Dynamical evolution of heavy quarks: Boltzmann vs Langevin

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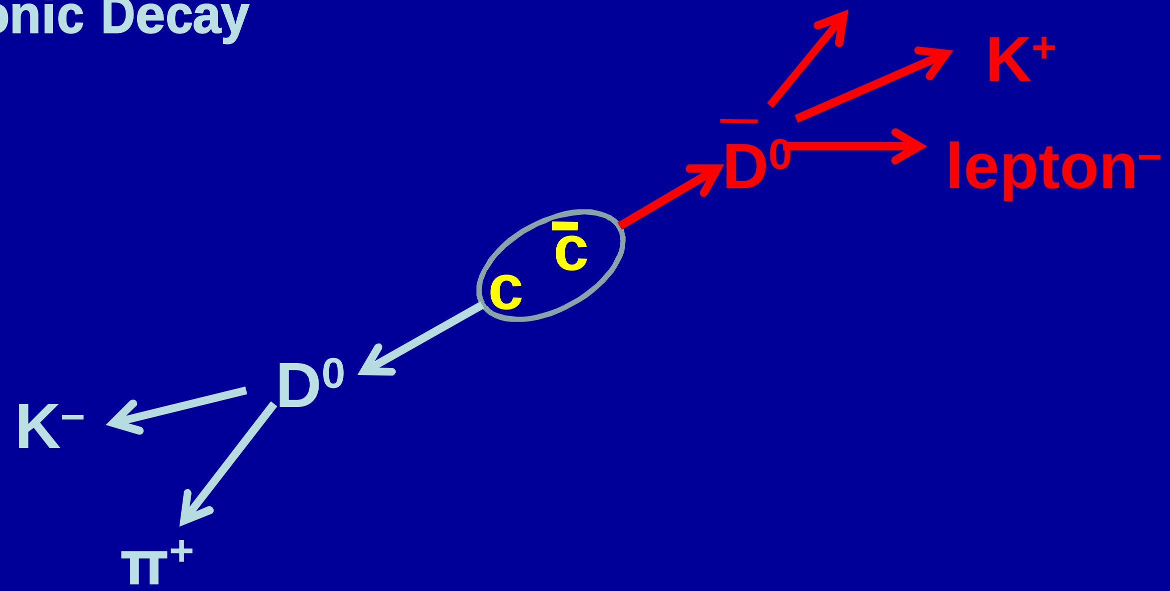
Outline

- Heavy Flavors
- Transport approach
- Fokker Planck approach
- Comparison
- Conclusions and future developments

Introduction Heavy Quarks

- ✓ Large mass $M_{HQ} >> \Lambda_{QDC}$ ($M_{charm} \cong 1.3$ GeV; $M_{bottom} \cong 4.2$ GeV)
- ✓ They are produced in the early stage
- ✓ They interact with the bulk and can be used to study the properties of the medium
- ✓ There are two way to detect charm:
  1) Single non-photonic electron and muon
  2) Hadronic Decay



R_{AA} of Heavy Quarks

QCD-based models describing collisional and radiative energy loss in the medium predict:

[Dokshitzer et al., PLB 519 (2001) 199], [Armesto et al., PRD 69 (2004) 114003], [Djordjevic et al., NPA 783 (2007) 493]

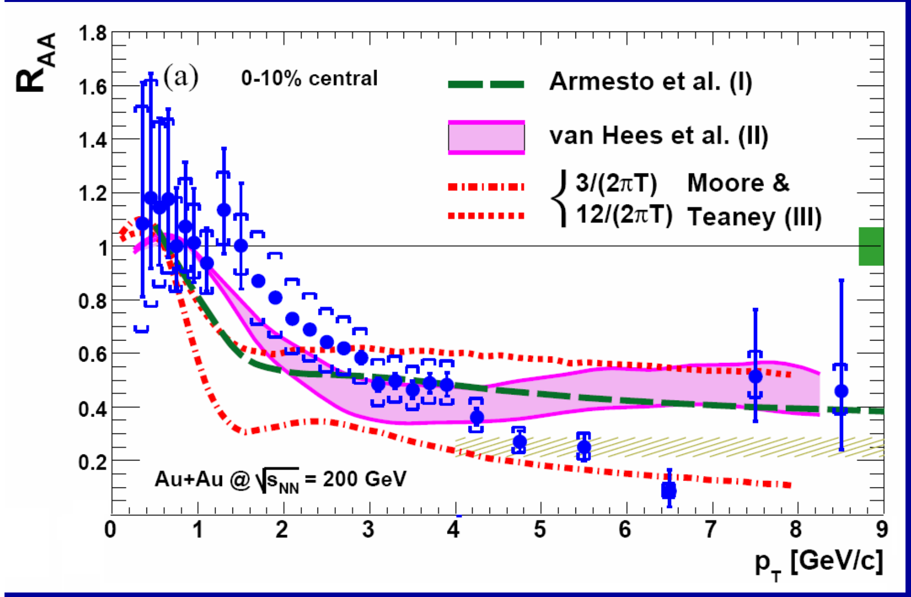
$$E_{loss}\left(light\ quark\right) > E_{loss}\left(Charm\right) > E_{loss}\left(Bottom\right)$$

Nuclear Modification factor:

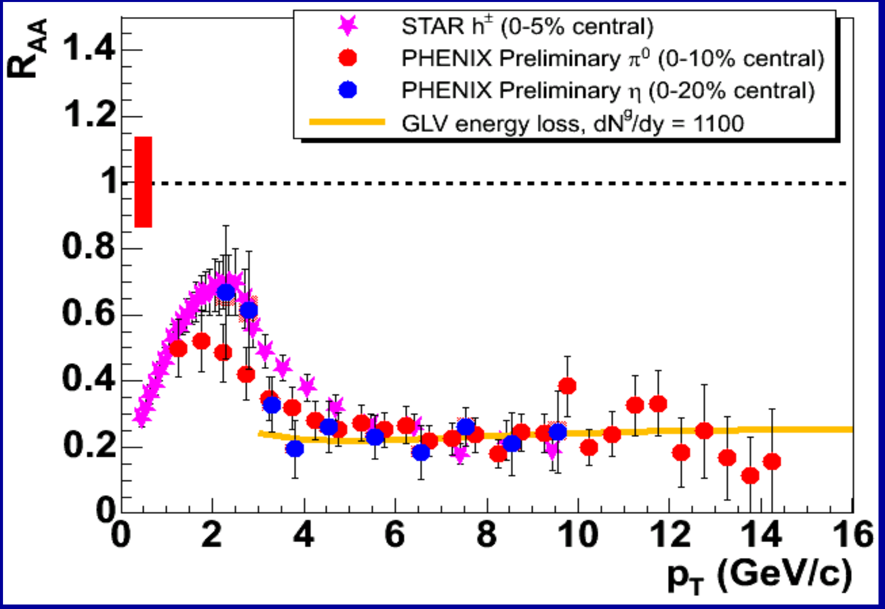
$$R_{AA}(p_T) = \frac{1}{N_{coll}} \frac{d^2N^{AA}/dp_T dy}{d^2N^{pp}/dp_T dy}$$

$$R_{AA}\left(light\ quark\right) < R_{AA}\left(Charm\right) < R_{AA}\left(Bottom\right)$$

R_{AA} at RHIC

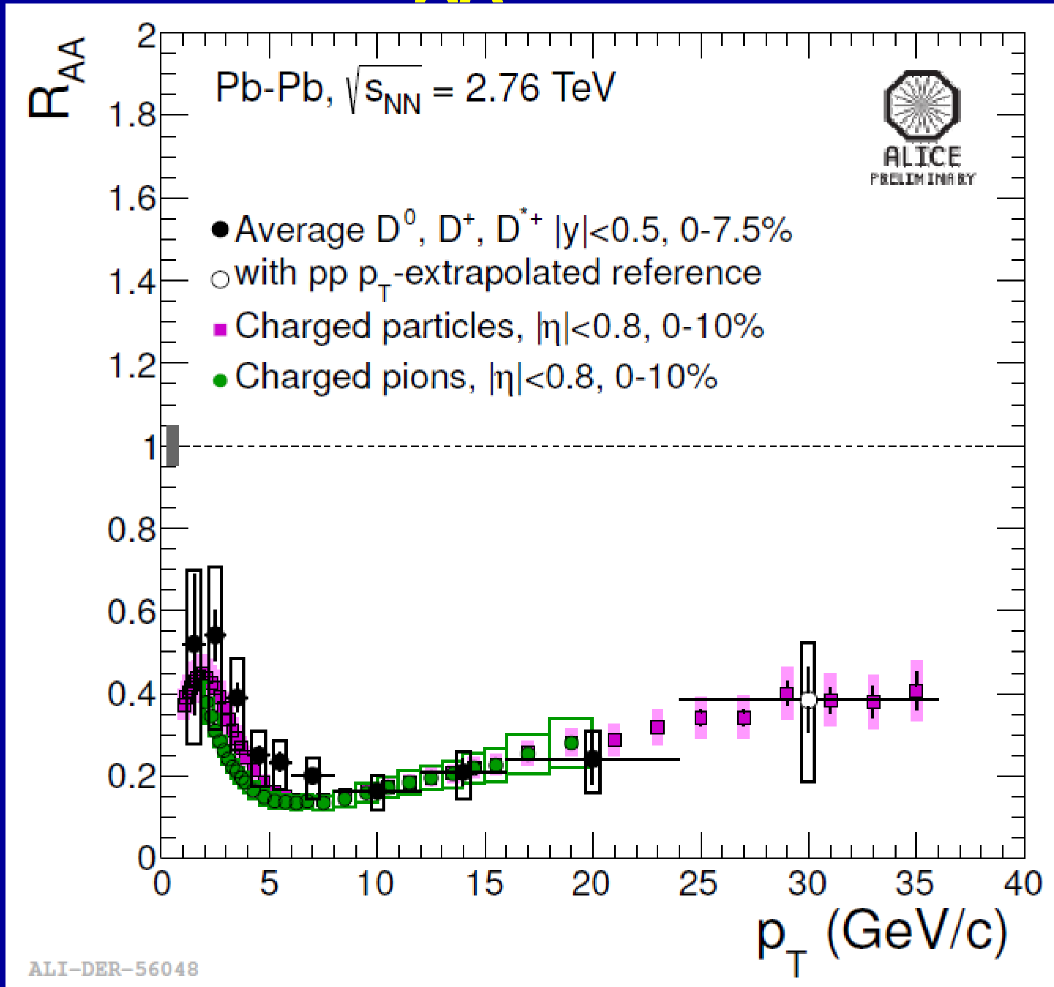


[PHENIX: *PRL98(2007)172301]*



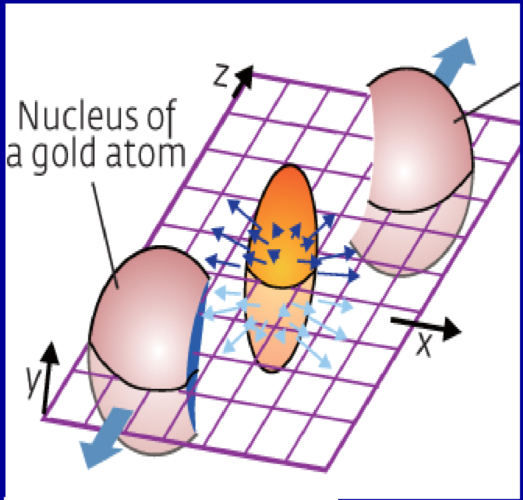
In spite of the larger mass at RHIC energy heavy flavor suppression is not so different from light flavor

R_{AA} at LHC



Again at LHC energy heavy flavor suppression is similar to light flavor especially at high p_T

Elliptic flow



$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

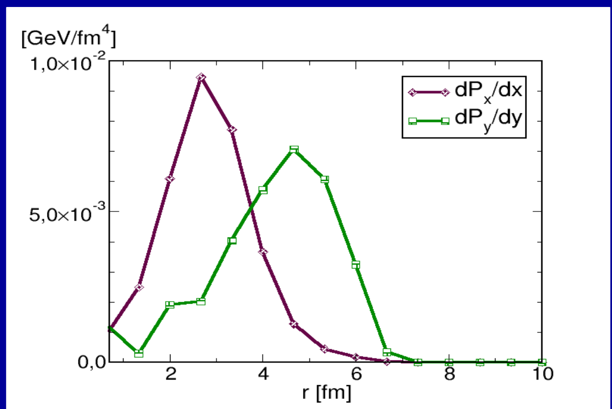
$$\frac{d^3N}{dyp_T dp_T d\phi} = \frac{1}{2\pi}\frac{d^2N}{dyp_T dp_T}\left[1 + 2v_2(y, p_T)\cos 2\phi\right]$$

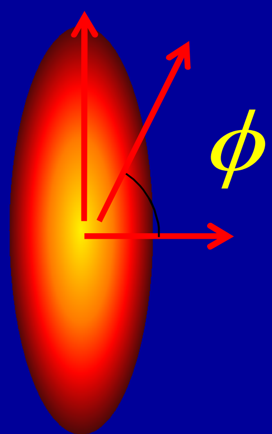
$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

Different origin of the v_2 of HQ with respect to v_2 of the bulk

Bulk: v_2 is generated by the different gradient pressure along x direction with respect to the y direction

HQ: v_2 is by the different path covered by heavy quark in the anisotropic medium





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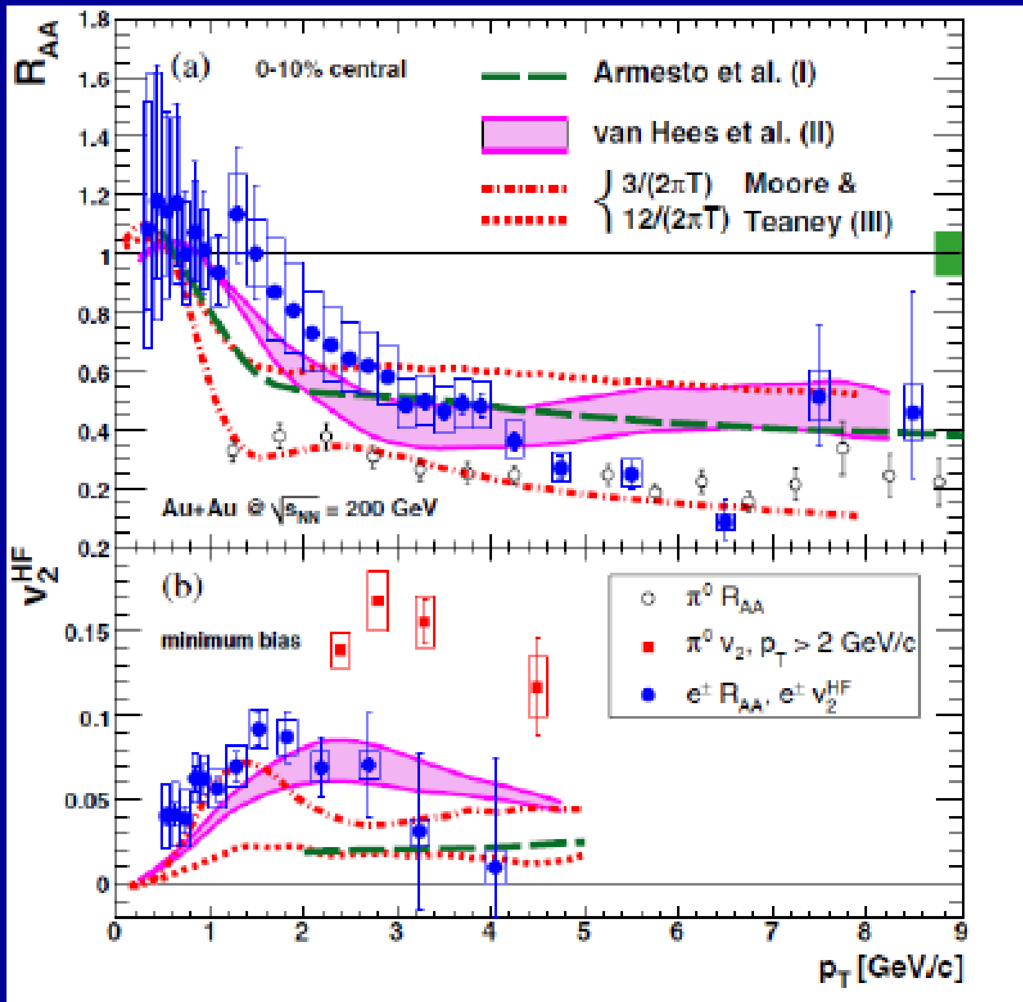
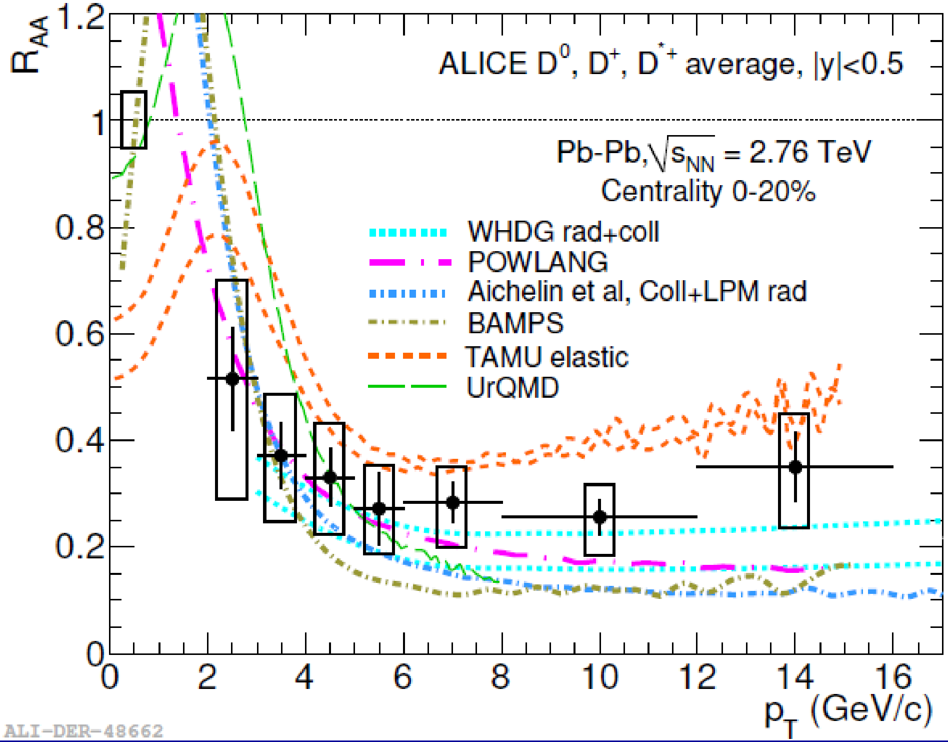


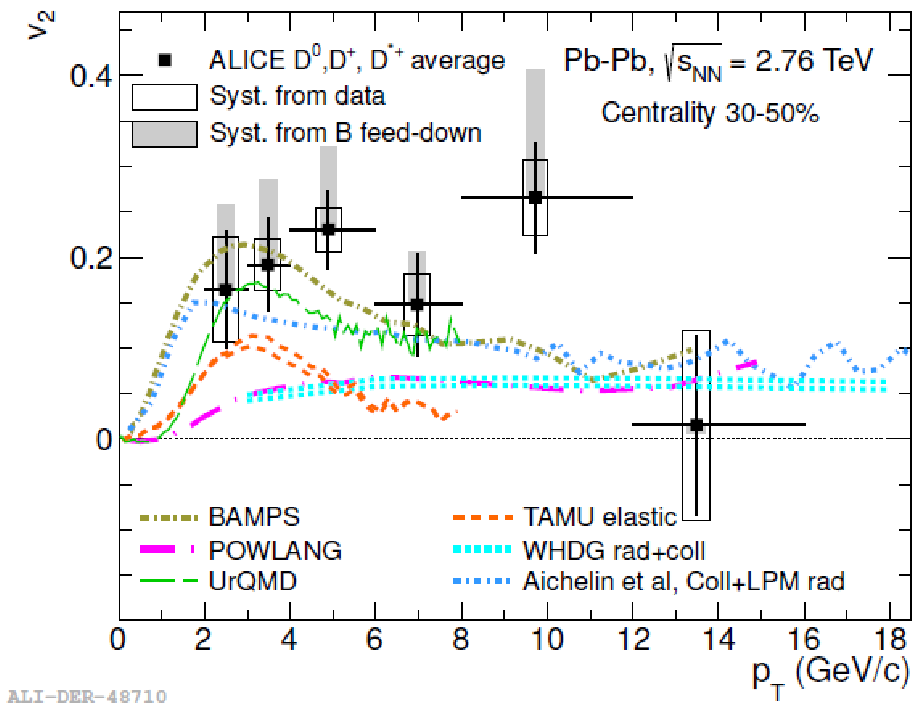
Simultaneous description of R_{AA} and v_2 is a tough challenge for all models

Elliptic flow at LHC

JHEP **1209** (2012) 112



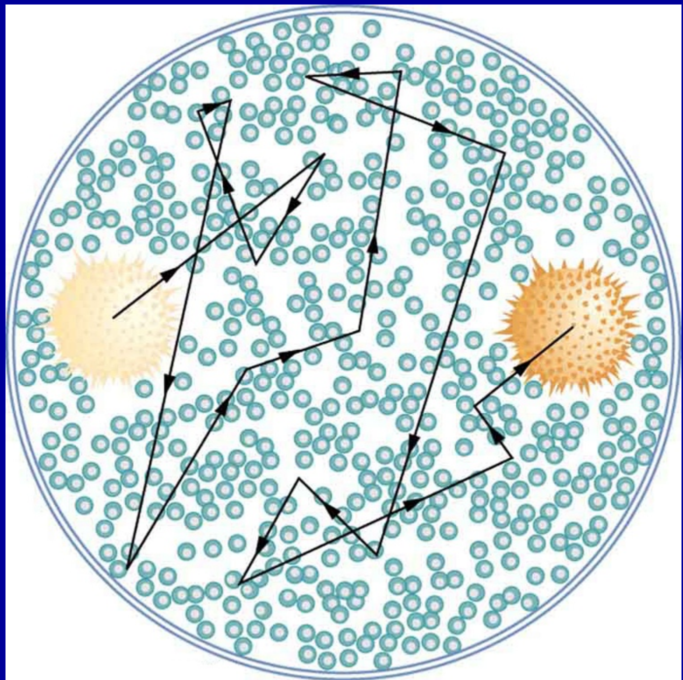
arXiv:1305.2707



Simultaneous description of R_{AA} and v_2 is a tough challenge for all models

Description of HQ propagation in the QGP

Brownian Motion



It is described by the Fokker-Planck equation

Transport theory

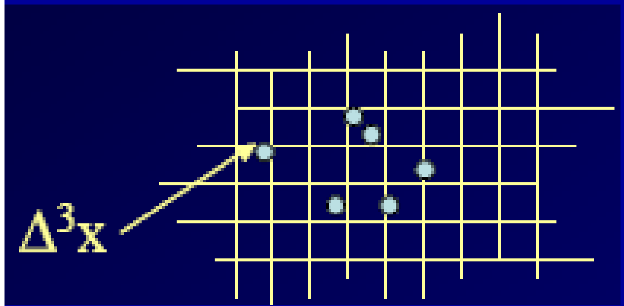
$$\underbrace{p^{\mu}\partial_{\mu} f(x,p)}_{\text{Free-streaming}} + \underbrace{M(X)\partial_{\mu} M(X)\partial_{p}^{\mu} f(X,p)}_{\text{Mean Field}} = \underbrace{C_{22}}_{\text{Collisions}}$$

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla_{r} f + \vec{F}\cdot\nabla_{p} f = C_{22}$$

Classic Boltzmann equation

It is valid to study the evolution of both bulk and Heavy quarks

Describes the evolution of the one body distribution function f(x,p)



To solve numerically the B-E we divide the space into a 3-D lattice and we use the standard test particle method to sample f(x,p)

[Z. Xhu, et al… PRC71(04)],[Ferini, et al. PLB670(09)], [Scardina,et al PLB724(13)],[Ruggieri,et al PLB727(13)]

Transport theory

✓ **Collision integral**

$$C_{22} = \int d^3k \left[\omega(p+k,k) f_{HQ}(p+k) - \omega(p,k) f_{HQ}(p)\right]$$

$$\omega(p,k) = g \int \frac{d^3q}{(2\pi)^3} f'(q) v_{rel} \sigma_{p,q \to p-k,q+k}$$

ω(p,k) is the transition rate for collisions of HQ with heath bath changing the HQ momentum from p to p-k

HQ with momentum p+k

HQ with momentum p-k

Gain term

$$\omega(p+k,k) f_{HQ}(p+k)$$

Loss term

$$\omega(p,k) f_{HQ}(p)$$

HQ with momentum p

Element of momentum space with momentum p

Transport theory

✓ **Collision integral (stochastic algorithm)**

Assuming two particle

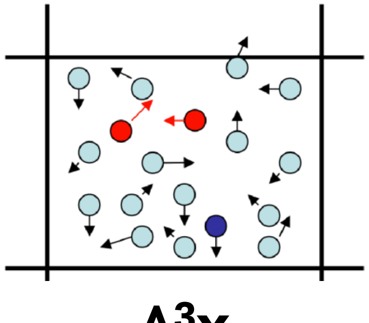
- **In a volume Δ^3x in space**
- **momenta in the range $(P,P+\Delta^3P)$; $(q,q+\Delta^3q)$**

$$\frac{(2\pi)^3\Delta N_{coll}}{\Delta t\Delta^3x\Delta^3p}=\frac{\Delta^3q}{(2\pi^3)}f_{HQ}(P)f_g(q)v_{rel}\sigma_{p,q\to p-k,q+k}$$

collision rate per unit phase space for this pair

$$\frac{(2\pi)^3\Delta N_{coll}}{\Delta t\Delta^3x\Delta^3P}=\frac{\Delta^3q}{(2\pi^3)}\frac{(2\pi)^3\Delta N_{HQ}}{\Delta^3x\Delta^3P}\frac{(2\pi)^3\Delta N_g}{\Delta^3x\Delta^3q}v_{rel}\sigma_{p,q\to p-k,q+k}$$

$$P_{22}=\frac{\Delta N_{coll}}{\Delta N_{HQ}\Delta N_g}=v_{rel}\sigma_{p,q\to p-k,q+k}\frac{\Delta t}{\Delta^3x}$$



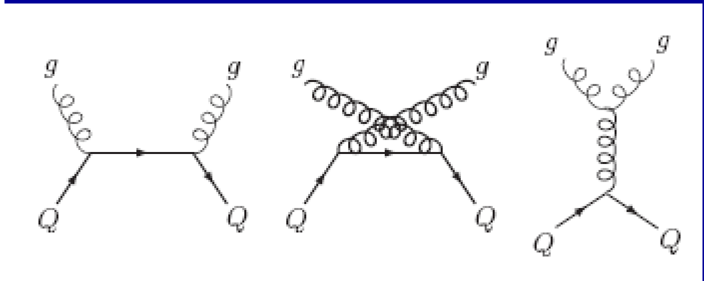
Δ^3x

$$\Delta t\to 0$$

$$\Delta^3x\to 0$$

Exact solution

Cross Section gc -> gc



The infrared singularity is regularized introducing a **Debye-screaning-mass m_D**

$$\frac{1}{t} \rightarrow \frac{1}{t - m_D}$$

$$m_D = \sqrt{4\pi\alpha_s}\,T$$

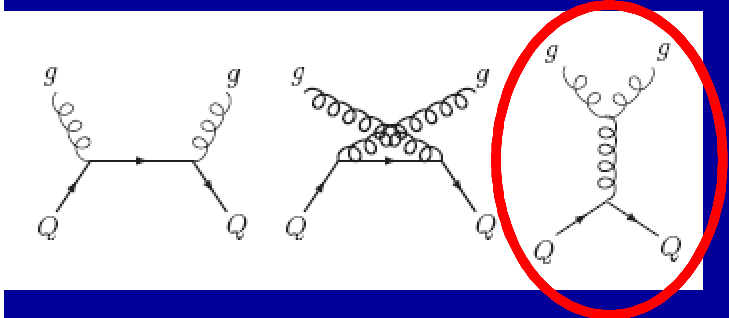
$$\sum|\mathcal{M}|^2 = \pi^2\alpha^2(Q^2)\left[\frac{32(s-M^2)(M^2-u)}{t^2} + \frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(s+M^2)}{(s-M^2)^2}\right.$$

$$+\frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(M^2+u)}{(M^2-u)^2} + \frac{16}{9}\frac{M^2(4M^2-t)}{(s-M^2)(M^2-u)}$$

$$\left.+16\frac{(s-M^2)(M^2-u)+M^2(s-u)}{t(s-M^2)} - 16\frac{(s-M^2)(M^2-u)-M^2(s-u)}{t(M^2-u)}\right].$$

[B. L. Combridge, Nucl. Phys. B151, 429 (1979)]
[B. Svetitsky, Phys. Rev. D 37, 2484 (1988)]

Cross Section gc -> gc



Dominant contribution

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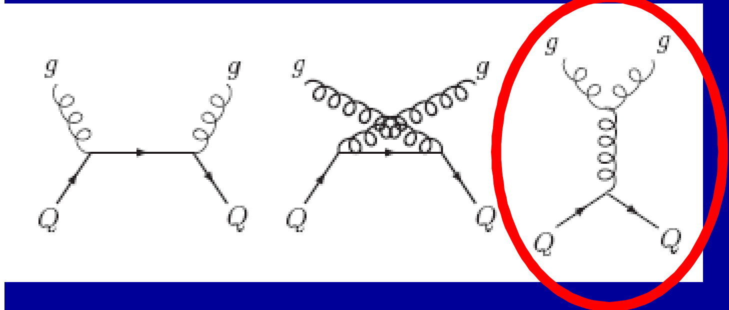

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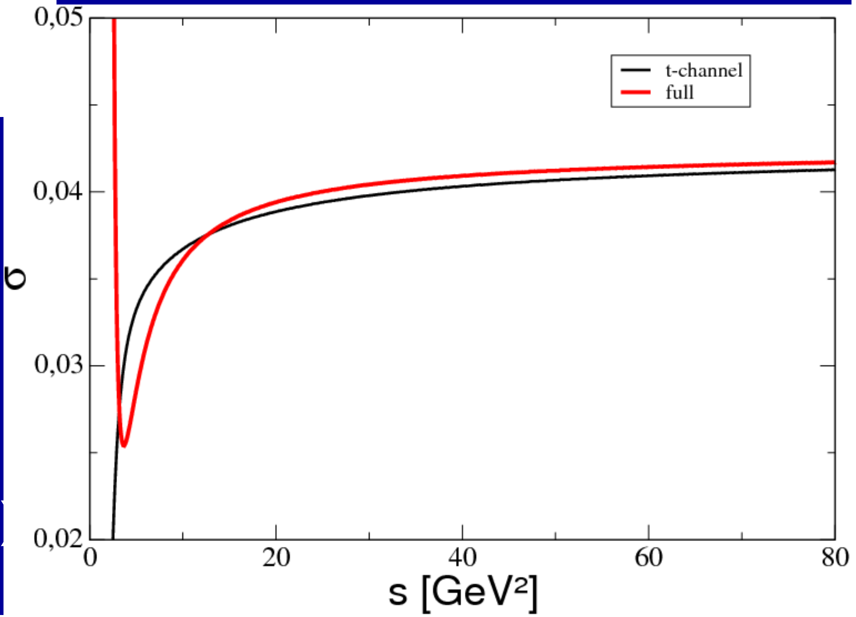
$$\sigma_{gc\to gc} = \frac{1}{16\pi\left(s-M_c^2\right)^2}\int_{-\left(s-M^2\right)^2/s}^{0} dt \sum |M|^2$$



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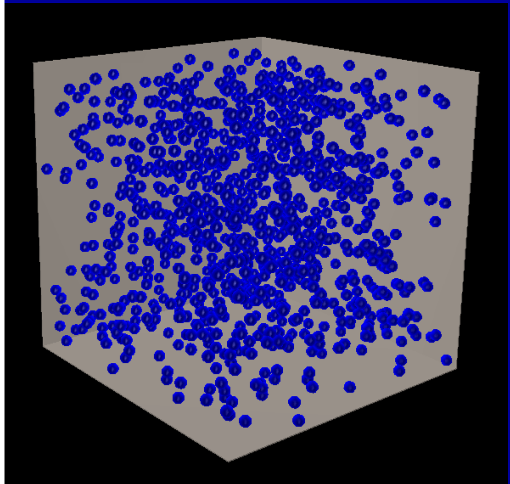
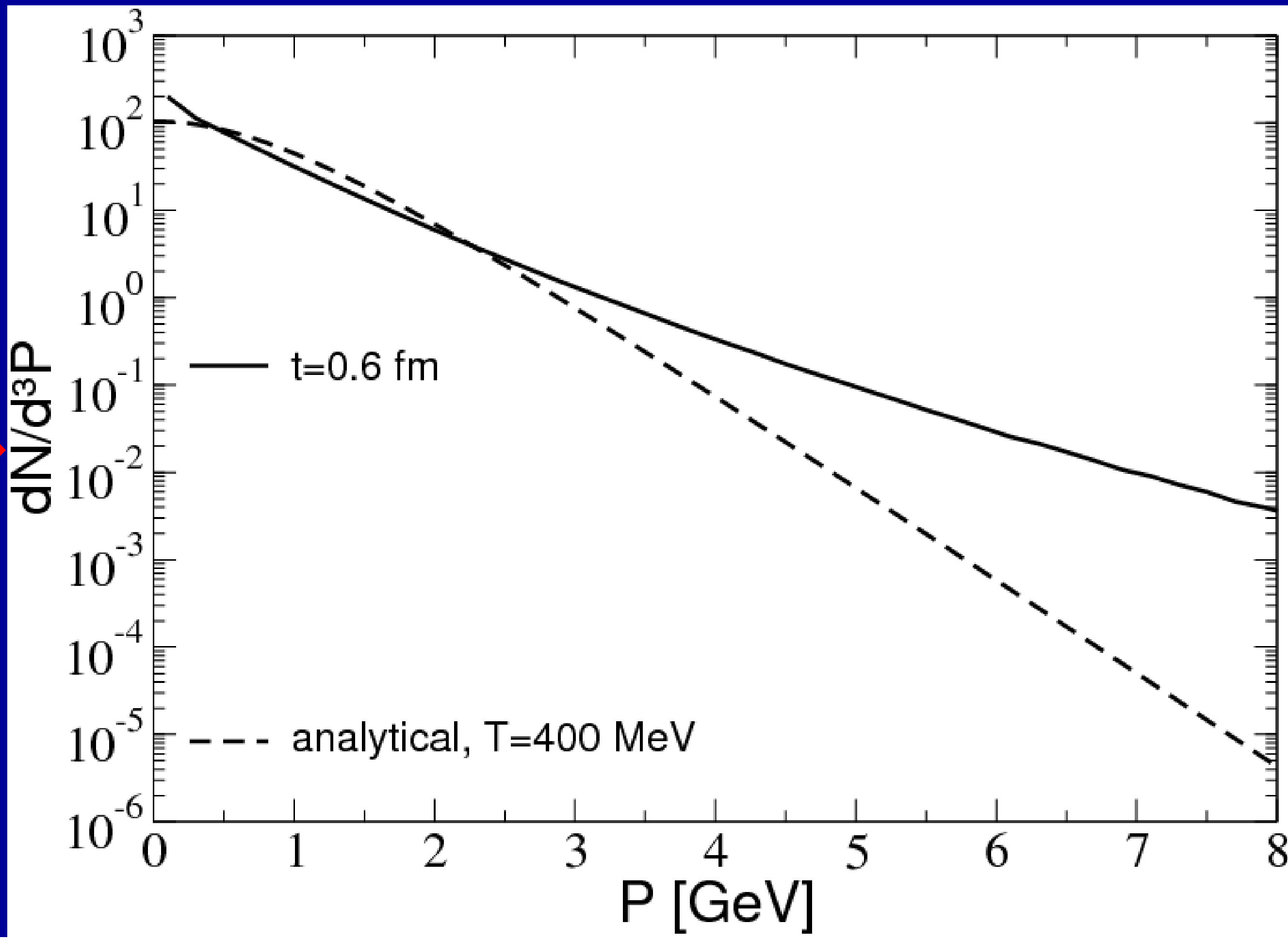
Charm evolution in a static medium



Simulations in which a particle ensemble in a **box** evolves dynamically

Bulk composed only by gluons in thermal equilibrium at T=400 MeV

C and $\overline{C}$ initially are distributed: uniformily in r-space, while in p-space



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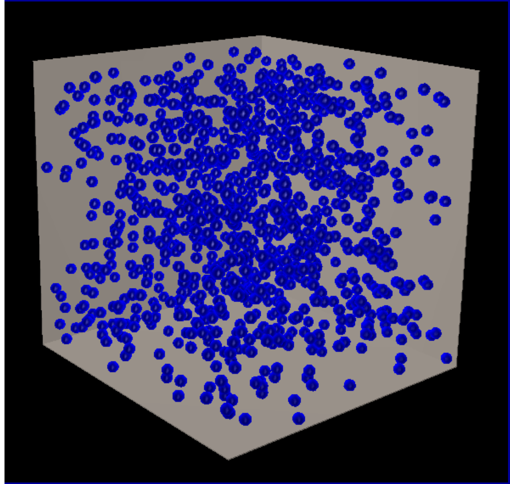

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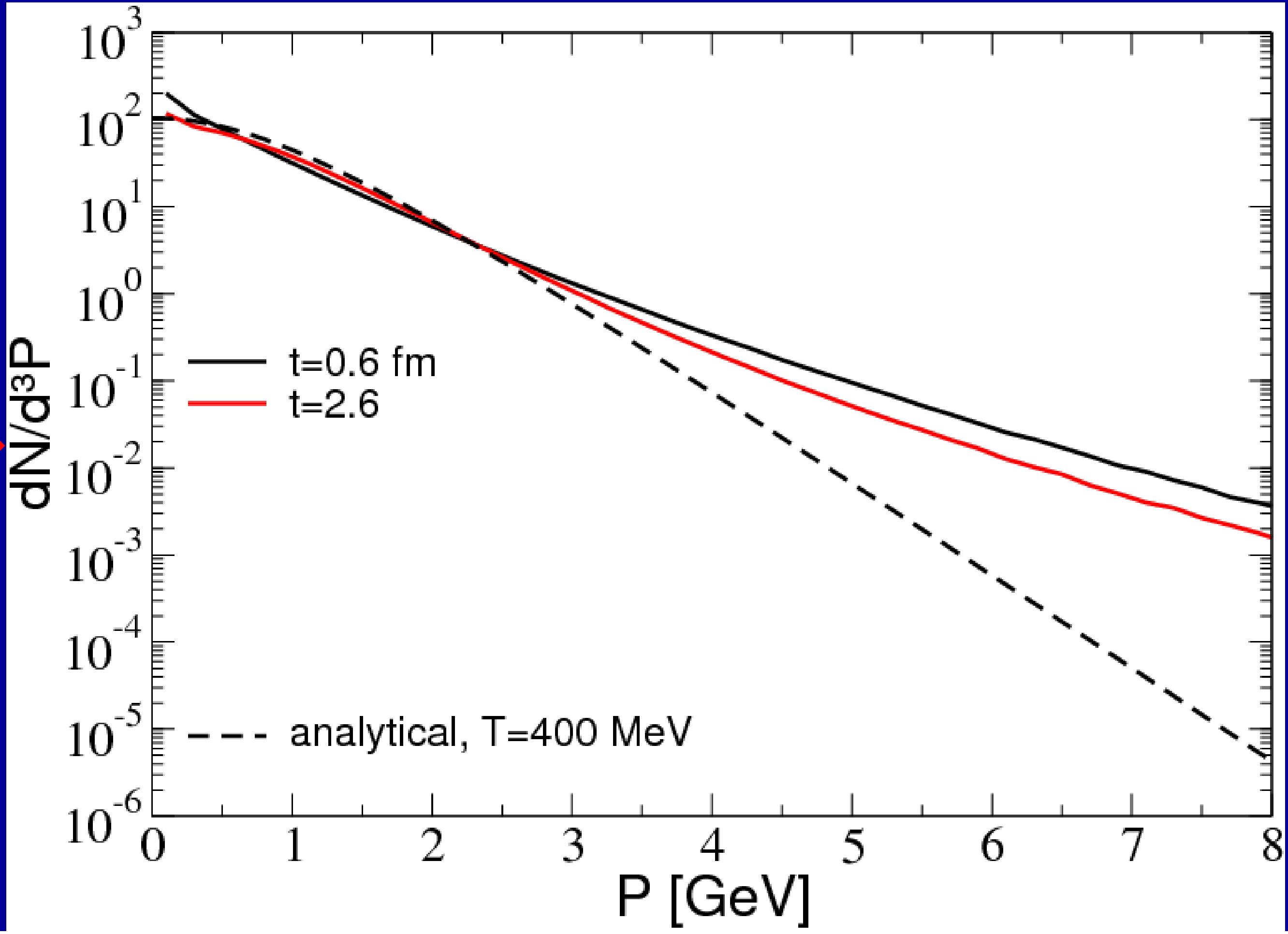

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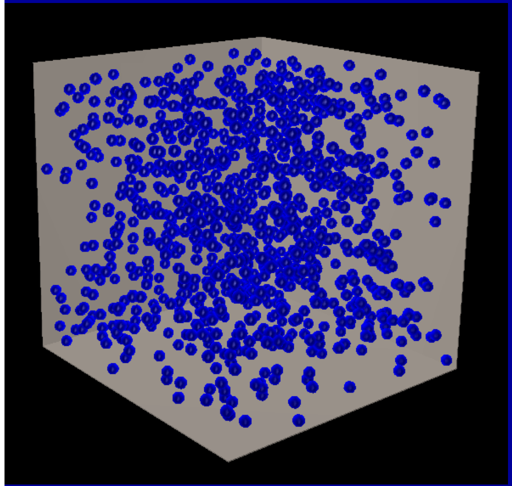

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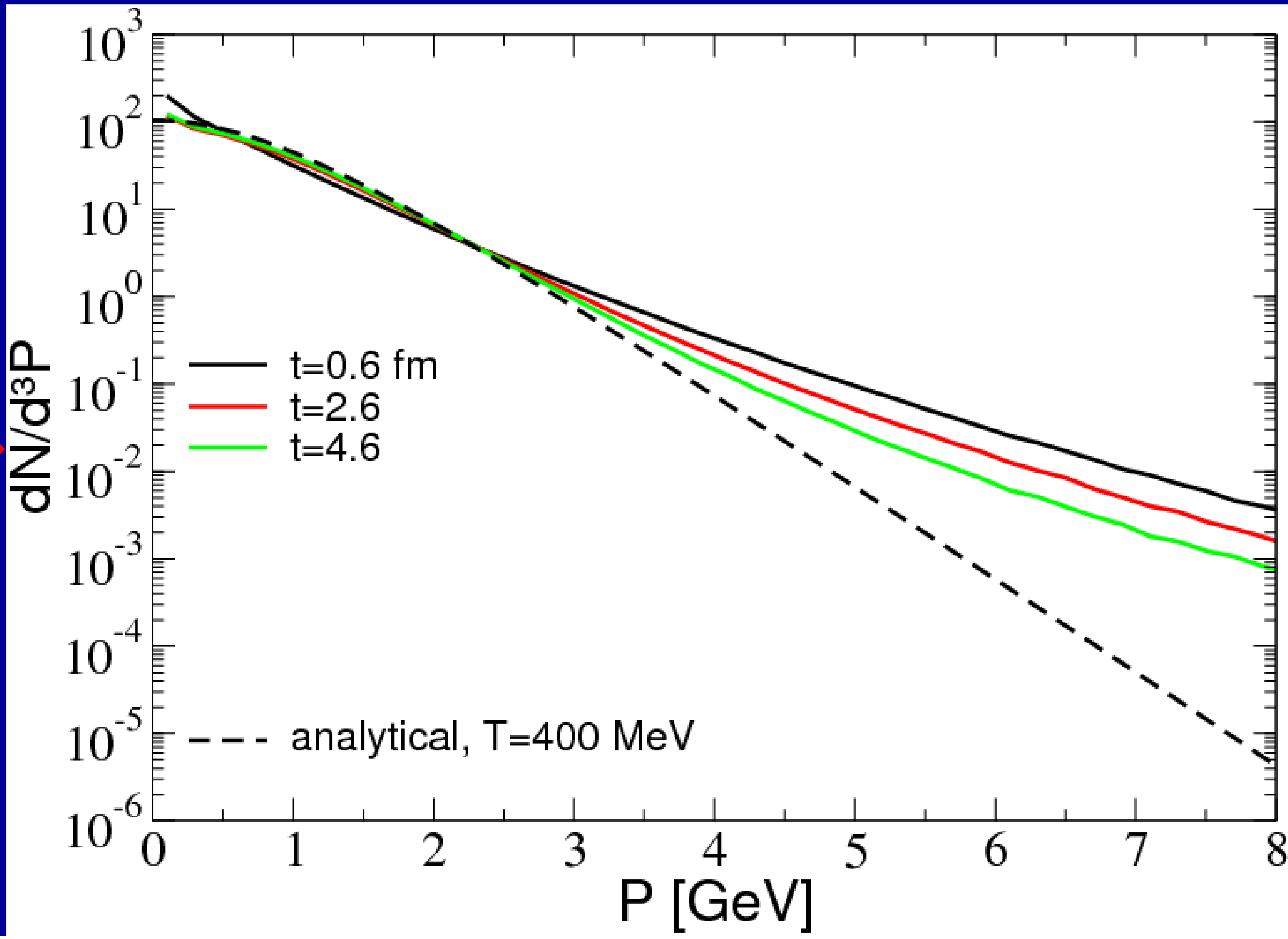

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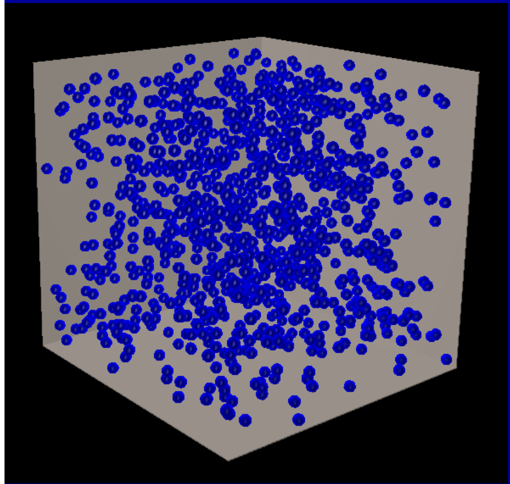

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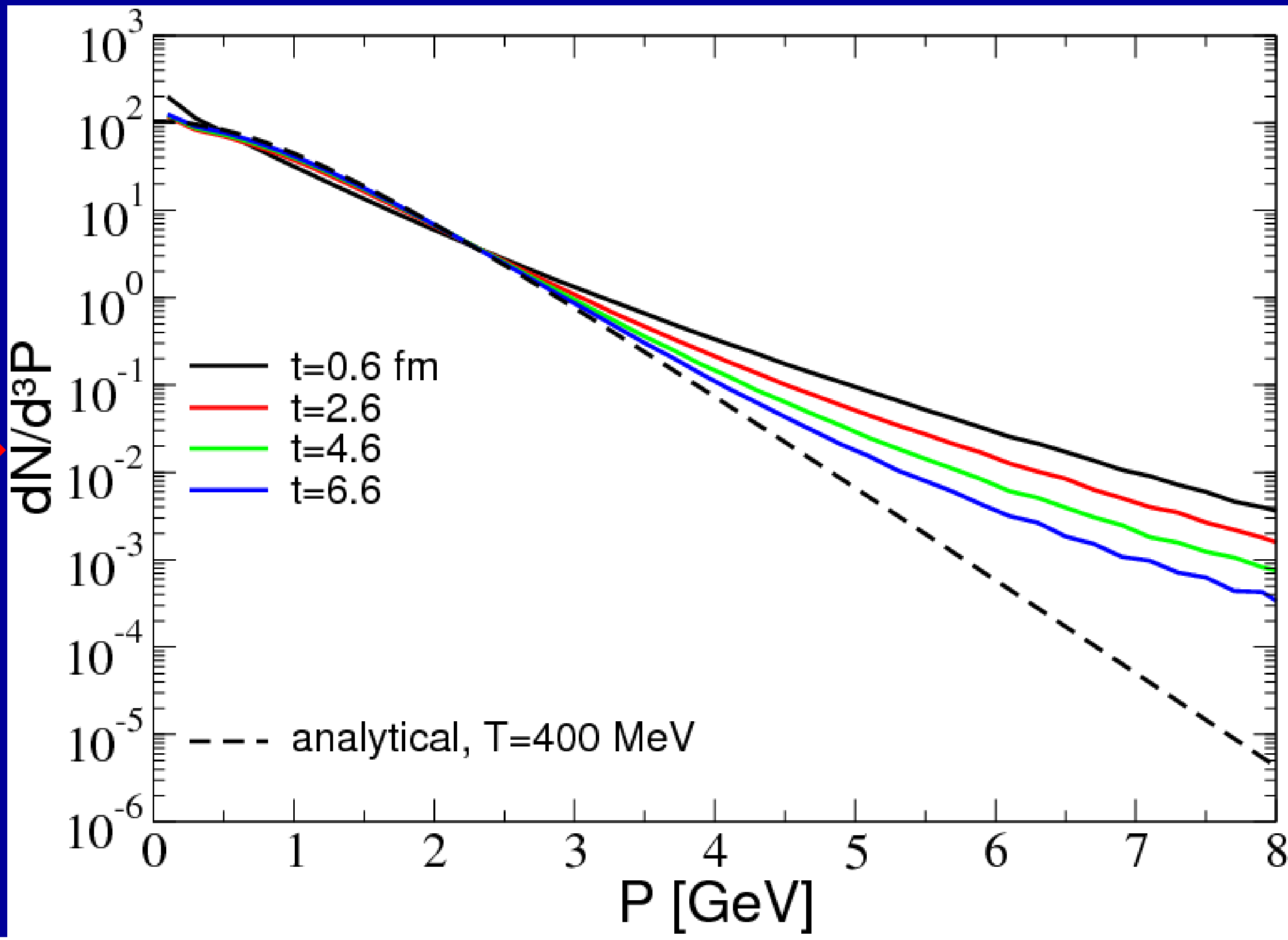

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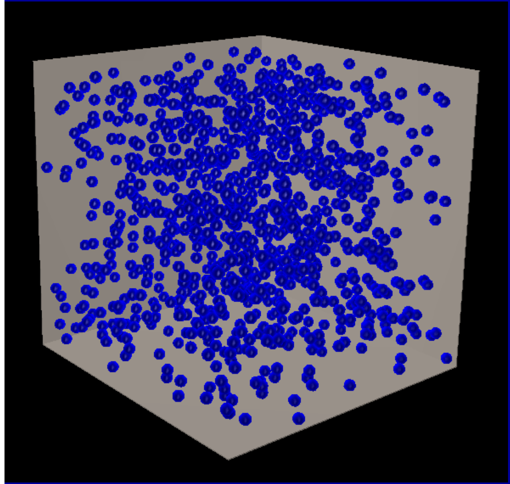

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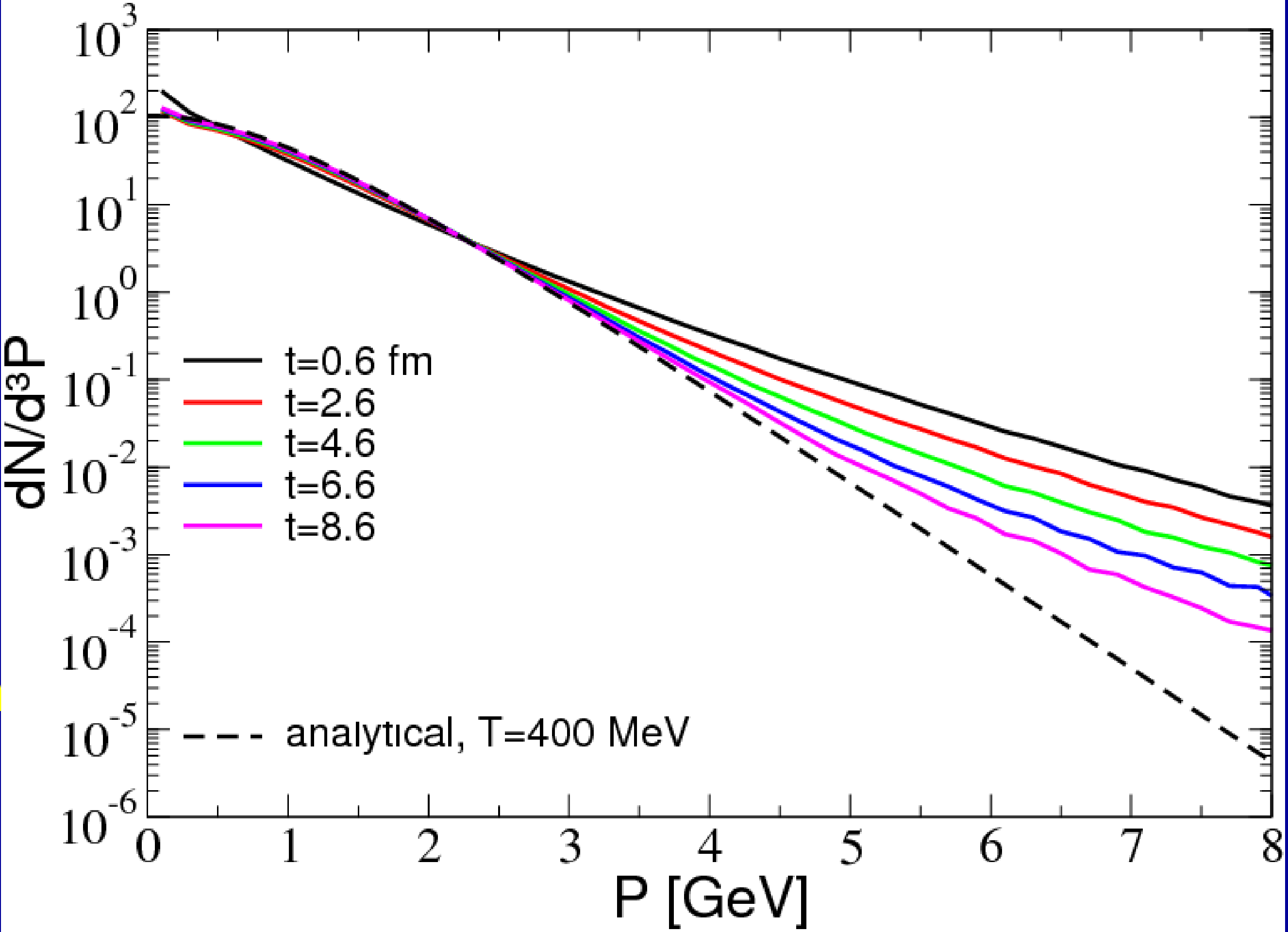
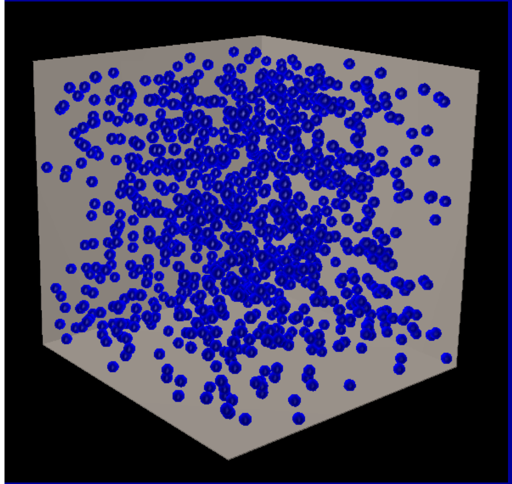


Due to collisions charm approaches to thermal equilibrium with the bulk

Charm evolution in a static medium



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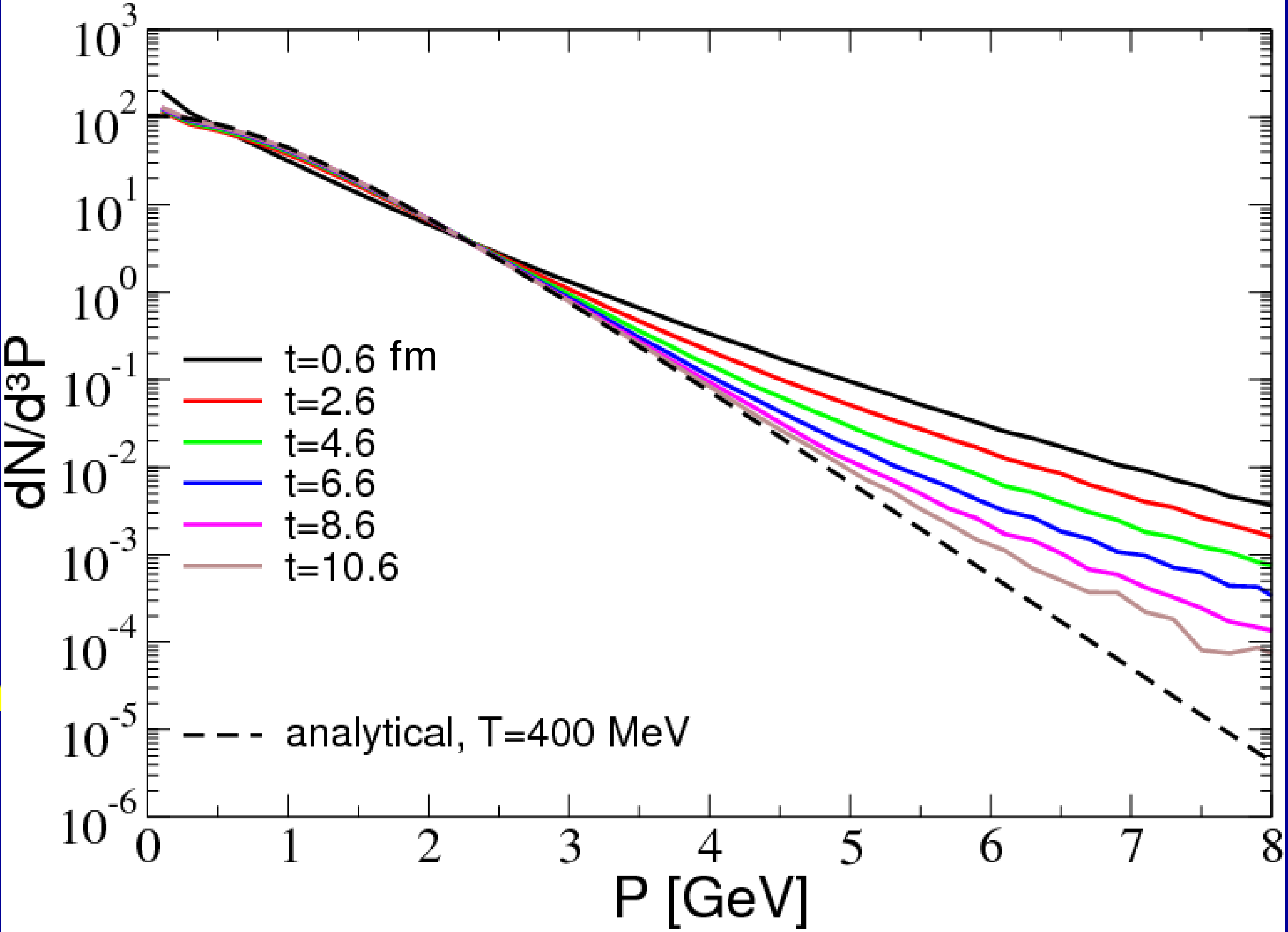
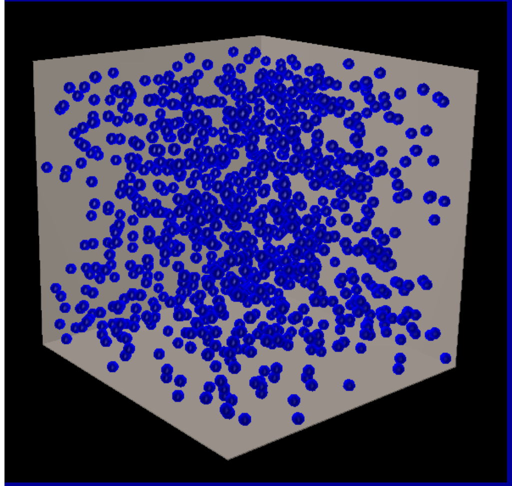
Bulk composed only by **gluons in thermal equilibrium at T=400 MeV**

C and $\bar{C}$ initially are distributed: **uniformly in r-space**, while in **p-space**



Due to **collisions charm** approaches to **thermal equilibrium** with the **bulk**

Charm evolution in a static medium



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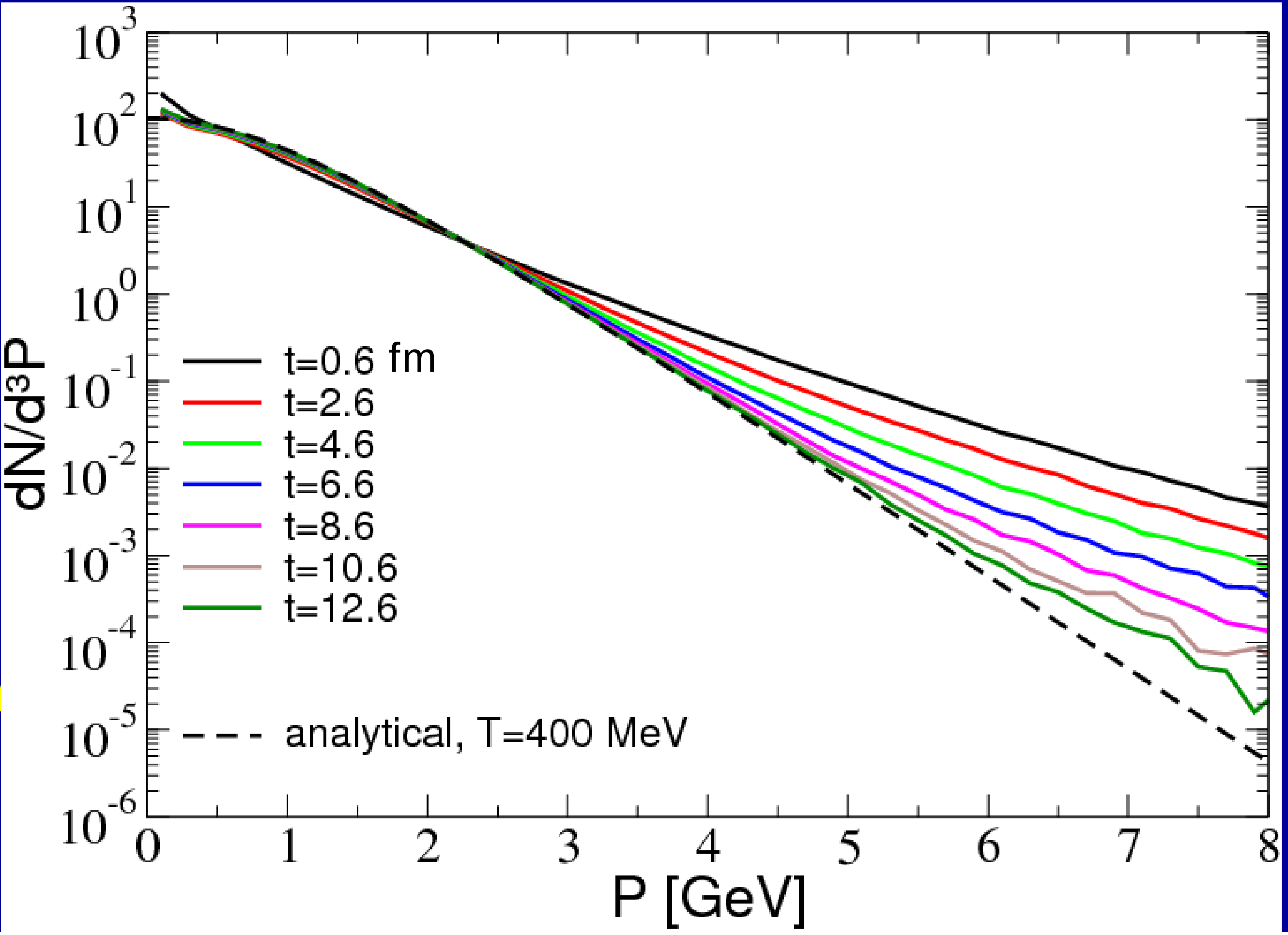

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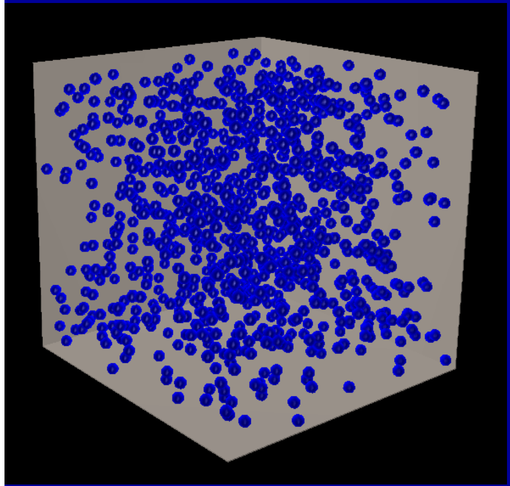

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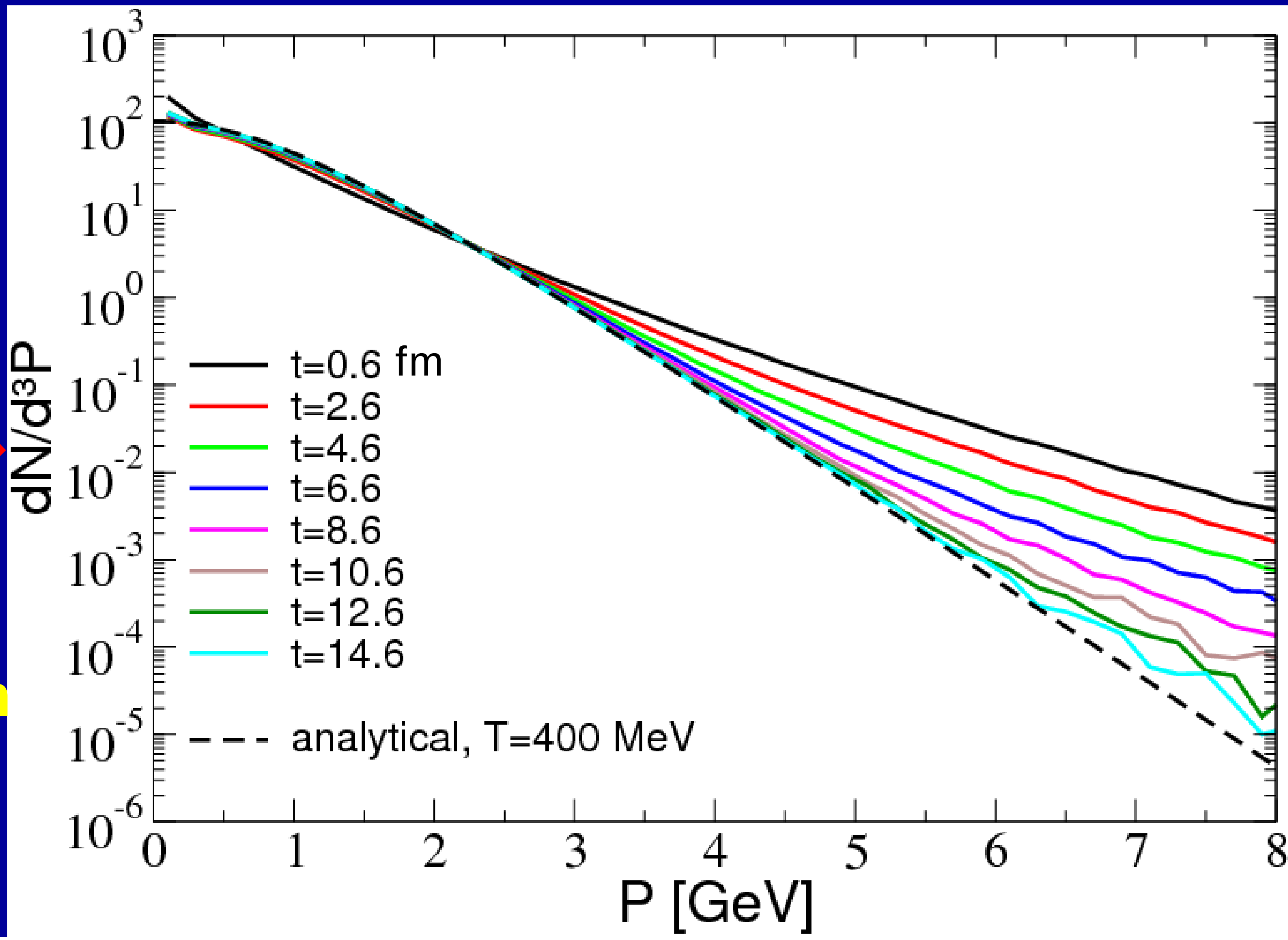

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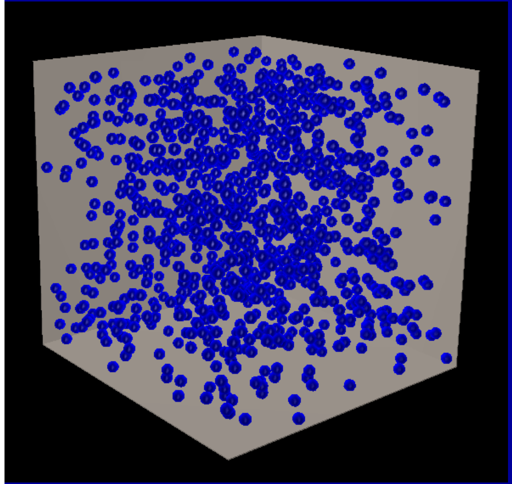

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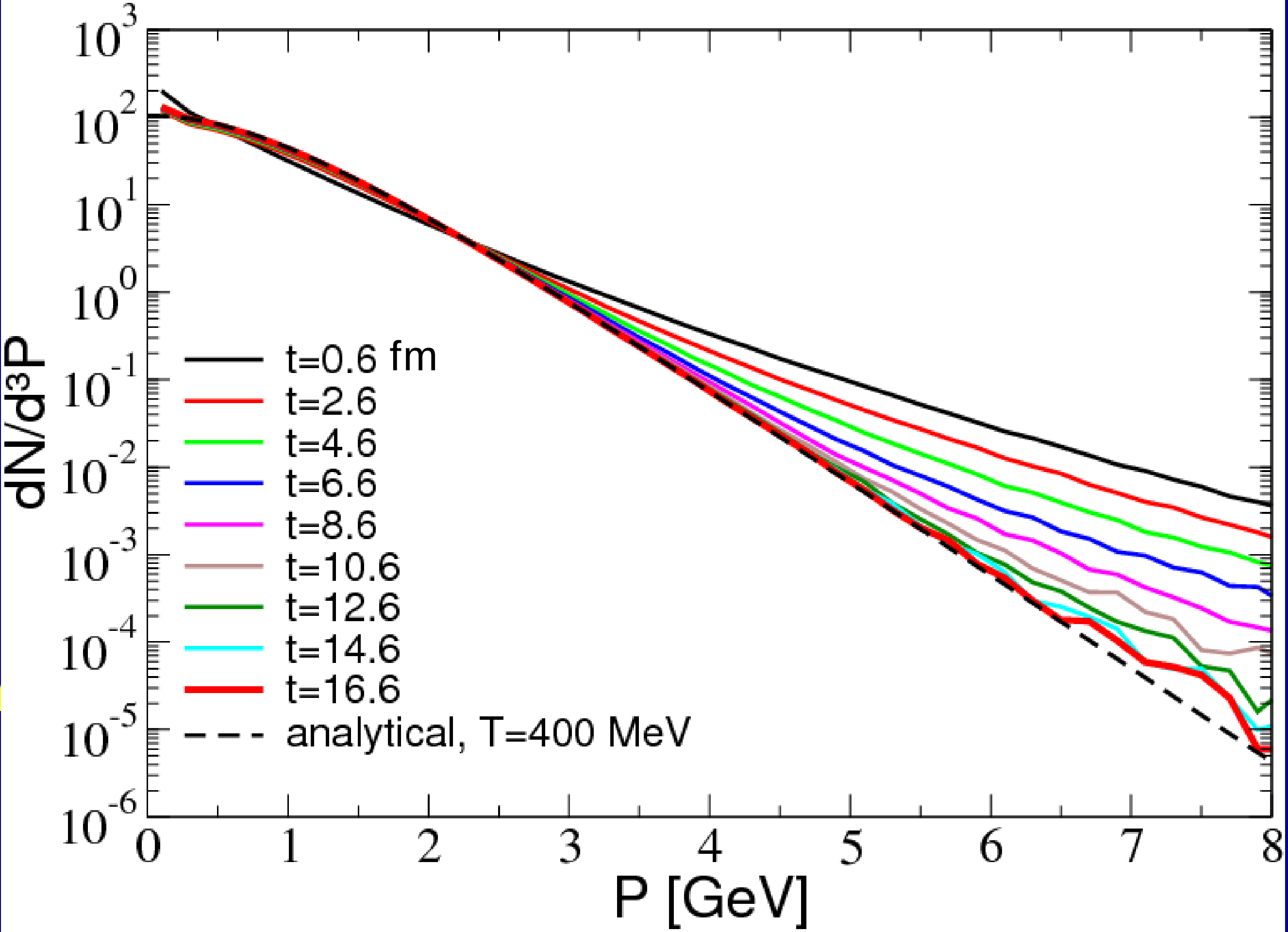
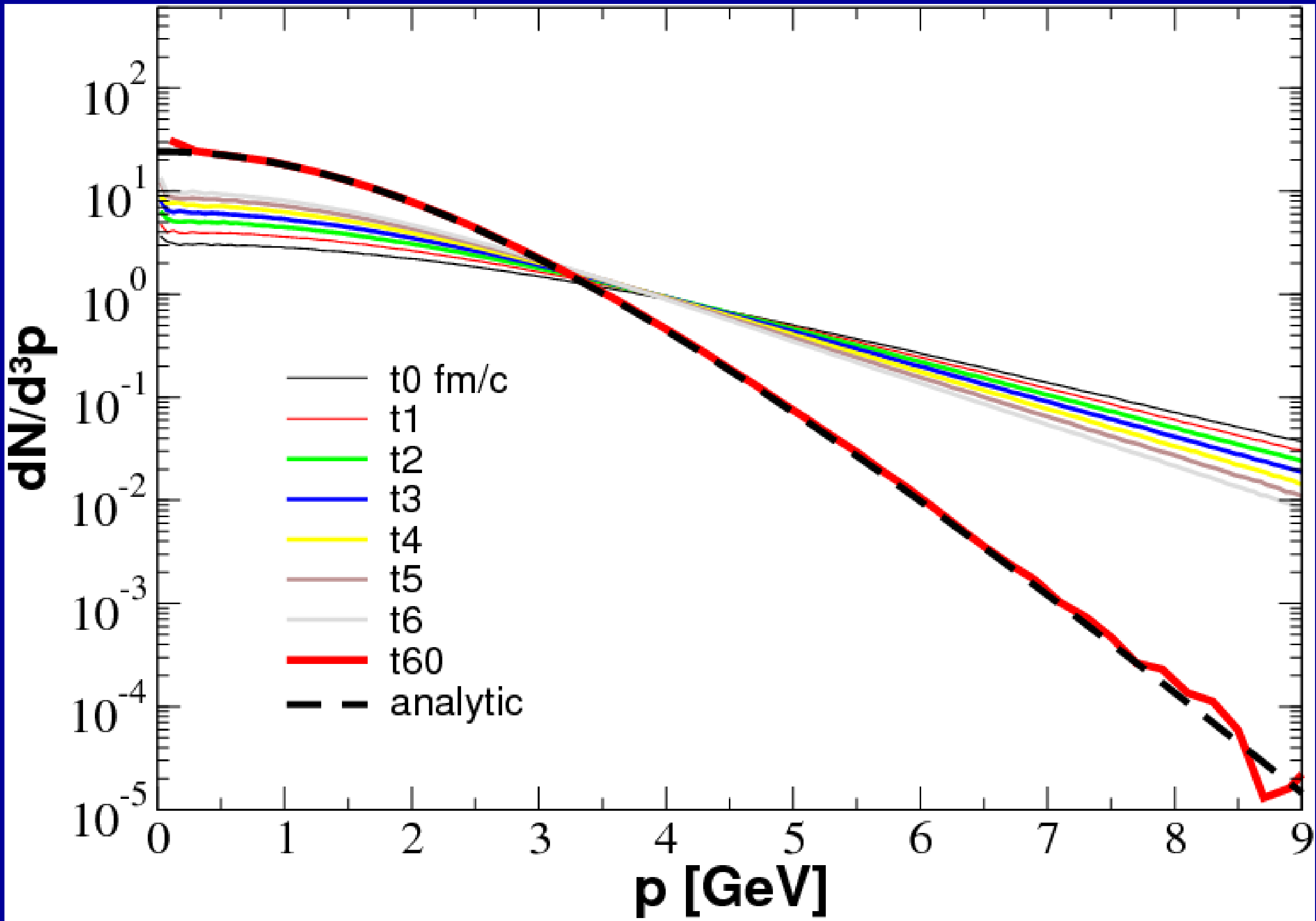


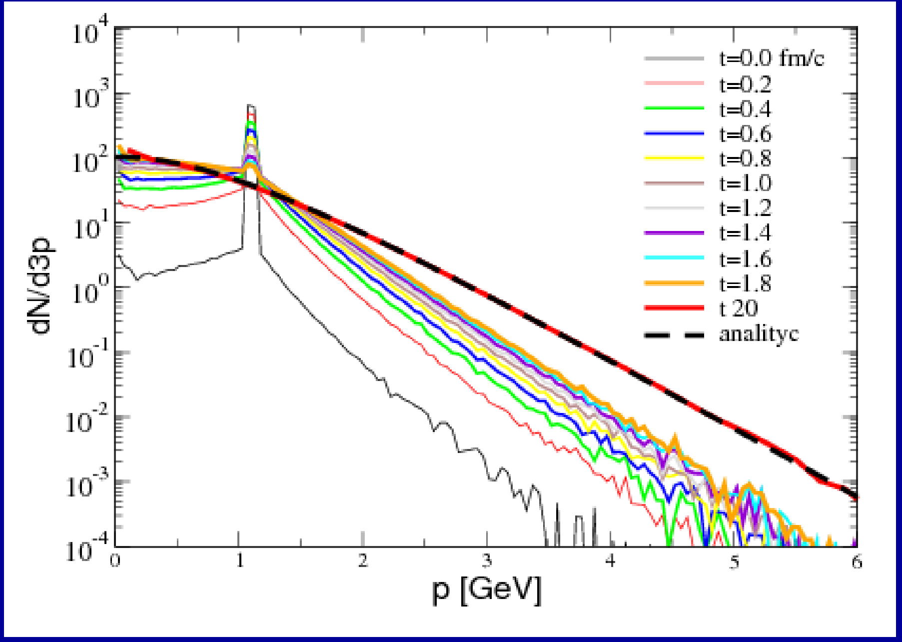
Due to collisions charm approaches to thermal equilibrium with the bulk

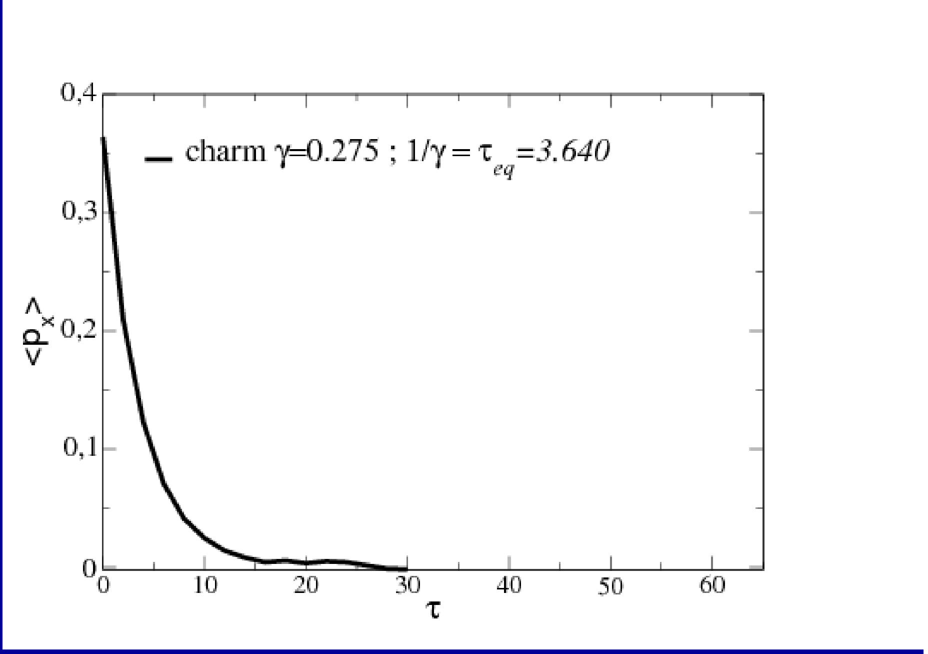
Bottom evolution in a static medium



mean momentum evolution in a static medium

We consider as initial distribution in p-space a $\delta(p-1.1GeV)$ for both C and B with $p_x=(1/3)p$





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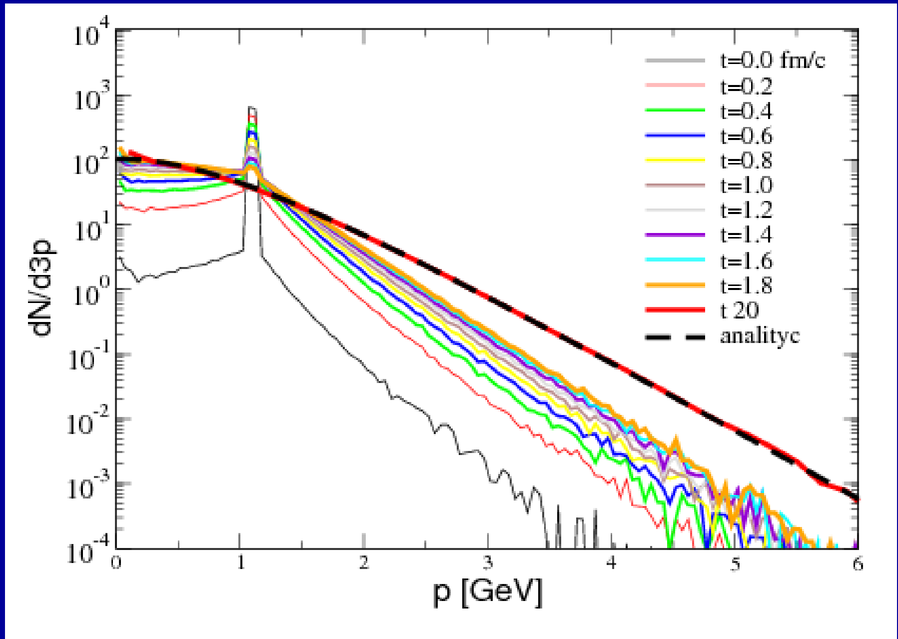
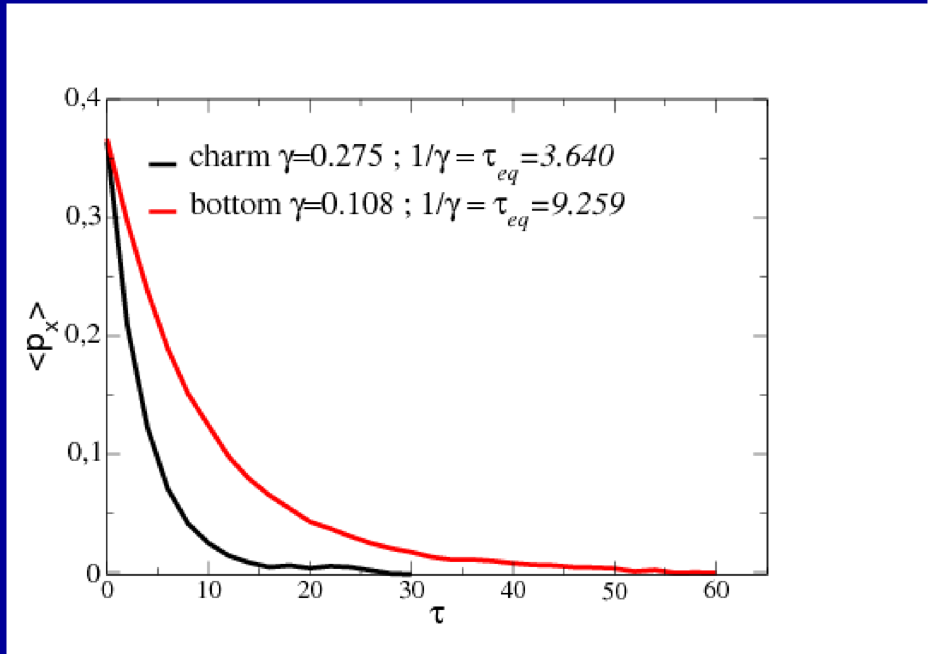




Each component of average momentum evolves according to $\langle p_i \rangle = p^0_i \exp(-\gamma t)$ where $1/\gamma$ is the relaxation time to equilibrium (τ)

$\tau_b/\tau_c = 2.55 \cong m_b/m_c$

Fokker Planck equation

The HQ propagation is usually described by the Fokker Planck approach where HQ interactions are conveniently encoded in transport coefficients that are related to elastic scattering matrix elements on light partons.

The Fokker Planck eq can be derived from the B-E

B-E $\left(\frac{\partial}{\partial t} + \frac{P}{E}\frac{\partial}{\partial x}\right) f(x,p,t) = C_{22}$ $\Rightarrow$ $\frac{\partial}{\partial t} f(p,t) = C_{22}$

$$C_{22} = \int d^3k \left[\omega(p+k,k) f(p+k) - \omega(p,k) f(p)\right]$$

If $|k| \ll |P|$

$$\omega(p+k,k) f(p+k) \approx \omega(p,k) f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

Fokker Planck equation

$$\omega(p+k,k)f(p+k) \approx \omega(p,k)f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

$$C_{22} \cong \int d^3k \left[k_i \frac{\partial}{\partial p_i}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j} \right] \omega(p,k) f(p)$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \frac{\partial}{\partial \mathbf{p}_i}\left[\mathbf{A}_i(\mathbf{p})\mathbf{f} + \frac{\partial}{\partial \mathbf{p}_j}[\mathbf{B}_{ij}(\mathbf{p})\mathbf{f}] \right]$$

where we have defined the kernels

$$\mathbf{A}_i = \int \mathbf{d}^3\mathbf{k}\, \omega(\mathbf{p},\mathbf{k})\mathbf{k}_i \rightarrow \textbf{Drag Coefficient}$$

$$\mathbf{B}_{ij} = \int \mathbf{d}^3\mathbf{k}\, \omega(\mathbf{p},\mathbf{k})\mathbf{k}_i\mathbf{k}_j \rightarrow \textbf{Diffusion Coefficient}$$

Where B_{ij} can be divided in a longitudinal and in a transverse component B_0 , B_1

[B. Svetitsky PRD 37(1987)2484]

Langevin Equation

The Fokker-Planck equation is equivalent to an ordinary stochastic differential equation

$$dx_j = \frac{p_j}{E} dt$$

$$dp_j = -\Gamma p_j dt + \sqrt{dt}\, C_{jk}(t, p + \xi dp)\rho_k$$

✓Γ is the deterministic friction (drag) force
✓ C_{ij} is a stochastic force in terms of independent Gaussian-normal distributed random variable

$$\rho = (\rho_x, \rho_y, \rho_z)$$

$$<\rho_i(t)> = 0$$

$$<\rho_i(t)\rho_k(t')> = \delta(t-t')\delta_{jk}$$

$$P(\rho) = \left(\frac{1}{2\pi}\right)^3 exp(-\frac{\rho^2}{2})$$

Interpretation of the momentum argument of the covariance matrix.

ξ=0 the pre-point Ito

ξ=1/2 mid-point Stratonovic-Fisk

ξ =1 the post-point Ito (or H¨anggi-Klimontovich)

Langevin Equation

Langevin process defined like this is equivalent to the Fokker-Planck equation:

$$\frac{\partial f}{\partial t} + \frac{p_j}{E}\frac{\partial f}{\partial x_j} = \frac{\partial}{\partial p_j}\left[\left(p_j\Gamma - \xi C_{lk}\frac{\partial C_{jk}}{\partial p_l}\right) f\right] + \frac{1}{2}\frac{\partial^2}{\partial p_j \partial p_k}(C_{jl}C_{kl} f)$$

the covariance matrix and Γ are related to the diffusion matrix and to the drag coefficient by

$$C_{jk} = \sqrt{2B_0}P^{\perp}_{jk} + \sqrt{2B_1}P^{\parallel}_{jk}$$

$$A_i = p_j\Gamma - \xi C_{lk}\frac{\partial C_{ij}}{\partial p_l}$$

For a process in which $B_0 = B_1 = D$

$$C_{jk} = \sqrt{2D(E)}\delta_{jk}$$

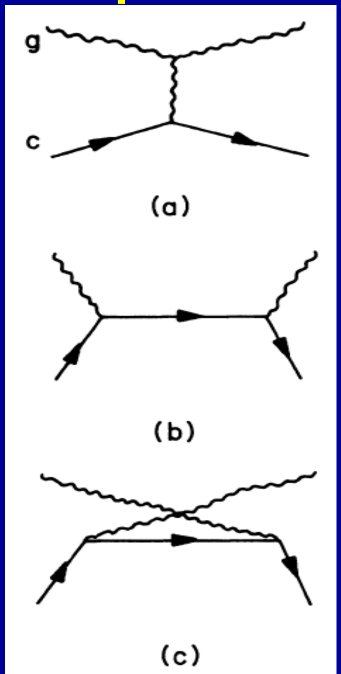
Evaluation of Drag and diffusion

For Collision Process the A_i and B_{ij} can be calculated as following :

$$A_i = \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_c}\sum|M|^2(2\pi)^4\delta^4(p+q-p'-q')f(q)[(p-p')_i] = \langle\langle(p-p')_i\rangle\rangle$$

$$B_{ij} = \frac{1}{2}\langle\langle(p-p')_i(p'-p)_j\rangle\rangle$$

Elastic processes



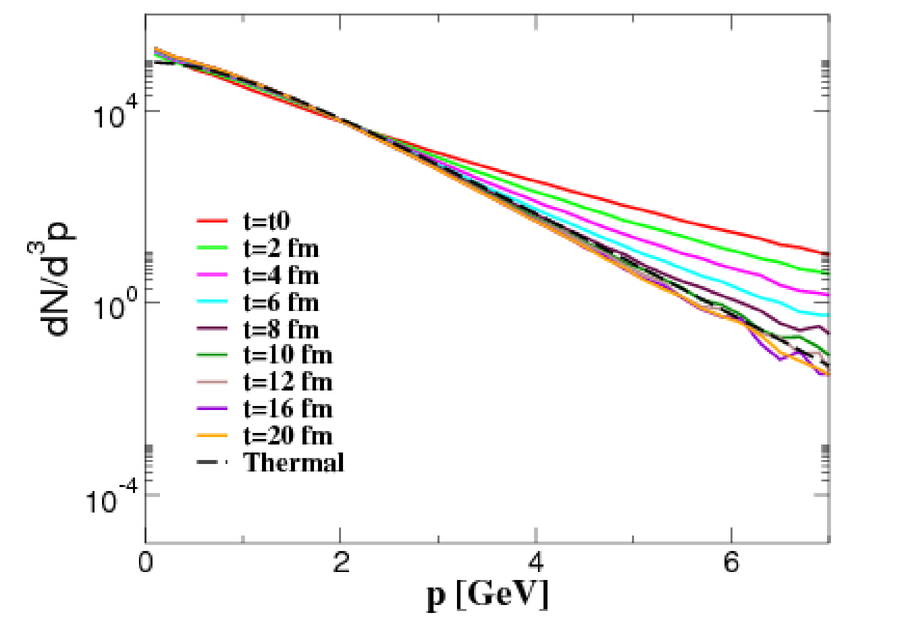
$gc \to gc$

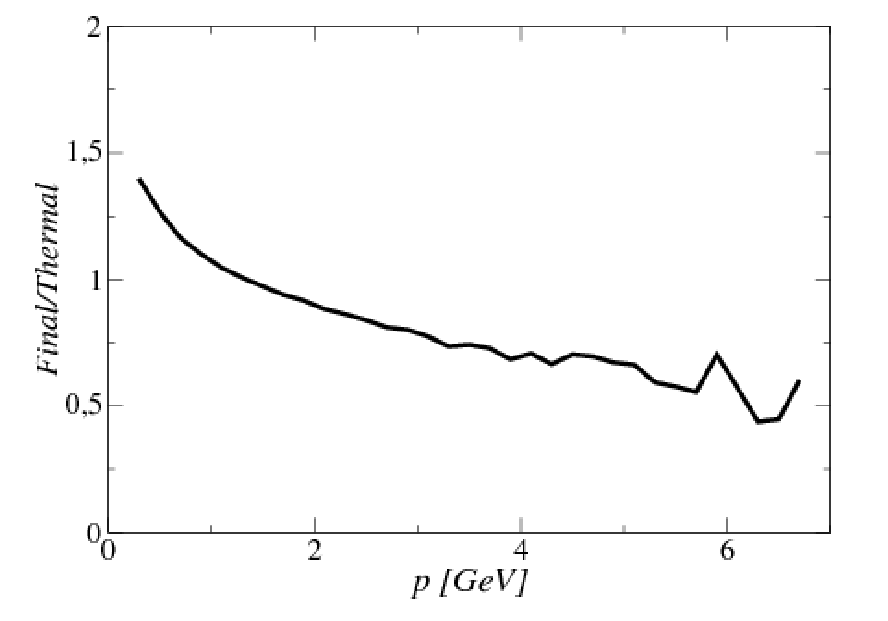
[B. Svetitsky PRD 37(1987)2484]

Charm propagation with the langevin eq

We solve Langevin Equation in a box in the identical environment of the B-E

Bulk composed only by gluon in Thermal equilibrium at T= 400 MeV.





The long-time solution of the Fokker Planck equation does not reproduces the equilibrium distribution (we are away from thermalization around 35-40 % at intermediate p_T).

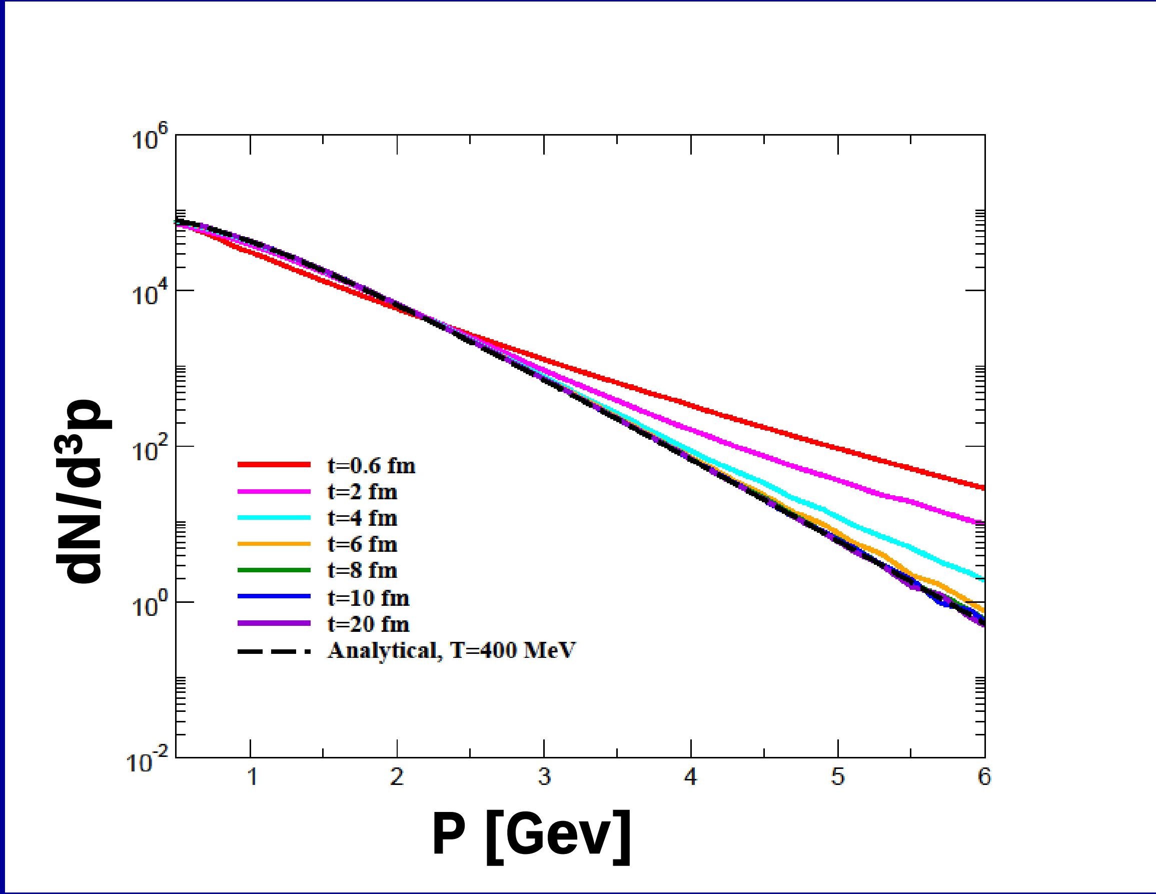
This is however a well-know issue related to the Fokker Planck

Charm propagation with the langevin eq

The long time solution is recovered relating the Drag and Diffusion coefficent by mean of the fluctuation dissipation relation $A(E)ET - D(E) + T(1-\xi)D'(E) = 0$

✓ **Imposing the simple relativistic dissipation-fluctuation relations**

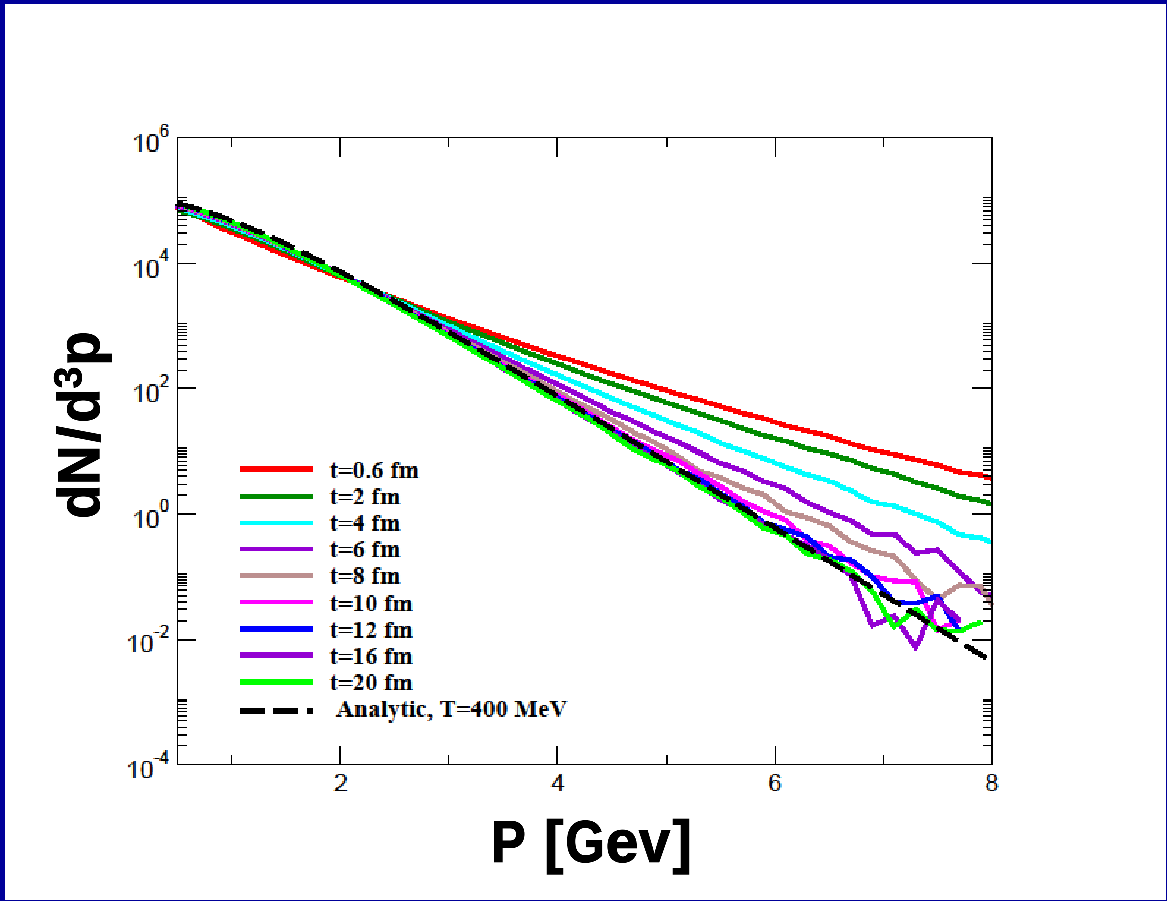
D=Constant
A= D/ET from FDT



Charm propagation with the langevin eq

✓ **Imposing the full relativistic dissipation-fluctuation relations**

D(E) $$A(E)ET - D(E) + T(1-\xi)D'(E) = 0$$

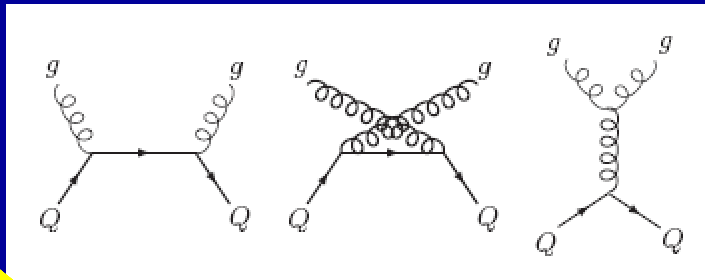


Charm propagation with the langevin eq

We are not interested into recover the long time solutions since the life-time of QGP is smaller then the thermalization time of the HQ

We want instead to relate the Drag and diffusion coefficent to the microscopic details of the collsion between HQ and the bulk

Langevin approach



Boltzman approach

M -> A_i, B_{ij}

M -> σ

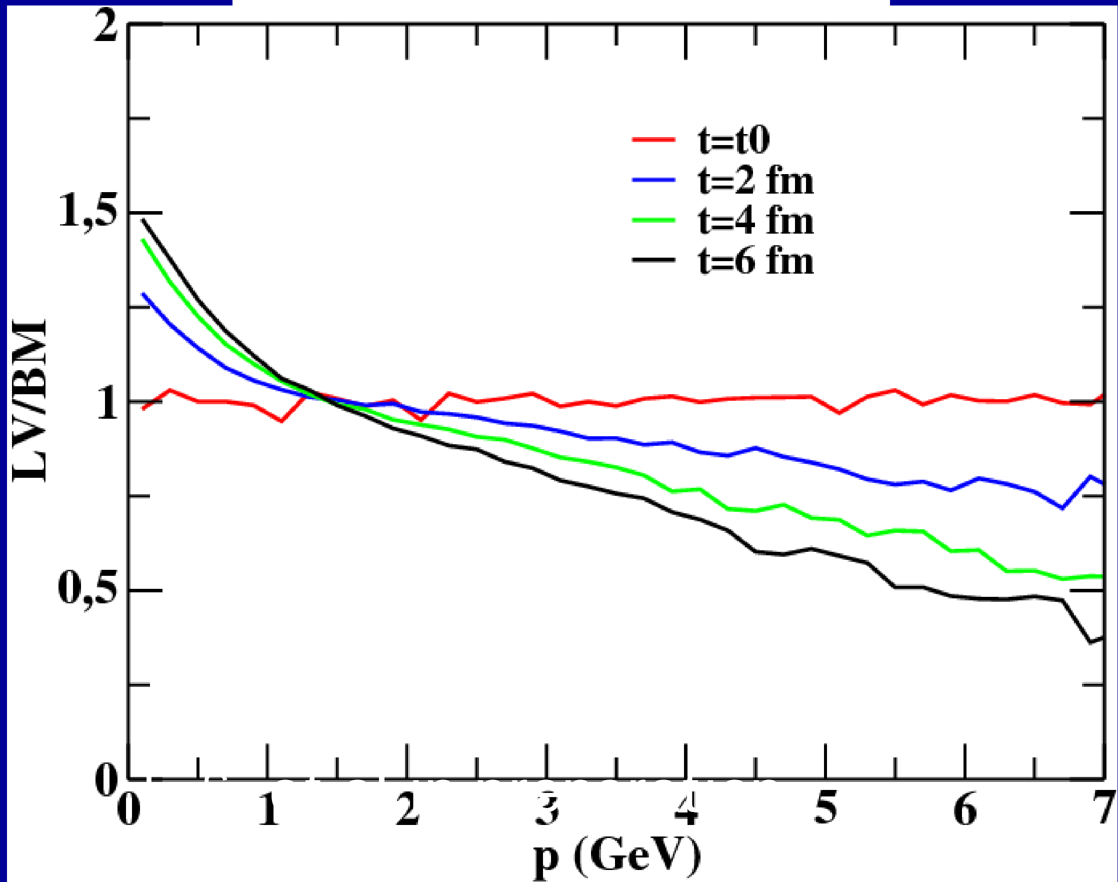
$$A_i = \frac{1}{2E_p}\int \frac{d^3q}{(2\pi)^3 2E_q}\int \frac{d^3q'}{(2\pi)^3 2E_{q'}}\int \frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_c}\sum|M|^2$$
$$(2\pi)^4\delta^4(p+q-p'-q')f(q)[(p-p')_i] = \langle\langle (p-p')_i\rangle\rangle$$

$$\sigma_{gc\to gc} = \frac{1}{16\pi\left(s-M_c^2\right)^2}\int_{-\left(s-M^2\right)^2/s}^{0} dt\sum|M|^2$$

$$B_{ij} = \frac{1}{2}\langle\langle (p-p')_i (p'-p)_j\rangle\rangle$$

Boltzmann vs Langevin (Charm)

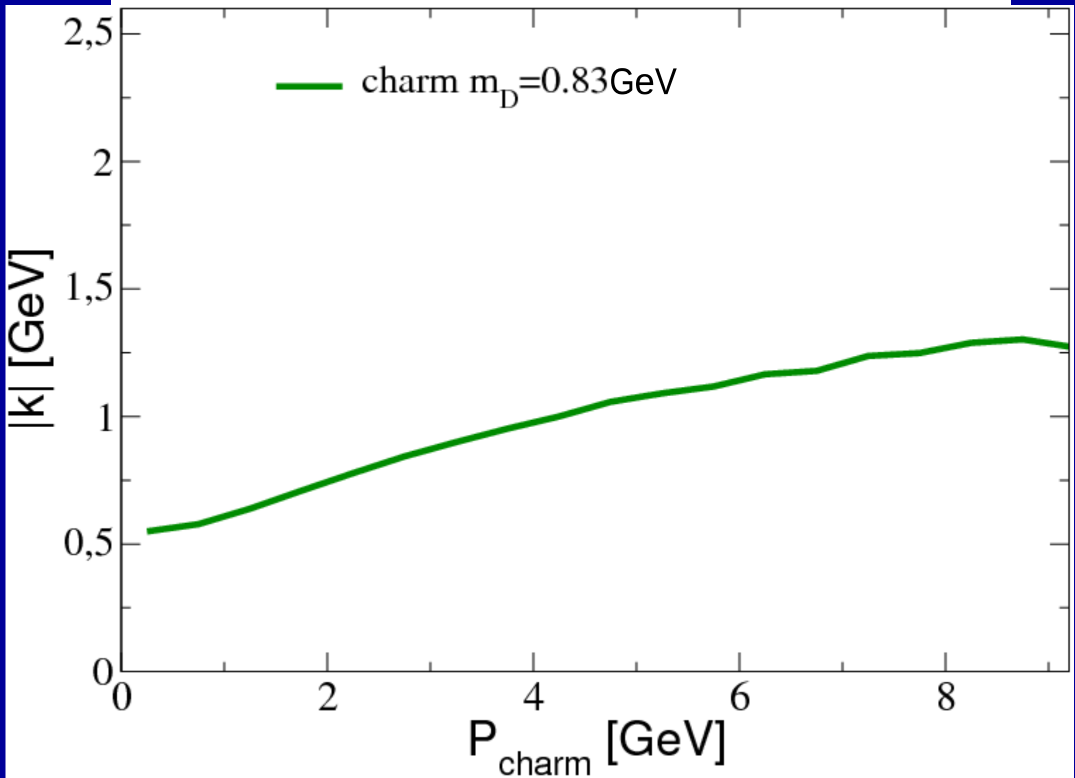
$$\left.\frac{dN}{d^3p}^{Langevin}\right/\frac{dN}{d^3p}^{Boltzmann}$$



[F. S. et all. in preparation]

Boltzmann vs Langevin (Charm)

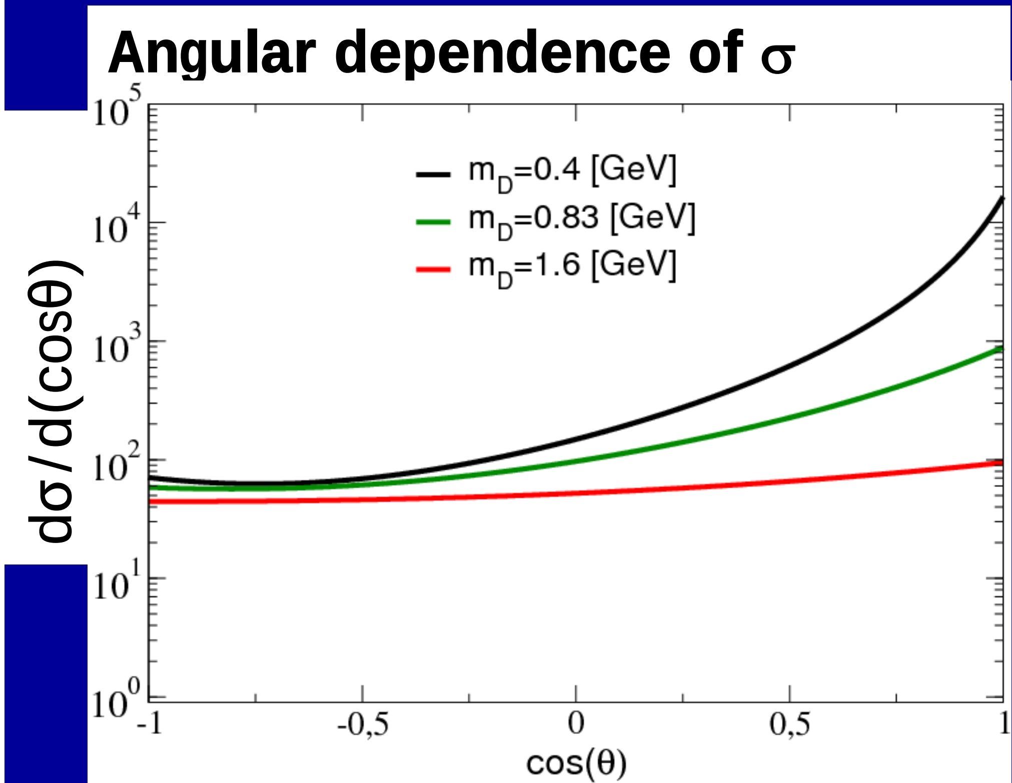
Mometum transfer vs P

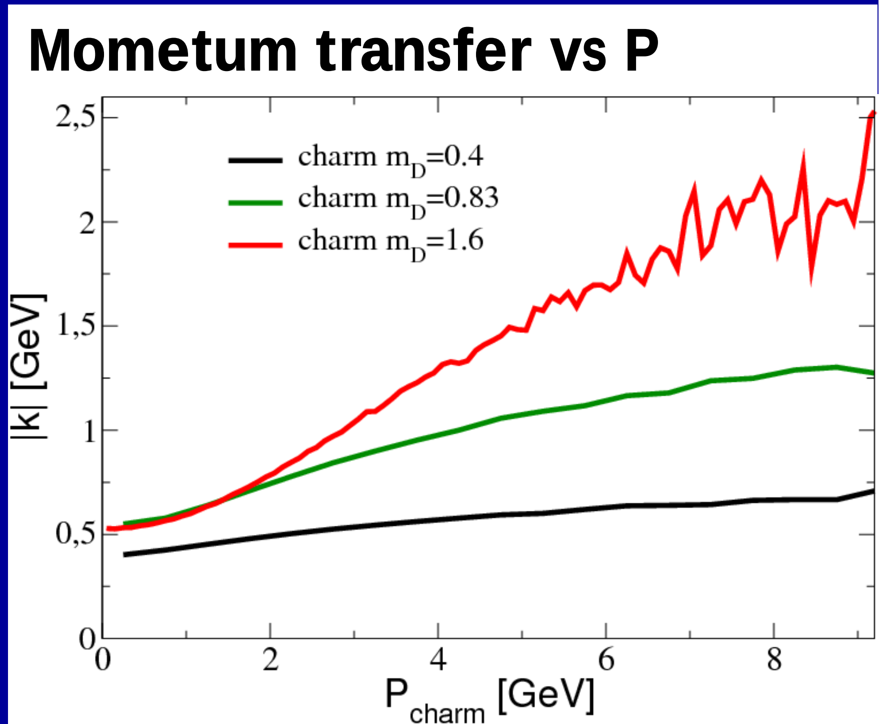


m_D=gt=0.83GeV

Boltzmann vs Langevin (Charm)

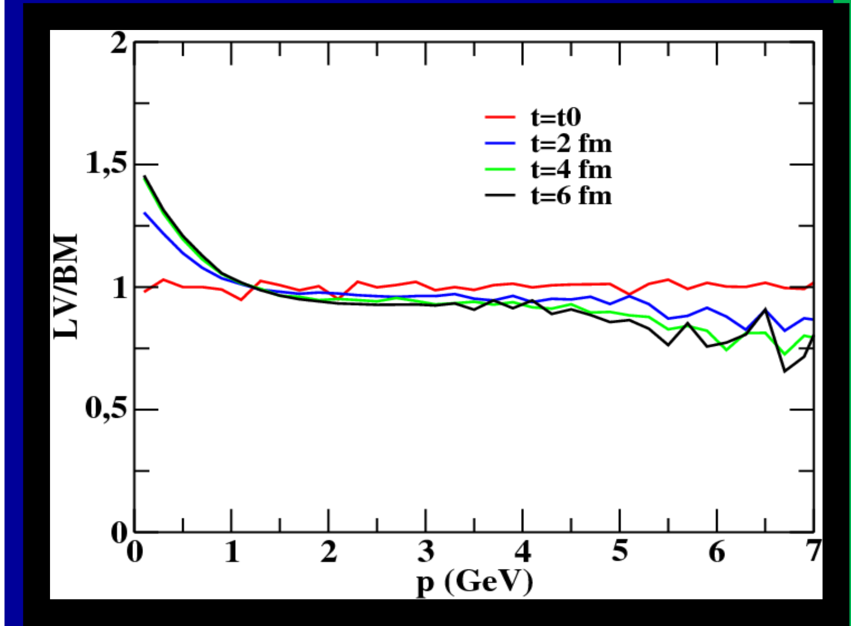
- simulating different average momentum transfer

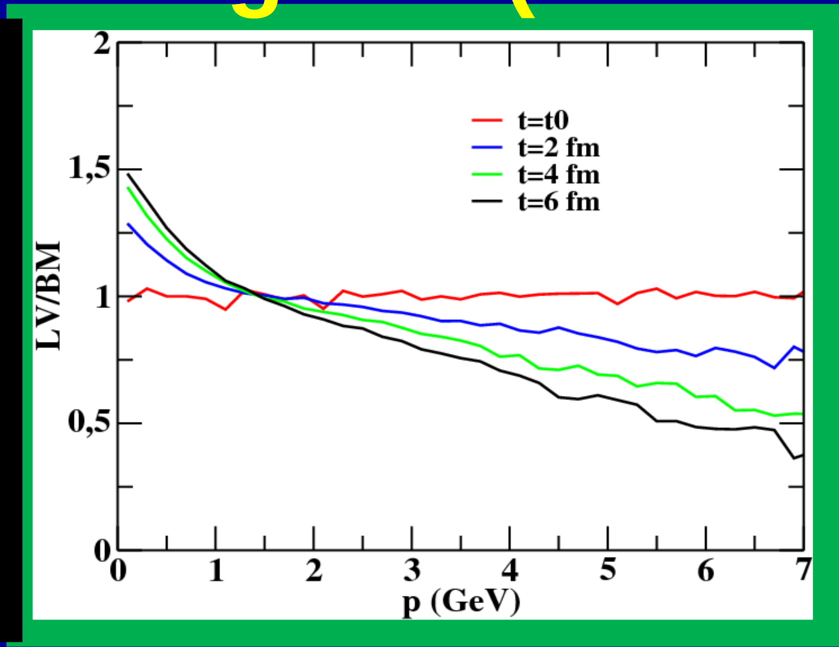


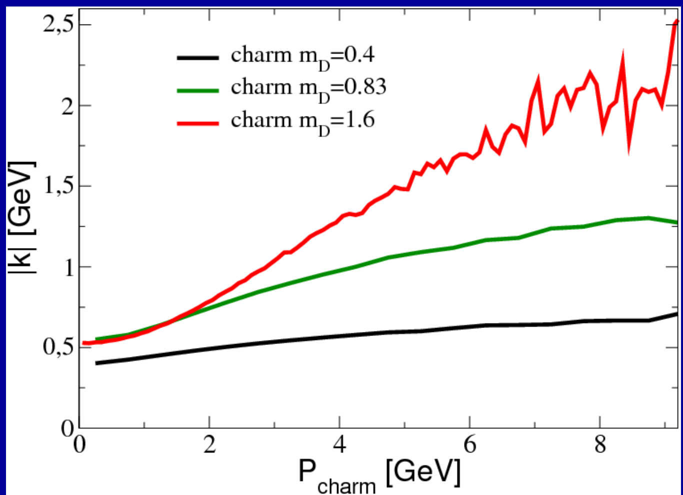


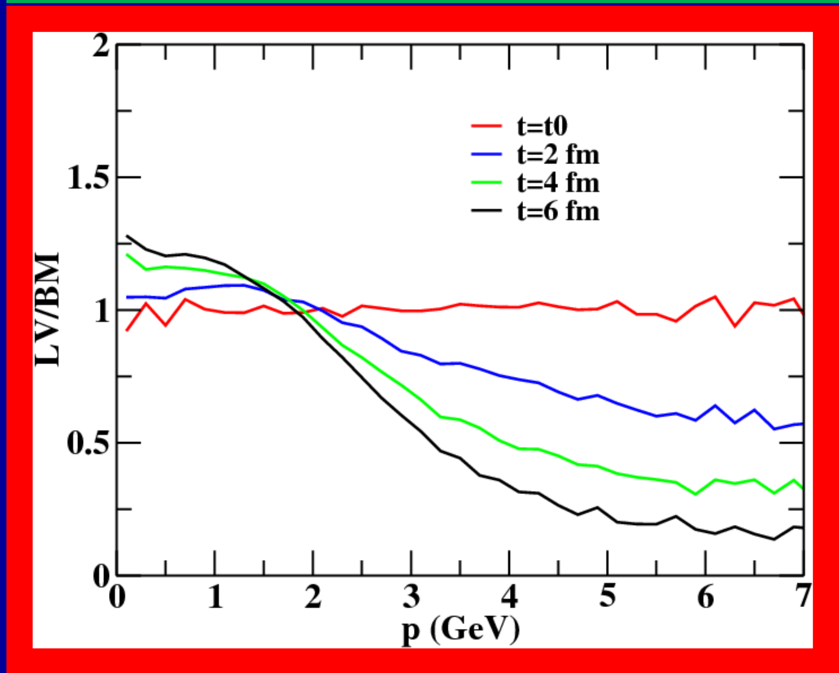
Decreasing m_D makes the σ more anisotropic → Smaller average momentum transfer

Boltzmann vs Langevin (Charm)



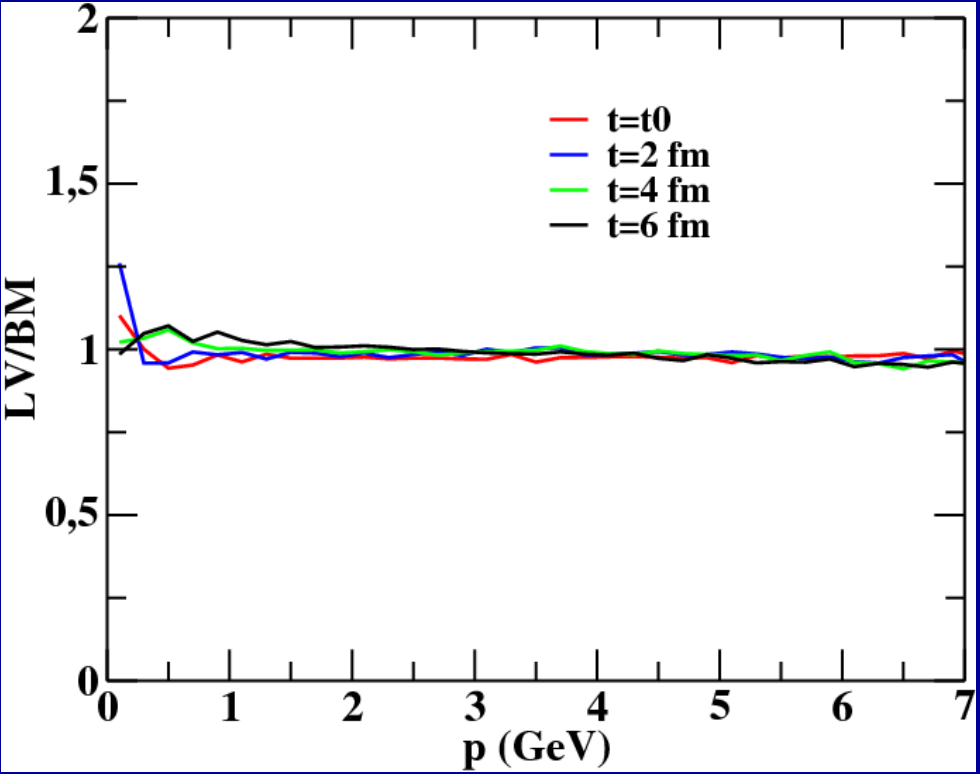






The smaller <k> the better Langevin approximation works

Boltzmann vs Langevin (Bottom)

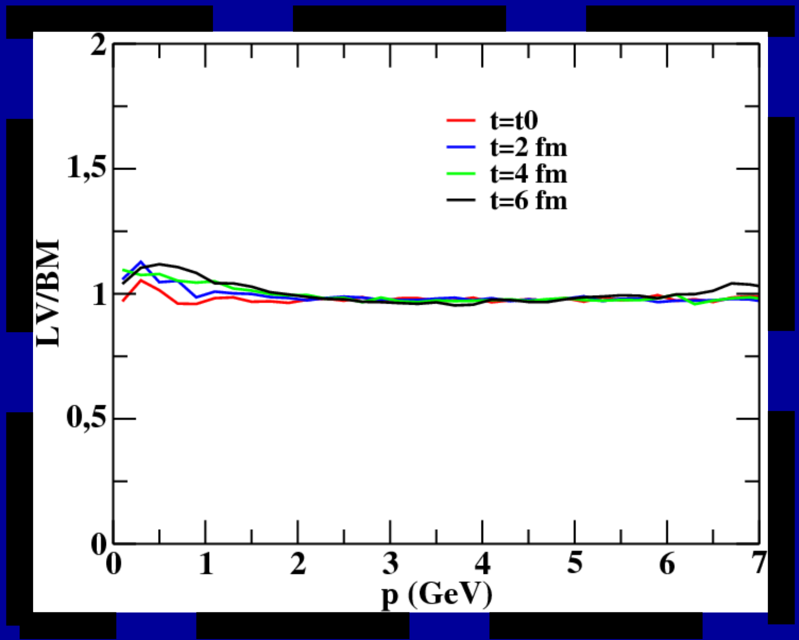


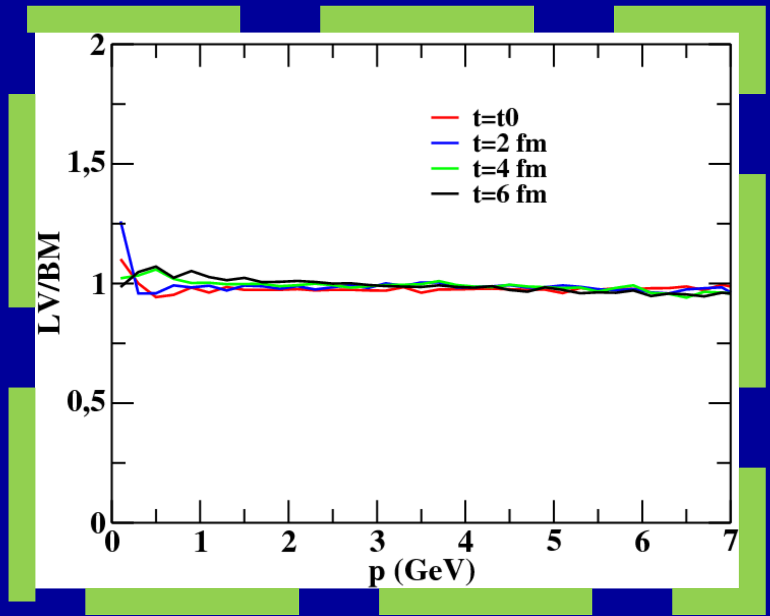
In bottom case Langevin approximation gives results similar to Boltzmann

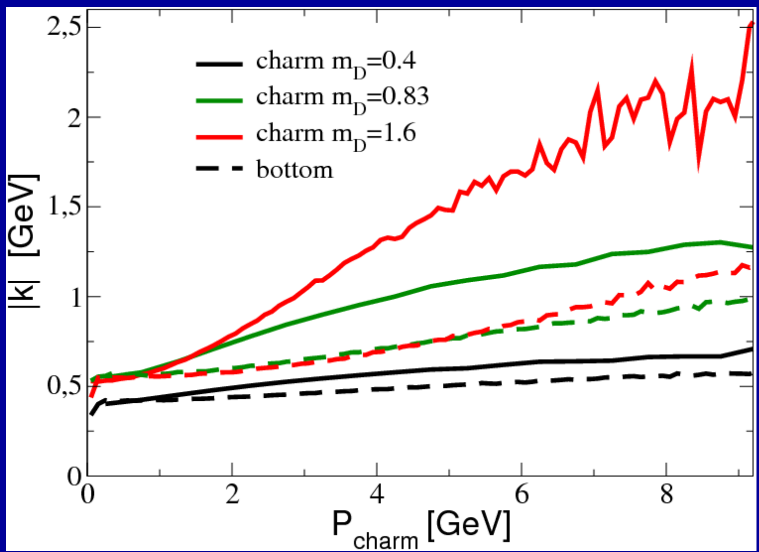
The Larger M the Better Langevin approximation works

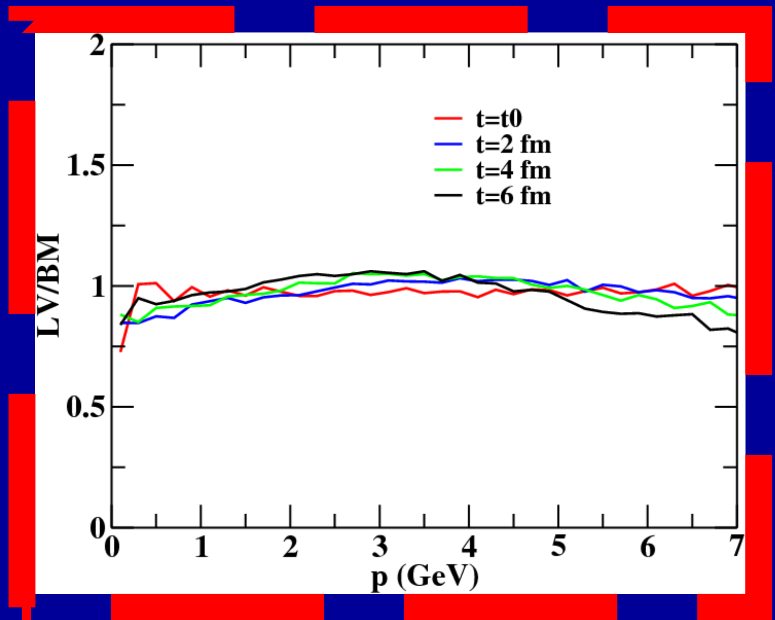
[F. S. et all. in preparation]

Boltzmann vs Langevin (Bottom)





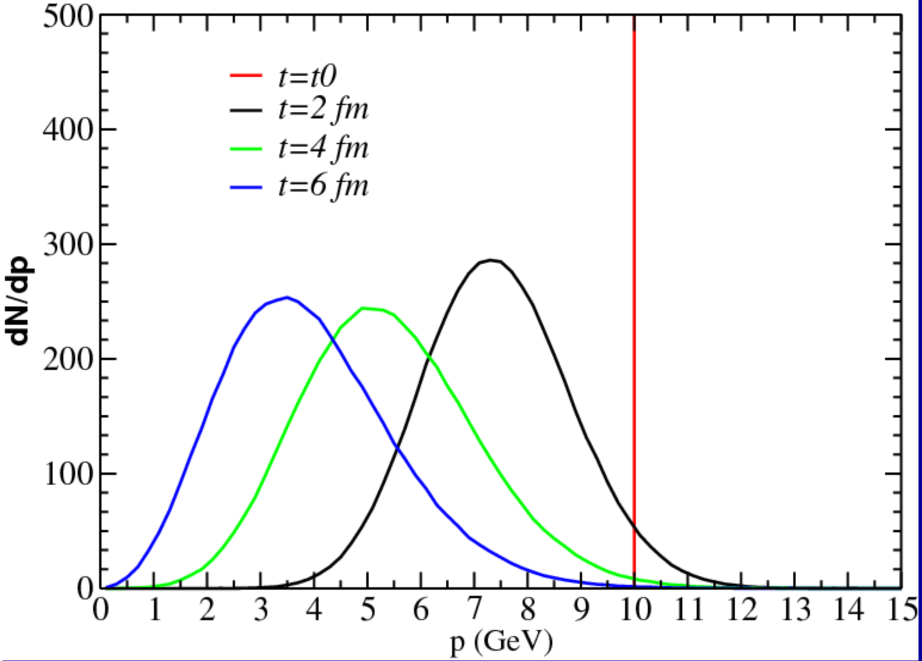




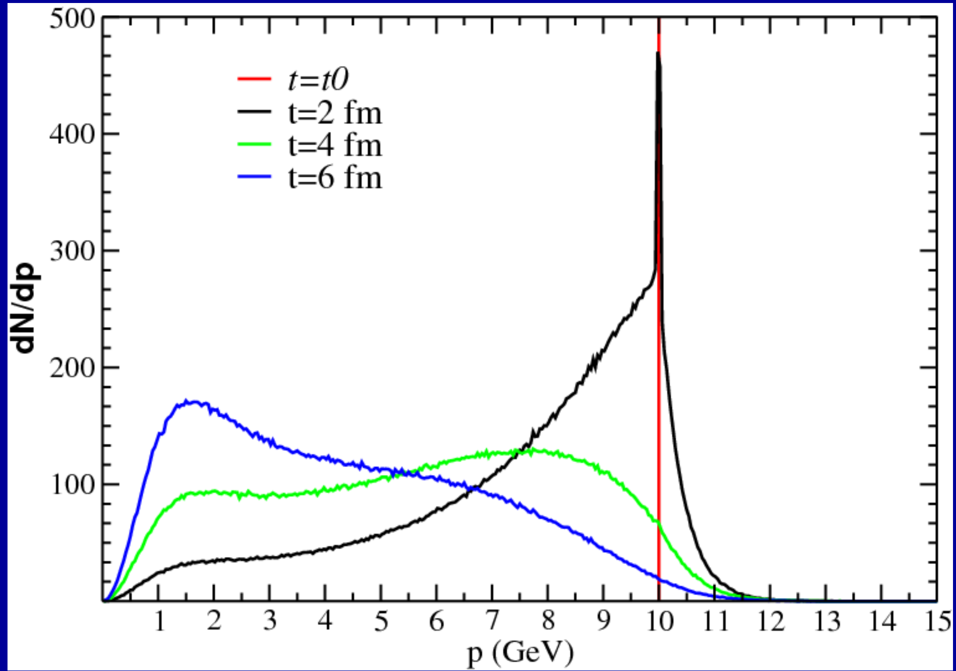
Momentum evolution starting from a δ (Charm)

$$\frac{dN}{d^3p}_{initial} = \delta\left(p - 10GeV\right)$$

Langevin



Boltzmann



- Clearly appears the shift of the average momentum with t due to the drag force
- The gaussian nature of diffusion force reflect itself in the gaussian form of p-distribution

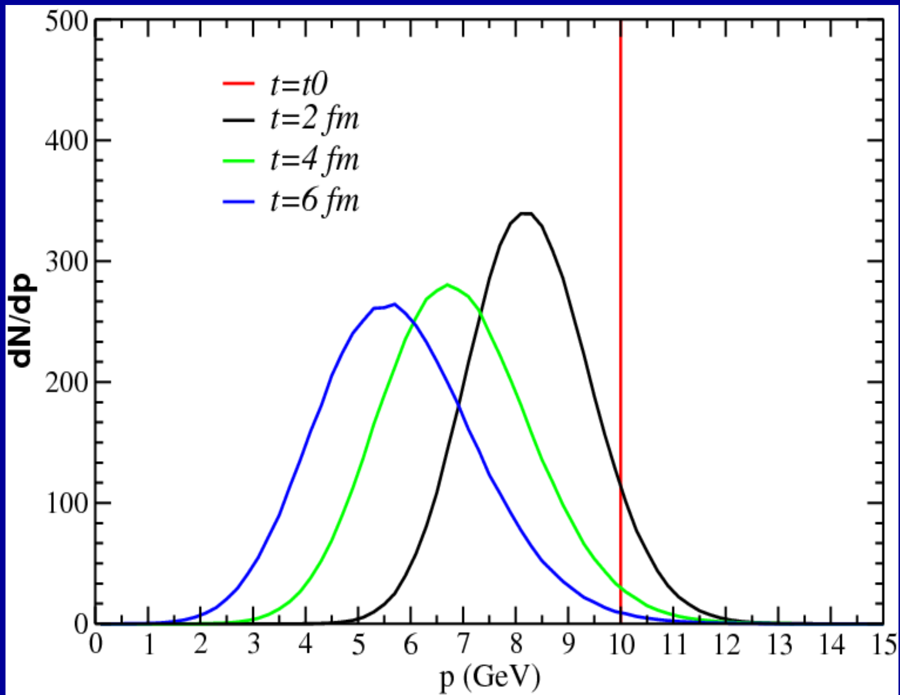
- Boltzmann approach can throw particle at low p instead Langevin can not
- A part of dynamic evolution involving large moment transferred is discarded with Langevin approach

[F. S. et all. in preparation]

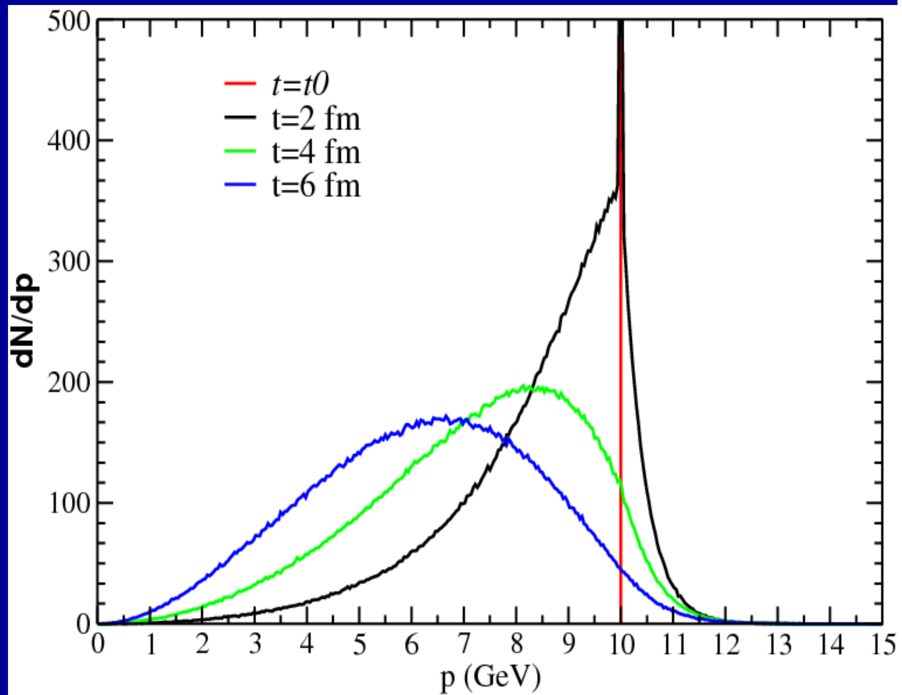
Momentum evolution starting from a δ (Bottom)

$$\left.\frac{dN}{d^3p}\right|_{initial} = \delta\left(p - 10GeV\right)$$

Langevin



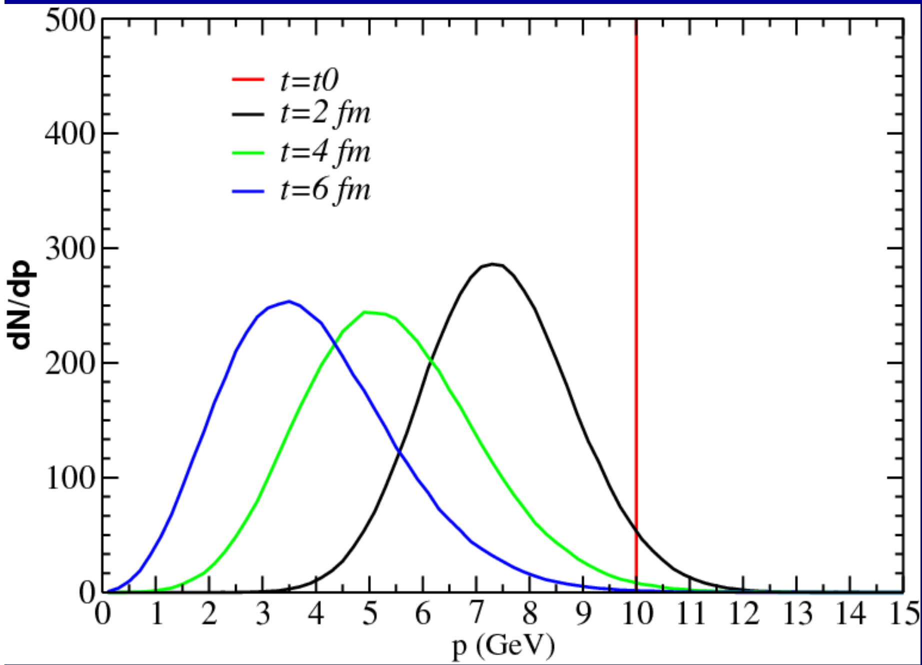
Boltzmann



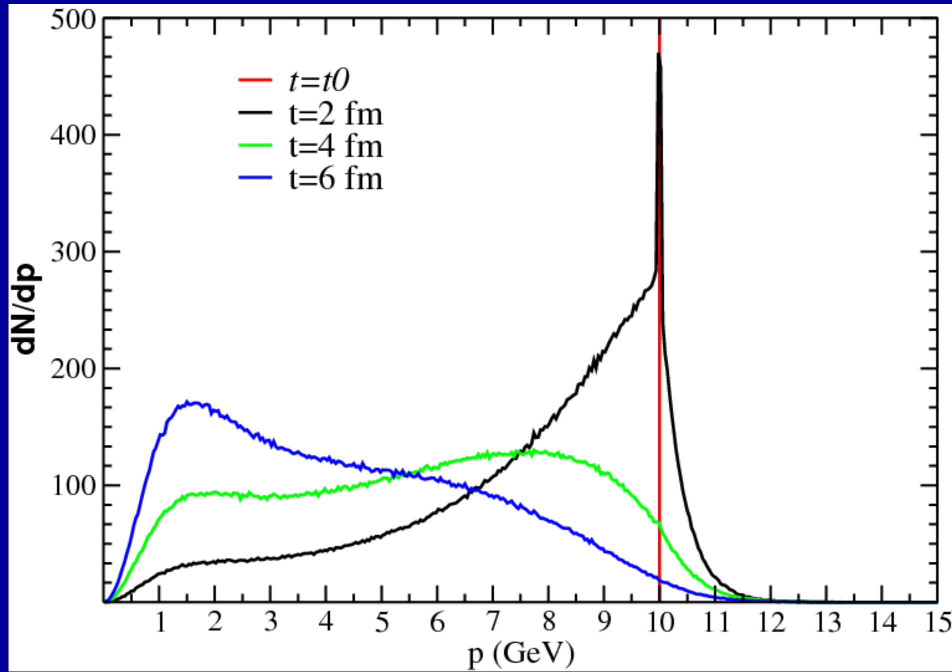
[F. S. et all. in preparation]

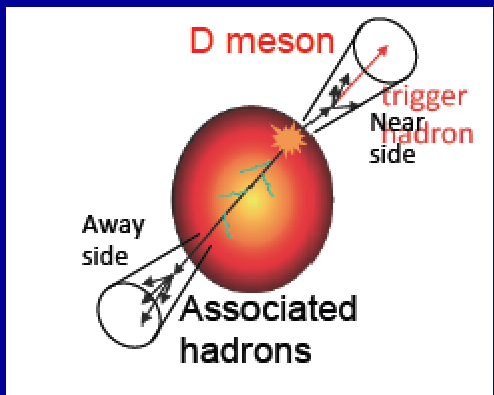
Back to Back correlation

Langevin



Boltzmann

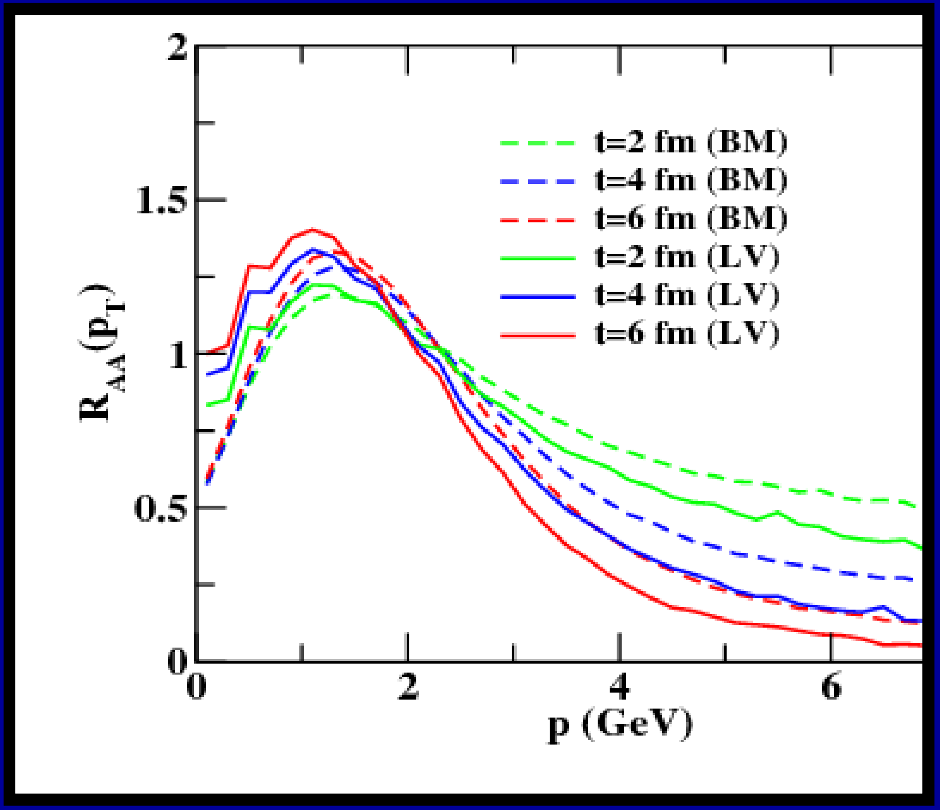




Such large spread of momentum implicates a large spread in the angular distributions of the charm that could be experimentally observed studying the back to back **Charm-antiCharm** angular correlation

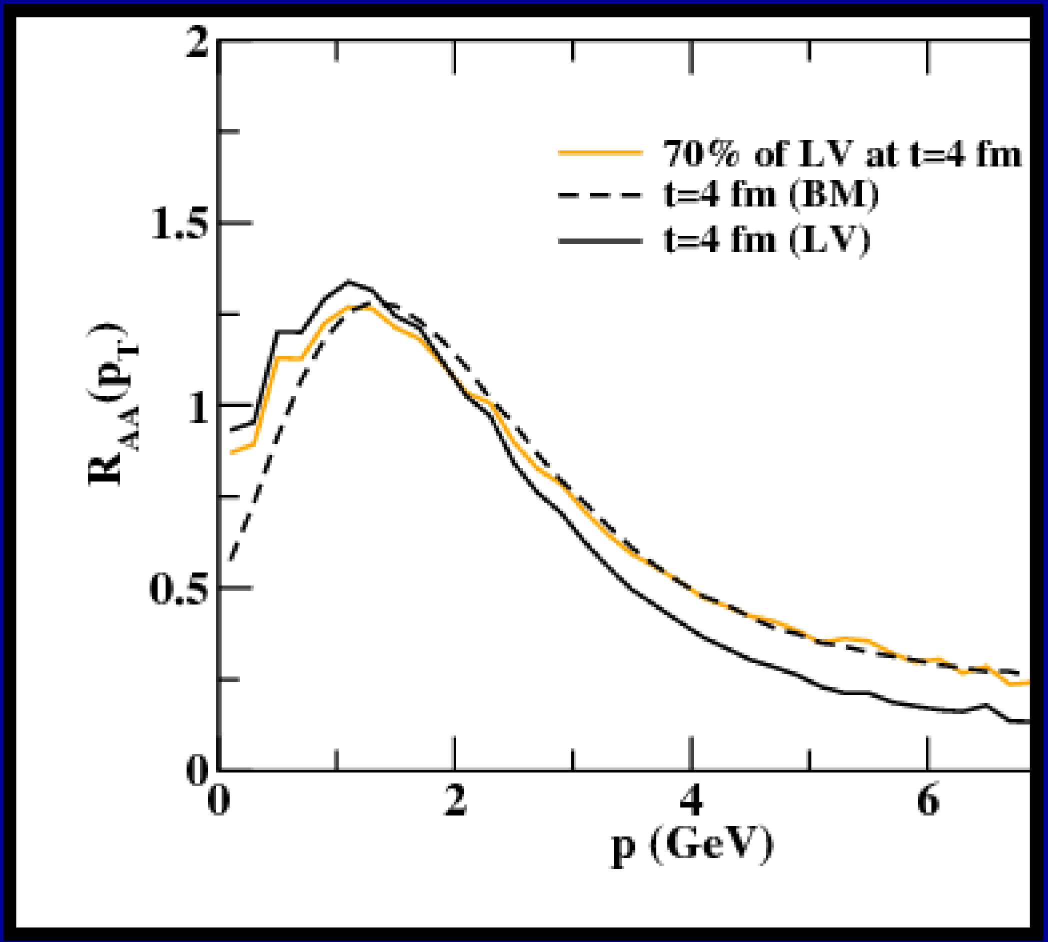
Nuclear Modification factor R_{AA}

$$R_{AA} = \frac{\left(\frac{dN}{d^3p}\right)_{output}}{\left(\frac{dN}{d^3p}\right)_{input}}$$



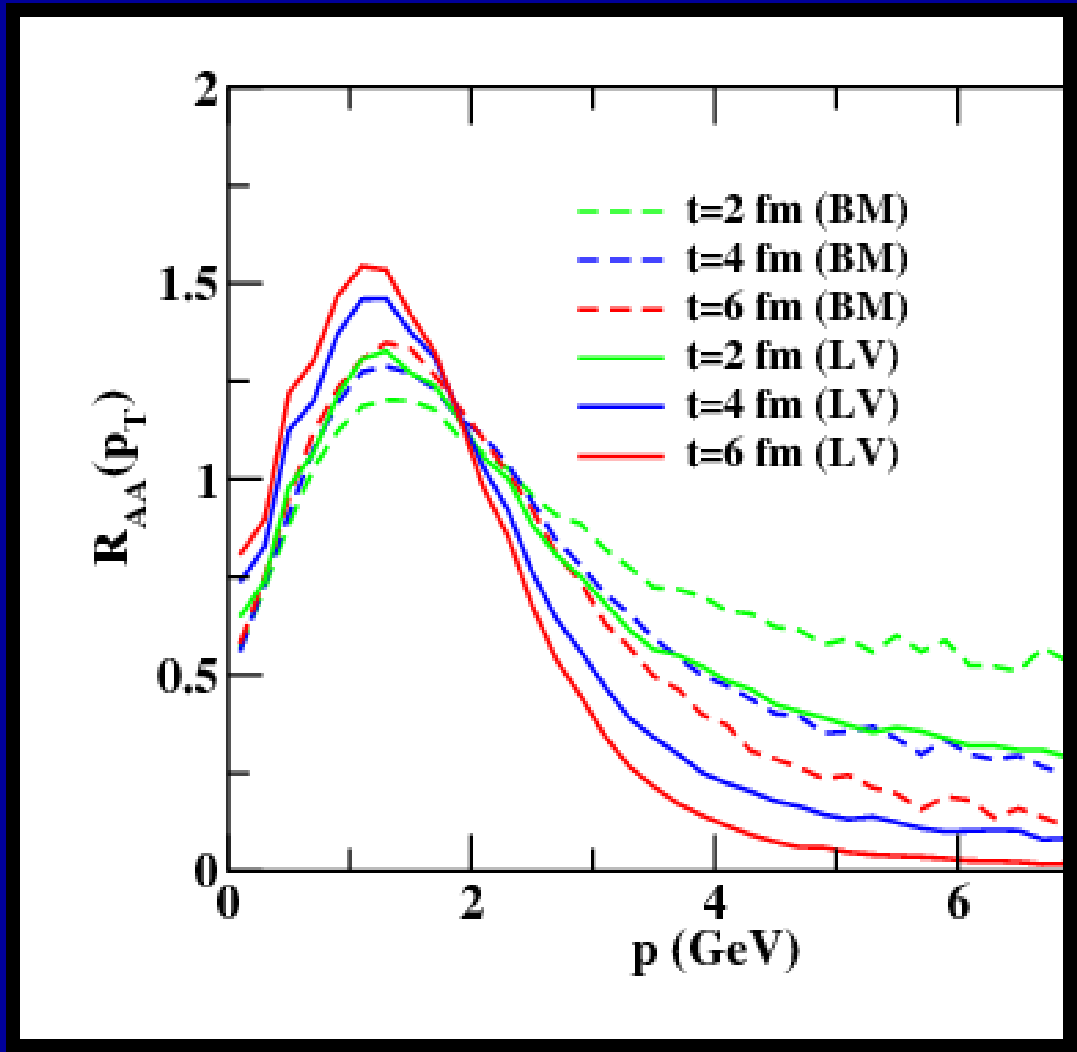
The Langevin approach indicates a smaller R_{AA} thus a larger suppression.

Nuclear Modification factor R_{AA}

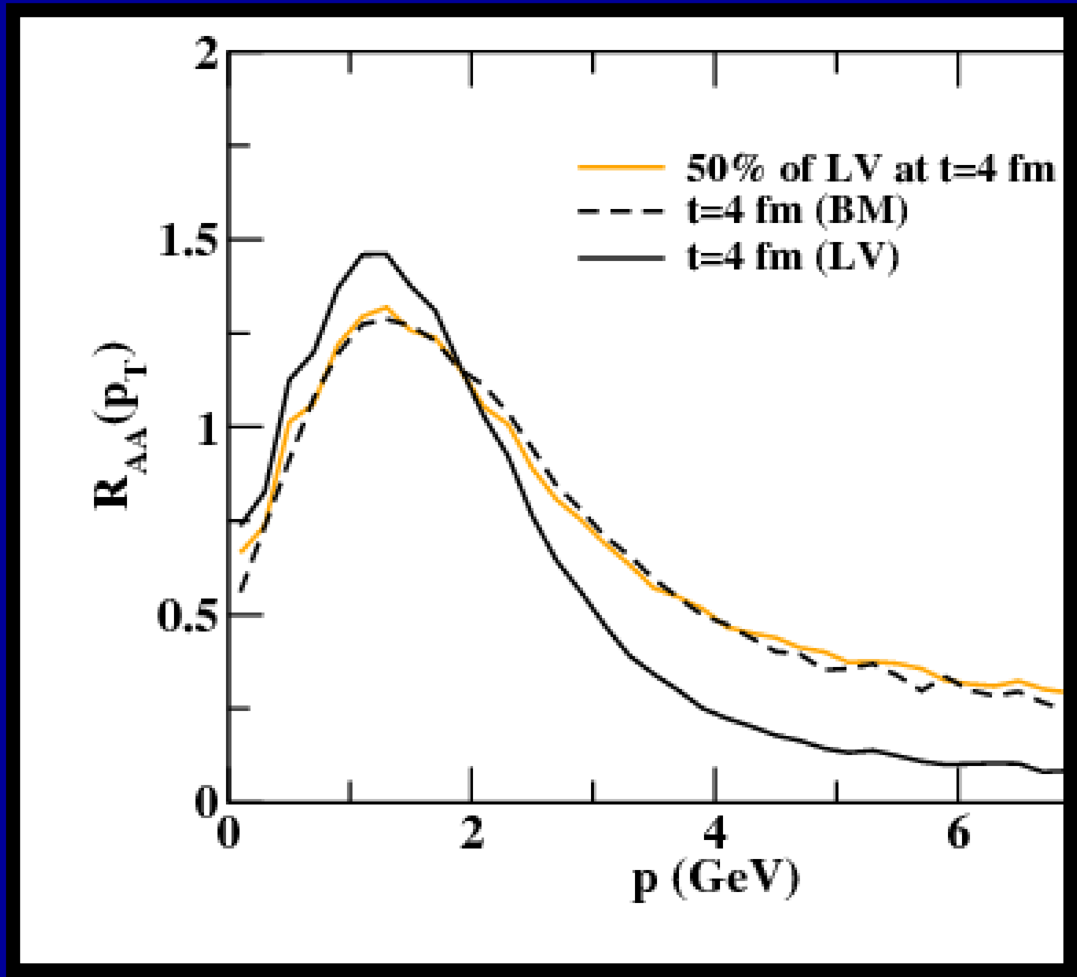


To reproduce the same nuclear suppression factor of Boltzmann equation we need to change D by the 30 %

Nuclear Modification factor R_{AA} m_D=1.6



Nuclear Modification factor R_{AA} m_D=1.6



To reproduce the same nuclear suppression factor of Boltzmann equation with m_D=1.6 we need to change D by the 50 %

Conclusions and perspective

- ✓ We have presented a comparison of the approximations involved by L-E by mean of a comparison with the full collision integral (B-E)

- ✓ Using Langevin we discard a part of the dynamical evolution

- ✓ The Langevin approximation is good for bottom whereas for charm it deviates of about 40-50 % at intermediate p. It discardes a part of the dynamic

- ✓ To get the similar RAA for both approaches we need to reduce the interaction of the Langevin approach by around 30%

- ✓ Calculations in a realistic background are under progress

- ✓ Comparison of the v_2 generated with Langevin and Boltzamann