# Chiral symmetry breaking patterns in the $U(n) \times U(n)$ meson model

Gergely Fejős

Theoretical Research Division
Nishina Center
RIKEN

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- $U(n) \times U(n)$ meson model is an effective theory of QCD (for $n = 3$):

$$\mathcal{L} = \partial_\mu M \partial^\mu M^\dagger - m^2 \,\mathrm{Tr}\,(MM^\dagger) - g_1 [\,\mathrm{Tr}\,(MM^\dagger)]^2 - g_2 \,\mathrm{Tr}\,(MM^\dagger MM^\dagger)$$

$\rightarrow$ two couplings $\Rightarrow$ no stable IR fixed point of the RG-flow
$\rightarrow$ famous example of fluctuation induced first order transition

- RG-analysis does not tell anything about the symmetry breaking pattern
$\longrightarrow$ it is usually *assumed* to be $U_L(n) \times U_R(n) \longrightarrow U_V(n)$
$\longrightarrow$ (probably) based on the Vafa-Witten theorem:
*„No vector symmetries can be broken spontaneously in vector-like gauge theories."*

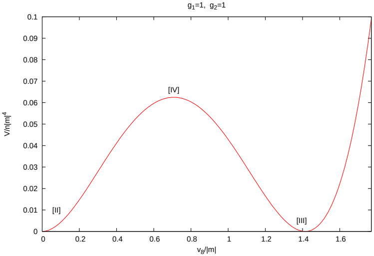
- Other scenarios also appeared in the literature:
$\longrightarrow U_L(3) \times U_R(3) \longrightarrow U_V(2)$
$\longrightarrow U_L(3) \times U_R(3) \longrightarrow U_L(2) \times U_R(2)$

- Searching for new local minima of the effective potential in an extended background: $<M>= v_0 T^0 + v_8 T^8$
- Calculation of the field equations at classical level

$$0 = \left.\frac{\partial V}{\partial M}\right|_{v_0, v_8}$$

$\longrightarrow$ 5 independent set of solutions for $v_0$, $v_8$
$\longrightarrow$ usual minimum: $v_0 \neq 0$, $v_8 = 0$ [$U_V(n)$ breaking]
$\longrightarrow$ new minimum: $v_0 \neq 0$, $v_8 \neq 0$ [? breaking]

- Surprisingly the spectrum is the same in both cases
  $\longrightarrow$ no new type of symmetry breaking!



- New local minima of the effective potential can always be generated by axial transformations from $<M>\sim \mathbf{1}$:

$$<M> \quad \longrightarrow \quad A^\dagger <M> A^\dagger \equiv U <M>$$

- One has to search for $U$ matrices with $\{\tilde{v}_i\}$ coefficients fulfilling the equation

$$U = \sum_{i \in \mathrm{diag}} \tilde{v}_i T^i$$

$\longrightarrow$ intesection of $U(n)$ and the center of its Lie-algebra

- A quite simple recursion can be derived for coefficients $\{\tilde{v}_i\}$
$\longrightarrow$ $2^n$ different vacua can be obtained
$\longrightarrow$ all show a breaking pattern $U_L(n) \times U_R(n) \longrightarrow U_V(n)$
- in the stability region relevant for QCD (i.e. $g_2 > 0$)
Vafa and Witten's theorem remains valid
- vacuum structure: more complicated than thought before