Dr. Marco Ruggieri
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ELLIPTIC FLOW FROM THERMAL AND KLN INITIAL CONDITIONS

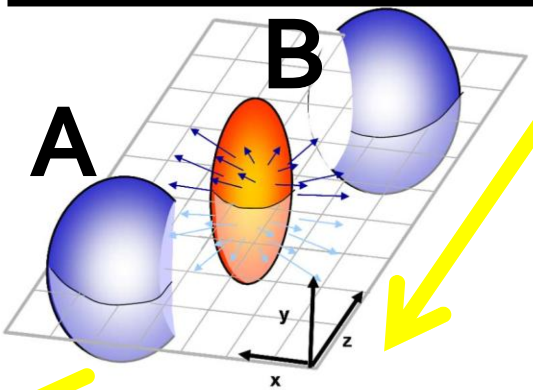
Based on collaboration with:
V. Greco, S. Plumari and F. Scardina

Kyoto, 2013 December 10

In this talk:

- Very short introduction to heavy ion collisions
- Transport theory and heavy ion collisions
- Thermalization
- Elliptic flow computation
- Conclusions and Outlook

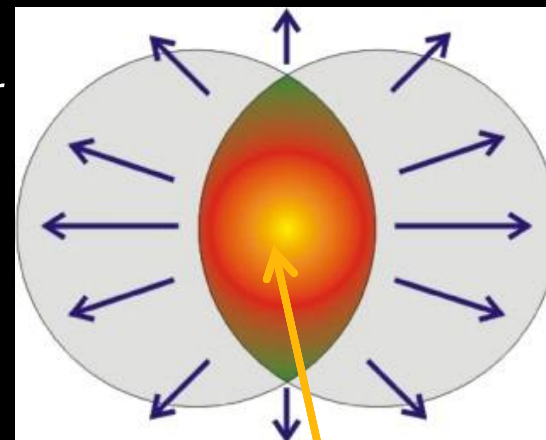
QGP in Heavy Ion Collisions



Impact parameter direction

Collision (flight) direction

Impact parameter direction



Collision direction

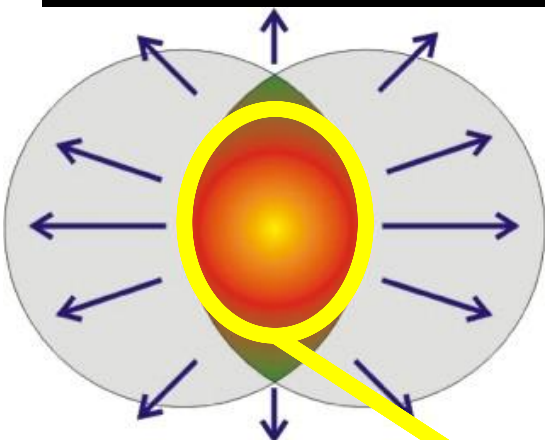
A,B: Cu, Au (RHIC@BNL)
Pb (LHC@CERN).

$$\sqrt{s}\ \text{up to}\ 200 \times A\ \text{GeV}\ , \qquad \text{RHIC}$$
$$\sqrt{s}\ \text{up to}\ 2.76 \times A\ \text{TeV}\ , \qquad \text{LHC}$$

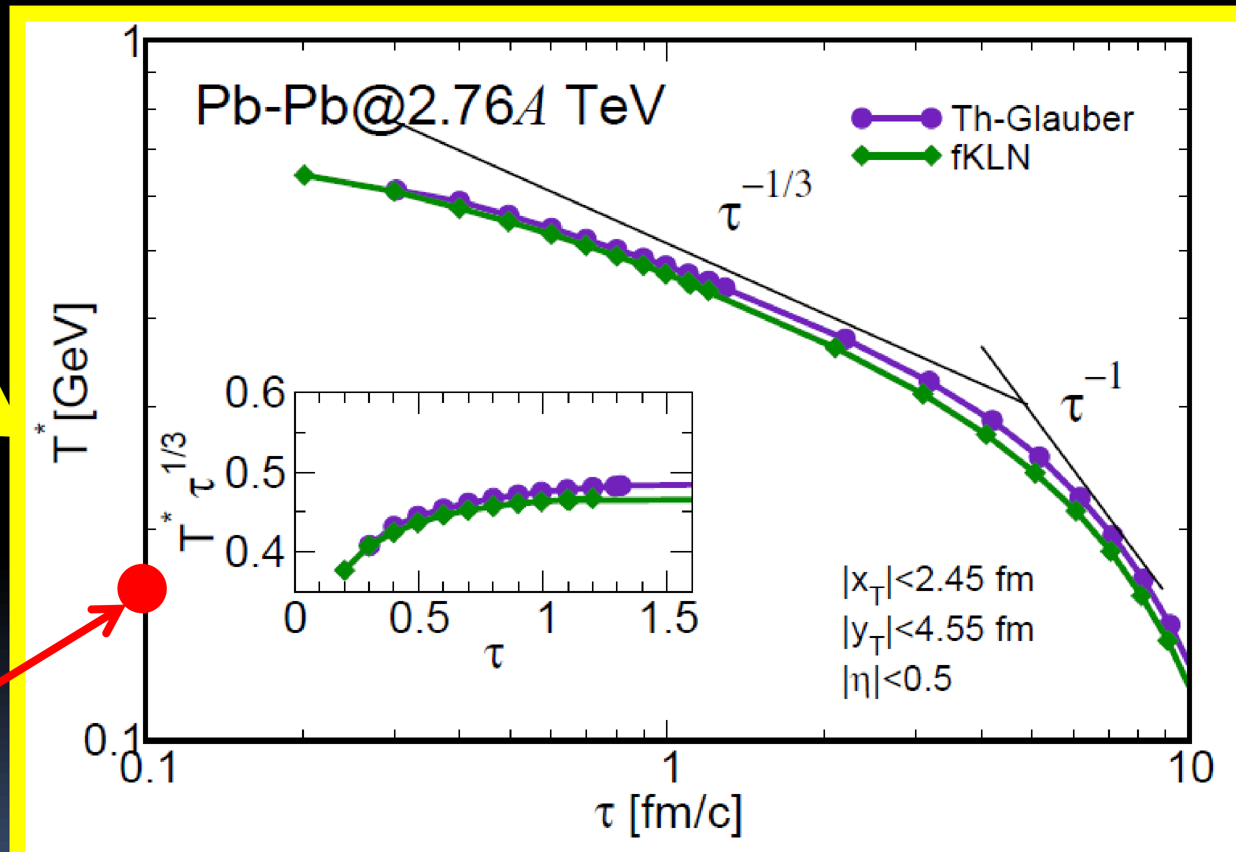
<u>FIREBALL:</u>
Hot and dense expanding
parton mixture:
QUARK-GLUON-PLASMA (QGP)
T about 10^{12} K,
t about 10^{-23} seconds

M. R. *et al.*, in preparation

QGP in Heavy Ion Collisions



Inner core temperature (simulation)



QCD-Tc

Initial temperature much larger than QCD critical temperature:

Description in terms of partons is appropriate.

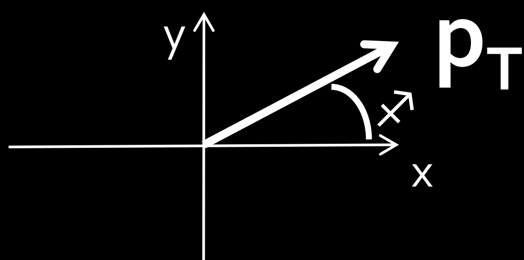
J. Y. Ollitraut, PRD46 (1992)

Elliptic flow

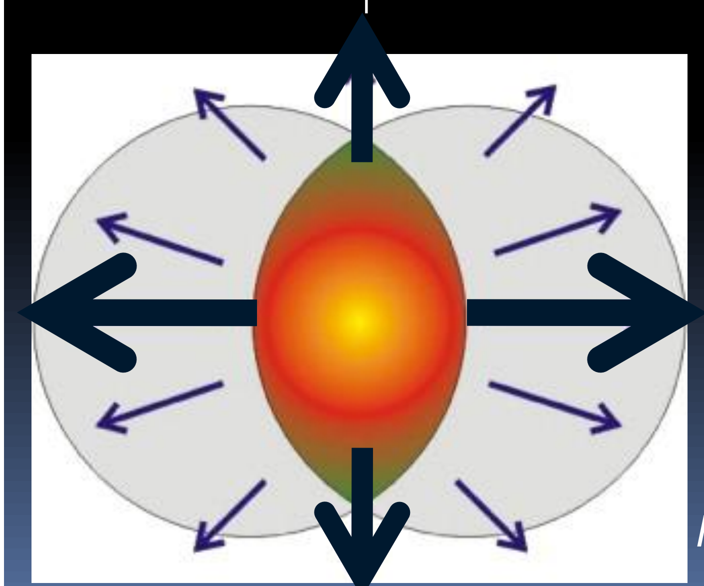
Particle multiplicity in momentum space

$$\frac{d^3N}{dy\,p_T\,dp_T\,d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy\,p_T\,dp_T}\left[1 + 2v_2(y,p_T)\cos 2\phi\right]$$

Elliptic flow:
leading contribution to anisotropy in momentum space



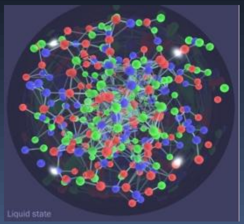
$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$



Immediately after the collision, ***pressure gradient*** *along* ***X*** *is larger than that along* ***Y****.*
As a consequence, ***the medium expands preferentially along the short axis of the ellipse,*** *creating a* ***flow.***

Collision direction

Impact parameter direction



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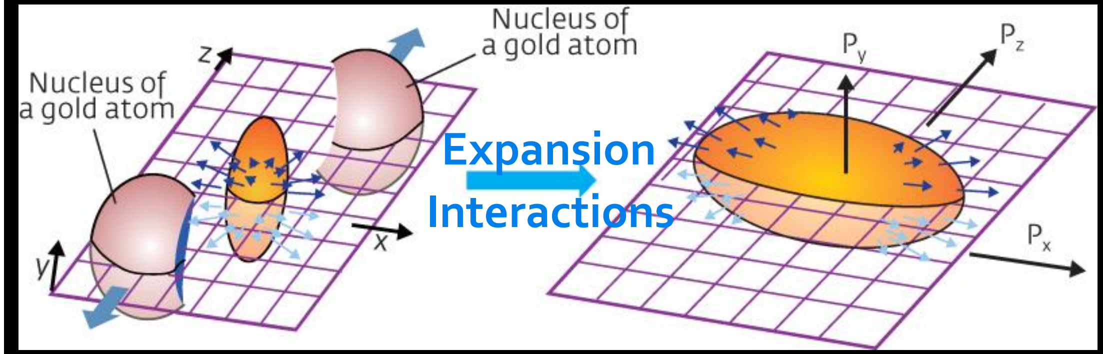
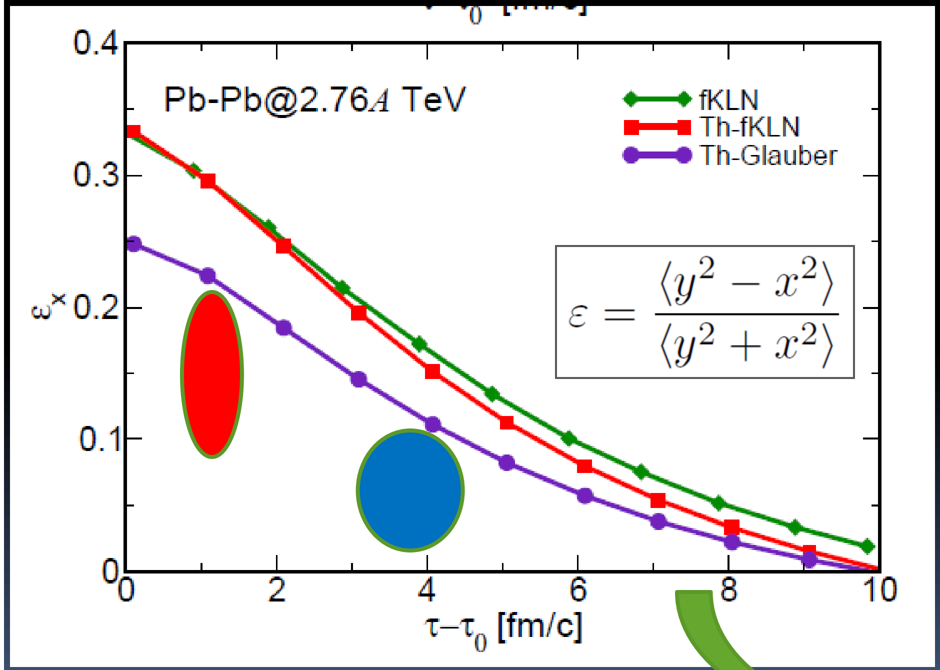
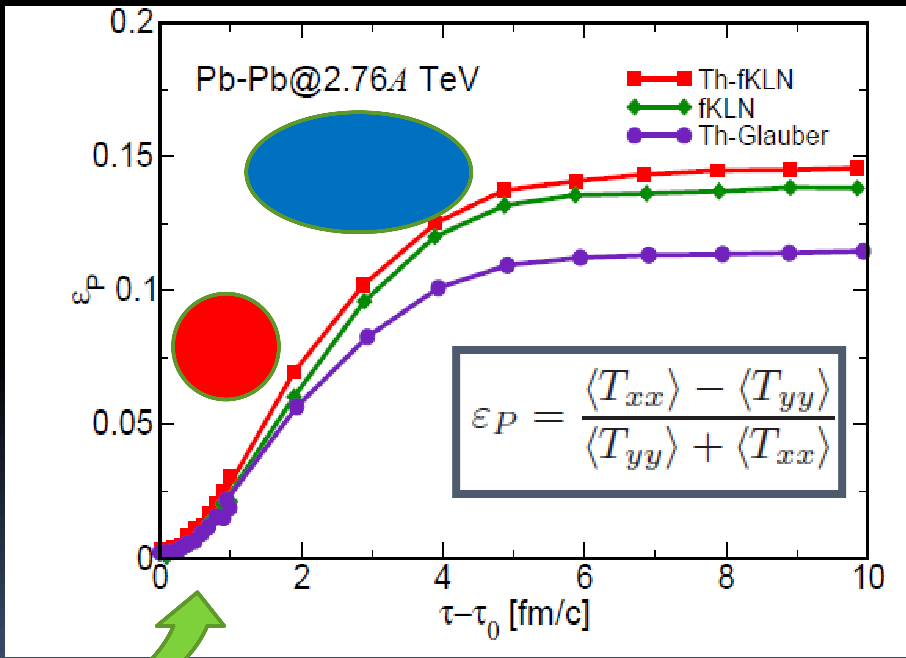






M. R. *et al.*, in preparation

Transfer of anisotropy

Boltzmann equation and QGP

In order to ***simulate*** the temporal evolution of the fireball we solve the ***Boltzmann equation*** for the parton distribution function $\boldsymbol{f}$:

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right) f(\boldsymbol{x},\boldsymbol{p},t) = C[f]$$

L. Boltzmann, 1872

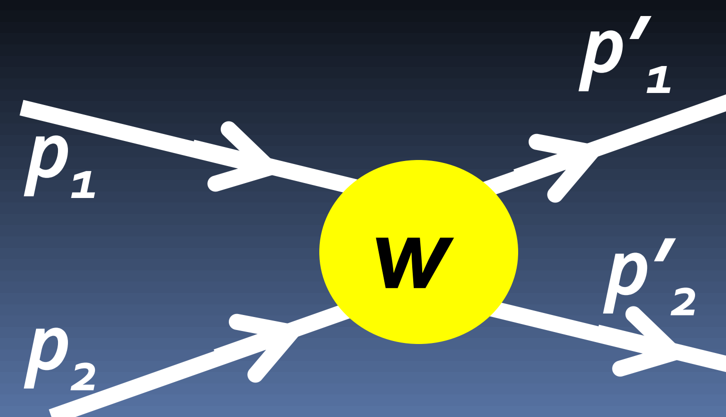
Drift term ***Collision integral***

Drift term: change of $\boldsymbol{f}$ due to particles flowing into and out of the phase space volume centered at ($\boldsymbol{x}$,$\boldsymbol{p}$).

Collision integral: change of $\boldsymbol{f}$ due to collision processes in the phase space volume centered at ($\boldsymbol{x}$,$\boldsymbol{p}$).

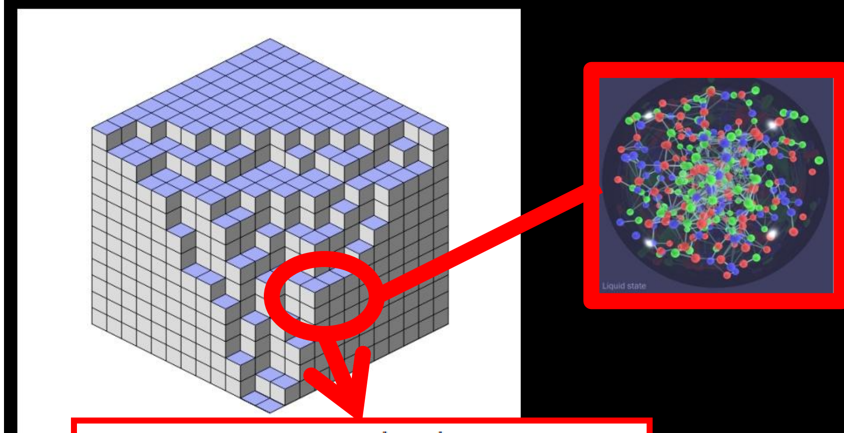
For the case of 12->1’2’ processes:

$$\begin{aligned} C[f] &= \frac{1}{2}\int d\boldsymbol{p}_2 \int d\boldsymbol{p}'_1 \int d\boldsymbol{p}'_2 \; w(12\to 1'2') \\ &\quad \times [f(\boldsymbol{x},\boldsymbol{p}'_1,t)f(\boldsymbol{x},\boldsymbol{p}'_2,t) - f(\boldsymbol{x},\boldsymbol{p}_1,t)f(\boldsymbol{x},\boldsymbol{p}_2,t)] \end{aligned}$$



eta/s: hydro “by” transport

We use ***Boltzmann equation*** to simulate a fluid at ***fixed eta/s***.
Total Cross section is ***computed*** in ***each configuration space cell*** according to ***Chapman-Enskog equation*** to give the ***wished value of eta/s***.



$$\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)\rho}\frac{1}{\sigma}$$

Plumari *et al.*, Phys. Rev. C86 (2012).
Greco *et al.*, Phys. Lett. B670 (2009).
Plumari *et al.*, J.Phys.Conf.Ser. 420 (2013).

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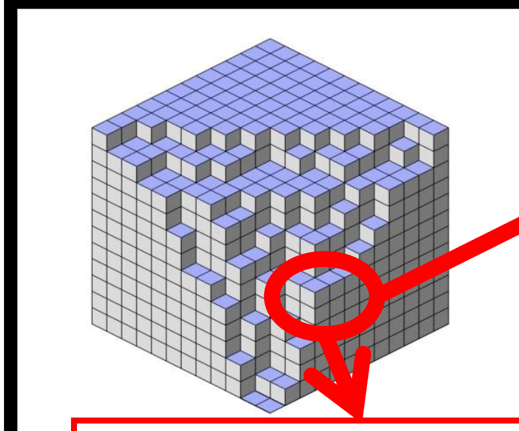

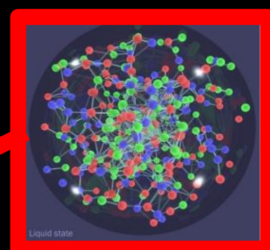

$$\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)\rho} \frac{1}{\sigma}$$

(.) Collision integral is gauged in each cell to assure that the fluid dissipates according to the desired value of eta/s.

(.) Microscopic details are not important: the specific microscopic process producing eta/s is not relevant, only macroscopic quantities are, in analogy with hydrodynamics.

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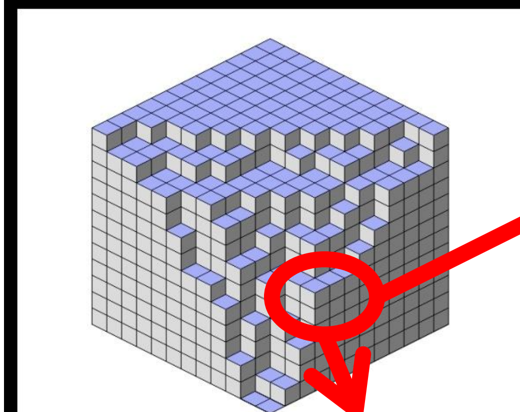

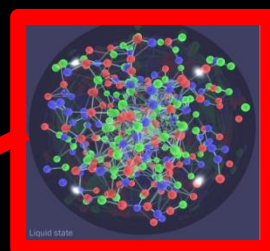

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Transport ⟷ ***“bridge”*** ⟷ ***Hydro***

Description in terms of parton distribution function

Dynamical evolution governed by macroscopic quantities

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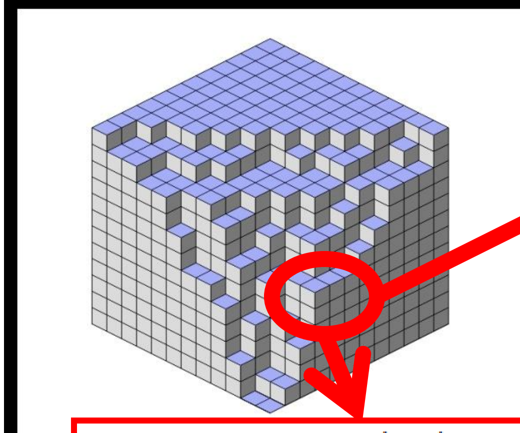

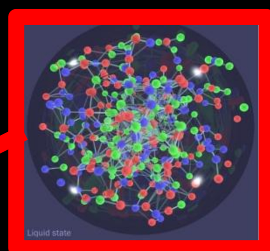

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Transport $\longleftrightarrow$ ***"bridge"*** ***Hydro***

Non perturbative description: *we never assume coupling is small.*

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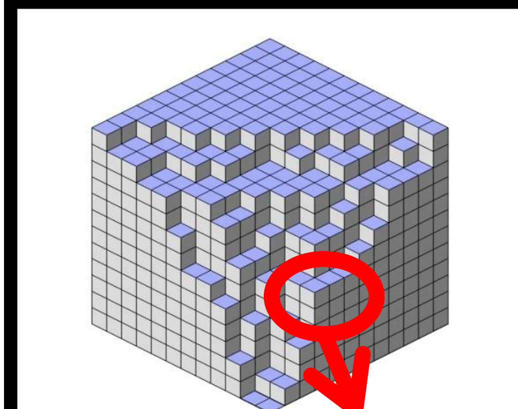
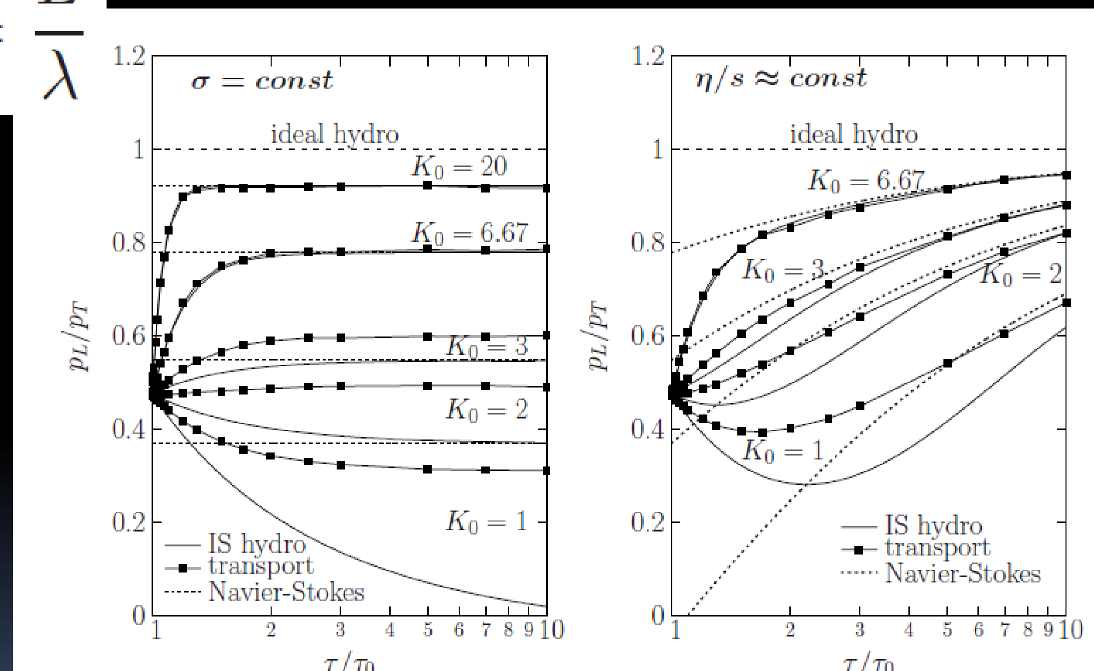


$$K_0 = \frac{L}{\lambda}$$

$$\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)\rho} \frac{1}{\sigma}$$



Huovinen and Molnar, PRC79 (2009)

There is agreement of hydro with transport also in the non dilute limit

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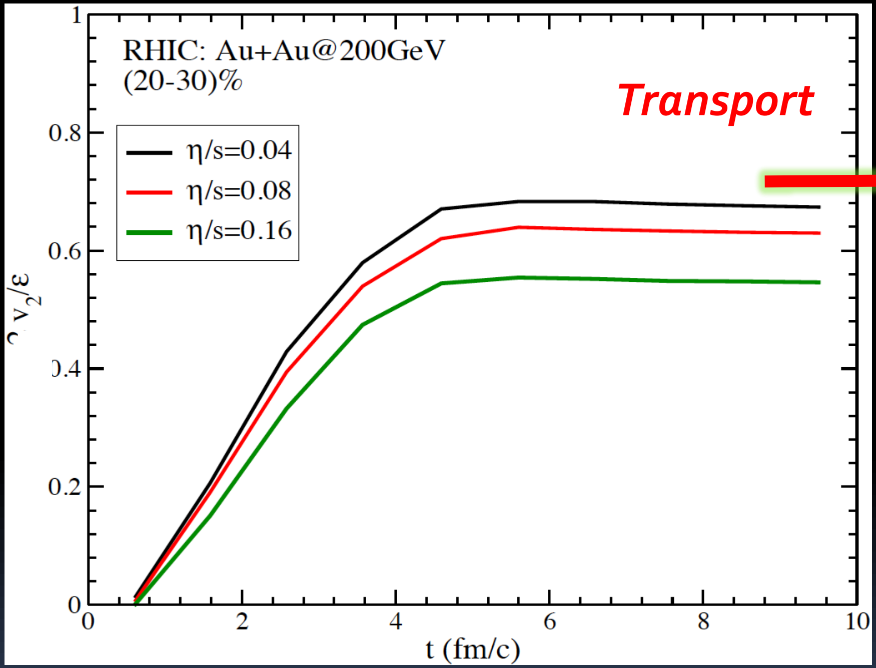

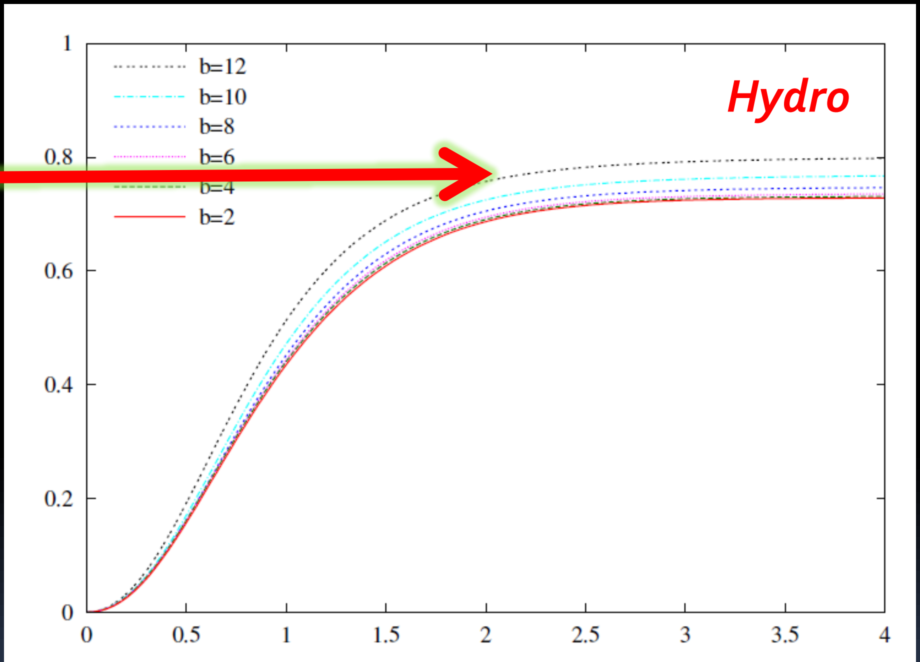

Bhalerao *et al.*, PLB627 (2005)

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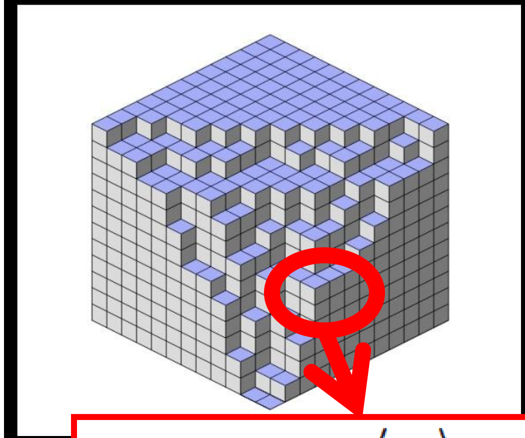

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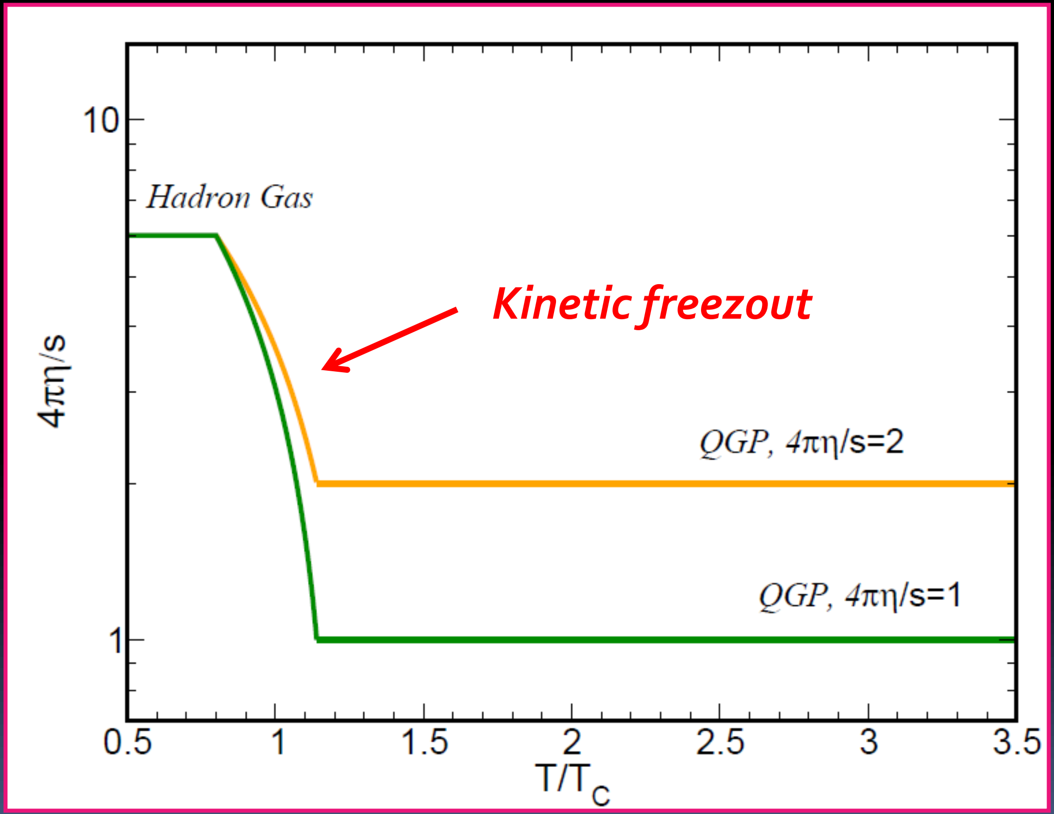
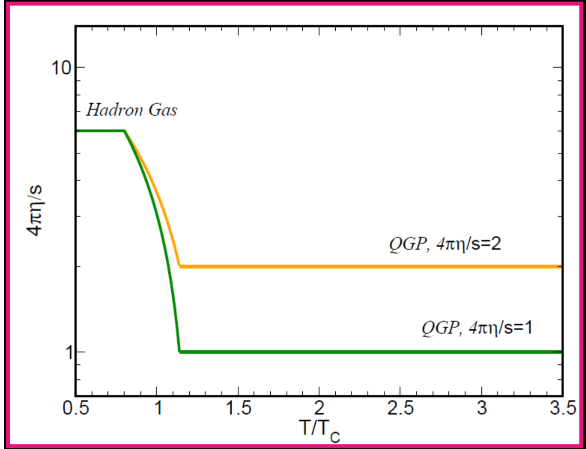
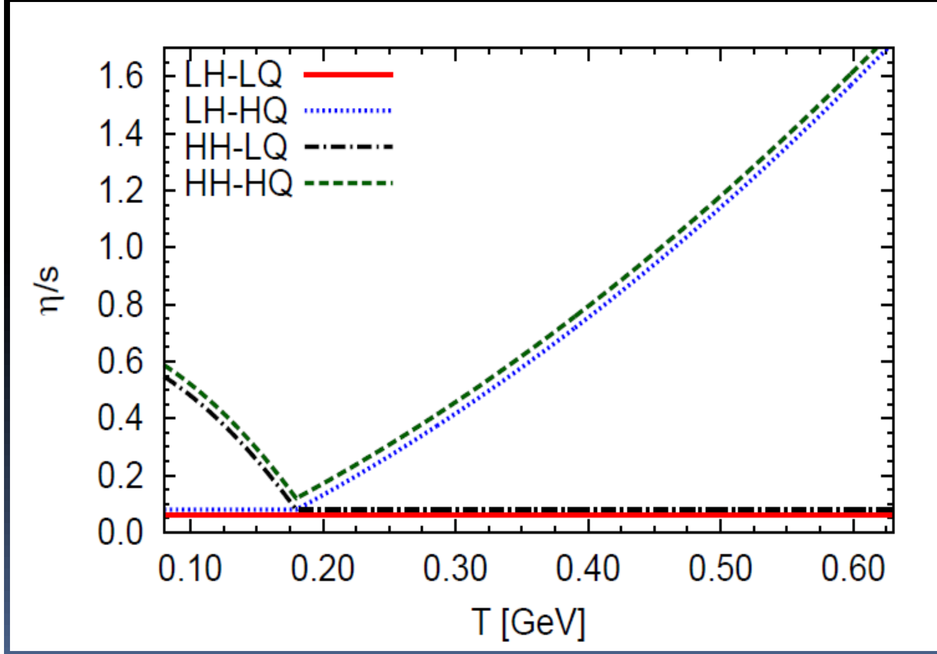


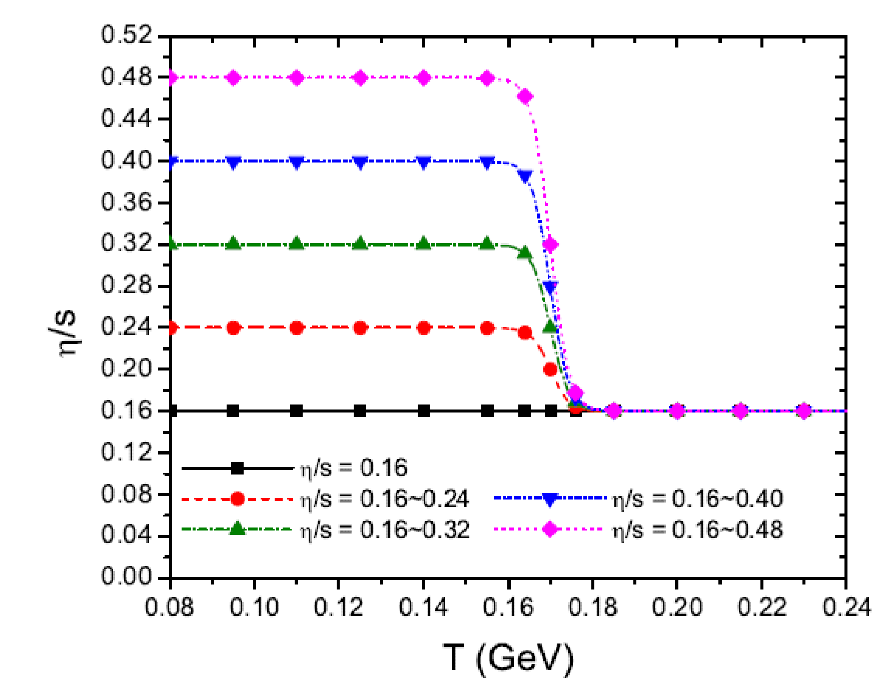
A ***smooth kinetic freezout*** is implemented in order to gradually reduce the strength of the interactions as the temperature decreases below the critical temperature.

eta/s: hydro "by" transport



Temperature dependence of eta/s already appeared in the literature recently.



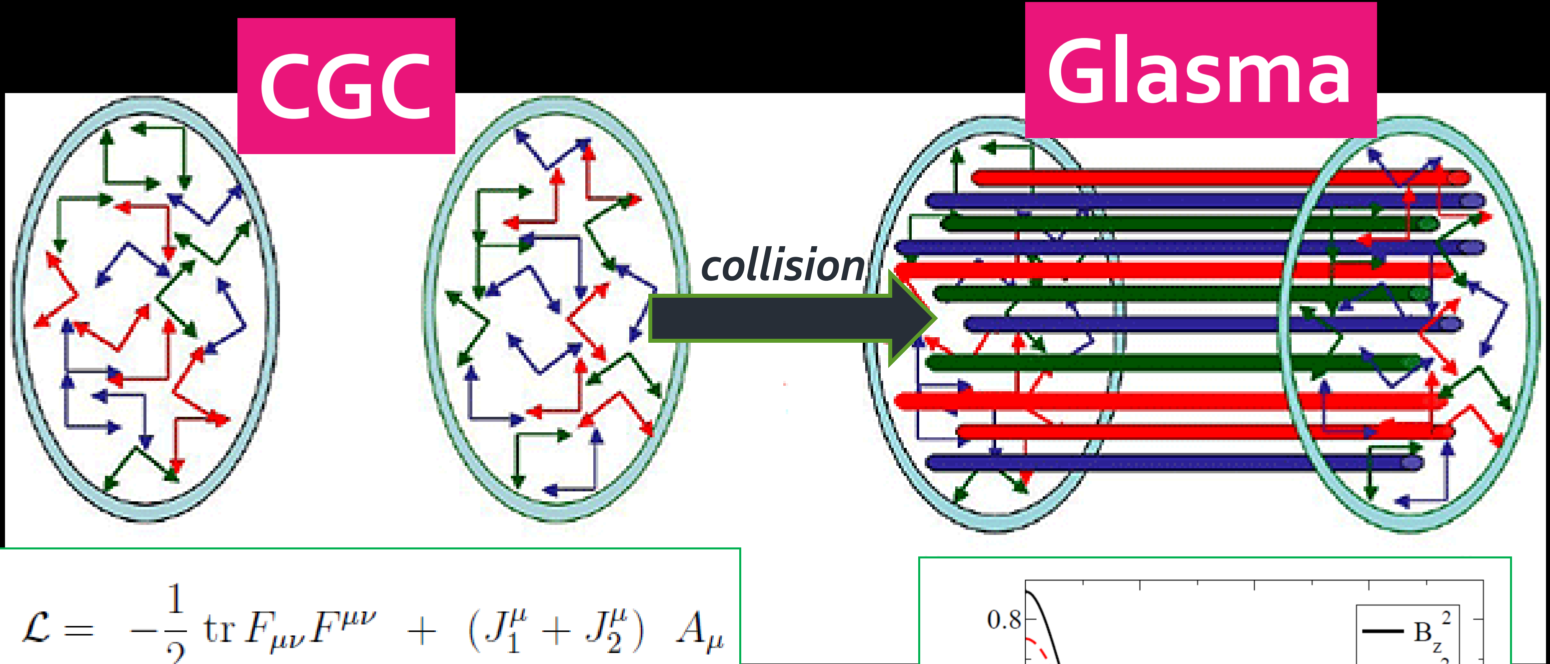


H. Niemi *et al.*, PRC86 (2012), PRL106 (2011)

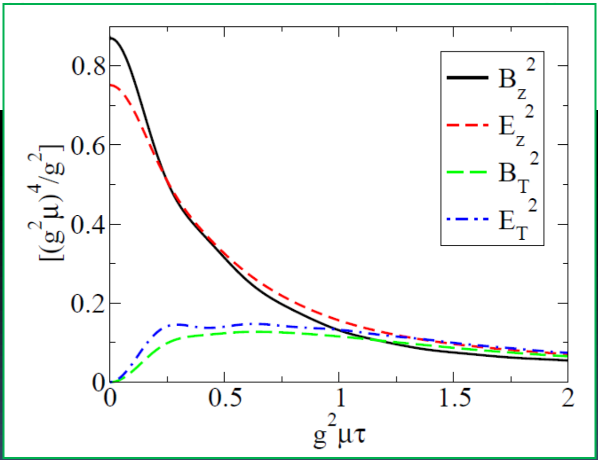
Shen and Heinz, PRC83 (2011)

McLerran and Venugopalan, PRD **49**, 2233 (1994)
McLerran and Venugopalan, PRD **49**, 3352 (1994)

Initial condition: Glasma



$$\mathcal{L} = \underbrace{-\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu}}_{\text{gluon dynamics}} + \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}}\, A_\mu$$



Lappi and McLerran, Nucl. Phys. **A772** (2006)

Reviews/Lectures

McLerran, 2011
Iancu, 2009
McLerran, 2009
Lappi, 2010
Gelis, 2010
Fukushima, 2011

Decay of flux tubes to parton liquid should occur on a timescale 1/Qs

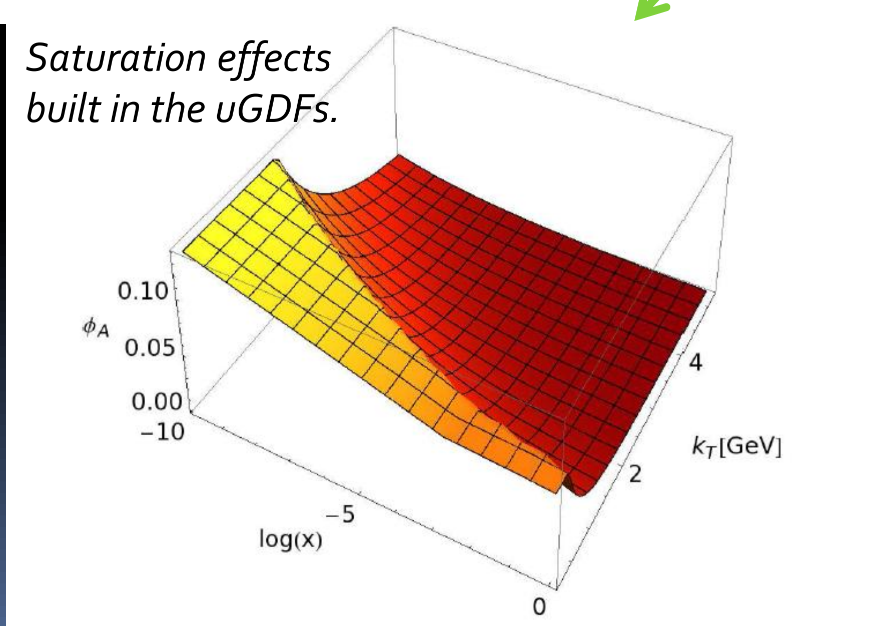
Initial condition: fKLN

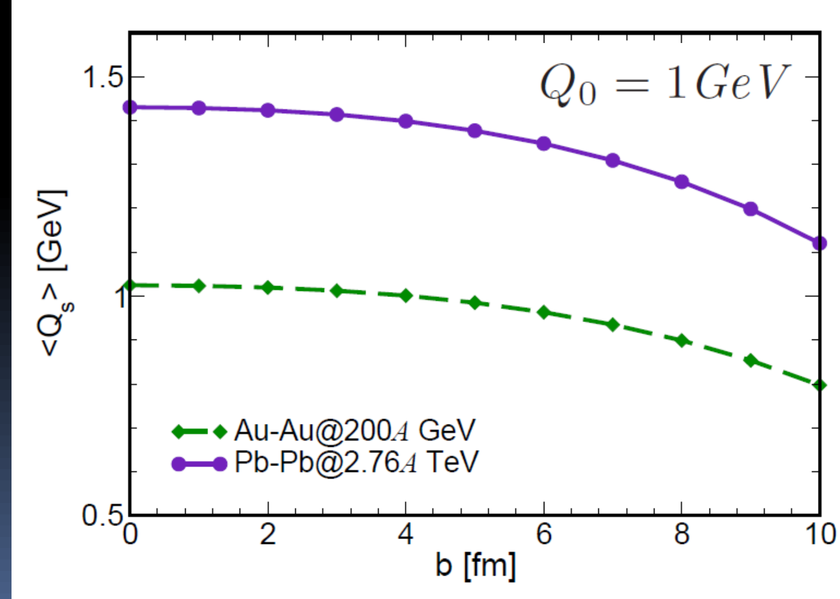
Nardi *et al.*, Nucl. Phys. A**747**, 609 (2005)
Kharzeev *et al.*, Phys. Lett. B**561**, 93 (2003)
Nardi *et al.*, Phys. Lett. B**507**, 121 (2001)
Drescher and Nara, PRC**75**, 034905 (2007)
Hirano and Nara, PRC**79**, 064904 (2009)
Hirano and Nara, Nucl. Phys. A**743**, 305 (2004)
Albacete and Dumitru, arXiv:1011.5161[hep-ph]

(f)KLN spectrum

$$\frac{dN_g}{d^2x_\perp dy} \propto \int \frac{d^2p_T}{p_T^2}\int_0^{p_T} d^2k_T \alpha_s(Q^2) \times\phi_A\left(x_A, \frac{(p_T+k_T)^2}{4};\boldsymbol{x}_\perp\right) \times\phi_B\left(x_B, \frac{(p_T-k_T)^2}{4};\boldsymbol{x}_\perp\right)$$

$$Q^2_{s,A}(x,\boldsymbol{x}_\perp) = Q_0^2\left(\frac{T_A(\boldsymbol{x}_\perp)}{1.53p_A(\boldsymbol{x}_\perp)}\right)\left(\frac{0.01}{x}\right)^\lambda$$





For Pb-Pb collision average Qs can be larger [Lappi, EPJC71 (2011)]

Few remarks on KLN

- fKLN is not glasma [Blaizot et al., NPA846 (2010)]
- It is not our purpose to insist on exact reproduction of experimental data

[Gale et al., PRL110 (2013)]

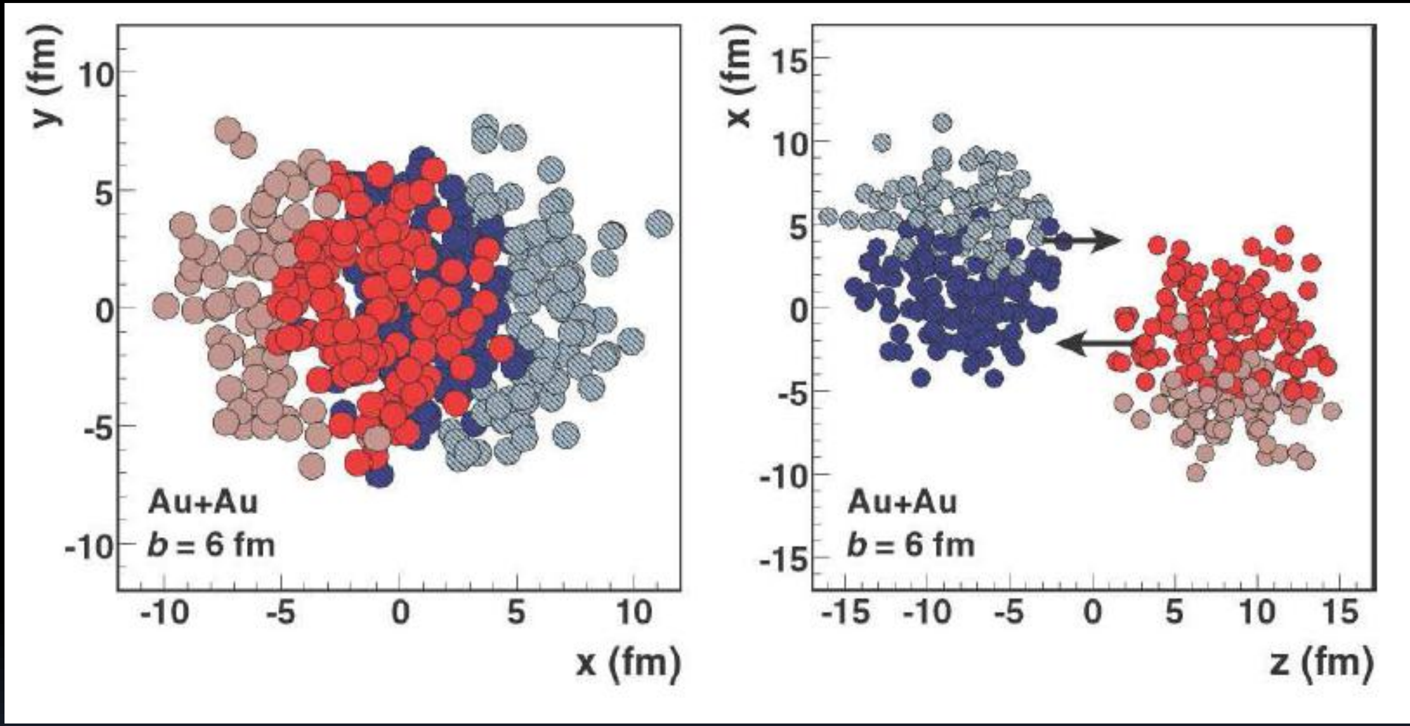
Rather we want to check the role of the initial distribution in momentum space

- Hydro widely uses KLN, and we are interested to compare the two approaches

Viscometer: Schen et al., arXiv1308:2111
Thermometer: Schen et al., arXiv1308:2440
Flow computations: Ollitrault et al., arXiv1311:5339
Drescher and Nara, PRC**75** (2007)
Hirano and Nara, PRC**79** (2009)
Hirano and Nara, NPA**743** (2004)

Initial condition: Th-Glauber

(Almost) Geometrical description of the fireball:

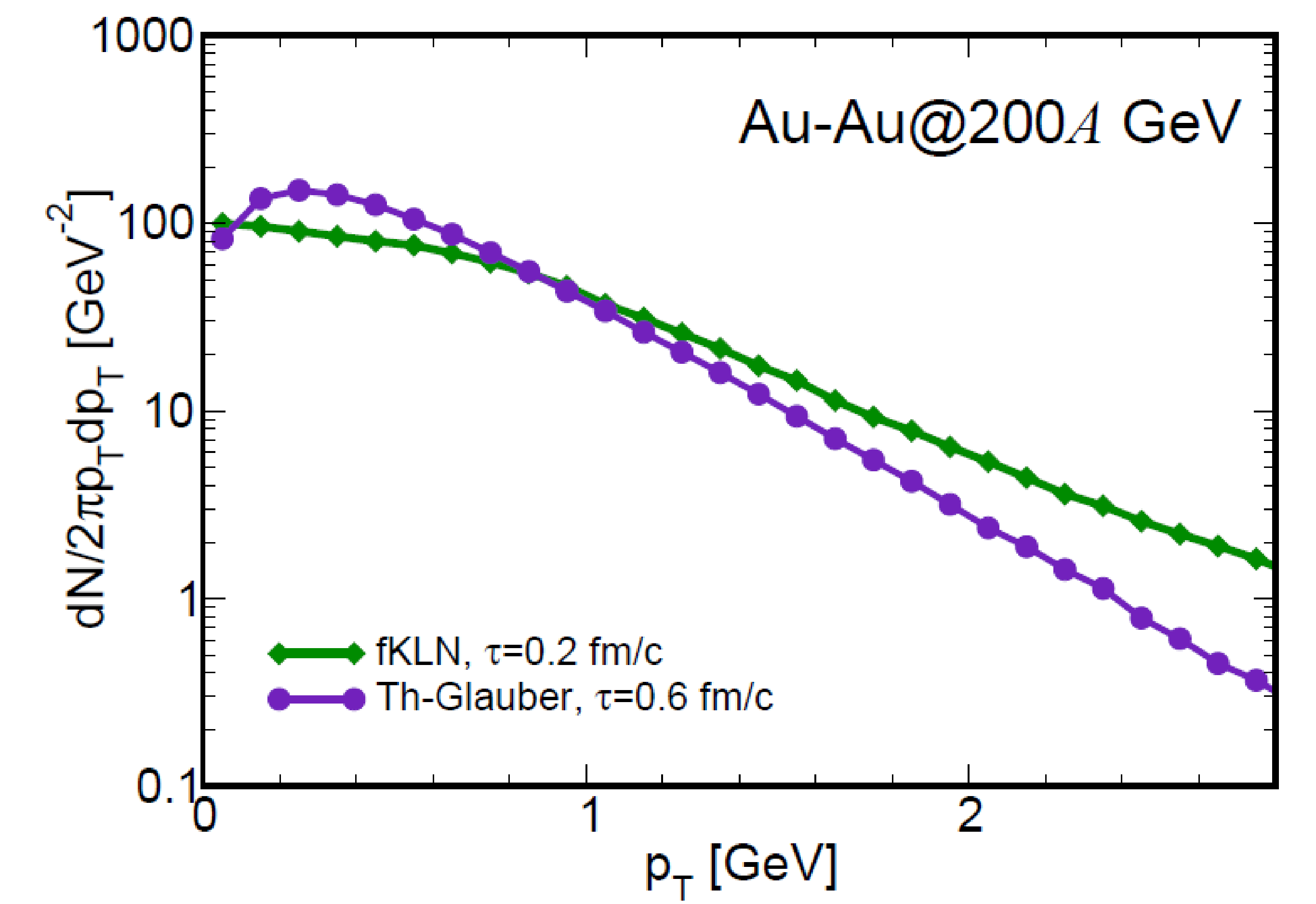


Assuming a nucleon distribution in the parents nuclei (typically a ***Woods-Saxon***), one counts ***how many particles*** from each nucleus are present in the ***overlap region***; among them, the ***participants*** are the nucleons that effectively can have an interaction (in fact, the particles that *are in the overlap region* but *do not interact*, are not considered).

For a review see: Miller *et al.*, Ann.Rev.Nucl.Part.Sci. **57, 205 (2007)**

M. R. *et al.*, PLB727 (2013)
M. R. *et al.*, in preparation

Initial spectra



Our novelty:

For fKLN we consider the ***initial spectrum given by the theory at small transverse momenta.***

M. R. *et al.*, PLB727 (2013)
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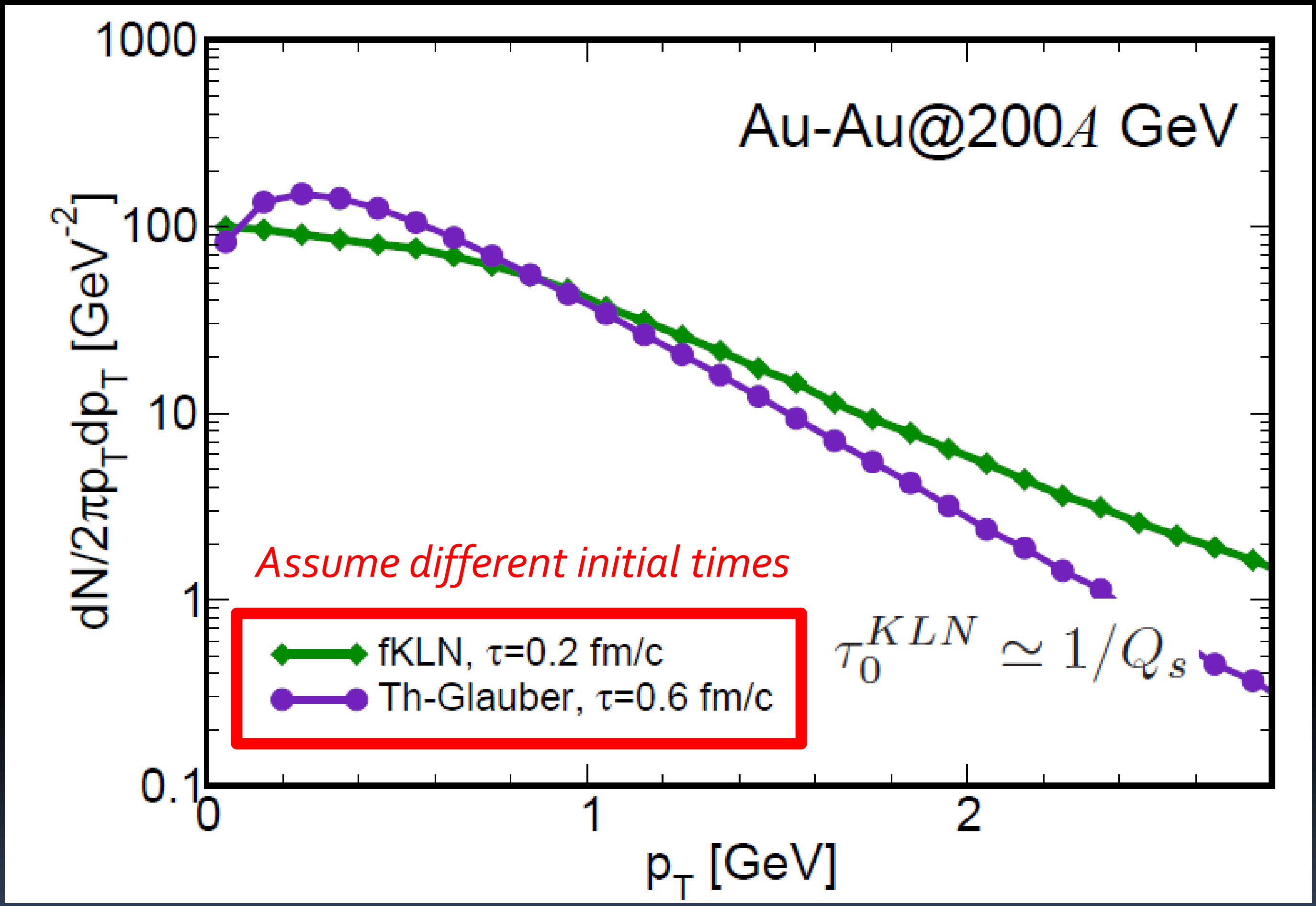

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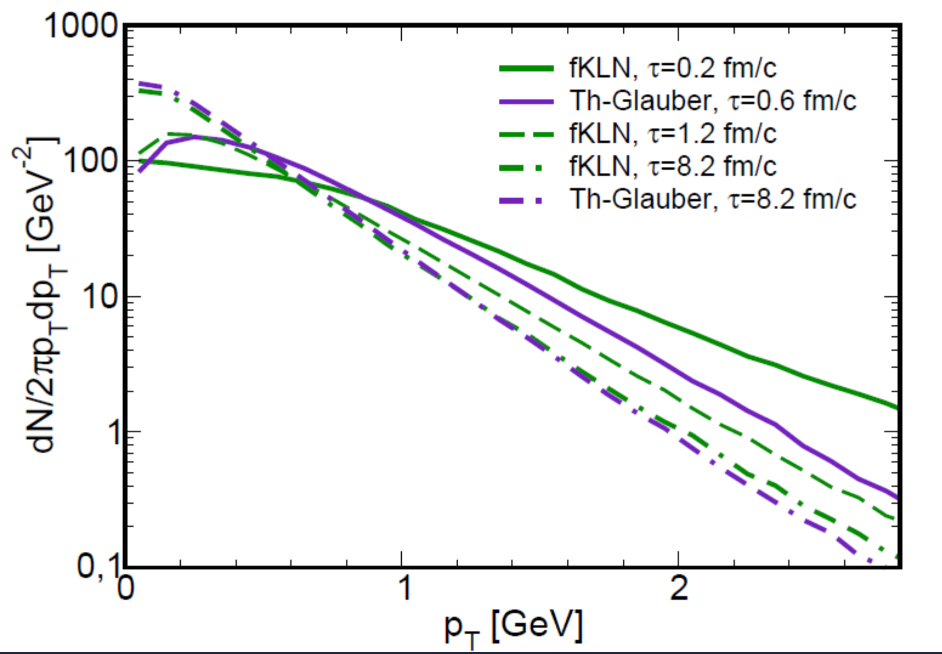
Thermalization

AuAu@200A GeV



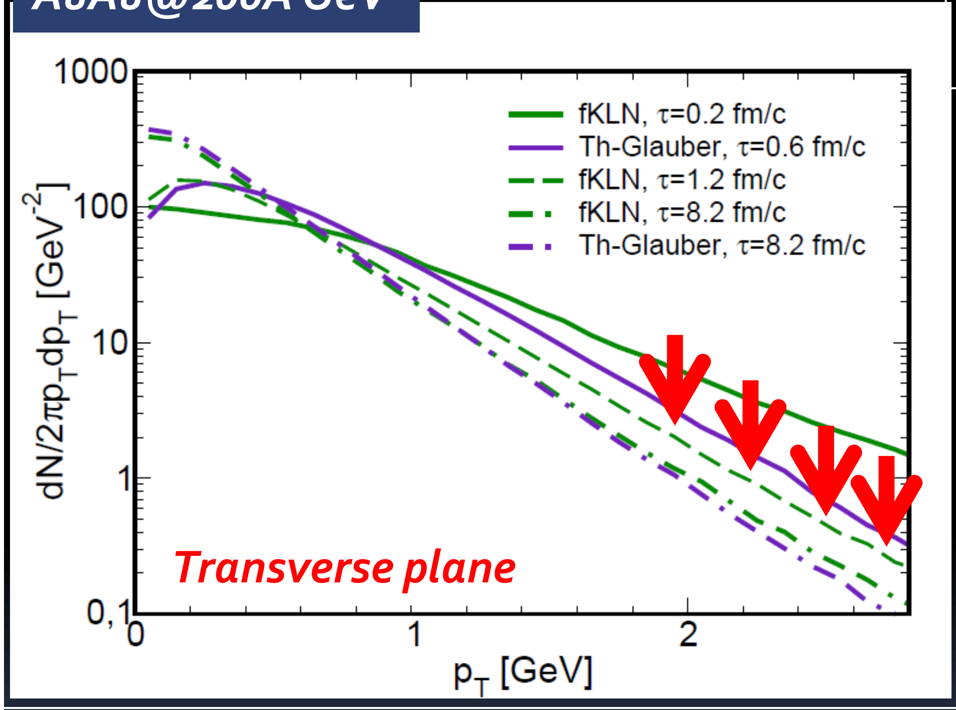
Final spectra of fKLN and Th-Glauber coincide

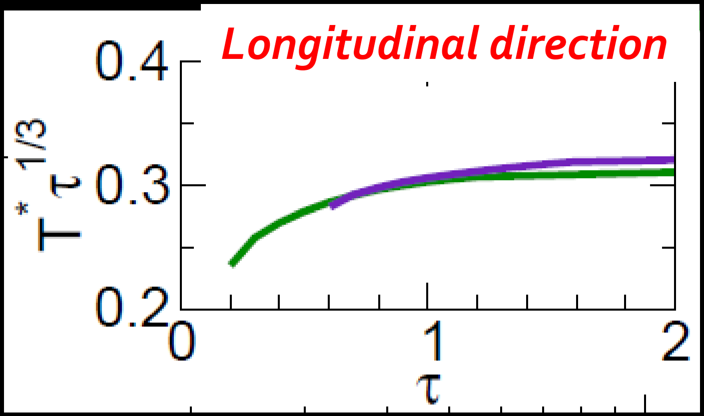
M. R. *et al.*, PLB727 (2013)
M. R. *et al.*, in preparation

Thermalization

Thermalization in less than 1 fm/c,
in agreement with:
Greiner *et al.*, Nucl. Phys. A**806**, 287 (2008).

AuAu@200A GeV





$$\sigma_{tot} = \frac{\langle p \rangle}{\rho\, g(a)} \frac{1}{\eta/s}$$

Not so surprising:
Because eta/s is small, large cross sections naturally lead to fast thermalization.
However, interesting:
We have dynamics in the early stages of the simulation, which prepares the momentum distribution to build up the elliptic flow.

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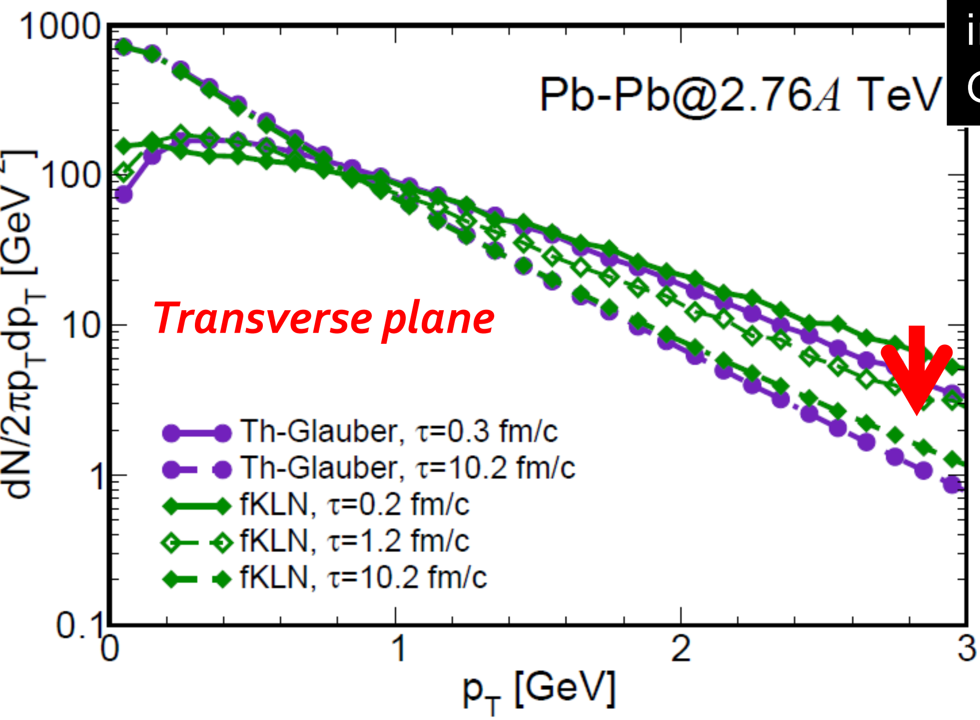

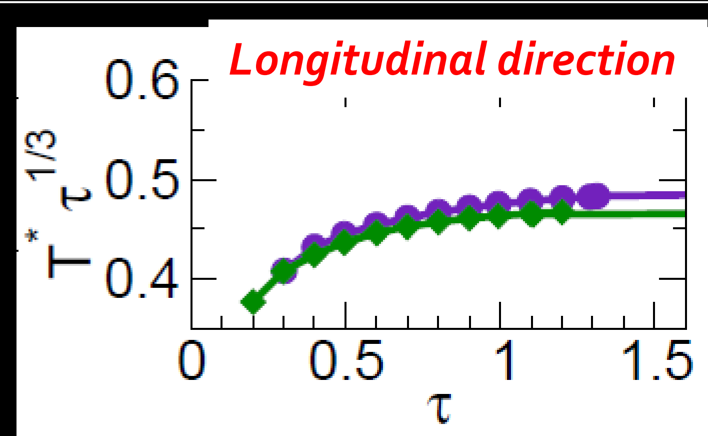

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M. R. *et al.*, PLB727 (2013)
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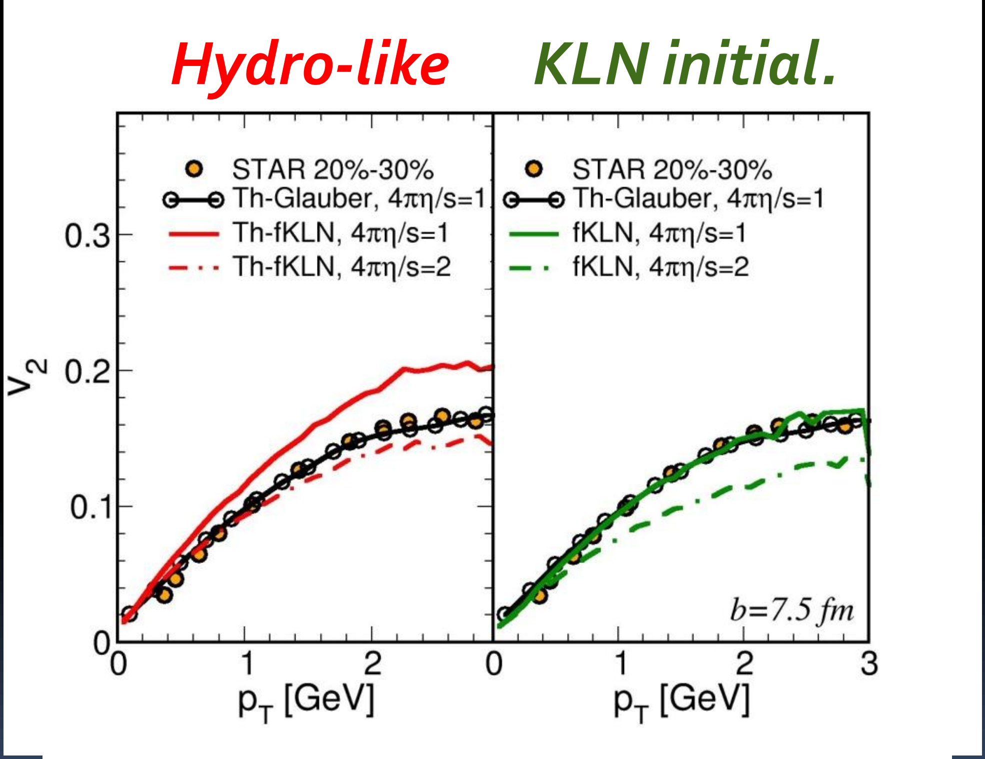
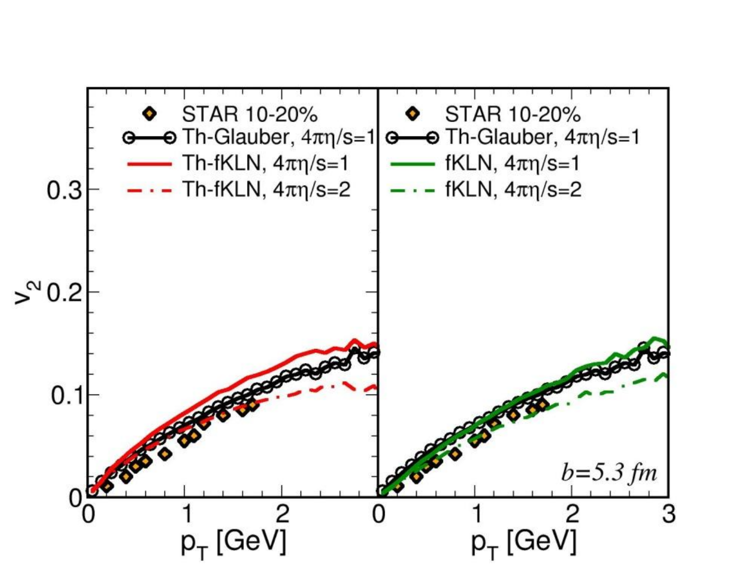
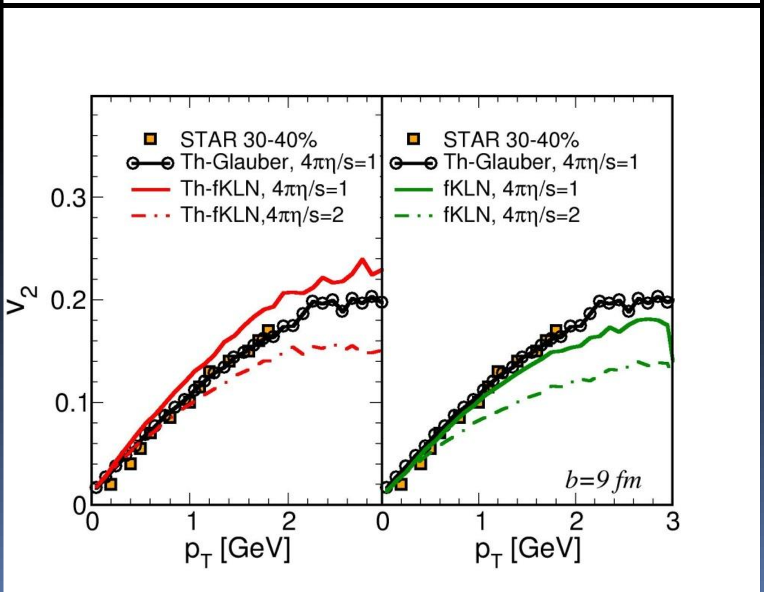
Elliptic flow from Transport

Au-Au collision
RHIC energy



Larger eccentricity of KLN implies larger v_2



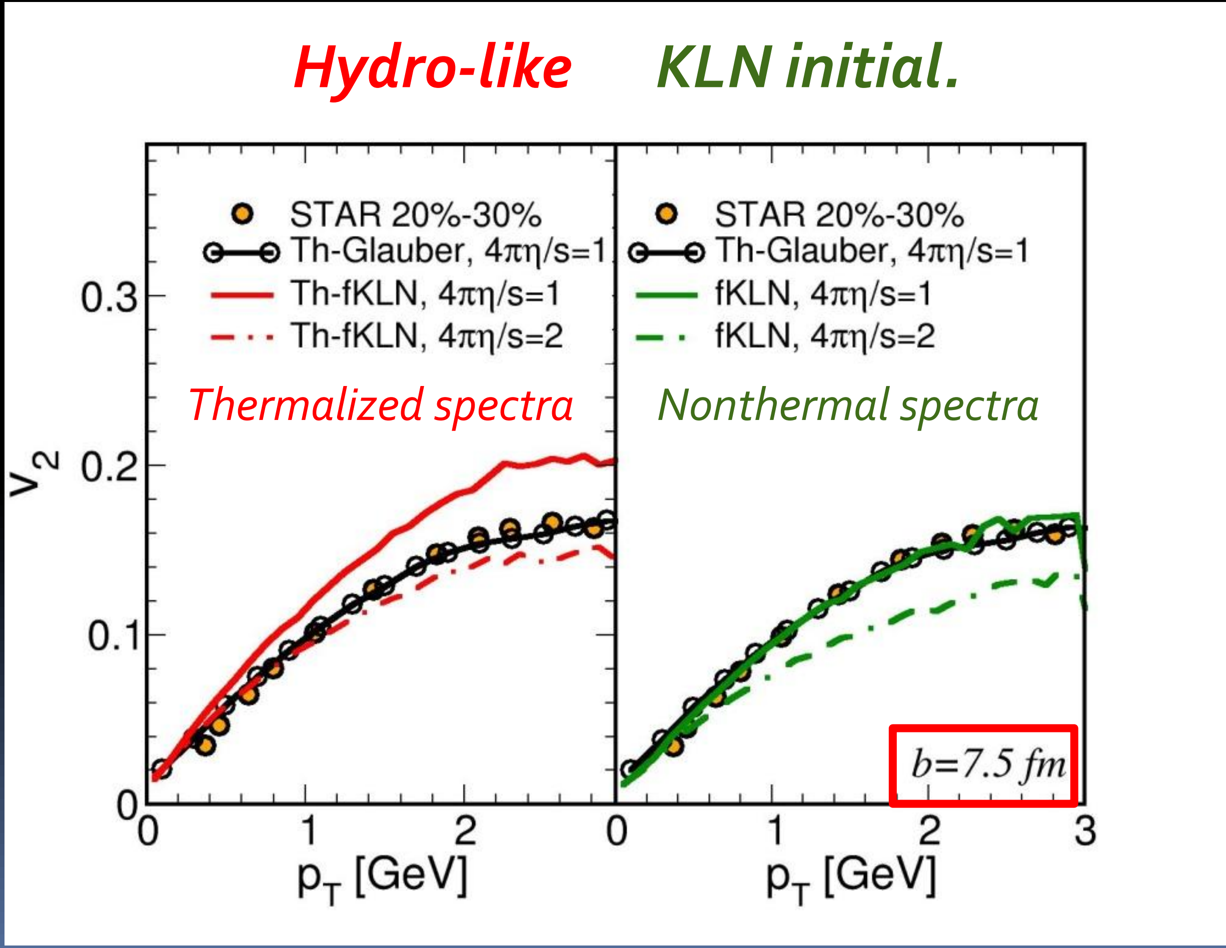


Results in fair agreement with hydro:
Song *et al.*, PRC83 (2011)

M. R. *et al.*, PLB727 (2013)
M. R. *et al.*, in preparation

Elliptic flow from Transport

Au-Au collision
RHIC energy



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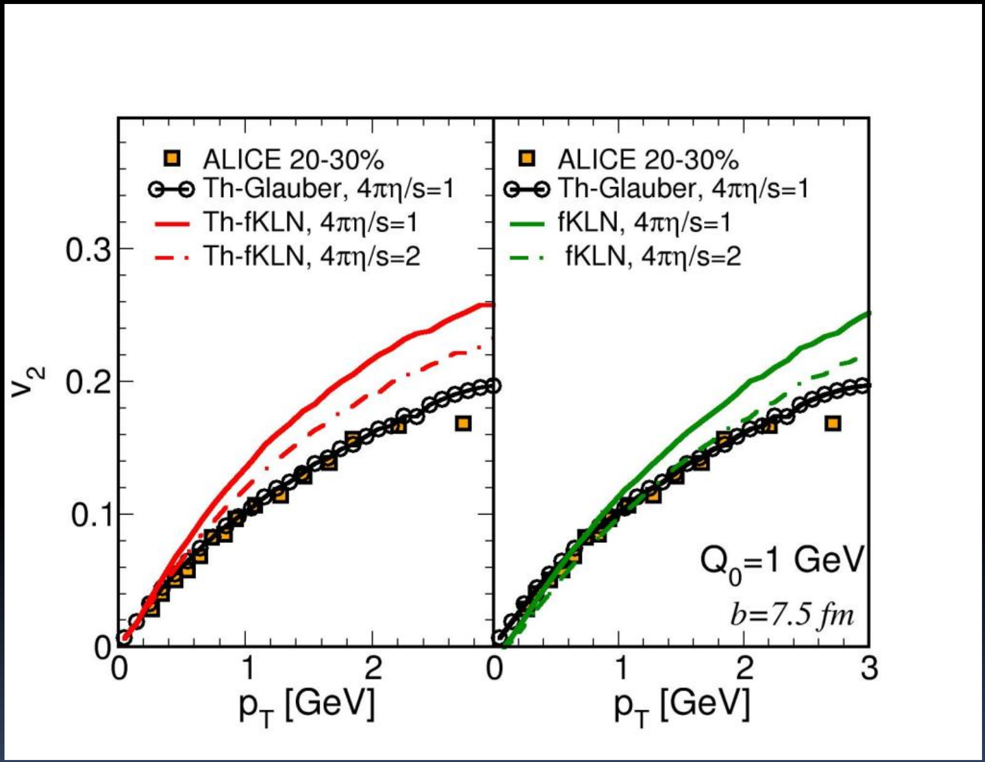
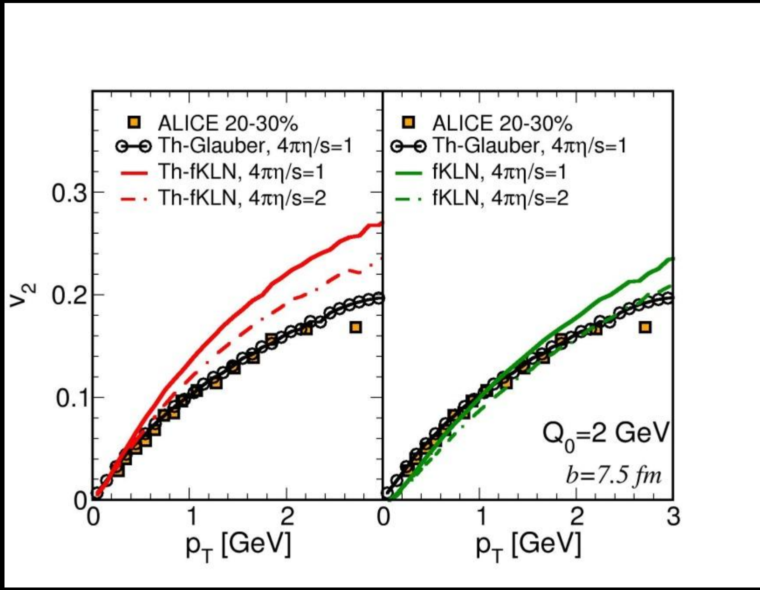
Elliptic flow from Transport

Pb-Pb collision
LHC energy





*Elliptic flow computations show this quantity is **very sensitive** to the **initial conditions:***
.) Initial anisotropy (eccentricity)
.) Initial momentum distribution

Measurements of elliptic flow in experiments might permit to identify the best theoretical initial conditions.

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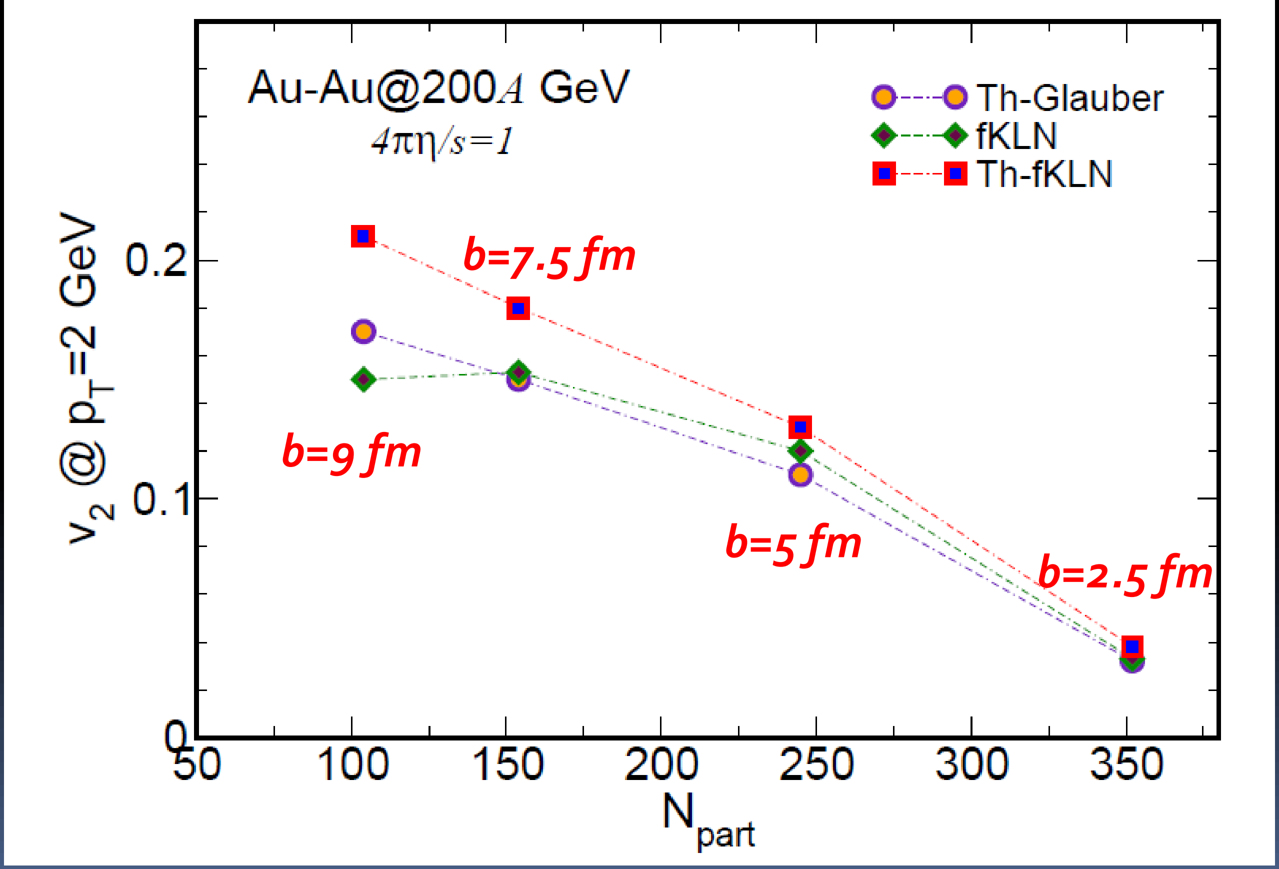
Elliptic flow from Transport

Au-Au collision

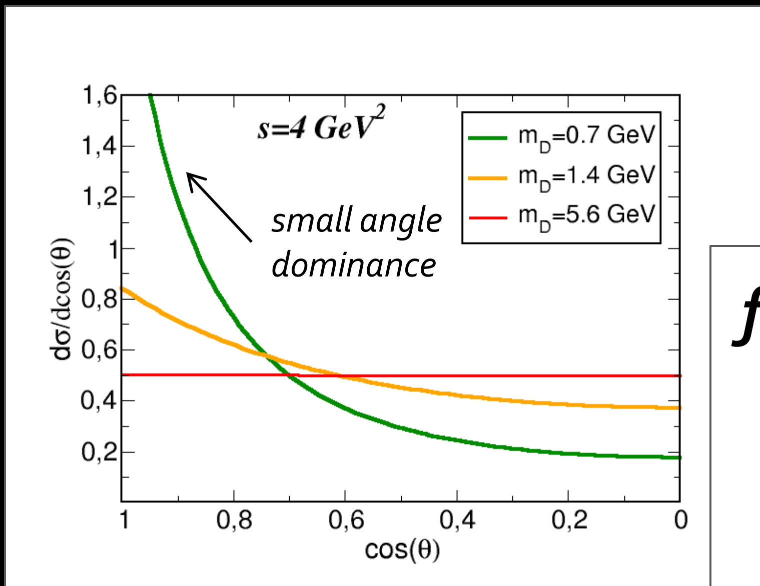
RHIC energy

Summary of the effect on differential v_2

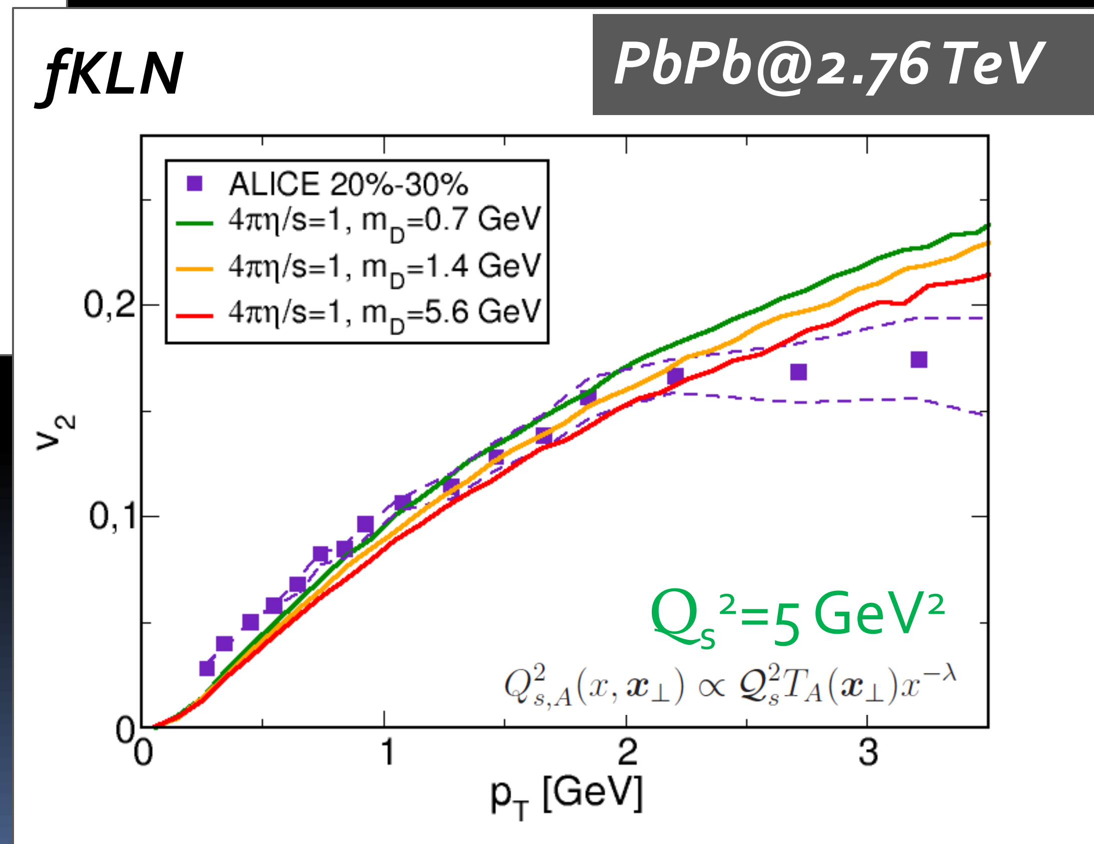


For more central collisions the effect on v2 becomes milder.

Are micro-details important?



$$\frac{d\sigma_{gg\to gg}}{dt} = \frac{9\pi^2\alpha_s^2}{2}\frac{1}{(t-m_D^2)^2}\left(1+\frac{m_D^2}{s}\right)$$



Same cross section used in:
Zhang *et al.*, PLB 455 (1999)
Molnar and Gyulassy, NPA 697 (2002)
Greco et al., PLB 670 (2009)

Increasing m_D makes the cross section isotropic. However: ***Strong change of the cross section does not result in a strong change of the elliptic flow.***

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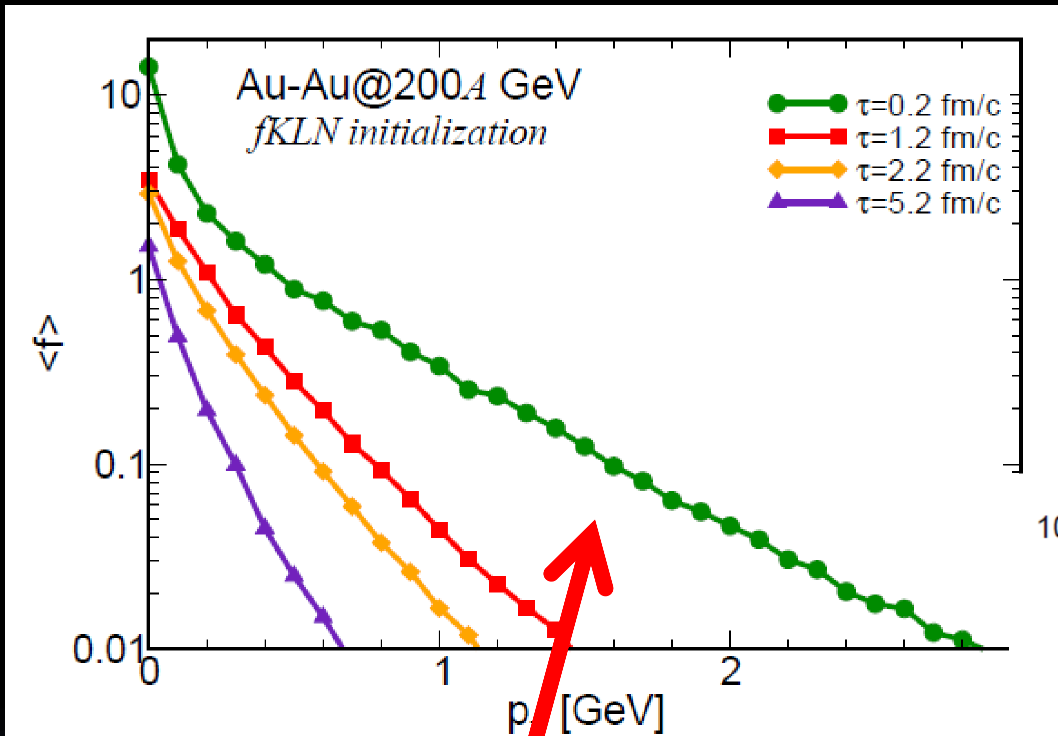
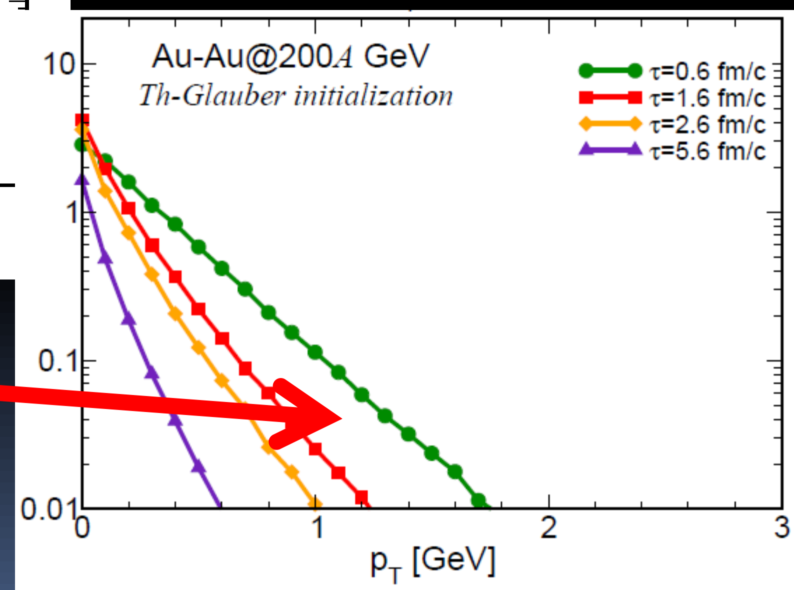
M. R. *et al.*, work in progress

Invariant distributions



$$f = \frac{(2\pi)^3}{g} \frac{\Delta N}{\Delta^2 x_\perp \Delta^2 p_T} \frac{1}{\Delta z \Delta p_z}$$

Longitudinal and transverse expansions help to dilute the system, lowering the invariant distribution functions



1+f factors in the collision integral, arising from the bosonic nature of gluons, should not modify in a substantial way our results on elliptic flow

Preliminary result: no change due to 1+f (at RHIC energy).

Conclusions

- QGP produced in heavy ion collisions behaves as a liquid rather than a gas, developing collective flows.
- ***Kinetic Theory*** permits to compute ***elliptic flow*** of plasma***,*** as well as its ***thermalization times*** and ***isotropization efficiency***.
- ***Initial distribution in momentum space affects the flow and the building up of momentum anisotropy.***

Outlook

(.)*Bose-Einstein condensate*

BE condensation, in particular at LHC energy

[Blaizot *et al.*, NPA920 (2013), NPA873 (2012)]

(.)Initial conditions from classical field dynamics

Implementation of the ***proper*** initialization from glasma spectrum&eccentricity

(.)Fluctuations in the initial condition

Systematic study of higher order harmonics

(.)Inelastic processes

Implementation of 2 to 3 and 3 to 2 processes in the collision integral

THANK YOU FOR YOUR ATTENTION

Freedom creates doubts.
(James Douglas Morrison)

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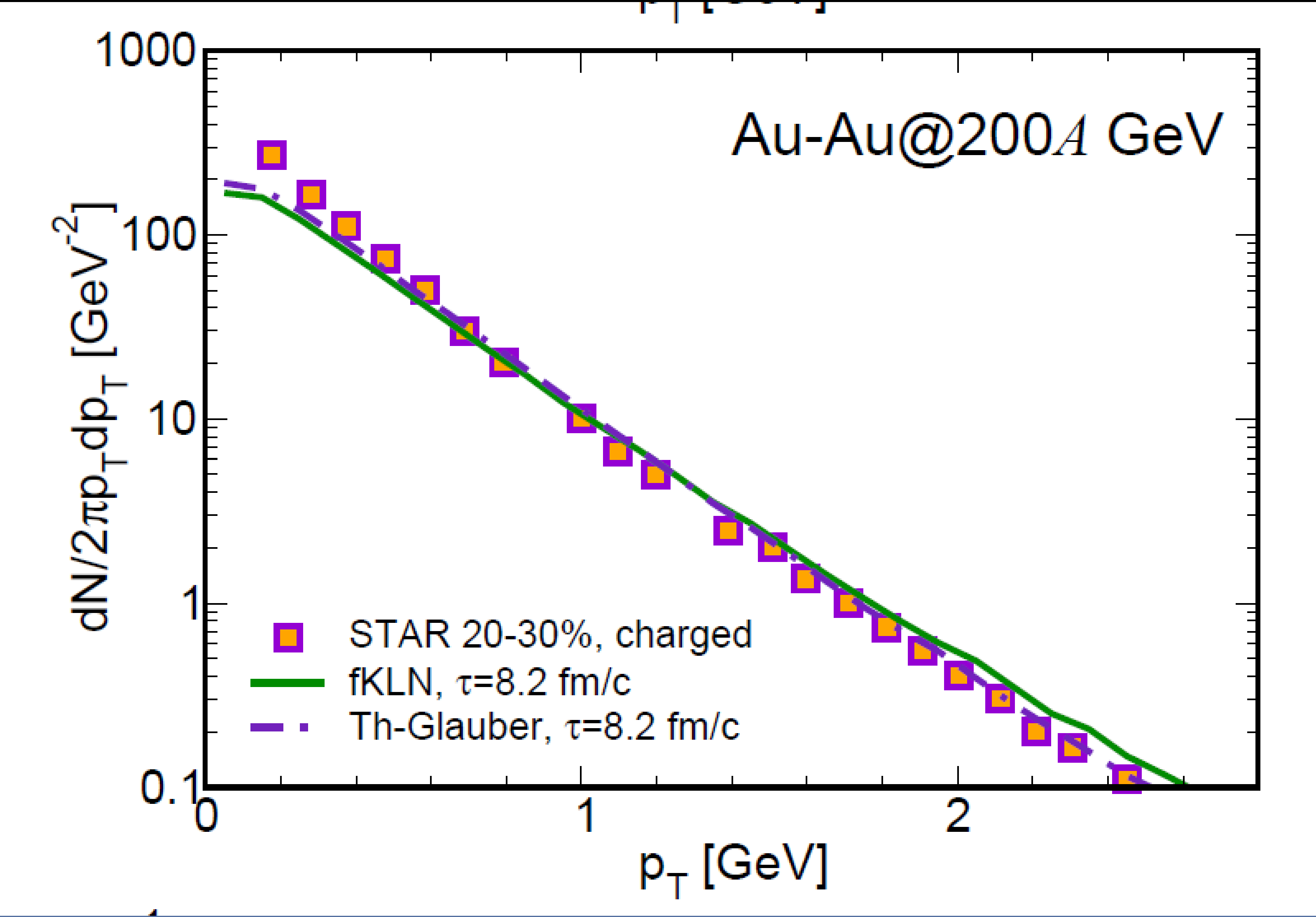
Spectra and data



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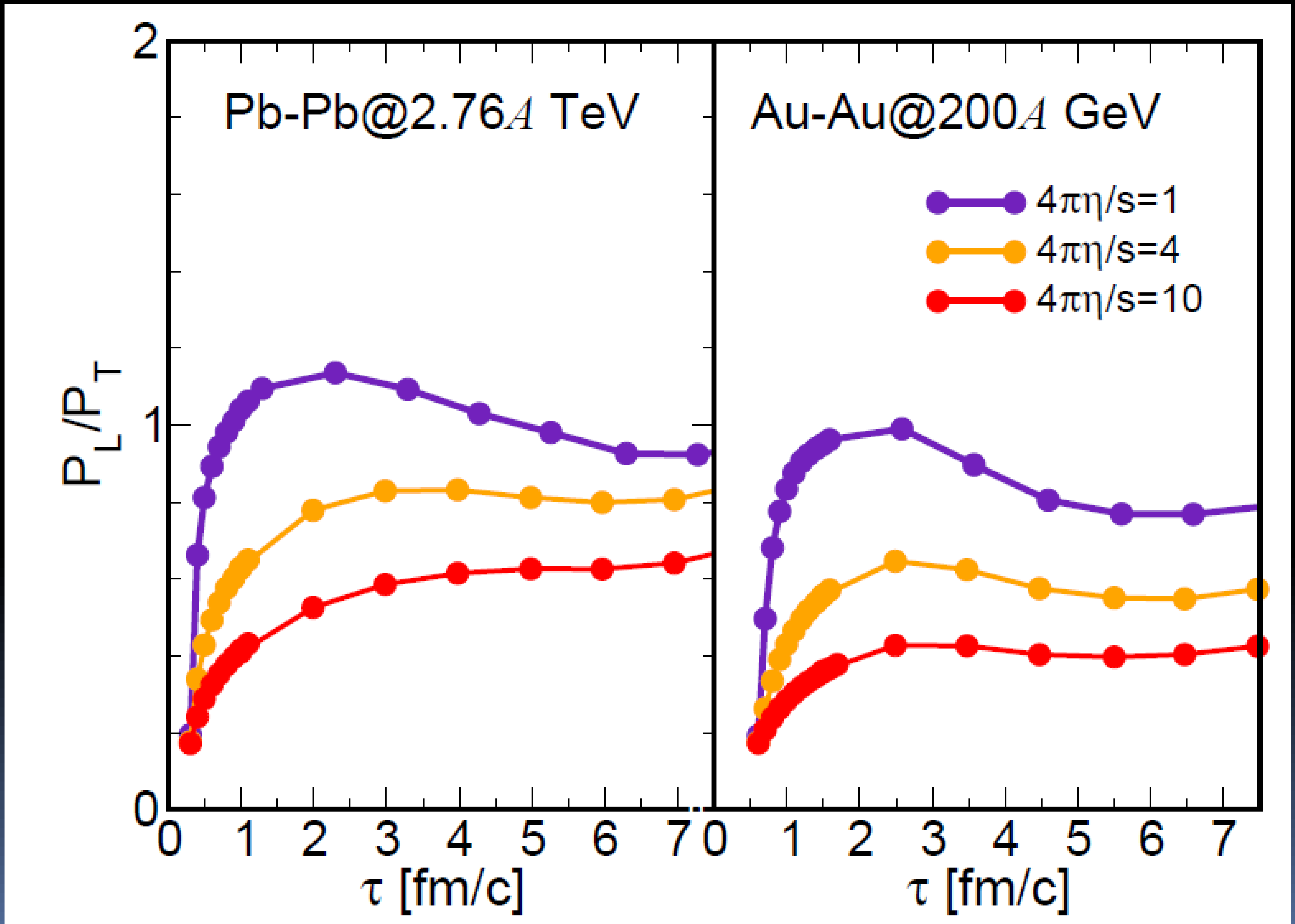
Pressures: weak coupling



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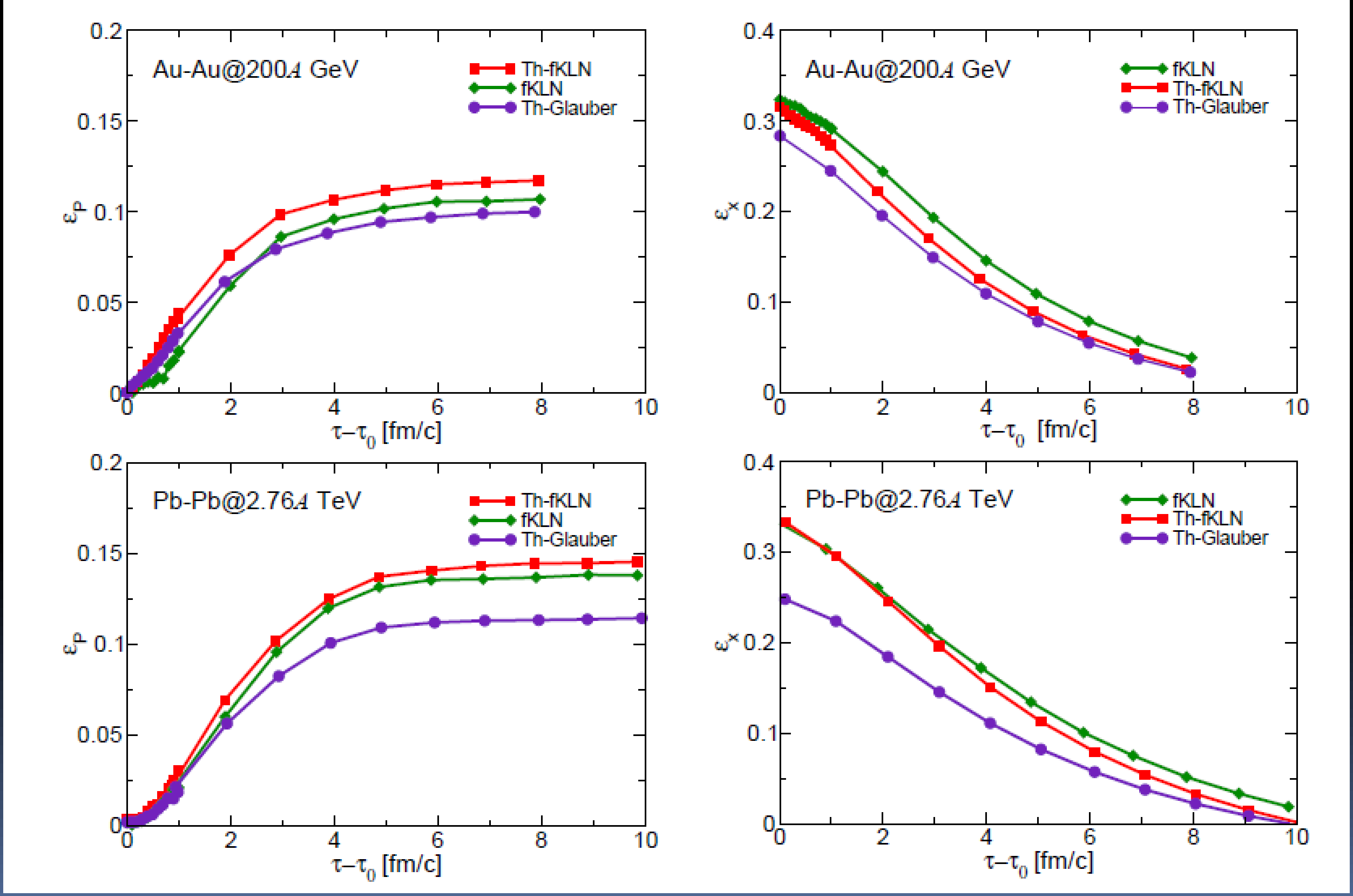
Eccentricities



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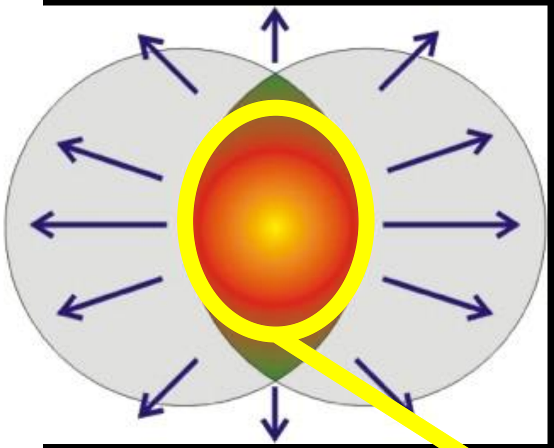
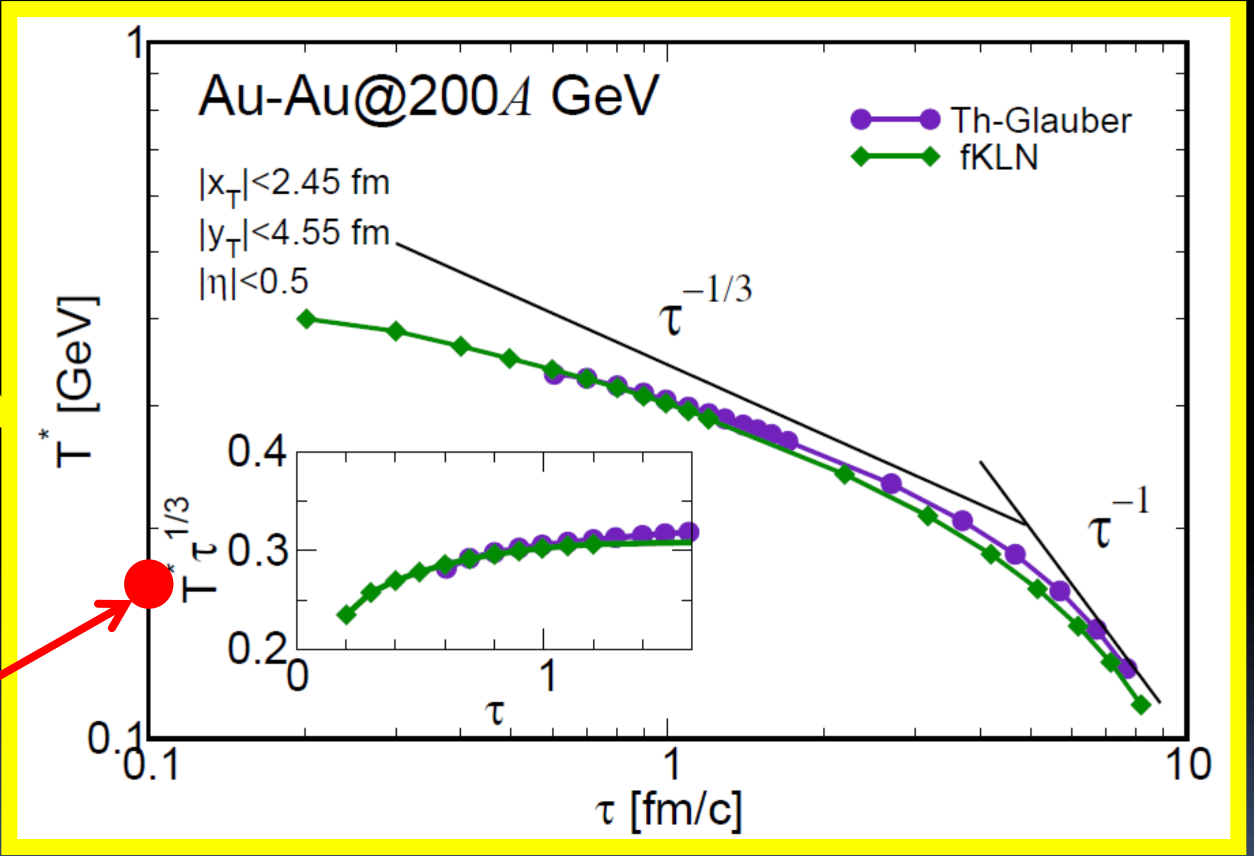
QGP in Heavy Ion Collisions



Inner core temperature (simulation)



QCD-Tc

Initial temperature much larger than QCD critical temperature:

Description in terms of partons is appropriate.

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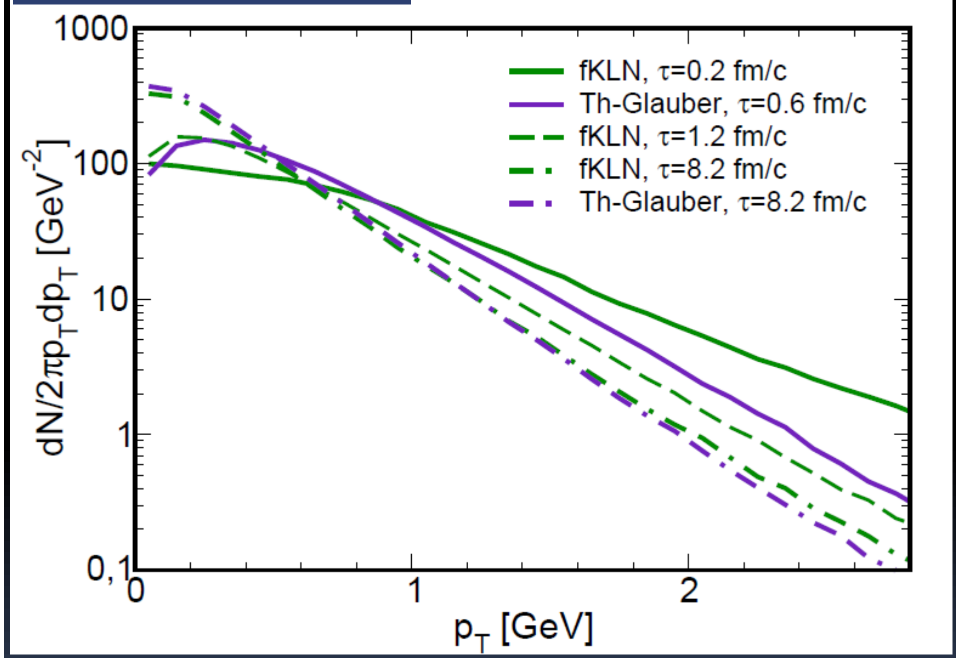
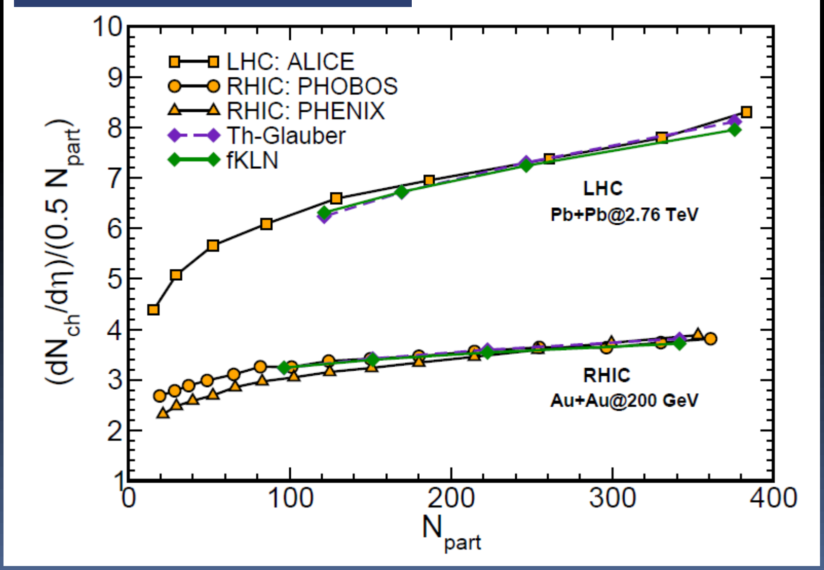
Thermalization

AuAu@200A GeV Spectra



AuAu@200A GeV Multiplicity



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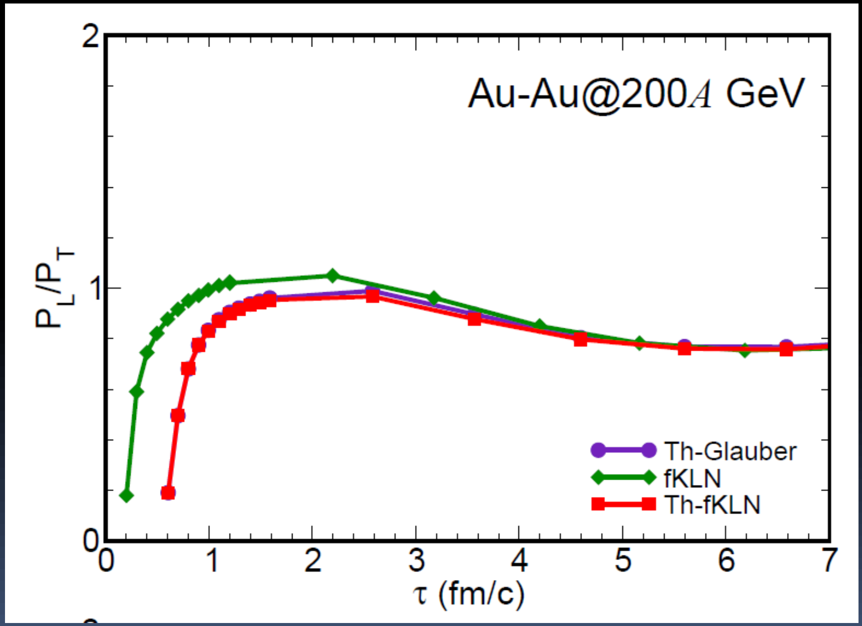
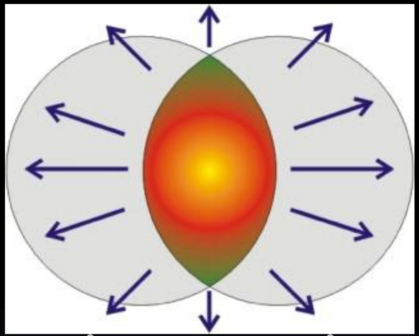
Fireball Isotropization

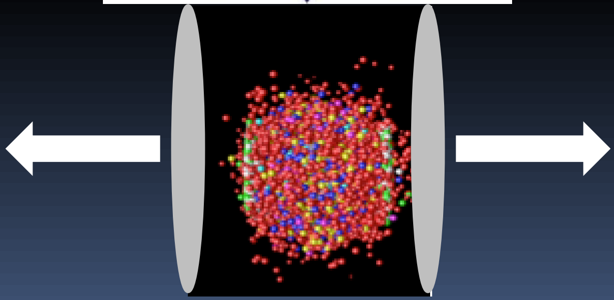
$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{E} f(x,p)$$

$$P_T = \frac{1}{V}\int_\Omega d^2x_\perp d\eta \, \frac{T_{xx}+T_{yy}}{2} \, ,$$

$$P_L = \frac{1}{V}\int_\Omega d^2x_\perp d\eta \, T_{zz} \, ,$$







Complete isotropization in strong coupling

(perfect gas would not be efficient to isotropize pressure)

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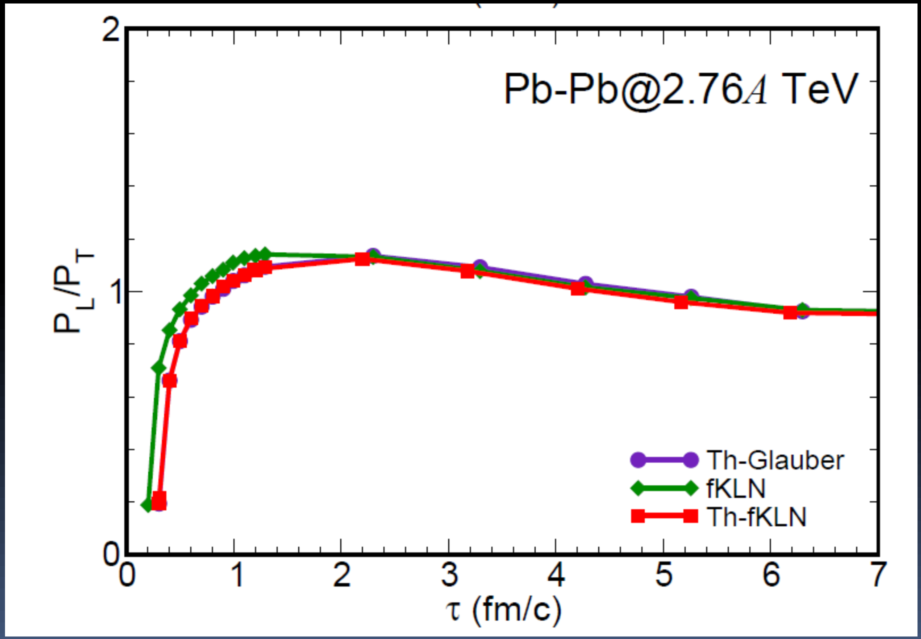

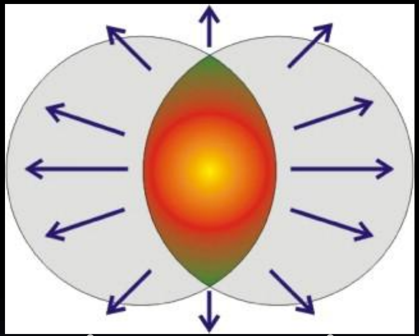

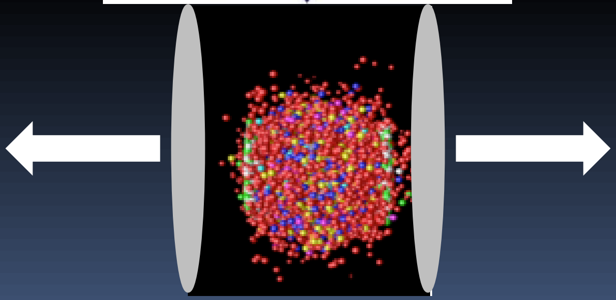

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