# QCD transition at finite temperature with Domain Wall fermions

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on behalf of the hotQCD and RBC/LLNL collaboration

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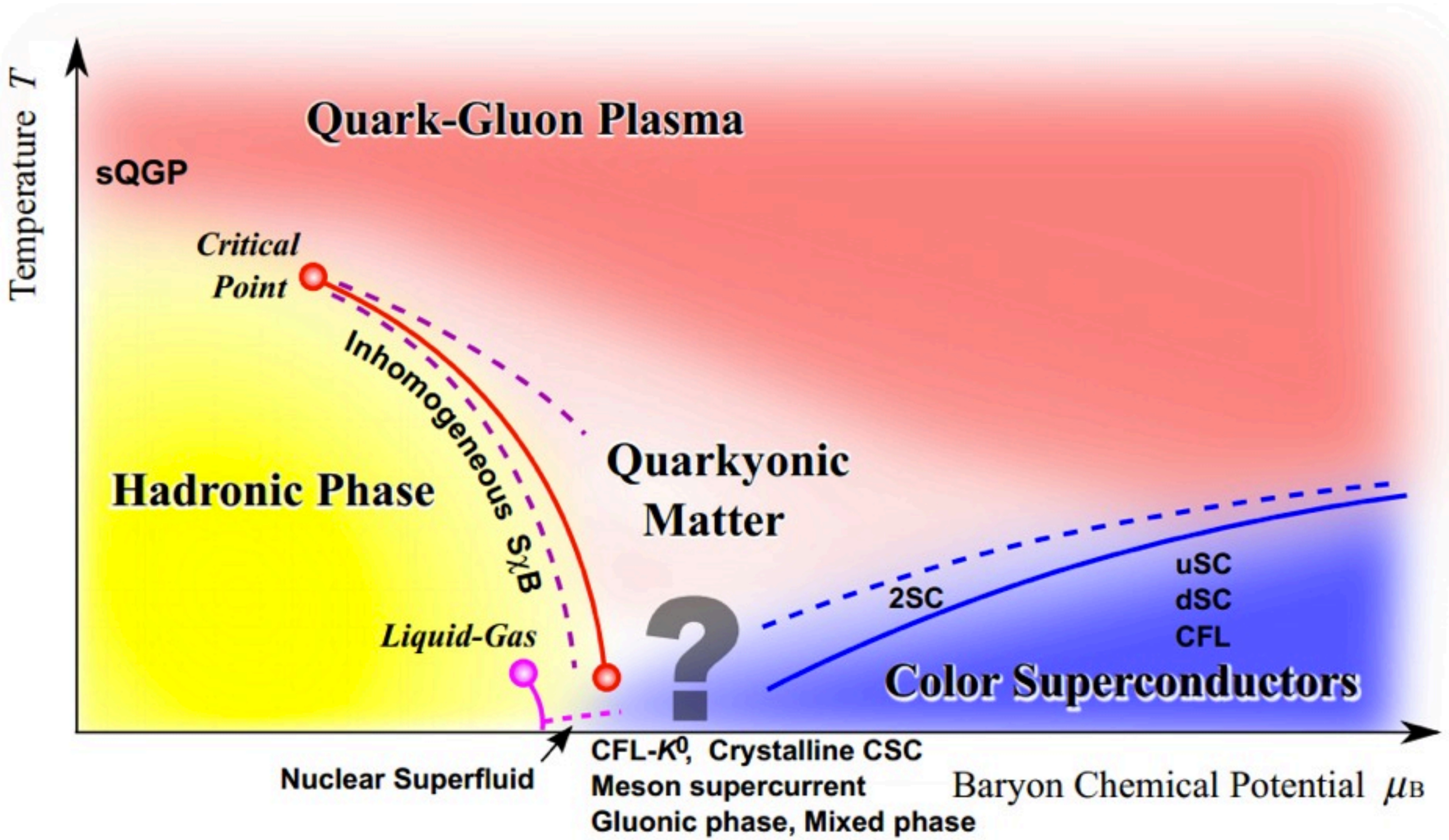
arXiv:1205.3535, arXiv:1309.4149

**New Frontiers in QCD 2013**

--- *Insight into QCD matter from heavy-ion collisions* ---

2013.11.28

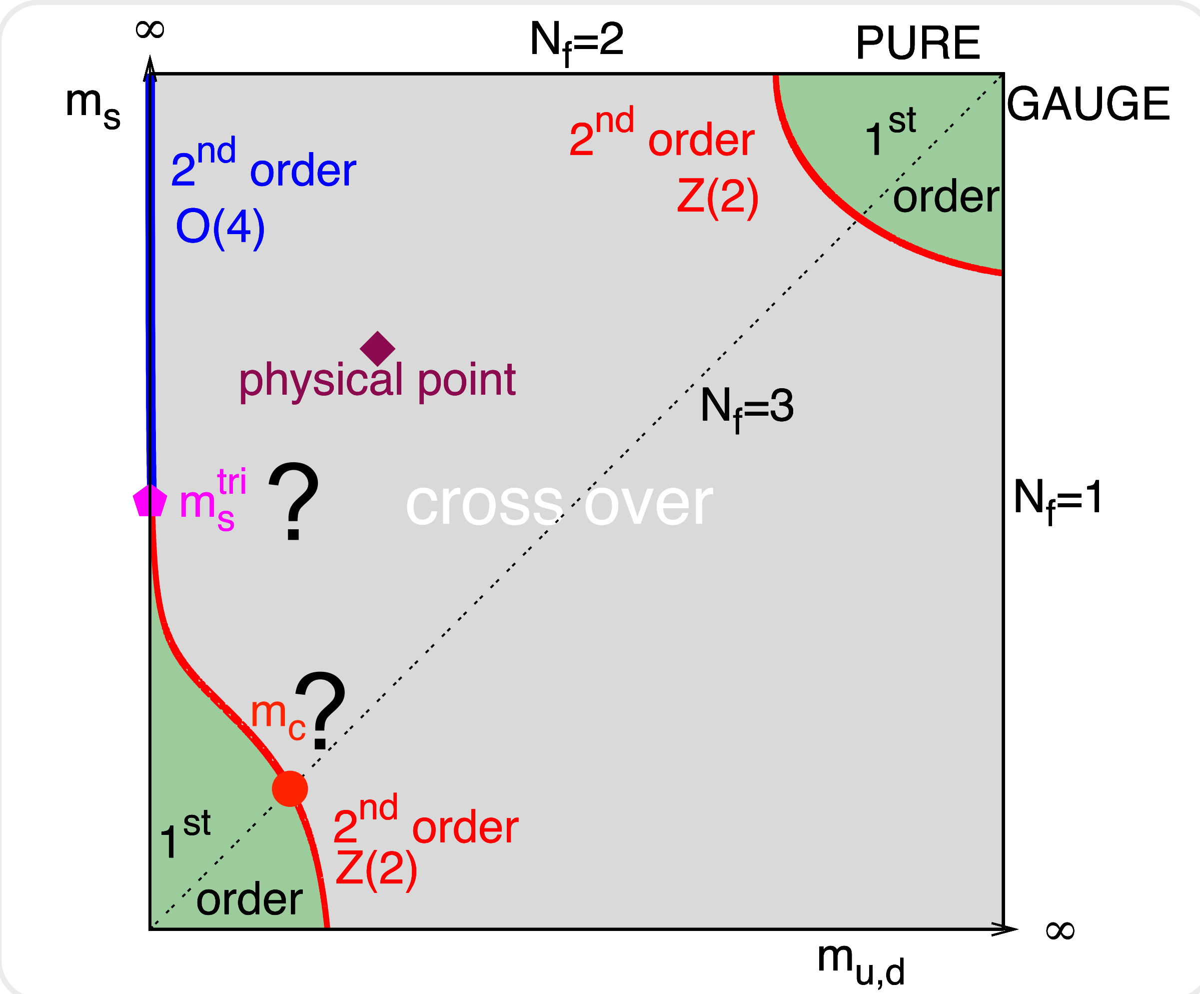
# sketched QCD phase diagram




Fukushima & Hatsuda '10

# QCD phase diagram at $\mu_B$=0

columbia plot:



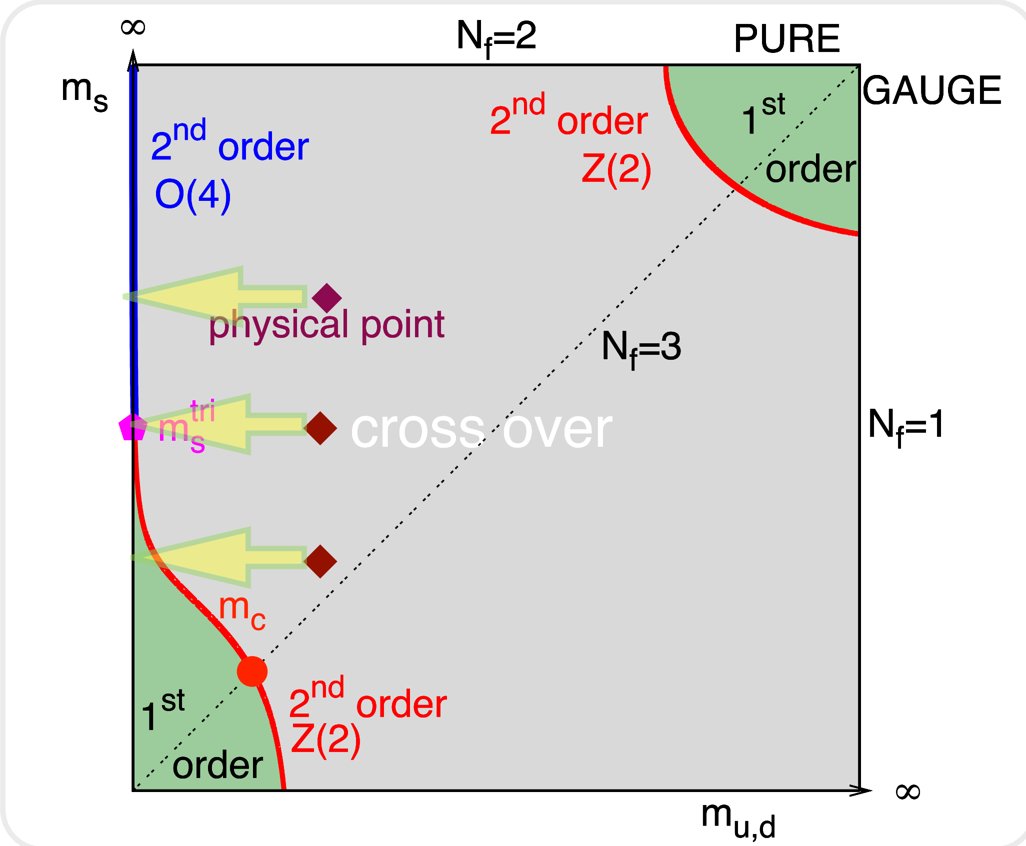

- $N_f$=2+1 theory: at m=0 or ∞ has a first order phase transition
- Intermediate quark mass region an analytic cross over is expected
- At physical quark masses, a cross over is confirmed
- Critical lines of second order transition
  $N_f$=2: O(4) universality class
  $N_f$=3: Ising universality class

★ The fundamental scale of QCD: chiral phase transition $T_c$ ?

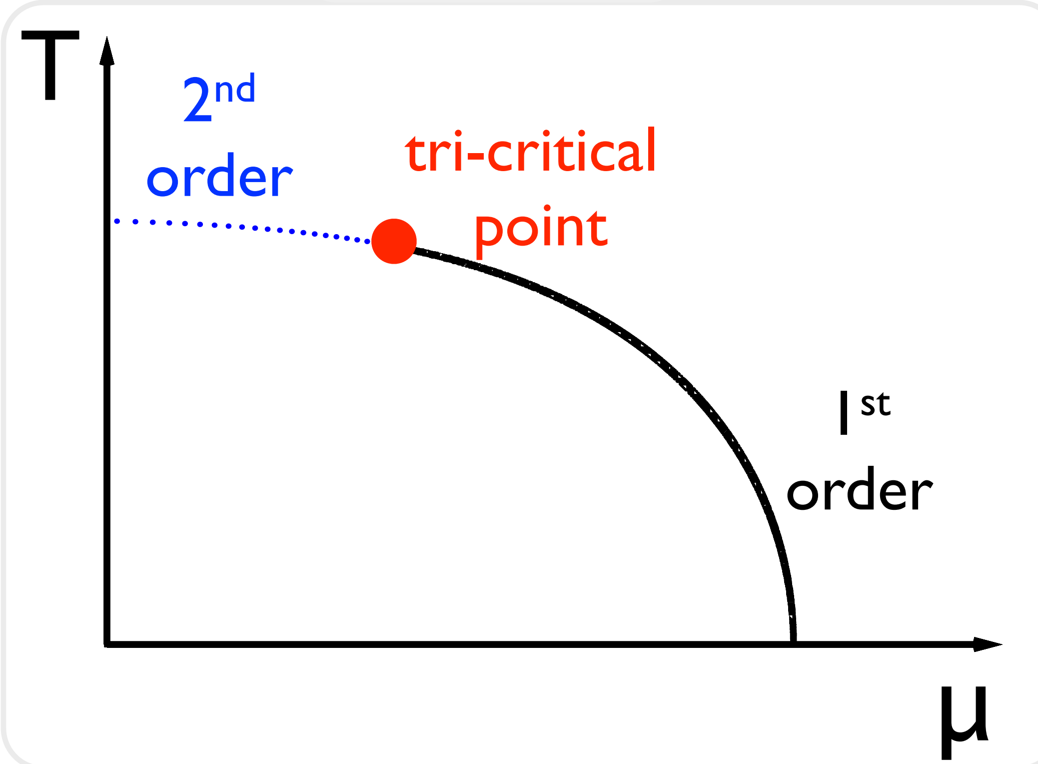
★ The value of tri-critical point ($m_s^{tri}$) ?

★ The location of $2^{nd}$ order Z(2) line ?

# scenarios of QCD phase transition at $m_l$=0




$m_s^{phy} > m_s^{tri}$

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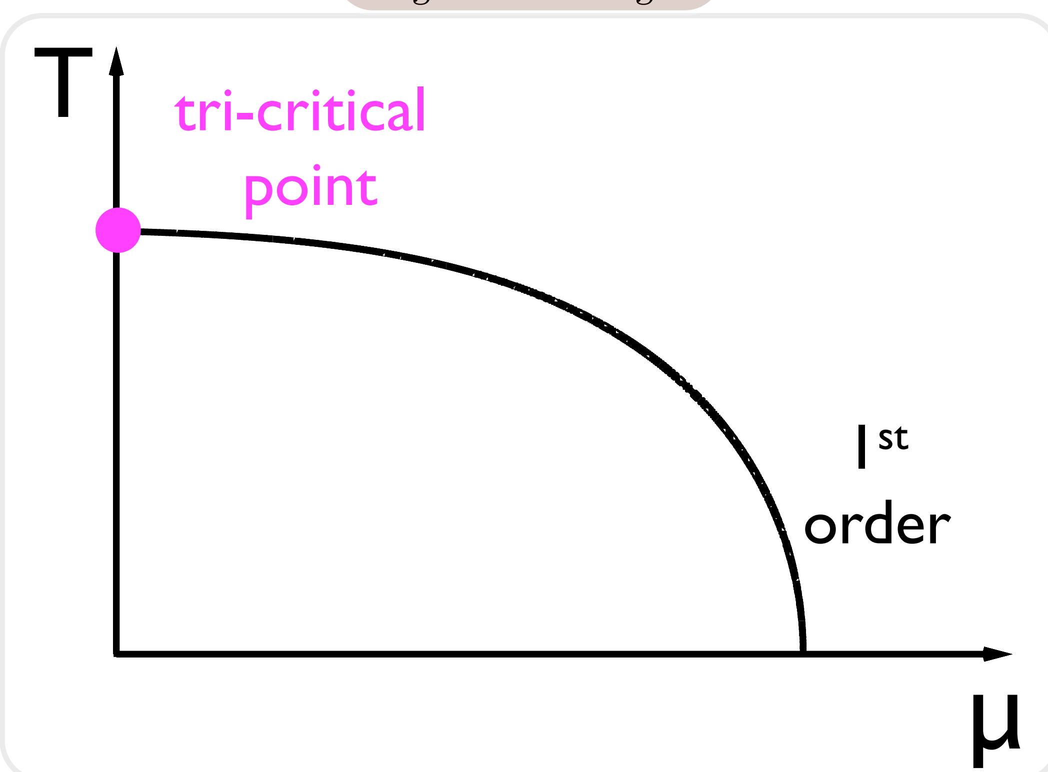


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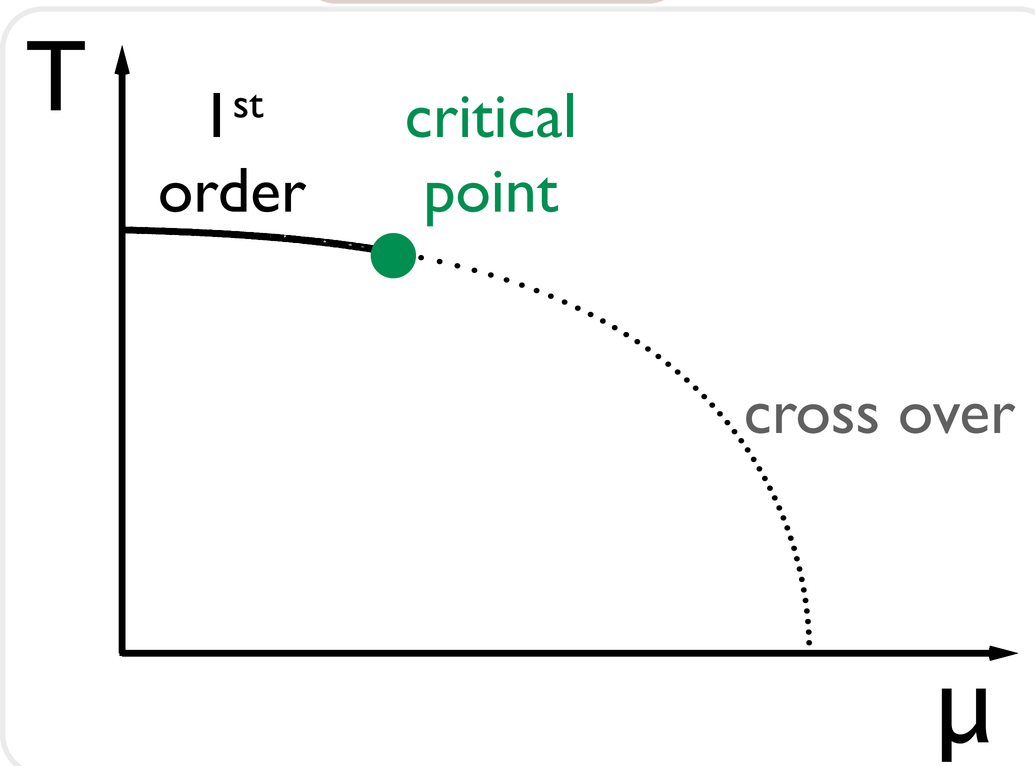
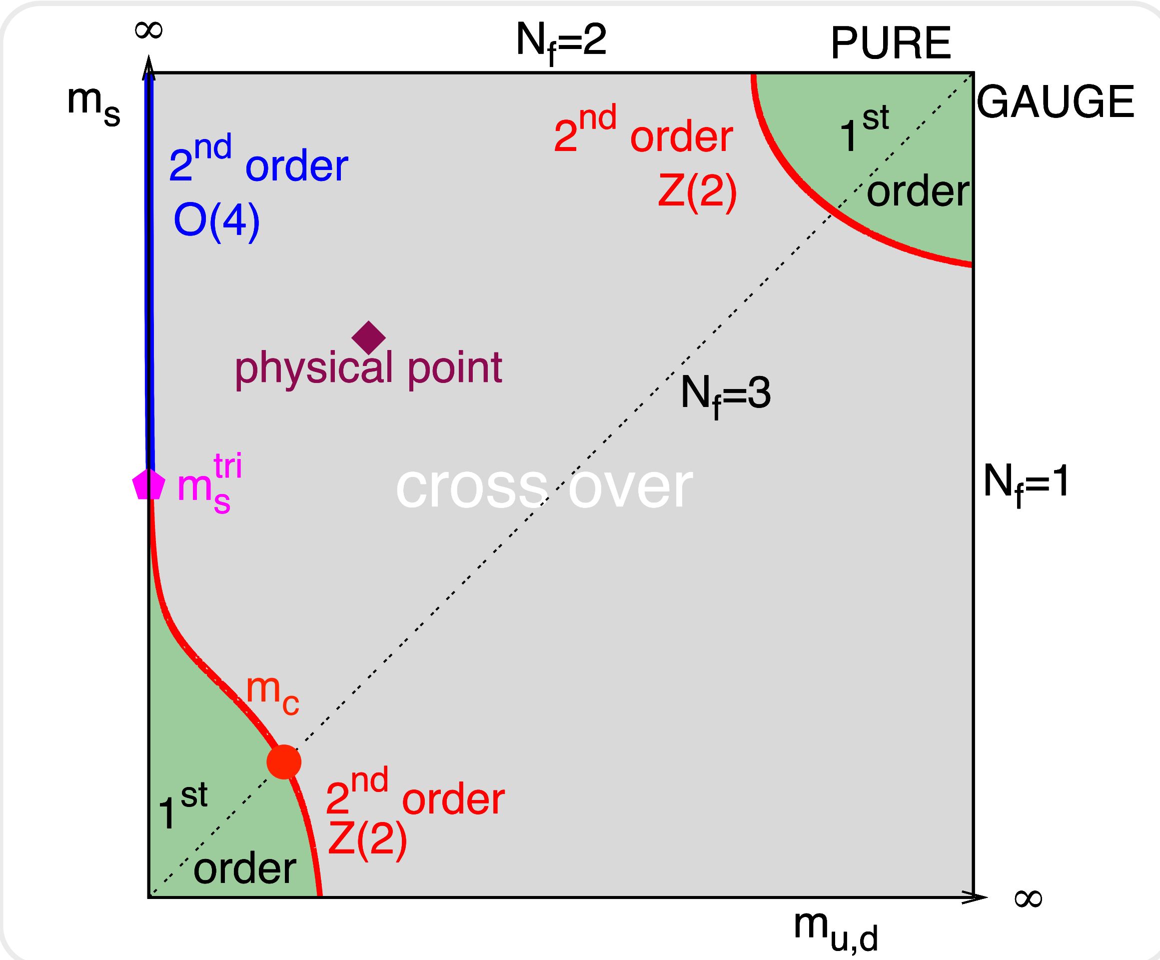
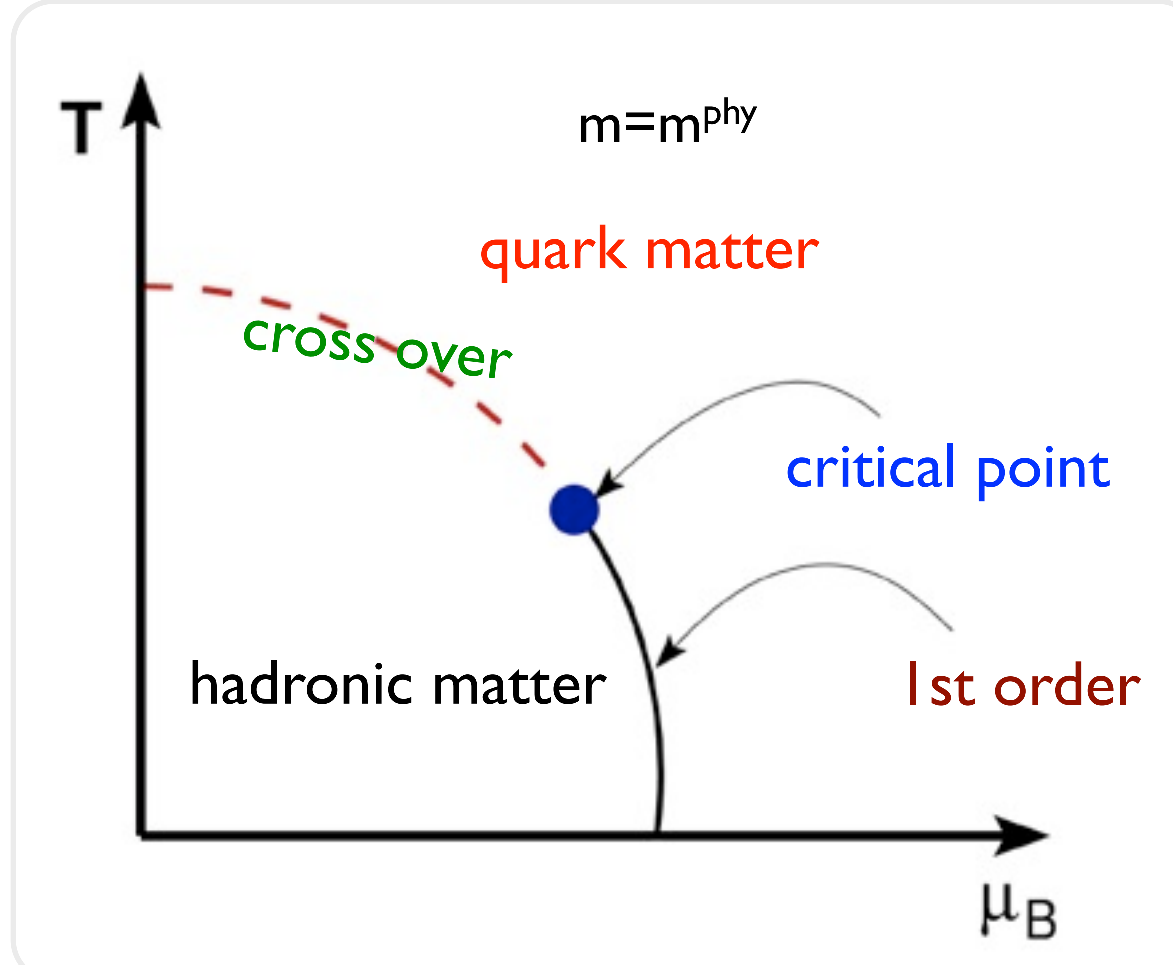
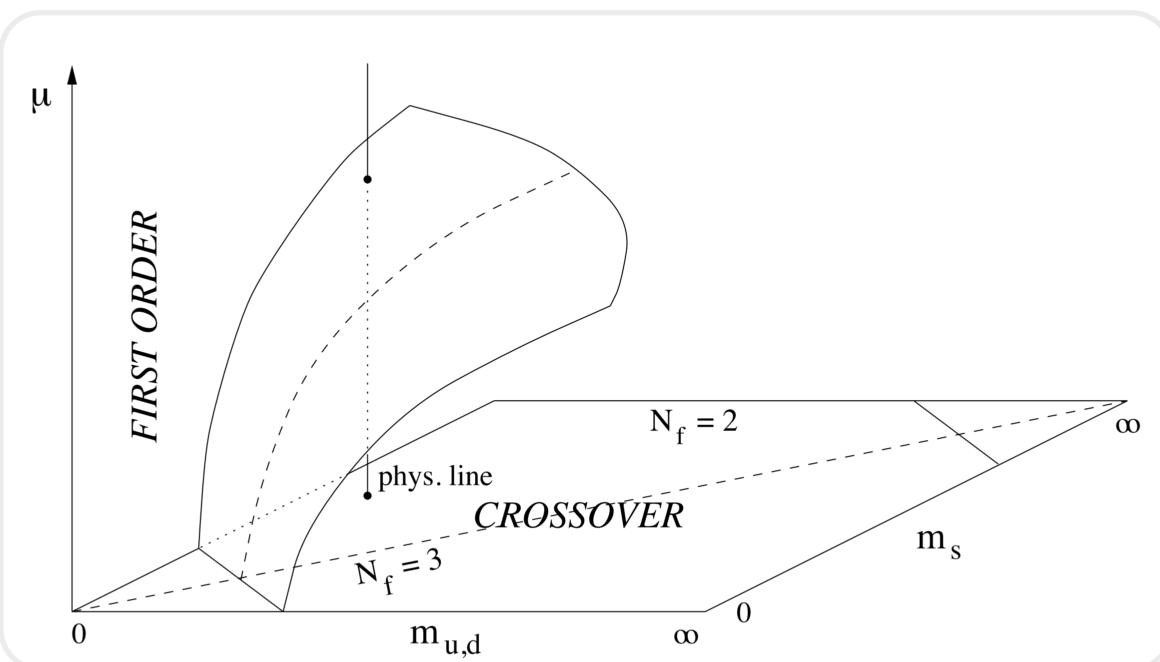


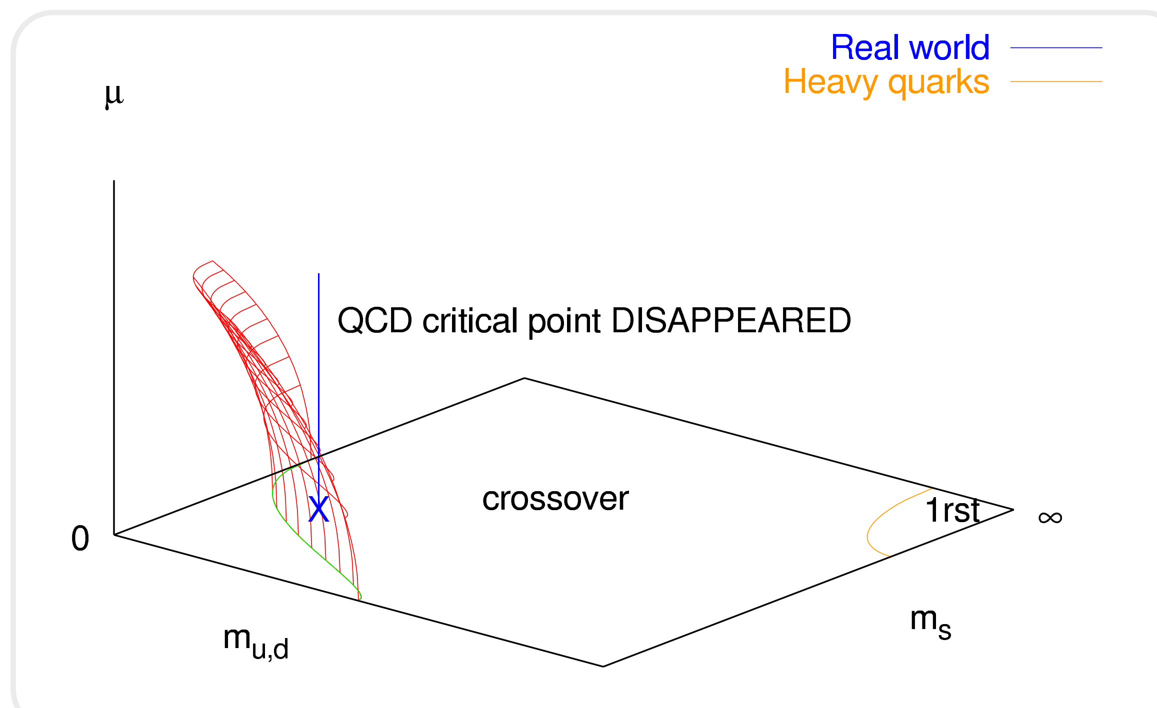
# QCD phase transition at the physical point



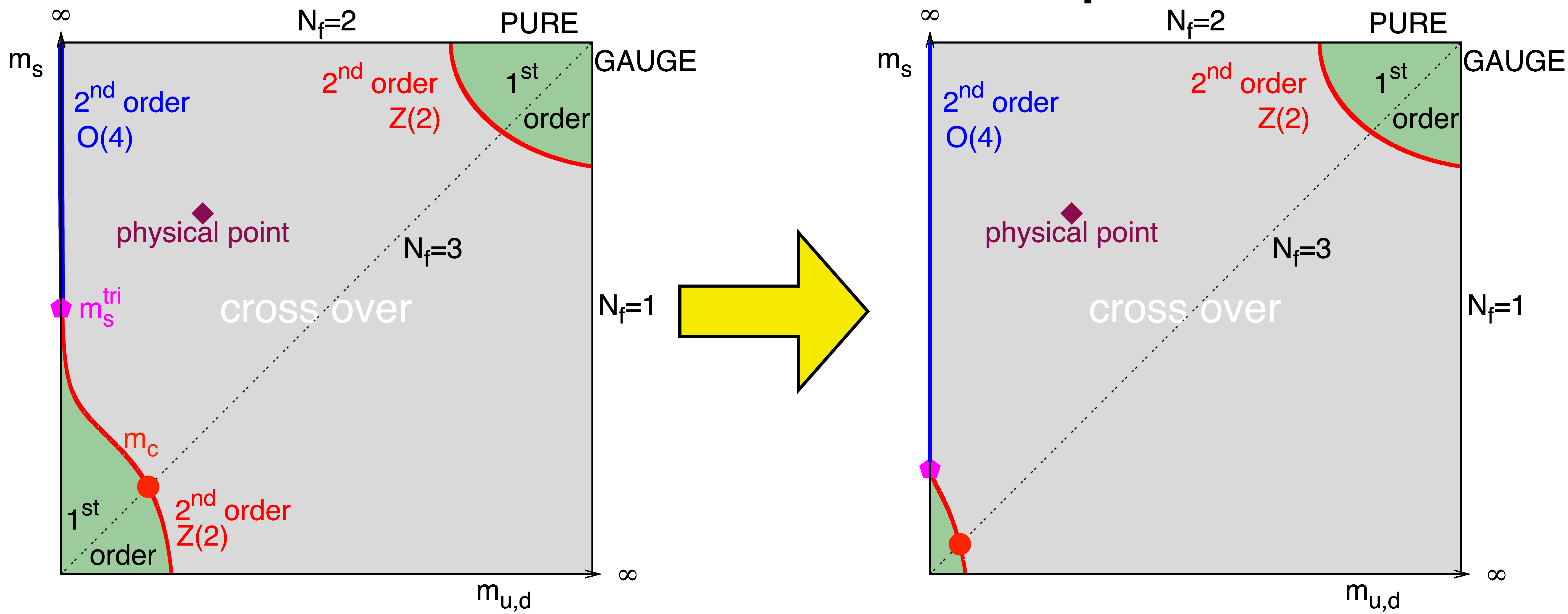




Karsch et al., '03




de Forcrand & Philipsen, '07

# QCD transition at finite temperature




Recent lattice QCD studies using Highly Improved Staggered Quarks with temporal extent of Nt=6 suggest

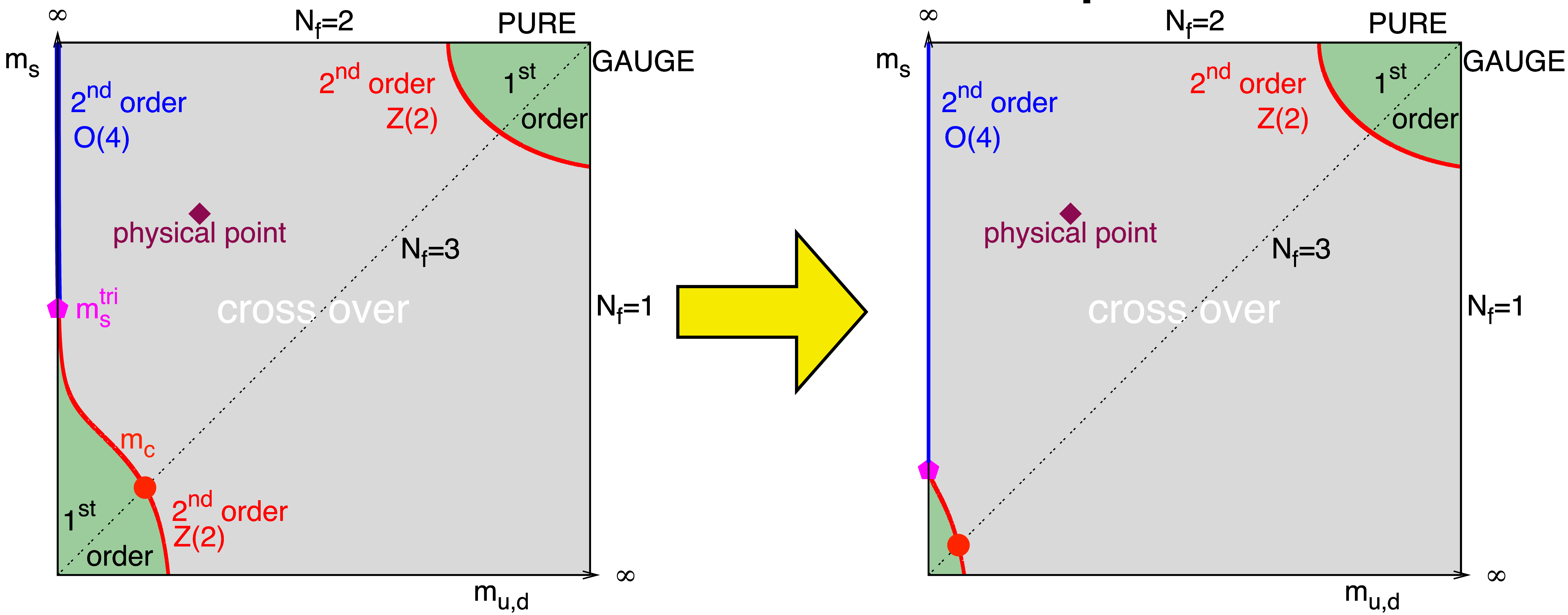
HTD lattice '11

coordinates of the physical point: $(\bar{m}_s/27, \bar{m}_s)$

coordinates of $m^c_{max} \approx (\bar{m}_s/270, \bar{m}_s/270)$

2nd order O(4) chiral phase transition seems to be more relevant to the physics at the physical point

# QCD transition at finite temperature

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$2^{nd}$ order O(4) chiral phase transition ?

# The symmetries of QCD

At the classical level, the symmetries of QCD with $N_f$ flavors of massless fermions:

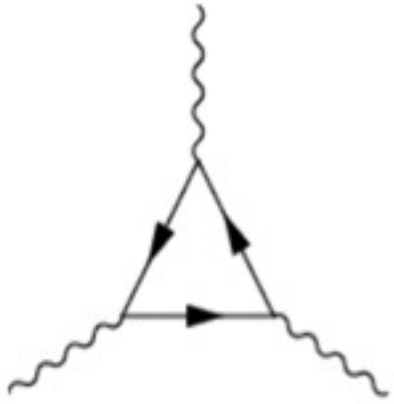
$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

- Spontaneous $SU(N_f)_L x SU(N_f)_R$ chiral symmetry breaking

  gives rise to 8 Goldstone bosons: the π, K, η

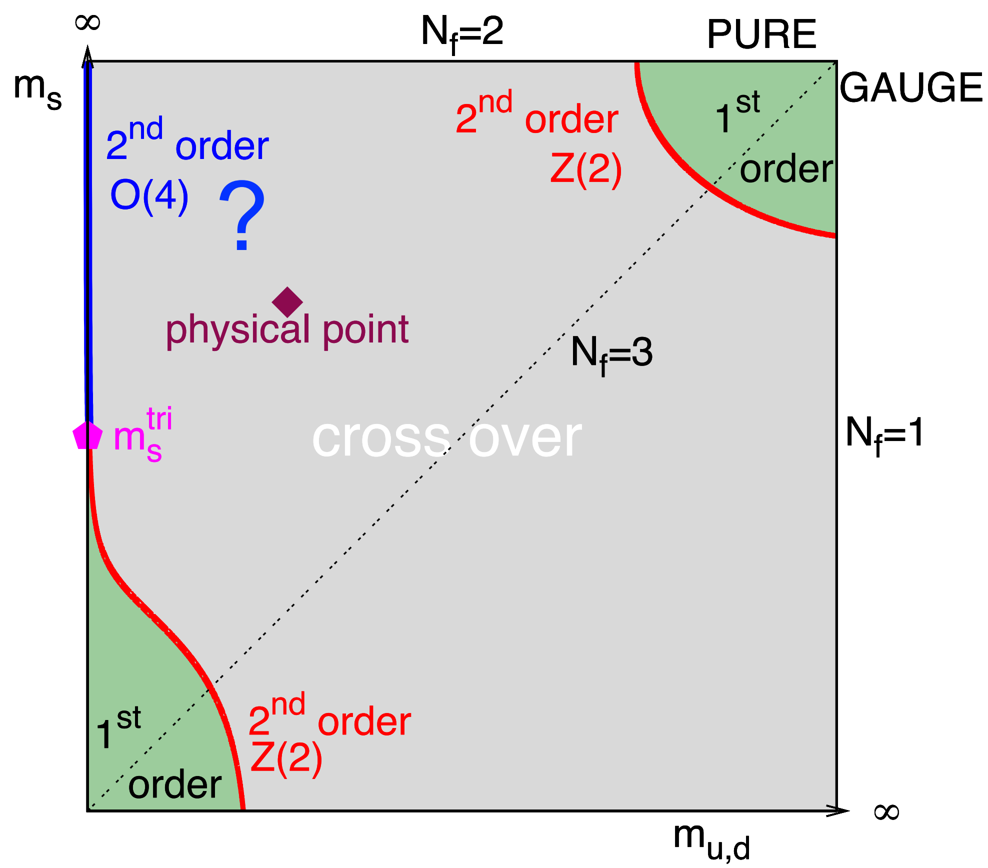
9th Goldstone boson η' ?

- $U(1)_A$ symmetry is violated by axial anomaly at the quantum level and is responsible for the η-η' mass splitting 't Hooft, Adler, Bell & Jackiw, Witten & Veneziano



$$\partial_\mu j_5^\mu = \frac{g^2 N_f}{16\pi^2} tr(\tilde{F}_{\mu\nu} F^{\mu\nu}), \qquad m_\eta^2 + m_{\eta'}^2 - 2m_K^2 = \frac{2N_f}{F_\pi^2}\chi_{top}^{N_f=0}$$

# $U(1)_A$ symmetry at finite T and its underlying mechanism



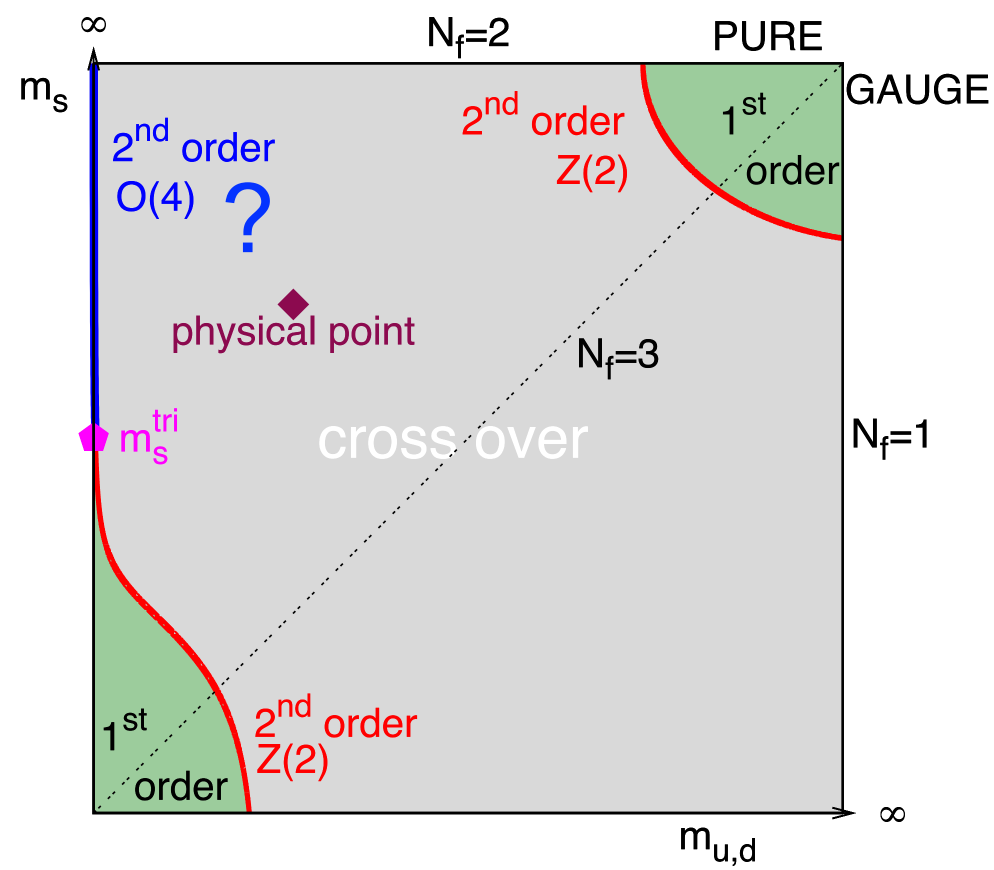

- Is $U(1)_A$ symmetry restored at or above $T_{\chi SB}$ ? Shuryak '94

- $N_f$=2 QCD at m=0 can be first order or second order $U(2)_L$x$U(2)_R$/$U(2)_V$ if $U(1)_A$ is effectively restored Pisarski & Wilczek '84 Butti, Pelissetto & Vicari '03

- Possible influence on the particle yield, reduction of η' mass ? Shuryak '94, Csorgo, Vertesi and Sziklai, PRL '10

- the fate of $U(1)_A$ symmetry at finite T and its underlying mechanism are not yet clear

# First principle calculations on the lattice




- Recent studies using non-chiral fermions, e.g. staggered fermions, give some evidence of O(N) scaling in the chiral limit of Nf=2+1 QCD

S. Ejiri et al., PRD '09, HTD, lattice '13

- Not yet conclusive due to the infamous taste symmetry breaking in the staggered discretization scheme with remnant $U(1)_A$ symmetry

# First principle calculations on the lattice

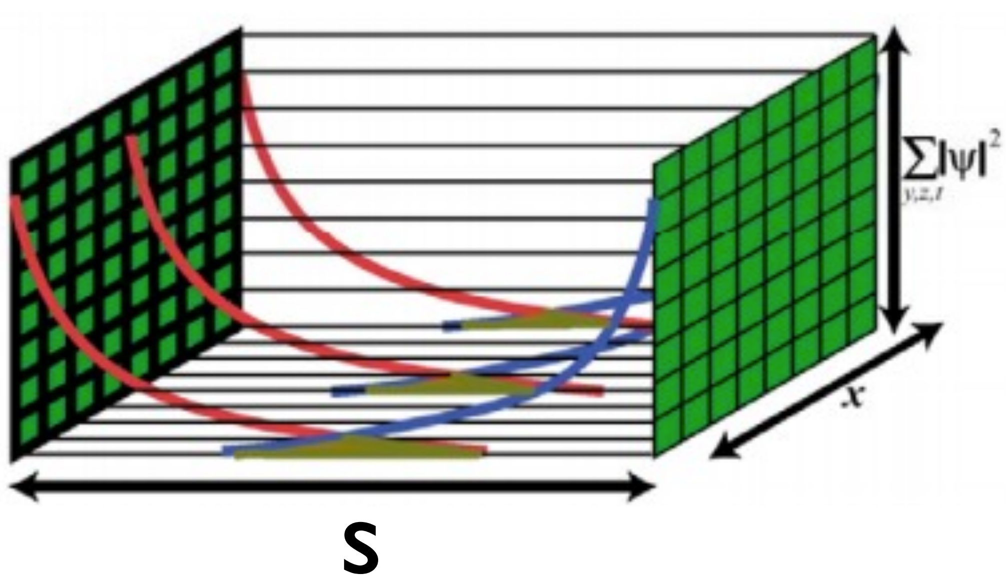
Difficulties:

- chiral fermions that preserve exact chiral symmetry and produce correct axial anomaly are needed

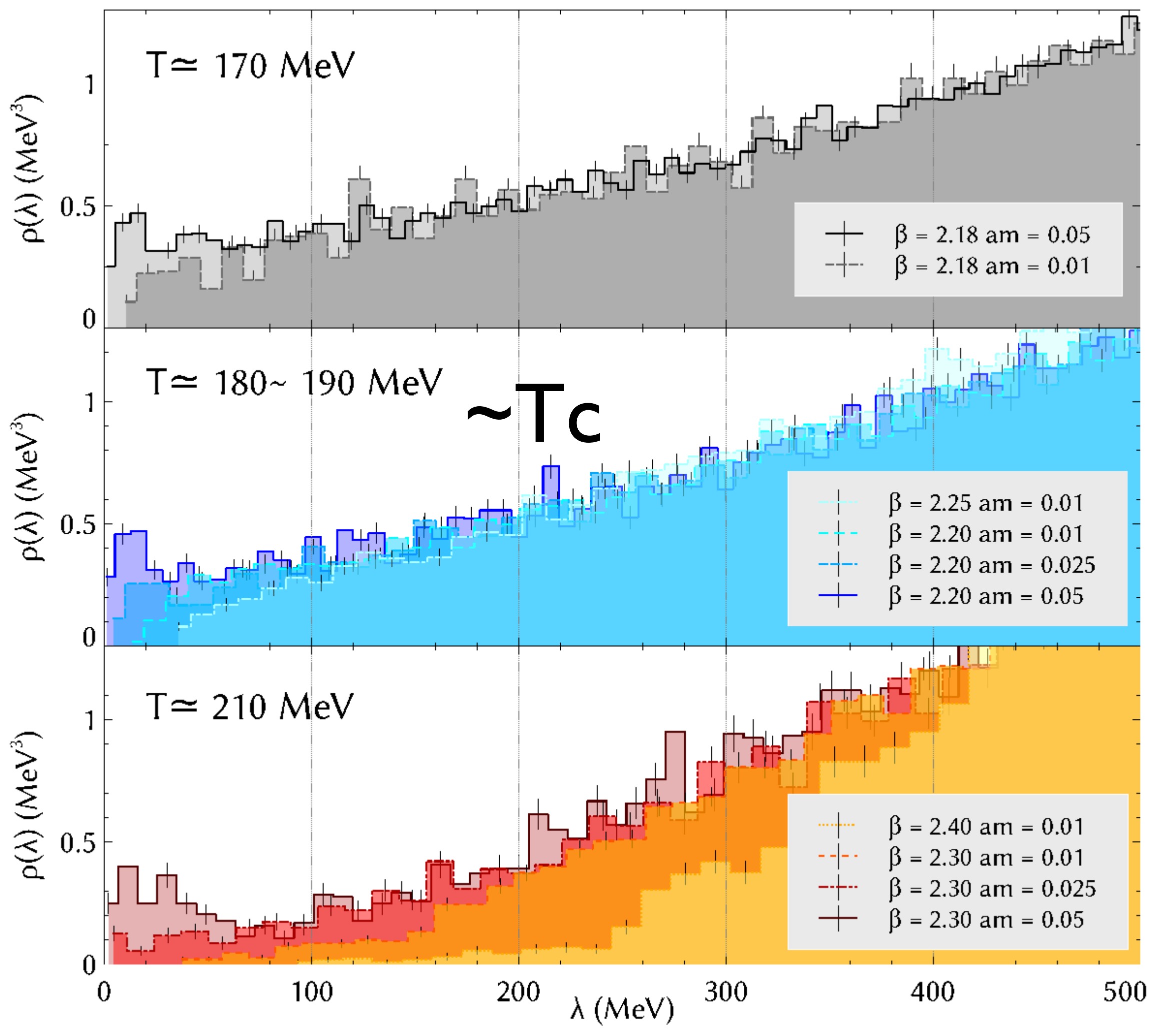
Ginsparg-Wilson relation: $\{D^{-1},\gamma_5\}=a\gamma_5$

- Overlap fermions: the only operator satisfies the Ginsparg-Wilson relation, however, there exists a "freezing" topology problem, more expensive than Domain Wall fermions

- Domain Wall fermions: preserve exact chiral symmetry and produce correct axial anomaly when the fifth dimension is sufficiently large. Residual symmetry breaking is quantified by the additive renormalization factor $m_{res}$ to the quark mass

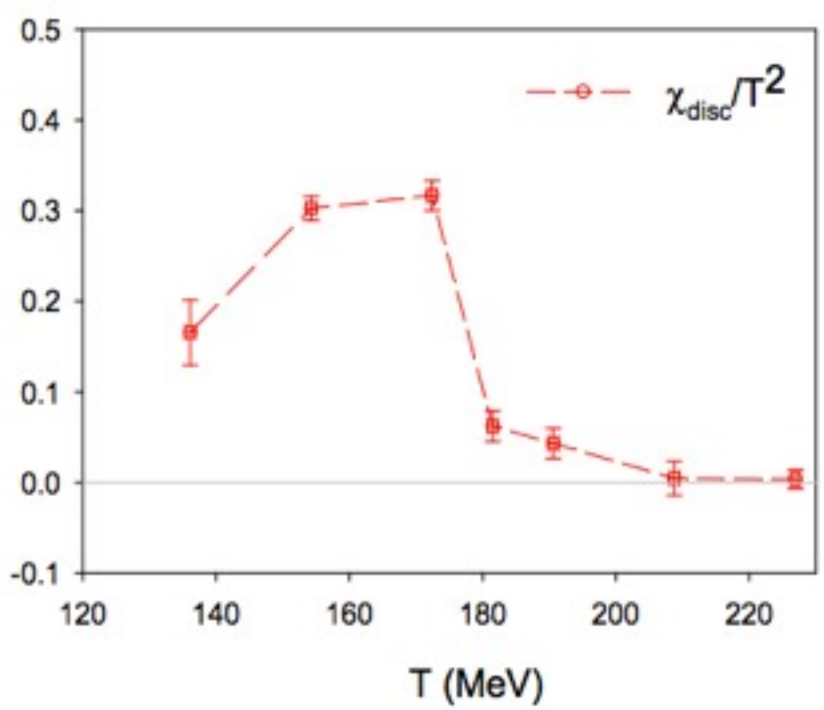


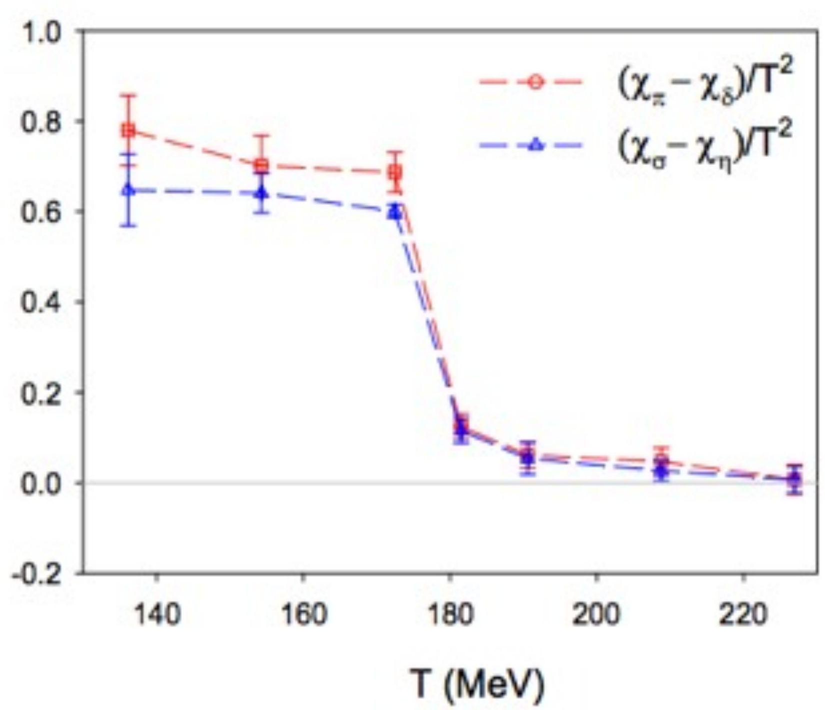
# studies on U(1)$_A$ symmetry at finite T using chiral fermions




$N_f$=2 QCD studies
using Overlap fermions
$16^3$x8 lattices
topology fixed to 0

Cossu et al., arXiv:1304.6145

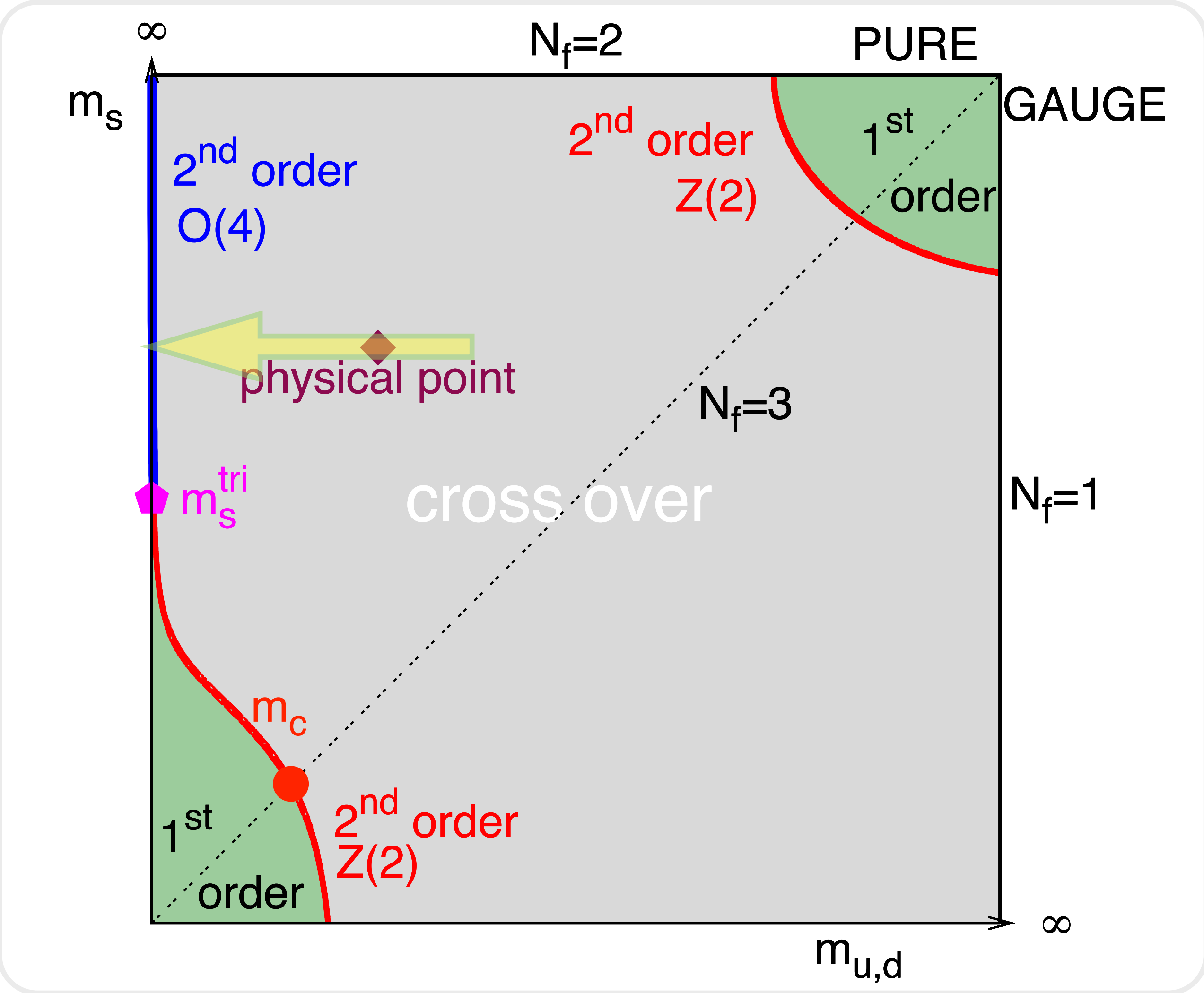



$N_f$=2 QCD studies
using Optimized
Domain Wall fermions
$16^3$x6 lattices

Chiu et al., Lattice 2013,
arXiv:1311.6220

# Nf=2+1 QCD studies using Domain Wall Fermions




simulations of 2+1flavor QCD using Domain Wall fermions on Nt=8 lattices with two pion masses:

$m_\pi$=140 MeV, Ns=32

$m_\pi$=200 MeV, Ns=32,24,16

based on arXiv:1205.3535, arXiv:1309.4149

results are shown for $m_\pi$=200 MeV if not mentioned explicitly

# hotQCD and RBC/LLNL collaboration

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- D. Toussaint, University of Arizona
- W. Unger, Frankfurt University
- P. Vranas, LLNL
- H. Yin, Columbia University

# signatures of chiral symmetry restoration

- $(\sigma, \pi^i)$ and $(\eta, \delta^i)$ each transform as an irreducible 4-dim. rep. of $SU(2)_L \times SU(2)_R$
- $(\sigma, \eta)$ and $(\pi^i, \delta^i)$ each transform as 2-dim. rep. of $U(1)_A$

iso-vector pseudoscalar

iso-scalar scalar

$$\begin{tikzcd}
\pi : \bar{q}\gamma_5 \frac{\tau}{2} q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \sigma : \bar{q} q \arrow[d, leftrightarrow, "U(1)_A"] \\
\delta : \bar{q}\frac{\tau}{2} q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"'] & \eta : \bar{q}\gamma_5 q
\end{tikzcd}$$

iso-vector scalar

iso-scalar pseudoscalar

restoration of $SU(2)_L \times SU(2)_R$:

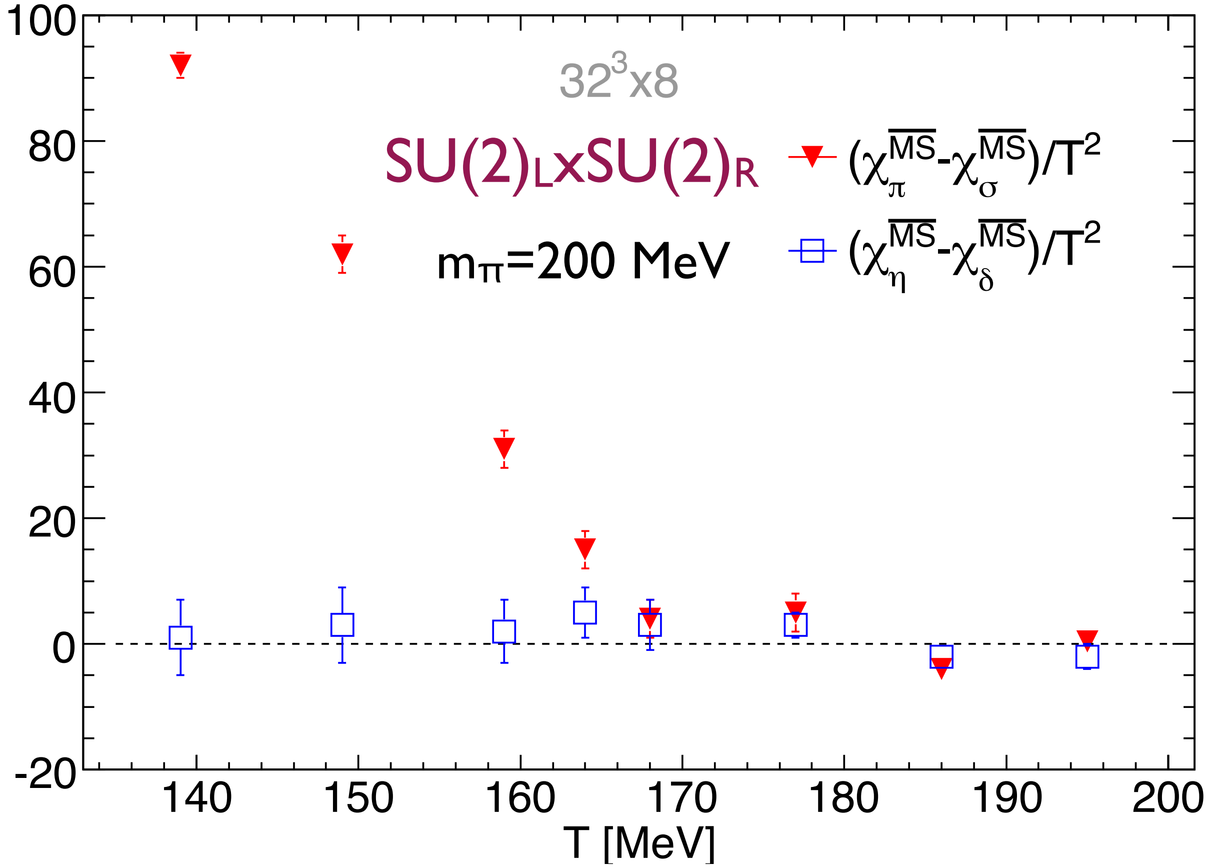
$$\chi_\pi - \chi_\sigma = 0$$
$$\chi_\delta - \chi_\eta = 0$$

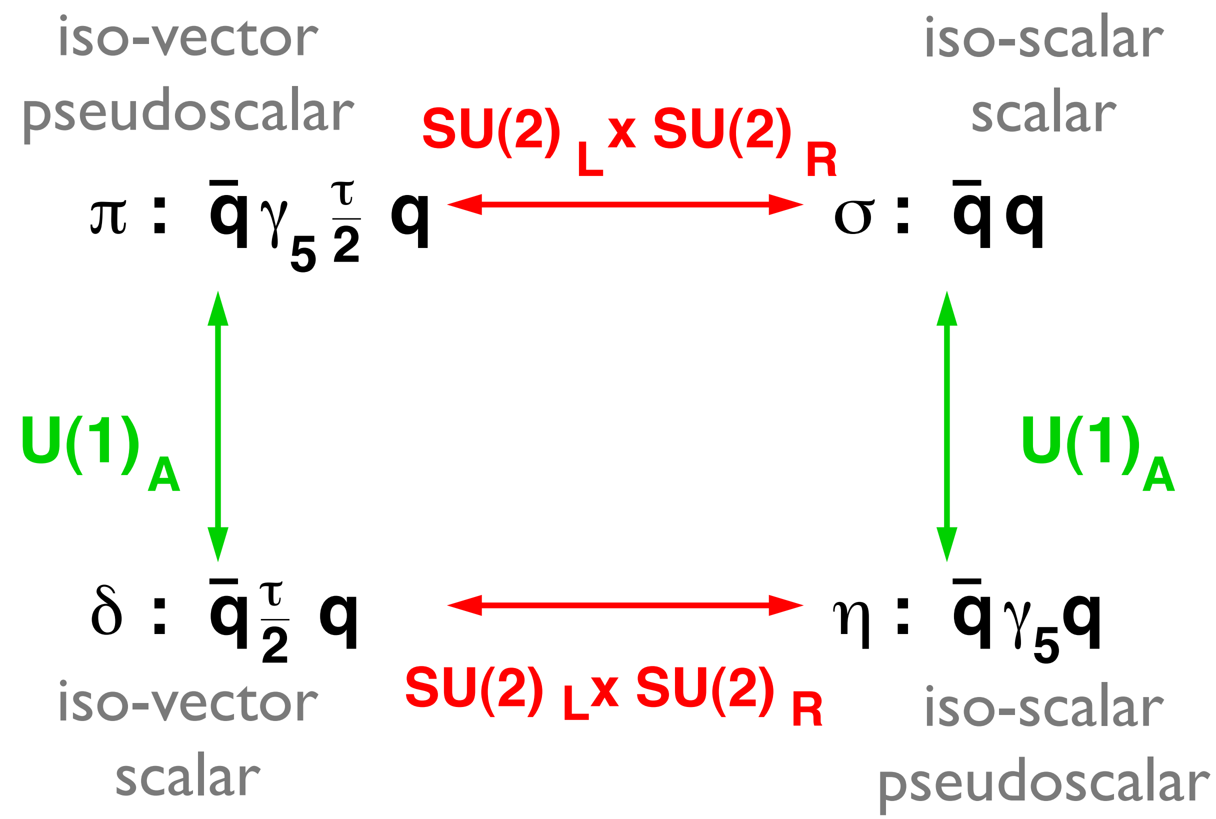
restoration of $U(1)_A$:

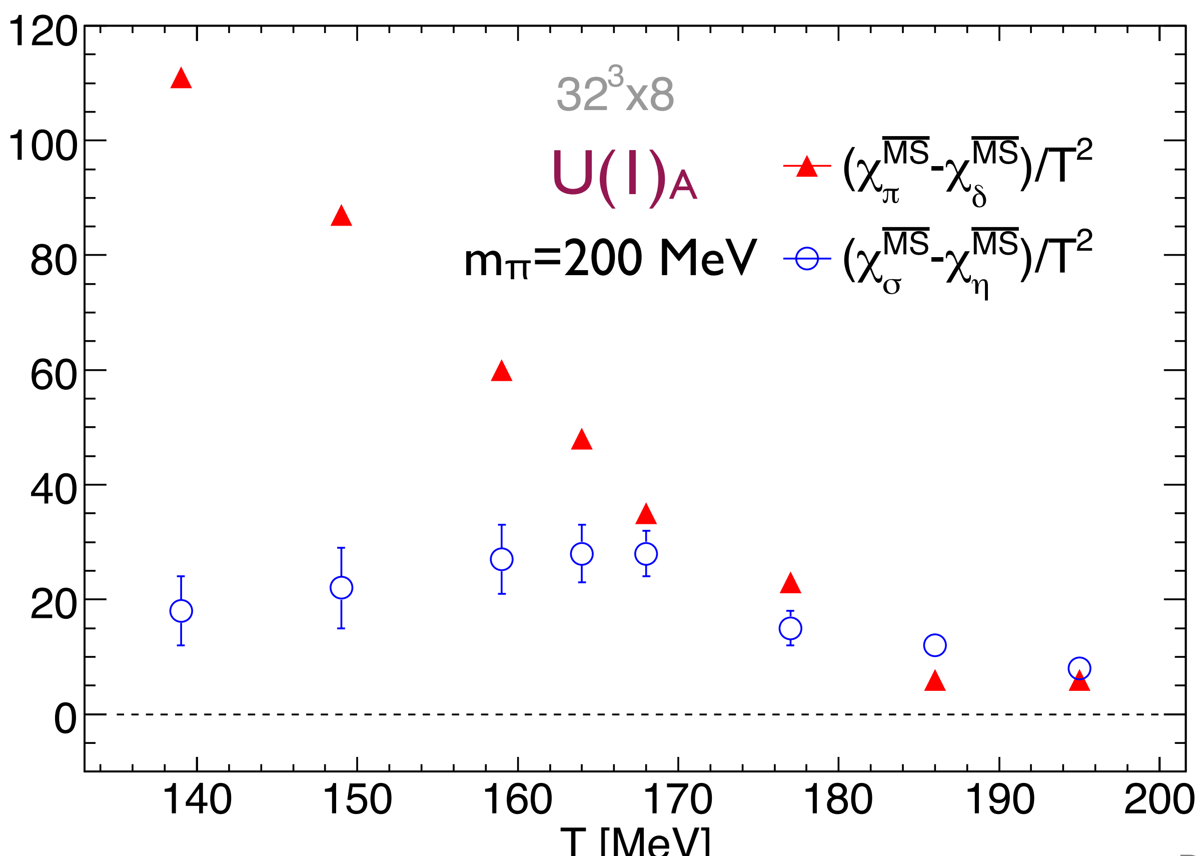
$$\chi_\pi - \chi_\delta = 0$$
$$\chi_\sigma - \chi_\eta = 0$$

- Susceptibilities defined as integrated two point correlation functions of the eight local operators, e.g. $\chi_\pi = \int d^4x \langle \pi^i(x) \pi^i(0) \rangle$

# fate of chiral symmetries at finite T




- $SU(2)_L \times SU(2)_R$ symmetry is restored at $T_{\chi SB} \sim 170$ MeV
- $U(1)_A$ symmetry remains broken up to 195 MeV $\sim 1.16\, T_{\chi SB}$

Buchoff et al., [HotQCD], arXiv:1309.4149

# Some issues

## LQCD calculations using Domain Wall fermions

- the finite lattice cutoff effects ?
- the finite volume effects ? contributions from exact zero modes ?
- the residual symmetry breaking effects ?

## Signatures of $U(1)_A$ restoration

- Can $U(1)_A$ restoration be signaled by two point correlation functions and their susceptibilities ? Aoki, Fukaya and Taniguchi, arXiv:1209.2061
- Dirac Eigenvalue spectrum

## Symmetry restorations in the chiral limit

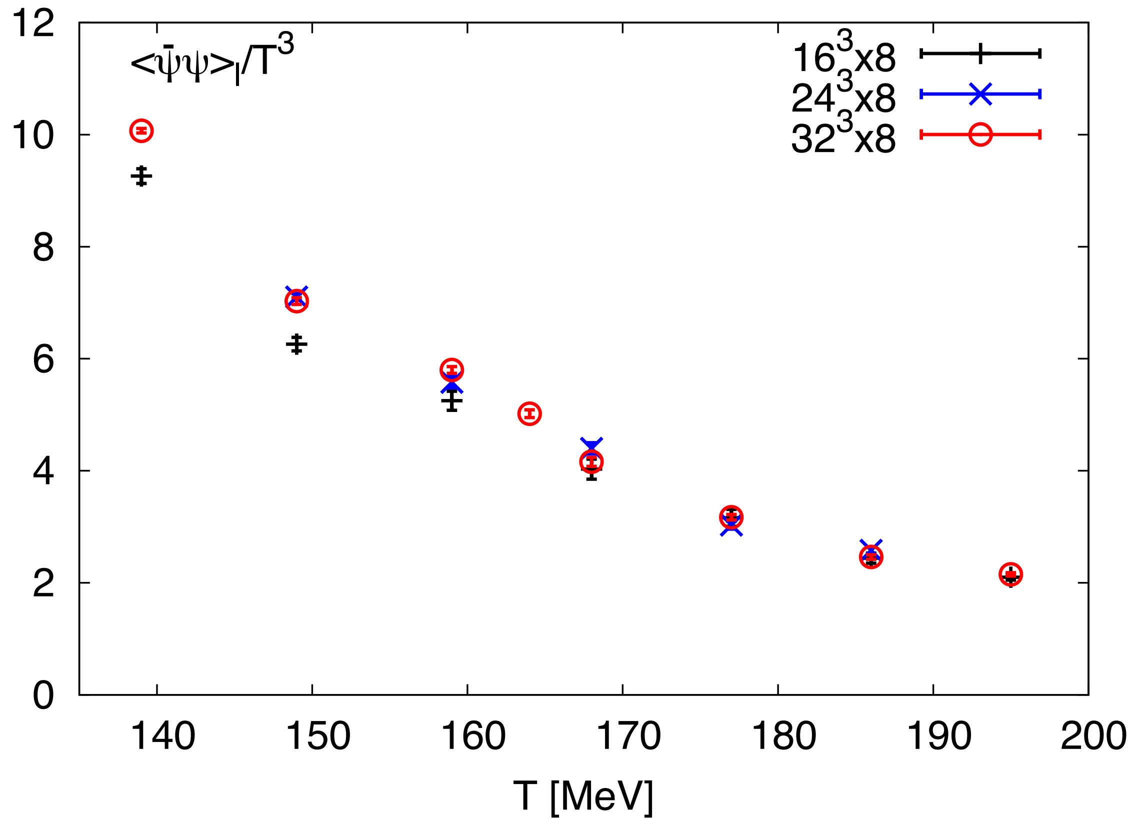
- explicit chiral symmetry breaking by the finite quark mass?

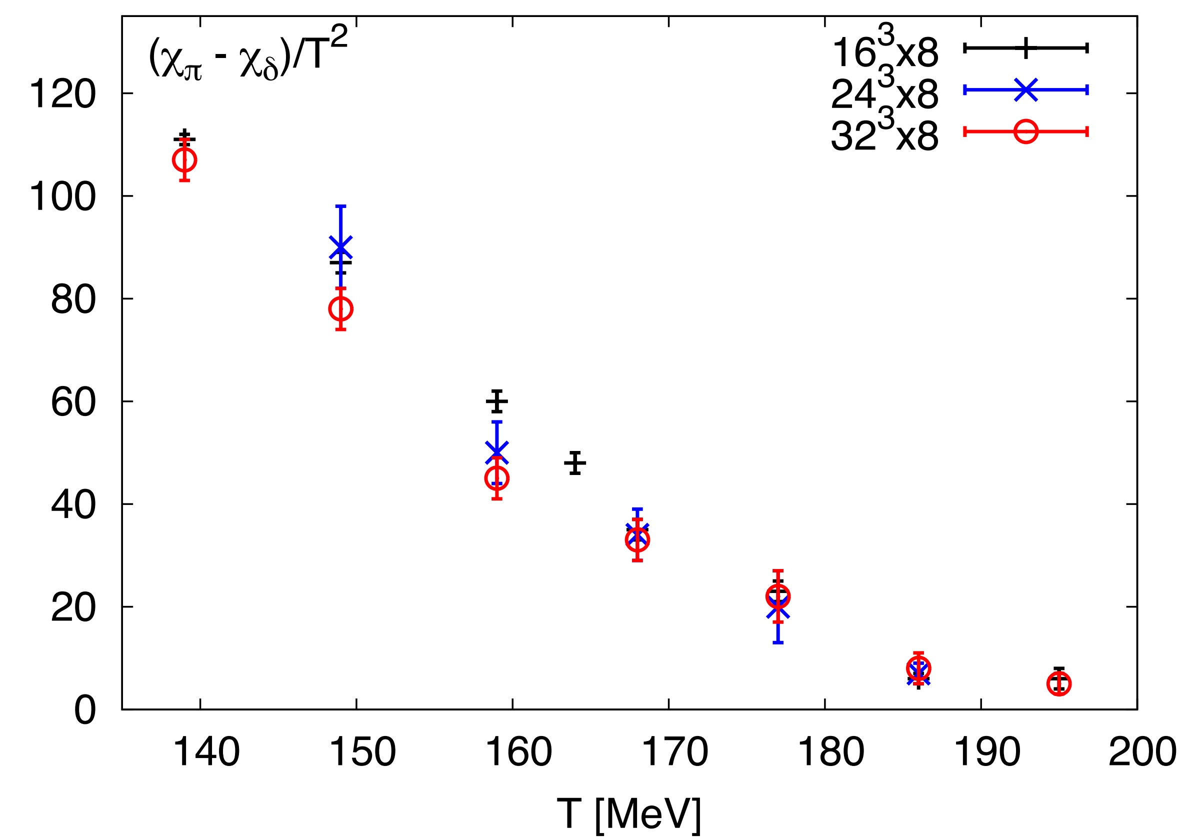
# contributions from exact zero modes

$$\langle\bar{\psi}\psi\rangle = \int_0^\infty d\lambda\,\rho(\lambda,\tilde{m})\,\frac{2\tilde{m}}{\tilde{m}^2+\lambda^2} + \frac{\langle|Q_{\rm top}|\rangle}{\tilde{m}V}$$

$$\chi_\pi - \chi_\delta = \int_0^\infty d\lambda\,\rho(\lambda,\tilde{m})\left(\frac{2\tilde{m}}{\tilde{m}^2+\lambda^2}\right)^2 + \frac{2\langle|Q_{\rm top}|\rangle}{\tilde{m}^2V}$$

The second terms are from exact zero mode contributions related to the non-zero topological charge and should vanish when V goes to infinity



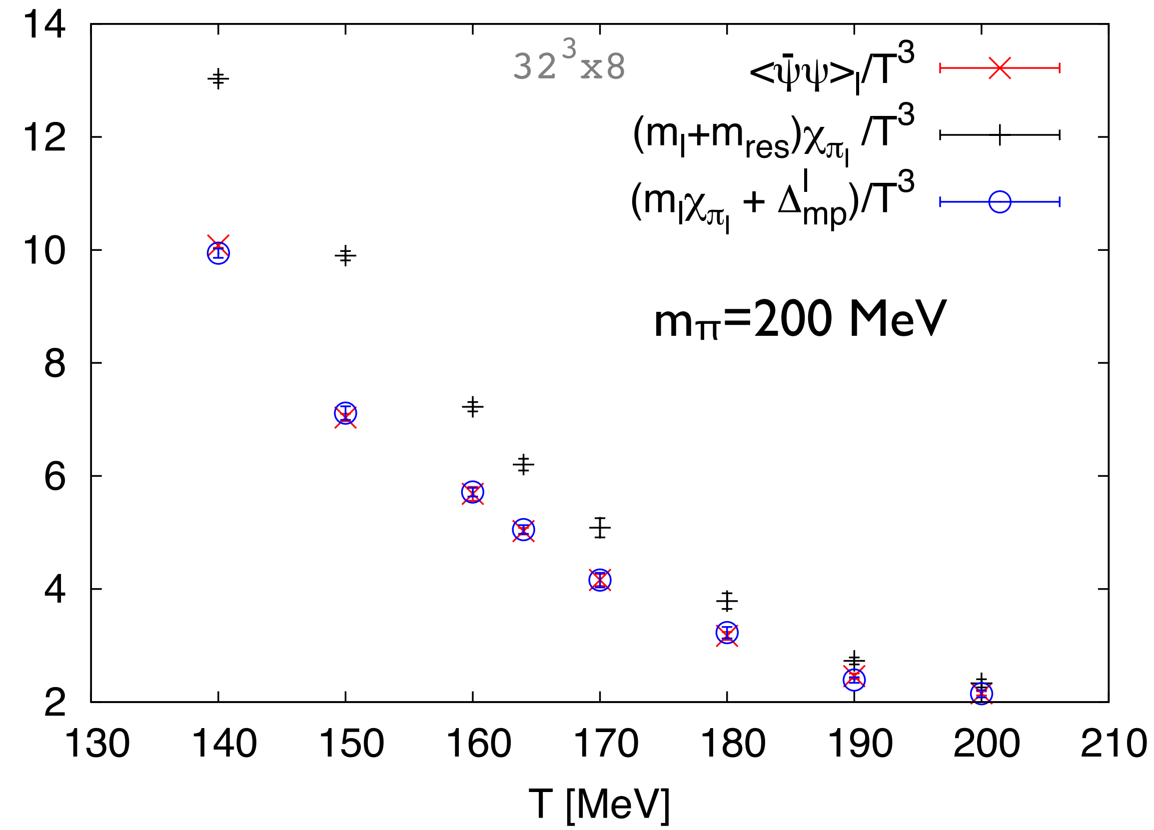

The mild volume dependence of chiral condensates and $\chi_\pi$ - $\chi_\delta$ at $T \gtrsim Tc$ indicates negligible exact zero mode contributions

# exact DWF ward Identity

Gell-Mann-Oakes-Renner relation: $\langle\bar\psi\psi\rangle = m\,\chi_\pi$

In DWF formalism:

$$\langle\bar\psi\psi\rangle_l = (m_l + m_{res})\chi_{\pi_l} + R^l_{5d}$$
$$= m_l\chi_{\pi_l} + \Delta^l_{\mathrm{mp}}$$
$$\langle\bar\psi\psi\rangle_s = (m_s + m_{res})\chi_{\pi_s} + R^s_{5d}$$
$$= m_s\chi_{\pi_s} + \Delta^s_{\mathrm{mp}}$$



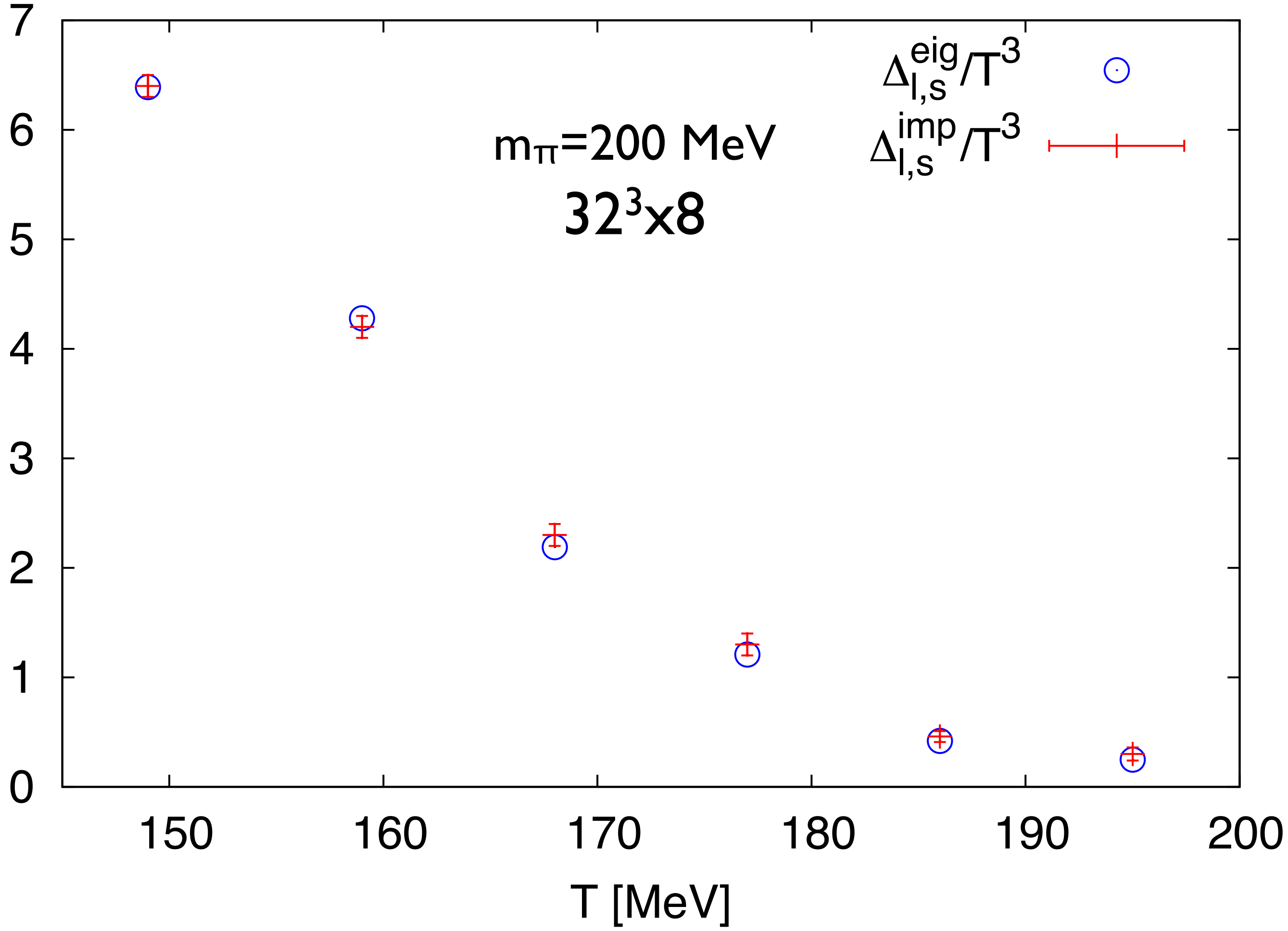

- subtracted pbp $\Delta_{l,s}$ to cancel the linear UV divergence in quark mass

$$\Delta_{l,s} = \langle\bar\psi\psi\rangle_l - \frac{m_l + m_{res}}{m_s + m_{res}}\langle\bar\psi\psi\rangle_s\ ,\quad \Delta_{l,s} = \Delta^{\mathrm{imp}}_{l,s} + R^l_{5d} - \frac{m_l + m_{res}}{m_s + m_{res}}R^s_{5d}$$

- improved subtracted pbp $\Delta^{\mathrm{imp}}_{l,s}$: suitable to DWF, cancel further residual chiral symmetry breaking effects

$$\Delta^{\mathrm{imp}}_{l,s} = (m_l + m_{res})(\chi_{\pi_l} - \chi_{\pi_s})$$

# reproduction of improved subtracted pbp from ρ(λ)



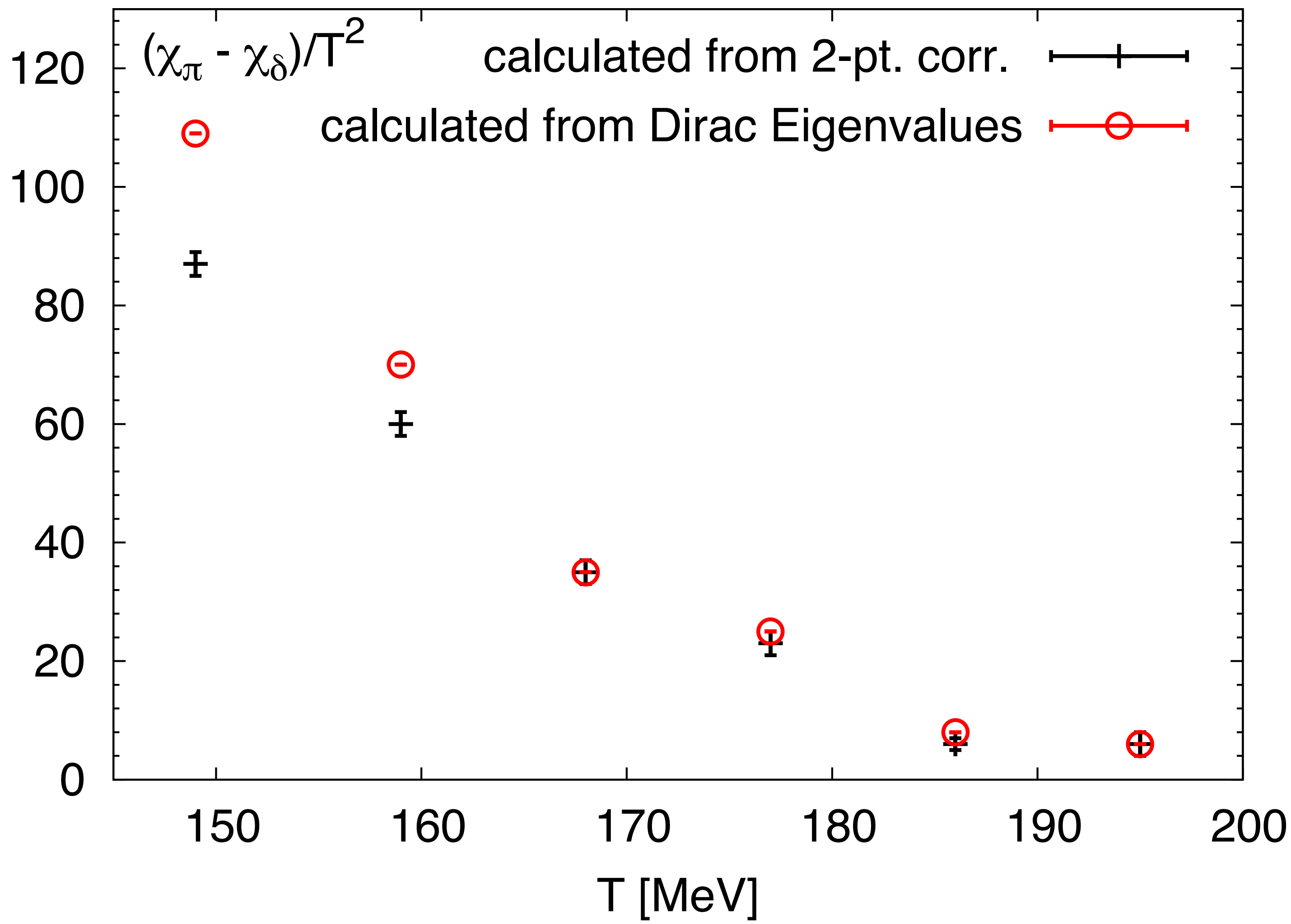

$$\Delta^{\rm eig}_{l,s} = \int_0^\infty {\rm d}\lambda\, \rho(\lambda)\, \frac{2m_l(m_s^2 - m_l^2)}{(\lambda^2 + m_l^2)(\lambda^2 + m_s^2)}$$

$$= \frac{1}{N_\sigma^3 N_\tau} \left\langle \sum_{n=1}^{100} \frac{m_l(m_s^2 - m_l^2)}{\Lambda_n^2(\Lambda_n^2 + m_s^2 - m_l^2)} \right\rangle$$

$$\Delta^{\rm imp}_{l,s} = (m_l + m_{res})(\chi_{\pi_l} - \chi_{\pi_s})$$

Subtracted chiral condensates can be reproduced well from Dirac Eigenvalue spectrum

# evaluation of $\chi_\pi$ - $\chi_\delta$ from 2-pt. corr. and Dirac Eigenvalues




$$\chi_\pi - \chi_\delta = \int_0^\infty \mathrm{d}\lambda\, \rho(\lambda, m) \left(\frac{2m}{m^2+\lambda^2}\right)^2$$

$$= \frac{1}{N_\sigma^3\, N_\tau} \left\langle \sum_{n=1}^{100} \frac{2m^2}{\Lambda_n^4} \right\rangle$$

Remarkable agreement between $\chi_\pi$ - $\chi_\delta$ evaluated from 2-point correlation functions and Dirac Eigenvalues

At two lowest T the discrepancy may come from the unphysical fluctuation of $\Lambda$ associated with the residual symmetry breaking

# signatures from Dirac eigenvalue spectrum ρ(λ)

$$\langle\bar{\psi}\psi\rangle = \int_0^\infty \mathrm{d}\lambda\,\rho(\lambda,\tilde{m})\,\frac{2\tilde{m}}{\tilde{m}^2+\lambda^2}\,, \qquad \chi_\pi - \chi_\delta = \int_0^\infty \mathrm{d}\lambda\,\rho(\lambda,\tilde{m})\left(\frac{2\tilde{m}}{\tilde{m}^2+\lambda^2}\right)^2$$

- the restoration of $SU(2)_L \times SU(2)_R$ symmetry

  Banks-Casher formula: $\langle\bar{\Psi}\Psi\rangle = \pi\rho(0)$

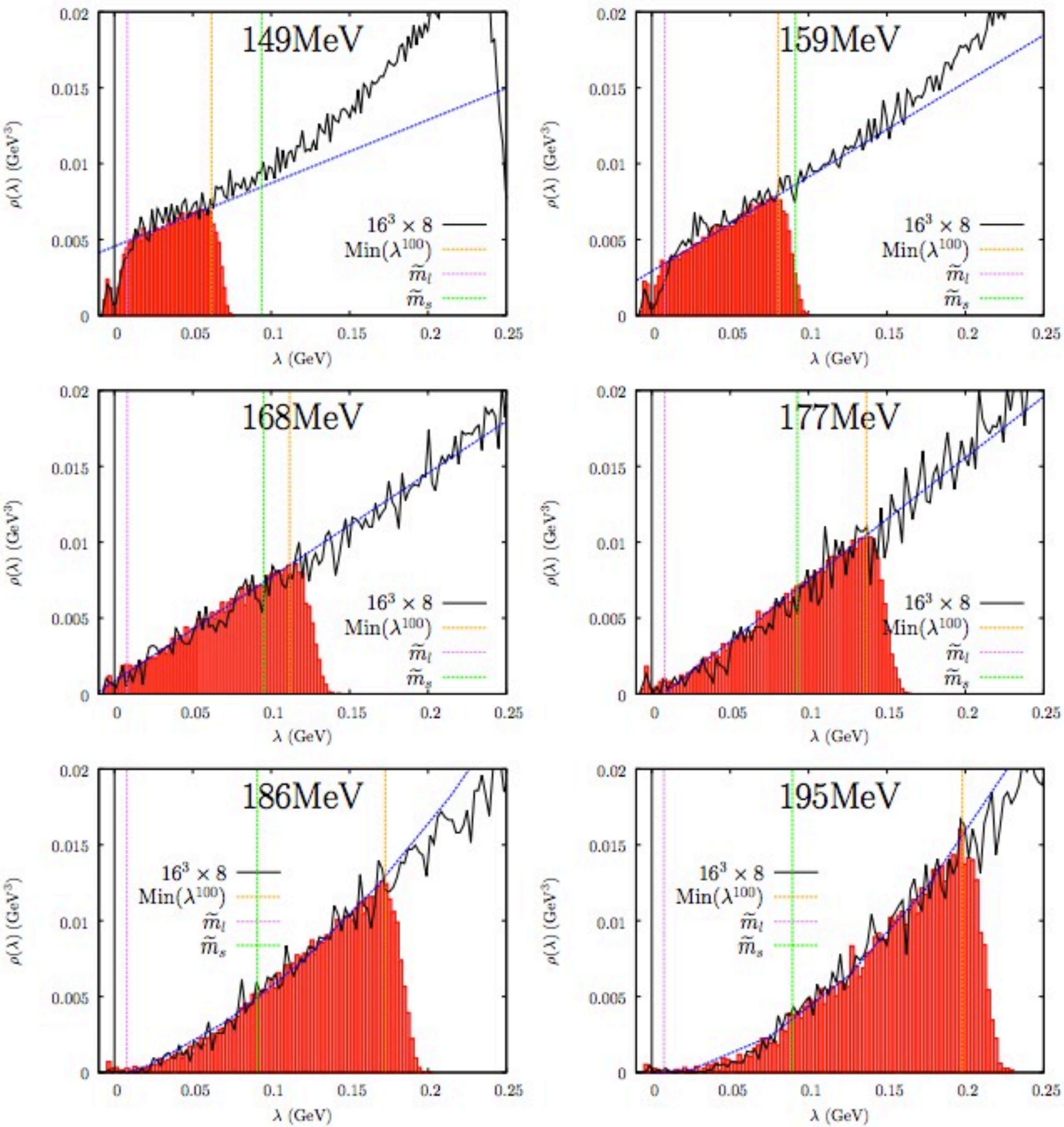
  - ρ(0)=0

- the restoration of $U(1)_A$ symmetry

  - ρ(λ) must go to zero faster than linearly
  - a sizable gap from zero, i.e. $\rho(\lambda<\lambda_c)=0$ Cohen, nucl-th/980106
  - $\rho(\lambda) = c\,|\lambda|^\alpha$ , $\alpha > 2$ if observables are analytic in $m^2$

Aoki, Fukaya and Taniguchi, arXiv:1209.2061

# Dirac Eigenvalue spectrum with $m_\pi$=200 MeV




black lines: $16^3$ results
red histograms: $32^3$ results

T<$T_c$

nonzero $\rho(0)$

T~$T_c$

vanishing $\rho(0)$

T>$T_c$

no gap from zero

U(1)$_A$ remains broken up to 195 MeV

# possible behaviors for $\rho(\lambda,m)$

Possible forms of $\rho(\lambda,m)$ making $\langle\bar{\Psi}\Psi\rangle=0$ and $\chi_\pi - \chi_\delta \neq 0$ ?

$$\rho(\lambda,m) = c_0 + c_1\lambda + c_2\, m^2\, \delta(\lambda) + c_3\, m + c_4\, m^2 + O(\lambda,m)$$

$c_0 + c_1\lambda$: low T chiral pertur. theory

Smilga & Stern, PLB '93

$c_2\, m^2\, \delta(\lambda)$: high T Dilute Instanton Gas Approx.

Gross, Yaffe & Pisarski, Rev. Mod. Phys. '81

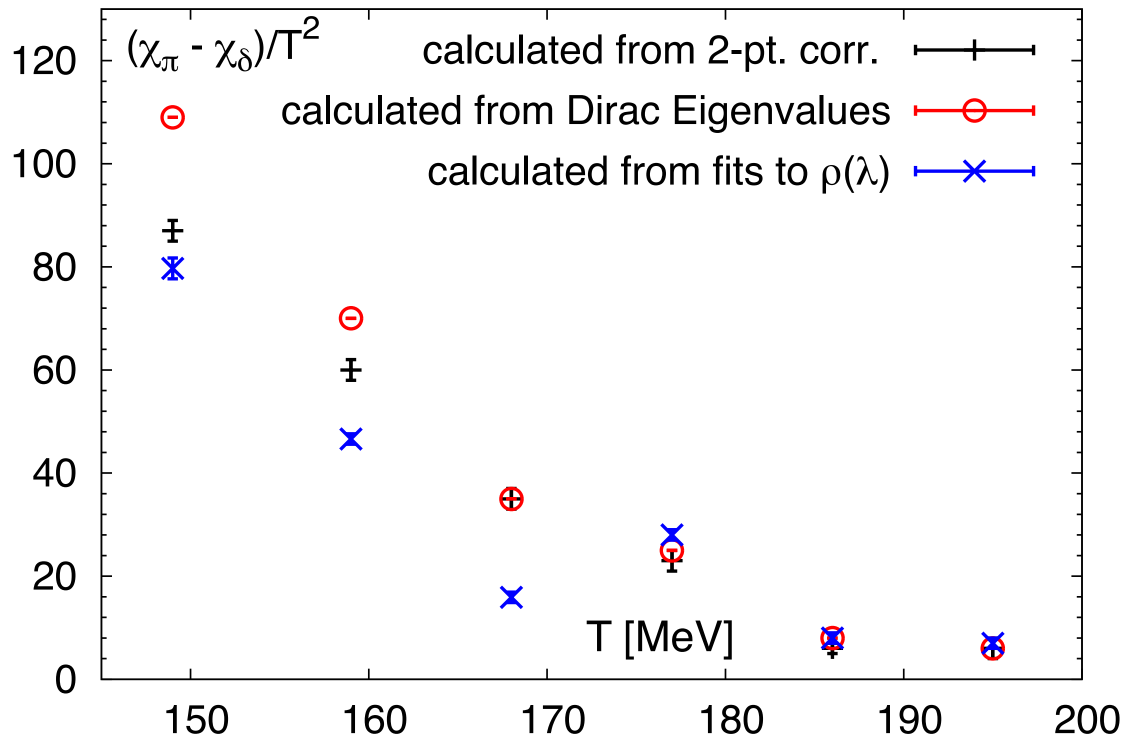
| Ansatz | $\langle\bar{\psi}\psi\rangle$ | $\chi_\pi$ | $\chi_\delta$ | $\chi_\pi - \chi_\delta$ | $2\chi_{\rm disc}$ |
|---|---|---|---|---|---|
| $c$ | $c\pi$ | $c\pi/m$ | $0$ | $c\pi/m$ | $0$ |
| $\lambda$ | $-2m\ln(m)$ | $-2\ln(m)$ | $-2\ln(m)$ | $2$ | $0$ |
| $m^2\delta(\lambda)$ | $m$ | $1$ | $-1$ | $2$ | $2$ |
| $m$ | $\pi m$ | $\pi$ | $0$ | $\pi$ | $\pi$ |
| $m^2$ | $\pi m^2$ | $\pi m$ | $0$ | $\pi m$ | $\pi m$ |

Bazavov et al., [HotQCD] arXiv:1205.3535

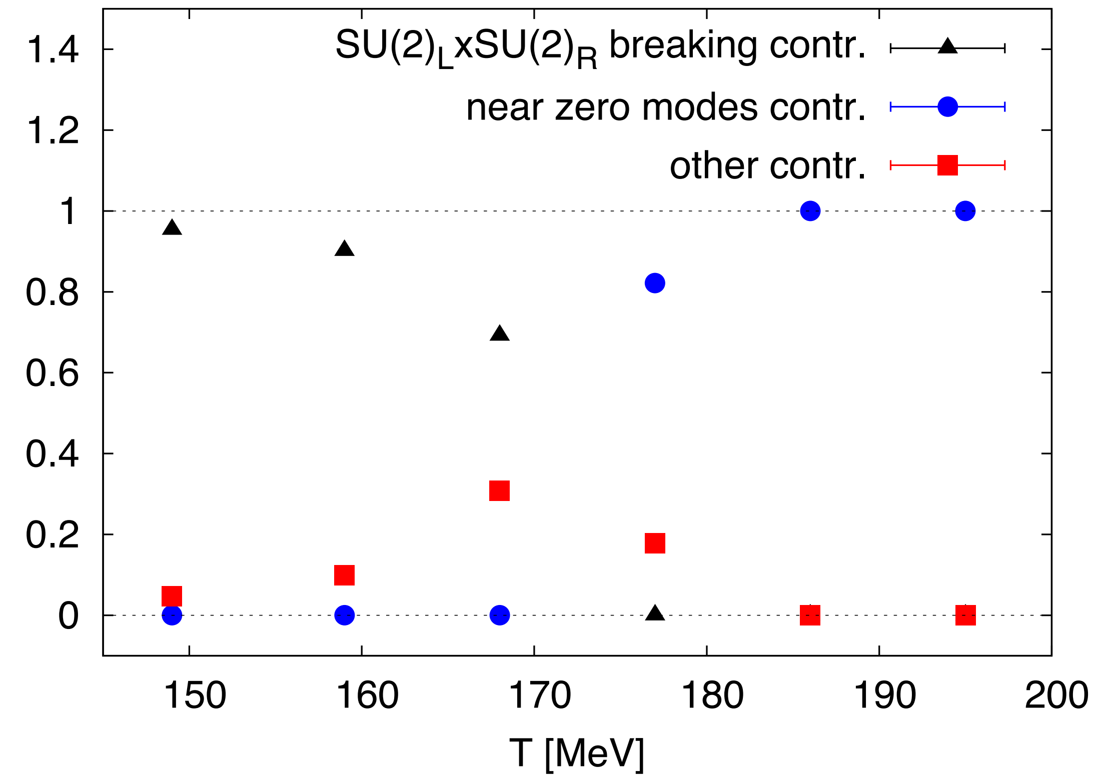
n>2 point correlation functions and their susceptibilities are needed to investigate U(1) breaking only if $\rho(\lambda)$ is analytic in $m^2$

Aoki, Fukaya and Taniguchi, arXiv:1209.2061

# fits to the Dirac Eigenvalue spectrum




fractions of different contributions




fitting ansatz to the Dirac Eigenvalue spectrum

$$\rho(\lambda, m) = a_0\, m + a_1\, m^2 \delta(\lambda) + a_2\, \lambda$$

$$\chi_\pi - \chi_\delta = a_0\, \pi + 2a_1 + 2a_2$$

a0: $SU(2)_L x SU(2)_R$ symmetry breaking contribution

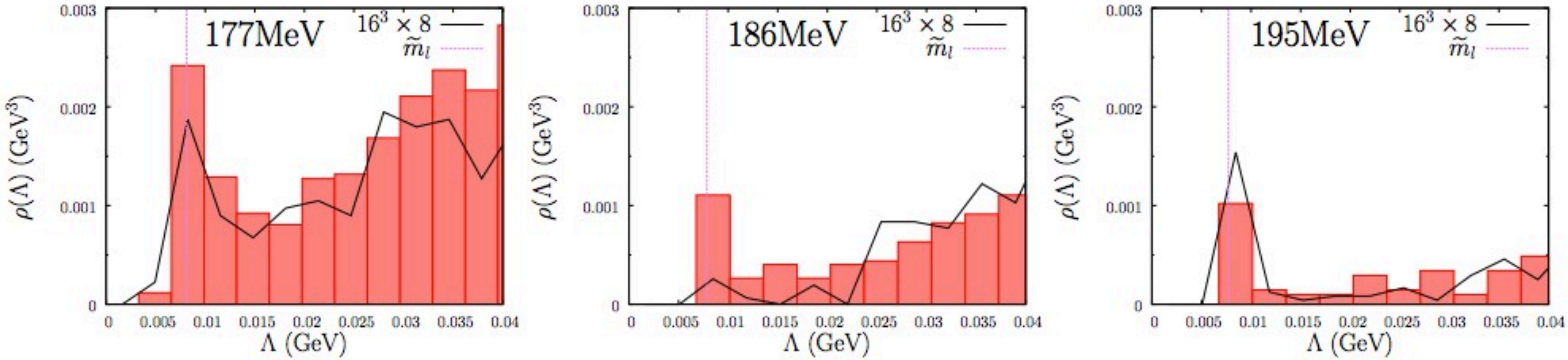
a1: near zero modes contribution

a2: linear infra. behavior

- current fitting ansatz to the Dirac Eigenvalue spectrum gives good description of $\chi_\pi$ - $\chi_\delta$ at three highest temperatures
- $SU(2)_L x SU(2)_R$ symmetry breaking term dominates below $T_c$ while near zero modes contribution dominates above $T_c$

# Underlying mechanism of U(1)$_A$ breaking

black lines: results from $16^3$ lattices, red histograms: results from $32^3$ lattices




- Resulting from non-zero global topology

  density of exact zero modes ~ $1/\sqrt{V}$

- In a relatively dilute gas of instantons and anti-instantons (DIGA)

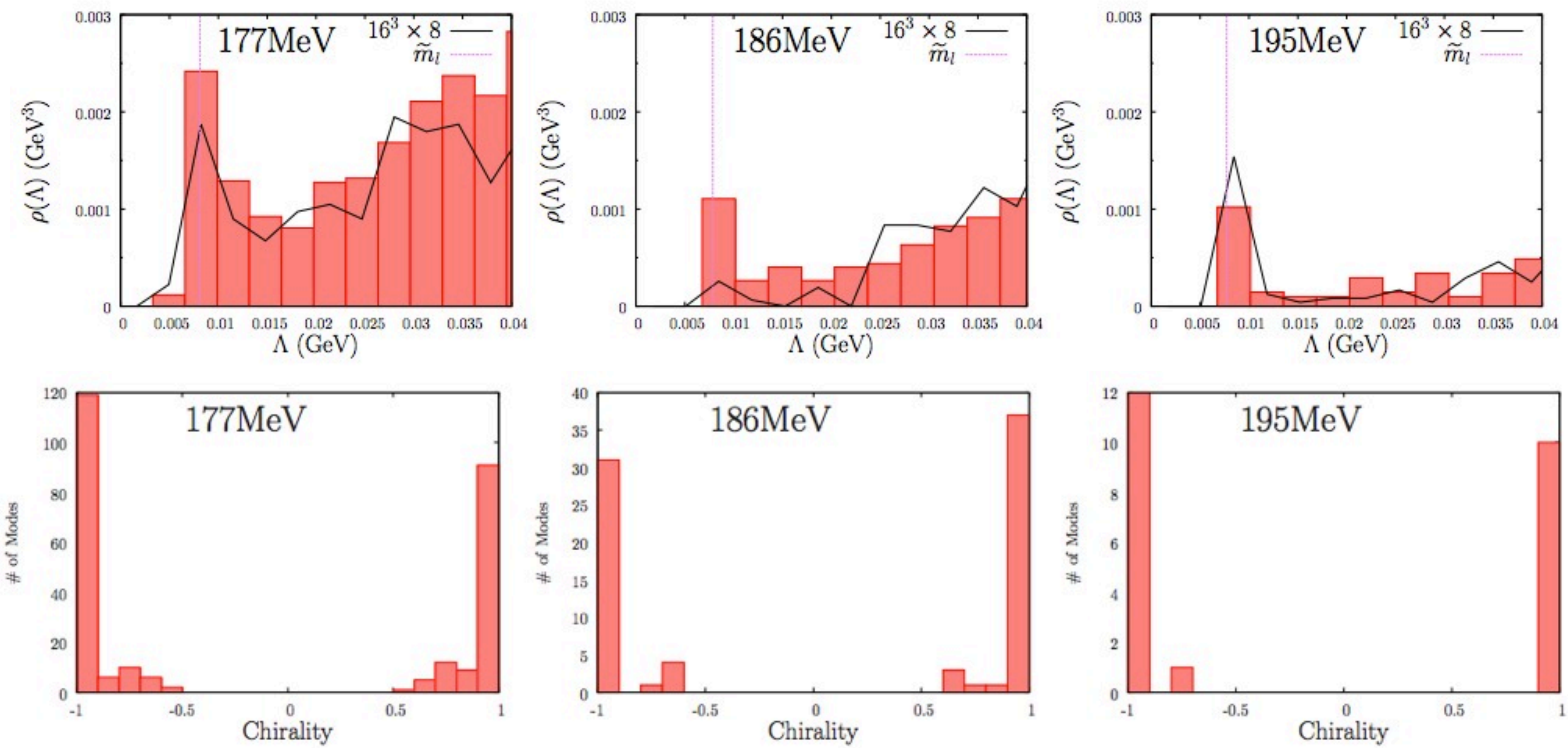
  density of near zero modes independent of V

No evidence of $\rho(\lambda)$ shrinking by a factor of sqrt(8) from $16^3$ to $32^3$ lattices is found, which favors DIGA

# Chirality of near zero modes

Distribution of chirality per configuration:

- obeys bimodal if coming from nonzero topology
- obeys binomial if coming from a dilute gas of instantons



# Distribution of chirality

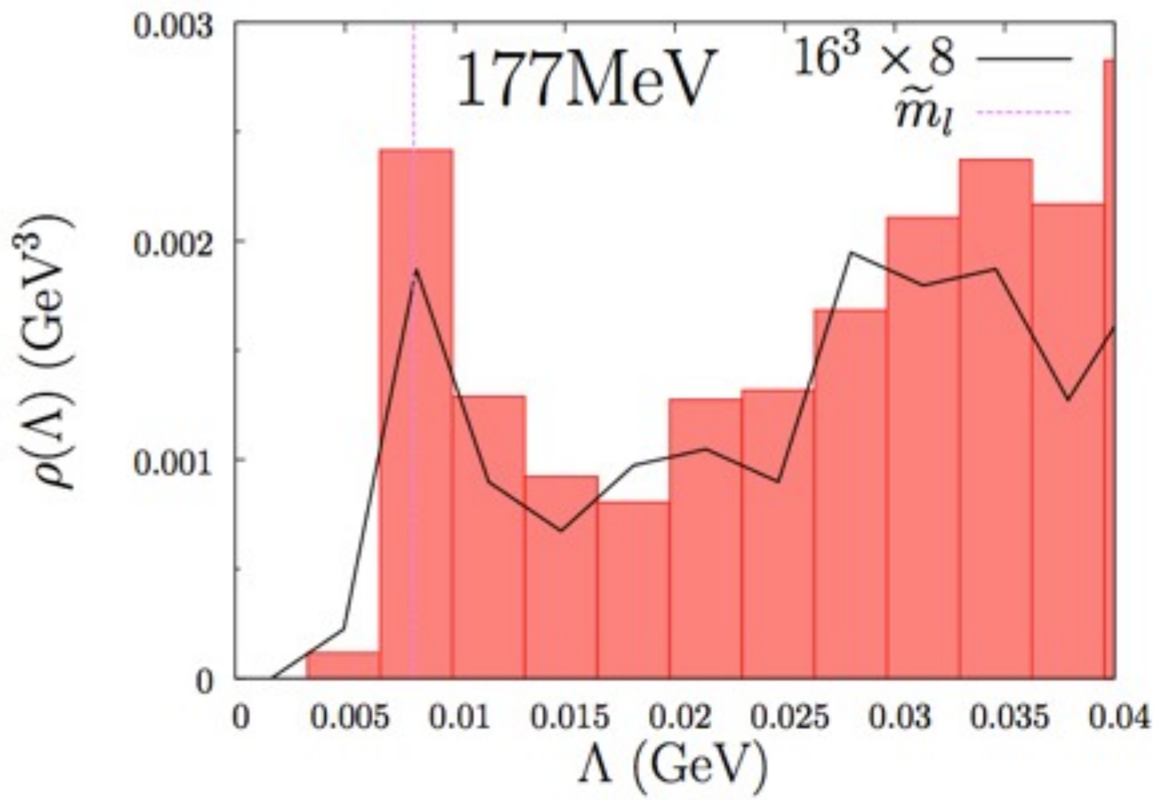
# of configurations with $N_0$ and $N_+$

$32^3$x8, T=177 MeV

| $N_+ \backslash N_0$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $N_0 = 1$ | 40 | 29 | - | - | - | - |
| $N_0 = 2$ | 11 | 20 | 12 | - | - | - |
| $N_0 = 3$ | 3 | 11 | 6 | 2 | - | - |
| $N_0 = 4$ | 0 | 1 | 2 | 1 | 0 | - |
| $N_0 = 5$ | 0 | 2 | 0 | 0 | 0 | 0 |

$N_0$: total # of near zero modes

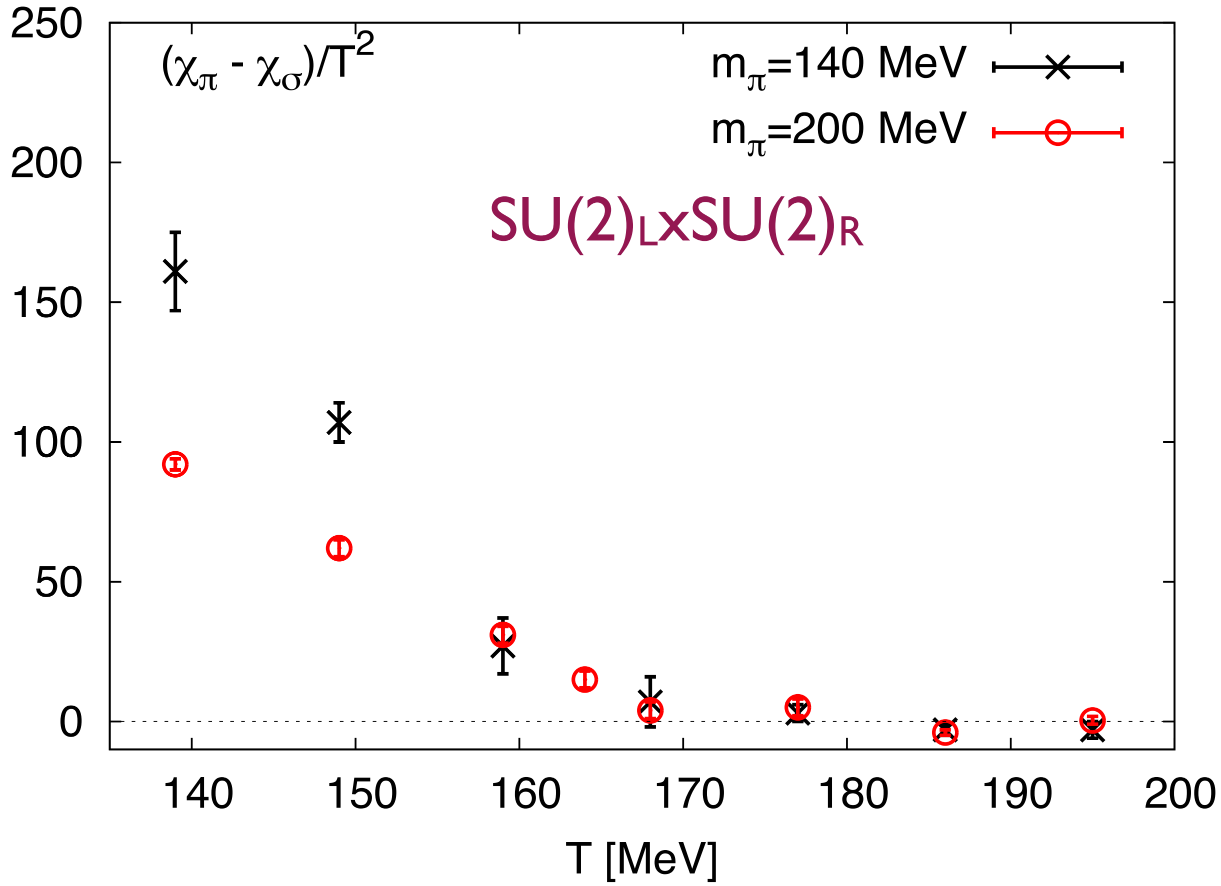
$N_+$: # of near zero modes with positive chirality

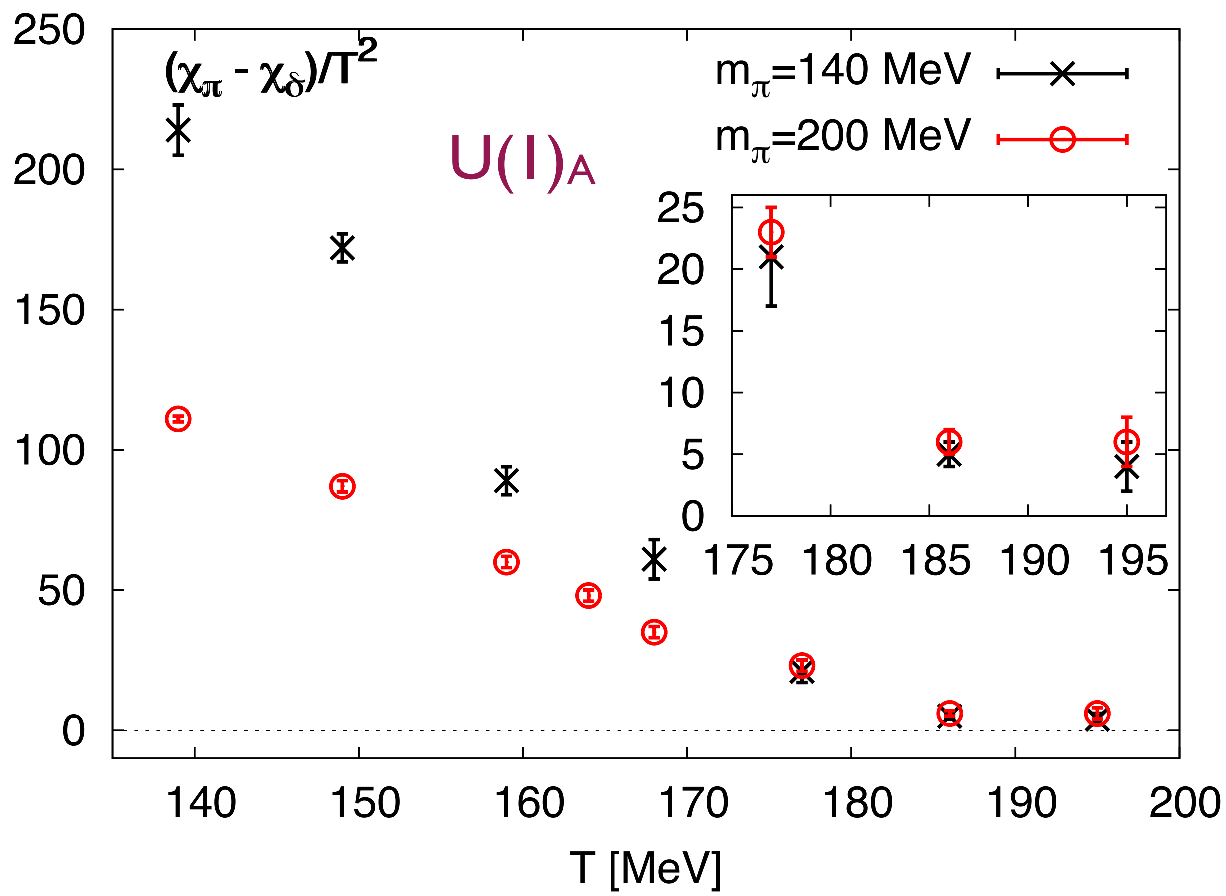


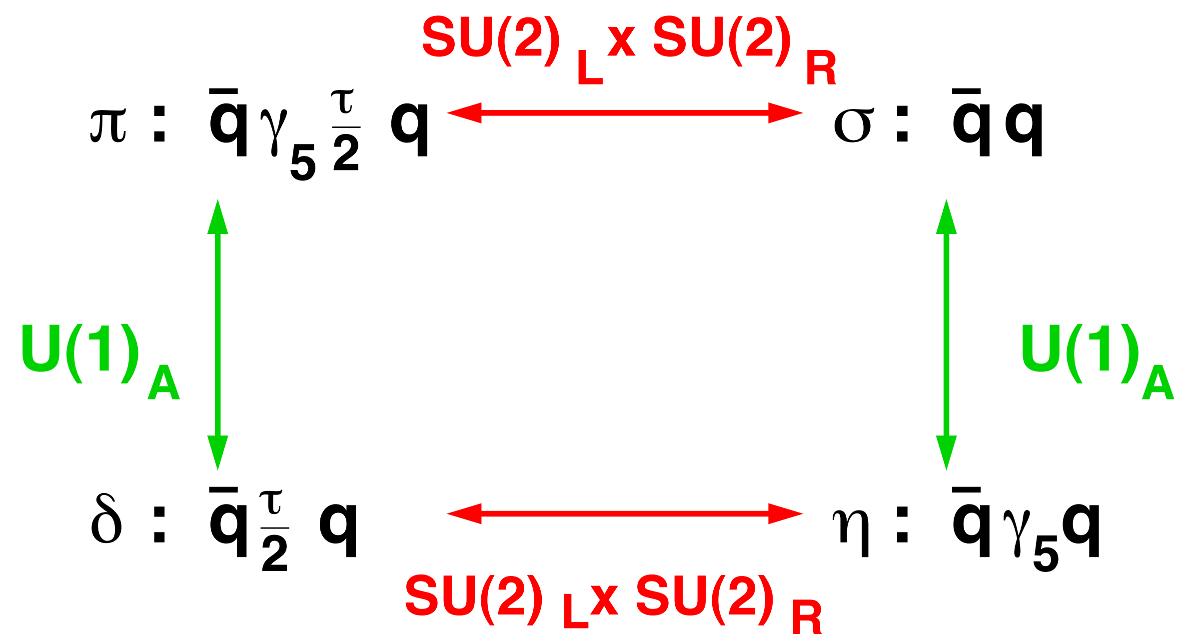

data behaviors more like a binomial contribution

a dilute instanton gas model can describe the non-zero $U(1)_A$ breaking above $T_c$ !

# mass dependence of chiral symmetry restorations



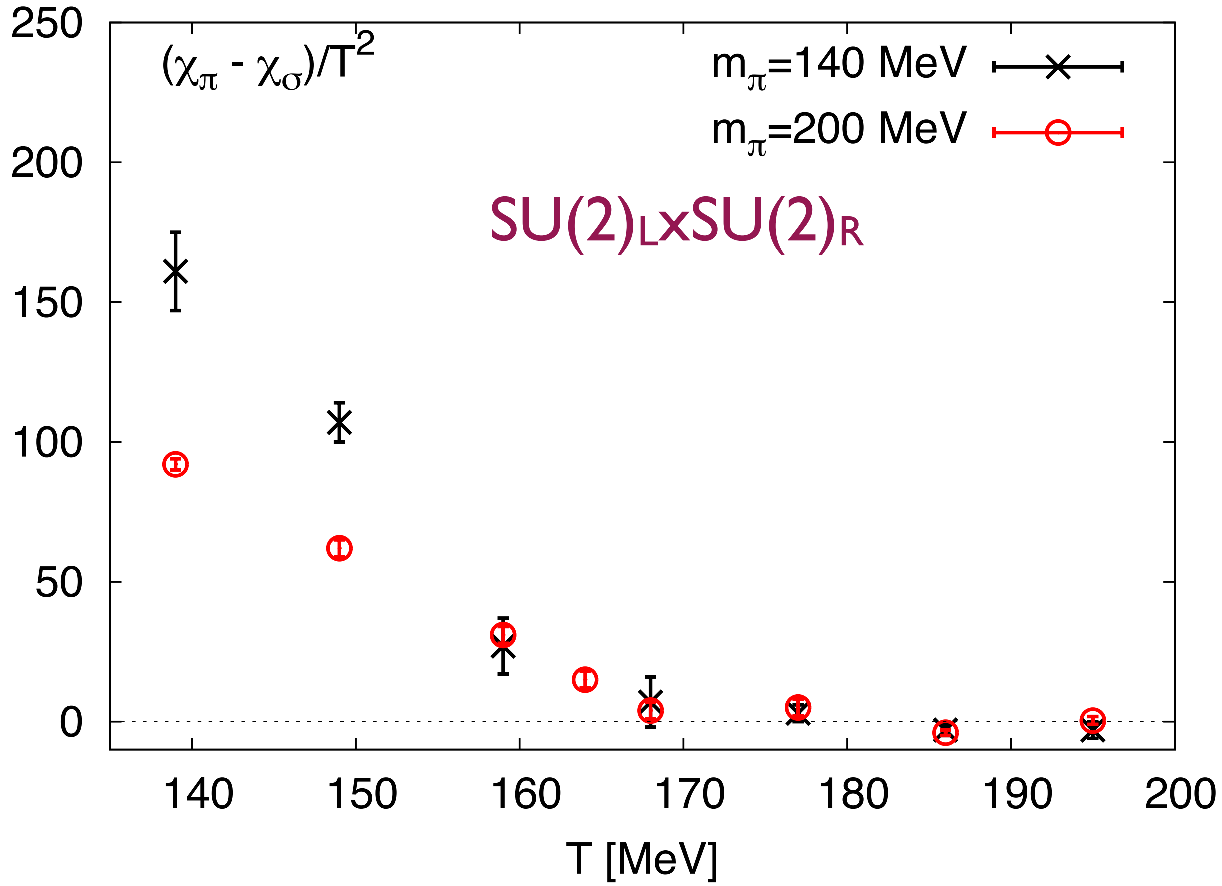



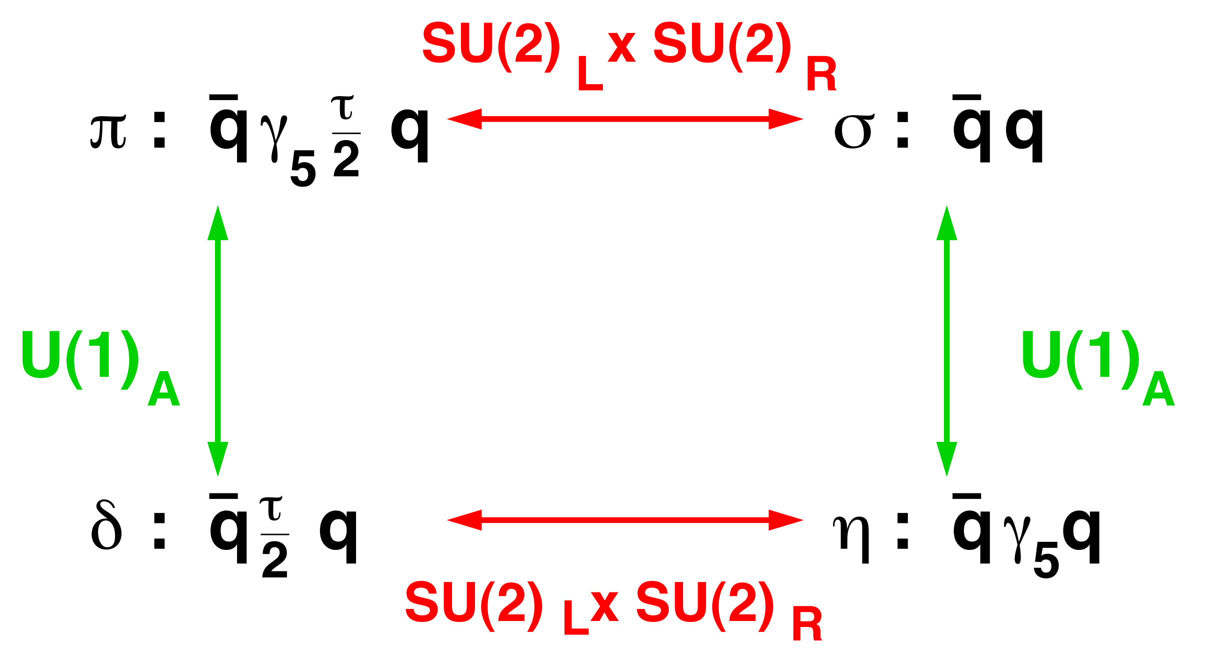



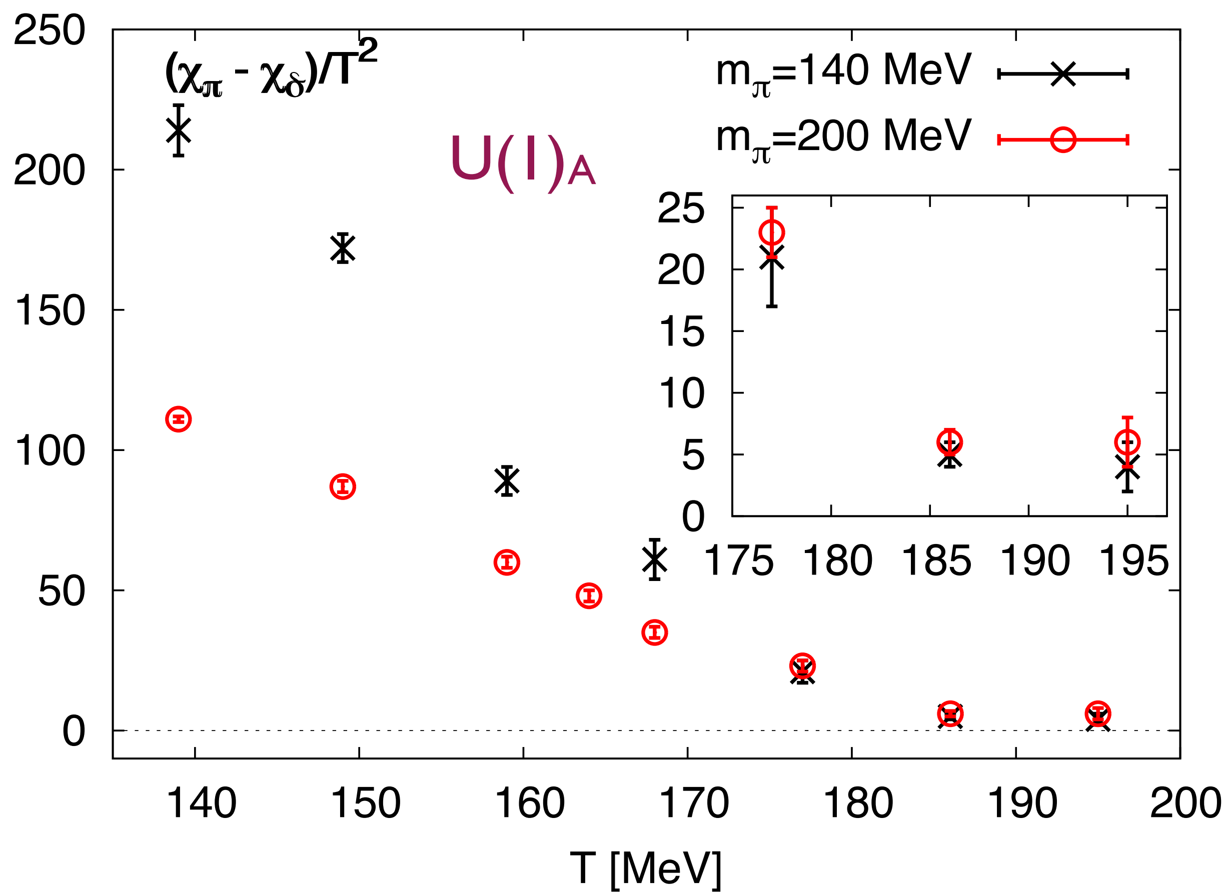

$(\chi_\pi - \chi_\sigma)$: finite quark mass effects negligible at $T \gtrsim 158$ MeV

$U(1)_A$ certainly does not restore at $T_{\chi SB}$~170 MeV,
remains broken up to
195 MeV ~ $1.16T_{\chi SB}$

# mass dependence of chiral symmetry restorations



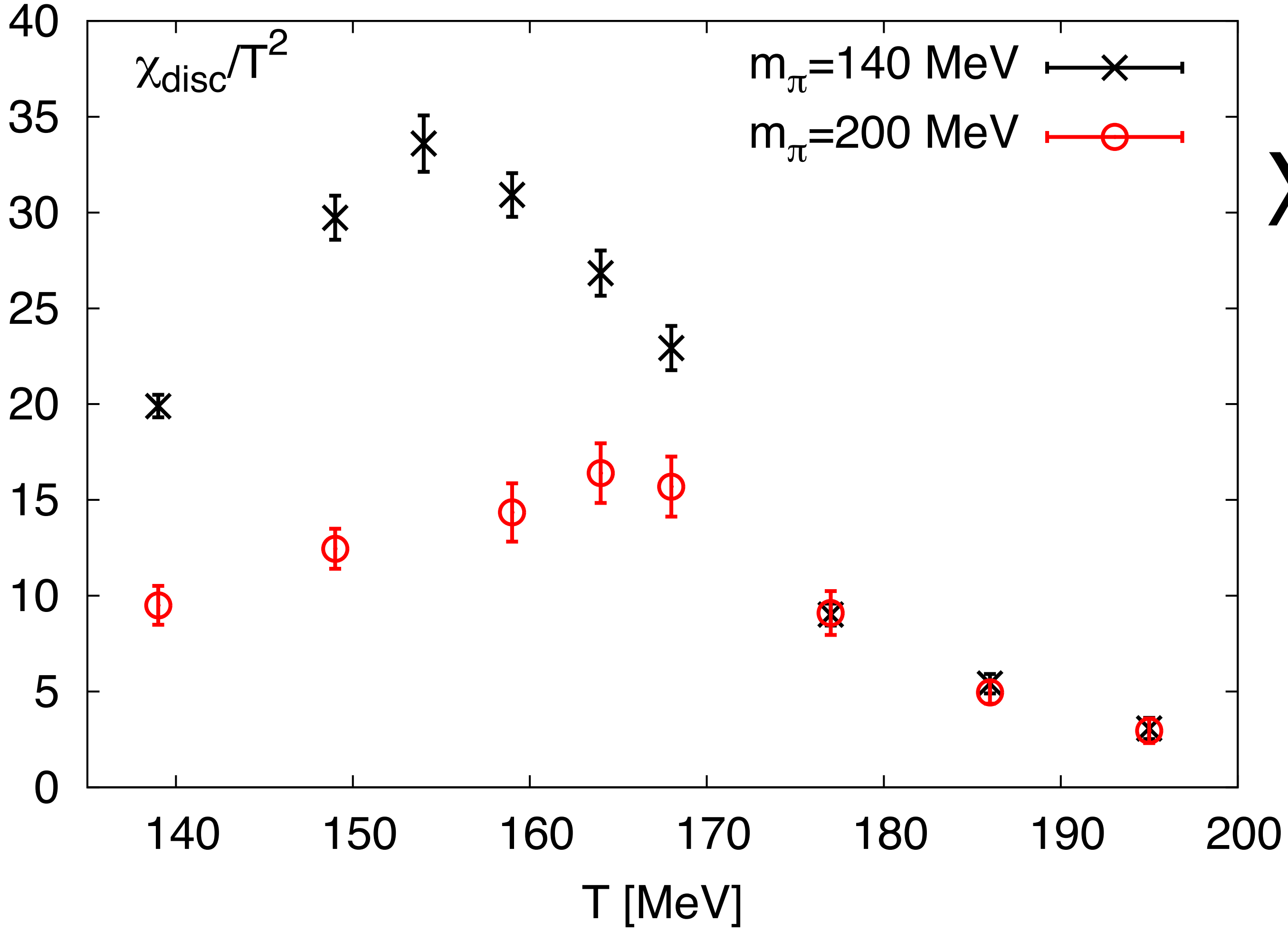




Differences in quark mass dependences:

$$\chi_\pi - \chi_\delta = (\chi_\pi - \chi_\sigma) + (\chi_\sigma - \chi_\delta)$$
$$= (\chi_\pi - \chi_\sigma) + 2\chi_{disc}$$

# disconnected susceptibilities




$$\chi_\pi - \chi_\delta = (\chi_\pi - \chi_\sigma) + (\chi_\sigma - \chi_\delta)$$
$$= (\chi_\pi - \chi_\sigma) + 2\chi_{disc}$$

restoration of $U(1)_A$ requires:

$$\chi_\pi - \chi_\delta = 2\chi_{disc} = 0$$

Non-zero values of $\chi_{disc}$ suggest the breaking of $U(1)_A$ symmetry in the current T window

$\chi_{disc} / T^2$ at the physical quark mass peaks at $T_{pc}$=154 MeV consistent with the result from staggered fermions

# O(N) scaling behavior in the high temperature region

Magnetic Equation of State (MEoS):

$$M = h^{1/\delta} f_G(z), \quad z = t/h^{1/\beta\delta}$$

external field: $h = \frac{1}{h_0}\frac{m_l}{m_s}$ reduced temperature: $t = \frac{1}{t_0}\frac{T-T_c}{T_c}$

$f_G(z)$: universal scaling function, O(N) etc $\beta,\delta$: universal critical exponents

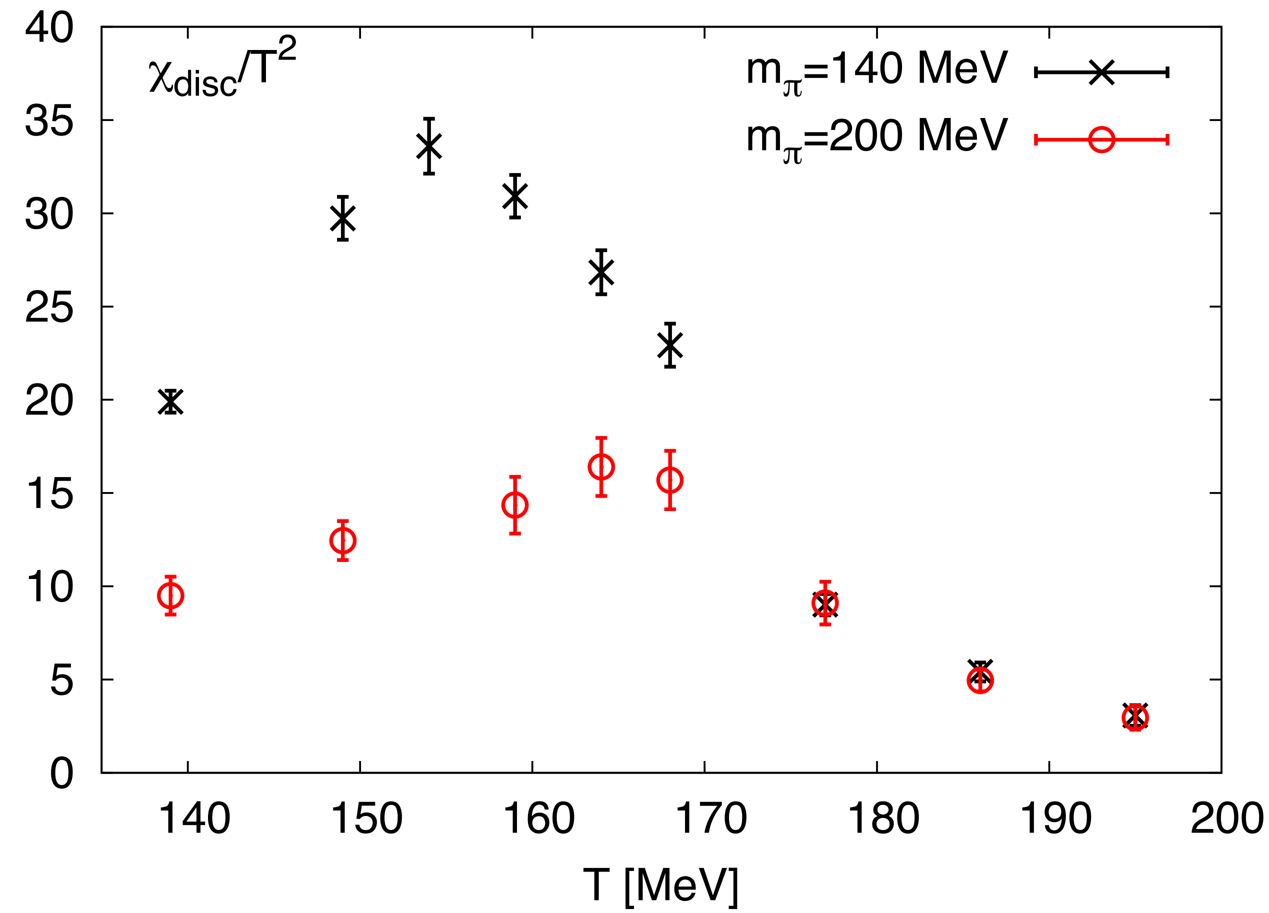
According to O(4) scaling for large positive values of z, i.e. high T

$$f_G(z) \sim R_\chi\, z^{-\beta(\delta-1)}$$

$$M = h^{1/\delta} f_G(z) \sim R_\chi\, t^{-\beta(\delta-1)}\, h$$

$$\chi_M \sim R_\chi\, t^{-\beta(\delta-1)}$$

Engels et al., NPB 675(2003)533



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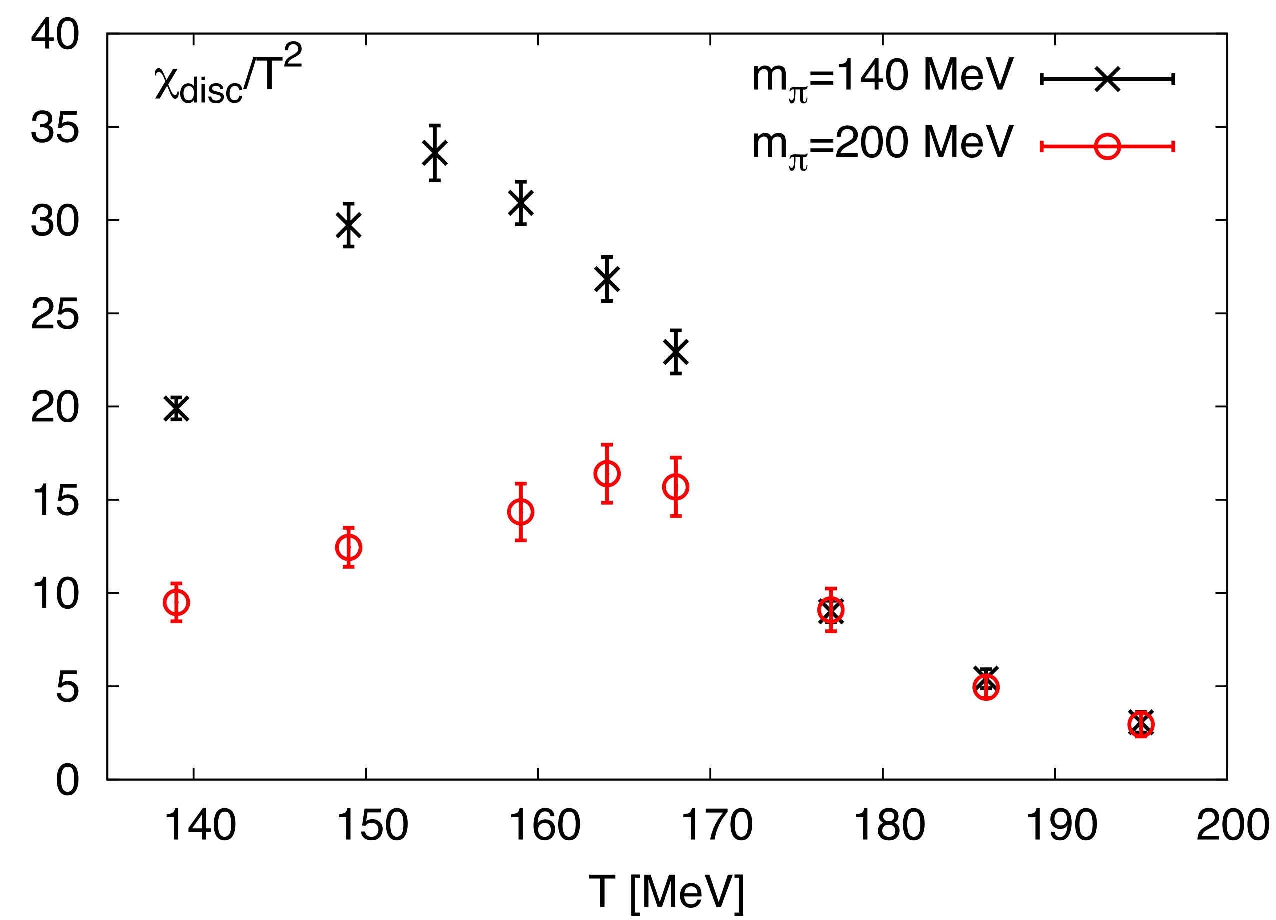
The mass independence of chiral susceptibility observed in the high temperature region indicates the O(4) scaling

Another evidence of the breaking $U(1)_A$ symmetry



# Summary

- Calculation with DWF at the physical pion mass on Nt=8 lattices
  - Crossover behavior
  - $T_{pc} \approx 154$ MeV
  - agreement with staggered results

- $U(1)_A$ symmetry of 2+1 flavor QCD on Nt=8 lattices
  - remain broken up to 195 MeV
  - breaking weakens rapidly after the restoration of $SU(2)_L \times SU(2)_R$
  - quantitatively explained by near zero modes
  - well described by a dilute gas of instantons and anti-instantons