Vector mesons in a strong magnetic field

Yoshimasa Hidaka
(RIKEN)

YH and Arata Yamamoto, 1209.0007,
Phys. Rev. D87 (2013) 094502

Orders of magnitude for magnetic fields

Wikipedia

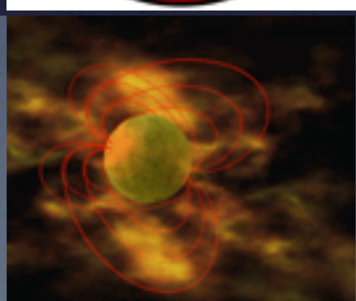
Typical magnet 50G

Neodymium magnet 12,500G

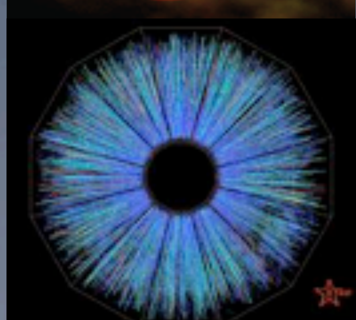
(strongest permanent magnet)

Strongest continuous magnetic field 450,000G

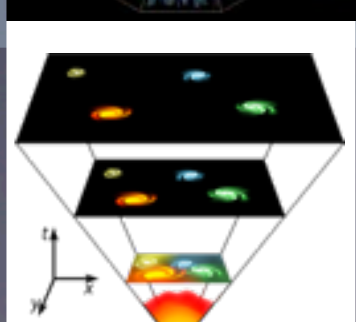
produced in a laboratory



Magnetars $\sim 10^{13}$ G



Heavy ion collisions $\sim 10^{4}\mathrm{MeV}^{2} \sim 10^{17}$ G



The early Universe $\sim 10^{22}$ G

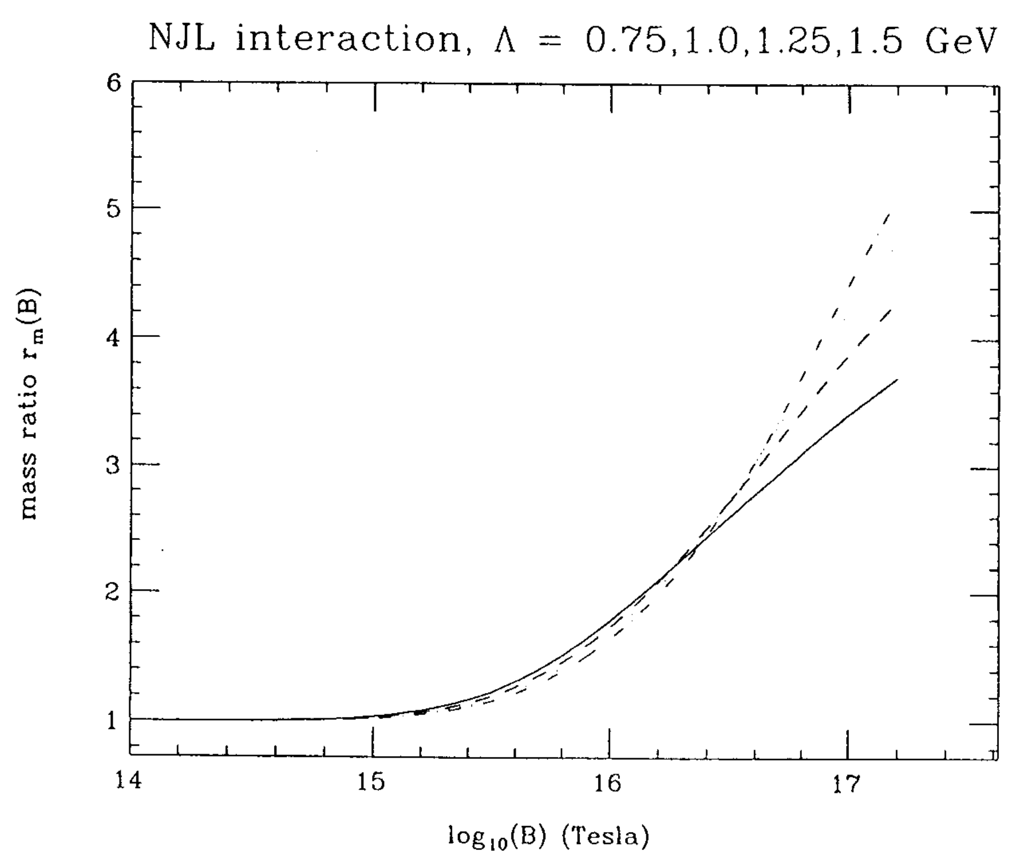
(Electroweak transition)

QCD + magnetic fields
Magnetic catalysis

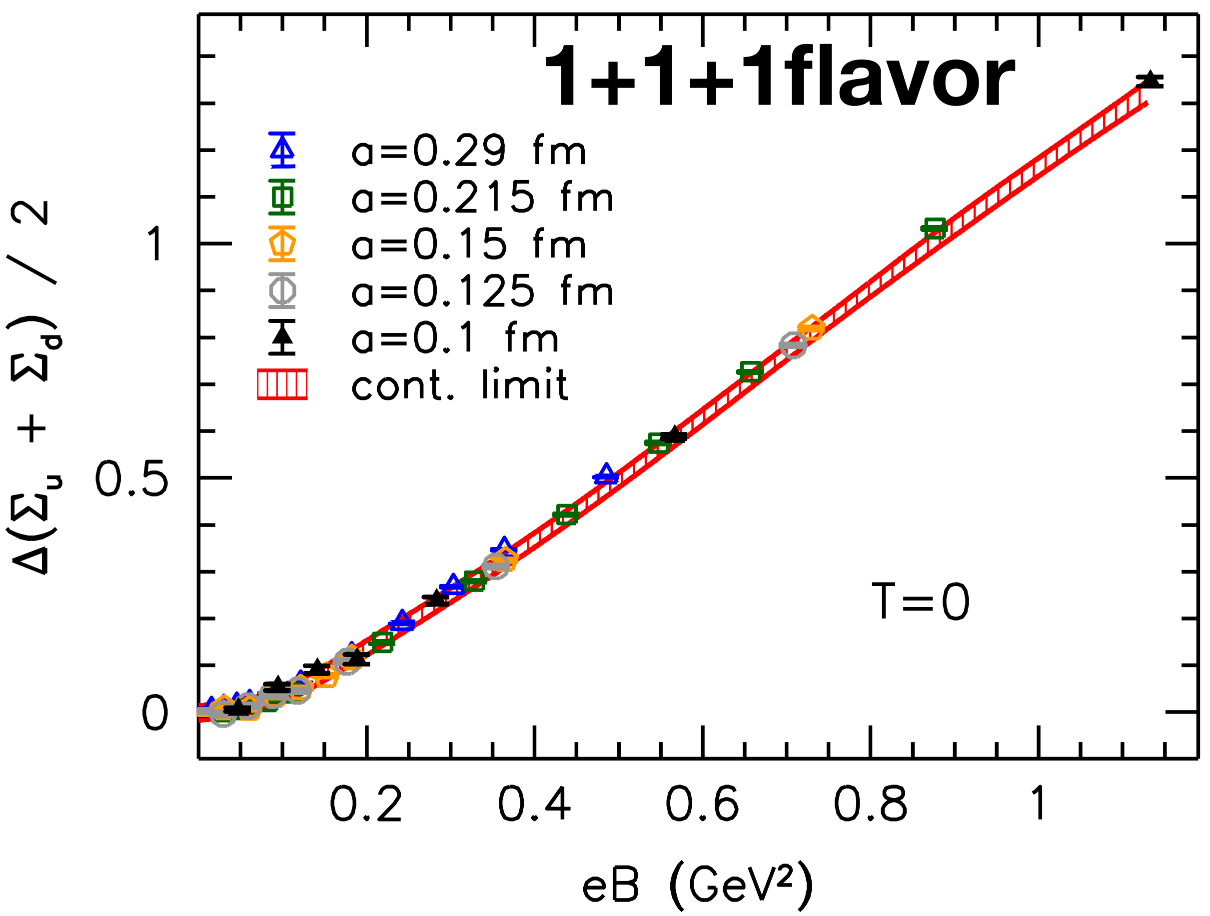
Gusynin, Miransky, Shovkovy('94), Review: Shovkovy, 1207.5081

Chiral symmetry breaking is enhanced in B.

Schramm, Muller, Schramm Mod. Phys. Lett. A 07, 973 (1992)



Bali, Bruckmann, Endrodi, Fodor, Katz, Schafer, JHEP 1202 (2012) 044



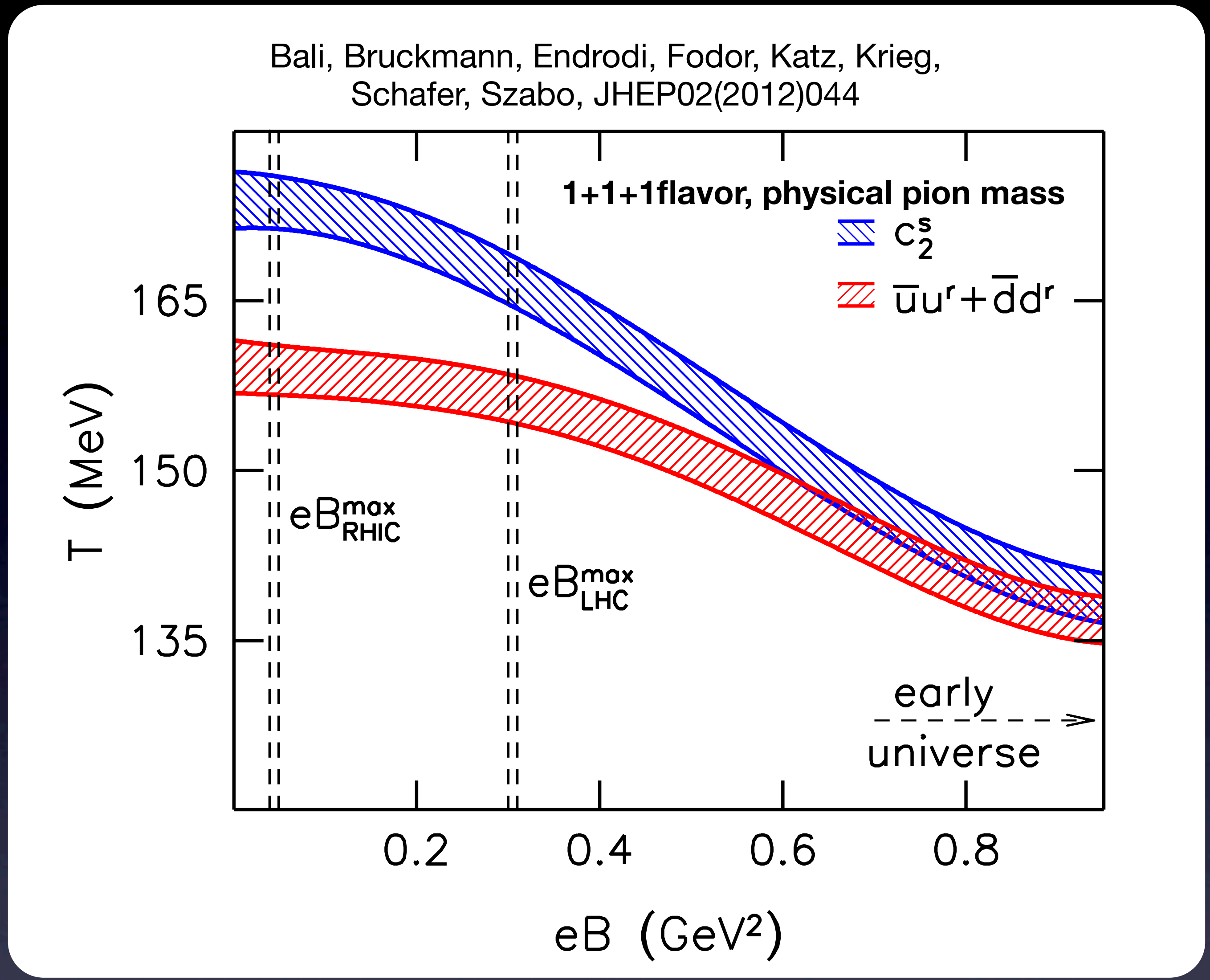
Model

Kawati, Konishi, Miyata('83), Klevansky, Lemmer('89), Suganuma, Tatsumi('91), Klimenko('92), Krive, Naftulin('92), Schramm, Muller, Schramm('92), ...

Lattice QCD

Buividovich, Chernodub, Luschevskaya, Polikarpov ('09) Braguta, Buividovich, Kalaydzhyan, Kuznetsov, Polikarpov('10) D'Elia, Mukherjee, Sanfilippo('10) D'Elia and Negro('11) Ilgenfritz, Kalinowski, Muller-Preussker, Petersson, and Schreiber('12), ...

QCD + magnetic field at finite T



Decreasing T_c!
"Inverse Magnetic Catalysis"

Possible explanations:

Magnetic inhibition, Fukushima, YH ('12) **Deconfining phase transition,** Fraga, Palhares ('12) **Quark mass gap, ...** Kojo, Su ('12), ('13)

QCD + magnetic fields

Chiral magnetic effect

Kharzeev, McLerran,Warringa ('07)   Fukushima, Kharzeev, Warringa ('08)

$$J_V^i = \sum_f \frac{q_f B^i N_c}{2\pi^2} \mu_A$$

Chiral separation effect

Son, Zhitnitsky('04), Metlitski, Zhitnitsky('05)

$$J_A^i = \sum_f \frac{q_f B^i N_c}{2\pi^2} \mu$$

closely related to chiral anomaly.

Today’s focus:

Hadron spectra in B
in particular,

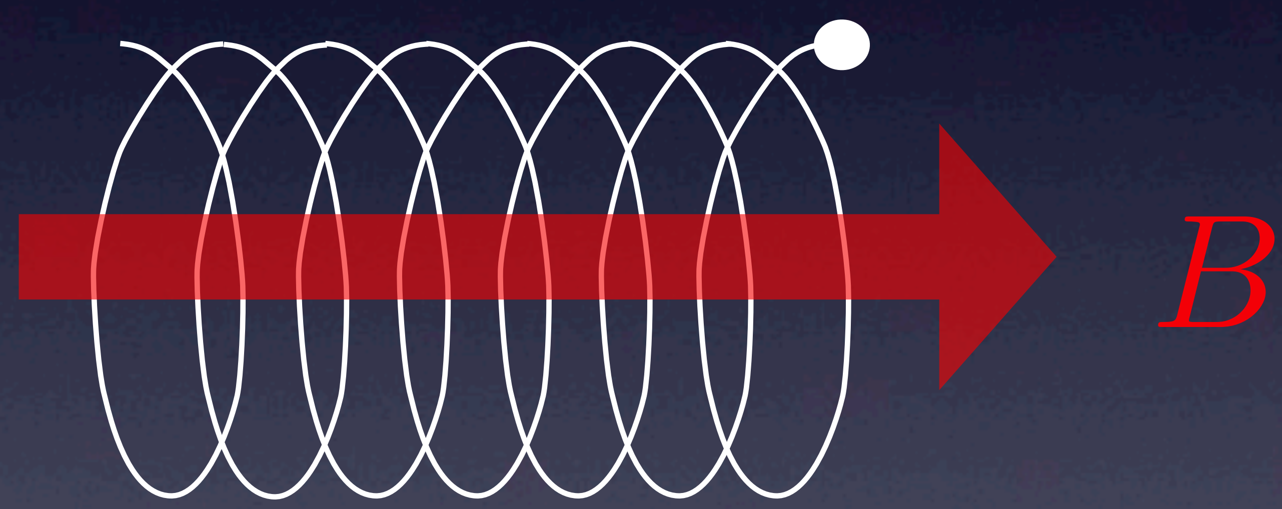
Vector mesons

Charged particle in a magnetic field

Classical equation of motion

$$H\ddot{\boldsymbol{x}} = e(\boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}),$$

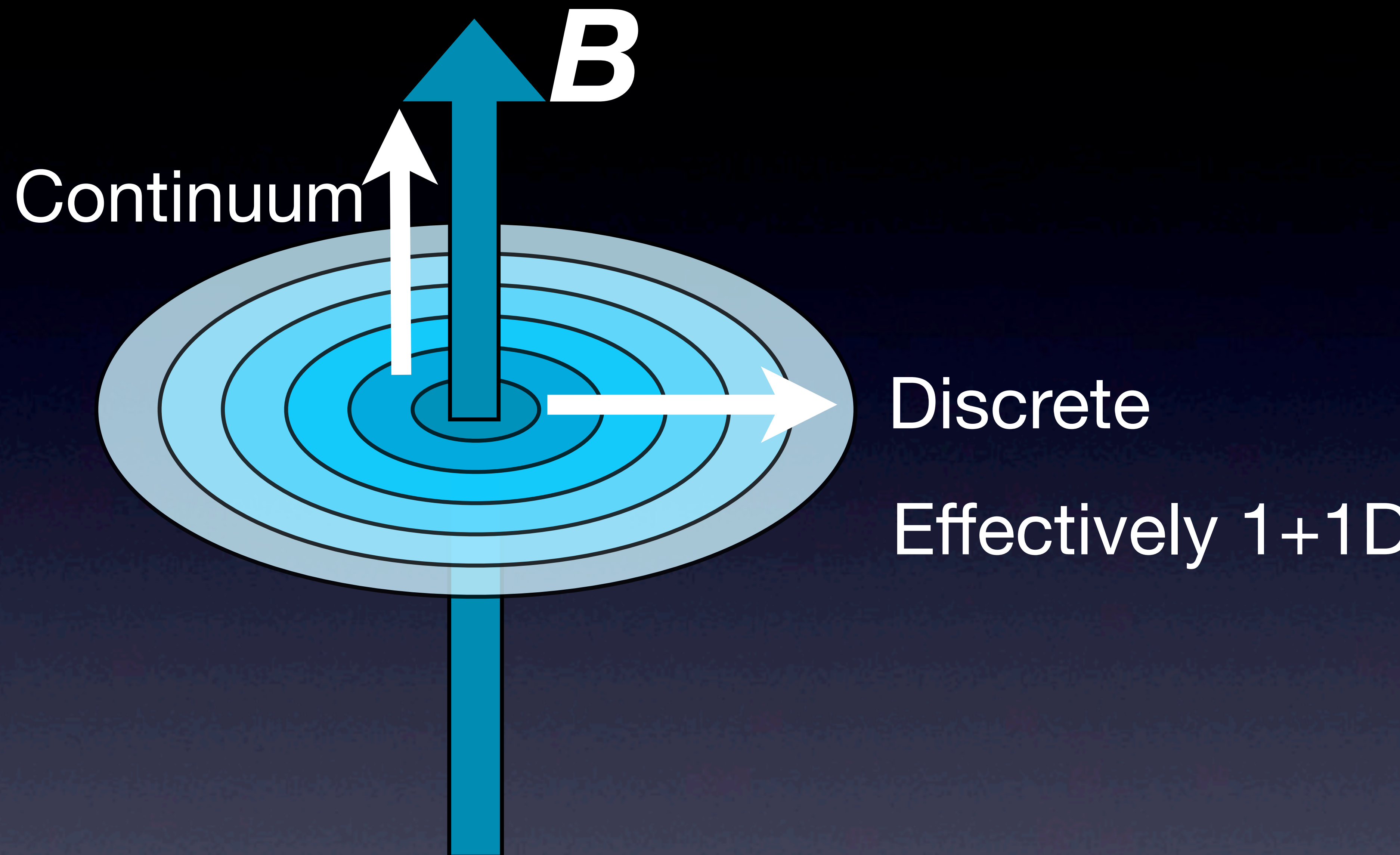
$$H = \sqrt{\boldsymbol{p}^2 + m^2}$$ Lorentz force



Closed orbital motion in the transverse plane

➡ **(Landau) quantization**

Landau quantization



$$E^2 = p_z^2 + m^2 + \underbrace{(2n+1)qB}_{\text{Landau quantization}} - \underbrace{g s_z qB}_{\text{Zeeman splitting}}$$

Why is a vector meson interesting?

Vector meson mass

$$m_\rho^2(B) \approx m_\rho^2 - eB$$

$$m_\rho^2(B = B_c) = 0$$

Vector meson condensation?

Schramm, Muller, Schramm ('92)

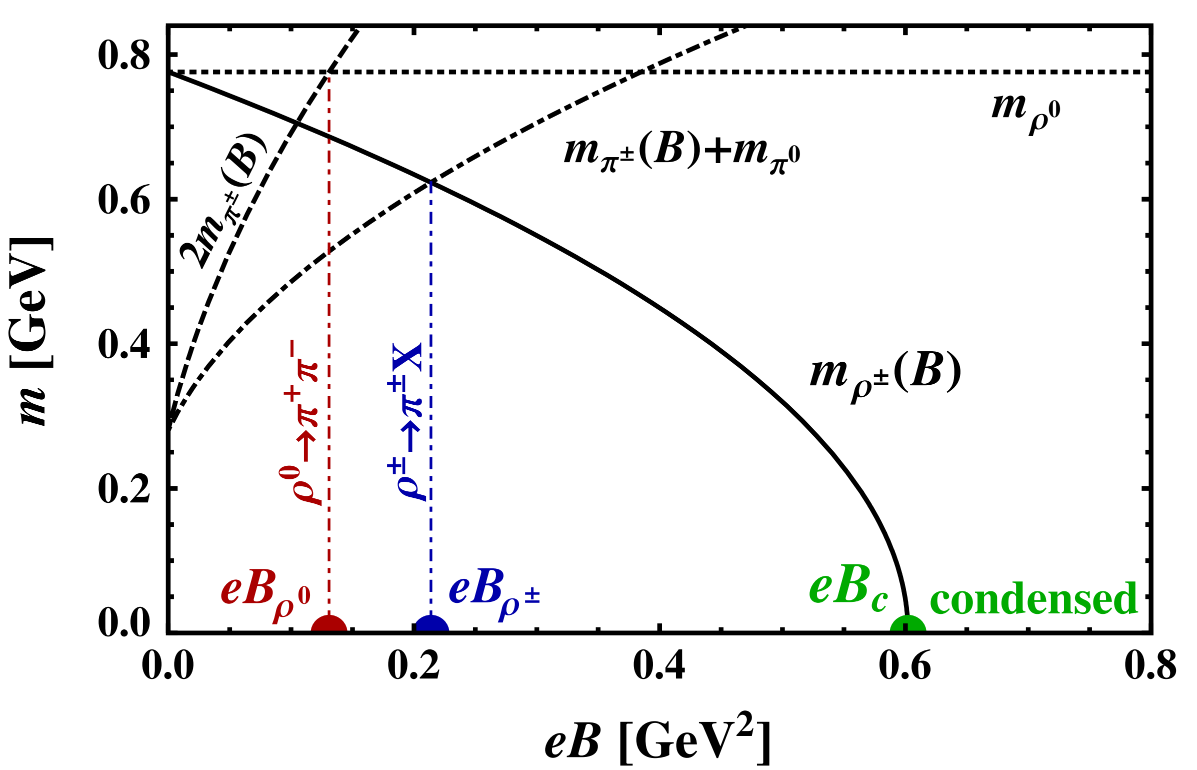
Does a charged vector meson condense in a strong B in QCD?

Model study I

Chernodub('10)('11), Chernodub, Doorsselaere, Verschelde ('11)

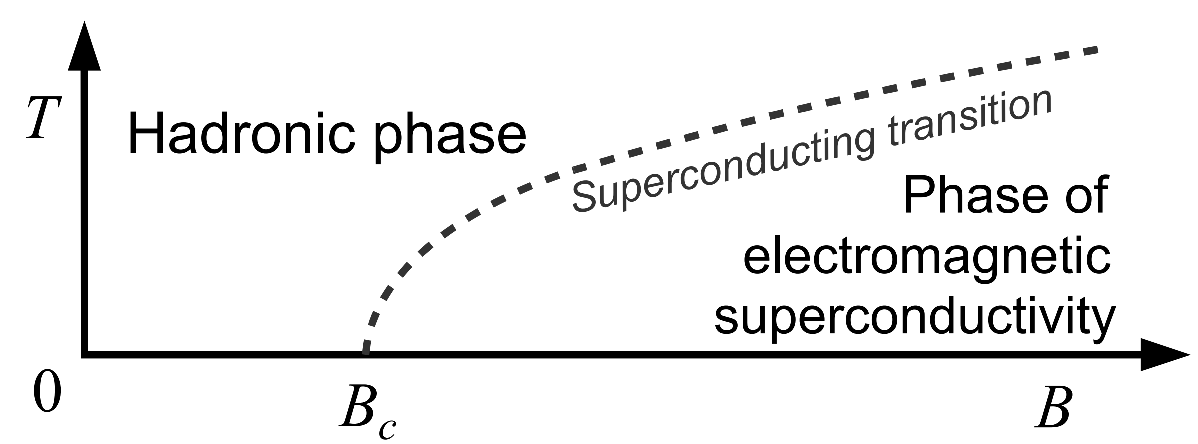
Hadronic model

Chernodub, Phys. Rev. D82 085011(2010)



Extended NJL model

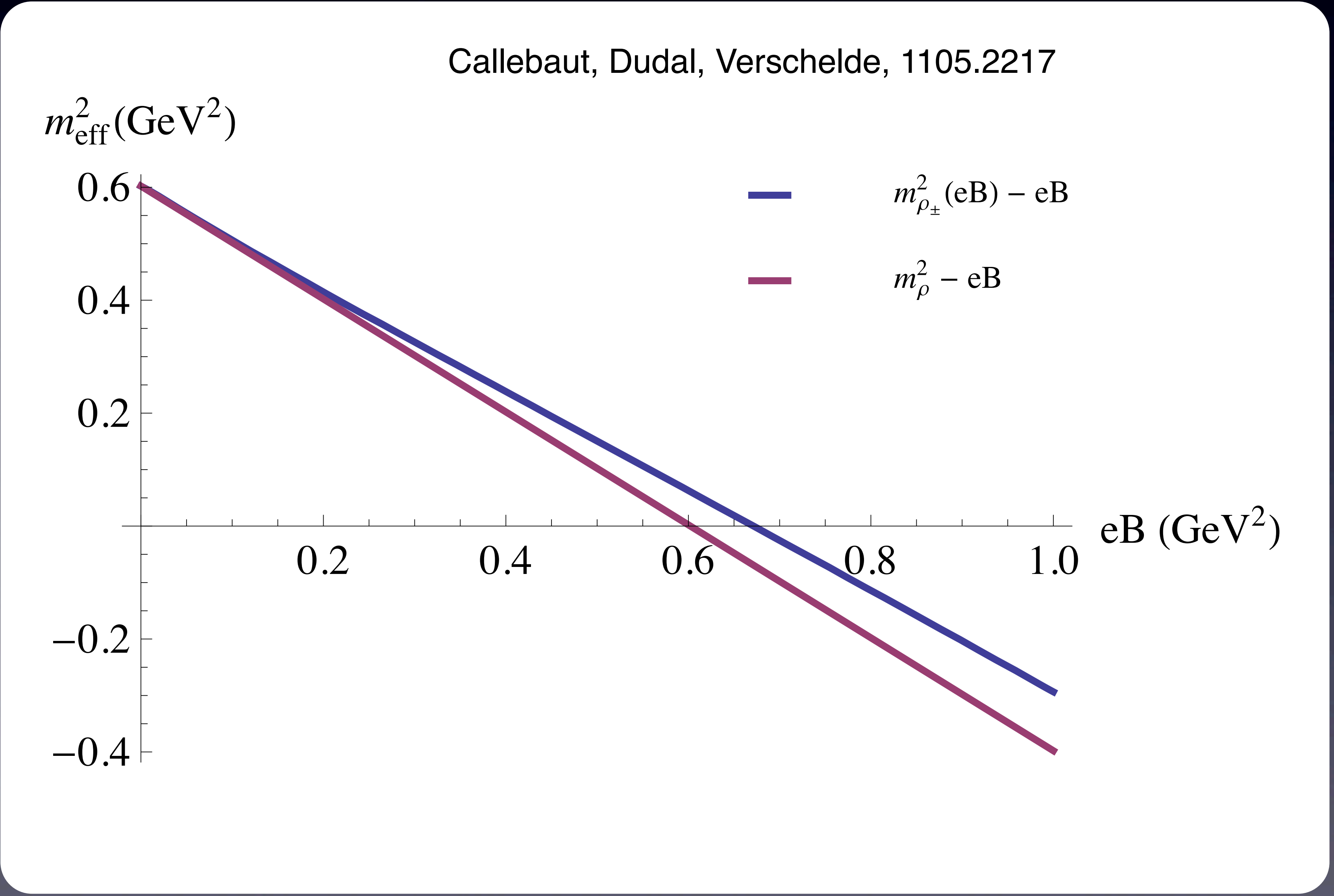
Chernodub, Phys. Rev. Lett. 106, 142003 (2011)



Model study II

Callebaut, Dudal, Verschelde ('10) ('11),
Ammon, Erdmenger, Kerner, Strydom ('11),
Bu, Erdmenger, Shock, Strydom('12),
Callebaut, Dudal ('13)

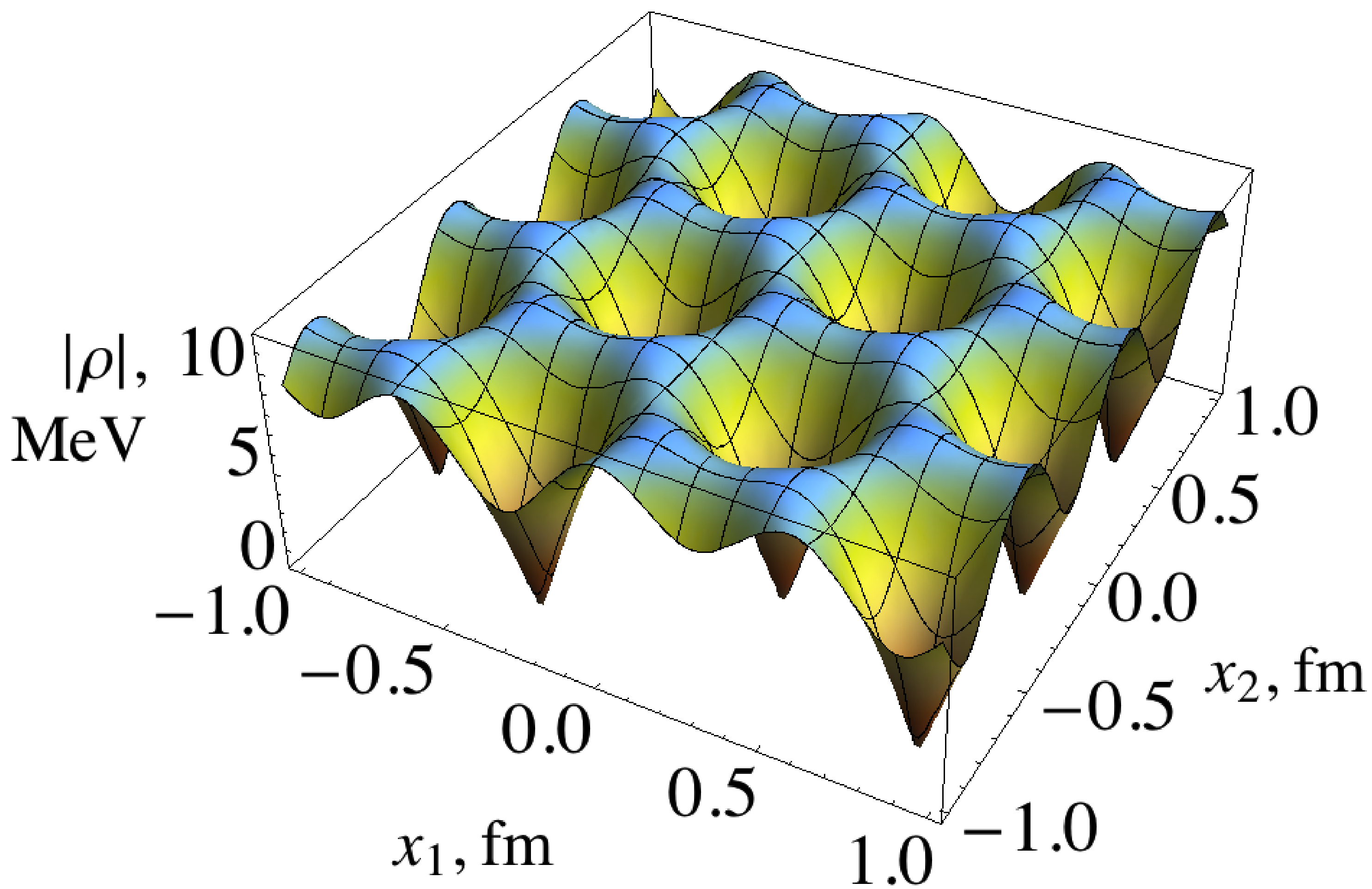
AdS/CFT models



Vacuum superconductivity

Inhomogeneous $\rho^{\pm}$ condensate

Chernodub, Van Doorsselaere, Verschelde, Phys. Rev. Phys. Rev. D 85, 045002 (2012)

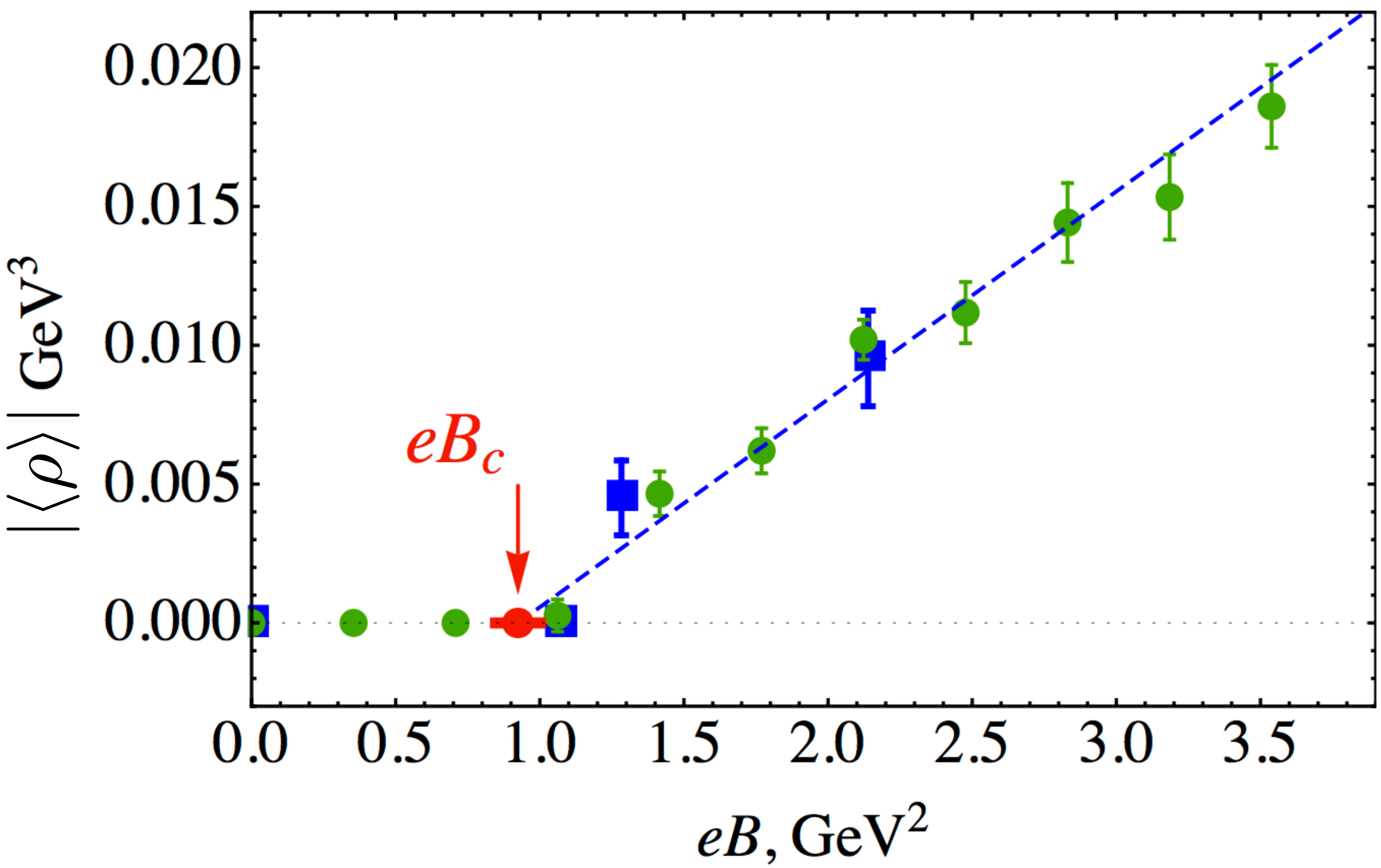


Similar result in holographic approach: Bu, Erdmenger, Shock, Strydom ('13)

Lattice study I

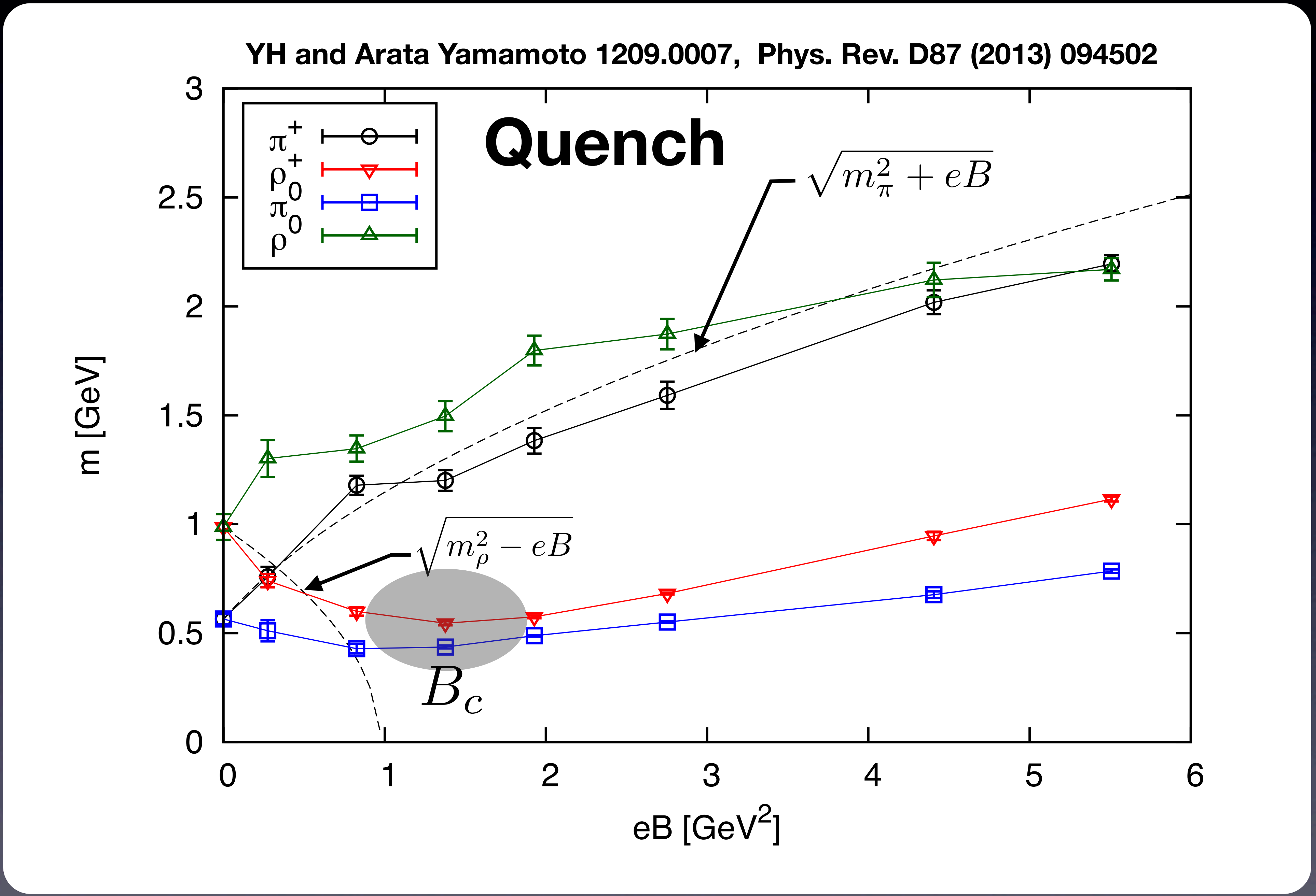
Bragutaa, Buividovichb,Chernodubd, Kotovb, Polikarpovb ('12)

Bragutaa, Buividovichb,Chernodubd, Kotovb, Polikarpovb, Phys. Lett. B718, 667 (2012)



Lattice Study II (our study)

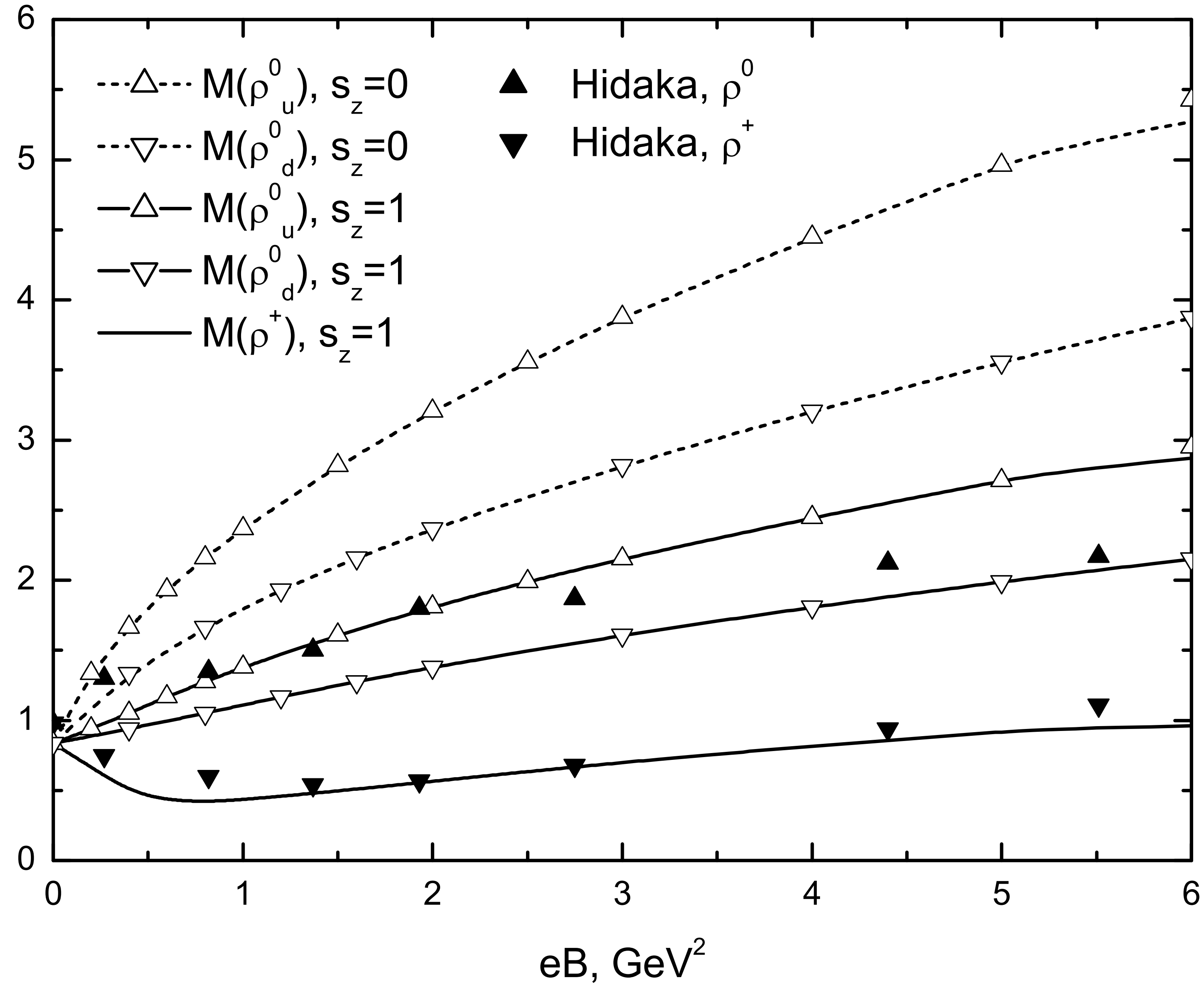
YH, Yamamoto ('12)



Model study III

$q\bar{q}$ system in confined potential + magnetic field

Andreichikov, Kerbikov, Orlovsky, Simonov, Phys. Rev. D 87, 094029 (2013)



Does the vector meson condensation really occur in QCD?

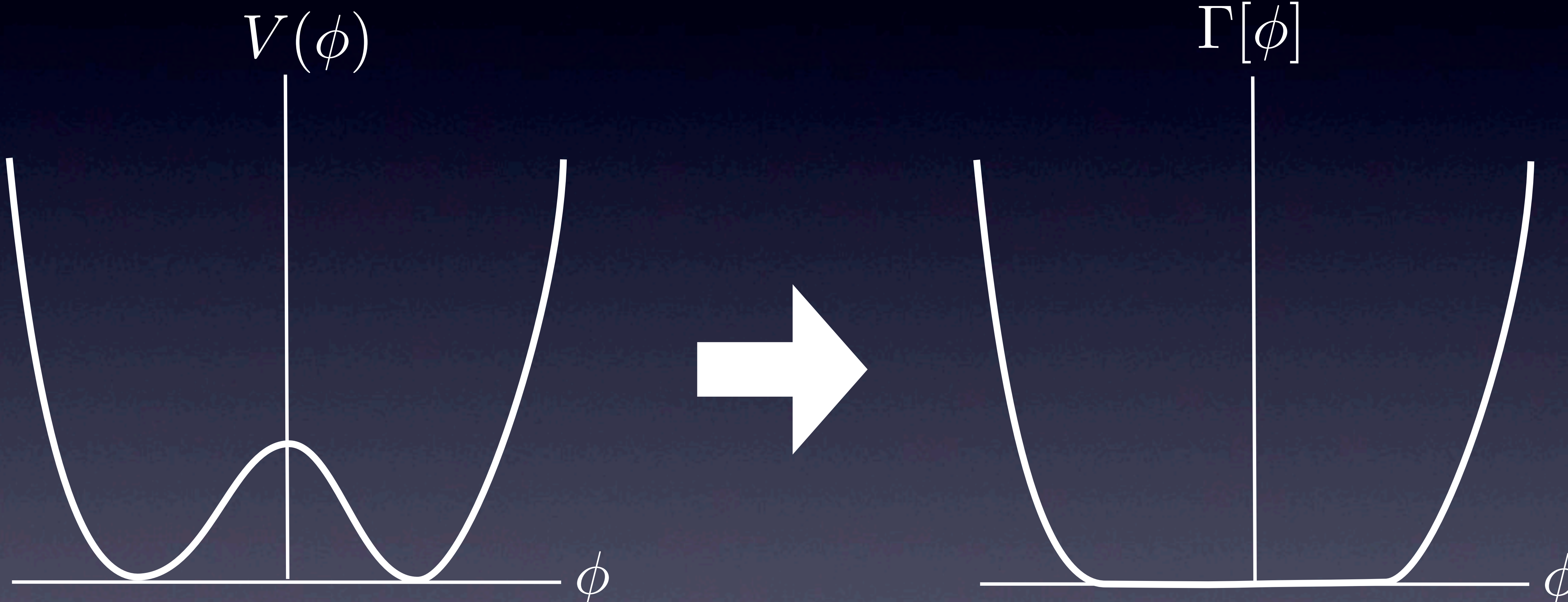
Our answer is NO.

I want to convince you this.

Theoretical analysis

Vafa-Witten theorem

Convexity of effective action



Potential in the Lagrangian

Effective action

Convexity

Generating functional: $e^{R[J]} \equiv \int \mathcal{D}\phi e^{-S[\phi]+J\phi}$

$$e^{R[J+\Delta J]} = \int \mathcal{D}\phi e^{-S[\phi]+J\phi+\phi\Delta J} = e^{R[J]}\langle e^{\phi\Delta J}\rangle_J$$

where $\langle \mathcal{O}\rangle_J \equiv \int \mathcal{D}\phi e^{-S[\phi]+J\phi-R[J]}\mathcal{O}$

$$e^{R[J+\Delta J]-R[J]} = \langle e^{\phi\Delta J}\rangle_J \geq e^{\langle\phi\rangle\Delta J}$$

$\langle e^{\mathcal{O}}\rangle \geq e^{\langle\mathcal{O}\rangle}$ Jensen's inequality

⇒ $R[J+\Delta J] - R[J] \geq \dfrac{\delta R[J]}{\delta J}\Delta J$ Convex

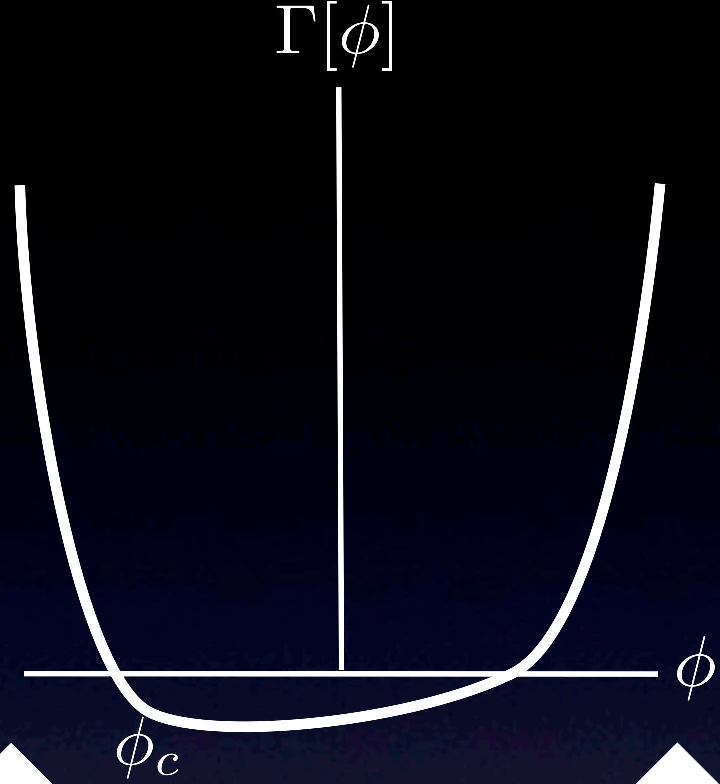
$\Gamma[\phi] = J\phi - R[J]$ is also convex.

Spontaneous symmetry breaking

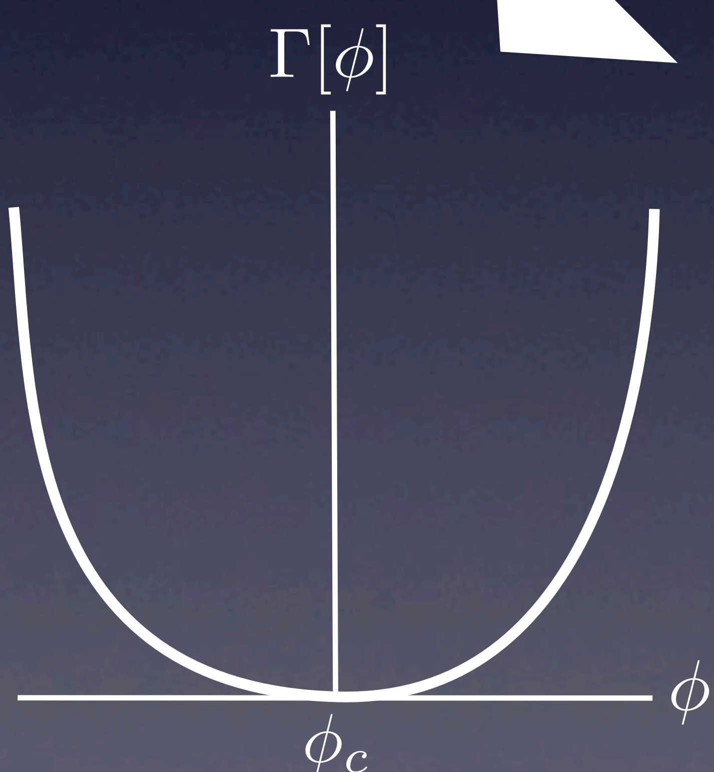
Explicit breaking term

$$\epsilon \neq 0$$

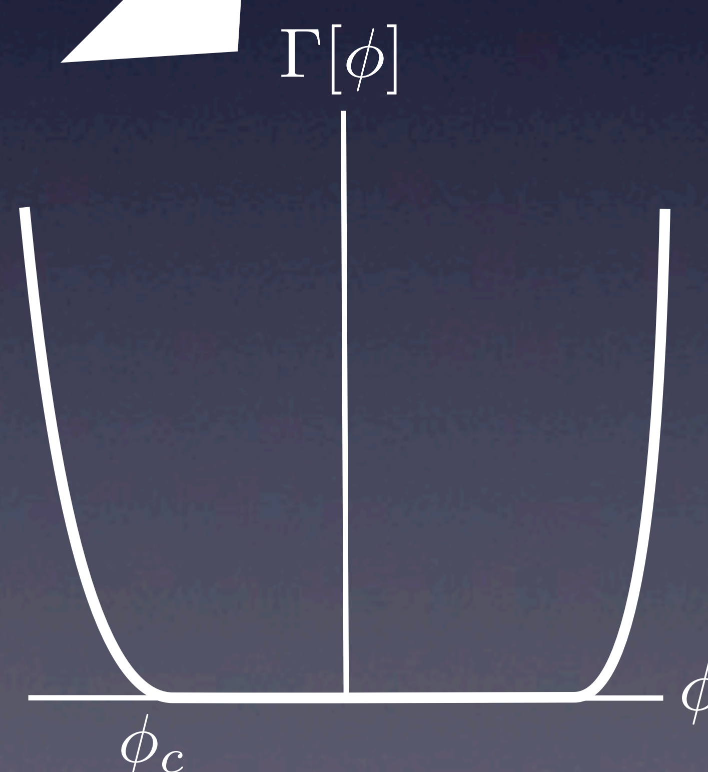
***Z*$_2$ symmetric at** $\epsilon = 0$



$$V \to \infty$$
$$\epsilon \to 0$$



$\phi_c = 0$ symmetric phase



$\phi_c \neq 0$ broken phase

Symmetry breaking

Add explicit breaking term to term.

$$S \to S + \int d^4x \epsilon \bar{\psi} \Gamma \psi \quad \text{e.g.,}\ \Gamma = \tau^3$$

Calculate the order parameter

$$\phi \equiv \frac{1}{\mathcal{N}} \int d^4x \bar{\psi}(x) F \psi(x)\ ,\ \text{e.g.,}\ F = \tau^3$$

$$\langle \phi \rangle_\epsilon = \langle \mathrm{Tr} F \frac{1}{\not{D} + m + \epsilon \Gamma} \rangle_{A,\epsilon}$$

$$\langle \mathcal{O} \rangle_{A,\epsilon} \equiv \frac{1}{\int d\mu} \int d\mu\ \mathcal{O}\ . \quad d\mu = \prod_{\mu,a,x} dA^a_\mu(x) \det(\not{D} + m + \epsilon \Gamma) e^{-S[A]}\ ,$$

Take $\epsilon \to 0$ **limit** **If** $\lim_{\epsilon \to 0} \langle \phi \rangle_\epsilon \neq 0$ **SSB!**

Vafa-Witten theorem (B=0, T=0)

No SSB occurs in the isospin channel.

$$\lim_{\epsilon \to 0} \langle \phi \rangle_\epsilon = 0$$

- **Fermion operator has no zero modes.**

  Fermion propagator is well defined.

- **Fermion determinant is nonnegative.**

  Schwarz inequality works.

- **Order parameter is nonsinglet.**

  Disconnected diagrams do not contribute.

Vafa-Witten theorem

for $\theta = 0$ vacuum

Dirac operator $D = \gamma_\mu(\partial_\mu + igA_\mu)$

Anti-Hermite $D^\dagger = -D$

Chiral symmetry $\gamma_5 D \gamma_5 = -D$

Eigenvalue of $\not{D} + m$: $\pm i\lambda_n + m \neq 0$

Positivity: $\det(\not{D} + m) = \prod_\lambda (i\lambda + m) = m^{n_0} \prod_{\lambda > 0} (\lambda^2 + m^2) > 0$

Upper bound of propagator: $\left\| \frac{1}{\not{D} + m} \right\|_{\mathrm{op}} = \frac{1}{m}$,

$$\langle \phi \rangle_\epsilon = \langle \mathrm{Tr} F \frac{1}{\not{D} + m + \epsilon \mathit{\Gamma}} \rangle_{A,\epsilon}$$

Expanding the order parameter with respect to ε.

$$\mathrm{Tr} F \frac{1}{\not{D} + m + \epsilon \mathit{\Gamma}} = \sum_{n=1}^{\infty} (-1)^n \epsilon^n \mathrm{Tr} F \frac{1}{\not{D} + m} \left(\mathit{\Gamma} \frac{1}{\not{D} + m} \right)^n$$

$$\left| (-1)^n \mathrm{Tr}\, F \frac{1}{\not{D} + m} \left(\mathit{\Gamma} \frac{1}{\not{D} + m} \right)^n \right| \leq \| \mathit{\Gamma} \|_{\mathrm{op}}^n \left\| \frac{1}{\not{D} + m} \right\|_{\mathrm{op}}^{n+1} \sqrt{\mathrm{Tr}\, F F^\dagger}$$

$$= \frac{C^n}{m^{n+1}}, \quad \text{where } C \equiv \| \mathit{\Gamma} \|_{\mathrm{op}}$$

$$\langle \phi \rangle_\epsilon \leq \left\langle \left| \mathrm{Tr}\, F \frac{1}{\not{D} + m + \epsilon \mathit{\Gamma}} \right| \right\rangle_{\epsilon, A} \leq \sum_{n=1}^{\infty} \frac{(\epsilon C)^n}{m^{n+1}} \xrightarrow{\epsilon \to 0} 0$$

No Spontaneously Symmetry Breaking!

Finite B

$$D_\mu = \partial_\mu - igA_\mu \to \partial_\mu - igA_\mu - iqA_\mu^{\mathrm{em}}$$

$$B^z = \partial_x A_y - \partial_y A_x \neq 0$$

Symmetry:

$$SO(3,1) \to SO(1,1)_{t,z} \times SO(2)_{x,y}$$

$$SU(2)_I \times U(1)_B \to U(1)_{I_3} \times U(1)_B = U(1)_{\mathrm{em}} \times U(1)_B$$

Positivity: OK

Possibility of inhomogeneous phase:

$$\phi \equiv \frac{1}{\mathcal{N}} \int d^4x \bar{\psi}(x) F \psi(x) \ ,$$

$$F = \tau_+ \gamma_+ f(x)$$ space dependent

If $\langle \bar{\psi}(x) \tau_+ \gamma_+ \psi(x) \rangle = g(x)$, we may choose $f(x) = g^*(x)$

Positivity, lower bound of quark propagator: OK
Order parameter: nonsinglet.

⇨ **No symmetry breaking.**

Counter arguments

- **QCD x QED should be considered**

  Vafa-Witten theorem, vector meson condensates, and magnetic-field-induced electromagnetic superconductivity of vacuum, Chernodub Phys. Rev. D86, 107703 (2012)

  Comment on "Charged vector mesons in a strong magnetic field"
  Chernodub, arXiv:1309.4071

- **Multivalued generating functional**

  Amending the Vafa-Witten Theorem, Li, Wang, Phys.Lett. B721, 141 (2013)

Counter arguments

- **QCD x QED should be considered**

Vafa-Witten theorem, vector meson condensates, and magnetic-field-induced electromagnetic superconductivity of vacuum, Chernodub Phys. Rev. D86, 107703 (2012)

His claim

Because of gauge symmetry, no NG mode appears (Higgs phase),
which is consistent with the Vaffa-Witten theorem,

Our claim

Our situation corresponds to a fixed U(1) gauge,
and no dynamical photons.

(Our result does not change in any gauge fixing conditions)

Technically, it corresponds to a fixed eB with $e \to 0$

In this case, the rho meson condensation is necessary in the Higgs phase.

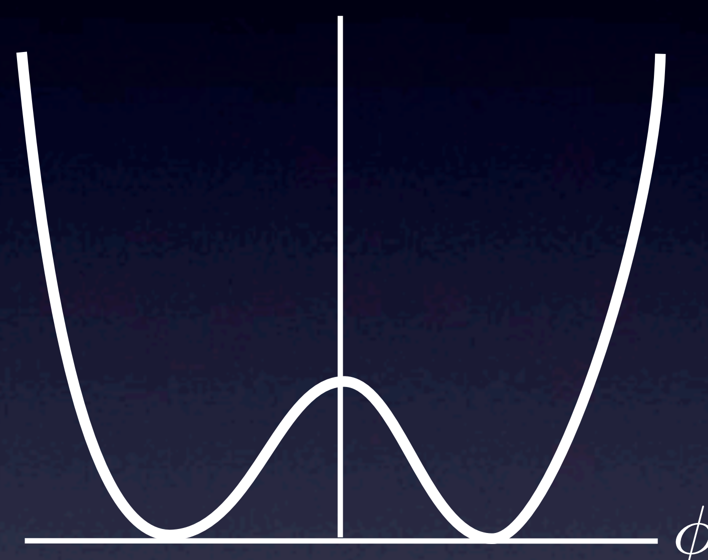
Counter arguments

- **Nontrivial generating functional**

Amending the Vafa-Witten Theorem, Li, Wang, Phys.Lett. B721, 141 (2013)

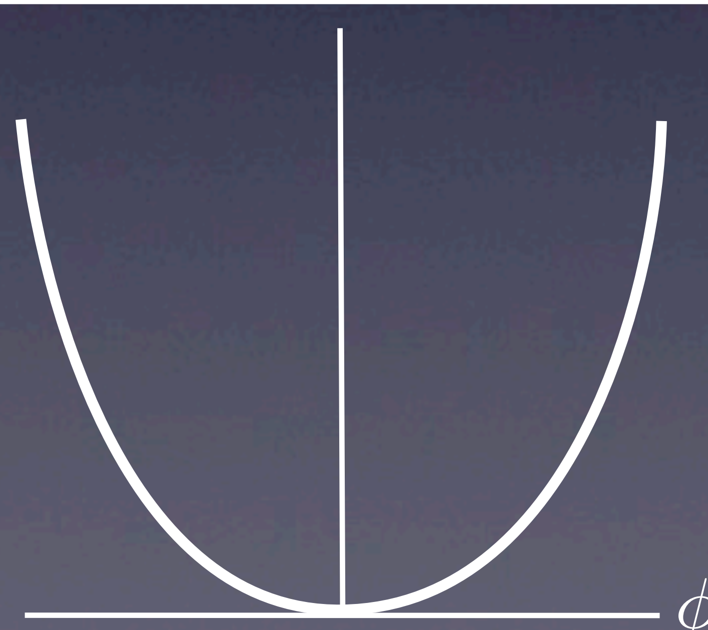
Their claim

If the generating functional is not single valued, the Vafa-Witten theorem may not hold.



Our claim

The generating functional is convex, so that it is single valued.



Comments
Does VW theorem work at

Finite T ? **OK!**

Finite μ_B ? **NO.**

Fermion determinant is complex.
No positivity.

Finite μ_I ? **NO.**

Fermion determinant is nonnegative.

$\frac{1}{\not{D} + m + \gamma_4 \tau_3 \mu_I}$ can be zero.

Generalized NJL model?   NO!

$$\mathcal{L} = \bar{\psi}(\not{D} + m)\psi + \frac{1}{2G}V_\mu^2$$

$$D_\mu = \partial_\mu - i\tau^a V_\mu^a - iqA_\mu^{\rm em}$$

Vector meson carries isospin, so that Disconnected diagrams also contributes the order parameter.

Supersymmetric model?   NO!

Fermion determinant has no positivity.

Summary

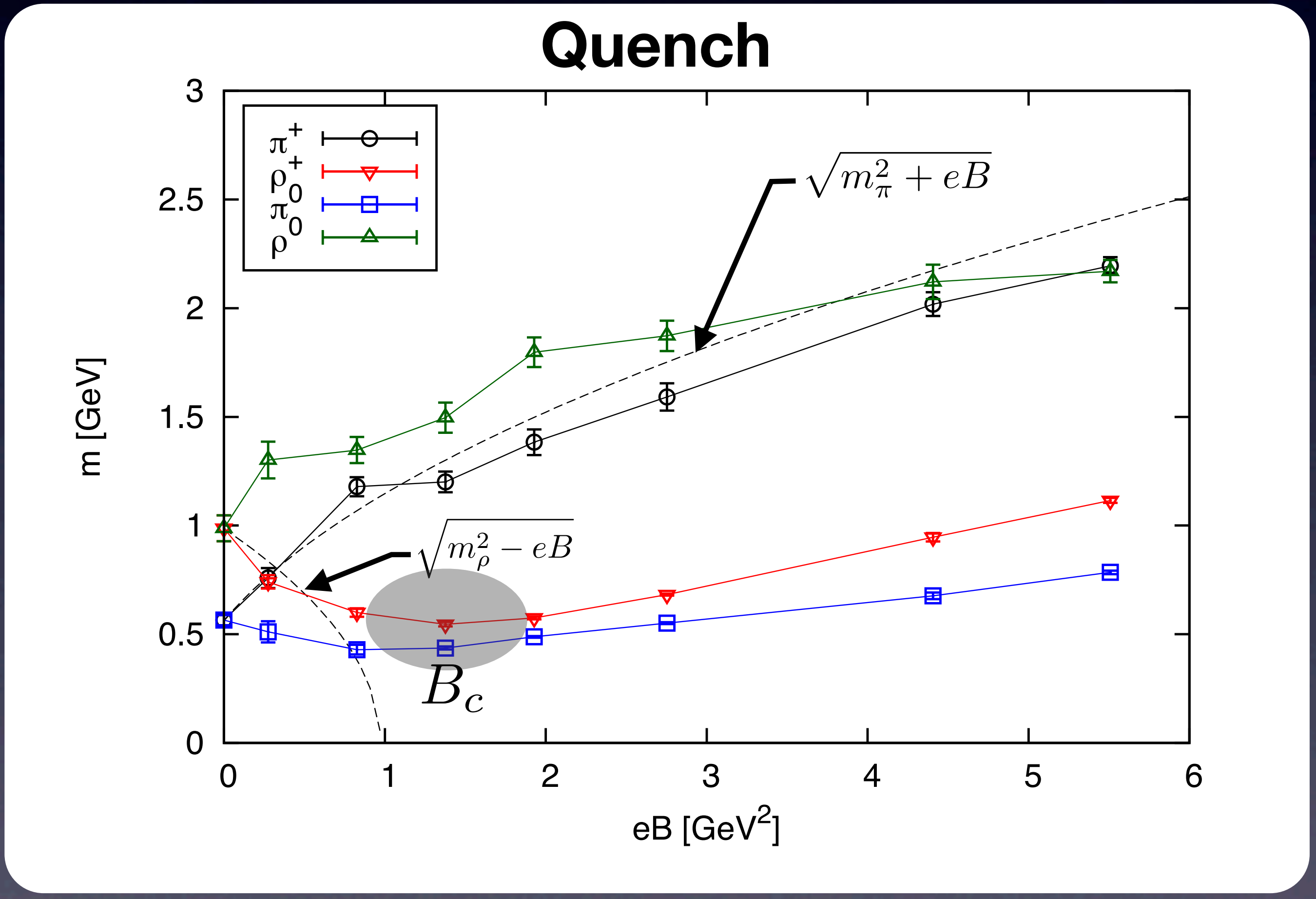
Vector meson condensation?

Our answer is no.

- **Vafa-Witten theorem**

Any models based on QCD should satisfy this theorem.

- **Lattice simulation**



If you find any loop hole, please let us know!