# Pre-equilibrium dynamics of QCD matter by gluon splitting

Akihiko Monnai (RIKEN BNL Research Center)
In collaboration with:
Berndt Müller (Duke University/BNL)

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# Overview

1. Introduction
   - The quark-gluon plasma
   - High-energy heavy ion collisions
2. Thermalization of gluons
   - Collinear gluon splitting and recombination
   - Fokker-Planck type diffusion
   - Numerical estimations: gluon distribution
3. Thermal & chemical equilibration of quarks
   - Quark and gluon splitting/recombination processes
   - Numerical estimations: quark and gluon distributions
4. Summary and outlook

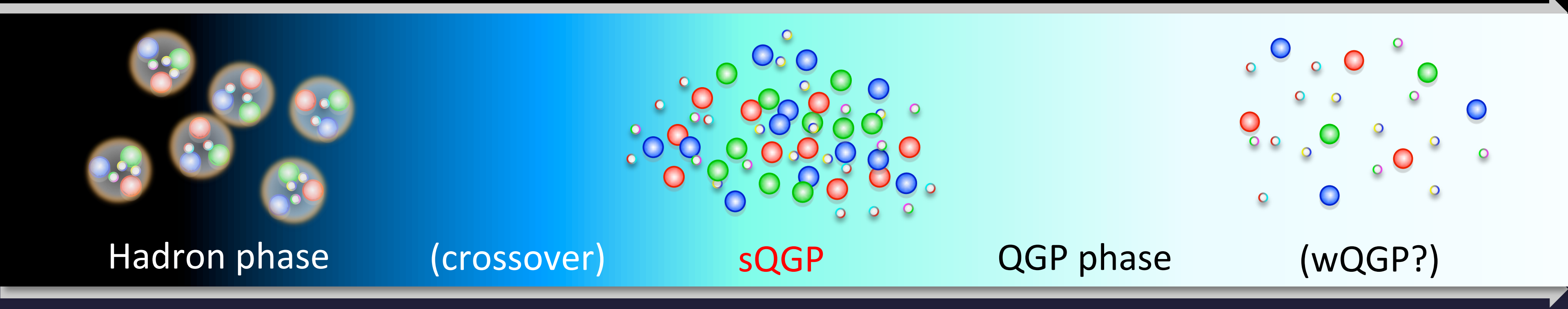
# 1. Introduction

Next: 2. Gluon splitting in early thermalization

# Introduction

- Quark-gluon plasma (QGP): many-body system of deconfined quarks and gluons

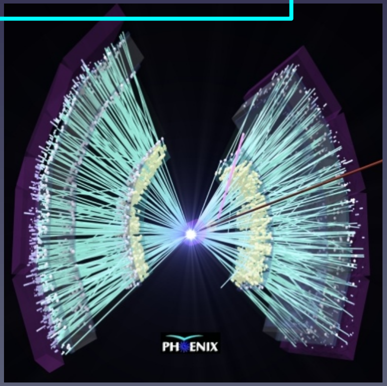
Graphics by AM



The QGP is supposed to have filled the early universe;
It can be produced in heavy ion experiments at RHIC & LHC

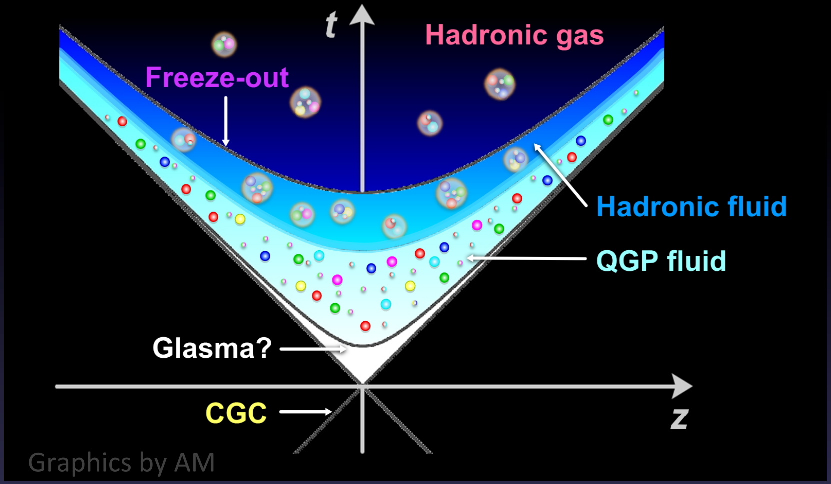
⇨ Heavy ion QGP is found to exhibit near-perfect fluidity

⇨ Early local equilibration ($\tau < 0.6$-$1$ fm) is implied



# Introduction

- Modeling a "Little Bang"



- τ > 10 fm/c: hadronic gas
  (weakly coupled)
- τ ~ 1-10 fm/c: QGP/hadronic fluid
  (strongly coupled)
- τ ~ 0-1 fm/c: CGC -> QGP ?
  (pre-equilibrated)
- τ < 0 fm/c: color glass condensate
  (saturated gluons)

Pre-equilibrated QCD medium is a high-energy frontier in hadron physics

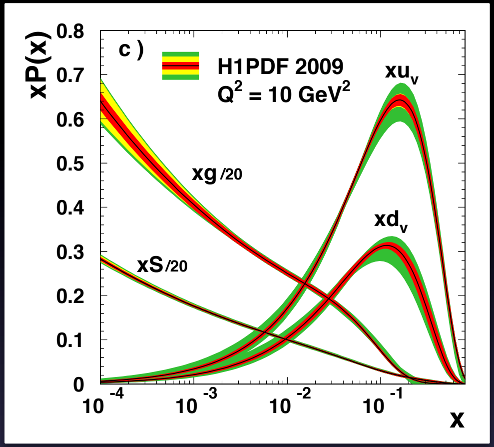
Dynamical description is required for understanding QCD matter

# Motivation

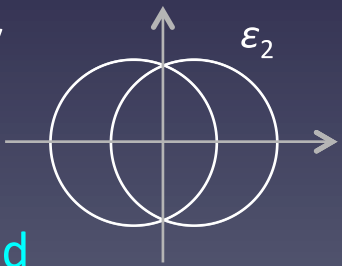
- Color glass condensate (CGC)
  - Gluons emitted from gluons emit gluons in a fast-travelling nucleon
  - ⇨ They start to overlap and saturated
  - ⇨ QCD matter at the initial stage of heavy ion collisions is dominated by gluons

gluon gluon
gluon gluon

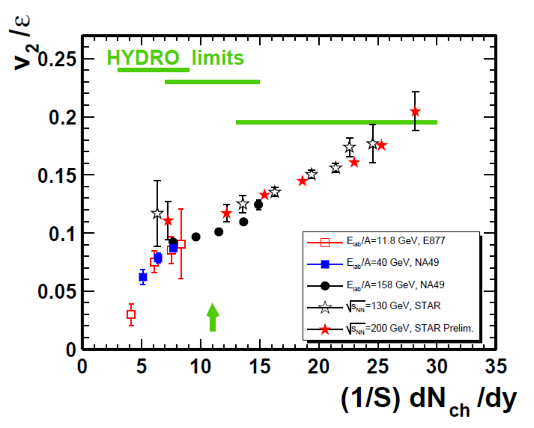
H1 Collaboration (2009)



- QGP fluid
  - Azimuthal momentum anisotropy $v_2$ is large compared with spatial one $\varepsilon_2$
  - ⇨ QCD matter is locally equilibrated at some point and behaves as a fluid

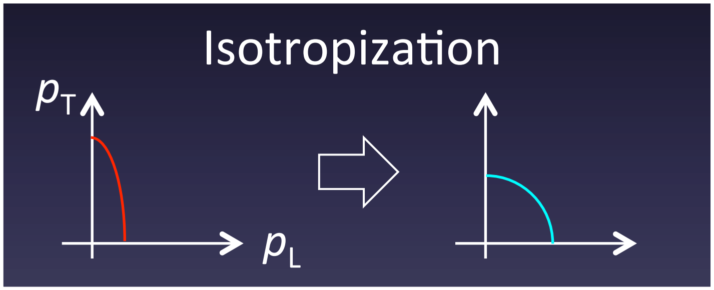


NA49 Collaboration (2003)

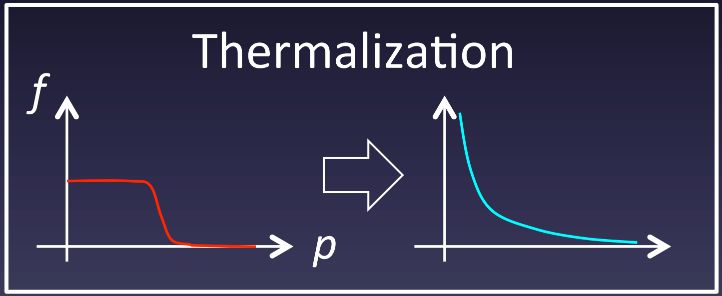


# Motivation

- Early (local) equilibration
  - One of the long standing issues in heavy ion physics
    - Naïve CGC and glasma lead to negative longitudinal pressure
    - CGC has large fraction of high-momentum gluons
  - Equilibration requires


Isotropization

&


Thermalization

⇨ We focus on thermalization and provide a model based on collinear splitting to introduce low momentum partons

| | |
|---|---|
| Cf: parton cascade picture | K. Geiger and B. Müller (1992) |
| bottom-up approaches | R. Baier et. al. (2001), P. Arnold et al. (2003) |
| Bose condensate | J.-P. Blaizot et al. (2012, 2013), X.-G. Huang and J. Liao (2013) |

# 2. Thermalization of gluons

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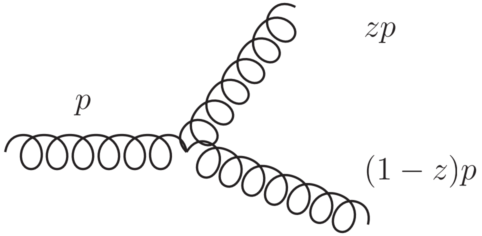
Next: 3. Thermal and chemical equilibration of quarks

# Collinear splitting model

## Gluon splitting



- A splitting modifies the parton distribution f as

$$f(p_i) \to z^{-d} f\left(\frac{p_i}{z}\right) + (1-z)^{-d} f\left(\frac{p_i}{1-z}\right)$$

as number is doubled while momentum is conserved:

$$2\int \frac{dp^d}{(2\pi)^d} f(p_i) = \int \frac{dp^d}{(2\pi)^d}\left[z^{-d} f\left(\frac{p_i}{z}\right) + (1-z)^{-d} f\left(\frac{p_i}{1-z}\right)\right]$$

$$\int \frac{dp^d}{(2\pi)^d} p_j f(p_i) = \int \frac{dp^d}{(2\pi)^d} p_j \left[z^{-d} f\left(\frac{p_i}{z}\right) + (1-z)^{-d} f\left(\frac{p_i}{1-z}\right)\right]$$

- Time evolution of gluon distribution by collinear splitting

$$\boxed{\frac{\partial f(p_i)}{\partial t}|_{\mathrm{sp}} = \frac{1}{2}\int_0^1 dz\, r(z)[f^z(p_i) + f^{1-z}(p_i) - f(p_i)] \equiv \mathcal{C}_{\mathrm{sp}}(p_i)}$$

where $f^z(p_i) \equiv z^{-d} f\left(\frac{p_i}{z}\right)$, $f^{1-z}(p_i) = (1-z)^{-d} f\left(\frac{p_i}{1-z}\right)$ and

$$r(z) = \Gamma R_{gg}(z) = 6\Gamma \frac{[1-z(1-z)]^2}{z(1-z)}$$ is the splitting rate

# Collinear splitting model

## Gluon splitting

- Rough estimation of parton emission rate $\Gamma$

  In static frame $\Gamma \sim \alpha_s Q$

  In boosted frame $\Gamma \sim \alpha_s Q(Q/p) \sim \alpha_s \hat{q} L/p$

  ⇨ $\Gamma \sim \alpha_s^{1/2} (\hat{q}/p)^{1/2}$ is implied

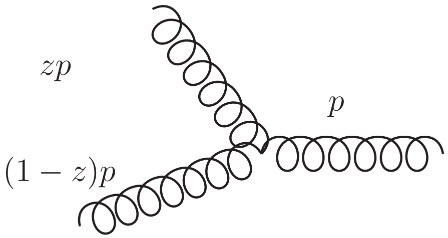
  (In thermal system $\Gamma_{\mathrm{th}} \sim \alpha_s^{3/2} T$ )

- The time evolution can violate the second law of thermodynamics if splitting continues on, i.e., not stable at thermal equilibrium

  ⇨ A mechanism that prevents “over-shrinking” in phase space should be present

# Collinear splitting model

## Gluon recombination



Time evolution should stop at local equilibrium (2nd law of thermodynamics)

⇨ Recombination should occur when density Is large (in analogy with Balitsky-Kovshegov equation)

- Parameterization of recombination process

$$\begin{aligned}\frac{\partial f(p_i)}{\partial t}\Big|_{\rm rc} &= -\frac{1}{2}\int_0^1 dz\ \tilde{r}(p_i,z)\frac{f_{\rm eq}^{z}(p_i)+f_{\rm eq}^{1-z}(p_i)-f_{\rm eq}(p_i)}{f_{\rm eq}(zp_i)f_{\rm eq}((1-z)p_i)}f(zp_i)f((1-z)p_i)\\ &\equiv \mathcal{C}_{\rm rc}(p_i)\end{aligned}$$

where $f_{\rm eq}(p)=\dfrac{d_g}{\exp(\sqrt{p^2+m_{\rm th}^2}/T)-1}$ is equilibrium distribution in medium

Degeneracy $d_g = 16$

Effective thermal mass $m_{\rm th}\sim \alpha_s^{1/2}T$

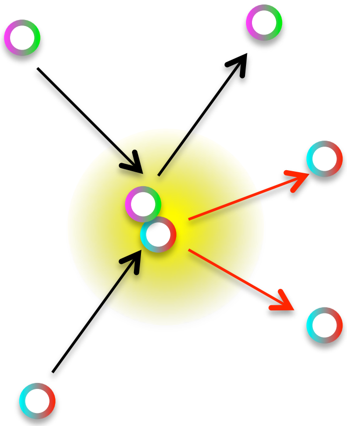
# Collinear splitting model

## Momentum smearing effect

- Splitting occurs when a parton becomes off-shell by interacting with medium

⇨ Elastic scattering is taken into account by relativistic Fokker-Planck equation

$$\boxed{\frac{\partial f}{\partial t}|_{\mathrm{FP}} = \frac{\partial}{\partial p_i}\left[A^i f + \frac{\partial}{\partial p_j}(B^{ij} f)\right] \equiv \mathcal{C}_{\mathrm{FP}}}$$



where $A^i \sim \nu p^i$ and $B^{ij} \sim D\delta^{ij}$ characterizes drag and diffusion effects

# Collinear splitting model

## ■ Momentum smearing effect

- Rough estimation of drag and diffusion coefficients

Analytic solution of a diffusion equation for a delta function $\delta(p)$

$\frac{1}{\sqrt{4\pi D t}} e^{-p^2/4Dt}$ ⇨ Standard deviation $\sigma = \sqrt{2Dt}$

⇔ Longitudinal momentum modification $\sigma \sim Q^2/2p$

⇨ $D \sim \hat{q}^2/\Gamma p^2$ and $D_{\mathrm{th}} \sim \alpha_s^{5/2} T^3$ are implied

Relativistic Einstein relation $D = \nu E T$ yields the drag coefficient

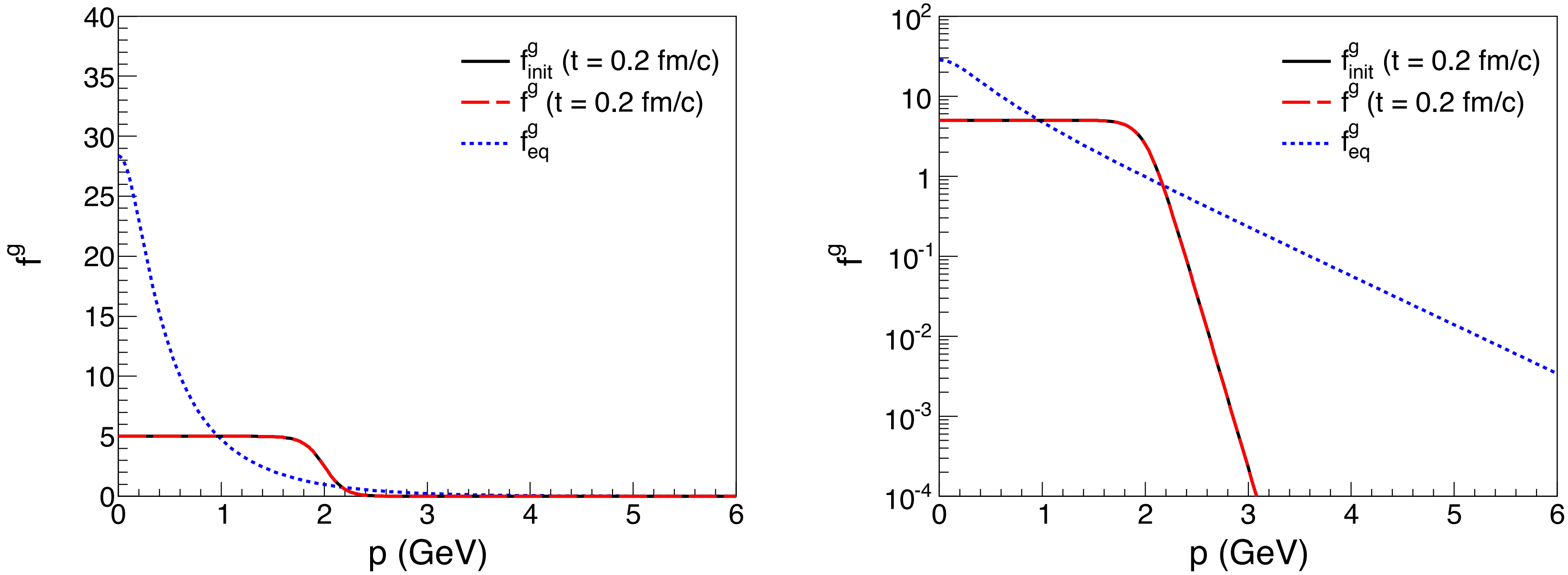
## ■ CGC-like initial condition

- Gluon distribution

$f_g(p < Q_s) \sim 1/\alpha_s$ and $f_g(p > Q_s) \sim 0$ $\qquad Q_s \sim 2 \ \mathrm{GeV}$

# Numerical analyses

- Time evolution in one-dimensional non-expanding system



# Numerical analyses

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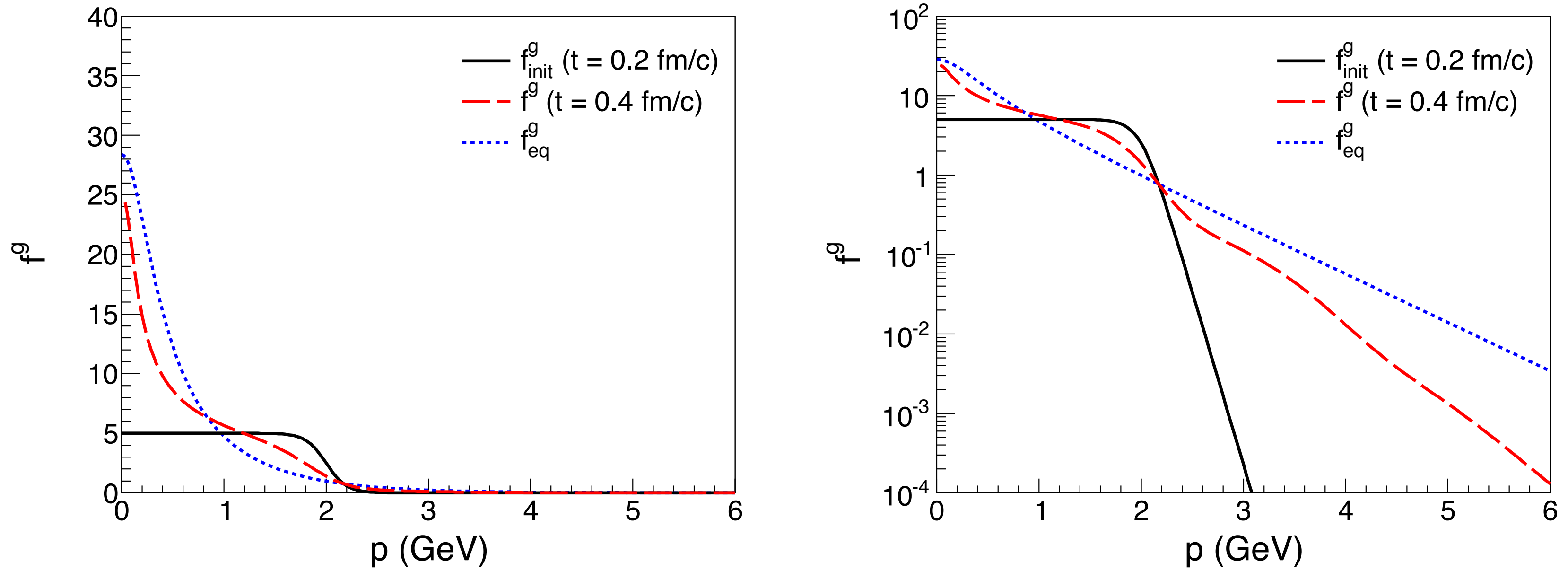
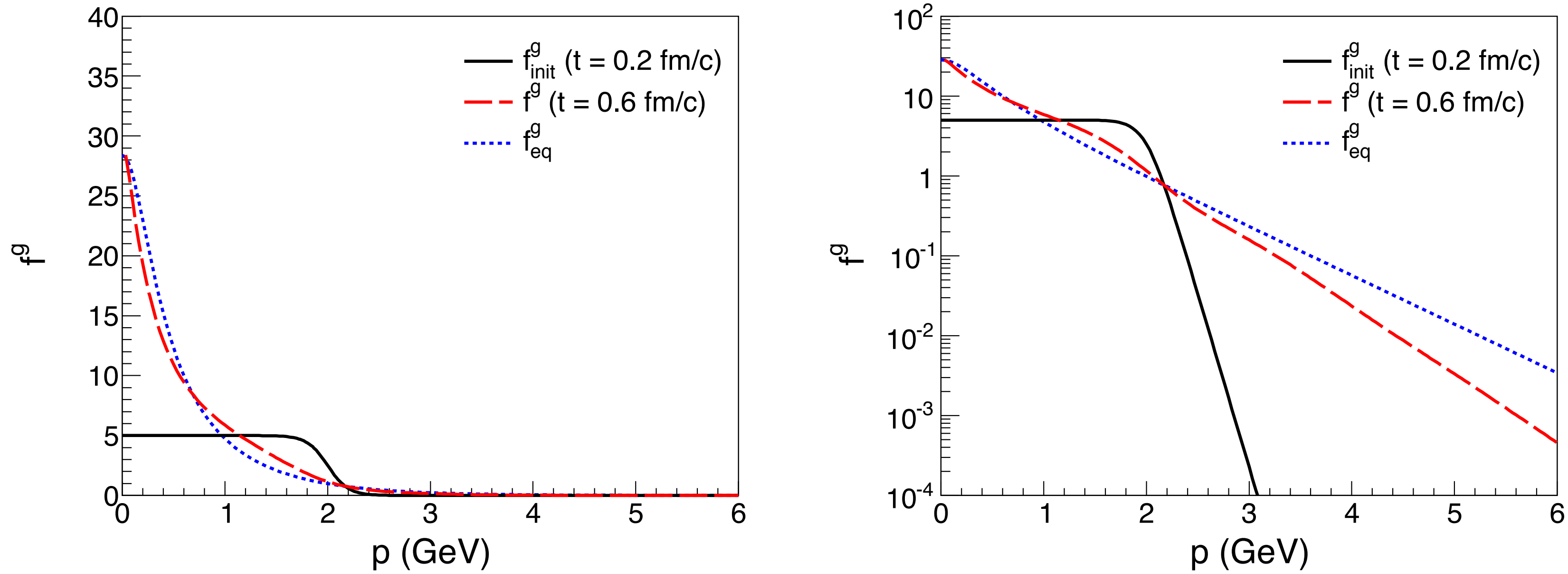


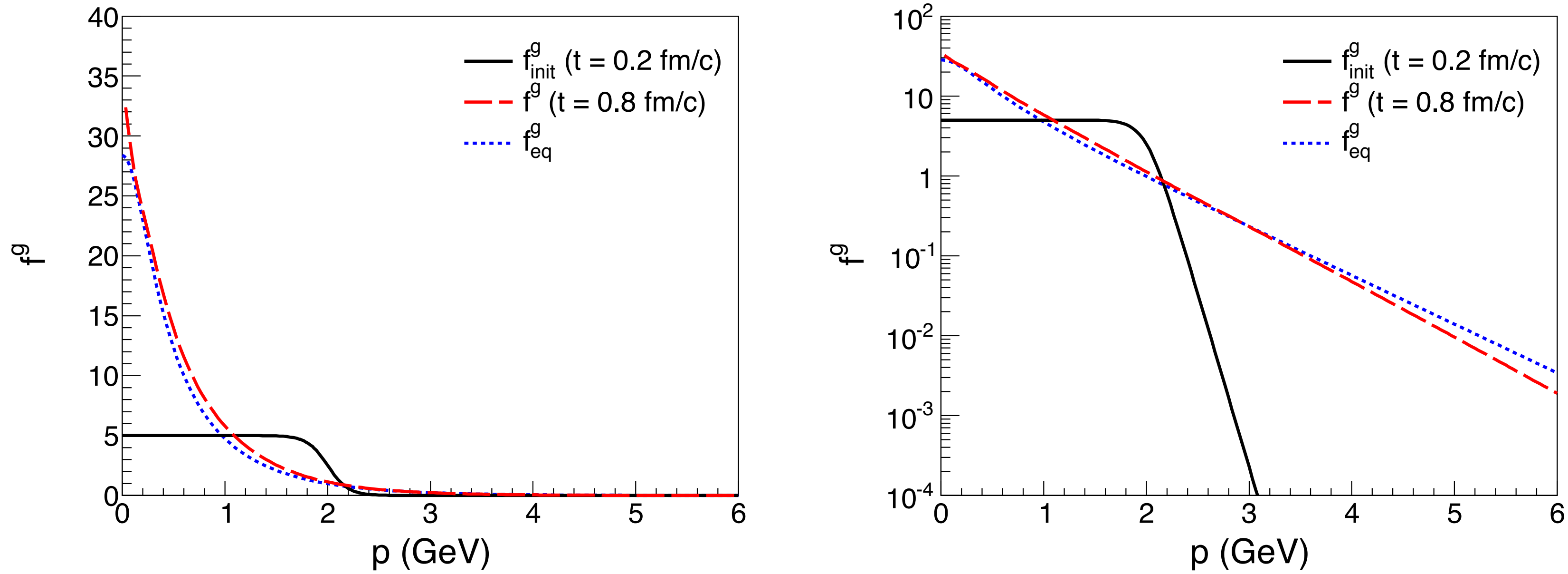
# Numerical analyses

- Time evolution in one-dimensional non-expanding system



# Numerical analyses

- Time evolution in one-dimensional non-expanding system



- Collinear gluon splitting contributes visibly to quick thermalization
- Entropy production is confirmed positive

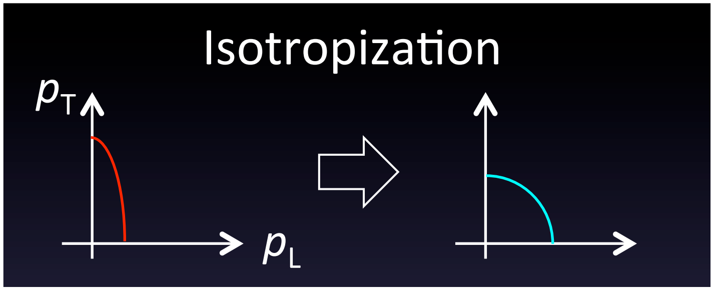
- Recombination is a key; Is the dynamics in dense region strong enough in 3D to enforce isotropization?

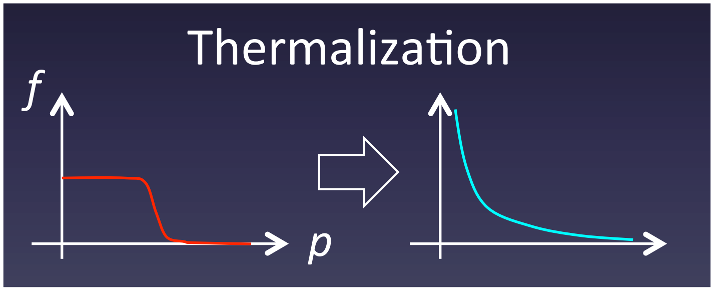
# 3. Thermal & chemical equilibration of quarks

Next: 4. Summary and outlook

# Motivation

- Early (local) equilibration
  - Equilibration requires more





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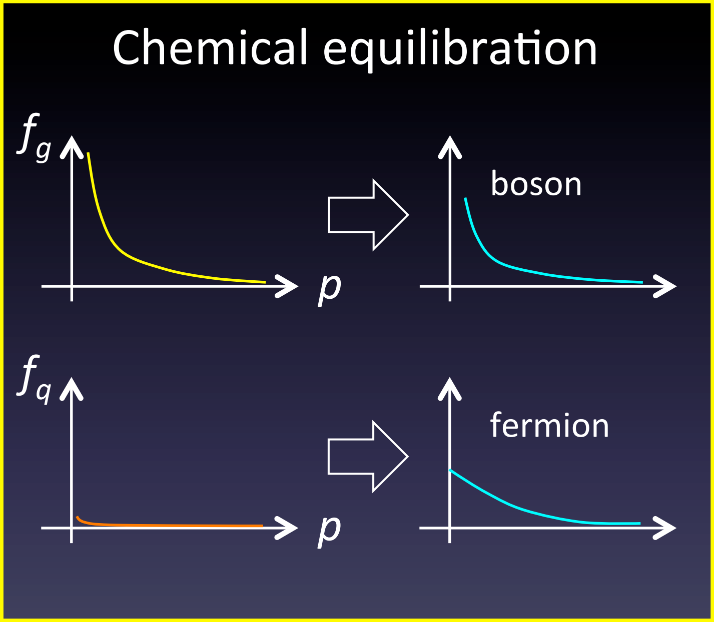


Gluon medium eventually becomes *quark*-gluon plasma
Is quark thermalization fast enough?
Is the QGP chemical equilibrated?

# Parton splitting model

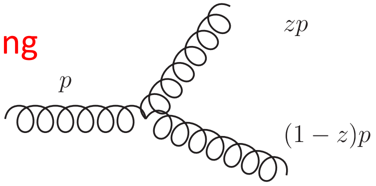
- Gluons + quarks

Time evolution of parton distributions by collinear processes

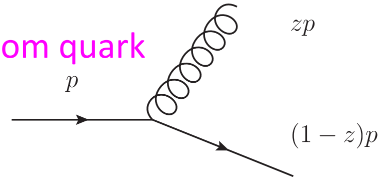
$$\frac{\partial f_g(p_i)}{\partial t}\Big|_{\rm sp} = \frac{1}{2}\int_0^1 dz\, r_{gg}(z)[f_g^z(p_i)+f_g^{1-z}(p_i)-f_g(p_i)] + \int_0^1 dz\, r_{gq}(z) f_q^z(p_i) - \int_0^1 dz\, r_{qg}(z) f_g(p_i) \equiv \mathcal{C}^g_{\rm sp}(p_i)$$

$$\frac{\partial f_q(p_i)}{\partial t}\Big|_{\rm sp} = \int_0^1 dz\, r_{gq}(z)[f_q^{1-z}(p_i)-f_q(p_i)] + \int_0^1 dz\, r_{qg}(z)[f_g^z(p_i)+f_g^{1-z}(p_i)] \equiv \mathcal{C}^q_{\rm sp}(p_i),$$

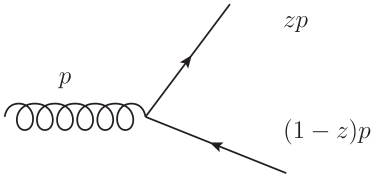
Gluon splitting



Gluon emission from quark



Quark pair production



Splitting functions

$$r_{gg}(z) = \Gamma R_{gg}(z) = 6\frac{[1-z(1-z)]^2}{z(1-z)}\Gamma$$

$$r_{gq}(z) = \Gamma R_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}\Gamma$$

$$r_{qg}(z) = 2N_f\Gamma R_{qg}(z) = 3[z^2+(1-z)^2]\Gamma \quad (N_f=3)$$

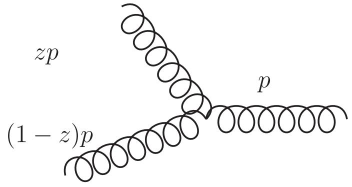
Quark-antiquark symmetry assumed

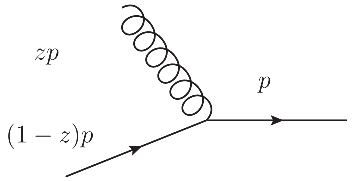
# Parton splitting model

- Gluons + quarks

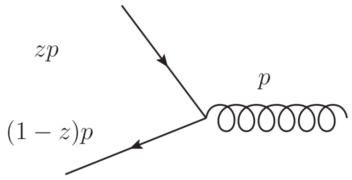
## Parameterization of recombination processes

$$
\begin{aligned}
\frac{\partial f_g(p_i)}{\partial t}\Big|_{\rm rc} = &-\frac{1}{2}\int_0^1 dz\, \tilde{r}_{gg}(p,z)\frac{f^z_{geq}(p_i)+f^{1-z}_{geq}(p_i)-f_{geq}(p_i)}{f_{geq}(zp_i)f_{geq}((1-z)p_i)} f_g(zp_i)f_g((1-z)p_i) \\
&-\int_0^1 dz\, \tilde{r}_{gq}(p,z)\frac{f^z_{qeq}(p_i)}{f_{geq}(zp_i)f_{qeq}((1-z)p_i)} f_g(zp_i)f_q((1-z)p_i) \\
&+\int_0^1 dz\, \tilde{r}_{qg}(p,z)\frac{f_{geq}(p_i)}{f_{qeq}(zp_i)f_{qeq}((1-z)p_i)} f_q(zp_i)f_q((1-z)p_i) \equiv \mathcal{C}^g_{\rm rc}(p_i)
\end{aligned}
$$





$$
\begin{aligned}
\frac{\partial f_q(p_i)}{\partial t}\Big|_{\rm rc} = &-\int_0^1 dz\, \tilde{r}_{gq}(p_i,z)\frac{f^{1-z}_{qeq}(p_i)-f_{qeq}(p_i)}{f_{geq}(zp_i)f_{qeq}((1-z)p_i)} f_g(zp_i)f_q((1-z)p_i) \\
&-\int_0^1 dz\, \tilde{r}_{qg}(p_i,z)\frac{f^z_{geq}(p_i)+f^{1-z}_{geq}(p_i)}{f_{qeq}(zp_i)f_{qeq}((1-z)p_i)} f_q(zp_i)f_q((1-z)p_i) \equiv \mathcal{C}^q_{\rm rc}(p_i)
\end{aligned}
$$



### Equilibrium distributions

$$f_{geq}(p) = \frac{d_g}{\exp(\sqrt{p^2+m_{\rm th}^2}/T)-1}$$

$$f_{qeq}(p) = \frac{d_q}{\exp(\sqrt{p^2+m_{\rm th}^2}/T)+1}$$

$d_g = 16$

$d_q = 31.5 \ (N_f = 3)$

$m_{\rm th} \sim \alpha_s^{1/2} T$

$T$ : fixed from momentum conservation

# Parton splitting model

## Momentum smearing effect

- Fokker-Plank equation holds for each component

⇨ $$\boxed{\frac{\partial f}{\partial t}\Big|_{\mathrm{FP}} = \frac{\partial}{\partial p_i}\left[A^i f + \frac{\partial}{\partial p_j}(B^{ij} f)\right] \equiv \mathcal{C}_{\mathrm{FP}}}$$

where $f = \{f_g, f_q\}$

## Initial condition (CGC-like)

- Gluon distribution

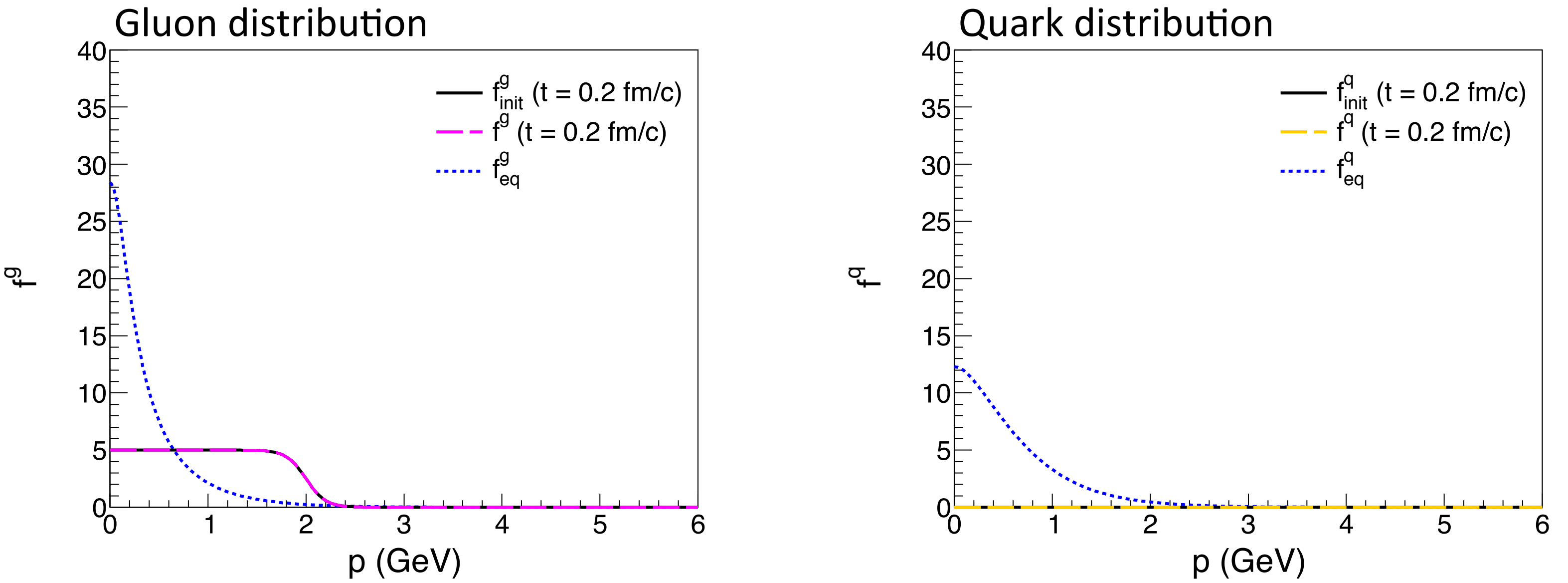
$f_g(p < Q_s) \sim 1/\alpha_s$ and $f_g(p > Q_s) \sim 0$ $Q_s \sim 2\ \mathrm{GeV}$

- Quark distribution

$f_q(p, t_0) = 0$

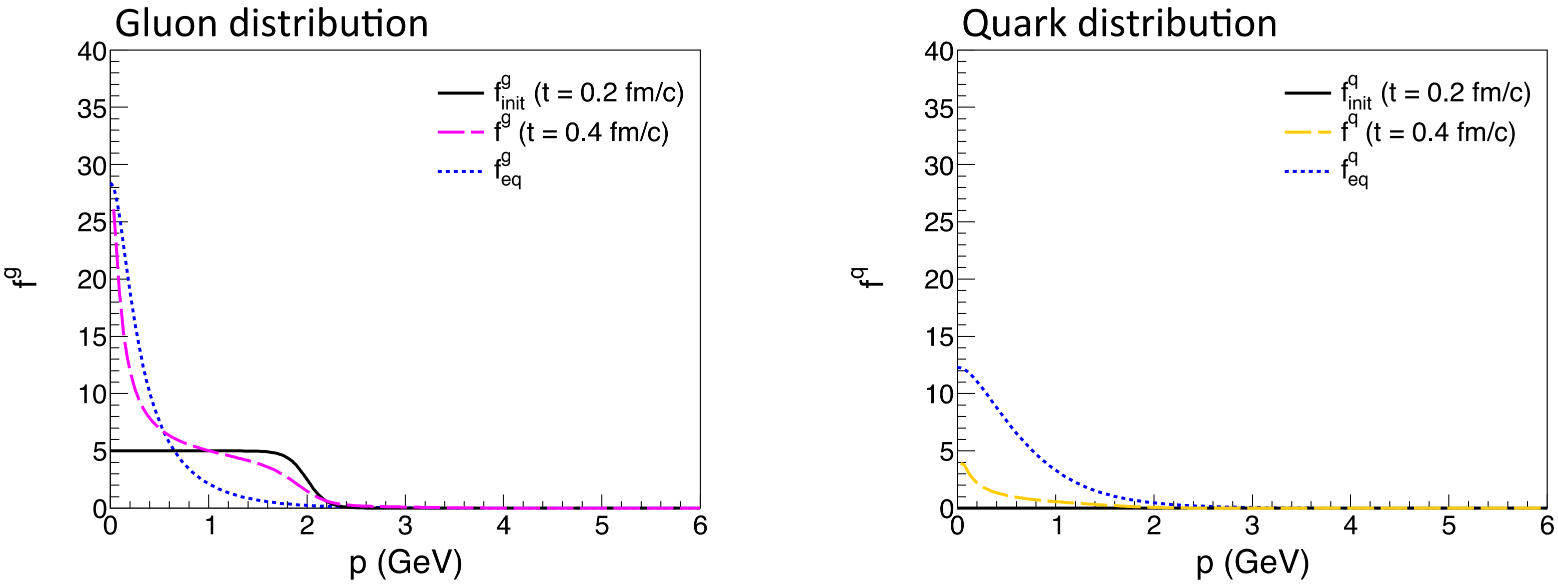
# Numerical analyses

- Time evolutions (t = 0.2 fm/*c*)



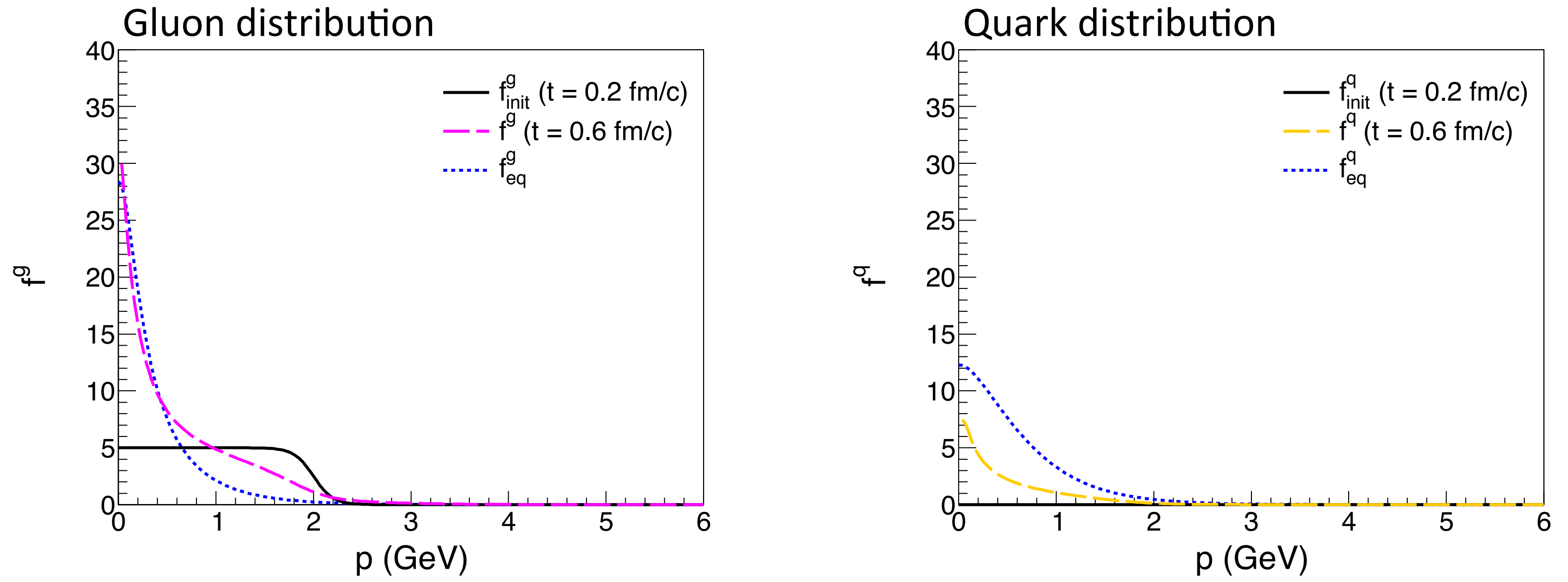
# Numerical analyses

- Time evolutions (t = 0.4 fm/*c*)



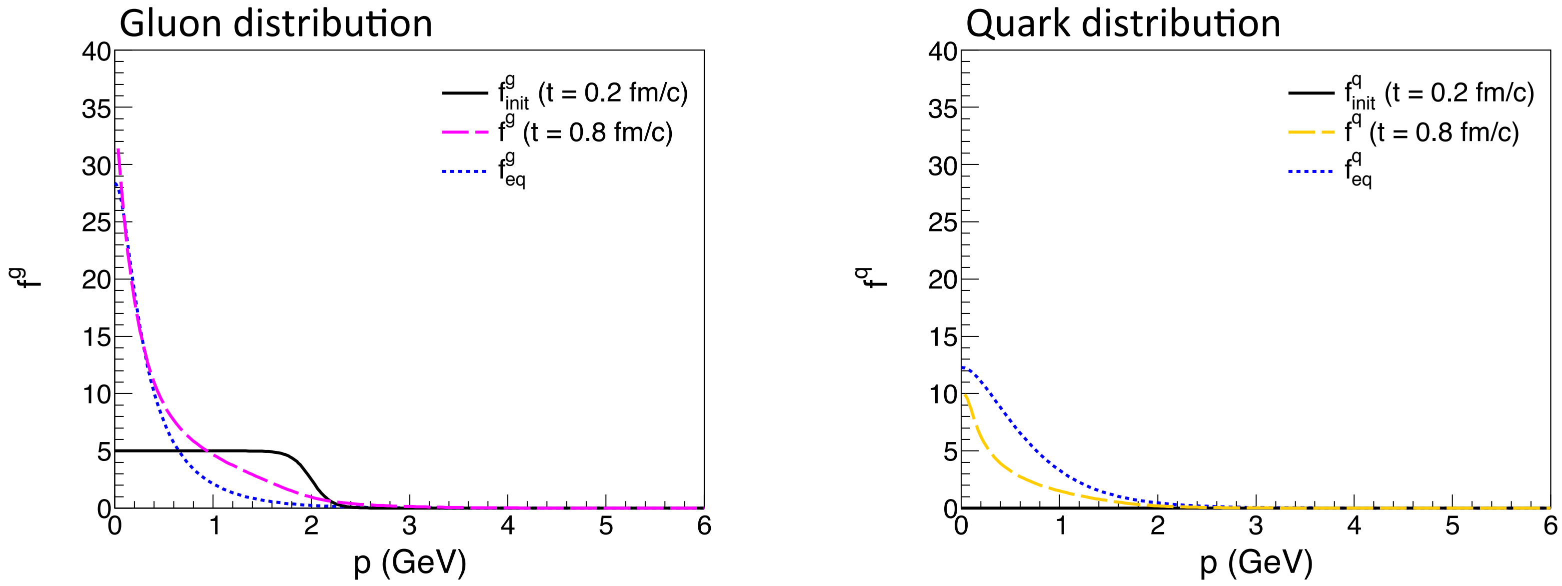
# Numerical analyses

- Time evolutions (t = 0.6 fm/*c*)



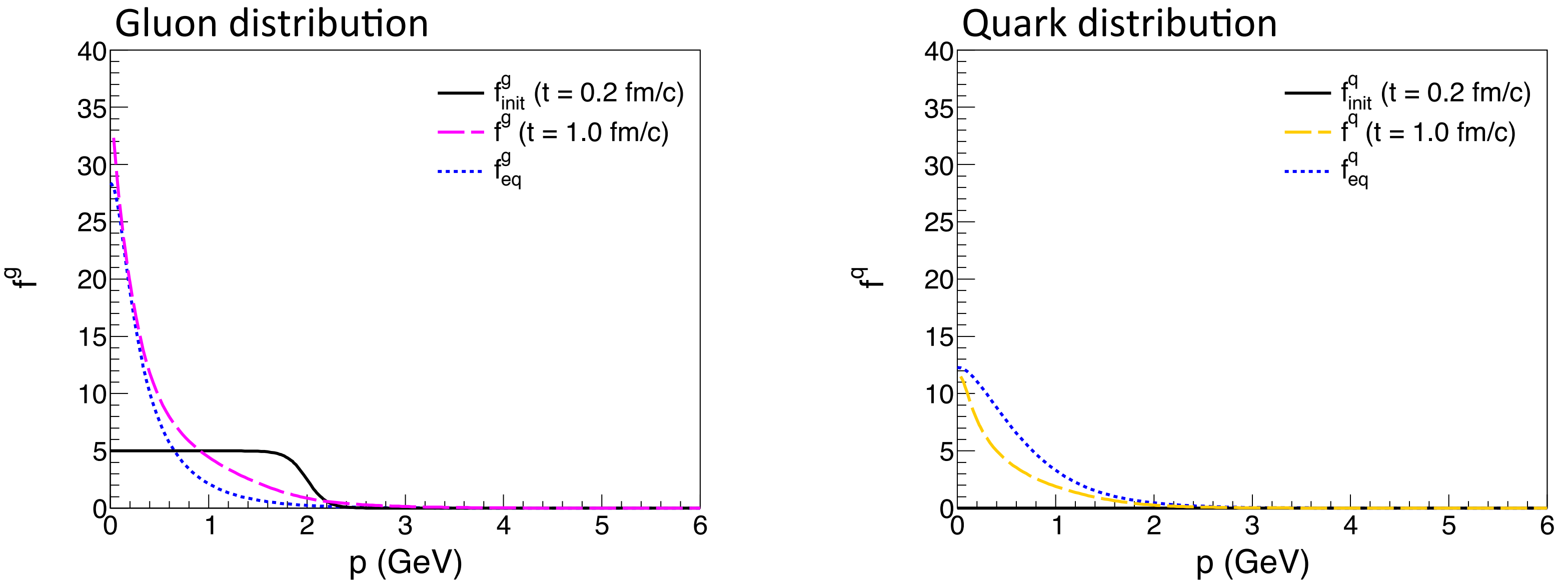
# Numerical analyses

- Time evolutions (t = 0.8 fm/*c*)



# Numerical analyses

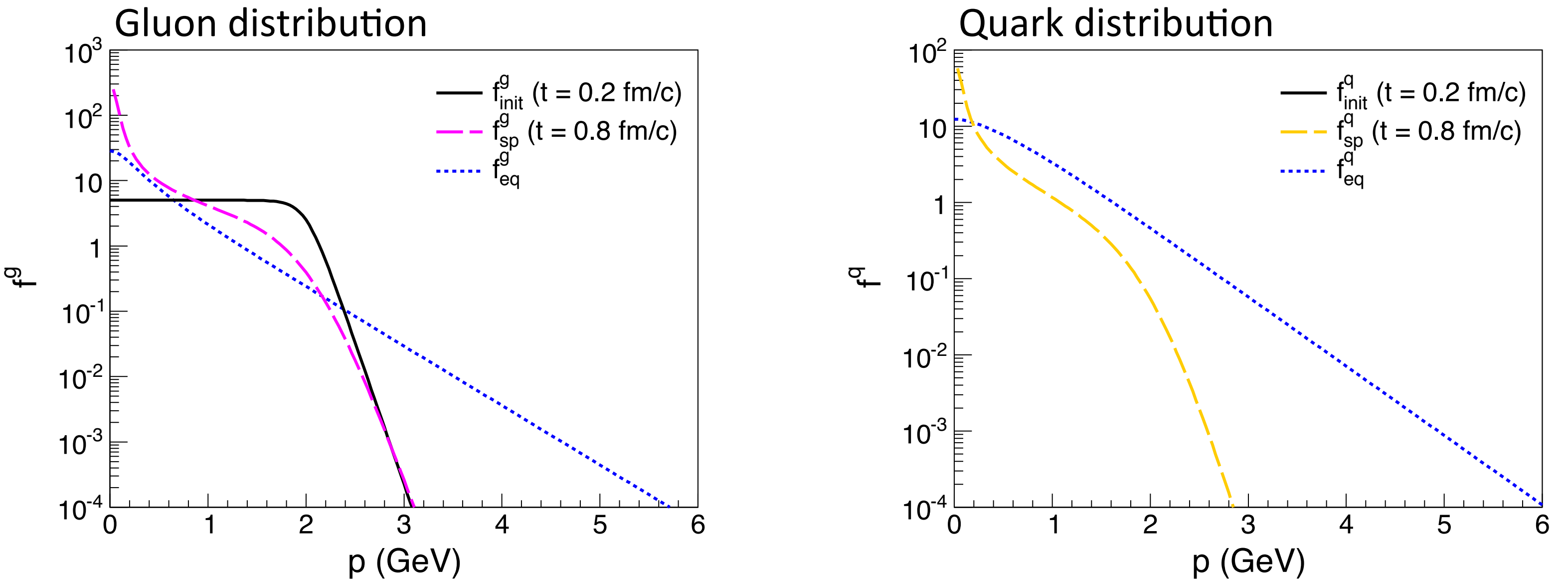
- Time evolutions (t = 1.0 fm/*c*)



- Distributions approach the thermal ones; chemical equilibration is relatively fast but would be slower than thermalization (~1.5-2.0 fm)
- Shape of quark distribution reflects that of gluon distribution as quarks are pair-created from gluons
- “Fermi pressure” would not develop as # of quarks are not enough

# Numerical analyses

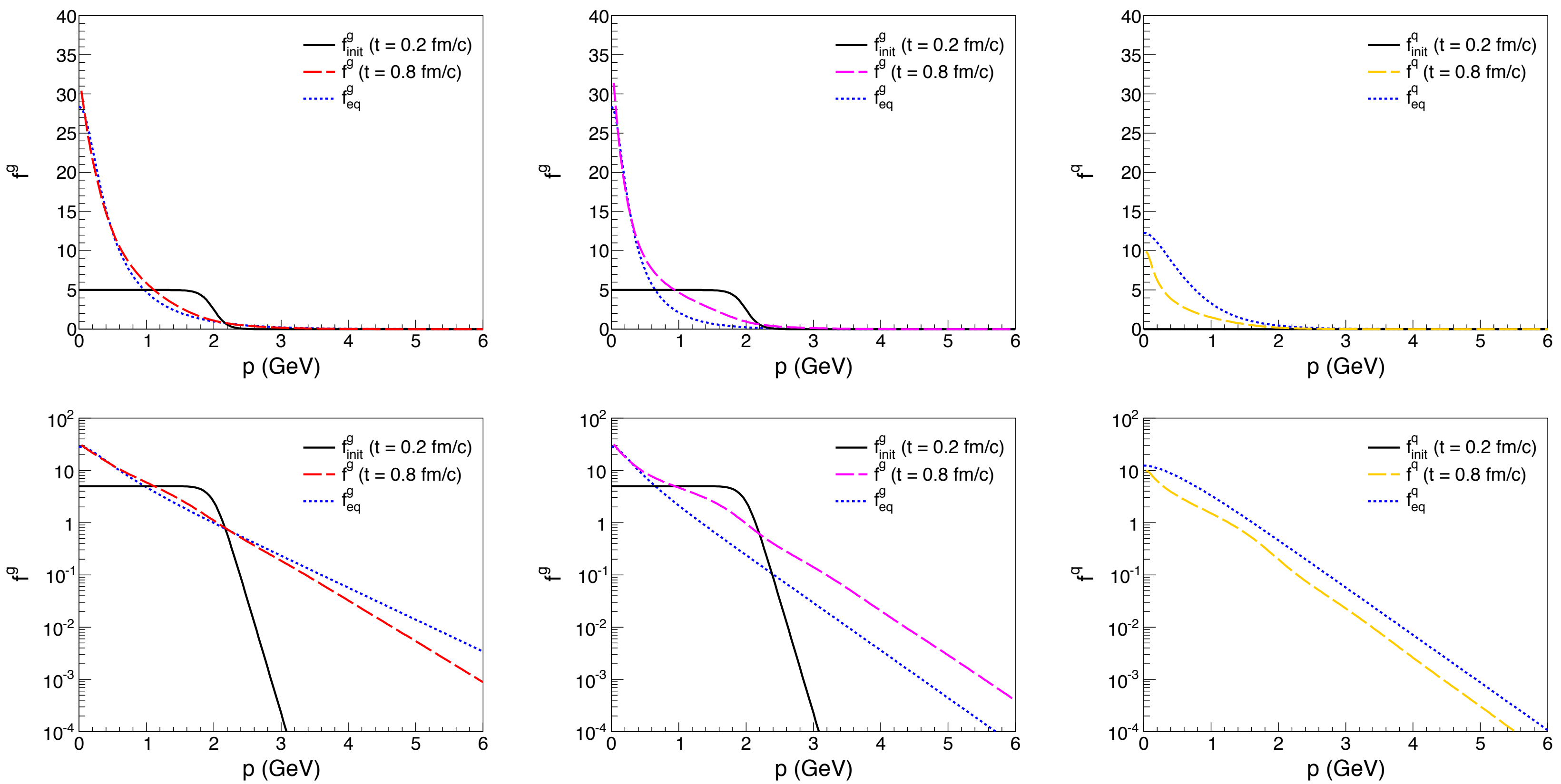
- Splitting with no recombination (logarithmic scale)



- The shape of quark distribution follows that of gluon distribution
- Quark number becomes too large for fermions near p = 0; splitting should be suppressed, leading to slower chemical equilibration

# Numerical analyses

- Comparison of $f_g$ (pure gauge), $f_g$ and $f_q$ ($N_f = 3$)



- Quark-gluon equilibration may be less “efficient” but more realistic

# Summary and outlook

- Collinear quark and gluon splitting in early thermalization
  - Low momentum gluons are quickly produced
  - Quark production is reasonable fast; recombination would be important for fermions

    ⇨ Describes transition from CGC to QGP
  - Thermalization might be faster than chemical equilibration
- Future prospects include
  - Three dimensional modeling for analyses on effects of expansion and isotropization
  - Non-thermalized partons lead to off-equilibrium energy-momentum tensor at the initial time of hydrodynamic stage

# The end

- Thank you for listening!
- Website: http://tkynt2.phys.s.u-tokyo.ac.jp/~monnai/