Particle production based on 2PI formalism

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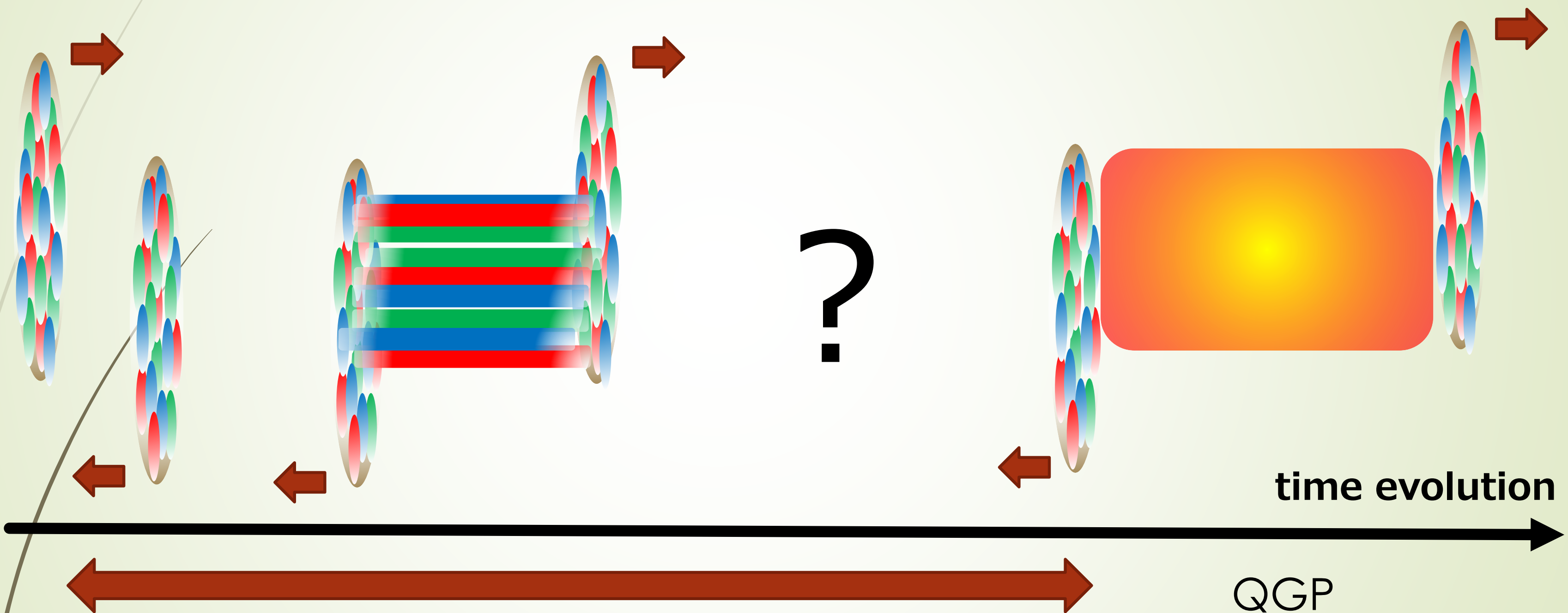
Contents

Introduction

Motivation
Early thermalization problem



$\tau_{\text{thermal}} \sim 0.6 - 1\,\text{fm/c}$

Essence of thermalization

What is the essence of thermalization process ?

far-from-equilibrium

$x \lesssim 10^{-2}$

high enegy scattering = small Bjorken x = **gluon** dominant

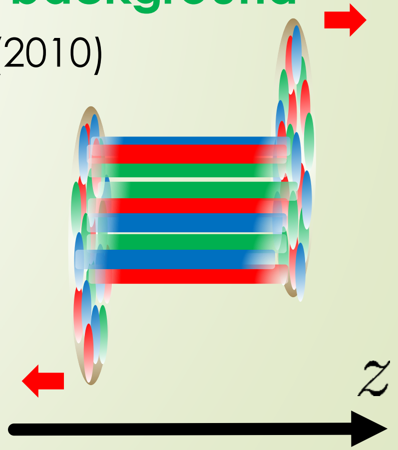
longitudinal color electromagnetic field as a **classical background**

review: McLerran (2010)

inhomogeneous color plasma => **unstable mode**

Mrowczynski (1988)

decay of field ,**particle production**
collisions , entropy production



thermalization = particle production in quantum field dynamics under the time dependent classical background fields

Approaches to thermalization

how to describe non-equilibrium system

- 1. Kinetic theory—described by 1-particle distribution function
  - solve Boltzmann eq.
  - 2->2 , 2->3 collision

  Baier, Mueller, Schiff, Son (2001)

- 2. make an effective model and solve it exactly
  - classical Yang-Mills eq. + (quantum) fluctuations
  - classical statistical approximation
  - chaoticity of CYM

  Rommatschke, Venugopalan (2006)

  Berges, Scheffler, Sexty (2008)

  Iida, Kunihiro, Muller, Ohnishi, Schafer, Takahashi (2013)

Approaches to thermalization

	merits	demerits
1. Boltzmann eq.	good for dilute system close to equilibrium	cannot describe far-from-equilibrium
2. classical statistical approximation	valid for far-from-equilibrium and unstable system	ignore a part of quantum effects
2PI	valid for **far-from-equilibrium** and **close-to-equilibrium** the **first-principle** calculations of quantum field theory	interactions **are underestimated** in unstable system **hard to solve**

Formalism

2PI formalism

review: Berges (2004)

2 Particle Irreducible (2PI) effective action

$$\Gamma[A,G] = W[J,R] - \int \frac{\delta W[J,R]}{\delta J} J - \int \frac{\delta W[J,R]}{\delta R} R$$

$$Z[J,R] = \int \mathcal{D}a \exp i \left(S[a] + \int J_a(x) a^a(x) + \frac{1}{2} \int R^{ab}(x,y) a^a(x) a^b(y) \right)$$

$$Z[J,R] = e^{iW[J,R]}$$

Defined as Legendre transform of generating functional
with respect to the sources J, R

2PI formalism

apply 2PI formalism to pure Yang-Mills theory

$$\Gamma[A,G] = \underbrace{S[A]}_{\text{CYM}} + \underbrace{\frac{i}{2}\,\mathrm{Tr}\ln G^{-1} + \frac{i}{2}\,\mathrm{Tr}\ln G_0^{-1}(A)G}_{\text{1-loop}} + \underbrace{\Gamma_2[A,G]}_{\text{2,3,..loop}}$$

$$S = \int d^4x \left[-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}\right]$$

temporal gauge : $A_t = 0$

2PI formalism

$$\Gamma[A,G] = \underbrace{S[A]}_{\text{CYM}} + \underbrace{\frac{i}{2}\,\mathrm{Tr}\ln G^{-1} + \frac{i}{2}\,\mathrm{Tr}\ln G_0^{-1}(A)G}_{\text{1-loop}} + \underbrace{\Gamma_2[A,G]}_{\text{2,3,..loop}}$$

define the effective action in Closed Time Path

Luttinger, Ward (1960)
Kadanoff, Baym (1962)
Cornwall, Jackiw, Tomboulis (1974)

$$\left.\frac{\delta\Gamma[A,G]}{\delta A}\right|_{J=R=0} = 0$$

EOM of classical background field

$$\left.\frac{\delta\Gamma[A,G]}{\delta G}\right|_{J=R=0} = 0$$

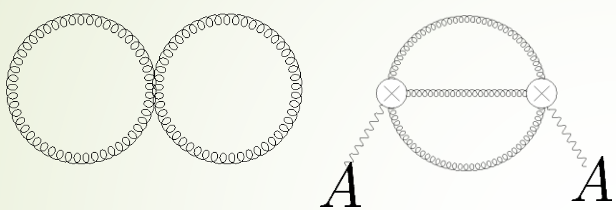
KB-CJT eq (Schwinger-Dyson eq)

$$\Rightarrow G_0^{-1}G - \Pi G = 1$$

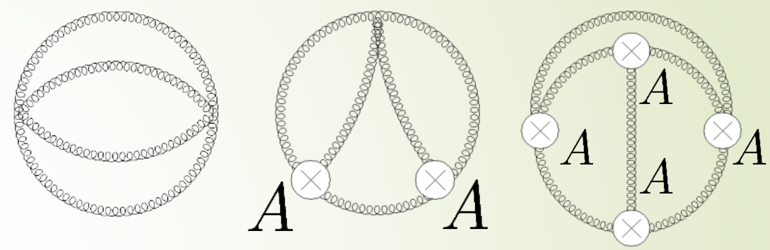
2-Particle Irreducible effective action

loop expansion of 2PI action

$$\Gamma_2 =$$

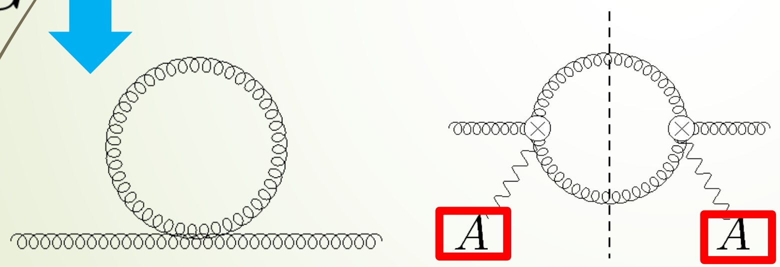


$\mathcal{O}(g^2)$



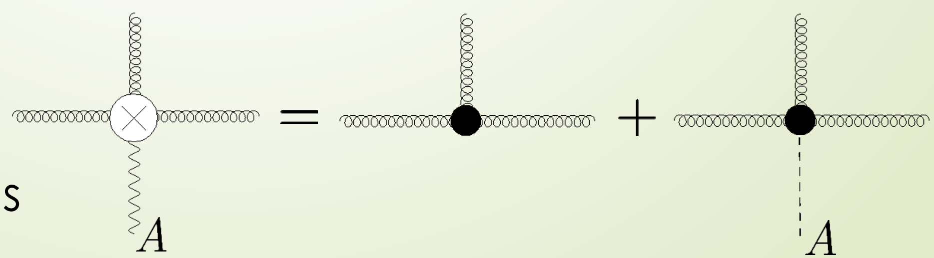
$\mathcal{O}(g^4)$

$\delta/\delta G$



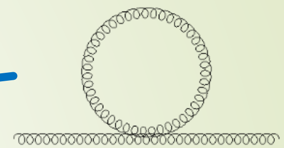
2-loop contains **1 to 2** process

3 point vertex
in background fields



KB-CJT eq of statistical function

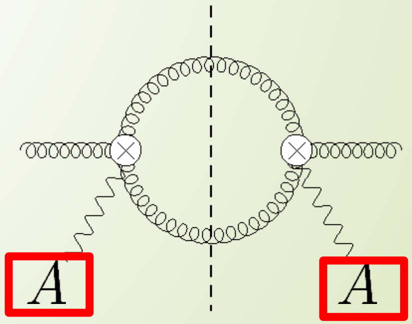
KB-CJT eq(real-time Schwinger-Dyson eq)



$$\left[\partial_t^2\delta_{ij} - (D^2\delta_{ij} - D_iD_j - 2igF_{ij} + g^2M^2(x))\right]^{ab}\mathcal{F}^{bc}_{jk}(x,y)$$

$$= -\int_{t_0}^{x_0}\Pi_{\rho\,ij}^{\ ab}(x,z)\mathcal{F}^{bc}_{jk}(z,y) + \int_{t_0}^{y_0} d^4z\,\Pi_{\mathcal{F}\,ij}^{\ ab}(x,z)\rho^{bc}_{jk}(z,y) + \text{3-loop}$$

memory integral
(causal **1 to 2 process**)

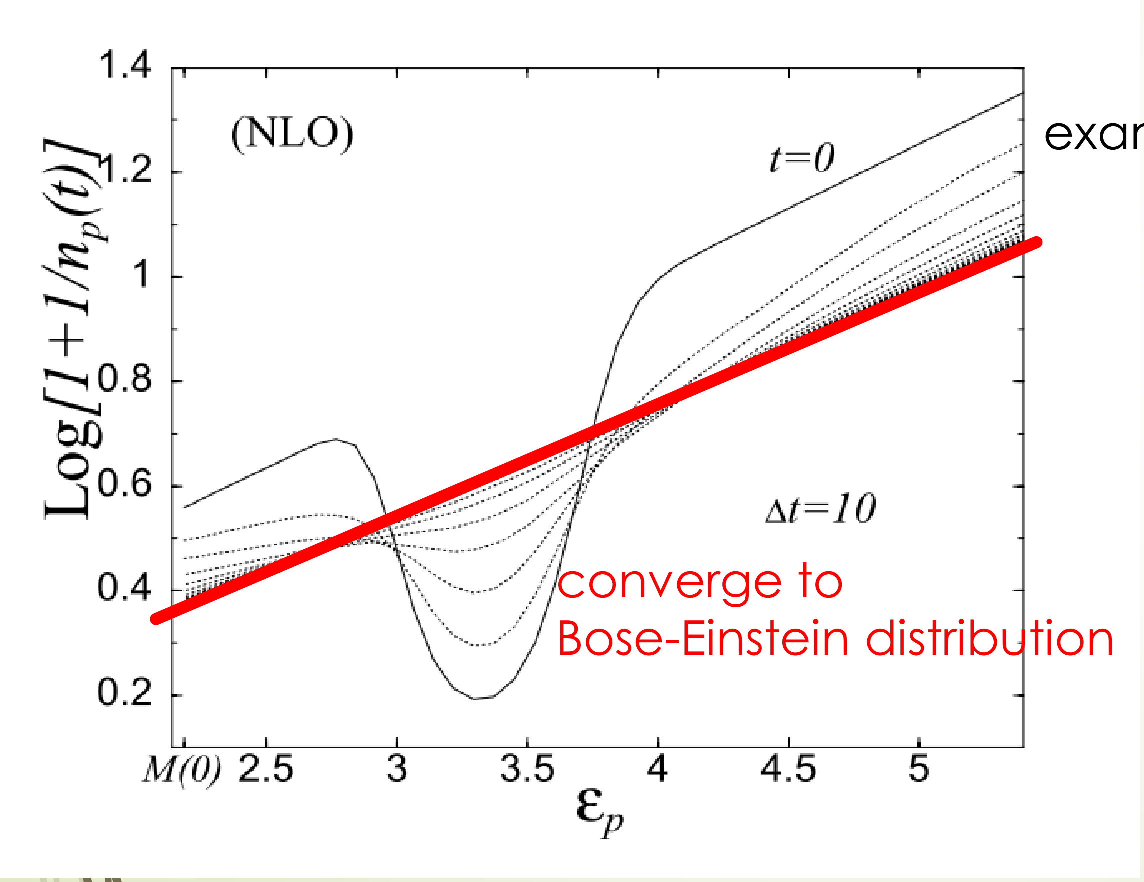


$\mathcal{F}$ is the real part of G

in an expanding geometry: Hatta, Nishiyama(2012)

Thermalization in the 2PI formalism

Berges (2002)
Arrizabalaga, Smit, Tranberg (2004)
Nishiyama, Ohnishi (2011)



example: O(N) scalar field theory

effective particle number

$$\mathcal{F}(t,t;\boldsymbol{p}) = \frac{1}{\omega_{\boldsymbol{p}}}\left(n(t)_{\boldsymbol{p}} + \frac{1}{2}\right)$$

arbitrary non-equilibrium initial condition

$$n(t=0)_{\boldsymbol{p}} \propto e^{-\frac{(|p|-p_{ts})^2}{2\sigma^2}}$$

Application to Glasma

set up

for simplicity :

flat metric
color SU(2)
background fields are homogeneous

$$A = \frac{1}{g} A^{(0)} + A^{(1)} + \mathscr{O}(g)$$

$$A_i^{a(0)} = \tilde{A}(t) \left(\delta^{a2} \delta_{ix} + \delta^{a1} \delta_{iy} \right)$$

$$\Rightarrow \begin{cases} B_z^3 \neq 0 \\ B_\perp^a = 0 \end{cases}$$

LO KB-CJT eq (*for a first study*)

$$\partial_t^2 \mathcal{F}(t, t', \boldsymbol{p}) = -\Omega[A^{(0)}] \mathcal{F}(t, t', \boldsymbol{p})$$

$$\tilde{A}(t) = \sqrt{B} \mathrm{cn} \left(t\sqrt{B}, 1/2 \right)$$

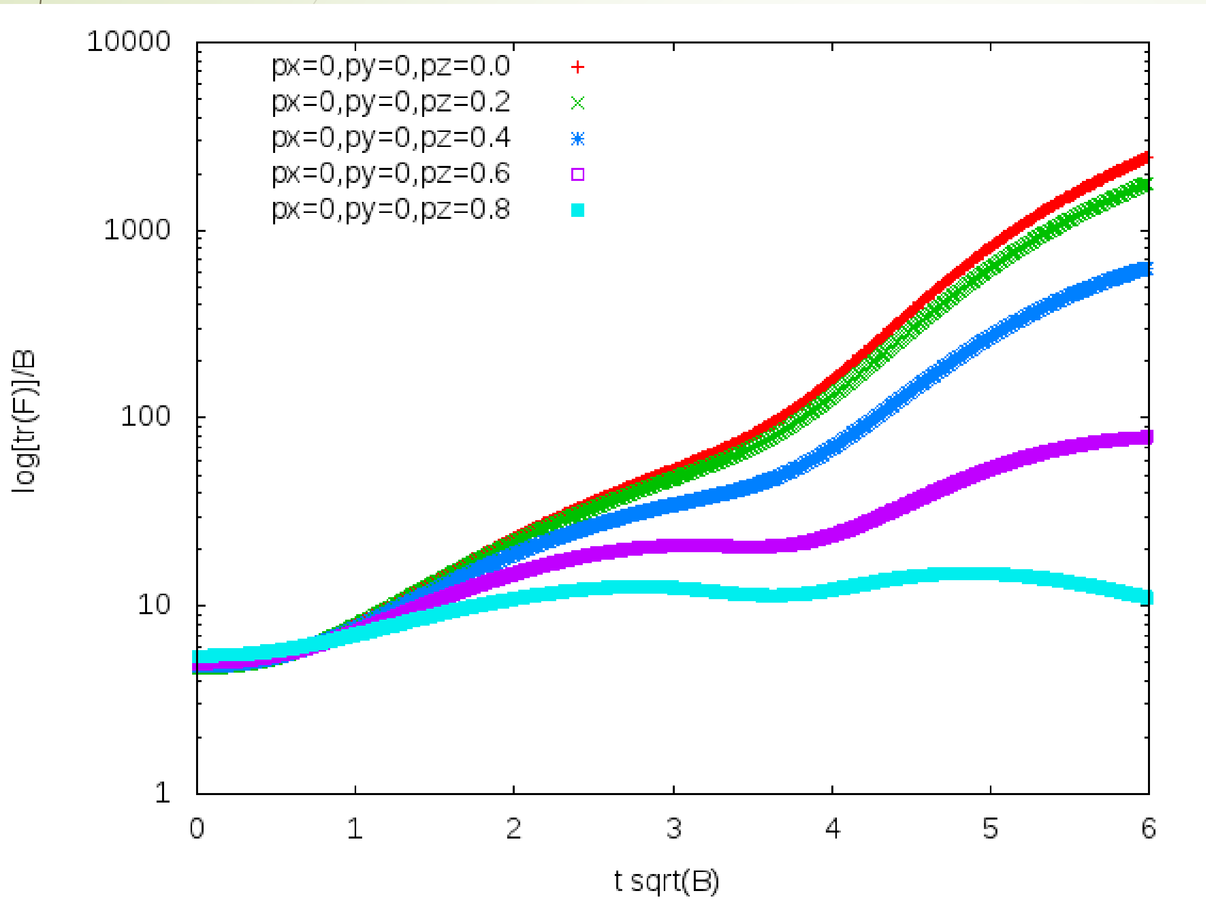
Ω 9×9 matrix(kinetic term)

one of the eigenvalues $\omega_{\mathrm{NO}}^2 = p_z^2 - B$

Nielsen-Olesen instability

- lower momentum modes are unstable
- $A_\perp^{1,2}$ are unstable

time evolution of statistical function



initial conditions

$$\tilde{A}(t=0) = \sqrt{B}$$

$$\partial_t \tilde{A}(t=0) = 0$$

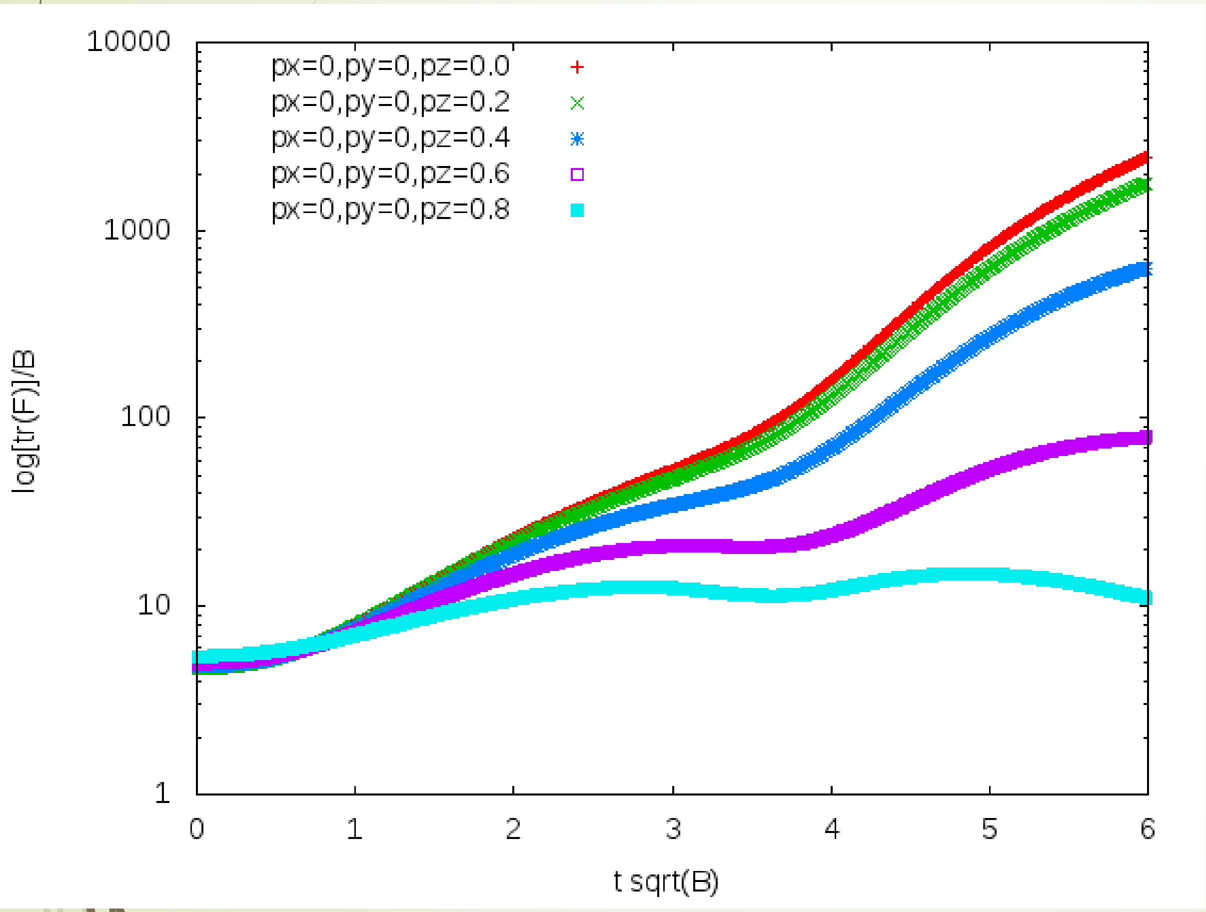
$$\mathcal{F}^{\alpha\beta}(0,0;\boldsymbol{p}) = \delta_{\alpha\beta}\frac{1}{\omega^{\alpha}_{\boldsymbol{p}}}\left(n_{\boldsymbol{p}} + \frac{1}{2}\right)$$

$$\partial_t \mathcal{F}^{\alpha\beta}(0,0;\boldsymbol{p}) = 0$$

$$\partial_t \partial_{t'} \mathcal{F}^{\alpha\beta}(0,0;\boldsymbol{p}) = \delta_{\alpha\beta}\omega^{\alpha}_{\boldsymbol{p}}\left(n_{\boldsymbol{p}} + \frac{1}{2}\right)$$

$$n_{\boldsymbol{p}} \propto e^{-\frac{p_\perp^2 + p_z^2}{2\sigma^2}}$$

time evolution of statistical function



lower momentum mode have larger growth rate

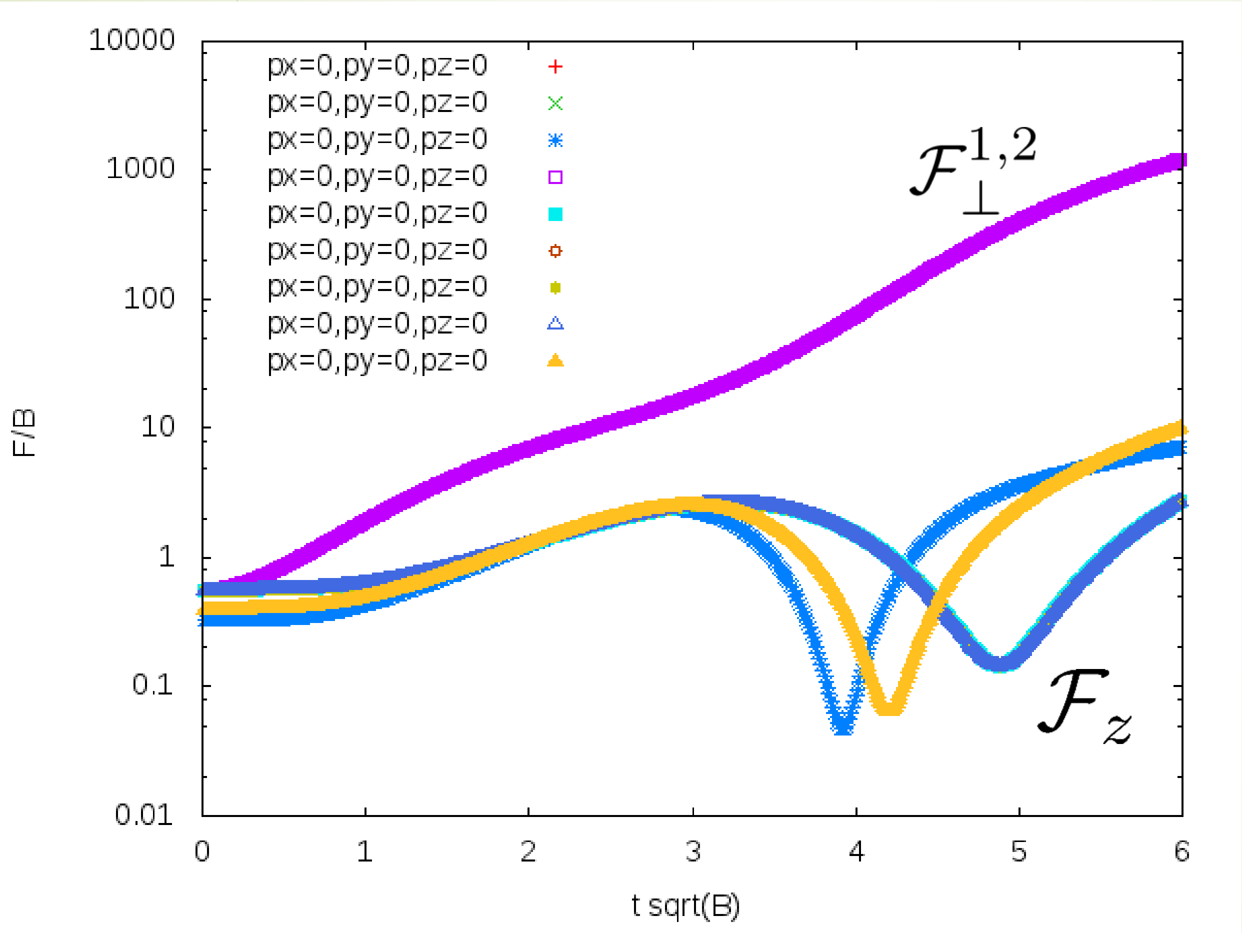
unstable when

$$p_z \lesssim 0.6\sqrt{B}$$

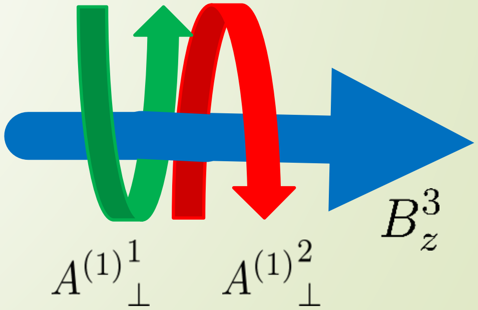
Nielsen-Olesen instability

$$\omega_{\rm NO}^2 = p_z^2 - \tilde{A}(t)^2 \xrightarrow{\text{time avg.}} p_z^2 - 0.5B$$

time evolution of statistical function



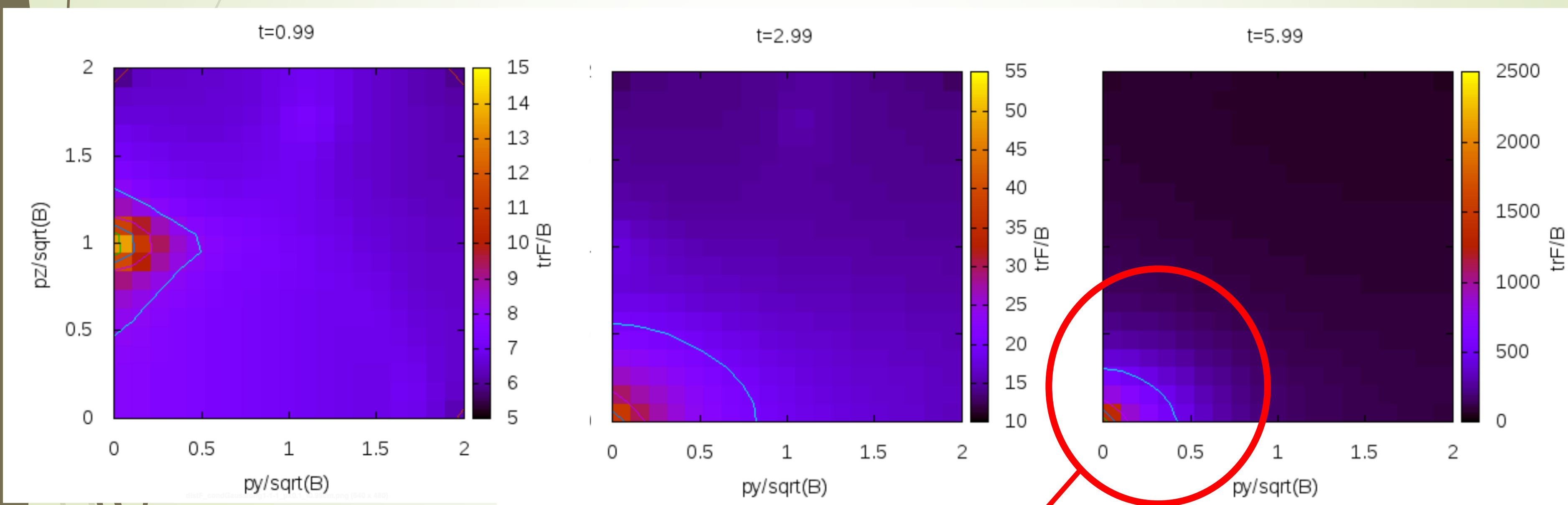
color-Lorentz components of zero mode



perpendicular components are unstable

time evolution of statistical function

time evolution

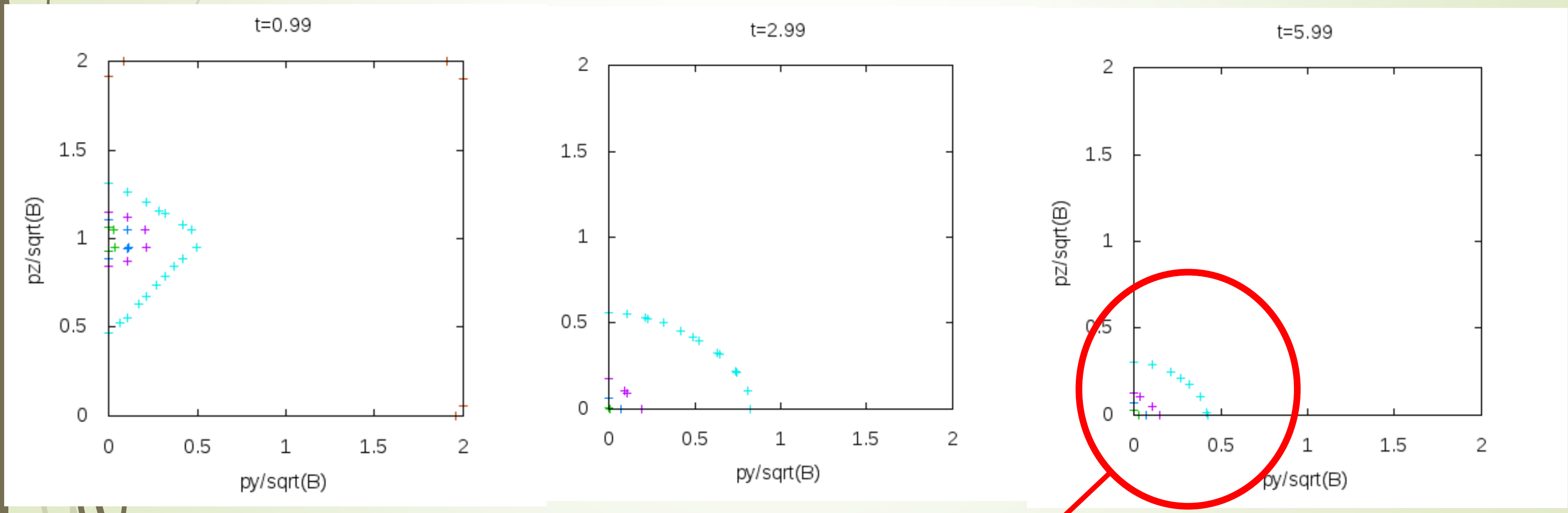


$p_x = 0.1\sqrt{B}$

there is a peak in low momentum region

time evolution of statistical function

time evolution



$p_x = 0.1\sqrt{B}$

anisotropic in py-pz plane

Dose this result means “thermalization” ?

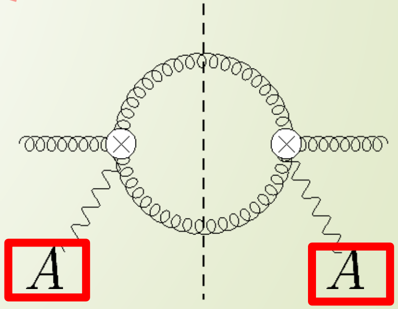
the answer is *NO*
due to the linearized approximation, there is no scattering

$$\left[\partial_t^2\delta_{ij} - (D^2\delta_{ij} - D_iD_j - 2igF_{ij} - g^2M^2(t))\right]^{ab}\mathcal{F}^{bc}_{jk}(x,y)$$

$$= -\int_{t_0}^{x_0}\Pi_{\rho\,ij}^{\ ab}(x,z)\mathcal{F}^{bc}_{jk}(z,y) + \int_{t_0}^{y_0} d^4z\,\Pi_{\mathcal{F}\,ij}^{\ ab}(x,z)\rho^{bc}_{jk}(z,y)$$

$$\partial_t n_{\boldsymbol{p}}(t) = 0$$

Kinetic entropy never increases



2-loop correction is need to entropy production

Summary

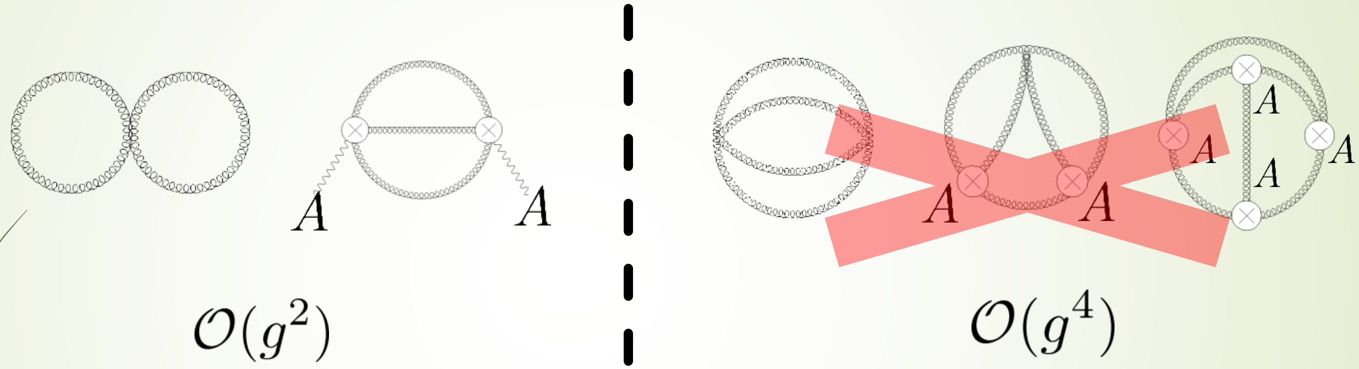
- thermalization=particle production in quantum field dynamics under the time dependent classical background fields
- 2PI formalism is best suited
- We assume spatial homogeneous background color magnetic field
- We showed that NO instability affect the statistical functions.
- It may affect particle distribution

- spatial localized background fields
- take into account NLO KB-CJT eq which describes collision under background fields

back up

Controlled gauge dependence

$\Gamma_2 =$



$\mathcal{O}(g^2)$ $\mathcal{O}(g^4)$

truncation

$$\text{gauge dep.} = \mathcal{O}(g^4)$$

A. Arrizabalaga, J. Smit (2002)

M.E. Carrington, G. Kunstatter, H. Zaraket (2005)

the gauge dependence appears at **higher order than the truncation order**

statistical and spectral function

decompose the Keldysh Green function

$$G(x,y) = \mathcal{F}(x,y) - \frac{i}{2}\rho(x,y)\left(\theta_C(x^0 - y^0) - \theta_C(y^0 - x^0)\right)$$

spectral function $\rho(x,y) \equiv i\langle[a(x), a(y)]\rangle$

statistical function $\mathcal{F}(x,y) \equiv \frac{1}{2}\langle\{a(x), a(y)\}\rangle$

statistical functions have information about ***particles***

$$\mathcal{F} \sim \frac{\cos(x^0 - y^0)\omega_{\boldsymbol{p}}}{\omega_{\boldsymbol{p}}}\left(n(\boldsymbol{p}) + \frac{1}{2}\right)$$

occupation number
(converge to the BE distribution)