Quark production in glasma

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collaboration with F. Gelis

NFQCD2013@YITP
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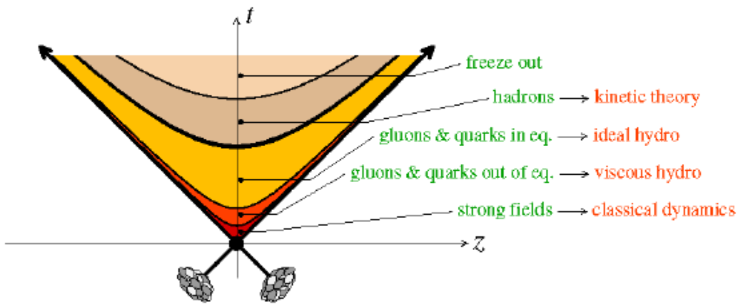
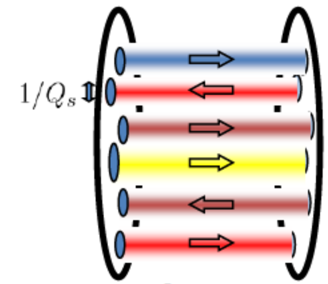
Introduction

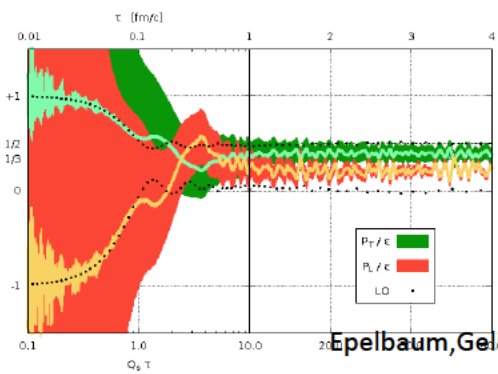
The time-evolution of Glasma toward isotropization and thermalization



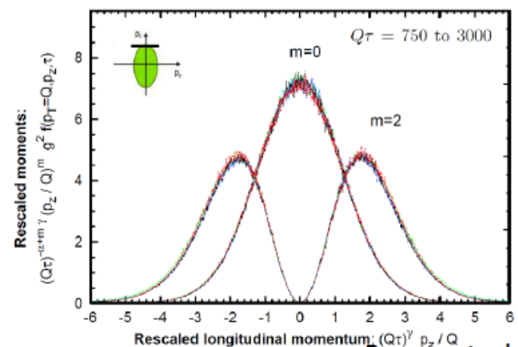


Glasma

large occupation number → classical dynamics



Epelbaum,Gelis (2013)



Berges et. al. (2013)

Classical statistical simulations of gluodynamics

Quark production is also important!

Quark production

CGC, glasma ← purely gluonic matter

How does the system reaches chemical equilibrium between light quarks and gluons?

Earlier works by F. Gelis, K. Kajantie and T. Lappi

PRC71, 024904(2005)
PRL96, 032304(2006)

- Limitation from numerical costs
- Treatment of the boost-invariance

There have been theoretical and technical advances on

- ➢ classical statistical method for over-occupied bosonic fields
- ➢ real-time lattice simulations of fermionic fields
- ➢ treatment of a boost-invariant system

Derivation of the classical statistical method

a) Diagrammatically sum up unstable modes. $\longrightarrow$ Francois's talk

b) From the Schwinger-Keldysh path-integral formalism.

S. Jeon, PRC72, 014907 (2005); arXiv:1308.0263.

Compute $\langle 0_{\rm in}|\mathcal{O}|0_{\rm in}\rangle$ $\Rightarrow$ the Schwinger-Keldysh (CTP) formalism

- ϕ^4-scalar theory as an example

  - Generating functional

$$Z[J_+, J_-] = \int \left[d\phi_+^i d\phi_-^i\right] \underbrace{\rho[\phi_+^i, \phi_-^i]}_{\text{density matrix}} \int \mathcal{D}\phi_+ \mathcal{D}\phi_- e^{i\int d^4x[\mathcal{L}_{\rm SK} + J_+\phi_+ - J_-\phi_-]}$$

  - SK Lagrangian

$$\mathcal{L}_{\rm SK} = \mathcal{L}[\phi_+] - \mathcal{L}[\phi_-]$$

$$= \left(\frac{1}{2}\partial_\mu\phi_+\partial^\mu\phi_+ - \frac{1}{2}m^2\phi_+^2 - \frac{\lambda}{4!}\phi_+^4\right) - \left(\frac{1}{2}\partial_\mu\phi_-\partial^\mu\phi_- - \frac{1}{2}m^2\phi_-^2 - \frac{\lambda}{4!}\phi_-^4\right)$$

- Keldysh transformation

$$\phi = \frac{\phi_+ + \phi_-}{2} \quad \chi = \phi_+ - \phi_-$$

$$\mathcal{L}_{\text{SK}} = -\left(\Box\phi + m^2\phi + \frac{\lambda}{3!}\phi^3\right)\chi - \frac{\lambda}{4!}\phi\chi^3$$

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classical field equation

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classical field equation

- Classical approximation

Strong fields ➡ $\phi \gg \chi$ ➡ Neglect the term $\frac{\lambda}{4!}\phi\chi^3$

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classical field equation

- Classical approximation

Strong fields $\Rightarrow$ $\phi \gg \chi$ $\Rightarrow$ Neglect the term $\frac{\lambda}{4!}\phi\chi^3$

The path integration over χ can be executed.

$$\int \mathcal{D}\phi\mathcal{D}\chi e^{i\int d^4x[\mathcal{L}_{\rm SK}+J_\chi\phi+J_\phi\chi]} = \int \mathcal{D}\phi e^{i\int d^4x J_\chi\phi}\,\delta\left(\Box\phi + m^2\phi + \frac{\lambda}{3!}\phi^3 + J_\phi\right)$$

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The delta function constraints the field trajectory to the classical path.

- The Wigner transform of the density matrix

$$W[\phi^i, \dot{\phi}^i] = \int [d\chi^i] e^{i\int d^3x \dot{\phi}^i \chi^i} \rho[\phi^i + \chi^i/2, \phi^i - \chi^i/2]$$

Perturbative vacuum $\Longrightarrow$ Gaussian distribution

$$W[\phi^i, \dot{\phi}^i] = \exp\left(- \int \frac{d^3k}{(2\pi)^3 \omega_k} \left[\omega_k^2 \phi^i(\mathbf{k})\phi^i(-\mathbf{k}) + \pi^i(\mathbf{k})\pi^i(-\mathbf{k}) \right] \right)$$

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Finally,

$$Z_{\text{class}}[J_\phi, J_\chi] = \int [d\phi^i d\dot{\phi}^i] W[\phi^i, \dot{\phi}^i] \int \mathcal{D}\phi e^{i\int d^4x J_\chi \phi}\, \delta\left(\Box\phi + m^2\phi + \frac{\lambda}{3!}\phi^3 + J_\phi\right)$$

1. **Generate an ensemble of initial values according to the Wigner distribution.**
2. **Solve the classical equation for each initial conditions.**
3. **Take the ensemble average.**

Quantum effects are incorporated only through the initial condition.
But the initial vacuum fluctuations contain rich physics.

Fermions and the classical statistical method

- gauge part

$$A^\mu = \frac{A_+^\mu + A_-^\mu}{2} \qquad \eta^\mu = A_+^\mu - A_-^\mu$$

$$\mathcal{L}_{\mathrm{gauge}}^{\mathrm{SK}} = \left([D_\mu, F^{\mu\nu}] - J_{\mathrm{ext}}^\nu\right)^a \eta_\nu^a + \frac{ig}{4}[D_\mu, \eta_\mu]^a [\eta^\mu, \eta^\nu]^a$$

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non-Abelian Maxwell eq.

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non-Abelian Maxwell eq.

- quark part

Fermion's occupation number ≤ 1 → Fermions are always quantum.

$$\mathcal{L}_{\text{matter}} = \bar{\psi}\,(i\gamma^\mu D_\mu - m)\psi$$

$$D_\mu = \partial_\mu + igA_\mu$$

Fermions and the classical statistical method

□ gauge part

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□ quark part

Fermion's occupation number ≤ 1 ➡ Fermions are always quantum.

$$\mathcal{L}_{\text{matter}} = \bar{\psi}\,(i\gamma^\mu D_\mu - m)\psi$$

Quadratic fields can be integrated out.

$$\int \mathcal{D}[\bar{\psi}_+, \bar{\psi}_-, \psi_+, \psi_-] e^{i\int d^4x \mathcal{L}_{\text{matter}}^{SK}} = \text{Det}\left[i(S_{\text{F}}^+)^{-1}\right] \text{Det}\left[i(S_{\text{F}*}^-)^{-1}\right]$$

$S_{\text{F}}^+(x,y)$: time-ordered propagator dressed by the gauge field A_+

$S_{\text{F}*}^-(x,y)$: anti-time-ordered propagator dressed by the gauge field A_-

- Expand the Dirac determinants w.r.t. η^μ

$$\mathrm{Det}\left[i(S_\mathrm{F}^{+})^{-1}\right]\mathrm{Det}\left[i(S_{\mathrm{F}*}^{-})^{-1}\right]$$
$$= \mathrm{Det}\left[i(S_\mathrm{F})^{-1}\right]\mathrm{Det}\left[i(S_{\mathrm{F}*})^{-1}\right] e^{-i\frac{g}{2}\mathrm{Tr}[(S_\mathrm{F}+S_{\mathrm{F}*})\gamma_\mu]\eta^\mu+\mathcal{O}(\eta^2)}$$

Propagator dressed by the averaged gauge field A^μ

- Expand the Dirac determinants w.r.t. η^μ

$$\mathrm{Det}\left[i(S_{\mathrm{F}}^{+})^{-1}\right]\mathrm{Det}\left[i(S_{\mathrm{F}*}^{-})^{-1}\right]$$
$$= \mathrm{Det}\left[i(S_{\mathrm{F}})^{-1}\right]\mathrm{Det}\left[i(S_{\mathrm{F}*})^{-1}\right] e^{\boxed{-i\frac{g}{2}\mathrm{Tr}[(S_{\mathrm{F}}+S_{\mathrm{F}*})\gamma_\mu]}\eta^\mu+\mathcal{O}(\eta^2)}$$

Propagator dressed by the averaged gauge field A^μ

- Integrate over η^μ

Maxwell equation which couples to the quark current

$$\boxed{[D_\mu, F^{\mu\nu}] = J^\nu_{\mathrm{ext}} + J^\nu_{\mathrm{quark}}}$$

$$J^\nu_{\mathrm{quark}}(x) = i\frac{g}{2}\mathrm{Tr}[(S_{\mathrm{F}} + S_{\mathrm{F}*})\gamma^\nu]$$

- Expand the Dirac determinants w.r.t. η^μ

$$
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Maxwell equation which couples to the quark current

$$
\boxed{[D_\mu, F^{\mu\nu}] = J^\nu_{\mathrm{ext}} + J^\nu_{\mathrm{quark}}}
$$

$$
\begin{aligned}
J^\nu_{\mathrm{quark}}(x) &= i\frac{g}{2}\mathrm{Tr}[(S_{\mathrm{F}} + S_{\mathrm{F}*})\gamma^\nu] \\
&= \frac{g}{2}\langle 0_{\mathrm{in}}|\left[\hat{\bar{\psi}}(x), \gamma^\nu\hat{\psi}(x)\right]|0_{\mathrm{in}}\rangle
\end{aligned}
$$

$\hat{\psi}(x)$ is a field operator satisfying the Dirac equation under the gauge field:

$$
\boxed{[i\gamma^\mu(\partial_\mu + igA_\mu) - m]\,\hat{\psi}(x) = 0}
$$

1. **Generate an ensemble of the initial gauge fields according to the Wigner function**

$$A^\mu(t_0,\mathbf{x}) = A^\mu_{\mathrm{coherent}}(t_0,\mathbf{x}) + \sum_n \left[a^\mu_n(t_0,\mathbf{x})c_n + a^{\mu\,*}_n(t_0,\mathbf{x})c^*_n\right]$$

Gaussian random number $\langle c_n c^*_{n'}\rangle_{\mathrm{ens}} = \frac{1}{2}\delta_{n,n'}$

2. **For each gauge configuration, solve the Maxwell equation and the Dirac equation as associated equations**

$$[D_\mu, F^{\mu\nu}] = J^\nu_{\mathrm{ext}} + J^\nu_{\mathrm{quark}}$$

$$\left[i\gamma^\mu(\partial_\mu + igA_\mu) - m\right]\hat{\psi}(x) = 0$$

3. **Take the ensemble average and the expectation values**

$$\langle \mathcal{O}(A)\rangle_{\mathrm{ens}} \qquad \langle 0_{\mathrm{in}}|\hat{\mathcal{O}}(\psi)|0_{\mathrm{in}}\rangle$$

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pure classical approximation for gauge fields as a first step

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Computing fermion contributions

- Mode functions approach

Aarts, Smit 1998

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m]\,\hat{\psi}(x) = 0$$

linear in the Dirac field

$$\hat{\psi}(x) = \sum_s \int d^3p\,[\psi^+_{\mathbf{p},s}(x)a_{\mathbf{p},s} + \psi^-_{\mathbf{p},s}(x)b^\dagger_{\mathbf{p},s}]$$

$\psi^\pm_{\mathbf{p},s}(x)$: mode functions, c-number solutions of the Dirac eq.

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m]\,\psi^\pm_{\mathbf{p},s}(x) = 0 \qquad \lim_{t\to-\infty}\psi^\pm_{\mathbf{p},s}(x) = \psi^{\mathrm{free},\pm}_{\mathbf{p},s}(x)$$

$$J^\mu_{\mathrm{quark}} = \frac{g}{2}\langle 0|\left[\hat{\bar{\psi}}(x), \gamma^\mu\hat{\psi}(x)\right]|0\rangle$$

$$= -\frac{g}{2}\sum_s \int d^3p\,\{\bar{\psi}^+_{\mathbf{p},s}(x)\gamma^\mu\psi^+_{\mathbf{p},s}(x) - \bar{\psi}^-_{\mathbf{p},s}(x)\gamma^\mu\psi^-_{\mathbf{p},s}(x)\}$$

With the c-number mode functions, expectation values can be computed.

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Numerical cost

$$N_t \times N^2_{\mathrm{latt}}$$

expensive in 3+1dim.

With the c-number mode functions, expectation values can be computed.

☐ Monte Carlo method with male and female stochastic fields

Borsanyi, Hindmarsh 2009

- Male and female fields
  - initial condition

$$\psi_{\mathrm{M}}(t_0,\mathbf{x}) = \sum_s \int d^3p \left[\psi^+_{\mathbf{p},s}(t_0,\mathbf{x}) c_{\mathbf{p},s} + \psi^-_{\mathbf{p},s}(t_0,\mathbf{x}) d_{\mathbf{p},s}\right]$$

$$\psi_{\mathrm{F}}(t_0,\mathbf{x}) = \sum_s \int d^3p \left[\psi^+_{\mathbf{p},s}(t_0,\mathbf{x}) c_{\mathbf{p},s} - \psi^-_{\mathbf{p},s}(t_0,\mathbf{x}) d_{\mathbf{p},s}\right]$$

  - evolution equation

$$\left[i\gamma^\mu(\partial_\mu + igA_\mu) - m\right] \psi_{\mathrm{M,F}}(x) = 0$$

- Gaussian random numbers

$$\langle c_{\mathbf{p},s} c^*_{\mathbf{p}',s'}\rangle = \langle d_{\mathbf{p},s} d^*_{\mathbf{p}',s'}\rangle = \frac{1}{2}\delta_{s,s'}\delta^3(\mathbf{p}-\mathbf{p}')$$

The expectation value can be reproduced by male-female ensemble average.

$$\begin{aligned} -g\langle\bar{\psi}_{\mathrm{M}}(x)\gamma^\mu\psi_{\mathrm{F}}\rangle &= -\frac{g}{2}\sum_s \int d^3p \left\{\bar{\psi}^+_{\mathbf{p},s}(x)\gamma^\mu\psi^+_{\mathbf{p},s}(x) - \bar{\psi}^-_{\mathbf{p},s}(x)\gamma^\mu\psi^-_{\mathbf{p},s}(x)\right\} \\ &= J^\mu_{\mathrm{quark}} \end{aligned}$$

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$$= J^\mu_{\mathrm{quark}}$$

The male-female combination is necessary to get this minus sign which originates in anti-commutativity.

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Numerical cost
$N_{\mathrm{config}} \times N_{\mathrm{latt}}^2$

  - evolution equation

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m]\, \psi_{\mathrm{M,F}}(x) = 0$$

$N_{\mathrm{config}} \times N_t \times N_{\mathrm{latt}}$

- Gaussian random numbers

$$\langle c_{\mathbf{p},s} c^*_{\mathbf{p}',s'} \rangle = \langle d_{\mathbf{p},s} d^*_{\mathbf{p}',s'} \rangle = \frac{1}{2} \delta_{s,s'} \delta^3(\mathbf{p} - \mathbf{p}')$$

The expectation value can be reproduced by male-female ensemble average.

Total cost
$N_{\mathrm{config}} \times N_{\mathrm{latt}} \times (N_{\mathrm{latt}} + N_t) \ll N_t \times N_{\mathrm{latt}}^2$
if $N_{\mathrm{config}} \ll N_{\mathrm{latt}}$ and $N_{\mathrm{config}} \ll N_t$

$-g\langle\bar{\psi}_{\mathrm{M}}($ … $\psi^-_{\mathbf{p},s}(x)\}$

… necessary to get this minus sign which originates in anti-commutativity.

Particle distribution function

- Particle definition is ambiguous in a background field or with interactions.
- But, it is informative to get physical insight on microscopic reactions.
- With some quasi-particle definition, momentum distribution can be computed.

$$f(\mathbf{p}, s) \equiv \langle 0_{\rm in}| a^\dagger_{\mathbf{p},s}(t) a_{\mathbf{p},s}(t) |0_{\rm in}\rangle \frac{(2\pi)^3}{V}$$

$$= \frac{1}{2\omega_p V} \sum_{s'} \int d^3p' \int d^3x \int d^3y \, \psi^{-\,\dagger}_{\mathbf{p}',s'}(t,\mathbf{x}) e^{i(\mathbf{p}+g\mathbf{A})\cdot\mathbf{x}} u(\mathbf{p},s) u^\dagger(\mathbf{p},s) e^{-i(\mathbf{p}+g\mathbf{A})\cdot\mathbf{y}} \psi^{-}_{\mathbf{p}',s'}(t,\mathbf{y})$$

valid only if the gauge potential is uniform

Another way is to introduce the Wigner distribution func.
e.g. Hebenstreit, Berges (2013)

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Another way is to introduce the Wigner distribution func.
e.g. Hebenstreit, Berges (2013)

the MC method

$$= -\frac{1}{2\omega_p V} \int d^3x \int d^3y \, \langle \psi^\dagger_{\rm M}(t,\mathbf{x}) e^{i(\mathbf{p}+g\mathbf{A})\cdot\mathbf{x}} u(\mathbf{p},s) u^\dagger(\mathbf{p},s) e^{-i(\mathbf{p}+g\mathbf{A})\cdot\mathbf{y}} \psi_{\rm F}(t,\mathbf{y}) \rangle + \frac{1}{2}$$

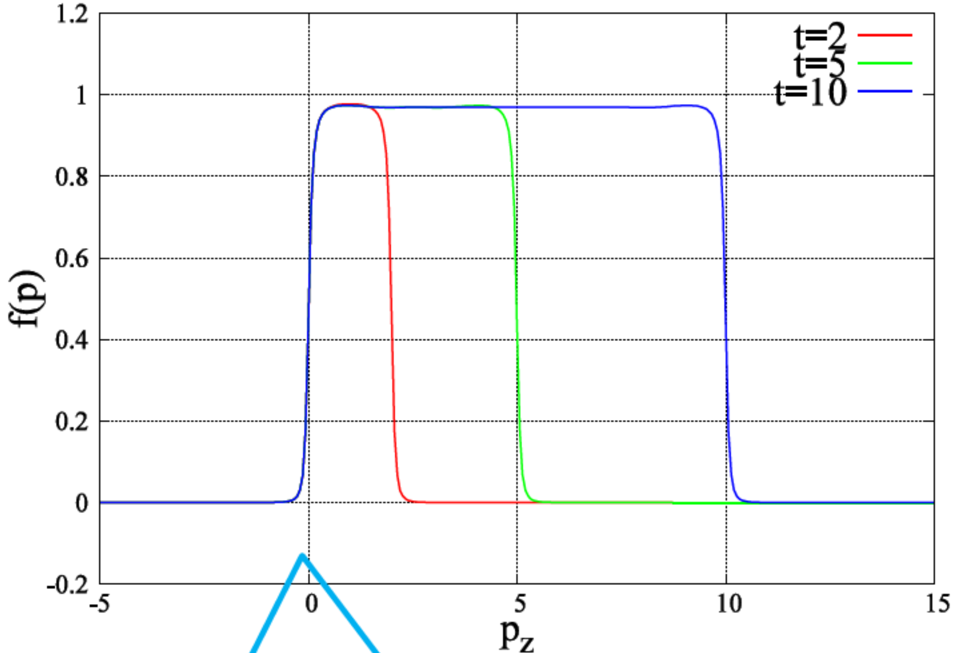
Benchmark --- QED uniform and constant electric field

Schwinger mechanism particle pair production

NT, Ann.Phys.324(2009)

$$eE = 1$$
$$m = 0.1$$

Analytic result



created with approximately
0 longitudinal momentum

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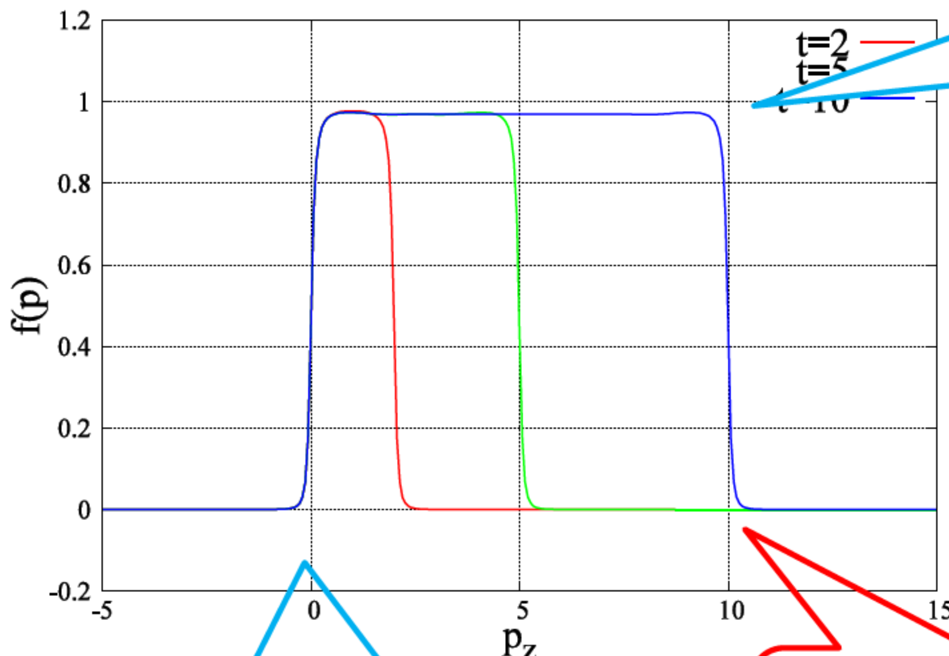


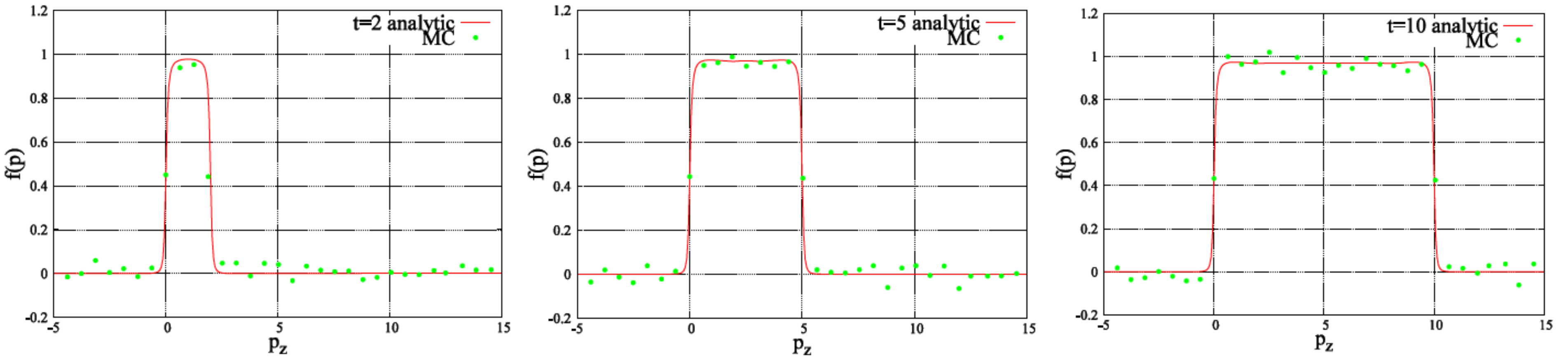
The height is given by
$$\exp\left(-\frac{\pi m_\perp^2}{eE}\right)$$

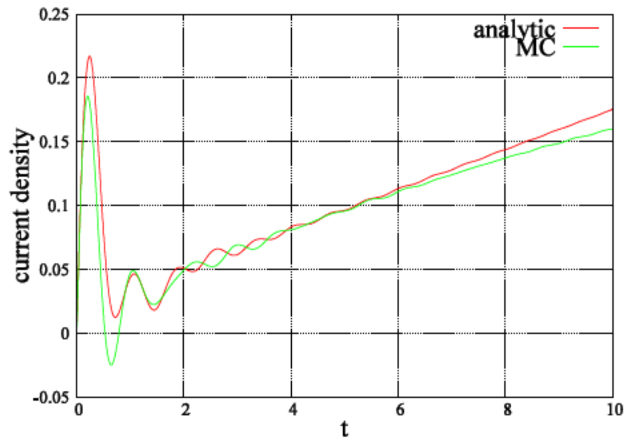
created with approximately
0 longitudinal momentum

accelerated according to
classical eq. of motion
$$p_z = eEt$$

Benchmark --- QED uniform and constant electric field

Comparison between the analytic and MC results





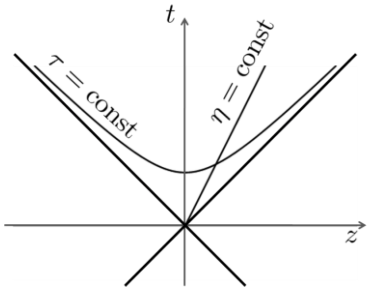
$$N_x = N_y = 32$$
$$N_z = 128$$
$$N_{\text{config}} = 256$$
$$L_x = L_y = L_z = 10.$$

The MC method well reproduces the analytic results.

Boost-invariant expansion

□ QFT in the τ-η coordinate system

ν : momentum conjugate to space-time rapidity η



➢ The relation between the particle mode having the quantum number ν and p_z.

$$a_{\mathbf{p}_\perp,\nu} = \frac{1}{\sqrt{2\pi}} \int \frac{dp_z}{\sqrt{\omega_p}} e^{-i\nu y_p} a_{\mathbf{p}}$$

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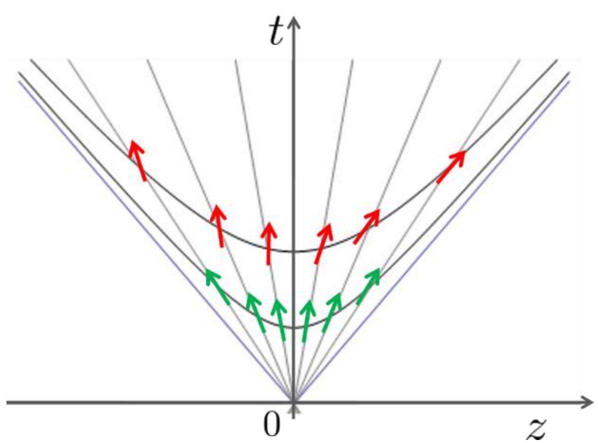
$\dfrac{\nu}{\tau}$: momentum observed in a frame moving with the velocity $v_z = z/t = \tanh \eta$

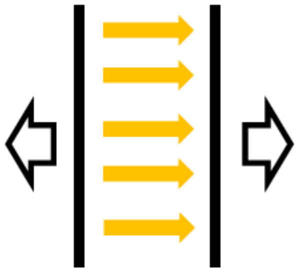


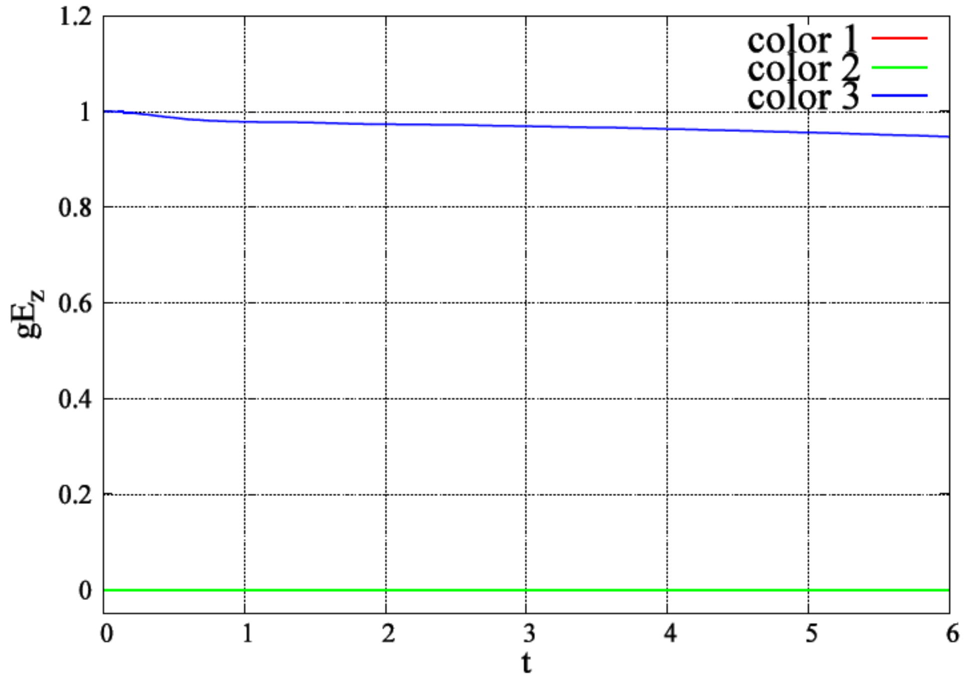
SU(2) expanding gauge fields – uniform

Uniform electric field in the z and the color 3 direction

$$E_i^a(\tau = 0^+, \vec{x}_\perp) = \delta_{i3}\delta^{a3}E_0$$

$$g = 1$$
$$m = 0.1$$
$$N_x = N_y = 32$$
$$N_\eta = 128$$
$$N_{\text{config}} = 128$$
$$L_x = L_y = 15$$
$$L_\eta = 10$$





scaled by gE_0

Time-evolution of the field strength

SU(2) expanding gauge fields – uniform

Uniform electric field in the z and the color 3 direction

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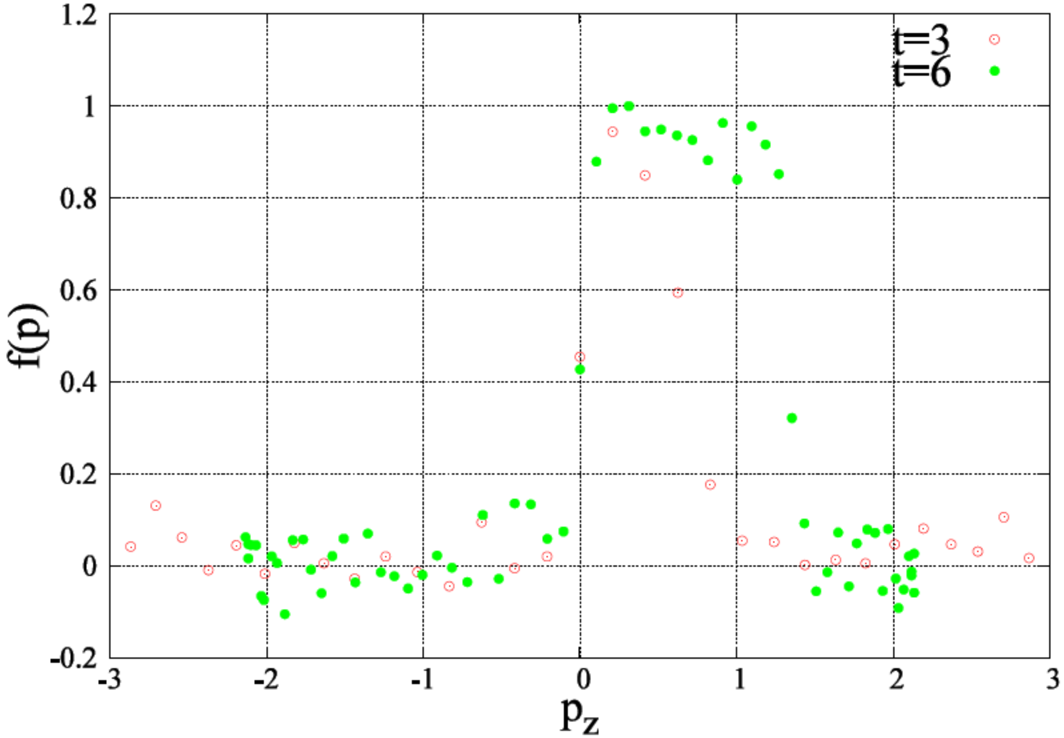

The longitudinal momentum distribution of “blue” quarks

SU(2) expanding gauge fields – uniform

Uniform electric field in the z and the color 3 direction

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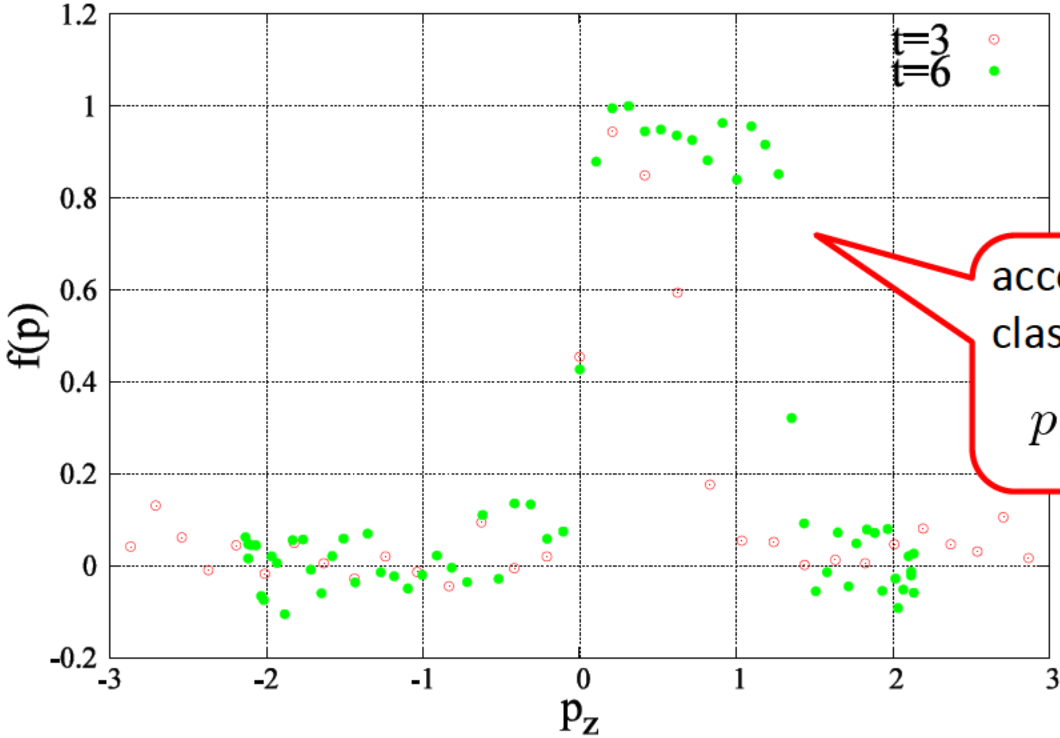

accelerated according to classical eq. of motion

$$p_z = \frac{\lambda}{\tau} = \frac{1}{2}g\alpha E\tau$$

The longitudinal momentum distribution of "blue" quarks

SU(2) expanding gauge fields – uniform

Uniform electric field in the z and the color 3 direction

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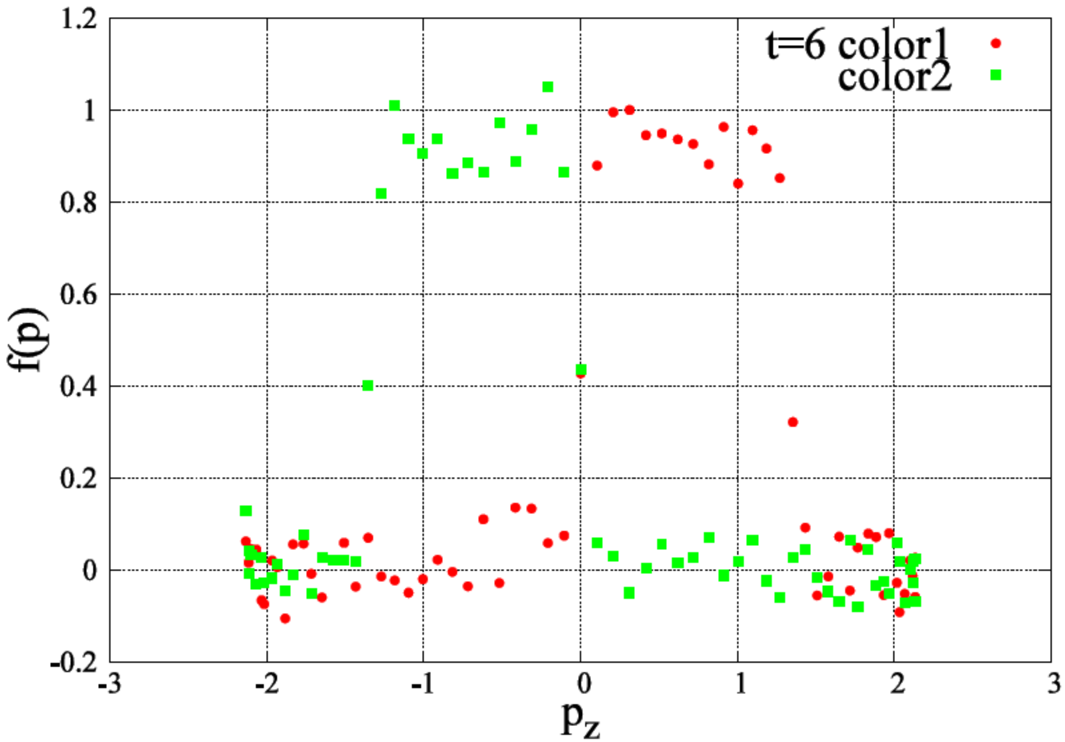


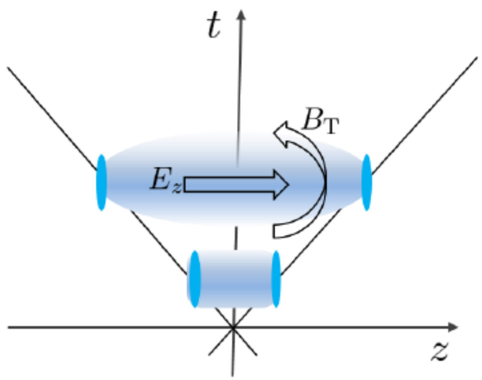
$$T^3 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

The distributions of “blue” and “red” quarks

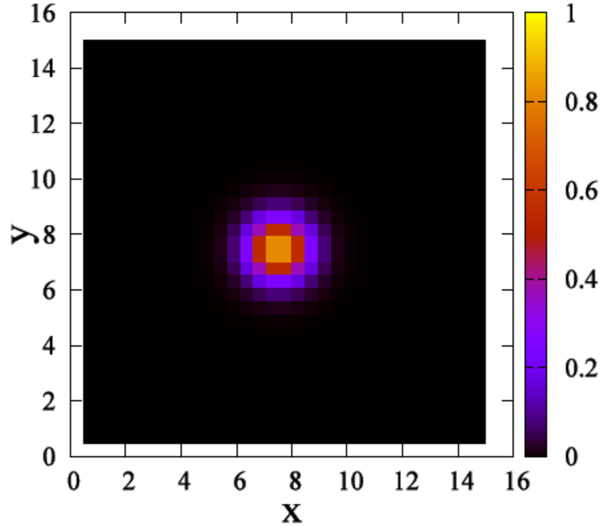
SU(2) expanding gauge fields – a flux tube

$$E_i^a(\tau = 0^+, \vec{x}_\perp) = \delta_{i3}\delta^{a3}E_0(x_\perp)$$

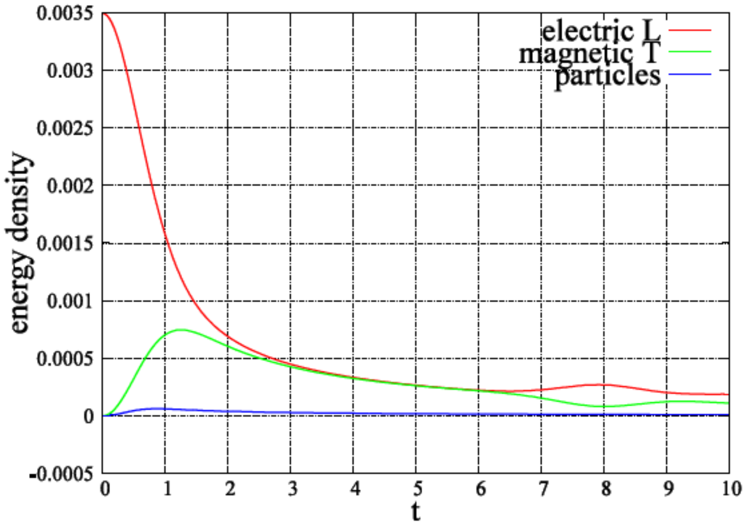
$$E_0(x_\perp) = E_0 e^{-(x_\perp/d)^2}$$



$$d = 1$$



Initial field configuration



The time-evolution of energy density

SU(2) expanding gauge fields – a flux tube

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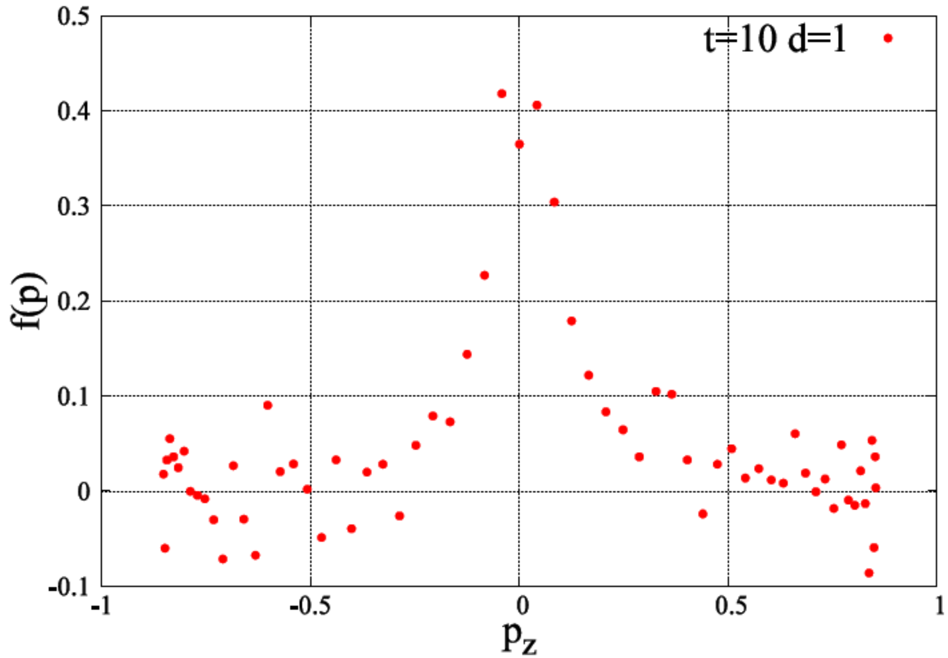
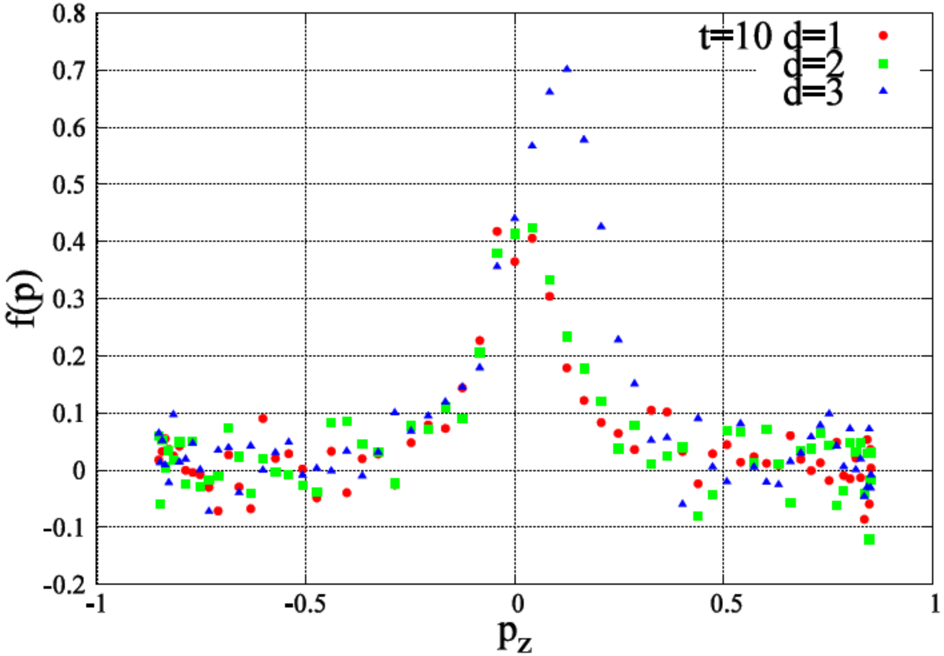
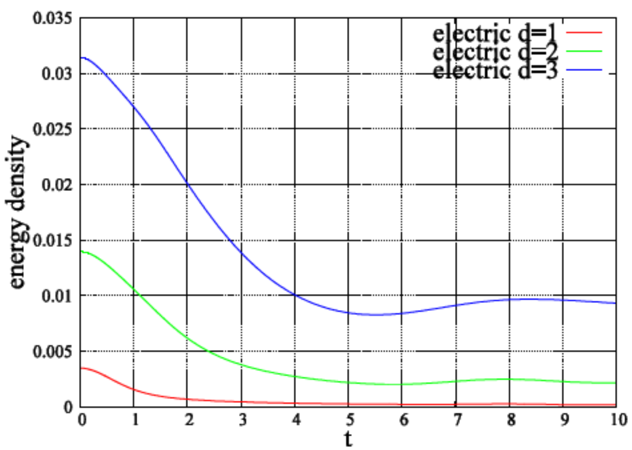


The longitudinal momentum distribution of “blue” quarks

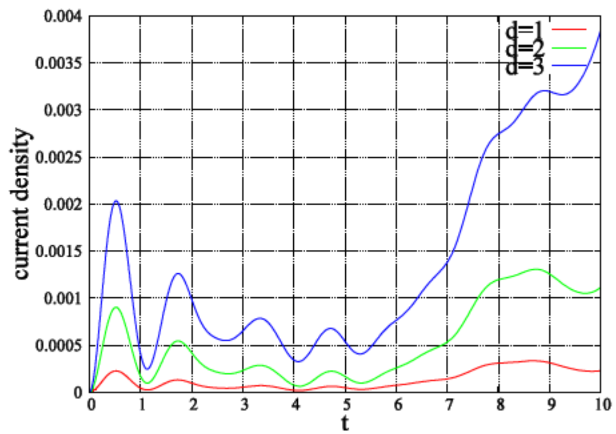
Dependence on the initial tube width



The longitudinal momentum distribution of “blue” quarks

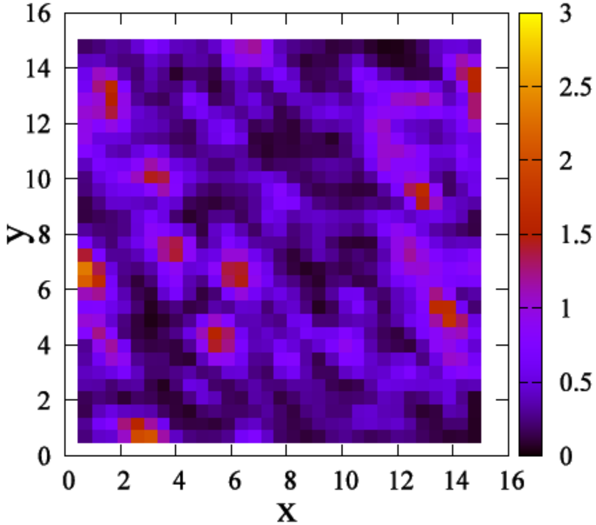


Time-evolution of the field strength

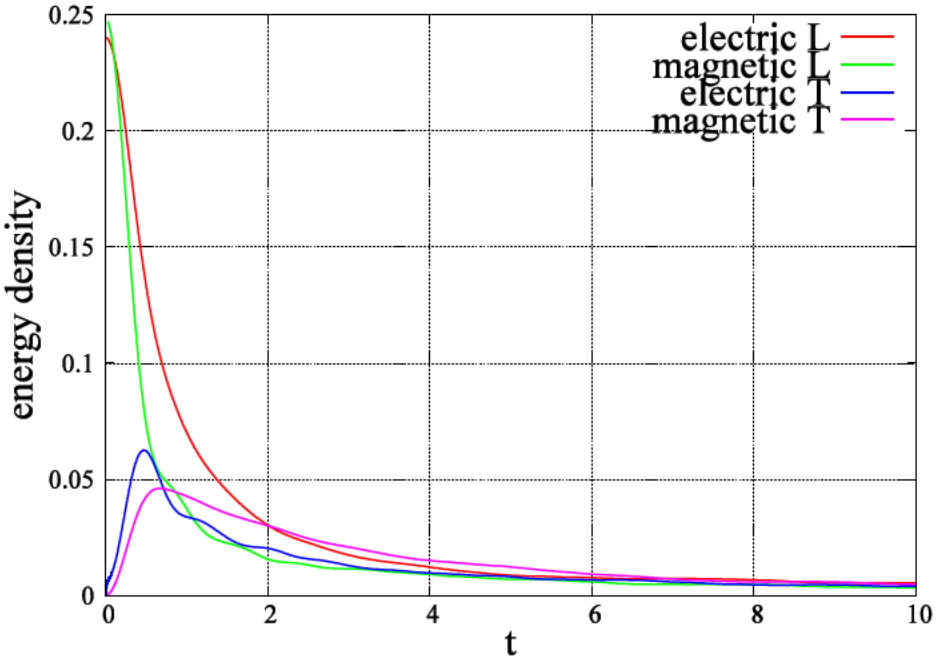


Time-evolution of the current

McLerran-Venugopalan initial condition

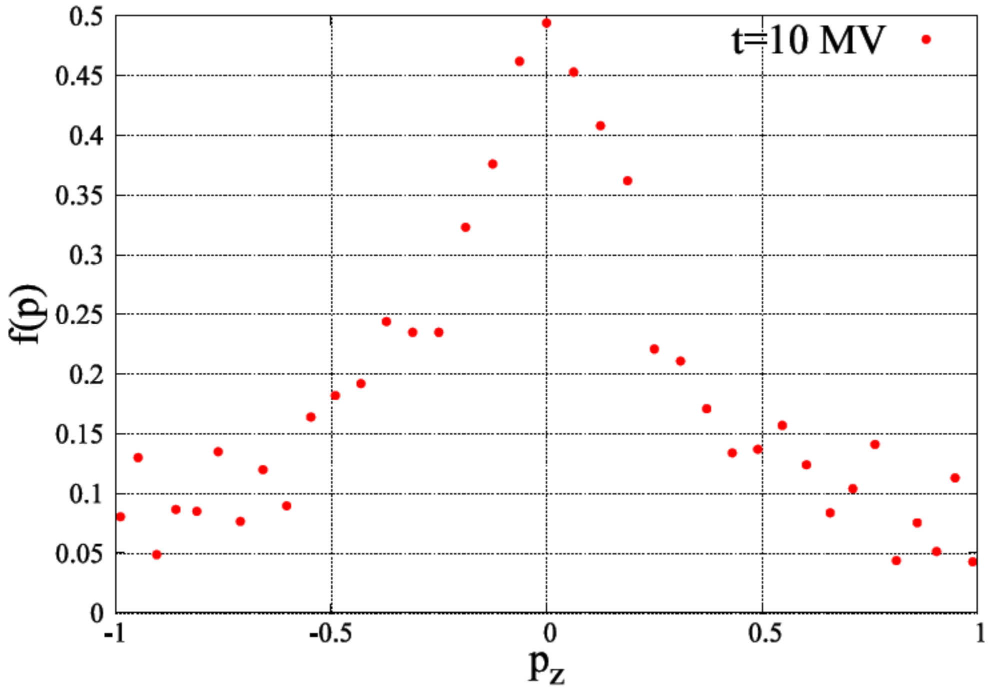


Initial electric field configuration



The time-evolution of energy density

McLerran-Venugopalan initial condition



scaled by $g\mu$

The longitudinal momentum distribution of “blue” quarks

Summary

- Fermion dynamics can be implemented in the classical statistical method.
- The MC method reduces the numerical cost for the computations of fermion production.
- The quark production in expanding gauge fields with the MV initial condition can be computed.