle \xi^2 \quad \Leftrightarrow \quad \text{3-dim YM: } \sqrt{\sigma_s} = c_s g^2(T)T$$

$$\langle B^2 \rangle = c_B \left(g^2(T)T\right)^4 \quad c_B = \frac{6}{\pi} c_s^2 c_m^2\,, \quad m_{\rm mag} = c_m g^2(T)T\,.$$

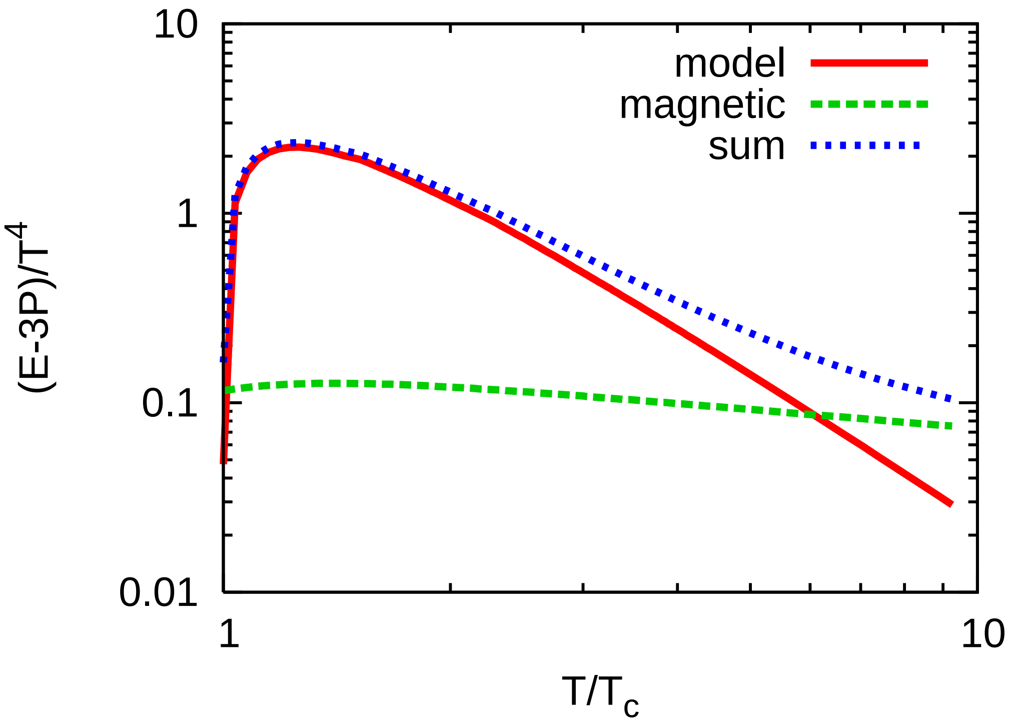
[Agasian (03)]

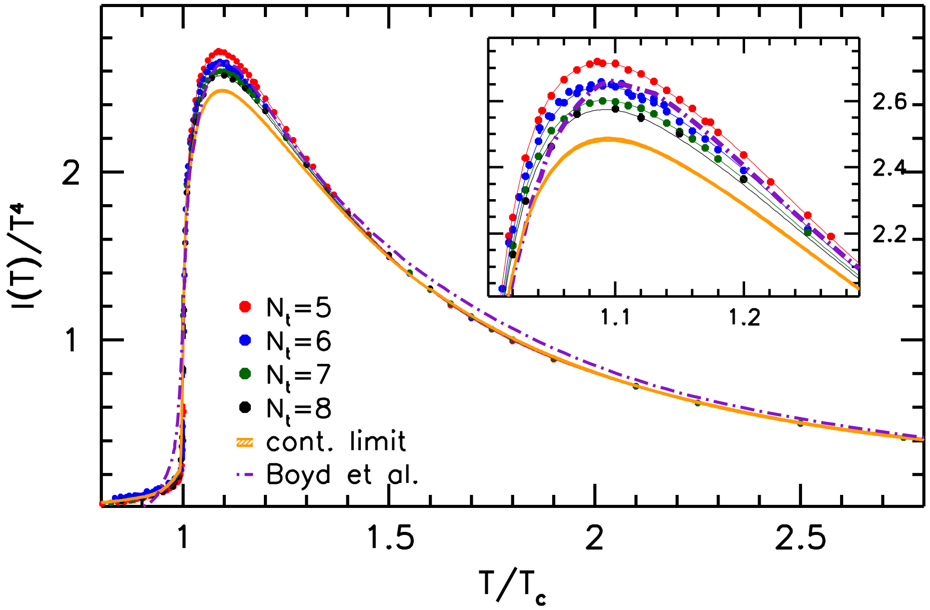
  - identify $\langle B^2 \rangle$ with $\langle \chi \rangle^4$ at "high" T: $V(\chi) \to V(g^2(T)T)$

$$\Omega = \underbrace{\Omega_g}_{\sim T^4} + \underbrace{\Omega_{\rm Haar}}_{\sim T} + \underbrace{V_{\chi+{\rm mix}}}_{\sim{\rm const} \Rightarrow \sim (g^2T)^4}$$

$$\begin{aligned} I &= \mathcal{E} - 3P \\ &= \underbrace{3\Omega_g - 2\int \frac{d^3p}{(2\pi)^3} \frac{p\sum_n nC_n e^{-np/T}}{1+\sum_n C_n e^{-np/T}}}_{\sim T^4 \Rightarrow 0 \times T^4} + \underbrace{3\Omega_{\rm Haar}}_{\sim T} + \underbrace{4\left(V_\chi + V_{\rm mix}\right)}_{\sim{\rm const} \Rightarrow \sim (g^2T)^4} \end{aligned}$$

- interaction measure: this model (L) & lattice [Borsanyi et al. (12)] (R)

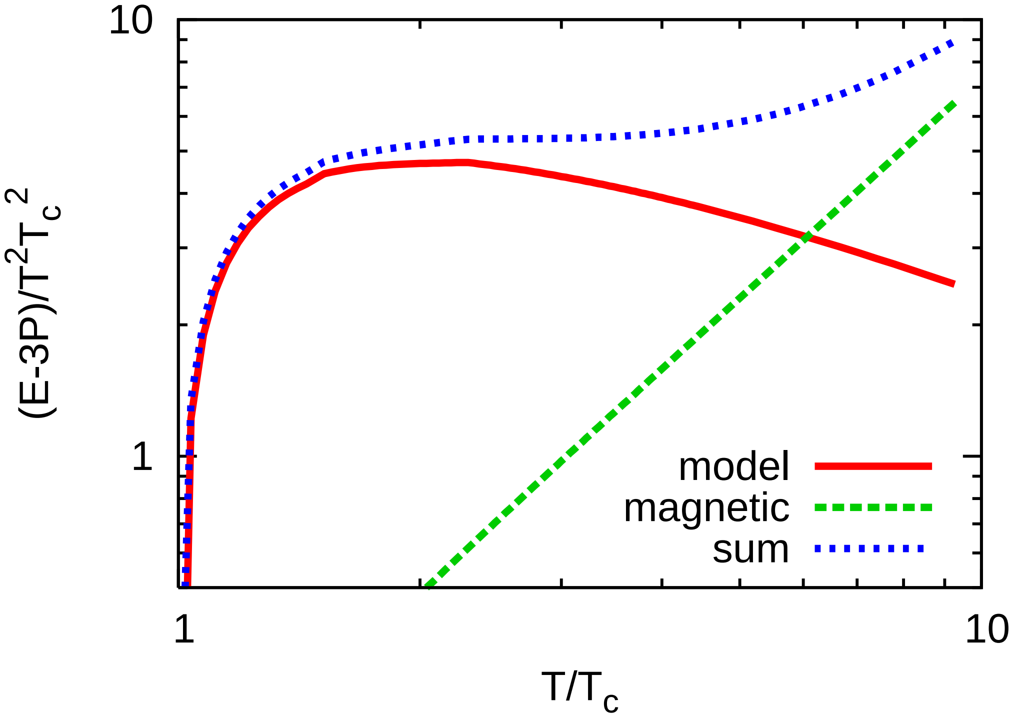


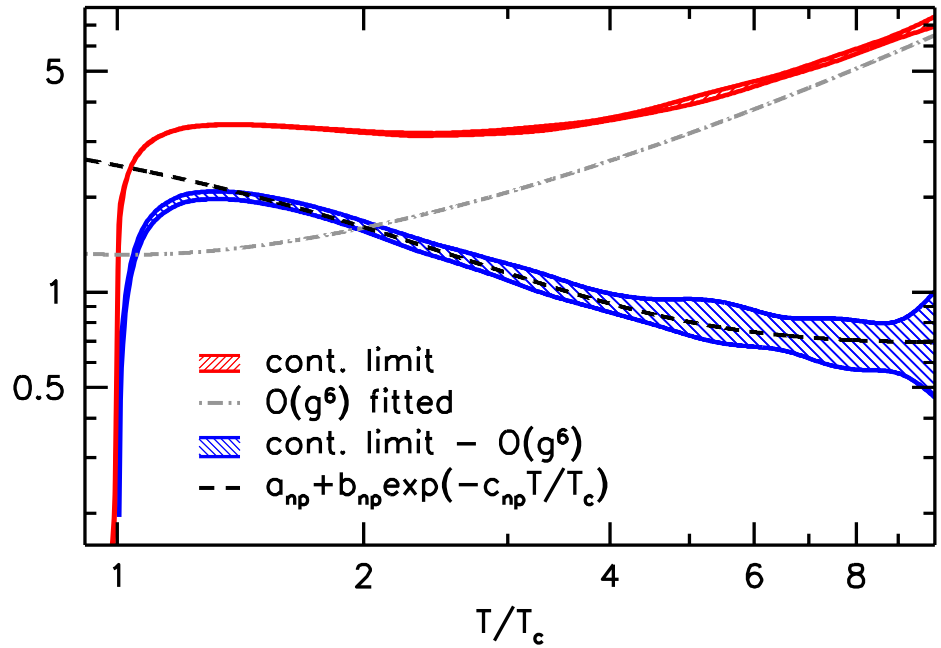


$$\Omega = \underbrace{\Omega_g}_{\sim T^4} + \underbrace{\Omega_{\rm Haar}}_{\sim T} + \underbrace{V_{\chi+{\rm mix}}}_{\sim{\rm const} \Rightarrow \sim (g^2 T)^4}$$

$$I_1(T < T_0) : V_{\chi+{\rm mix}} \sim {\rm const} \;\Rightarrow\; I_2(T_0 < T) : V_{\chi+{\rm mix}} \sim \# T^4$$

• interaction measure: this model (L) & lattice [Borsanyi et al. (12)] (R)





$$\Omega = \underbrace{\Omega_g}_{\sim T^4} + \underbrace{\Omega_{\mathrm{Haar}}}_{\sim T} + \underbrace{V_{\chi+\mathrm{mix}}}_{\sim\mathrm{const} \Rightarrow \sim(g^2T)^4}$$

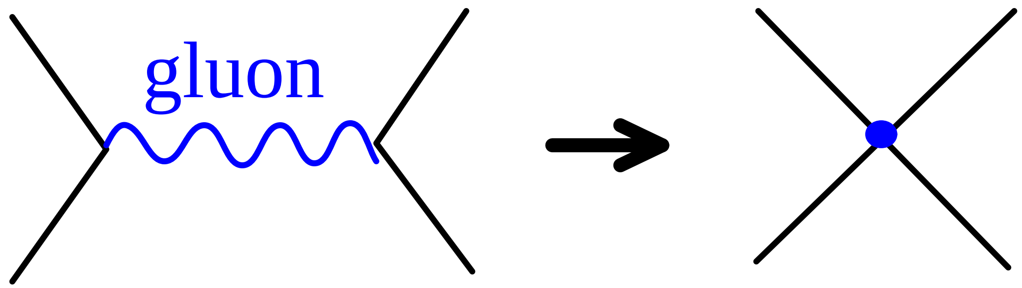
$$I_1(T < T_0) : V_{\chi+\mathrm{mix}} \sim \mathrm{const} \;\Rightarrow\; I_2(T_0 < T) : V_{\chi+\mathrm{mix}} \sim \#T^4$$

– decreasing I_1/T^2 + increasing $I_2/T^2 \Rightarrow I/T^2 \sim$ constant

– tendency of I^{lat}/T^2 is reproduced. *alternative to HTL!*

How to introduce quarks?

- non-pert. part $\sim$ **non-local** (T, Φ) effective 4-fermi interaction



ref1. Kondo (2010):
SU(2) formulation using Faddeev-Niemi decomposition and FRG
$\Rightarrow$ SU(3)???

ref2. Haas et al. (2013):
quark back-reaction to the gluon potential in PQM plus FRG

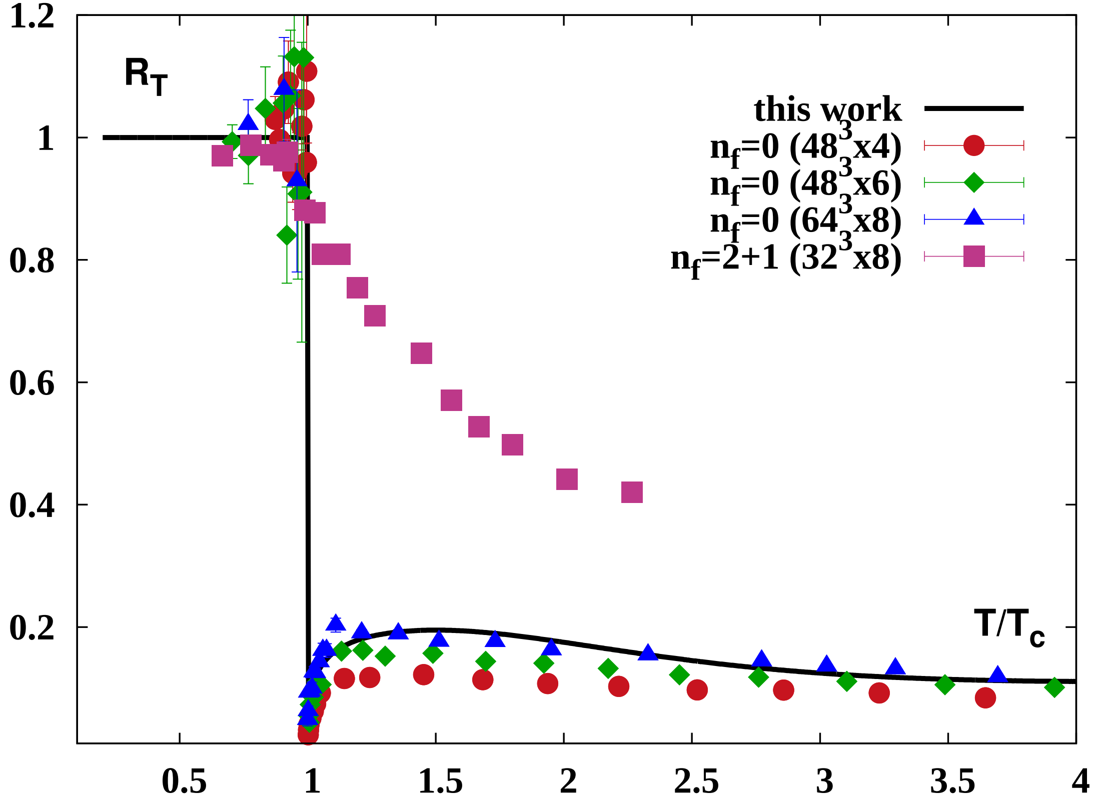
- NP/microscopic derivation w/ RG needs to be done in future.

III. Fluctuations of Φ and n_B

characterizing deconfinement of quarks and gluons in QCD?

$\Rightarrow$ Polyakov loop ...?

$\Rightarrow$ fluctuations!

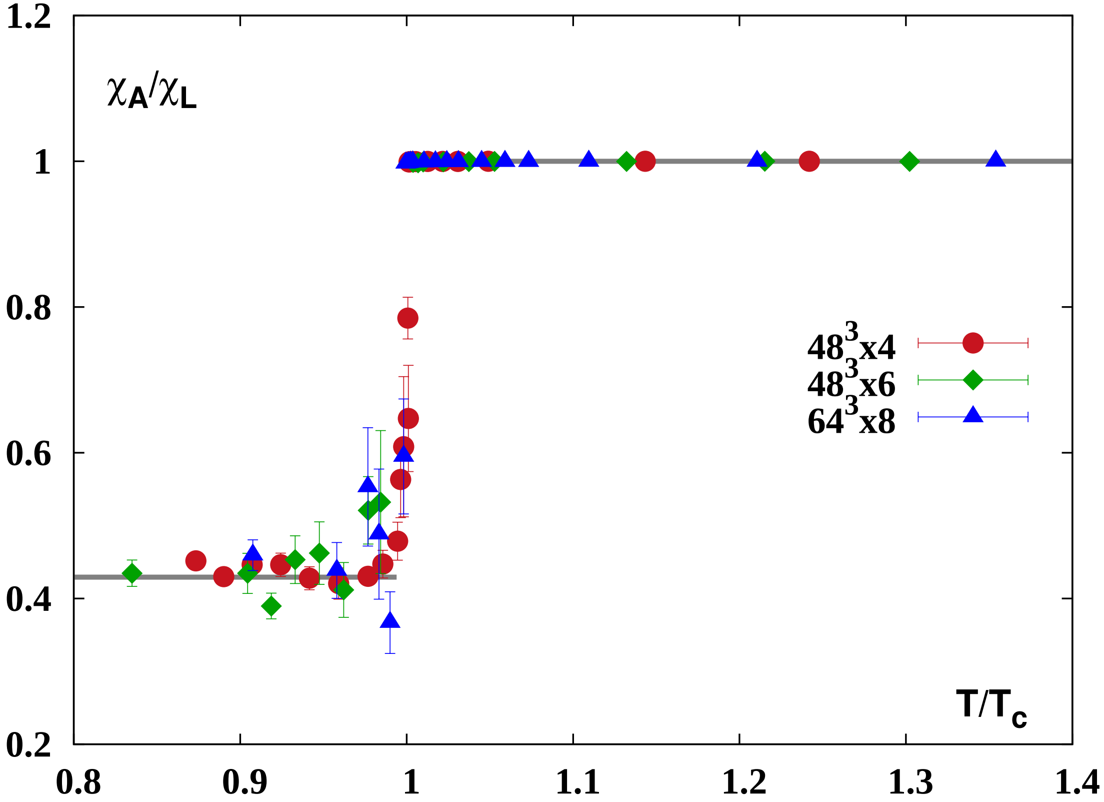


... but not all of them usefull.

$\Rightarrow$ What needs to be looked at?

Probing deconfinement with Polyakov loop susceptibilities (YM)

[Lo-Friman-Kaczmarek-Redlich-CS (2013)]



$N_c = 3 : L = L_R + iL_I$
susceptibilities of modulus, real and imaginary part of L:

$$T^3\chi_A = \frac{N_\sigma^3}{N_\tau^3}\left(\langle|L|^2\rangle - \langle|L|\rangle^2\right) \; ,$$

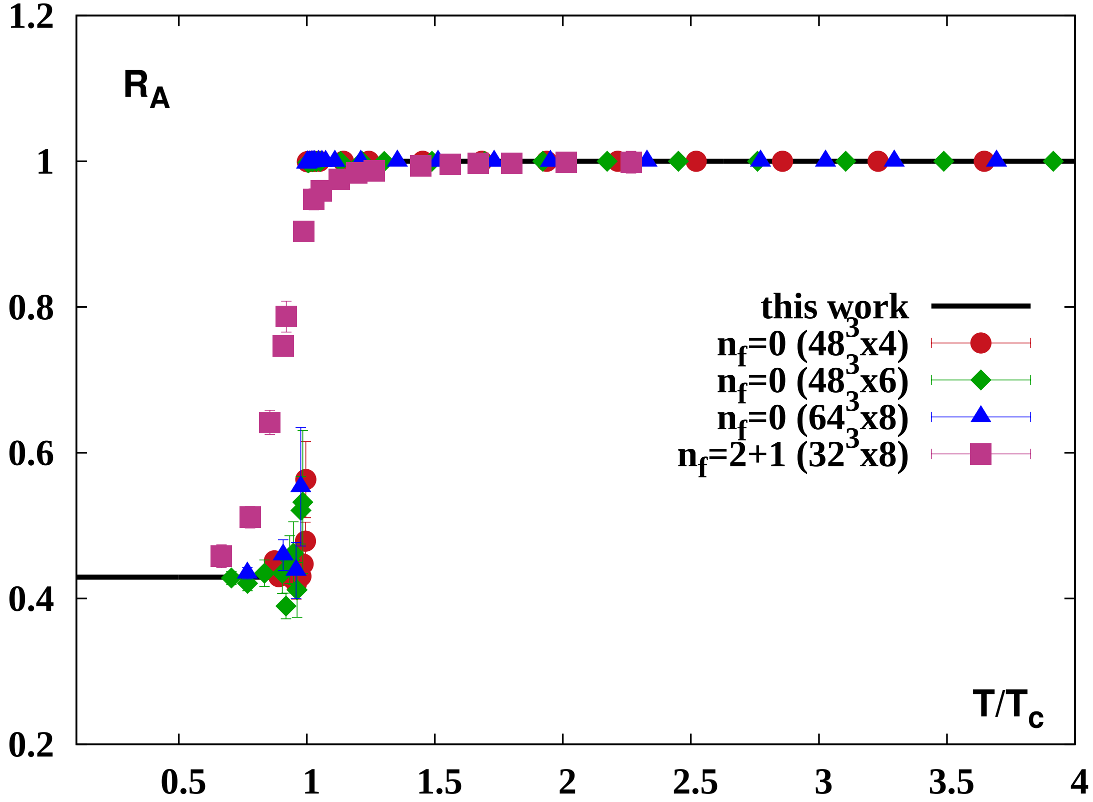
$$T^3\chi_{R,I} = \frac{N_\sigma^3}{N_\tau^3}\left(\langle L_{R,I}^2\rangle - \langle L_{R,I}\rangle^2\right) \; .$$

ratio $R_A = \chi_A/\chi_R$

- deconf. phase: $R_A \simeq 1$ from spontaneous $Z(3)$ breaking with $\langle L_I\rangle = 0$
- conf. phase: $R_A \simeq 0.43$ from unbroken $Z(3) \Leftarrow R_A^{\mathrm{Gaussian}} = 2 - \frac{\pi}{2}$

Probing deconfinement with Polyakov loop susceptibilities (QCD)

[Lo-Friman-Kaczmarek-Redlich-CS (2013)]



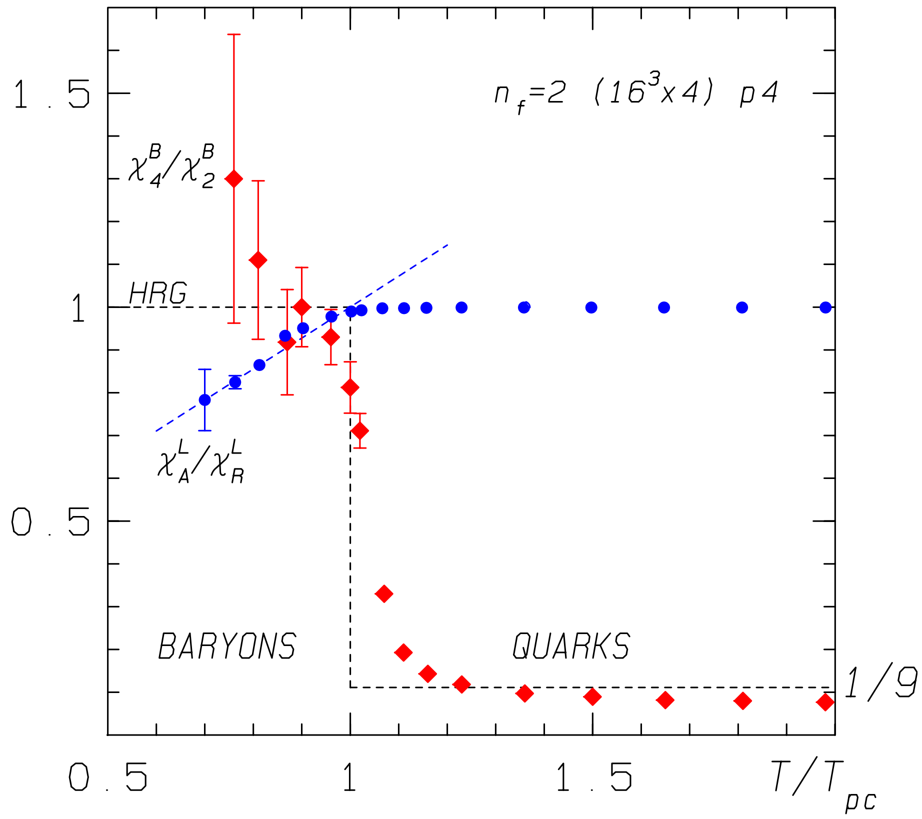
$N_f = 2+1$: HISQ

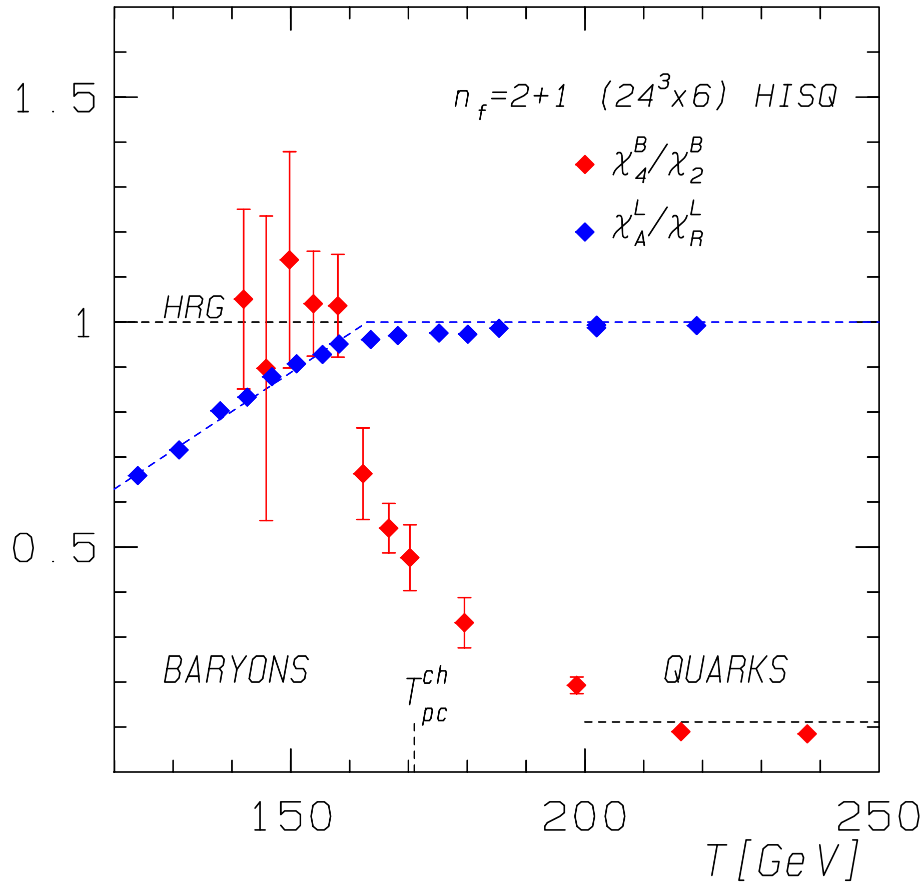
[Bazavov et al. ('12)]

$T_c = T_{\rm ch} \sim 155$ MeV

- dynamical quarks smoothen susceptibilities.
- a clear remnant of $Z(3)$ in R_A
- R_A characterizing gluon deconfinement: $T_{\rm ch} \simeq T_{\rm g-deconf}$!

How to characterize deconfinement? [Friman-Kaczmarek-Karsch-Lo-Redlich-CS]





- quark number susceptibilities: kurtosis measures fermion number B^2
- ratios χ_B^4/χ_B^2 and $R_A = \chi_A/\chi_R \Rightarrow$ quark and gluon conf.
- $T_{\rm ch} \simeq T_{\rm q-deconf} \simeq T_{\rm g-deconf}$ at $\mu = 0$!

Summary

- **derivation of gluon partition function from YM Lagrangian**
  - Polyakov loops naturally appear representing group character.
  - gluons are forbidden below T_c dynamically.
- **interplay between chromoelectric and chromomagnetic gluon dynamics**
  - identify $\langle B^2 \rangle$ with $\langle \chi^4 \rangle$
  - matching to 3-dim theory: magnetic scale $g^2(T)T$ comes in.
  - consistent with I/T^2 in LQCD

strong gauge dynamics: fluctuations and correlations for better discrimination