Jets in quark-gluon plasmas

New Frontiers in QCD 2013

--- *Insight into QCD matter from heavy-ion collisions* ---

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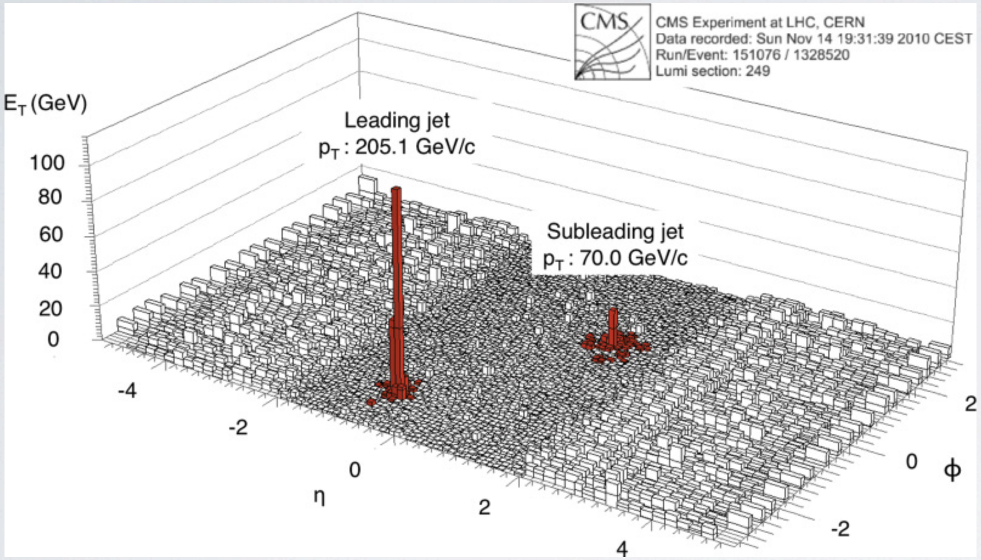
Outline

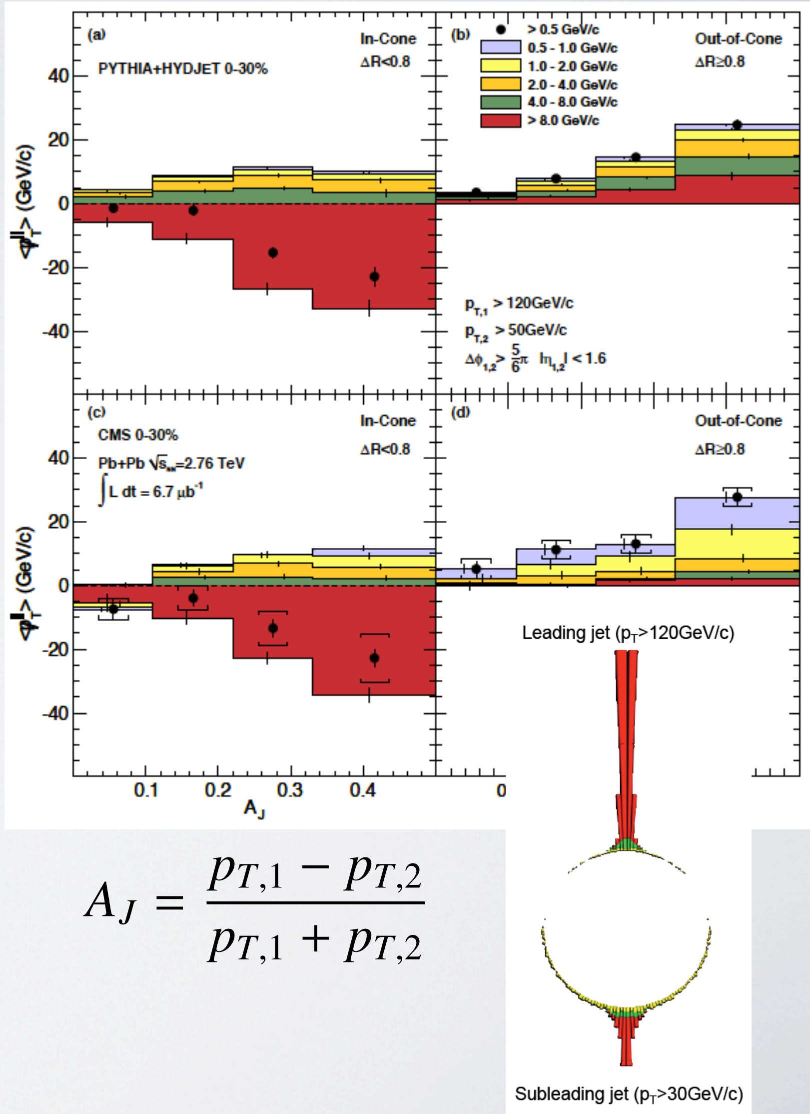
- Phenomenological motivations
- In-medium gluon branching (BDMPSZ mechanism)
- Multiple branching, (de)coherence, in-medium cascade
- Radiative corrections to the jet quenching parameter
- Turbulent cascade
- Summary

Work done in collaboration with F. Dominguez, E. Iancu and Y. Mehtar-Tani (arXiv:1209.4585, 1301.6102, 1311.5823)

Di-jet asymmetry

there is more to it than just 'jet quenching'...





$$A_J = \frac{p_{T,1} - p_{T,2}}{p_{T,1} + p_{T,2}}$$

Missing energy is associated with additional **radiation of many soft quanta at large angles**

We argue that this reflects a **genuine feature of the in-medium QCD cascade** (JPB, E. Iancu and Y. Mehtar-Tani, arXiv: 1301.6102)

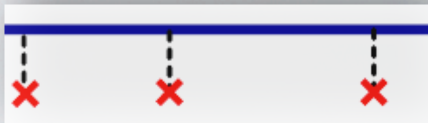
In-medium parton branching
BDMPSZ mechanism

(Baier, Dokshitzer, Mueller, Peigné, Schiff; Zakharov ~ 1996)

First order perturbation theory
in a random external field

Momentum broadening

$\mathcal{P}(\boldsymbol{k}-\boldsymbol{p}_0;t_L,t_0)$ probability to acquire transverse momentum $\mathbf{k}-\mathbf{p}_0$ when propagating in medium from t_0 to t_L



Evolution equation

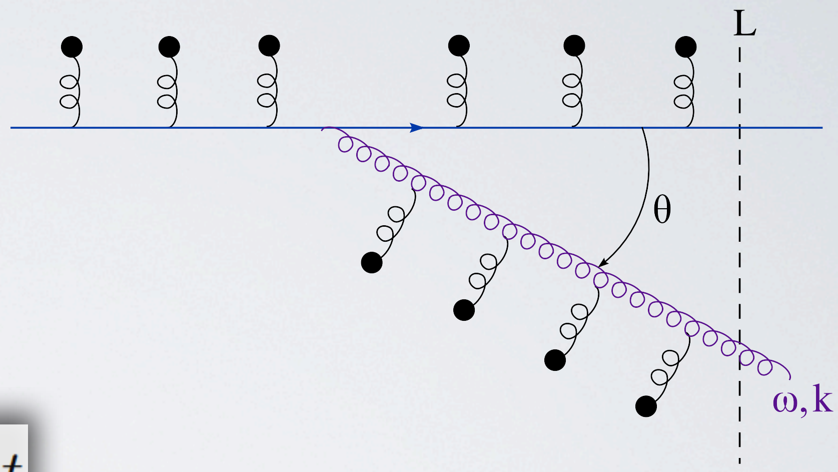
$$\frac{\partial}{\partial t}\mathcal{P}(\boldsymbol{k}-\boldsymbol{p}_0;t,t_0)=\int_{\boldsymbol{l}}\mathcal{C}(\boldsymbol{l},t)\,\mathcal{P}(\boldsymbol{k}-\boldsymbol{p}_0-\boldsymbol{l};t,t_0)$$

Diffusion approximation

$$\frac{\partial}{\partial t}\mathcal{P}(\boldsymbol{k}-\boldsymbol{p}_0;t,t_0)=\frac{1}{4}\frac{\partial^2}{\partial \boldsymbol{k}^2}\left[\hat{q}(t,\boldsymbol{k}^2)\,\mathcal{P}(\boldsymbol{k}-\boldsymbol{p}_0;t,t_0)\right]$$

$$\Delta k_\perp^2=\hat{q}\Delta t$$

The BDMPSZ mechanism



Gluon emission is linked to momentum broadening

$$\frac{1}{\tau_{\rm f}} \sim \frac{k_\perp^2}{2\omega}$$

$$\Delta k_\perp^2 = \hat{q}\Delta t$$

Time scale for the branching process

$$\tau_{\rm br}(\omega) \sim \sqrt{\frac{2\omega}{\hat{q}}}$$

Medium of finite extent

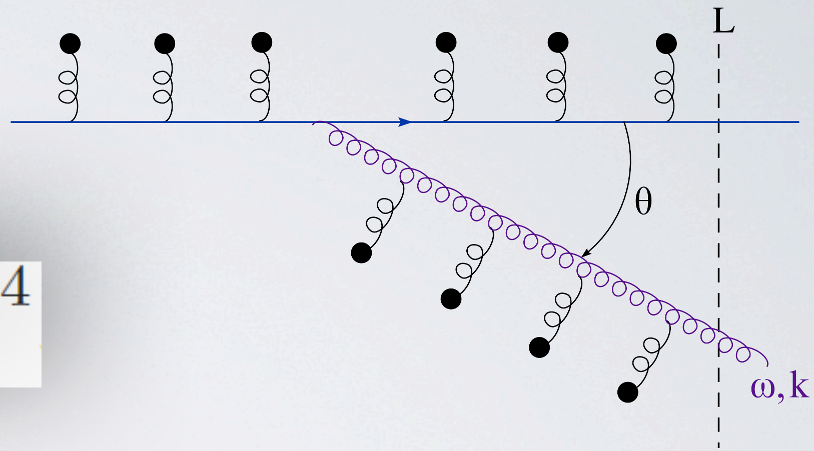
$$\tau_{\rm br} \lesssim L \Rightarrow \omega \lesssim \omega_c \qquad \omega_c \sim \hat{q}L^2$$

Formation time and emission angle

Typical branching kT and angle

$$k_{\mathrm{br}}^2 = \hat{q}\tau_{\mathrm{br}}$$

$$\theta_{\mathrm{br}} \sim k_{\mathrm{br}}/\omega \sim \left(\hat{q}/\omega^3\right)^{1/4}$$



Hard gluon: small angle, long time

$$\tau_{\mathrm{br}} \lesssim L \qquad \omega \lesssim \omega_c \qquad \theta_{\mathrm{br}} \gtrsim \theta_c$$

Soft gluon: large angle, short time

$$\tau_{\mathrm{br}} \ll L \qquad \omega \ll \omega_c \qquad \theta_{\mathrm{br}} \gg \theta_c$$

BDMPSZ spectrum

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\omega_c}{\omega}} \equiv \bar{\alpha} \sqrt{\frac{\omega_c}{\omega}} = \bar{\alpha} \frac{L}{\tau_{\mathrm{br}}(\omega)}$$

Hard emissions

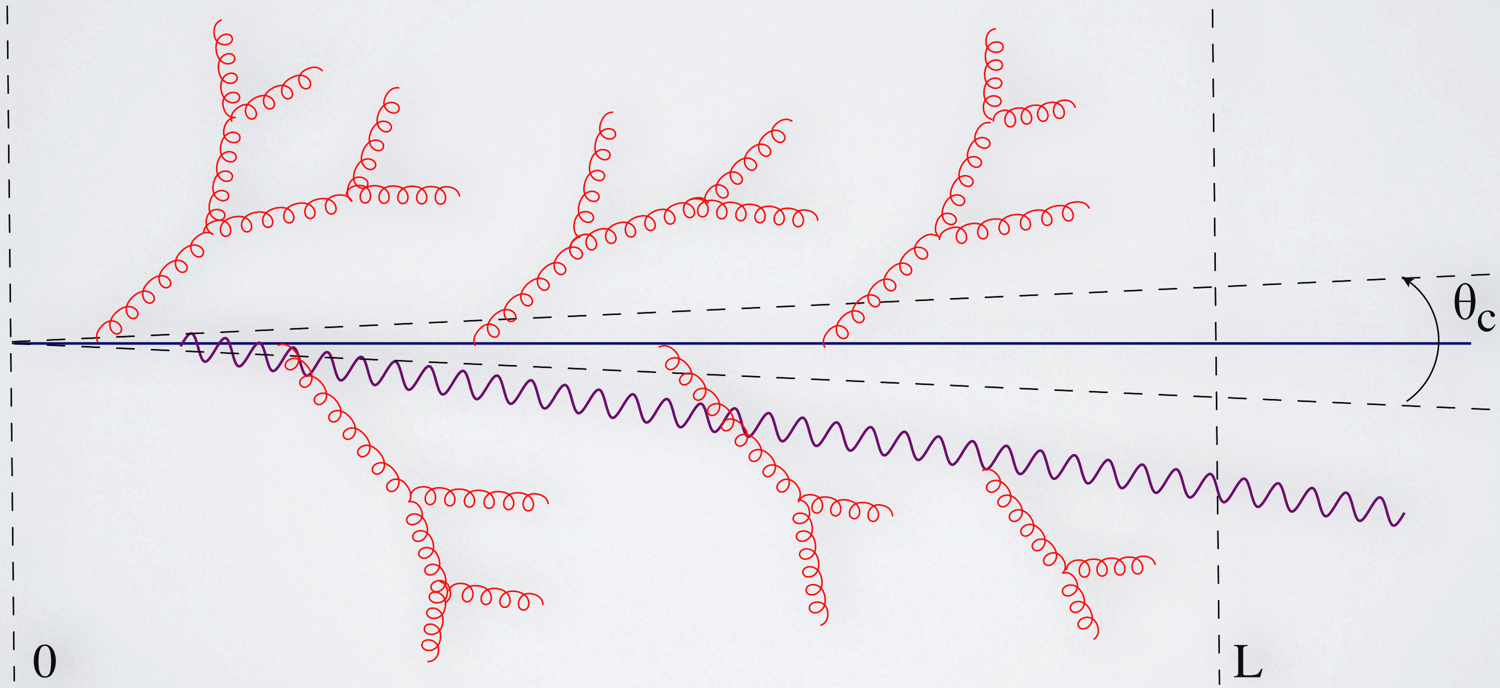
- rare events, with probability $\sim \mathcal{O}(\alpha_s)$
- dominate energy loss: $E_{\mathrm{hard}} \sim \alpha_s \omega_c$
- small angle, not important for di-jet asymmetry

Soft emissions

- frequent, with probability $\sim \mathcal{O}(1)$
- weaker energy loss: $E_{\mathrm{soft}} \sim \alpha_s^2 \omega_c$
- but arbitrary large angles: control di-jet asymmetry

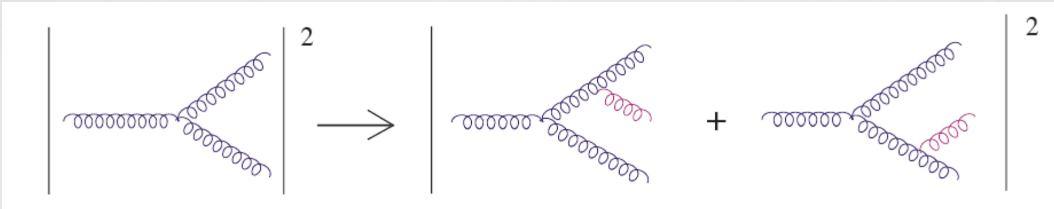
large angles emissions are dominated by soft multiple branchings

Multiple branchings
(de)-coherence
in-medium cascade



Multiple emissions

A priori complicated by interferences



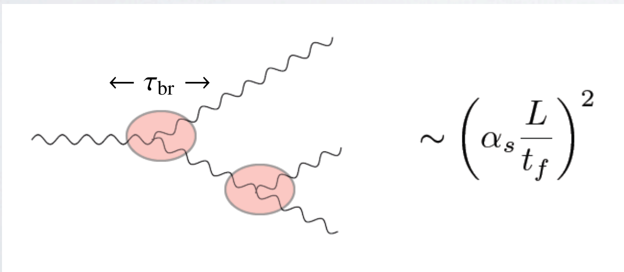
In vacuum, these interferences lead to angular ordering

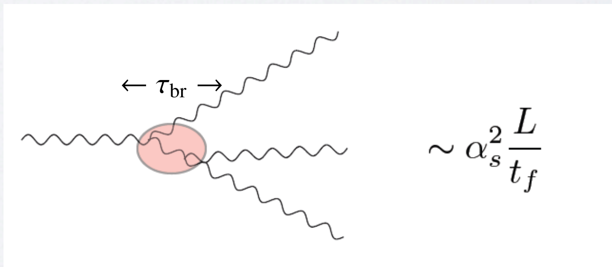
In medium color coherence is rapidly lost via rescattering

Mehtar-Tani, Salgado, Tywoniuk (1009.2965; 1102.4317)
Iancu, Casalderey-Solana (1106.3864)

In medium, interference effects are subleading

Independent emissions are enhanced by a factor L/τ_f



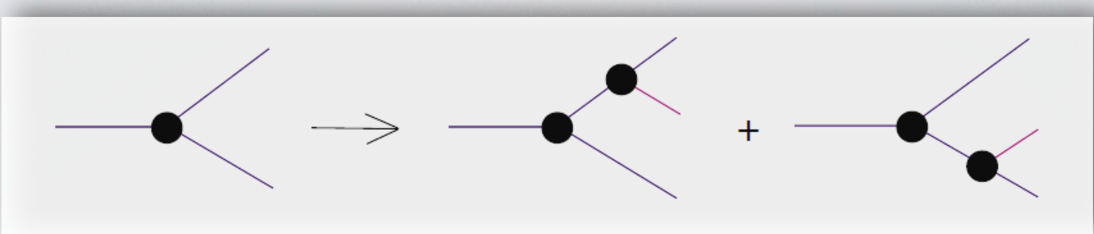


JPB, F. Dominguez, E. Iancu, Y. Mehtar-Tani, arXiv: 1209.4585

Resumming the leading terms

When $\bar{\alpha}L/\tau_{br} \sim 1$ all powers of $\bar{\alpha}L/\tau_{br} \sim 1$ need to be resummed.

Since independent emissions dominate, the leading order resummation is equivalent to a probabilistic cascade, with nearly local branchings



Blob: BDMPSZ spectrum

Line: momentum broadening

JPB, Dominguez, Iancu and Mehtar-Tani (arXiv:1209.4585)

Note: already implemented in Monte Carlo codes

MARTINI (Jeon, Gale, Schenke)
Q_Pythia (Armesto, Salgado et al)
Stachel, Wiedemann, Zapp

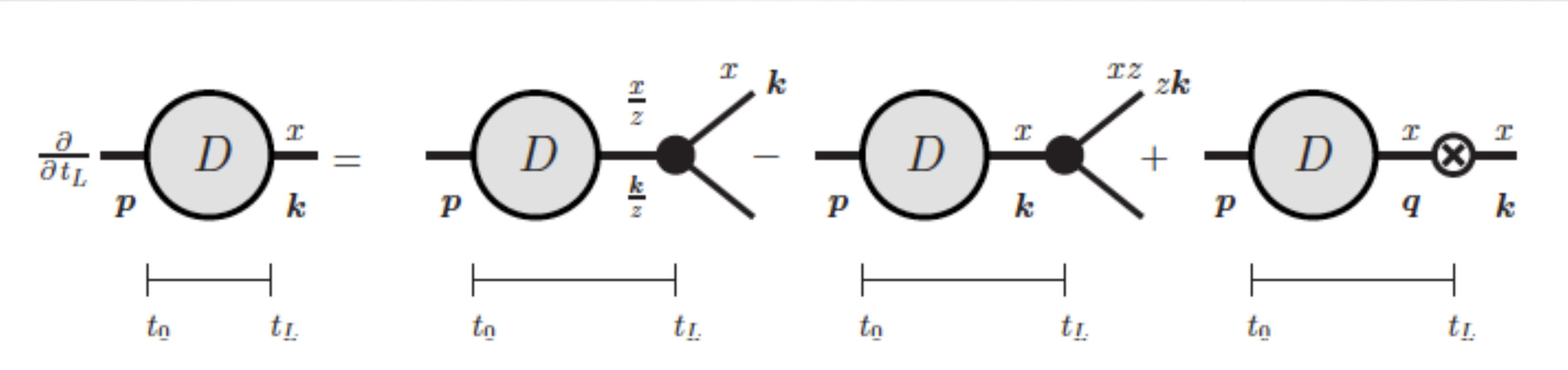
Inclusive one-gluon distribution

Initial parton $(p_0^+, \mathbf{p} = 0)$

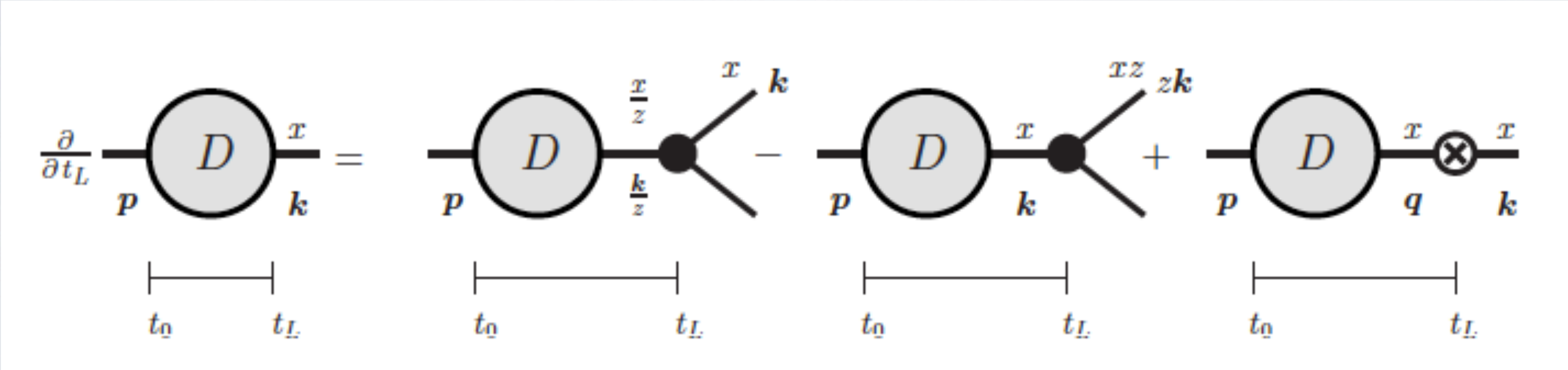
Probability to find a parton with $(k^+ = xp_0^+, \mathbf{k})$

at (light-cone) time t

$$D(x, \mathbf{k}, t)$$



Inclusive one-gluon distribution



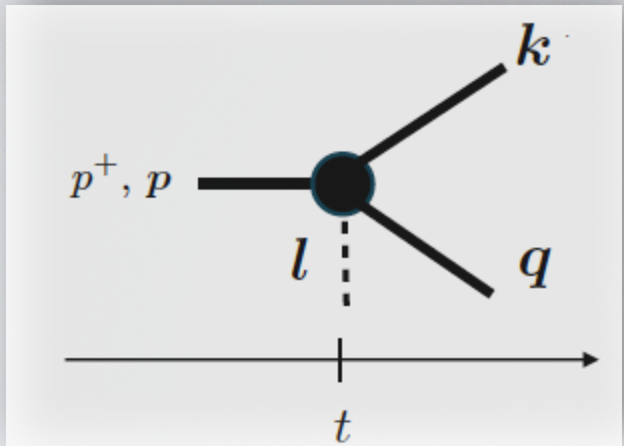
Leading order equation

$$\frac{\partial}{\partial t} D(x, \boldsymbol{k}, t) = \int_{\boldsymbol{l}} \mathcal{C}(\boldsymbol{l}, t) D(x, \boldsymbol{k} - \boldsymbol{l}, t)$$
$$+ \alpha_s \int_0^1 \mathrm{d}z \left[\frac{2}{z^2} \mathcal{K}\left(z, \frac{x}{z} p_0^+; t\right) D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t\right) - \mathcal{K}\left(z, x p_0^+; t\right) D(x, \boldsymbol{k}, t) \right]$$

Radiative correction to $\hat{q}$

Beyond leading order

$$\frac{\partial}{\partial t_L} D(x, \boldsymbol{k}, t_L) = \alpha_s \int_0^1 \mathrm{d}z \left[2\mathcal{K}\left(z, \frac{x}{z} p^+, t_L\right) D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t_L\right) - \mathcal{K}\left(z, x p^+, t_L\right) D\left(x, \boldsymbol{k}, t_L\right) \right]$$
$$- \frac{1}{4} \left[\hat{q}_0(t_L) + \hat{q}_1(t_L) \right] \left(\frac{\partial}{\partial \boldsymbol{k}} \right)^2 D\left(x, \boldsymbol{k}, t_L\right)$$



$$\boldsymbol{Q} \equiv \boldsymbol{k} - z(\boldsymbol{q} + \boldsymbol{l})$$

$z \simeq 1$ $\quad$ $\boldsymbol{Q} + \boldsymbol{l} \simeq \boldsymbol{k} - \boldsymbol{q}$

$$\hat{q}_0(t) \equiv \int_{\boldsymbol{q}} \boldsymbol{q}^2 \, \mathcal{C}(\boldsymbol{q}, t)$$

$$\hat{q}_1(t, \boldsymbol{k}^2) \equiv 2\alpha_s \int \mathrm{d}z \int_{\boldsymbol{Q}, \boldsymbol{l}}^{\boldsymbol{k}^2} \left[(\boldsymbol{Q} + \boldsymbol{l})^2 - \boldsymbol{l}^2 \right] \mathcal{K}\left(\boldsymbol{Q}, \boldsymbol{l}, z, p^+, t\right)$$

Radiative correction to $\hat{q}$

$$\hat{q}_1(t, \boldsymbol{k}^2) \equiv 2\alpha_s \int \mathrm{d}z \int_{\boldsymbol{Q},\boldsymbol{l}}^{\boldsymbol{k}^2} \left[(\boldsymbol{Q}+\boldsymbol{l})^2 - \boldsymbol{l}^2\right] \mathcal{K}\left(\boldsymbol{Q}, \boldsymbol{l}, z, p^+, t\right)$$

$$(\omega\,,\,\boldsymbol{k}) \quad \rightarrow \quad \left(\Delta t = \frac{\omega}{\boldsymbol{k}^2}\,,\,\boldsymbol{k}\right)$$

Double logarithmic correction (large)

$$\boxed{\hat{q}_1 = \frac{\alpha_s C_A}{\pi} \int \frac{\mathrm{d}\Delta t}{\Delta t} \int \frac{\mathrm{dk}^2}{\mathrm{k}^2} \hat{q}_0}$$

[A. H. Mueller, B. Wu, T. Liou arXiv: 1304.7677]

Correction to interaction with medium constituents

Energy flow through democratic branching

Integrating over transverse momentum yields equation for energy flow

$$\frac{\partial D(x,\tau)}{\partial \tau} = \int \mathrm{d}z\, \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z},\tau\right) - \frac{z}{\sqrt{x}} D(x,\tau) \right]$$

$$\mathcal{K}(z) = \frac{\bar{\alpha}}{2} \frac{f(z)}{[z(1-z)]^{3/2}}, \qquad f(z) = [1 - z(1-z)]^{5/2}$$

Similar eq. postulated: R. Baier, A. H. Mueller, D. Schiff, D.T. Son (2001) S. Jeon, G. D. Moore(2003)

Formally analogous to DGLAP. But very different kernel... and physics.

$\Longrightarrow$ A QCD cascade of a new type

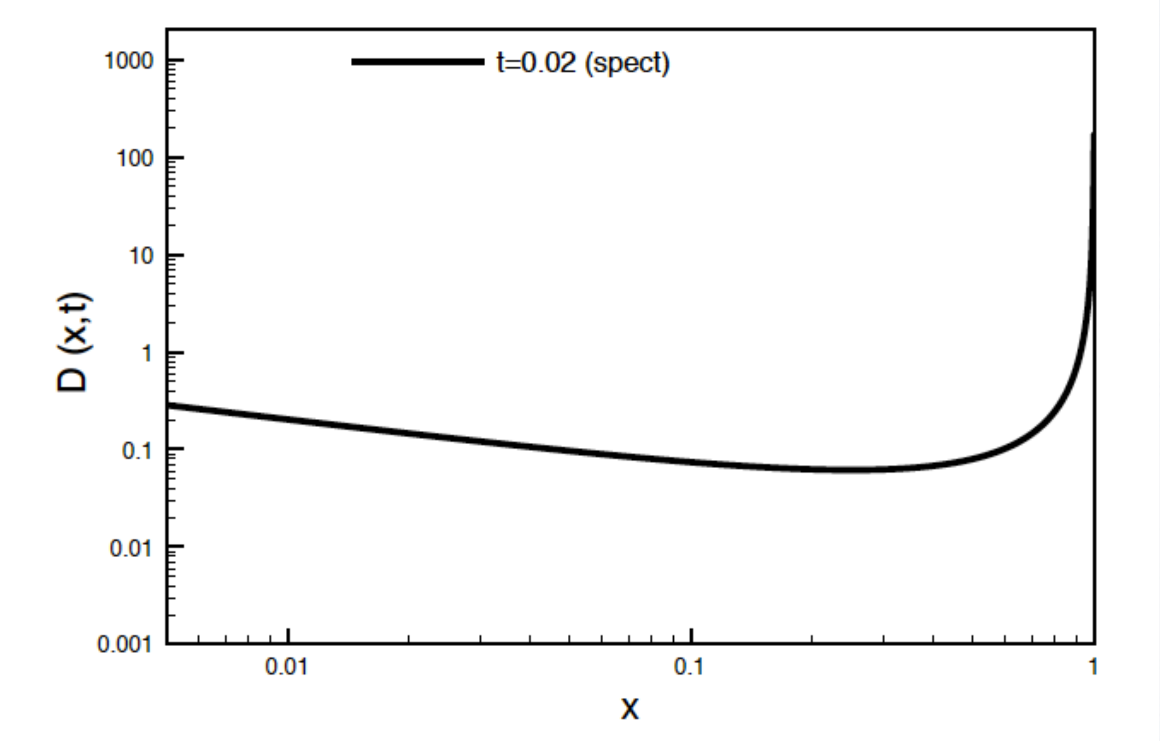
Exhibits wave turbulence

Short times

$$\frac{\partial D(x,\tau)}{\partial \tau} = \int \mathrm{d}z\,\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\Big(\frac{x}{z},\tau\Big) - \frac{z}{\sqrt{x}}\,D\big(x,\tau\big)\right]$$

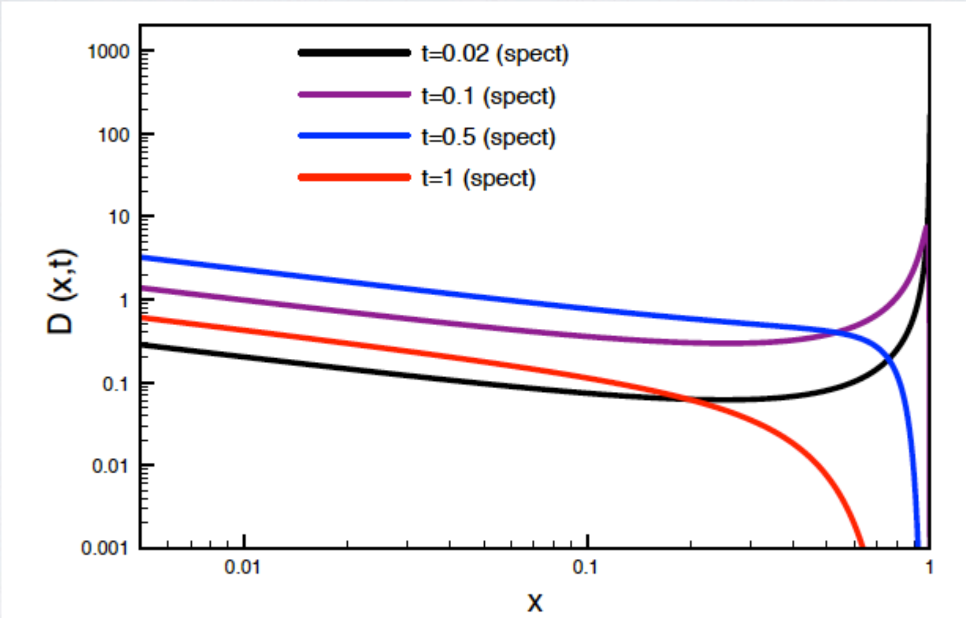
At short time, single emission by the leading particle $(D_0(\tau=0,x)=\delta(x-1))$

D is the BDMSZ spectrum



How do multiple branchings affect this spectrum ?

One finds (exact result) $$D(x,t) \simeq \frac{t}{\sqrt{x}}\, e^{-\pi t^2} \qquad \text{for } x \ll 1$$



Fine (local) cancellations between gain and loss terms

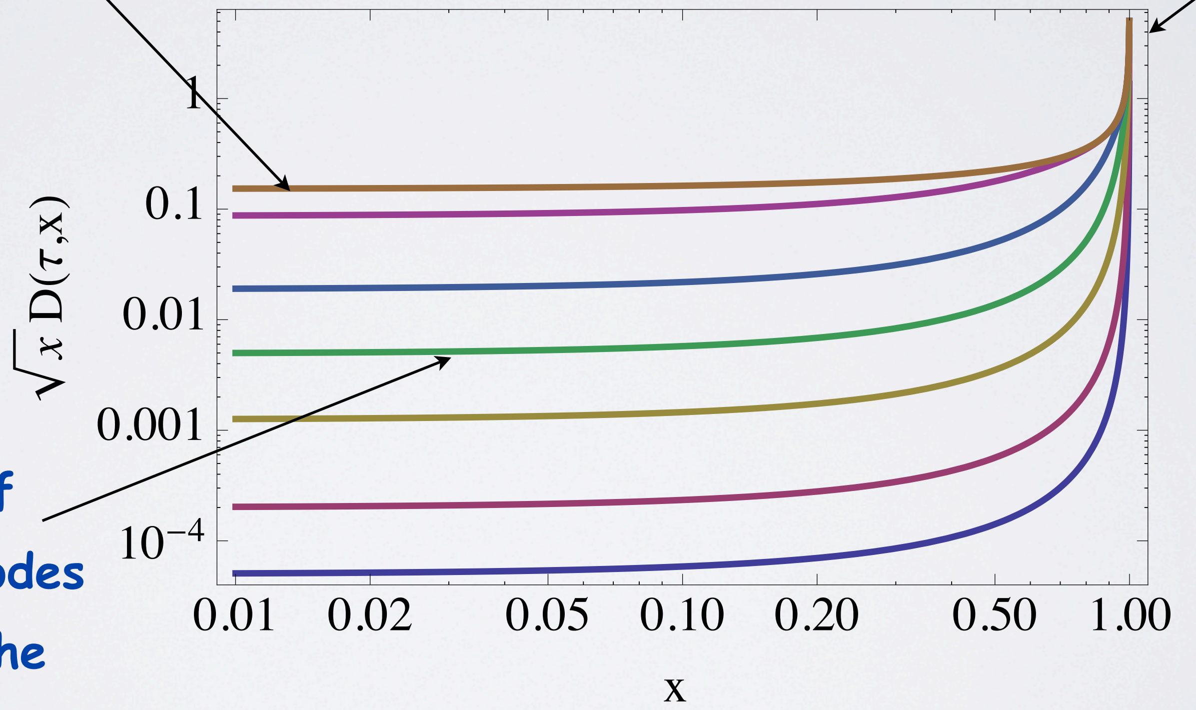
BDMPS spectrum emerges as a fixed point, scaling, spectrum

Characteristic features of wave turbulence (Kolmogoroz, Zakharov)

Digresssion: source problem

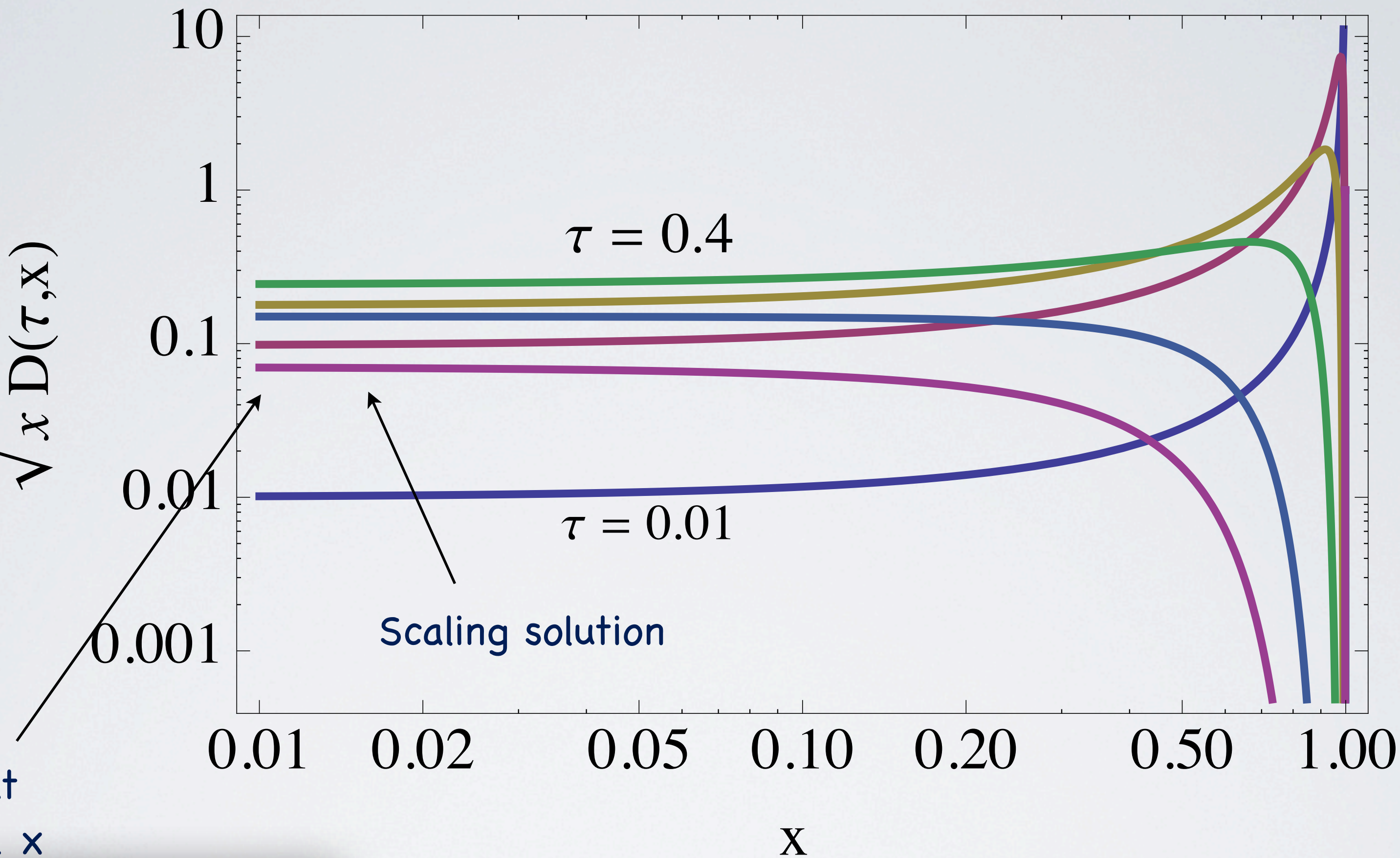
At this (fixed) point
ALL the energy flows
through the whole
system

Energy is injected
at x=1, at a
constant rate



The population of
the various x-modes
grows, keeping the
shape of the
spectrum at small x

Relevance to di-jet asymmetry



Flow of energy at very small x

$$\mathcal{E}_{\rm flow} = E\frac{\upsilon\tau^2}{2} = \frac{\upsilon}{2}\,\bar{\alpha}^2\omega_c \qquad \omega_c \equiv \frac{\hat{q}L^2}{2} \qquad \upsilon \simeq 5$$

Estimate $\hat{q} = 1\,\mathrm{GeV}^2/\mathrm{fm}$ $\quad \omega_c \simeq 40$ Gev $\quad \bar{\alpha}^2 \simeq 0.1 \quad \mathcal{E}_{\rm flow} \simeq 15$ GeV

$L = 4$ fm

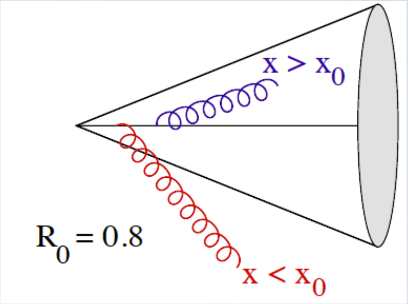
J.-P. B., E. Iancu, Y. Mehtar-Tani, arXiv: 1301.6102

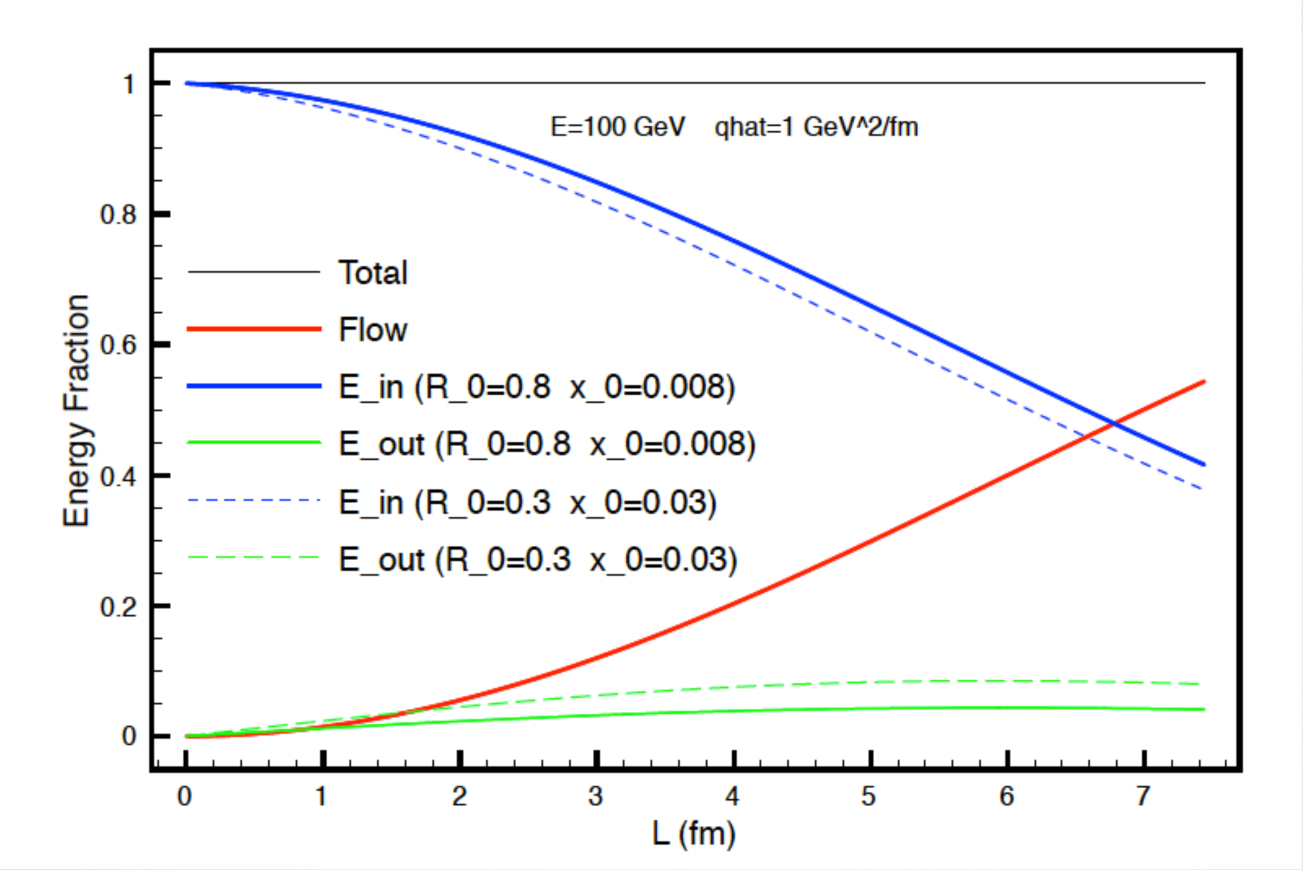
Energy flow at large angle

$E_{\rm in}$ energy in the jet with x>xo

$E_{\rm out}$ energy in the spectrum with x<xo

$E_{\rm out} + E_{\rm flow}$ energy out of the jet cone





Summary

In a medium of large size, the successive branchings can be treated as independent

Large radiative corrections can be absorbed in a renormalization of the jet quenching parameter

In-medium cascade is very different from the in-vacum cascade (no angular ordering, turbulent flow)

This turbulent cascade provides a simple and natural mechanism for the transfer of jet energy towards very large angles