The Quark Mass Gap in Strong Magnetic Fields

Toru Kojo

with Nan Su

(University of Illinois, Urbana-Champaign)

T.K. & Nan Su arXiv: 1211.7318

T.K. & Nan Su arXiv: 1305.4510

Why QCD in Magnetic Fields ??

There are at least two reasons to study :

1, QCD in strong magnetic fields may be realized in Nature.

"Core" of compact stars, Quark-Gluon Plasma (QGP)

Why QCD in Magnetic Fields ??

There are at least two reasons to study :

1, QCD in strong magnetic fields may be realized in Nature.

"Core" of compact stars, Quark-Gluon Plasma (QGP)

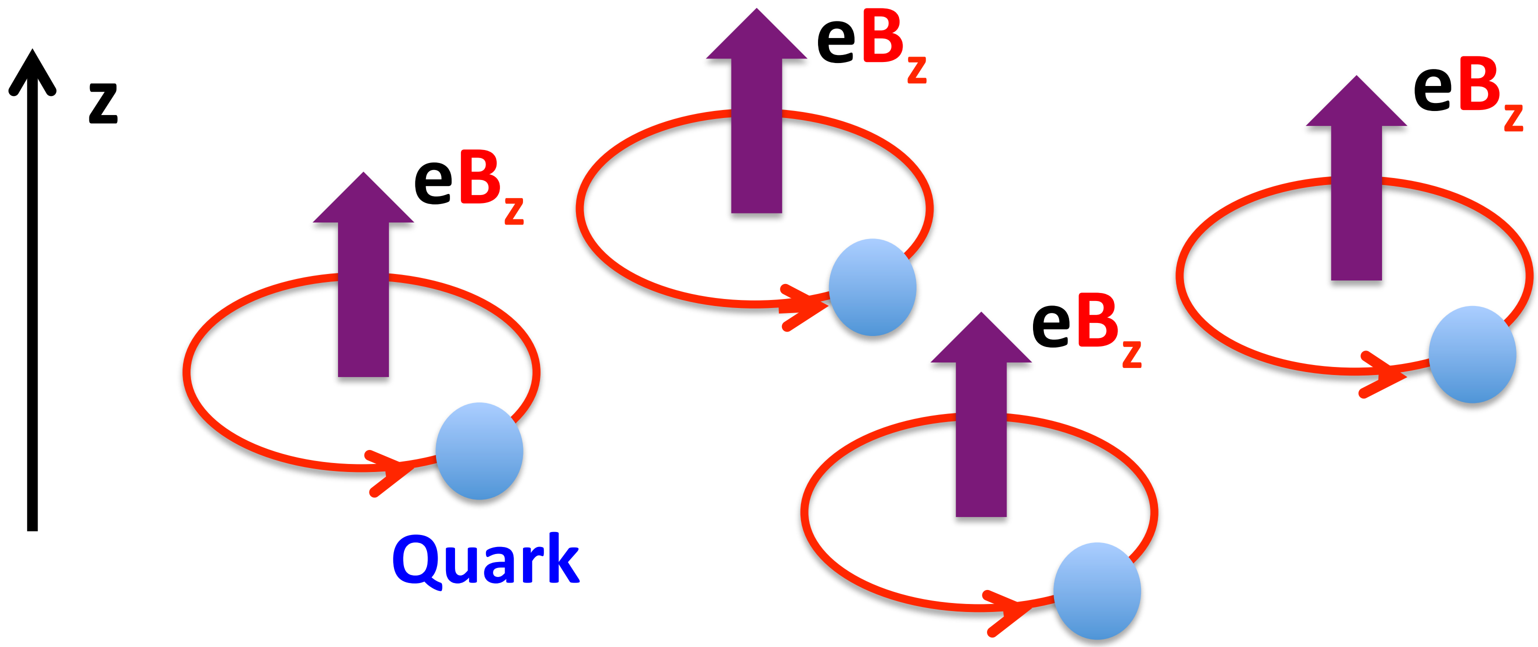
2, We can do "*$B \neq 0$ experiment*" on the lattice:

(No sign problem)

Excellent "*laboratory*" to study the *interplay* between *quarks* and *gluons.*

This talk

Classical mechanics in mag. fields



Quarks have **electric** charges: ***wrap*** *around* ***B***

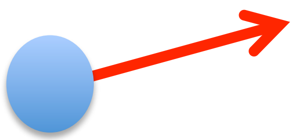
(**Gluons do not**) (Lorenz force)

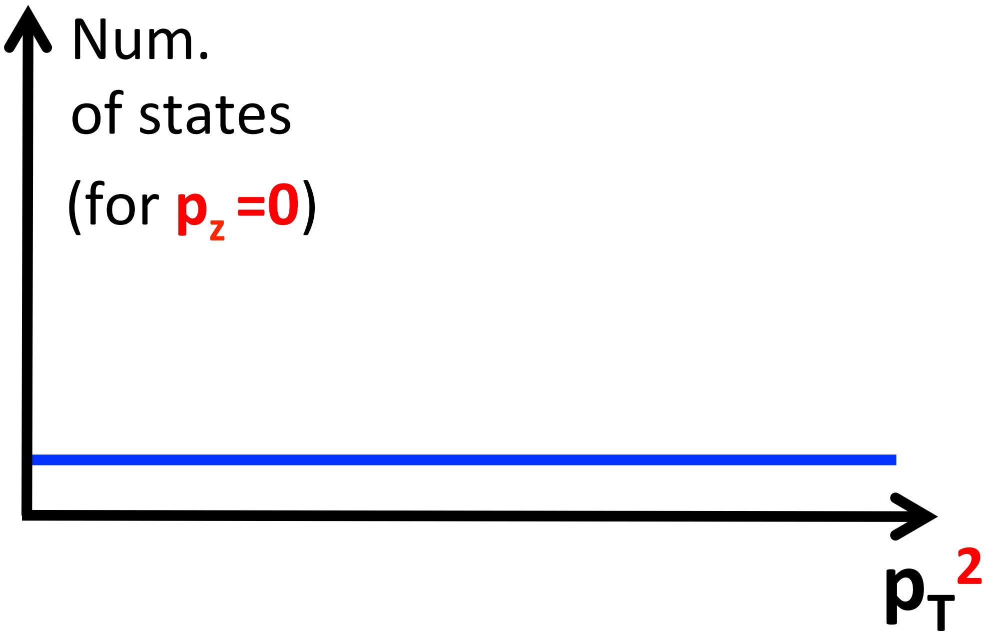
Free Quark's motion in **z & t-directions**.

Quantum mechanics in mag. fields

(spinless, free particles)

B = 0

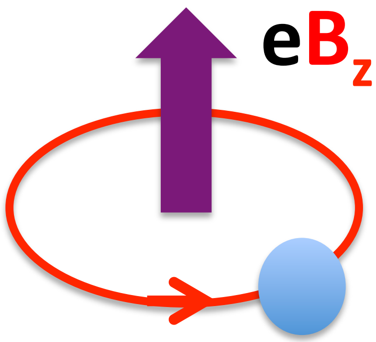




Quantum mechanics in mag. fields

(spinless, free particles)

B ≠ 0

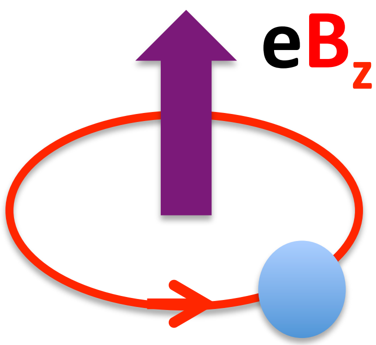


periodic → *discretization*

Quantum mechanics in mag. fields

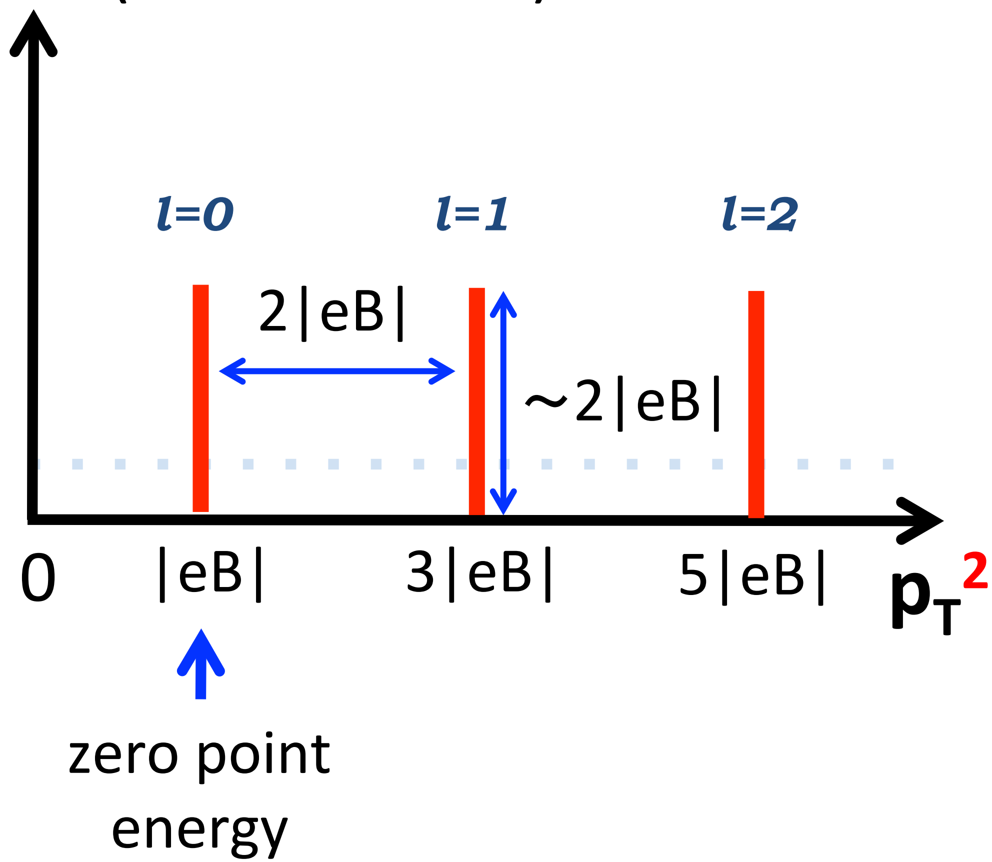
(spinless, free particles)

B ≠ 0



periodic → *discretization*

(orbital levels)

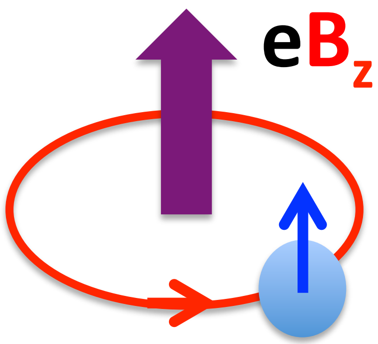


Quantum mechanics in mag. fields

(spin 1/2, free particles)

(orbital + Zeeman splitting)

B ≠ 0

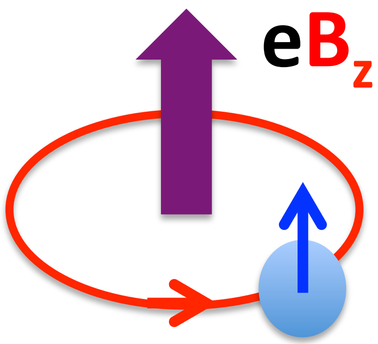


periodic → *discretization*

Quantum mechanics in mag. fields

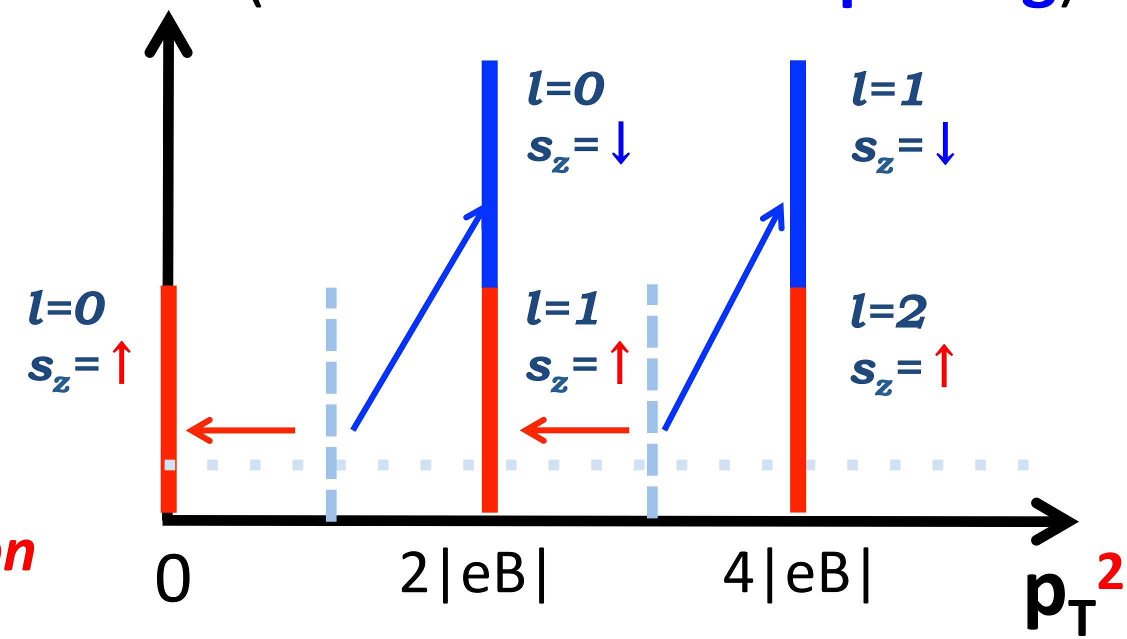
(spin 1/2, free particles)

B ≠ 0

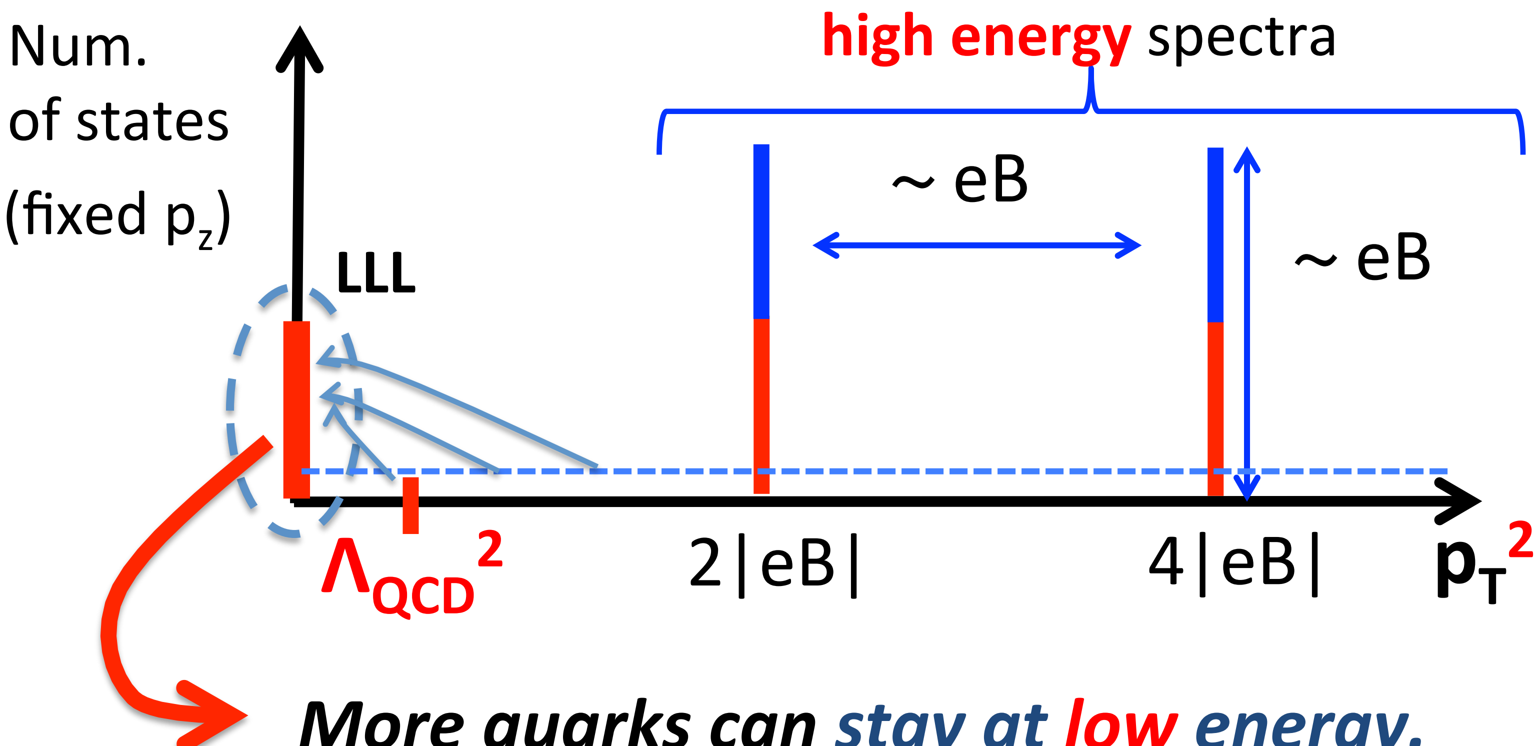


periodic → *discretization*

(orbital + Zeeman splitting)



The IR phase space for quarks



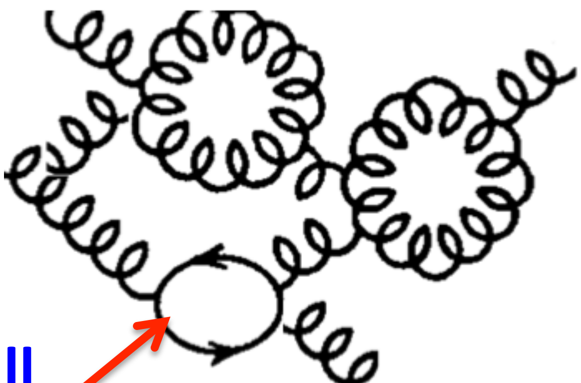
More quarks can stay at low energy.

Size of IR phase space of quarks can be controlled by B.

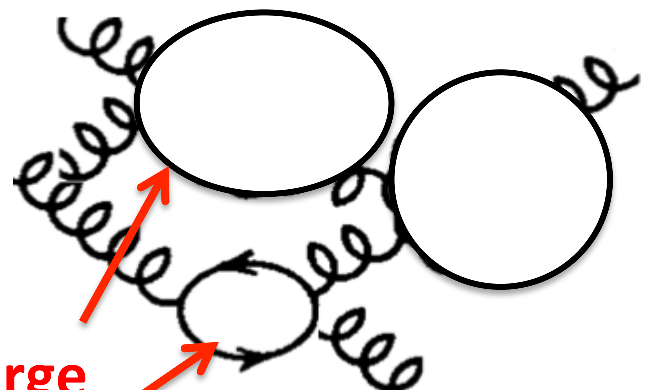
Quarks as probes of gluodynamics

A "*naive*" picture (*weak coupling, pert.*)

small B ⟷ ***large B***



small fraction



large fraction

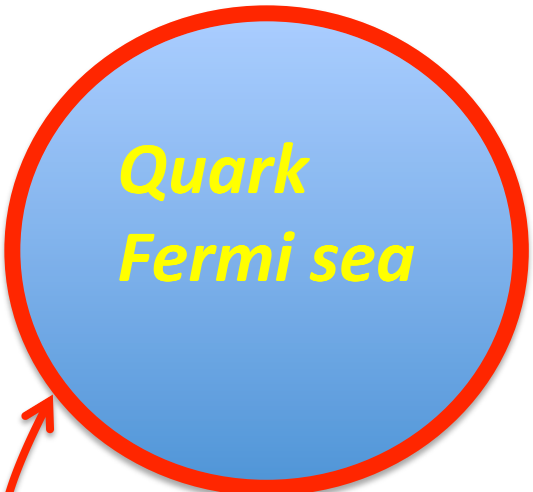
We need non-pert. version of this

for most of phenomenologically interesting region

Applications in mind : *Dense* QCD

A vital question in dense QCD:

Which μ **turns gluodynamics into *weak coupling* regime?**



"size" of IR phase space

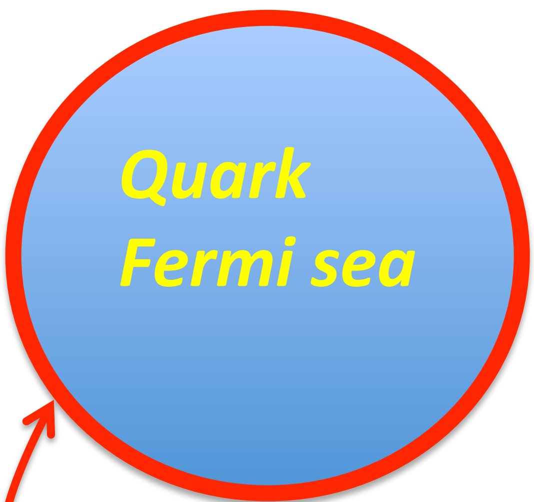
Area for E $\sim$ 0 :

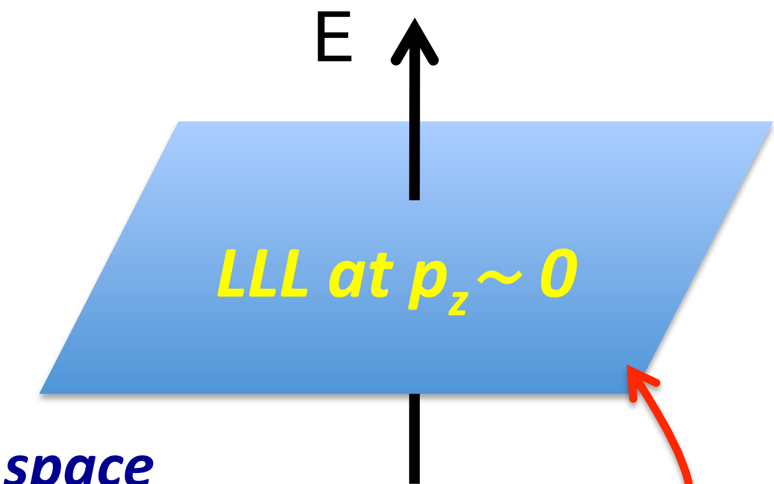
$$4\pi\,\mu_q^2$$

Applications in mind : *Dense* QCD

A vital question in dense QCD:

Which μ **turns gluodynamics into** ***weak coupling*** **regime?**





"size" of IR phase space

Area for E $\sim$ 0 :

$$4\pi\, \mu_q^2$$

Area for E $\sim$ 0 :

$$2\pi\, |eB|$$

Applications in mind : *Dense* QCD

***A vital question* in dense QCD:**

***Which μ* turns gluodynamics into *weak coupling* regime?**

Quark Fermi sea

E

LLL at $p_z \sim 0$

"size" of IR phase space

Area for E ∼ 0 :

$$4\pi\,\mu_q^2$$

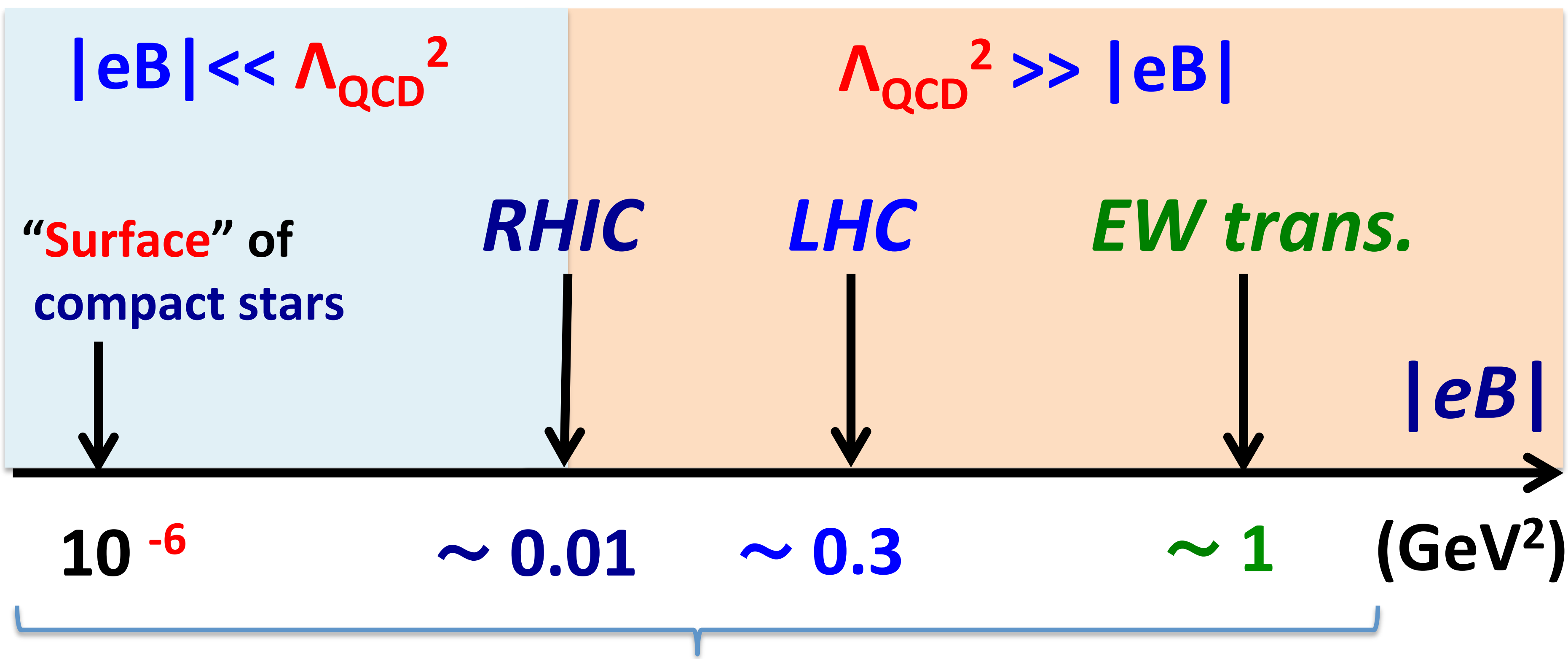
Area for E ∼ 0 :

$$2\pi\,|eB|$$

Hopefully, we may get its rough estimate from "B-exp."

What's current situations of the QCD in B?

Strength of B-field



Lattice data are available (both for *full* and *quenched*)

History (within my best knowledge)

1) *ChSB in mag. fields (concept) : 1989 -*

Klevansky-Lemmer (89), Suganuma-Tatsumi (90),

Gusynin-Miransky-Shovkovy (94-), ... (for NJL, QED,...)

(*Not specific to QCD*, "*universal* aspects" of fermions in B)

History (within my best knowledge)

1) *ChSB in mag. fields (concept) : 1989 -*

Klevansky-Lemmer (89), Suganuma-Tatsumi (90),

Gusynin-Miransky-Shovkovy (94-), ... (for NJL, QED,...)

(*Not specific to QCD*, "*universal* aspects" of fermions in B)

2) *QCD in mag. fields (paradigm shift) : 2007 -*

Kharzeev-McLerran-Warringa (07), Fukushima-Kharzeev-Warringa (08),..

(*QCD topology* & *Its phenomenological applications*)

History (within my best knowledge)

1) *ChSB in mag. fields (concept)* : *1989 -*

Klevansky-Lemmer (89), Suganuma-Tatsumi (90),

Gusynin-Miransky-Shovkovy (94-), ... (for NJL, QED,...)

(*Not specific to QCD*, "*universal* aspects" of fermions in B)

2) *QCD in mag. fields (paradigm shift)* : *2007 -*

Kharzeev-McLerran-Warringa (07), Fukushima-Kharzeev-Warringa (08),..

(*QCD topology* & *Its phenomenological applications*)

3) *Lattice* studies on *ChSB* & *Deconf.* : *2008 -*

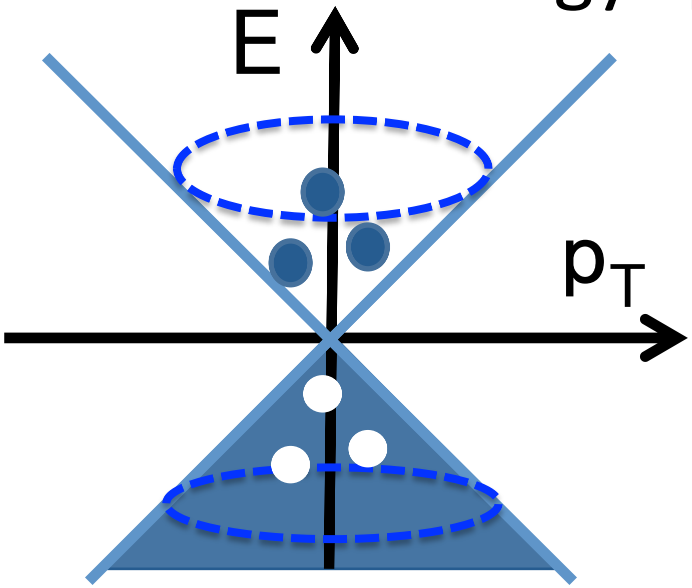
Buividovich et al. (2008) (quenched)

D'Elia-Muckherjee-Sanflippo (2010) (full, heavy pion)

Bali et al. (2012) (full, physical pion)

Enhanced ChSB in mag. fields

Energy spectra (for **fixed p_z**)

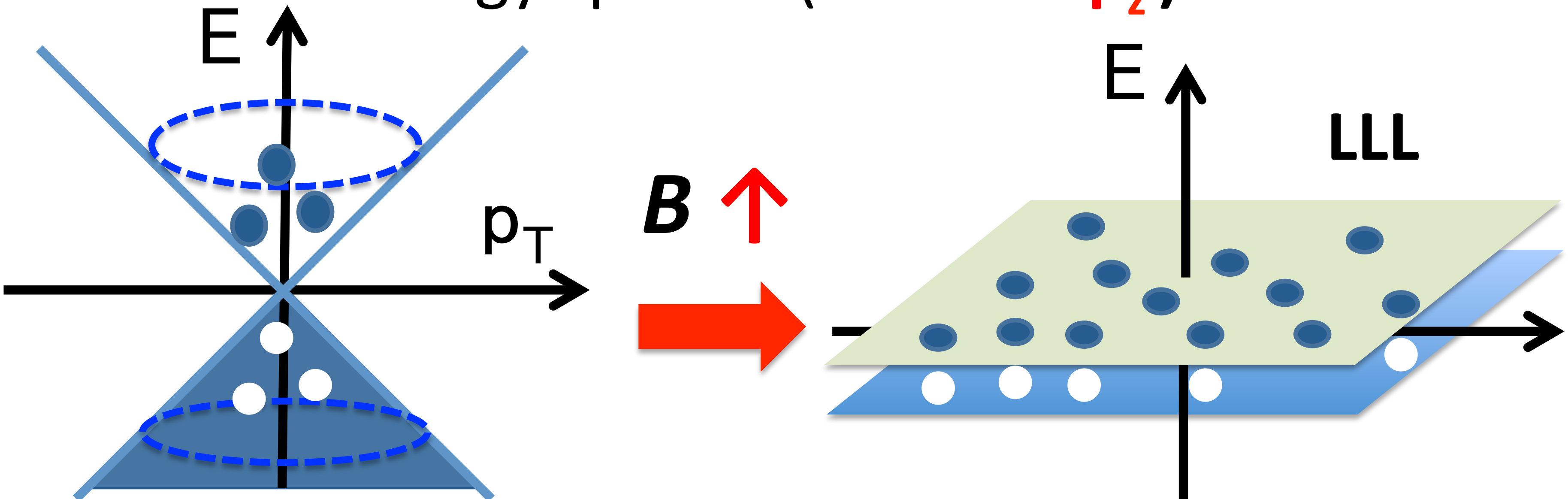


Area ～ Λ^2_{QCD}

(transverse phase space)

Enhanced ChSB in mag. fields

Energy spectra (for **fixed p_z**)



Area ~ Λ^2_{QCD}

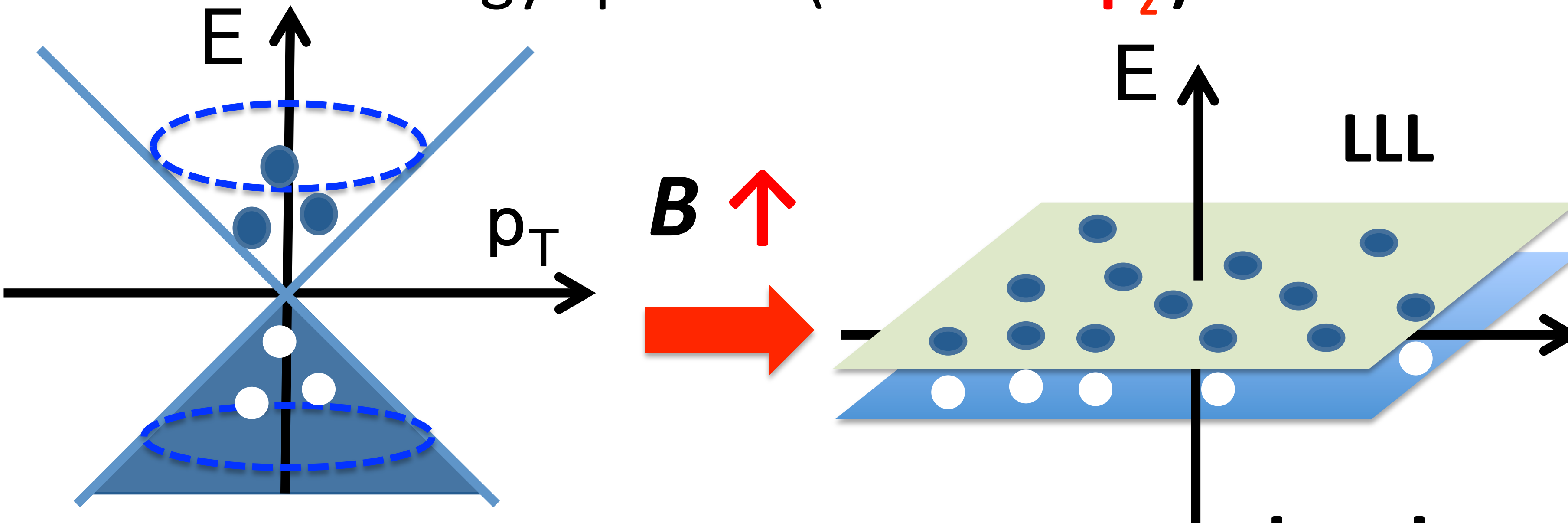
(transverse phase space)

Area ~ |eB|

(More quarks at low energy)

Enhanced ChSB in mag. fields

Energy spectra (for **fixed p_z**)



Area ~ Λ^2_{QCD} **Area ~ |eB|**

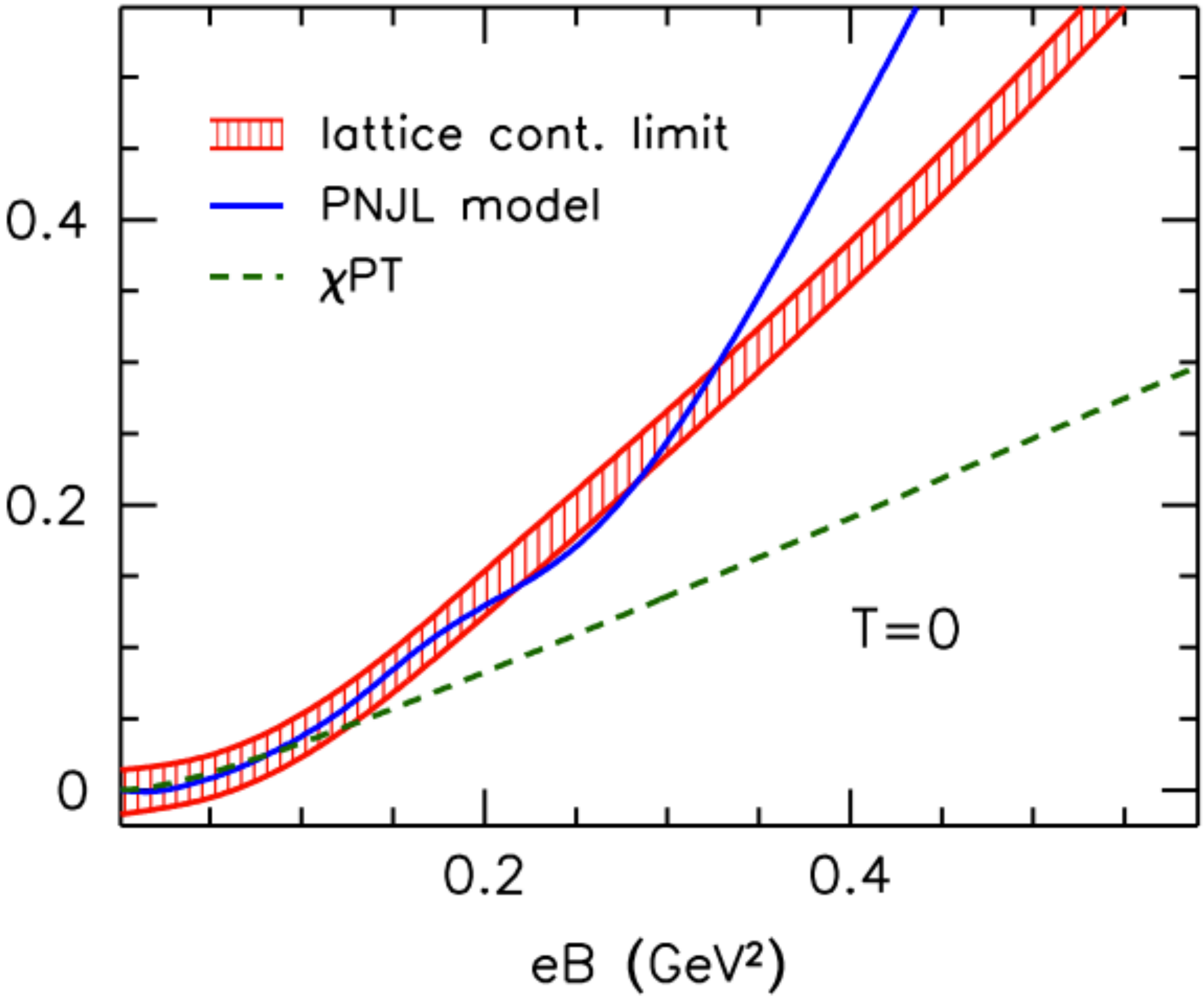
(transverse phase space) **(More quarks at low energy)**

Bigger Chiral Condensate ~ *Magnetic Catalysis*

Models vs Lattice, 1 : *ChSB*

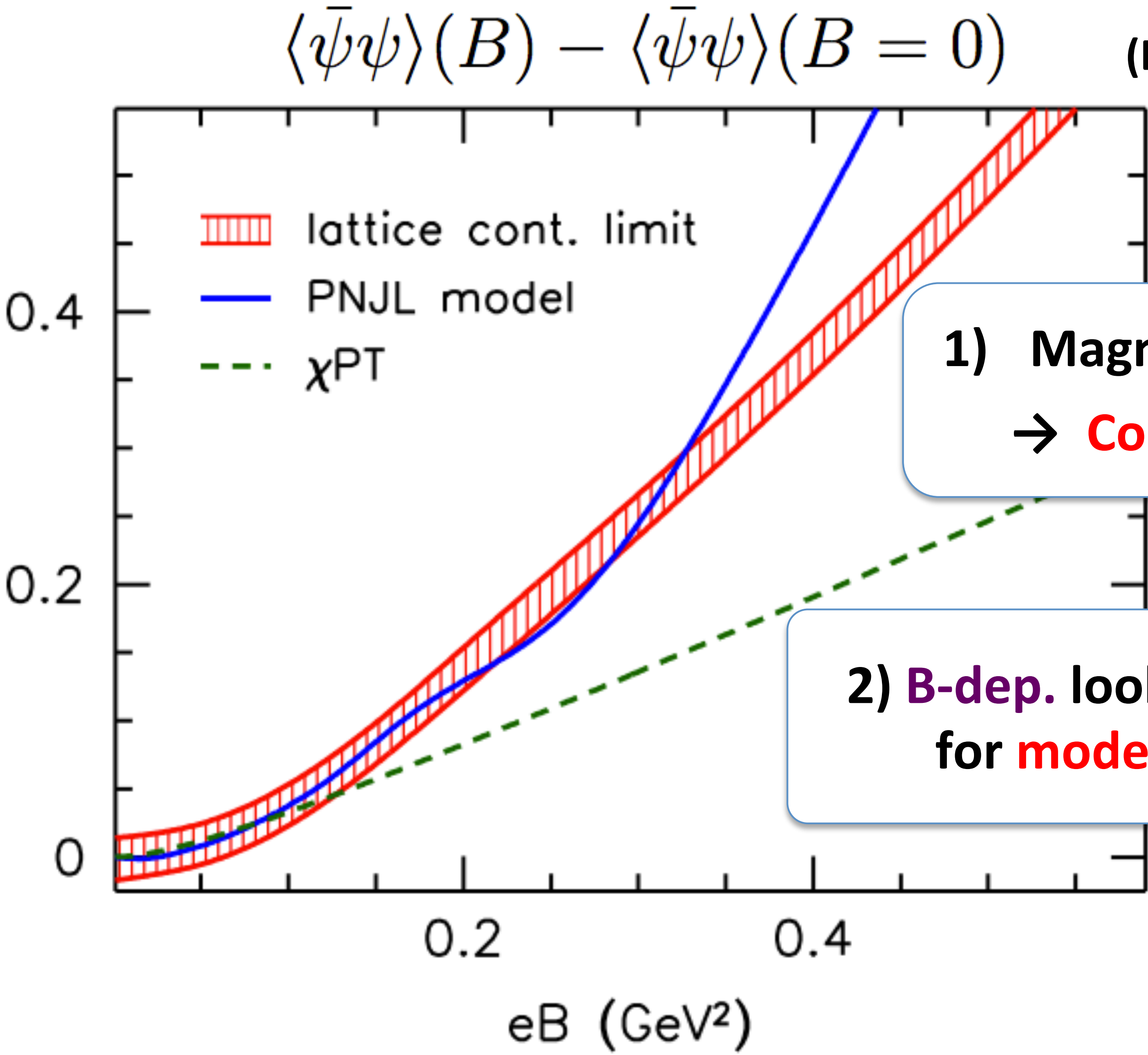
$$\langle\bar{\psi}\psi\rangle(B) - \langle\bar{\psi}\psi\rangle(B=0)$$

(Bali et al, 2012)



Models vs Lattice, 1 : *ChSB*

(Bali et al, 2012)

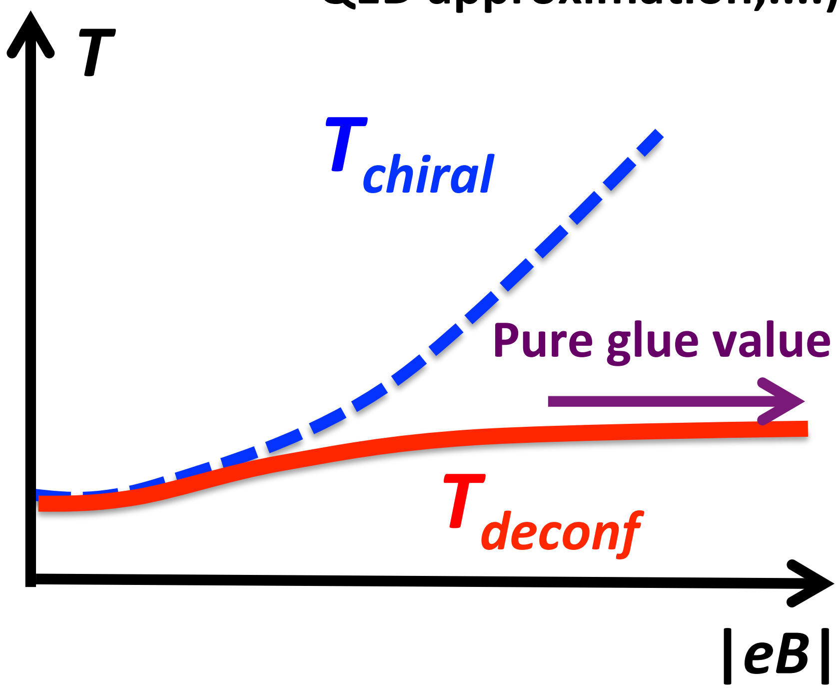


1) Magnetic catalysis
→ Confirmed

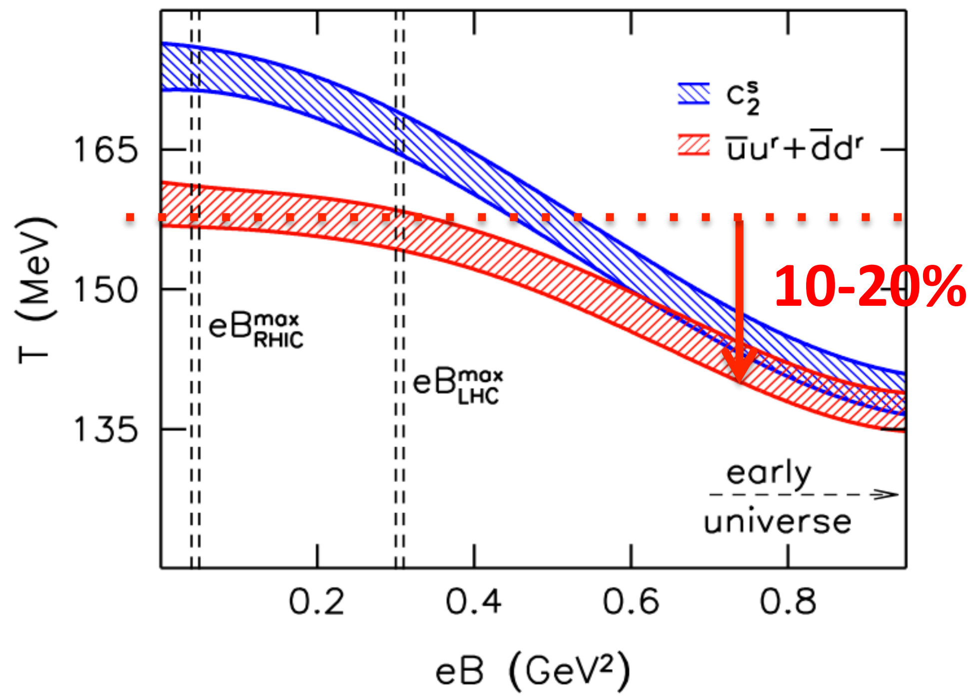
2) B-dep. looks different
for models and lattice

Models vs Lattice, 2: $T_c(B)$

Models (PNJL, PQM, QED approximation,....)



Lattice phys. pion mass (Bali et al, 2012)



Inverse mag. catalysis

(Mag. inhibition)

Qualitative discrepancy....

Claim: 1

Discrepancies b.t.w. models & lattice data

comes from

misidentification of the zero-th order effects.

Discrepancies in B-dep. of the chiral condensate

& qualitative behavior of Tc(B) have the same root.

Fluctuation effects are NLO issues. (see below)

Contents

1, Fermions in strong mag. fields

(Some relevant formula)

Field theory bases : quark part

" Ritus bases for non-int. fermions in B "

1) Choose the gauge for EM fields : e.g.) $A_2^{\mathrm{em}} = Bx_1$

Field theory bases : quark part

" Ritus bases for non-int. fermions in B "

1) Choose the gauge for EM fields : e.g.) $A_2^{\mathrm{em}} = Bx_1$

2) Apply "*spin projection*" :

$$\psi_\pm \equiv \mathcal{P}_\pm \psi \qquad \mathcal{P}_\pm = \frac{1 \pm \mathrm{i}\gamma_1\gamma_2\,\mathrm{sgn}(e_f B)}{2}$$

Field theory bases : quark part

" Ritus bases for non-int. fermions in B "

1) Choose the gauge for EM fields : e.g.) $A_2^{\mathrm{em}} = Bx_1$

2) Apply "*spin projection*" :

$$\psi_\pm \equiv \mathcal{P}_\pm \psi \qquad \mathcal{P}_\pm = \frac{1 \pm \mathrm{i}\gamma_1\gamma_2\,\mathrm{sgn}(e_f B)}{2}$$

3) Expand by proper spatial wavefunctions :

$$\psi_\pm(x) = \sum_{l=0} \int \frac{\mathrm{d}^2 p_L \mathrm{d}p_2}{(2\pi)^3}\, \psi^\pm_{l,p_2}(p_L)\, \underline{H_l\Big(x_1 - \frac{p_2}{B}\Big)}\, \mathrm{e}^{-\mathrm{i}p_2 x_2}\, \mathrm{e}^{-\mathrm{i}p_L x_L}$$

$$p_L \equiv (p_0, p_z)$$

Harmonic oscillator w.f. with

$$m\omega = |eB|$$

Field theory bases : quark part

The action for the LLL (n=0): $\chi = \psi_+^{l=0}$

$$\mathcal{S}_{\rm LLL} = \int_{p_L, p_2} \bar{\chi}_{p_2}(p_L)\left(-\mathrm{i}\not{p}_L + m\right)\chi_{p_2}(p_L)$$

(No B-dep. !)

for the n-th hLL : $\psi_n = \psi_+^{l=n} + \psi_-^{l=n-1}$

$$\mathcal{S}_{\rm nLL} = \int_{p_L, p_2} \bar{\psi}_{n,p_2}(p_L)\left(-\mathrm{i}\not{p}_L + \underline{\mathrm{i}\,\mathrm{sgn}(eB)\sqrt{2n|eB|}\gamma_2} + m\right)\psi_{n,p_2}(p_L)$$

Field theory bases : quark part

The action for the LLL (n=0): $\chi = \psi_{+}^{l=0}$

$$\mathcal{S}_{\rm LLL} = \int_{p_L,p_2} \bar{\chi}_{p_2}(p_L)\,(-\mathrm{i}\not{p}_L + m)\,\chi_{p_2}(p_L)$$

(No B-dep. !)

for the n-th hLL : $\psi_n = \psi_{+}^{l=n} + \psi_{-}^{l=n-1}$

$$\mathcal{S}_{\rm nLL} = \int_{p_L,p_2} \bar{\psi}_{n,p_2}(p_L)\left(-\mathrm{i}\not{p}_L + \underline{\mathrm{i}\,\mathrm{sgn}(eB)\sqrt{2n|eB|}\gamma_2} + m\right)\psi_{n,p_2}(p_L)$$

The propagator :

$$\langle \psi_{n,p_2}(p_L)\bar{\psi}_{n',p_2'}(p_L')\rangle = \underline{S_n^{\rm 2D}(p_L)} \times \delta_{nn'}\delta(p_2 - p_2')\delta^2(p_L - p_L')$$

(1+1)-dimensional for each index "n"

(No p_2 – dep.)

Important formula

from integration of x_1 & p_2

$$\langle \bar{\psi}(x)\psi(x)\rangle_{\underline{\mathrm{4D}}} = -\frac{|eB|}{2\pi}\int_{p_L} \mathrm{tr}\left(S_\chi^{\mathrm{2D}}(p_L) + \sum_{n=1} S_n^{\mathrm{2D}}(p_L) \right)$$

Important formula

from integration of x_1 & p_2

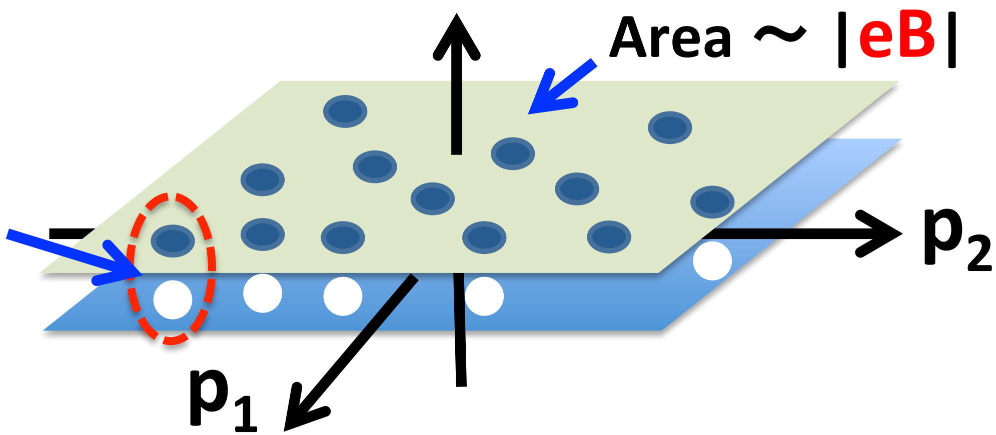
$$\langle \bar{\psi}(x)\psi(x)\rangle_{\underline{4\mathrm{D}}} = -\frac{|eB|}{2\pi}\int_{p_L} \mathrm{tr}\left(S^{2\mathrm{D}}_{\chi}(p_L) + \sum_{n=1} S^{2\mathrm{D}}_{n}(p_L)\right)$$

$$\boxed{\langle \bar{\psi}(x)\psi(x)\rangle_{4\mathrm{D}} = \frac{|eB|}{\underline{2\pi}}\langle \bar{\psi}(x_L)\psi(x_L)\rangle_{\underline{2\mathrm{D}}}}$$

(*also holds for interacting fermions*)

Intuitively,

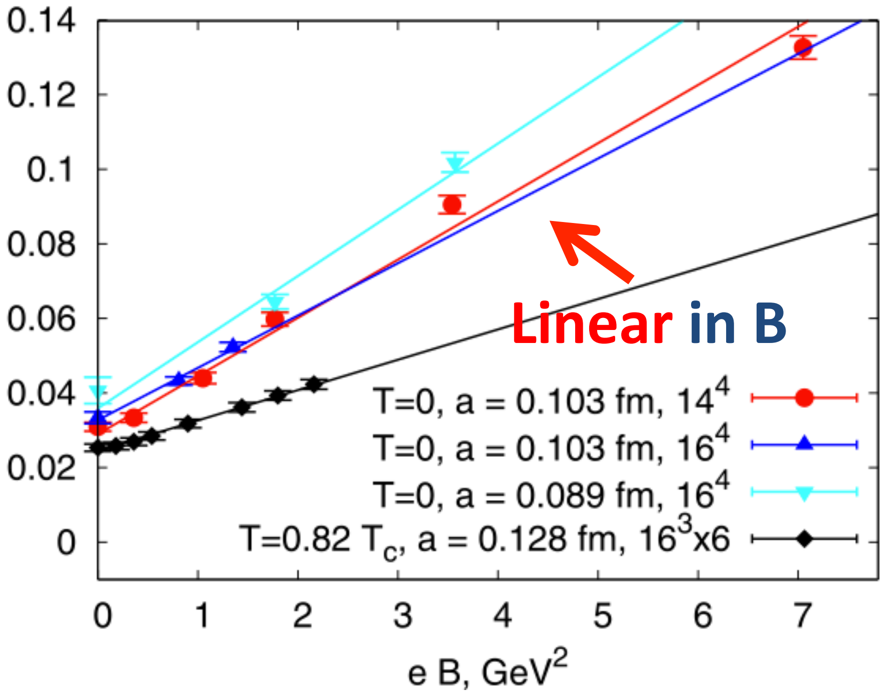
$$\langle \bar{\psi}\psi\rangle_{\underline{2\mathrm{D}}}$$



Chiral condensate on the lattice

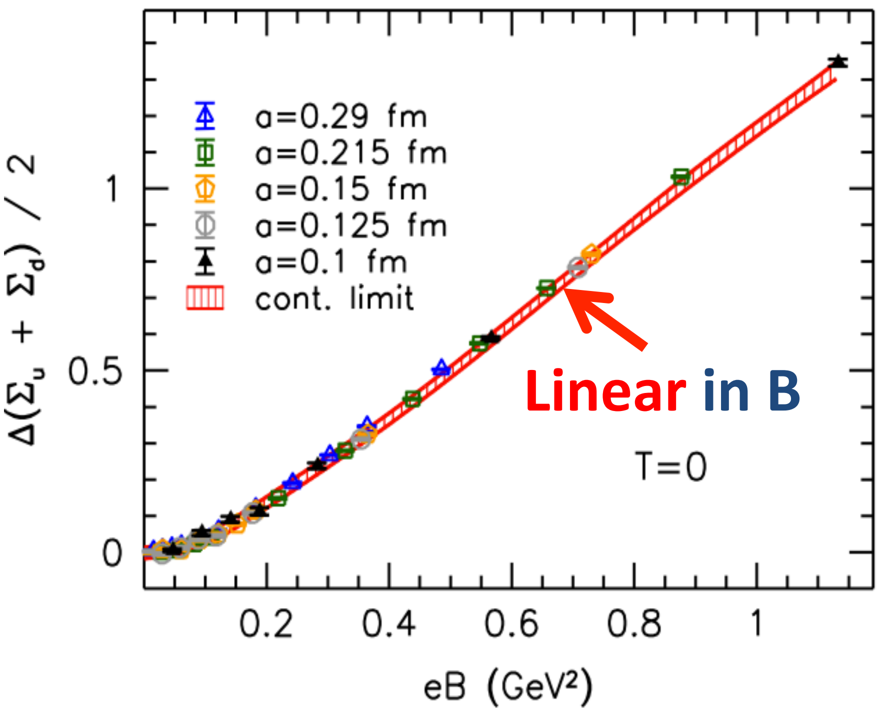
Quenched $SU_c(2)$

Buividovich et al, 2010



Full $SU_c(3)$

Bali et al, 2011, 2012

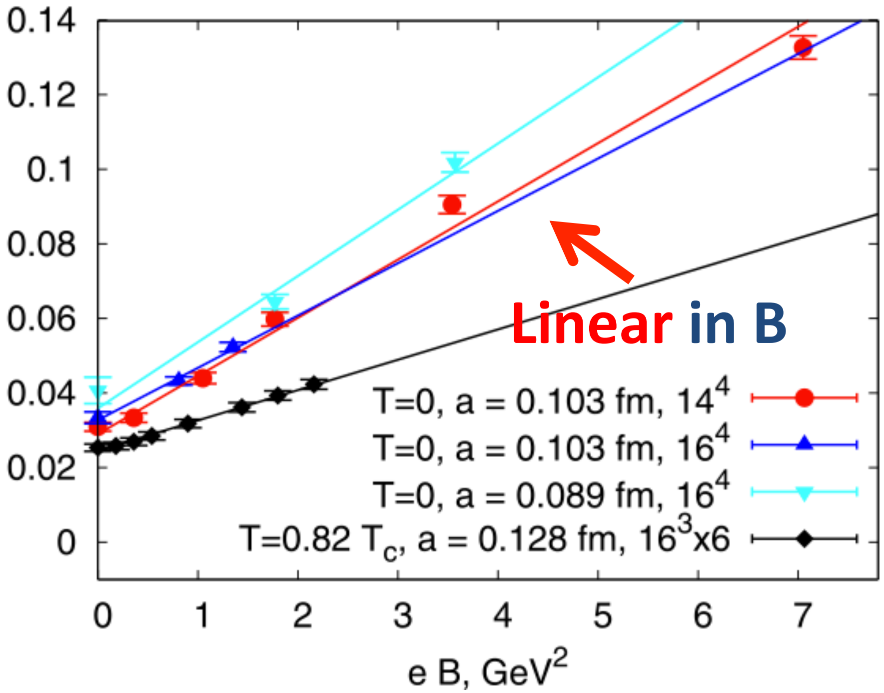


$$\langle \bar{\psi}\psi \rangle_{4D} = \frac{|eB|}{2\pi} \langle \bar{\psi}\psi \rangle_{2D}$$

Chiral condensate on the lattice

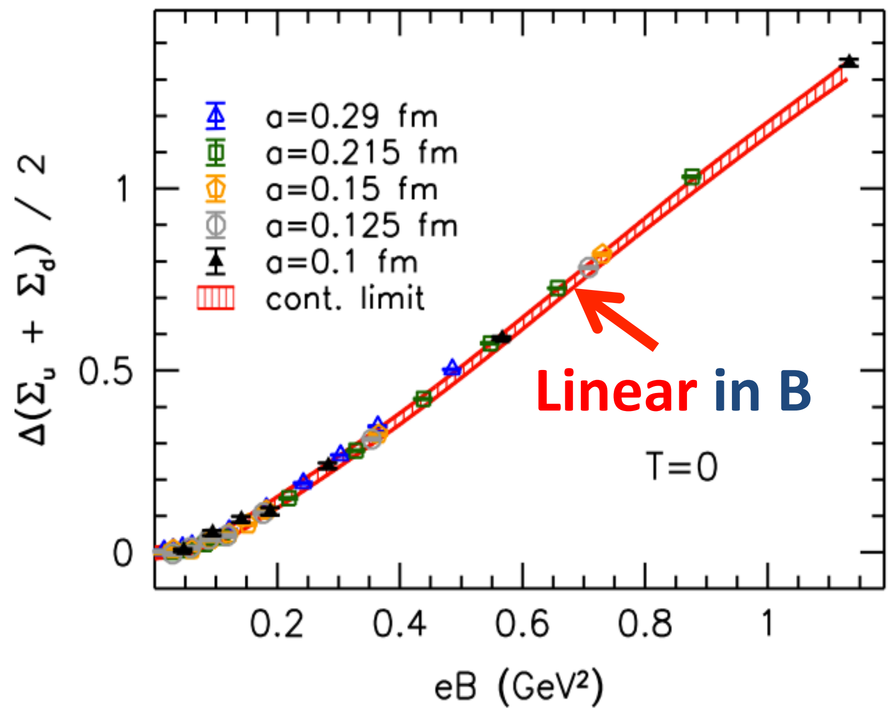
Quenched $SU_c(2)$

Buividovich et al, 2010



Full $SU_c(3)$

Bali et al, 2011, 2012



$$\langle\bar{\psi}\psi\rangle_{4D} = \frac{|eB|}{2\pi}\langle\bar{\psi}\psi\rangle_{2D} \quad \Rightarrow \quad \boxed{\langle\bar{\psi}\psi\rangle_{2D} \sim c_0\Lambda_{QCD} + \cdots}$$

Problems in most theories...

(The NJL, QED-like treatments, Sakai-Sugimoto models,....)

Problems in most theories...

(The NJL, QED-like treatments, Sakai-Sugimoto models,....)

Problem 1) ***B-dep. of the chiral condensate***

$$M_q \sim |eB|^{1/2} \Rightarrow \langle \bar{\psi}\psi \rangle_{\mathrm{2D}} \sim |eB|^{1/2}$$

$$\langle \bar{\psi}\psi \rangle_{\mathrm{4D}} \sim |eB|^{3/2}$$ ≠ **lattice data**

Problems in most theories...

(The NJL, QED-like treatments, Sakai-Sugimoto models,....)

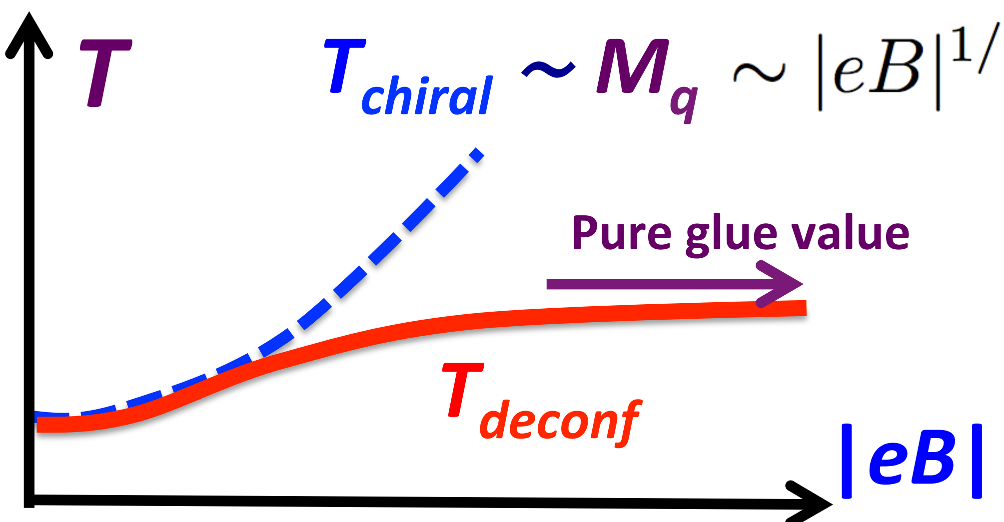
Problem 1) *B-dep. of the chiral condensate*

$$M_q \sim |eB|^{1/2} \quad \Rightarrow \quad \langle \bar{\psi}\psi \rangle_{\mathrm{2D}} \sim |eB|^{1/2}$$

$$\langle \bar{\psi}\psi \rangle_{\mathrm{4D}} \sim |eB|^{3/2}$$ ≠ **lattice data**

Problem 2) *"B-T" phase diagram*

$$T_{chiral} \sim M_q \sim |eB|^{1/2}$$



≠ **lattice data**

Claim: 2

Within the domain of B *studied on the lattice*,

the quark mass gap should be : $\boxed{M_q \sim \Lambda_{\mathrm{QCD}}}$

If so,

$$T_{\mathrm{chiral}}(B) \sim M \sim \Lambda_{\mathrm{QCD}}$$

(instead of $\sim |eB|^{1/2}$)

Claim: 2

Within the domain of B *studied on the lattice*,

the quark mass gap should be : $\boxed{M_q \sim \Lambda_{\mathrm{QCD}}}$

If so,

$$T_{\mathrm{chiral}}(B) \sim M \sim \Lambda_{\mathrm{QCD}}$$

(instead of $\sim |eB|^{1/2}$)

Then we have a better chance to explain

reduction of T_c & other gluonic quantities

(Quarks do not decouple from gluons)

2, Quenched QCD in strong mag. fields

We *separate* issues of *fluctuations* such as

back reaction from quark to gluon sector,

mesonic fluctuations,

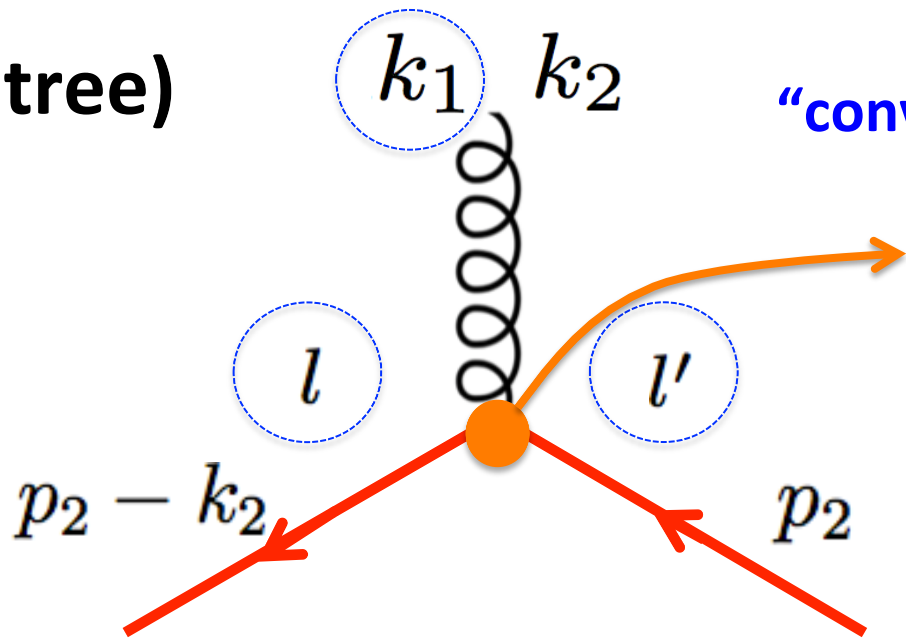
etc., etc.,.......

Quark-gluon vertex

$$S_{\text{int}} = \int_x \bar{\psi}(x)\gamma_\mu t_a \psi(x)\, A_\mu^a(x)$$

4D Gluons couple to different LLs.

tree)



"conversion" from (*l*, *l'*) to k_1

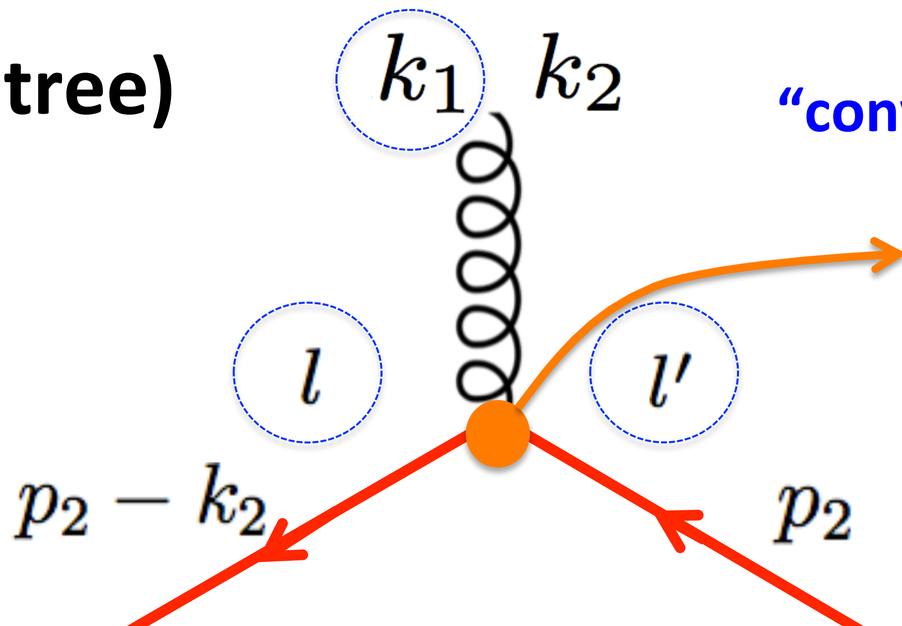
***Ritus* bases**

Quark-gluon vertex

$$S_{\rm int} = \int_x \bar{\psi}(x)\gamma_\mu t_a \psi(x)\, A_\mu^a(x)$$

4D Gluons couple to different LLs.

tree)



"conversion" from (*l*, *l'*) to k_1

"Form factor"

$$I_{l,l'}(k_1, k_2)$$

(including "***Schwinger phase***")

Ritus **bases**

Form factor

$$\Delta l = |l - l'|$$

$$I_{l,l'}(\vec{k}_\perp) \propto \left(\frac{k_\perp^2}{2|eB|}\right)^{\frac{\Delta l}{2}} \mathrm{e}^{-k_\perp^2/4|eB|}$$

For couplings b.t.w. different "orbital" levels

Form factor

$$\Delta l = |l - l'|$$

$$I_{l,l'}(\vec{k}_\perp) \propto \left(\frac{k_\perp^2}{2|eB|}\right)^{\frac{\Delta l}{2}} \mathrm{e}^{-k_\perp^2/4|eB|}$$

For couplings b.t.w. different "orbital" levels

➡ ***Soft gluon contributions are suppressed***

Form factor

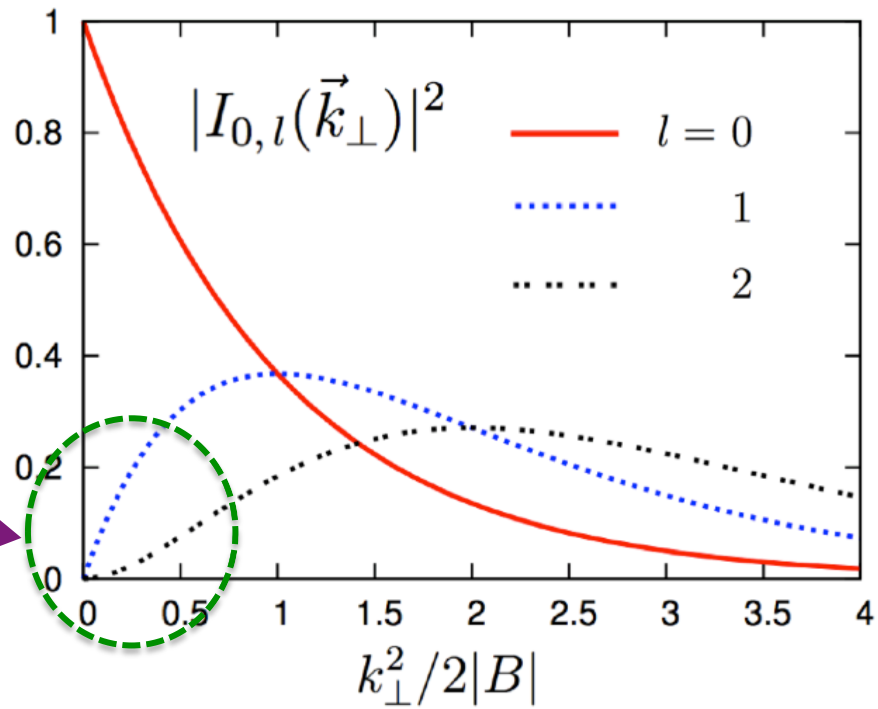
$$\Delta l = |l - l'|$$

$$I_{l,l'}(\vec{k}_\perp) \propto \left(\frac{k_\perp^2}{2|eB|}\right)^{\frac{\Delta l}{2}} \mathrm{e}^{-k_\perp^2/4|eB|}$$

For couplings b.t.w. different "orbital" levels

➡ ***Soft gluon contributions are suppressed***

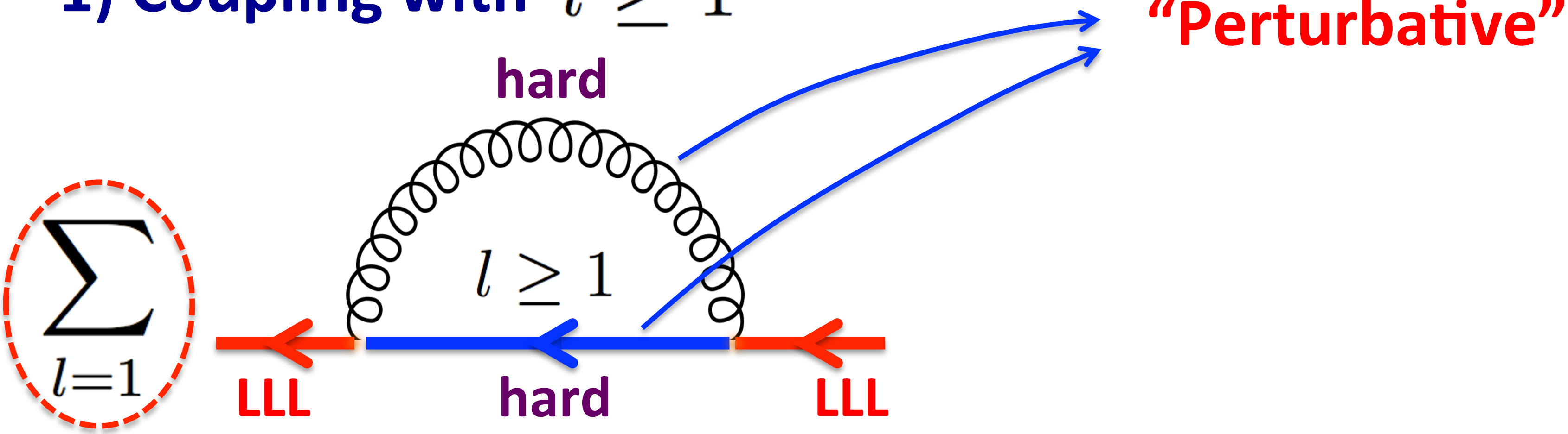
e.g.) LLL to l-th orbital



IR suppression

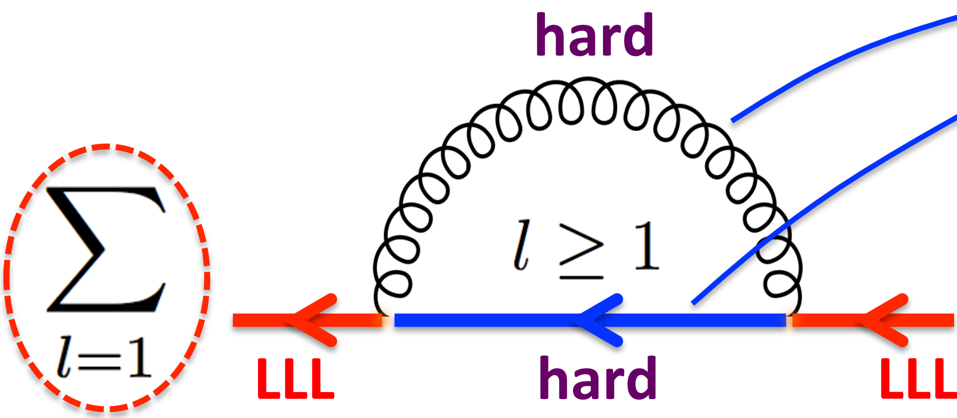
LLL mass gap : 3-distinct contributions

1) Coupling with $l \geq 1$



LLL mass gap : 3-distinct contributions

1) Coupling with $l \geq 1$



"Perturbative"

under control for

$$|eB| \geq (0.1 - 0.3)\,\mathrm{GeV}^2$$

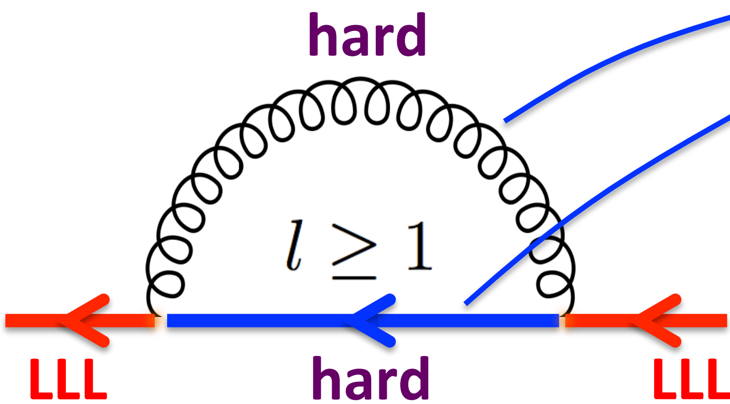
& very small B-dep.

T.K., Nan Su (2013)

LLL mass gap : 3-distinct contributions

1) Coupling with $l \geq 1$

$$\sum_{l=1}$$



"Perturbative"

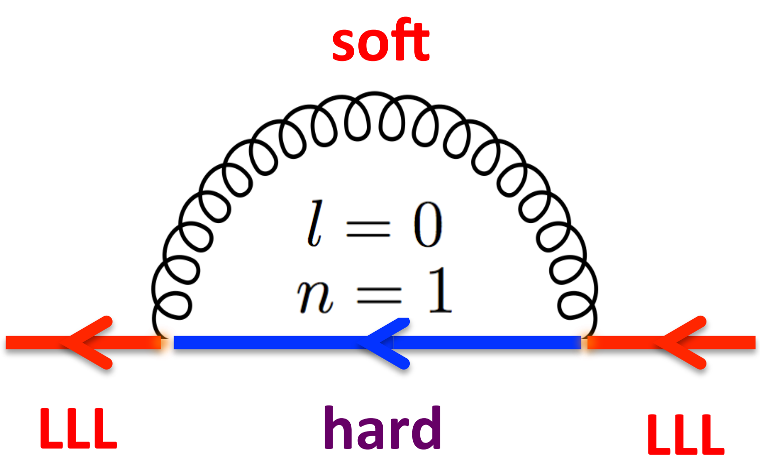
under control for

$$|eB| \geq (0.1 - 0.3)\,\mathrm{GeV}^2$$

& very small B-dep.

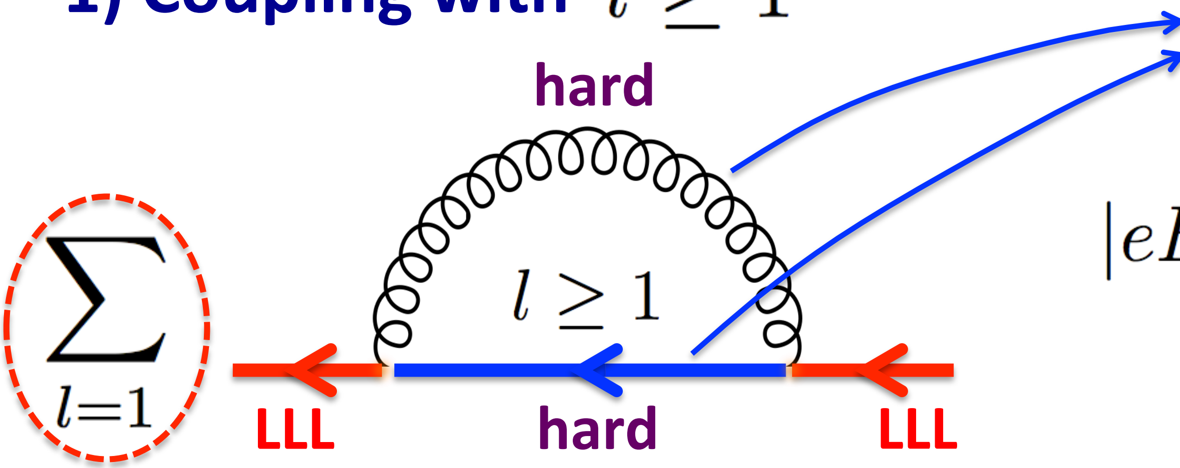
T.K., Nan Su (2013)

2) Coupling with $l = 0$ & $n = 1$



LLL mass gap : 3-distinct contributions

1) Coupling with $l \geq 1$



"Perturbative"

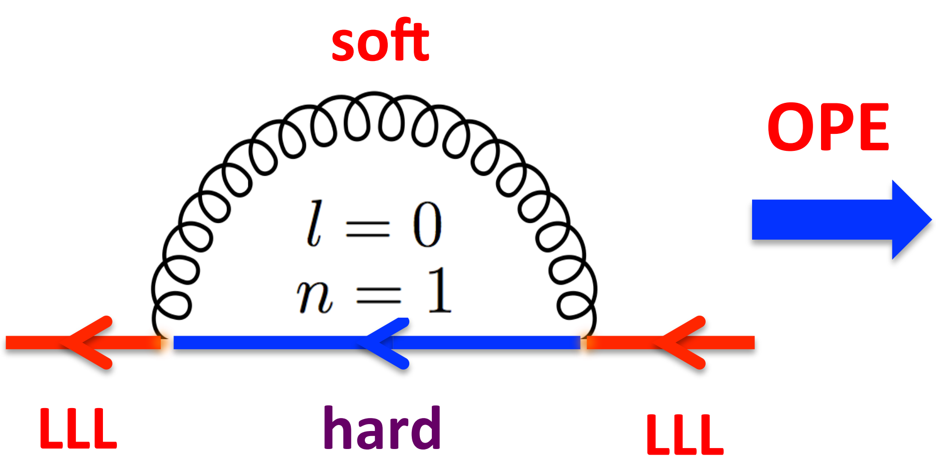
under control for

$$|eB| \geq (0.1 - 0.3)\,\mathrm{GeV}^2$$

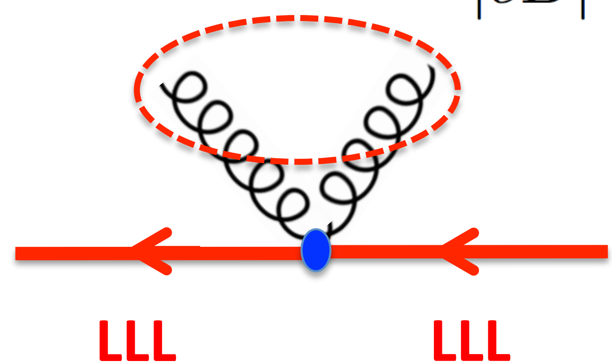
& very small B-dep.

T.K., Nan Su (2013)

2) Coupling with $l = 0$ & $n = 1$

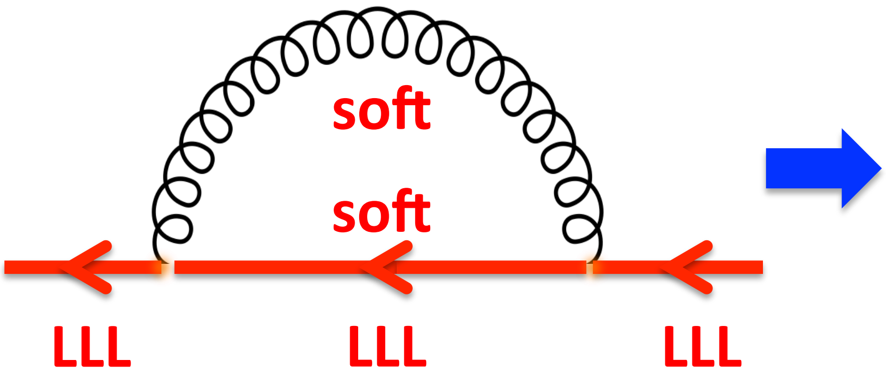


$$\sim \frac{m\langle G^2 \rangle}{|eB|^2} \ll \Lambda_{\mathrm{QCD}}$$



LLL mass gap : 3-distinct contributions

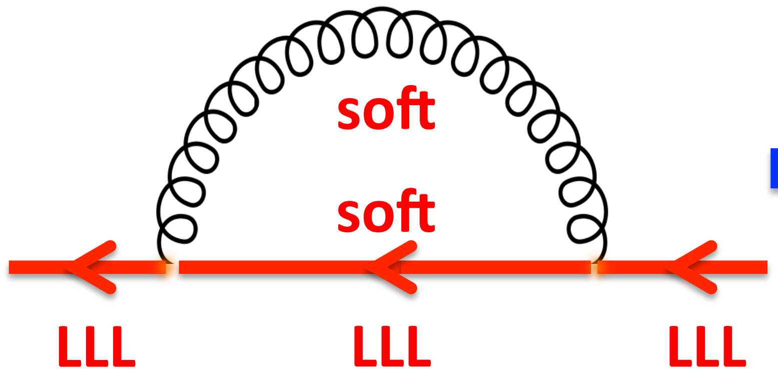
3) Couplings within LLLs



Everything must be treated "Non-perturbatively"

LLL mass gap : 3-distinct contributions

3) Couplings within LLLs



Everything must be treated "Non-perturbatively"

Natural framework → Schwinger-Dyson eq.

with

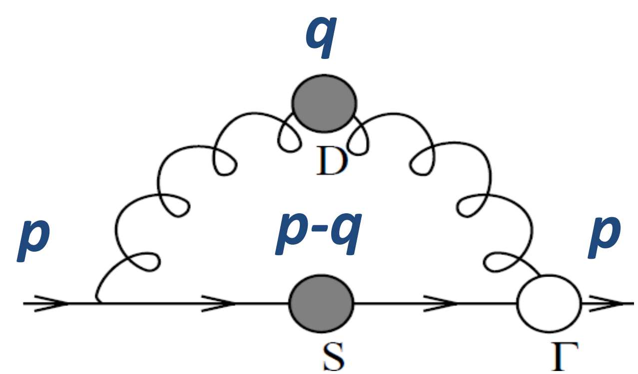
Non-perturbative " *force* "

e.g.) full gluon propagator & vertex for quenched QCD

Structure of the Schwinger-Dyson eq.

1) No explicit B-dep. for the LLL

2) No p_T-dep. → " factorization "



$$M(p_L) \sim \int_{q_L} S^{2D}_{LLL}(p_L - q_L; M) \boxed{\int_{q_\perp} e^{-\frac{q_\perp^2}{2|eB|}} D^{4D}_{NP}(q_L, q_\perp)}$$

Form factor
for "ΔL = 0 process"

2D " *smeared* " force

(origin of B-dep.)

Comparison of forces, 1

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D_{\mathrm{NP}}^{\mathrm{4D}}(q) \sim \int_0^{\underline{\sim|eB|}} \mathrm{d}q_\perp^2\ D_{\mathrm{NP}}^{\mathrm{4D}}(q)$$

origin of B-dep.

Comparison of forces, 1

$$\int_{q_\perp} \underline{e^{-\frac{q_\perp^2}{2|eB|}}}\, D^{4D}_{NP}(q) \sim \int_0^{\underline{\sim|eB|}} dq_\perp^2\ D^{4D}_{NP}(q)$$

origin of B-dep.

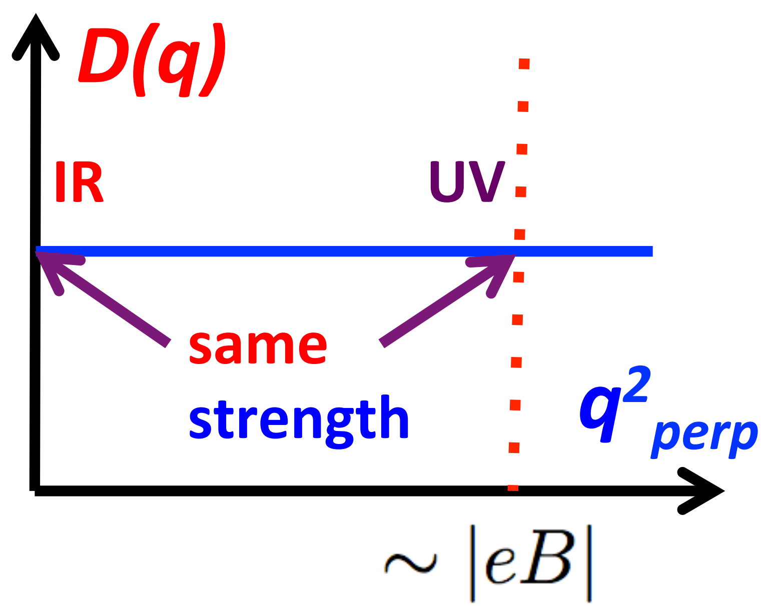
1) Contact interactions (NJL, etc.)

$$\sim \int_0^{\sim|eB|} dq_\perp^2 \ \text{const.}$$

$$\boxed{\sim \underline{|eB|} \times \text{const.}}$$

2D Force is strongly B-dep.

$$M \sim |eB|^{1/2}$$



Comparison of forces, 2

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D_{\mathrm{NP}}^{\mathrm{4D}}(q) \sim \int_0^{\underline{\sim|eB|}} \mathrm{d}q_\perp^2\ D_{\mathrm{NP}}^{\mathrm{4D}}(q)$$

origin of
B-dep.

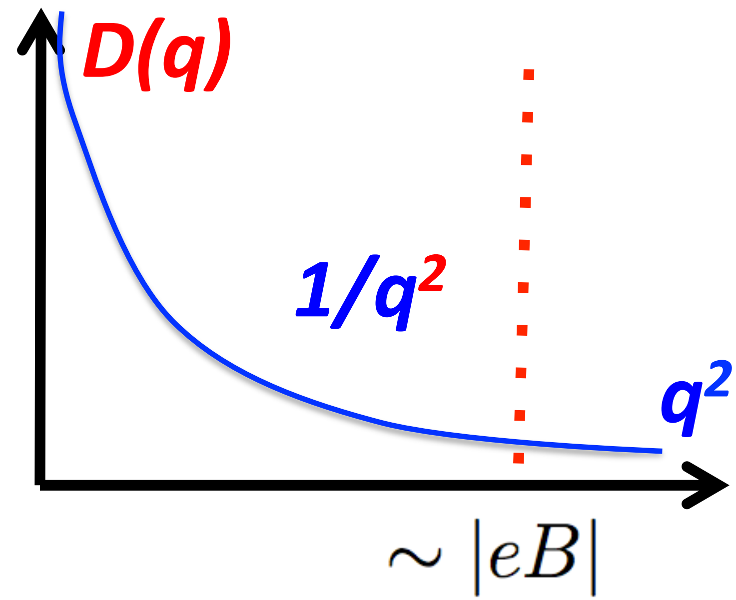
2) QED case ($1/q^2$ force : *running is negligible*)

$$\sim \int_0^{\sim|eB|} \mathrm{d}q_\perp^2\ \frac{1}{q_\perp^2 + q_L^2}$$

$$\Rightarrow \boxed{\sim \ln \frac{q_L^2}{\underline{|eB|}}}$$

2D Force is still marginally B-dep.

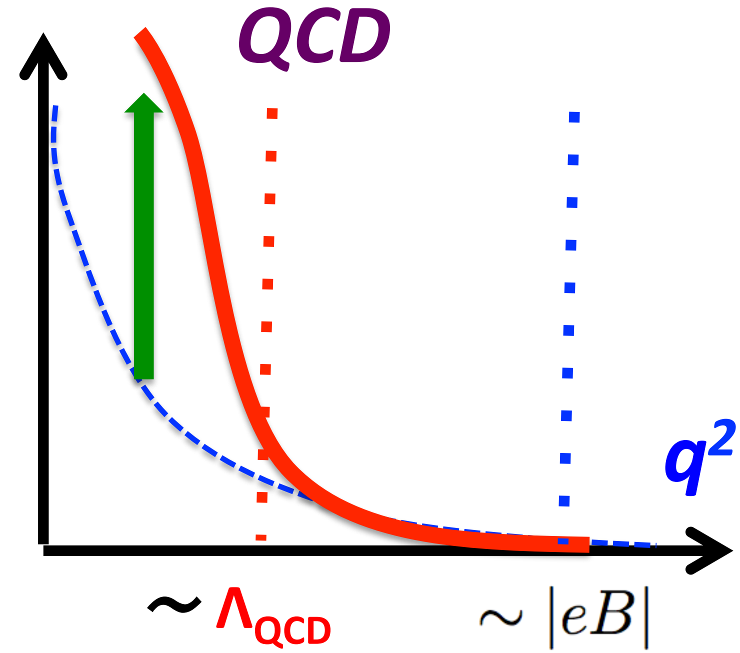
$$M \sim |eB|^{1/2}$$



Comparison of forces, 3

Now suppose: QCD force has strong "*IR enhancement*"

$$\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} D^{4\mathrm{D}}(q_L, q_\perp)$$



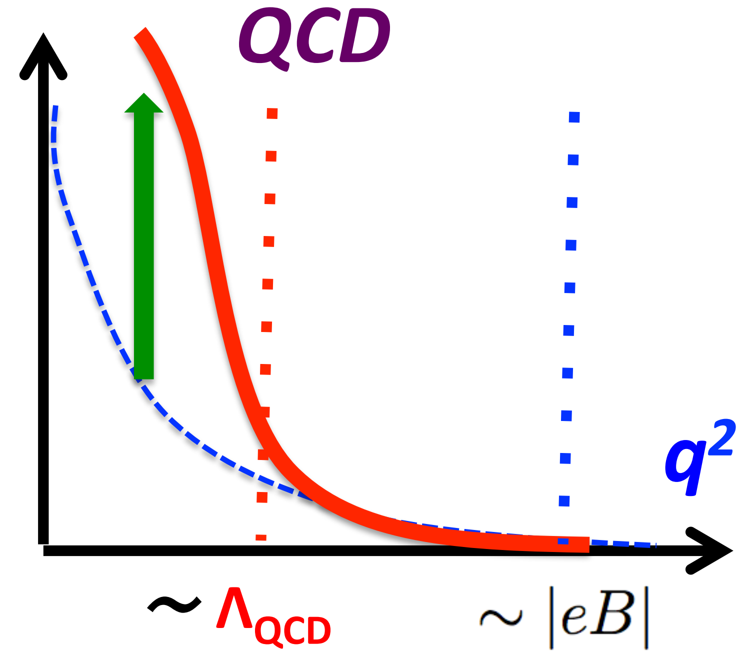
Comparison of forces, 3

Now suppose: QCD force has strong "*IR enhancement*"

$$\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} D^{\mathrm{4D}}(q_L, q_\perp)$$

For small $q_{perp} \sim \Lambda_{QCD}$:

we can set : $\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \sim 1$



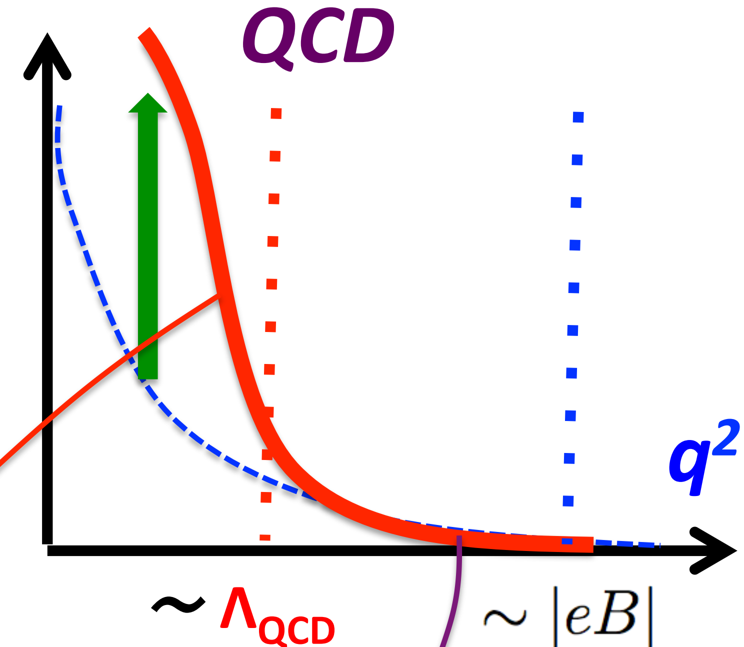
Comparison of forces, 3

Now suppose: QCD force has strong "*IR enhancement*"

$$\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} D^{4\mathrm{D}}(q_L, q_\perp)$$

For small $q_{perp} \sim \Lambda_{QCD}$:

we can set : $\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \sim 1$



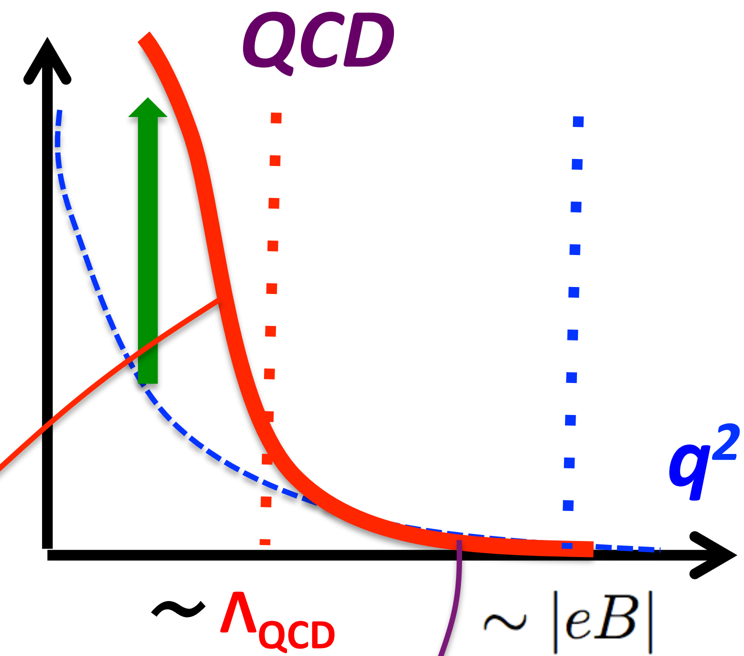
$$\sim \int_0^{\sim \Lambda_{\mathrm{QCD}}^2} \mathrm{d}q_\perp^2 D^{4\mathrm{D}}(q_L, q_\perp)$$

+ *small B-dep. corrections*

Comparison of forces, 3

Now suppose: QCD force has strong "*IR enhancement*"

$$\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} D^{4\mathrm{D}}(q_L, q_\perp)$$



For small q_{perp} $\sim \Lambda_{QCD}$:

we can set : $\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \sim 1$

$$\sim \int_0^{\sim \Lambda_{\mathrm{QCD}}^2} \mathrm{d}q_\perp^2 \, D^{4\mathrm{D}}(q_L, q_\perp)$$

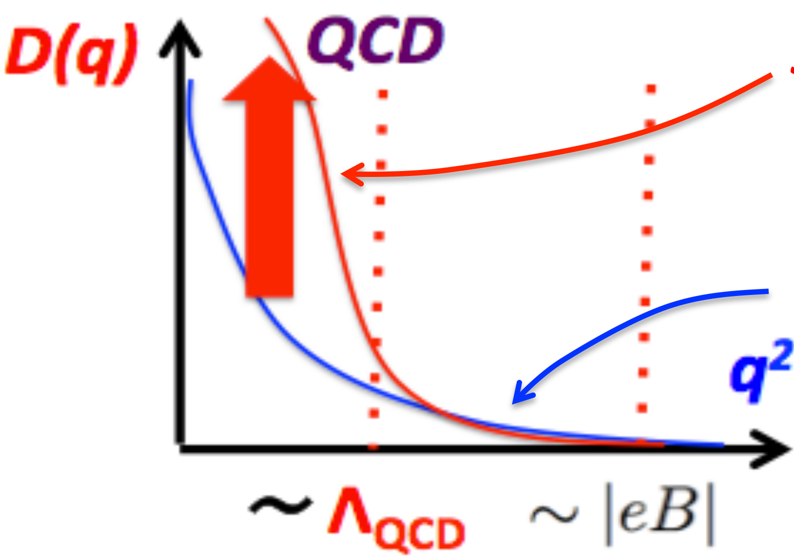
+ *small B-dep. corrections*

The dominant part :
"*nearly B-indep.*"

$$M \sim \Lambda_{\mathrm{QCD}}$$

Claim: 3

More and more "*IR enhancement*" *of forces*, then less and less *B-dep.* of *the quark mass gap*.



strong (IR)

weak (UV)

$$M \sim \Lambda_{\mathrm{QCD}}$$

The key is "*contrast*" between *IR* and *UV* forces.

Playing with a *toy* model

" *Linear rising* " *potential* for color charges

$$D_{\mu\nu} = C_F \times g_{\mu 0} g_{\nu 0} \times \frac{\sigma}{(\vec{p}^2)^2}$$ *string tension*

- **Motivated by Coulomb gauge studies.**

(ref: Gribov, Zwanziger)

- **The model has " *IR enhancement* ".**

- ***Confining*, in the sense that**

"No $q\bar{q}$ continuum in the meson spectra."

- ***Oversimplifications*** **:** *No $1/p^2$ tail, No color mag. int.*, etc.

- ***We will solve eqs. within " rainbow ladder "***

Schwinger-Dyson eq. for the LLL

e.g.) scalar part

$$M(p_L) = \int_{q_L} \gamma_0 \, S^{\mathrm{2D}}_{\mathrm{LLL}}(p_L - q_L; M)\, \gamma_0 \otimes \boxed{\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}\, D^{\mathrm{4D}}_{00}(q)}$$

Schwinger-Dyson eq. for the LLL

e.g.) scalar part

$$M(p_L) = \int_{q_L} \gamma_0 \, S^{2\mathrm{D}}_{\mathrm{LLL}}(p_L - q_L; M)\, \gamma_0 \otimes \boxed{\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \, D^{4\mathrm{D}}_{00}(q)}$$

for large B

$$\int_0^\infty \mathrm{d}q_\perp^2 \, \frac{\sigma \, \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}{(q_\perp^2 + q_z^2)^2} \;\Rightarrow\; \frac{\sigma}{q_z^2} - \cancel{\frac{\sigma}{q_z^2 + \underline{2|eB|}}}$$

(confining in 2D)

Schwinger-Dyson eq. for the LLL

e.g.) scalar part

$$M(p_L) = \int_{q_L} \gamma_0\, S^{\mathrm{2D}}_{\mathrm{LLL}}(p_L - q_L; M)\, \gamma_0 \otimes \boxed{\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}\, D^{\mathrm{4D}}_{00}(q)}$$

$$\int_0^\infty \mathrm{d}q_\perp^2\, \frac{\sigma\, \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}{\left(q_\perp^2 + q_z^2\right)^2} \;\Rightarrow\; \frac{\sigma}{q_z^2} - \cancel{\frac{\sigma}{q_z^2 + \underline{2|eB|}}}$$

for large B

(confining in 2D)

The *B-dependence* dropped out, and we get

$$M(p_L) \simeq \int_{q_L} \gamma_0\, S^{\mathrm{2D}}_{\mathrm{LLL}}(p_L - q_L; M)\, \gamma_0 \times \frac{\sigma}{q_z^2}$$

SD-eq. for *'t Hooft model* (QCD_2) in A_z =0 gauge

(whose properties are *known*)

Summary

1) Magnetized QCD is a good "laboratory".

2) To explain data for chiral condensate & $T_c(B)$,

$M_q(B)$ should be $\sim \Lambda_{QCD}$, instead of $\sim |eB|^{1/2}$.

3) The key is IR enhancement of QCD forces.

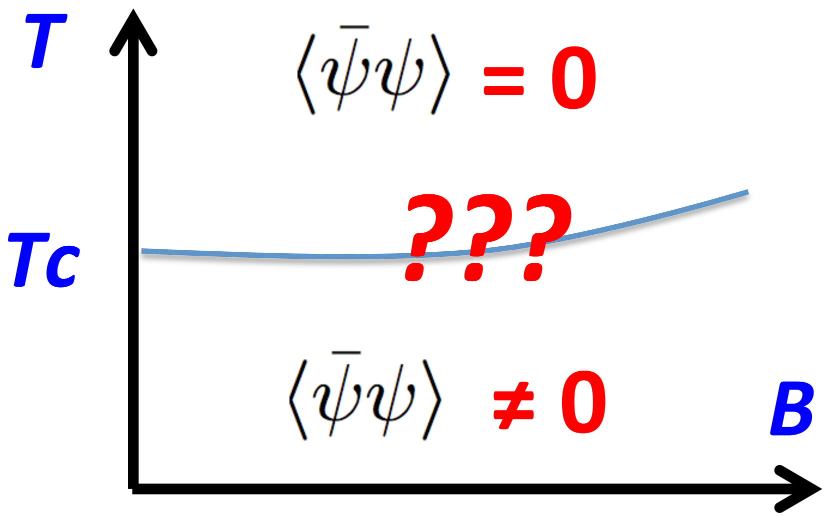
4) With $M_q(B) \sim \Lambda_{QCD}$, fluctuations are now operative.

see) Fukushima-Pawlowski (12), Fukushima-Hidaka (12)→ mesonic flucs.

Effects on gluonic sector → Ozaki's talk (this workshop)

Magnetized QCD : Basics, 8

The QCD phase diagram in (B-T) planes.



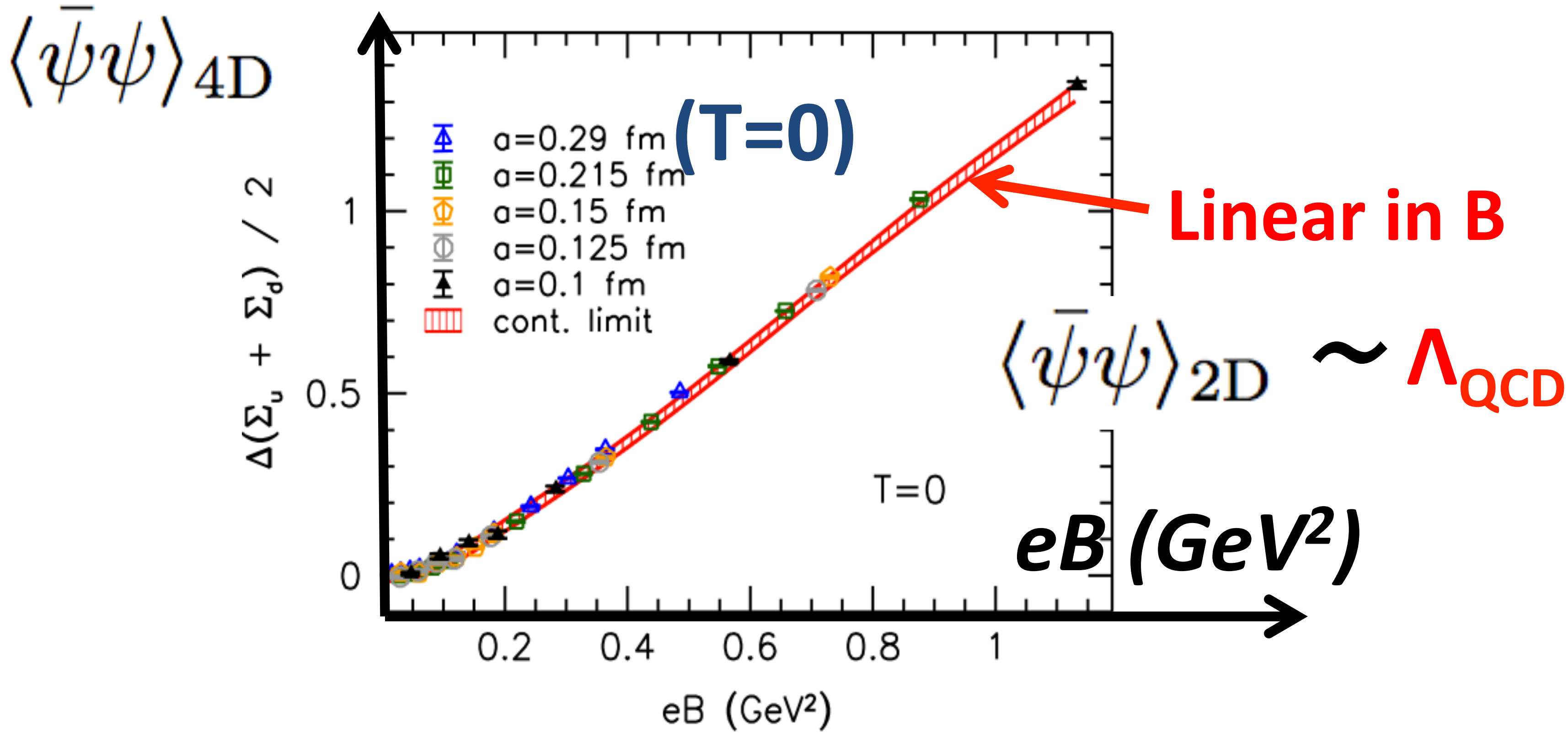
***Tc* : *Dissociation* temperature of *condensate*.**

$$\langle\bar{\psi}\psi(x)\rangle^{\mathrm{4D}}_{B} \sim \underline{|eB|} \times \underline{\langle\bar{\psi}\psi(t,z)\rangle^{\mathrm{2D}}_{B}}$$

***Tc* is determined by dissociation of** $\langle\bar{\psi}\psi\rangle_{\mathrm{2D}}$

On *Magnetic Catalysis*

Lattice simulations indeed confirmed.



However, its B-dep. is different from theories

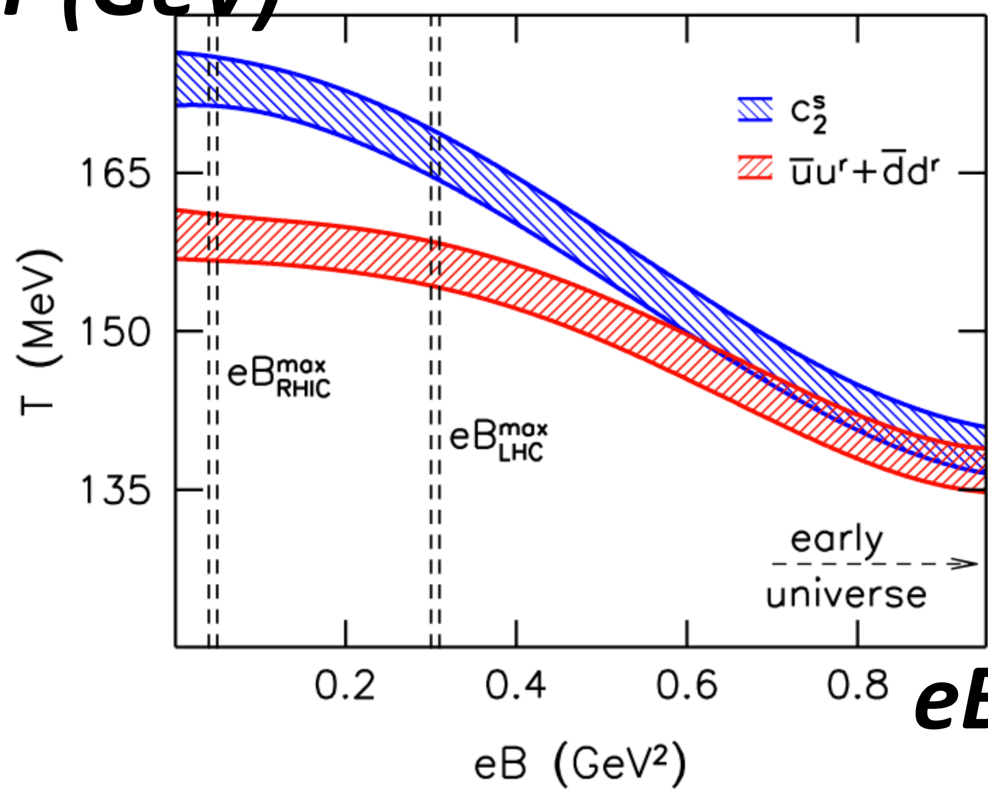
On *B-T phase diagram*

Theories: At larger B,

Dissociation of Chiral condensates

& De-confinement happen at larger Tc

T (GeV)



eB (GeV^2)

However

B ↑ → Tc ↓

Opposite !!!

Origins of contradictions ?

Most Theories predict : $\langle\bar{\psi}\psi\rangle_{2\mathrm{D}}$ **~ |eB|$^{1/2}$**

(NJL model or QED like calculations)

Then Tc behaves like : ***Tc*** **~ |eB|$^{1/2}$**

(|eB|$^{1/2}$ condensate does not easily dissociate)

Instead we need : $\langle\bar{\psi}\psi\rangle_{2\mathrm{D}}$ **~ Λ_{QCD}**

Then Tc behaves like : ***Tc*** **~ Λ_{QCD}**

(We have better chance to explain reduction of Tc)

Competing effects for larger IR phase space

$B \uparrow$

Magnetic Catalysis

for ChSB

(Review: Shovkovy 12)

$\langle\bar{\psi}\psi\rangle \uparrow$ $\quad M_q \uparrow$

suppress

$T_c \uparrow$

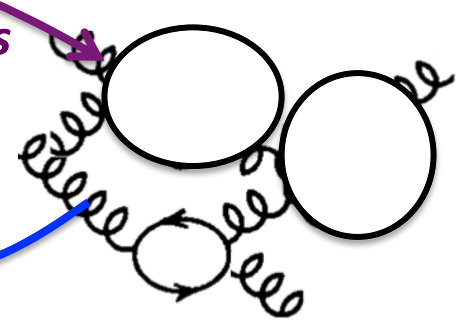
(larger gap → approach pure glue results)

Larger Screening effects

for gluons

(Miransky-Shovkovy 02)

suppress



$T_c \downarrow$

(due to reduction of non-pert. force)

Instead of solving this highly nonlinear problem,

we suggest the regime:

$$\Lambda_{QCD} \ll |eB|^{1/2} \ll N_c^{1/2}\,\Lambda_{QCD}$$

separate IR from UV

regard screenings
as secondary effects

and ***consider large Nc value of the quark mass gap.***

It should be regarded as the **upper bound**:

1/Nc corrections just **reduce the gap**.

(gluon screening, hadronic fluctuations)

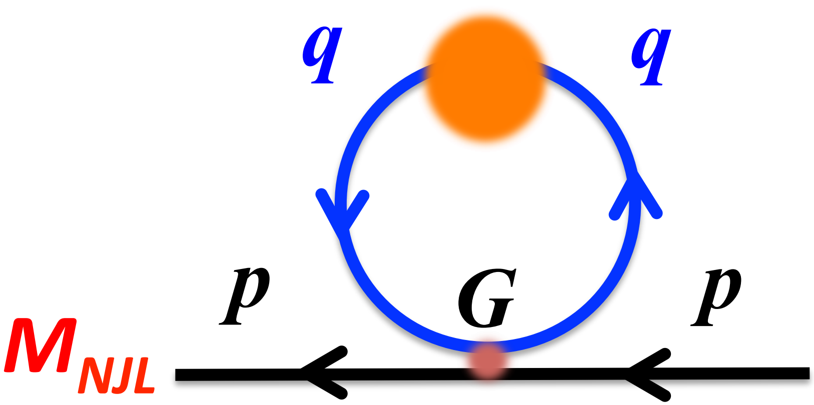
(Fukushima-Pawlowski 12, Fukushima-Hidaka 12)

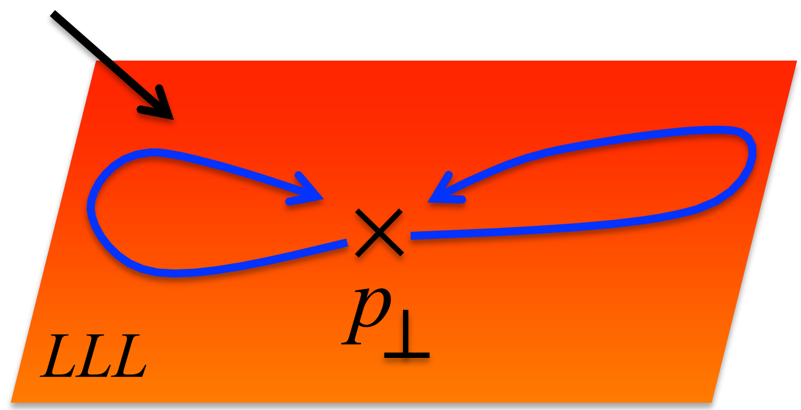
Gap eq. (NJL case, T=0)

Gap eq.

$$M_{\rm NJL}(B) = G\,{\rm tr}\,S(x,x) \rightarrow G\,\frac{|eB|}{2\pi}\underline{\int\frac{d^2q_L}{(2\pi)^2}\,{\rm tr}\,S_{\rm 2D}(q_L)}$$

area $\sim |eB|$ ***2D quark propagator***





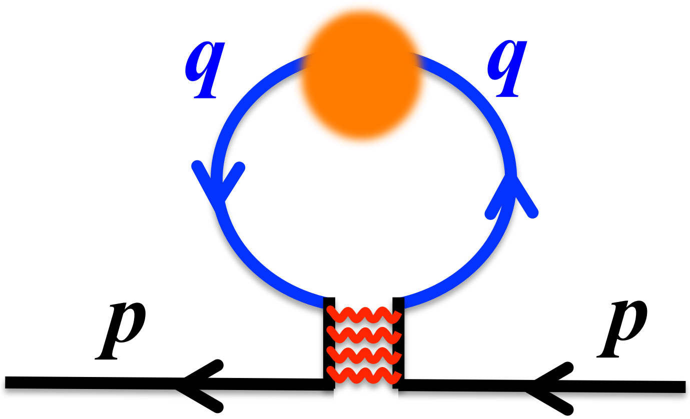
Problem : **all the self-interactions have equal strength *G*.**

$$\underline{\langle\bar{\psi}\psi\rangle^B_{\rm NJL}} \simeq -\frac{1}{G}\,\underline{M_{\rm NJL}(B)}$$

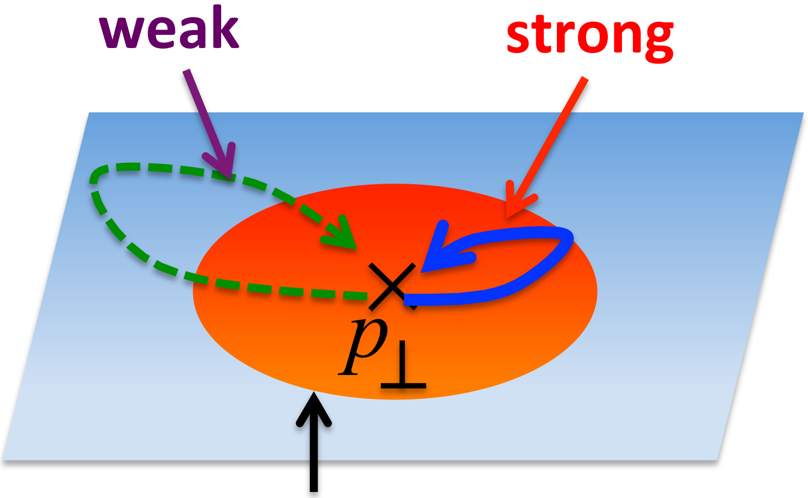
same B-dep.

Gap eq. (QCD, T=0)

Key features : strong in IR & weak in UV



$M(p)$



area $\sim \Lambda_{QCD}^2 << |eB|$

The Gap eq. does **NOT** pick up the factor $|eB|$.

(modulo weak coupling corrections)

The Gap is solely determined by the scale Λ_{QCD} .

Condensate (QCD, T=0)

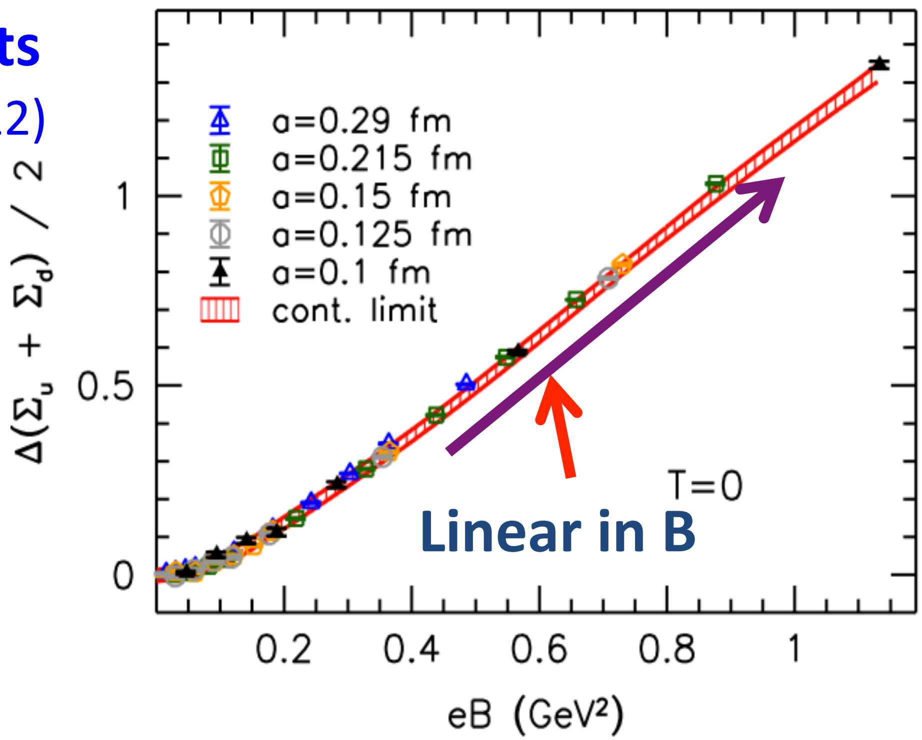
$$\langle\bar{\psi}\psi\rangle_{4\mathrm{D}} = \mathrm{tr}\, S(x,x) \rightarrow \frac{|eB|}{2\pi}\int\frac{d^2q_L}{(2\pi)^2}\,\mathrm{tr}\, S_{2\mathrm{D}}(q_L)$$

$$\Rightarrow \langle\bar{\psi}\psi\rangle_{4\mathrm{D}} \sim |eB| \times \langle\bar{\psi}\psi\rangle_{2\mathrm{D}}$$

$\sim \Lambda_{QCD}$

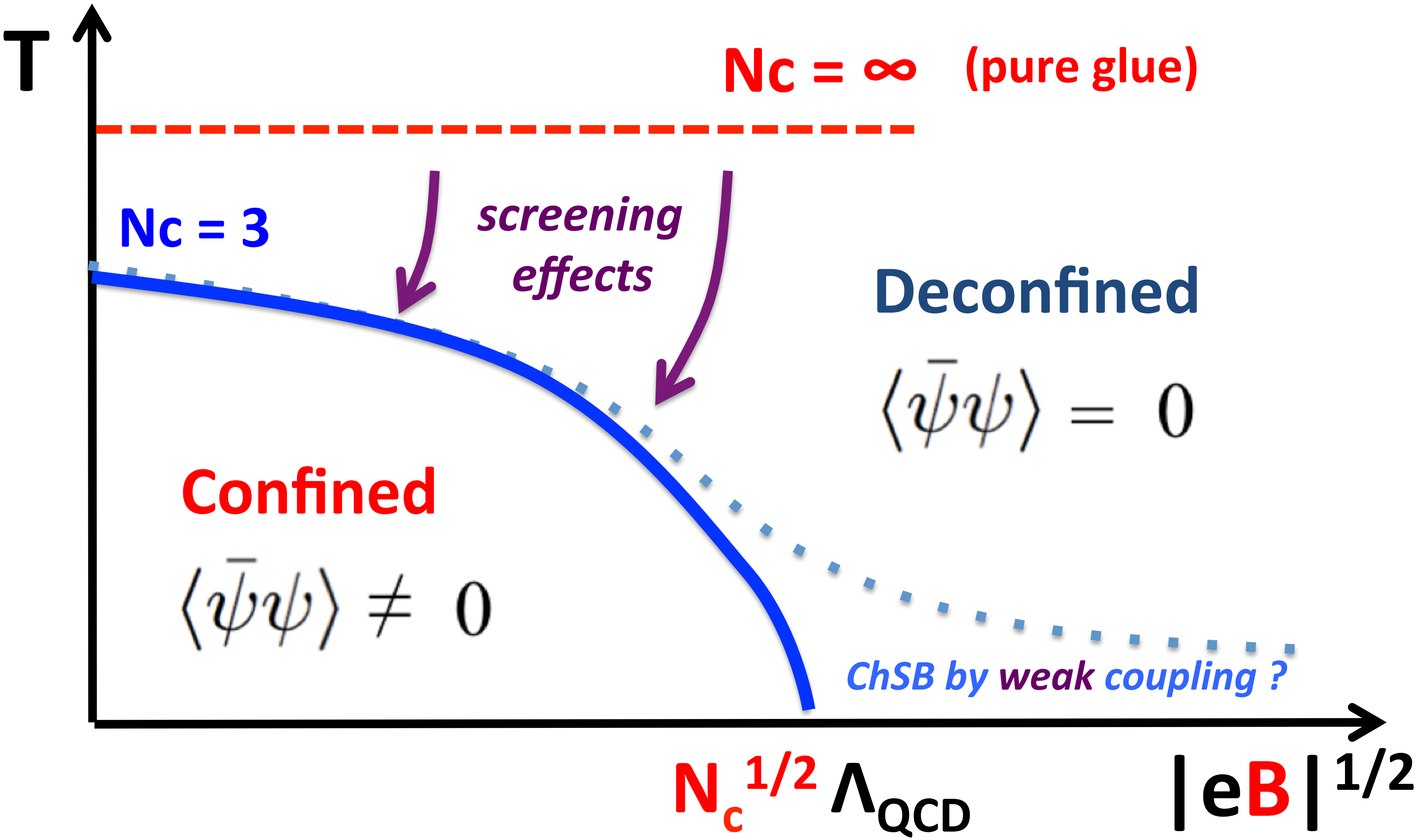
Lattice results

(Bali et al. 11,12)



B - *T* phase diagram ?

(see also Fraga-Noronha-Palhares 12)



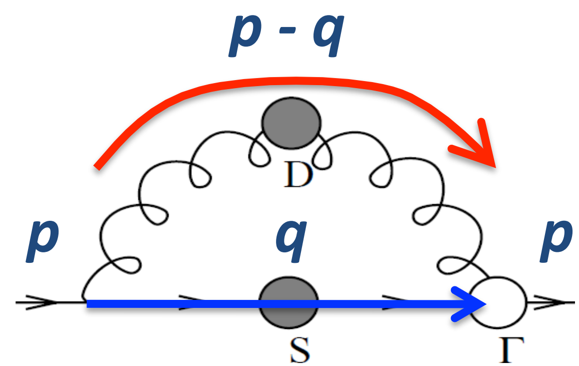
3, Dim. reduction of a confining model

(*as an explicit example*)

within the regime :

$$\Lambda_{QCD} \ll |eB|^{1/2} \ll N_c^{\ 1/2}\,\Lambda_{QCD}$$

Schwinger-Dyson eq. for the LLL



$$\Sigma(p) = \int \frac{\mathrm{d}^4 q}{(2\pi)^4}\, \gamma_4\, S(q;\Sigma)\, \gamma_4\, D_{44}(p-q)$$

quarks in LLL : P_T - independent

factorization

$$\Sigma_{\mathrm{2D}}(\underline{p_L}) = \int \frac{\mathrm{d}^2 q_L}{(2\pi)^4}\, \gamma_4\, S_{\mathrm{2D}}(\underline{q_L};\Sigma_{\mathrm{2D}})\, \gamma_4 \otimes \underline{\int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2}\, D_{44}(p-q)}$$

2D conf. gluon propagator

4D self-consistent eq. → *2D self-consistent eq.*

(Gribov-Zwanziger model) ('t Hooft model in axial gauge)

B does not appear : self-energies are functions of Λ_{QCD}

Bethe-Salpeter eq. for the LLLs

Consider ***meson currents*** for which

both quark & anti-quark can couple to the ***LLL states***.

(Some currents **CAN NOT**, see next slide.)

Dim. reduction can be carried out in the same way :

*Both **total** & **relative** momenta are **indep. of trans. momenta**.*

- *Quark & anti-quark **align** in the z-direction.*

Classifications of Mesons (2-flavor case)

Expanding quark fields by the Landau levels: $\psi^f = \psi^f_{\rm LLL} + \sum_{n=1} \psi^f_n$

we can pick out currents $\bar{\psi}\Gamma\psi$ for which

both quark & anti-quark can decay to the **LLL.**

List of light mesons:

neutral $(u\bar{u}\,, d\bar{d}) \otimes (1\,, \gamma_5\,, \gamma_L\,, \gamma_L\gamma_5\,, \sigma_{LL'}\,, \sigma_{\perp\perp'})$

charged $(u\bar{d}\,, d\bar{u}) \otimes (\gamma_\perp\,, \gamma_\perp\gamma_5\,, \sigma_{L\perp})$

e.g.)

- **Neutral** pion (charged pions do NOT).
- **Neutral**, **longitudinal** part of **vector** mesons.
- **Charged, transverse** part of **vector** mesons.

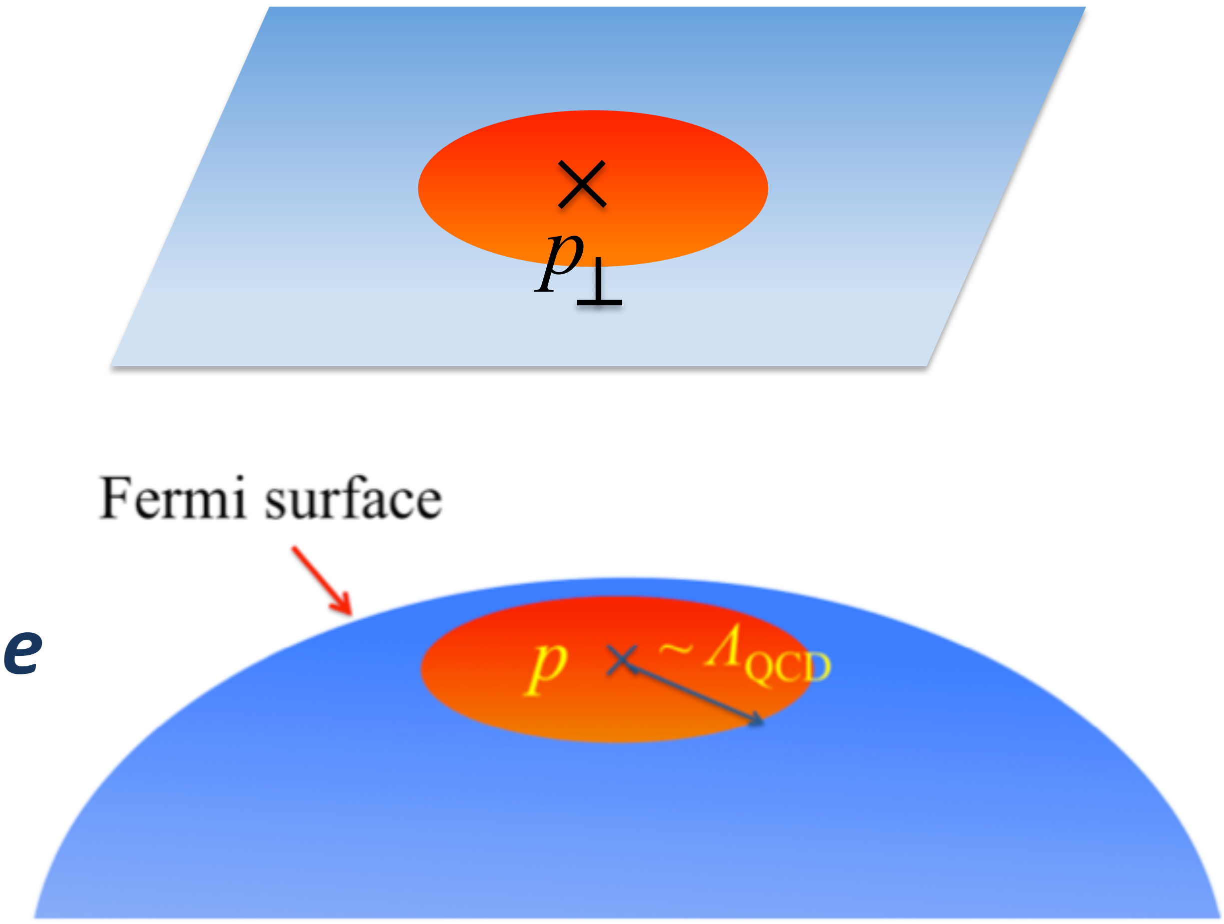
....

(***seems to be consistent with known lattice results.***)

Implications for dense QCD ?

Physics of the LLL

Physics near the Fermi surface

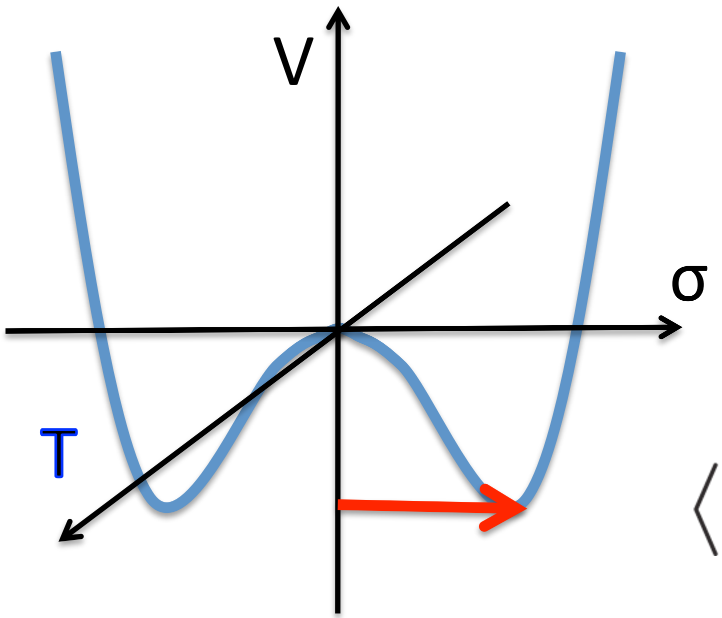


Similar modulo Fermi surface curvature

Backup

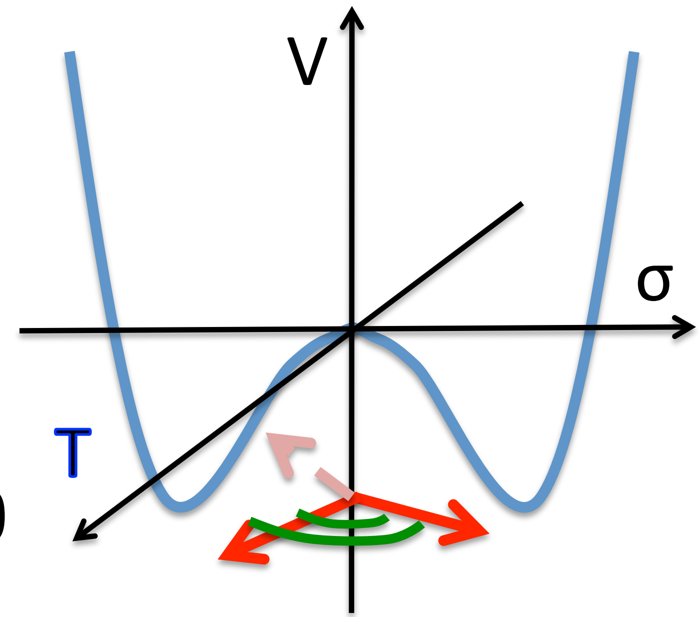
Coleman's theorem ?

- Coleman's theorem: No Spontaneous sym. breaking in 2D



$\langle e^{i\theta} \rangle \neq 0$ (SSB)

$\langle \rho \rangle \neq 0$



$\langle e^{i\theta} \rangle = 0$ (No SSB)

IR divergence in (1+1)D phase dynamics

- Phase fluctuations belong to:

Excitations (physical pion spectra)	ground state properties (No pion spectra)

Quasi-long range order & large Nc

- Local order parameters:

gapless modes gapped modes

$$\langle \bar{\Psi}_+ \Psi_- \rangle \sim \langle e^{i\sqrt{4\pi/N_c N_f}\phi} \rangle \otimes \langle \mathrm{tr} g \rangle \otimes \langle \mathrm{tr} h \rangle$$

due to IR divergent phase dynamics → 0 0 finite

But this does not mean the system is in the usual symmetric phase!

- Non-Local order parameters:

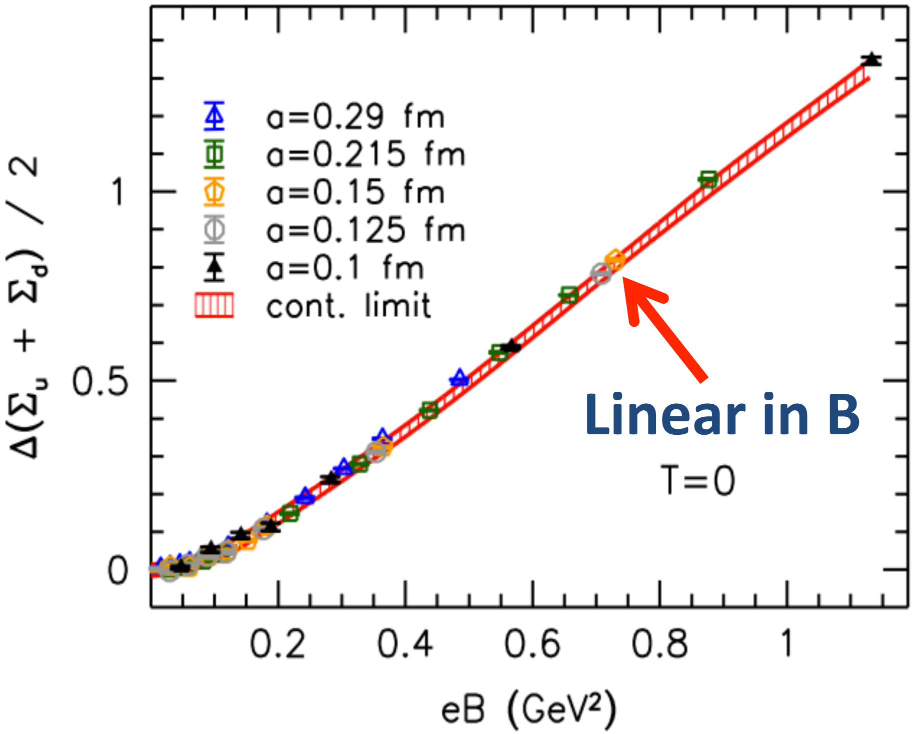
$$\langle \bar{\Psi}_+ \Psi_-(x) \bar{\Psi}_- \Psi_+(0) \rangle \sim \begin{cases} e^{-m|x|} & \text{: symmetric phase} \\ \langle \bar{\Psi}_+ \Psi_- \rangle^2 & \text{: long range order} \\ |x|^{-C/N_c} & \text{: quasi-long range order} \end{cases}$$

(including disconnected pieces)

(power law)

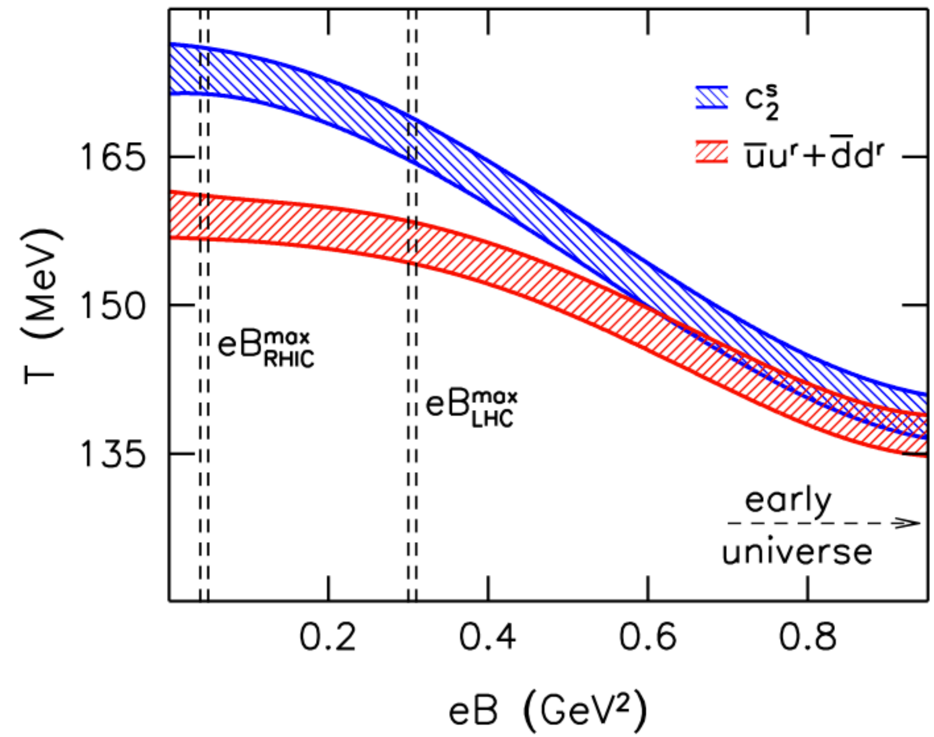
Lattice Results (Bali et al. 11, 12)

Chiral condensate (T=0)



$$|\langle\bar{\psi}\psi\rangle| \sim \underline{|eB|}\Lambda_{\mathrm{QCD}}$$

Tc (B)



B ↑ → ***Tc*** ↓

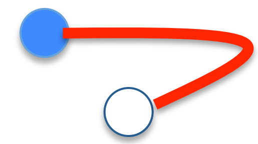
On the IR prescription

$$\frac{\sigma}{(\vec{k}^2)^2} \xrightarrow{\text{IR cut}} \frac{\sigma}{(\vec{k}^2+\Lambda_{\rm IR}^2)^2} \xrightarrow{\text{F.T.}} \underline{-\frac{\sigma}{\Lambda_{\rm IR}}} + \overbrace{\sigma r}^{\text{linear potential}} + O(\Lambda_{\rm IR} r^2)$$

- Probe colored objects:

IR div.: const. from naïve IR cutoff

- Color singlet sector:



IR const. $\longrightarrow$ irrelevant.
(Linear conf. without IR const.)

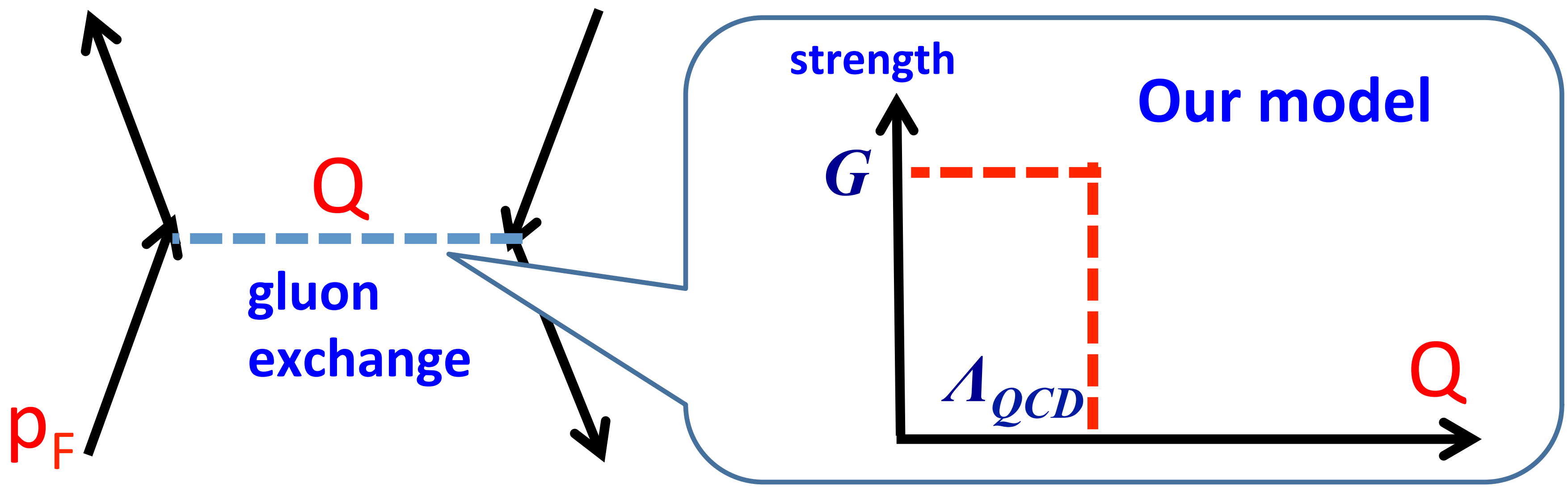
- As far as color-singlet sector is concerned,
  we can get the same results even if we drop off div. const.
  (principal value IR regulation; e.g., Coleman, Aspects of Symmetry)

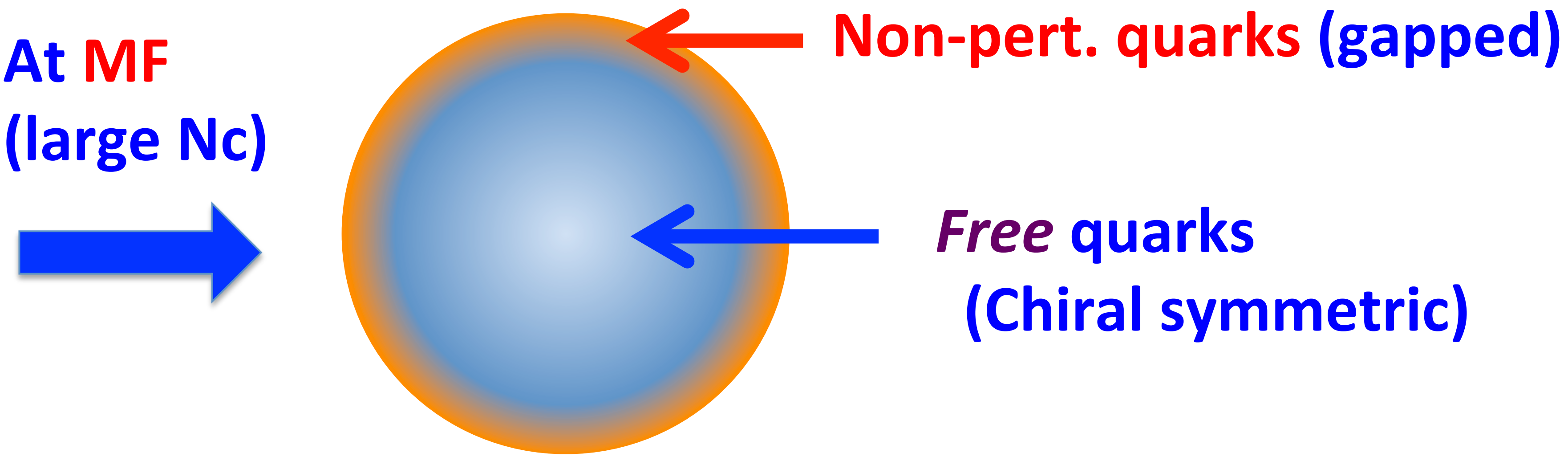
- S-D eqs. → just sub-diagrams in B-S eqs.

- Div. of poles will be used as color selection rules at best.

  (Actually div. of poles may not be necessary condition: Callan-Coote-Gross76)

Model & consequences



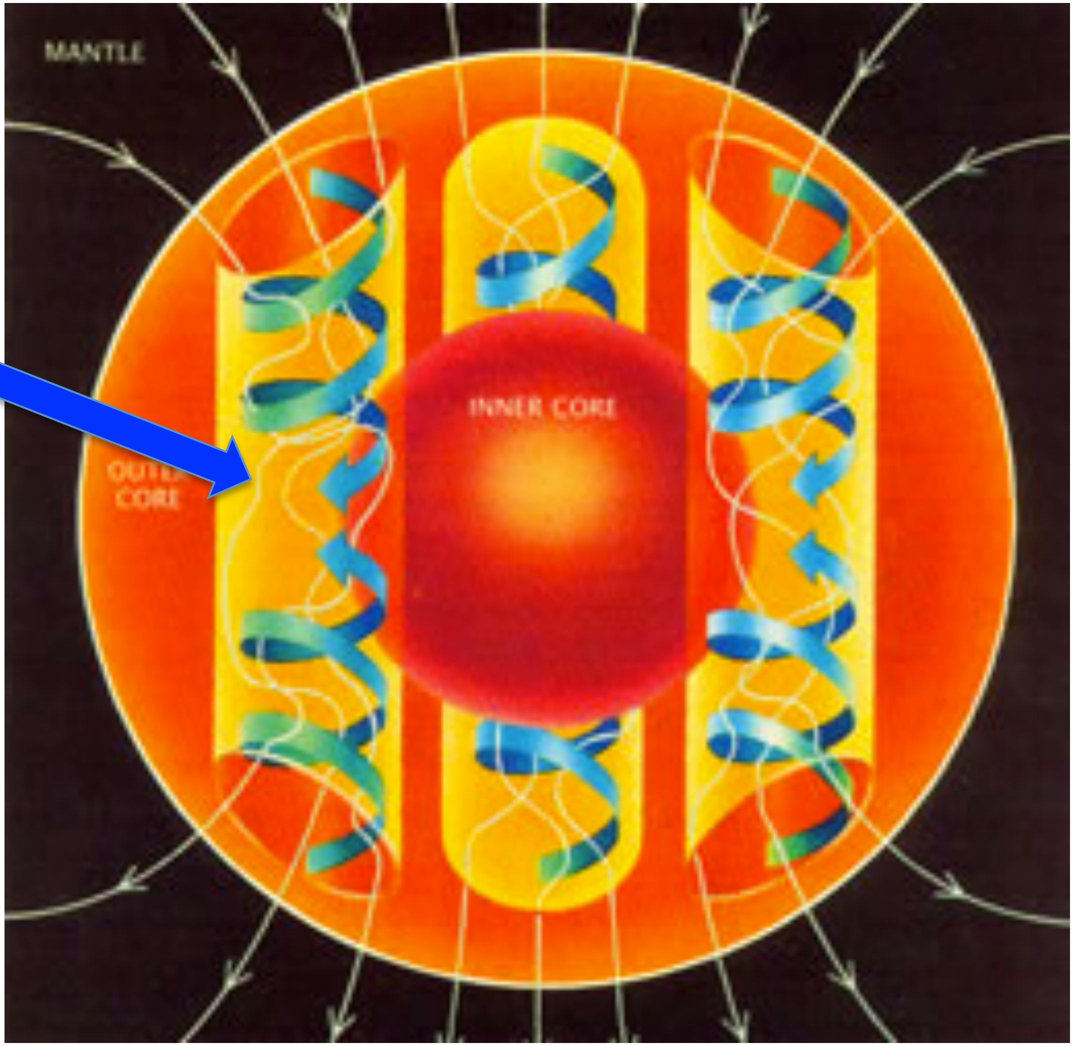


Magnetic fields in Nature: 1

Earth (surface) ～ 10^{-6} T

motion of charged fluids

magnetic fields



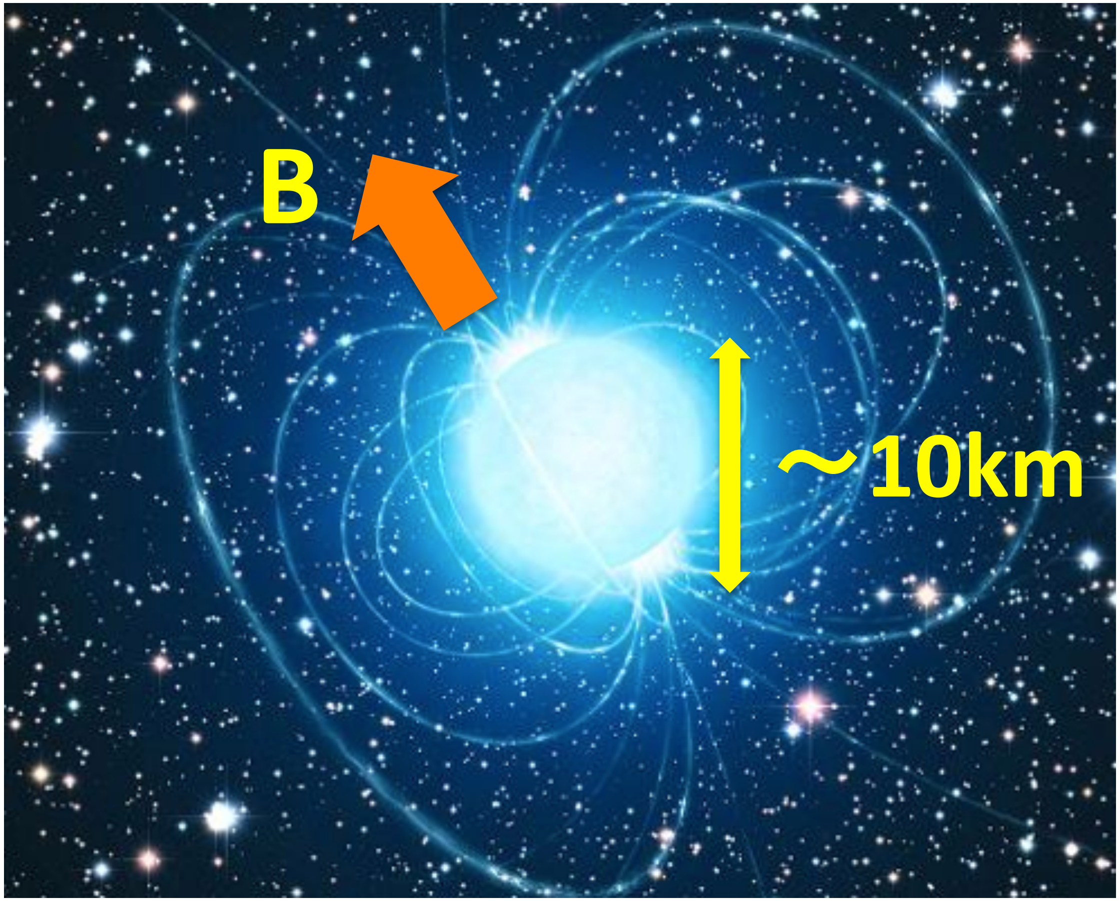
Magnetic fields in Nature: 2

Compact stars (surface) $\sim 10^{8} - 10^{11}$ T

earth's x $(10^{14} - 10^{17})$

charged fluids move very fast

Very Large magnetic fields



Magnetic fields in Nature: 3

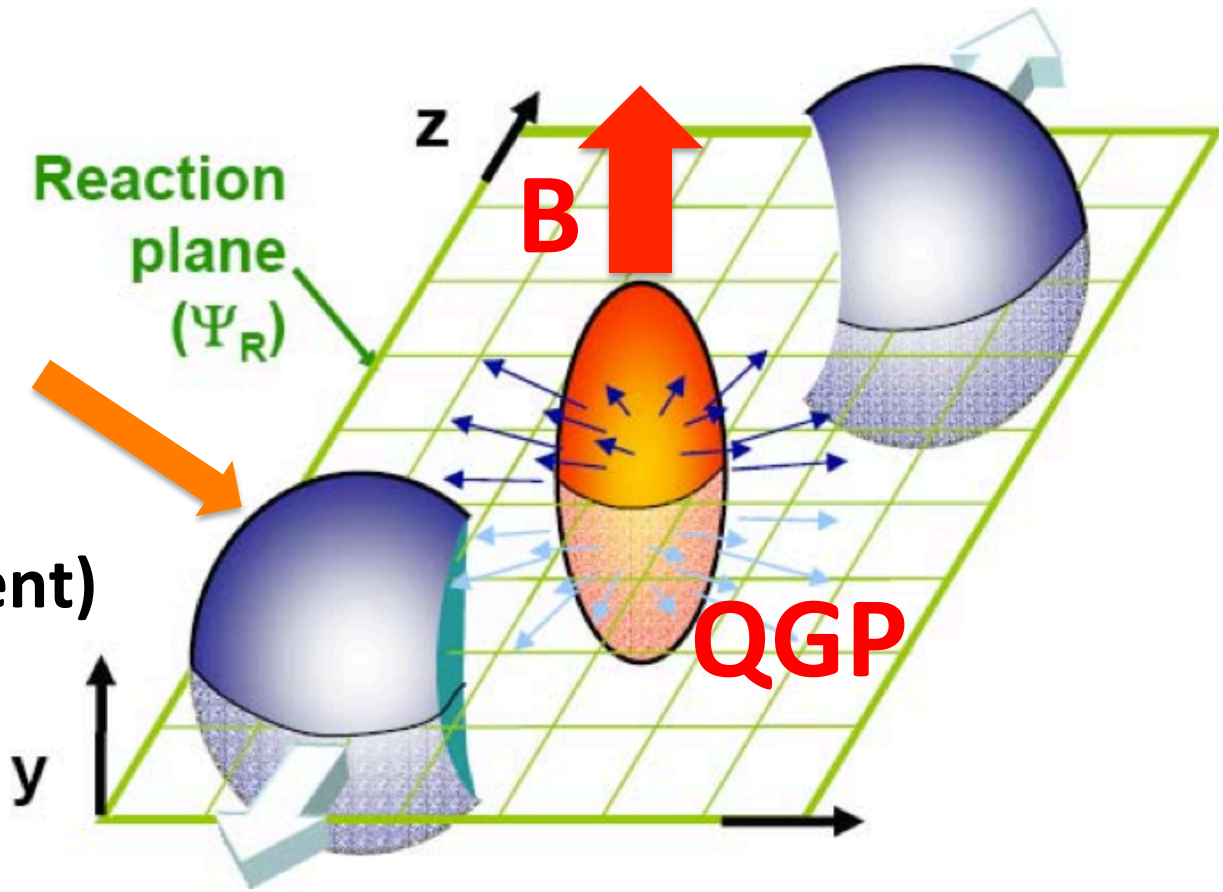
Relativistic Heavy Ion Collisions

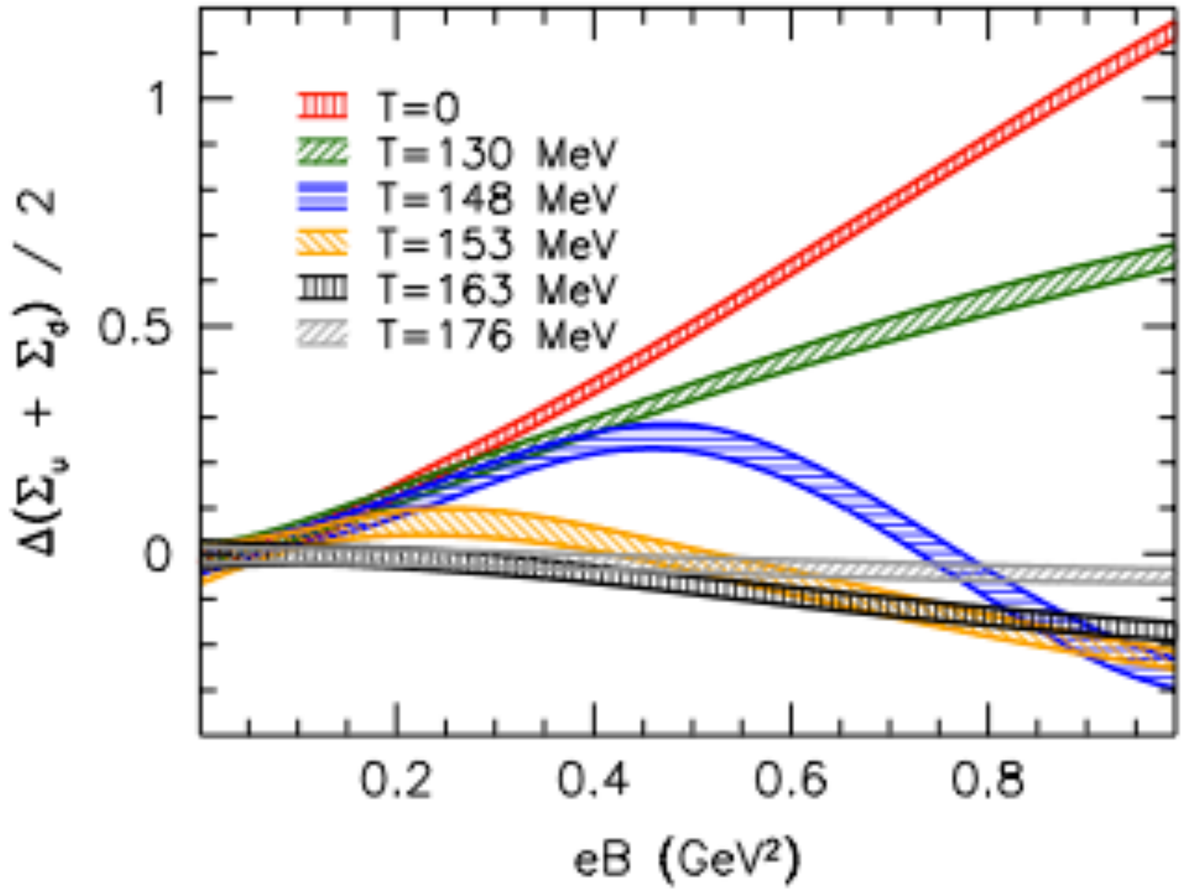
(RHIC, LHC) $\sim 10^{16}$ T $\sim \Lambda_{QCD}{}^2$ (Big!!)

Ion (Charged)

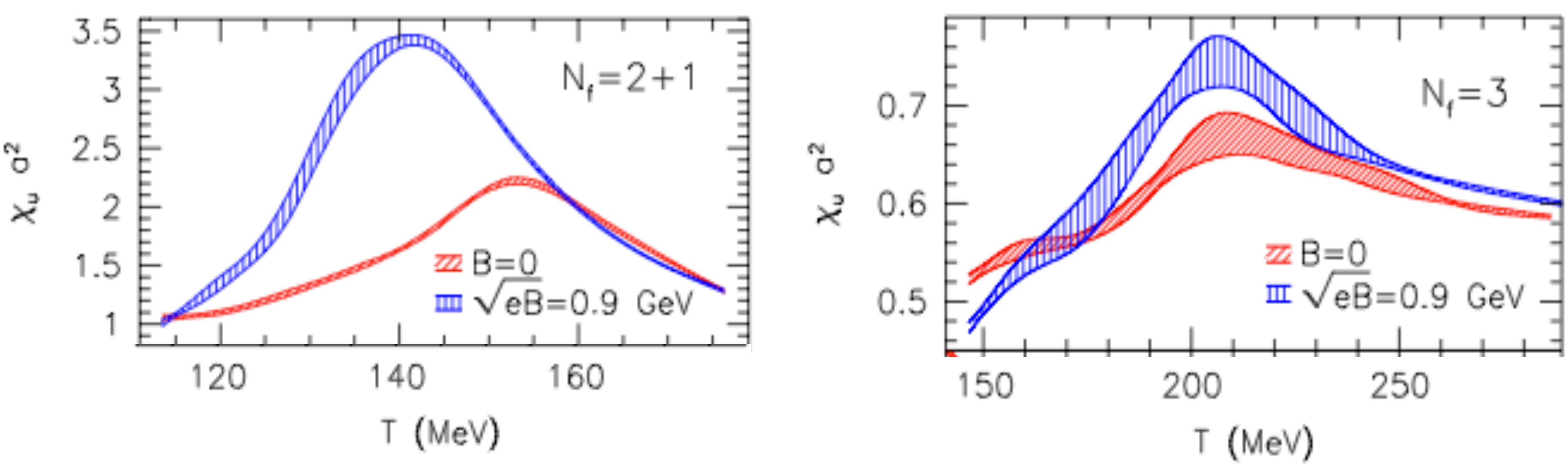
at high speed

(Large electric current)

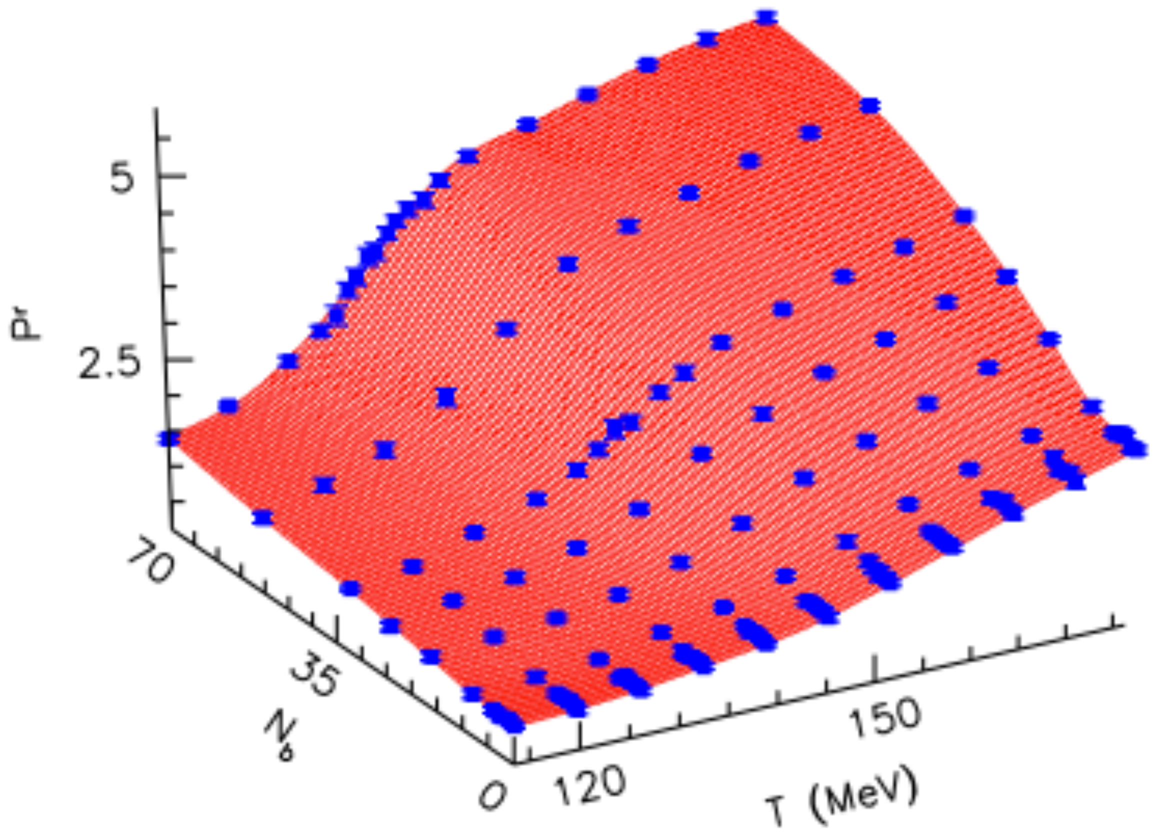




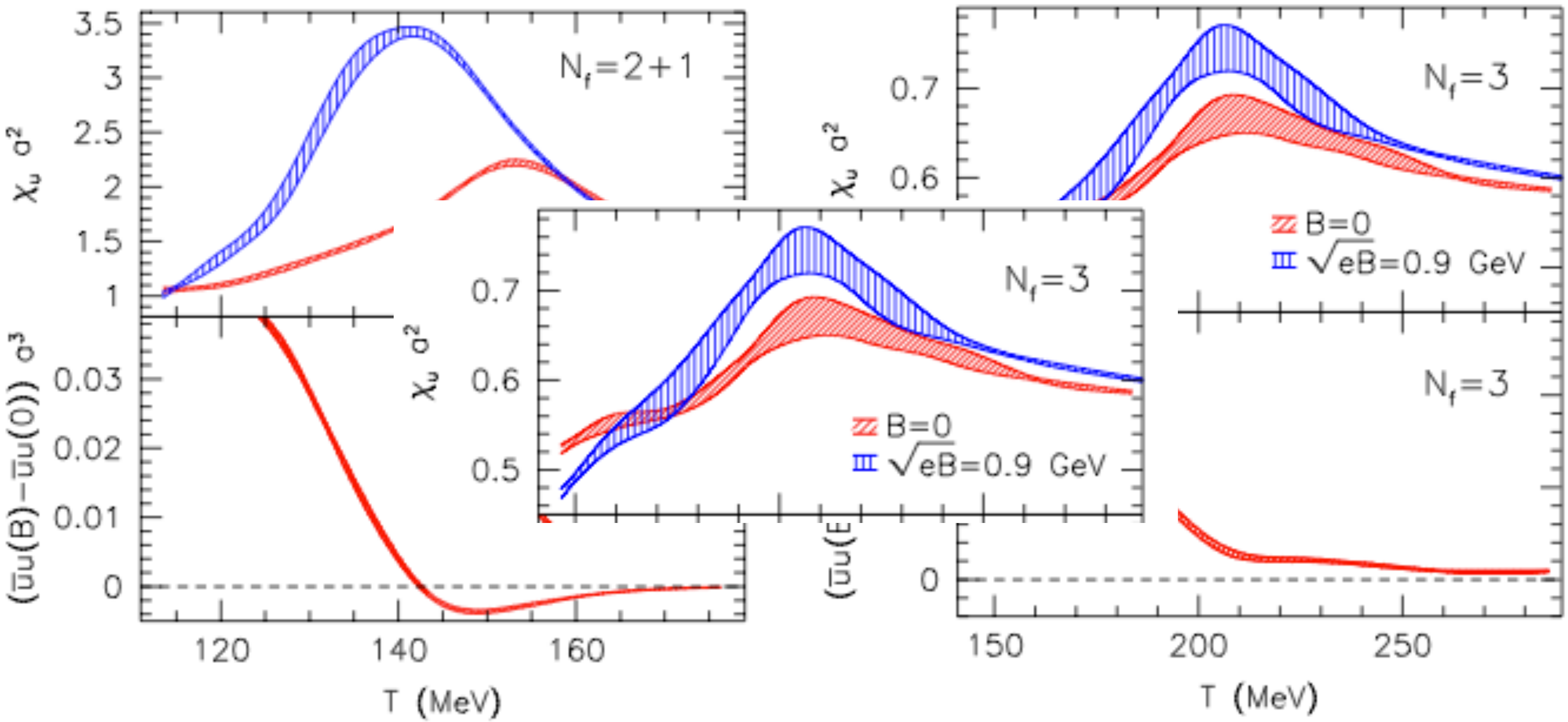
T=0
T=130 MeV
T=148 MeV
T=153 MeV
T=163 MeV
T=176 MeV
1
0.5
0
0.2
0.4
0.6
0.8
eB (GeV²)
Δ(Σ_u + Σ_d) / 2



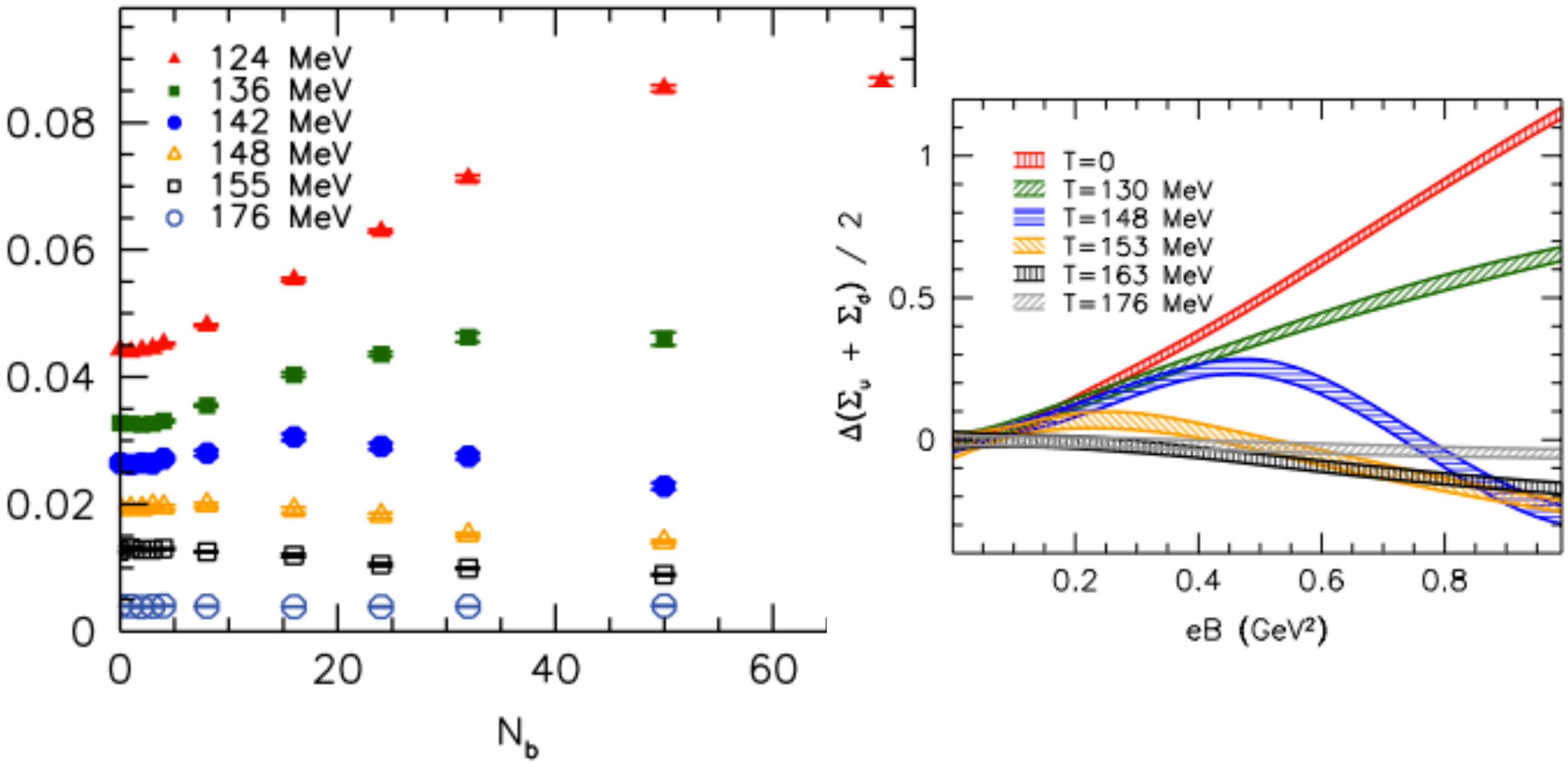
$N_f=2+1$
$\chi_u\ a^2$
B=0
$\sqrt{eB}$=0.9 GeV
T (MeV)
$N_f=3$
$\chi_u\ a^2$
B=0
$\sqrt{eB}$=0.9 GeV
T (MeV)



5
2.5
Pr
70
35
N_6
0
120
150
T (MeV)



N_f=2+1
χ_u a^2
(ūu(B)−ūu(0)) a^3
T (MeV)
N_f=3
B=0
√eB=0.9 GeV



124 MeV
136 MeV
142 MeV
148 MeV
155 MeV
176 MeV
0
0.02
0.04
0.06
0.08
0
20
40
60
N_b
T=0
T=130 MeV
T=148 MeV
T=153 MeV
T=163 MeV
T=176 MeV
$\Delta(\Sigma_u + \Sigma_d) / 2$
0
0.5
1
0.2
0.4
0.6
0.8
eB (GeV²)