Bottomonium and transport in the QGP

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(FASTSUM collaboration)

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Introduction

QCD thermodynamics

- euclidean formulation
- lattice QCD
- pressure, entropy, fluctuations, . . .

this talk: QCD real-time dynamics

- in or close to thermal equilibrium (linear response)
- lattice QCD
- analytical continuation to real time
- Green functions, in particular spectral functions

$\Rightarrow$ quarkonium spectral functions
$\Rightarrow$ transport coefficients from the lattice

Outline

quarkonia

- bottomonium spectral functions in the QGP

  S waves: Υ at rest, moving
  P waves: melting

light quarks

- transport: electrical conductivity

conclusion

FASTSUM collaboration

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bottomonium:

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PRL  (2011) 1010.3725 [hep-lat]
JHEP (2011) 1109.4496 [hep-lat]
JHEP (2013) 1210.2903 [hep-lat]
JHEP (2014) 1310.5467 [hep-lat]
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conductivity:

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PRL  (2013) 1307.6763 [hep-lat]
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Quarkonia and the QGP

quarkonia as a thermometer for the quark-gluon plasma

Matsui & Satz 86

- tightly bound states of charm quarks ($J/\psi, \ldots$) or bottom quarks ($\Upsilon, \ldots$) survive to higher temperatures
- broader states melt at lower temperatures

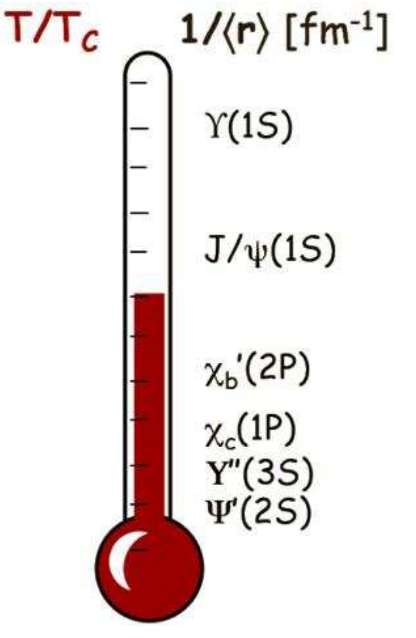


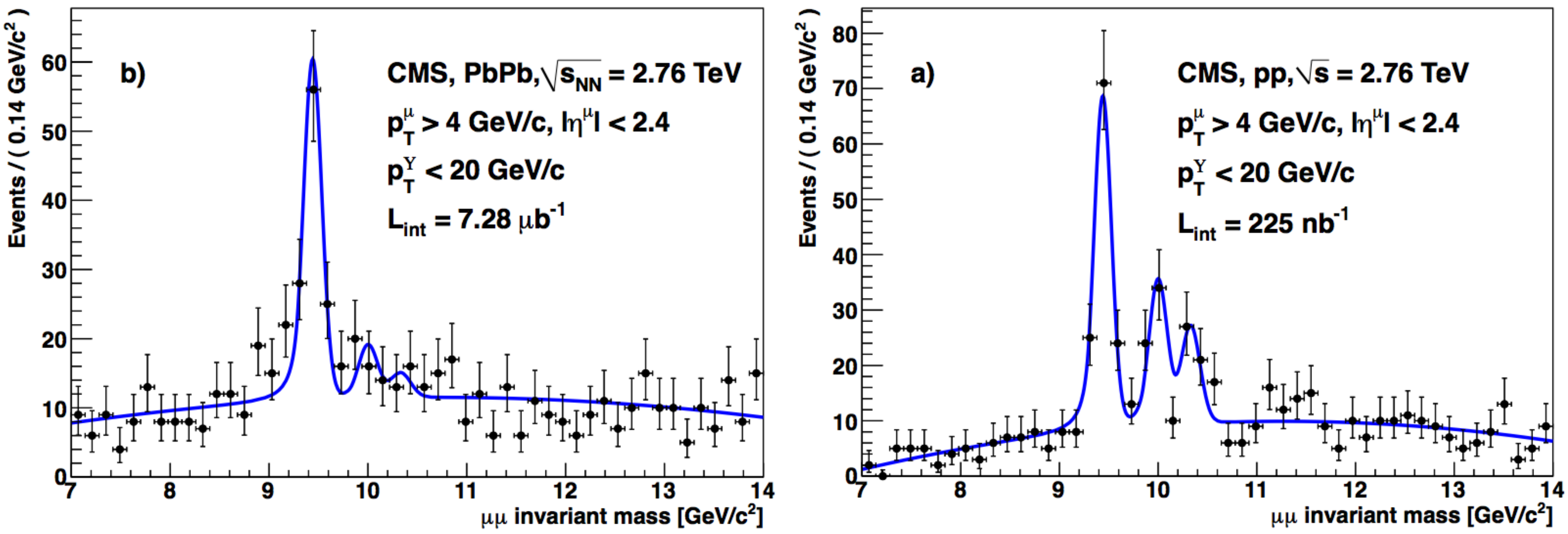
figure by A. Mocsy

melting pattern informs about temperature of the QGP

- relevant for heavy-ion collisions
- quantitative predictions required

Quarkonia and the QGP

- CMS results at the LHC: Υ spectrum
- compare PbPb collisions (left) and pp collisions (right)



- Υ(1S) survives – Υ(2S,3S) suppressed
- sequential melting

Quarkonia and the QGP

how to find the response of quarkonia to the QGP?

- potential models
- lattice QCD
- $\dots$

at $T > 0$:

- plethora of potential models: (seemingly) conflicting results
- interpretation of lattice correlators hindered by thermal (periodic) boundary conditions

re-addressed recently using first-principle approach:

- effective field theories (EFTs) and separation of scales

Quarkonia and EFTs

$M \gg T > \ldots$

hierarchy of scales:

- heavy quark mass M
- temperature T
- inverse size Mv
- Debye mass gT
- binding energy Mv^2

$\longrightarrow$ weak coupling

corresponding EFTs:

- NRQCD
- NRQCD + HTL
- pNRQCD
- pNRQCD + HTL
- $\ldots$

Laine, Philipsen, Romatschke & Tassler 07
Laine 07-08 Burnier, Laine & Vepsäläinen 08-09
Beraudo, Blaizot & Ratti 08 Escobedo & Soto 08
Brambilla, Ghiglieri, Vairo & Petreczky 08
Brambilla, Escobedo, Ghiglieri, Soto & Vairo 10
Escobedo, Soto & Mannarelli 11

...

Non-relativistic QCD

this talk:

- use NRQCD, one of the EFTs, nonperturbatively
- no potential model / no weak coupling

lattice QCD:

- heavy quarks with NRQCD

  requirement $M \gg T$

  bottomonium: $M_b \sim 4.5$ GeV $\qquad T \sim 150 - 400$ MeV

use of NRQCD very well motivated

Lattice QCD

- QGP with two light flavours (Wilson-like)
- many time slices: highly anisotropic lattices ($a_s/a_\tau = 6$)
- lattice spacing: $a_\tau^{-1} \simeq 7.35$ GeV, $a_s \simeq 0.162$ fm
- lattice size: $12^3 \times N_\tau$

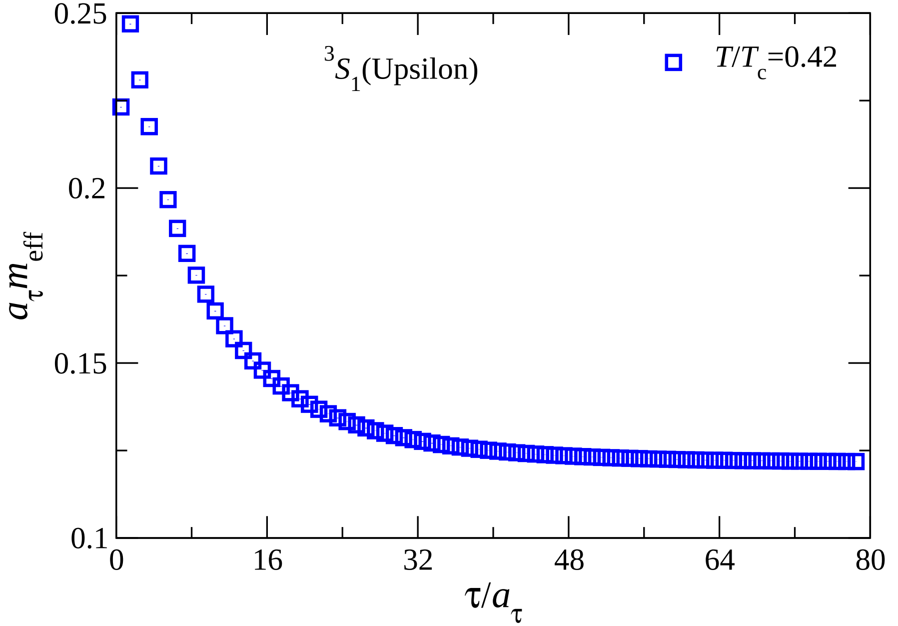
N_τ	80	32	28	24	20	18	16
T/T_c	0.42	1.05	1.20	1.40	1.68	1.86	2.09
$N_{\rm cfg}$	250	1000	1000	500	1000	1000	1000

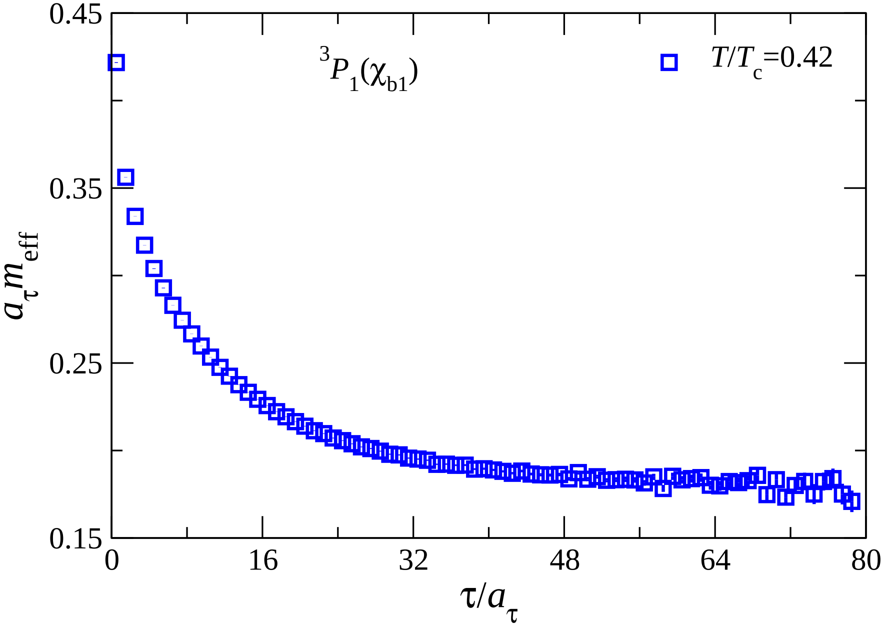
- bottom quark: NRQCD

  mean-field improved action with tree-level coefficients, including up to $\mathcal{O}(v^4)$ terms Davies et al 94
- in progress: extension to $N_f = 2+1$ 1311.3208 [hep-lat]

Spectrum at zero temperature

- exponential decay $G(\tau) \sim \exp\left(-m_{\rm eff}\tau\right)$
- no periodicity in euclidean time: initial-value problem





Υ (S wave)

χ_{b1} (P wave)

- effective mass plot $m_{\rm eff} = -\log\left[G(\tau)/G(\tau - a_\tau)\right]$

Spectrum

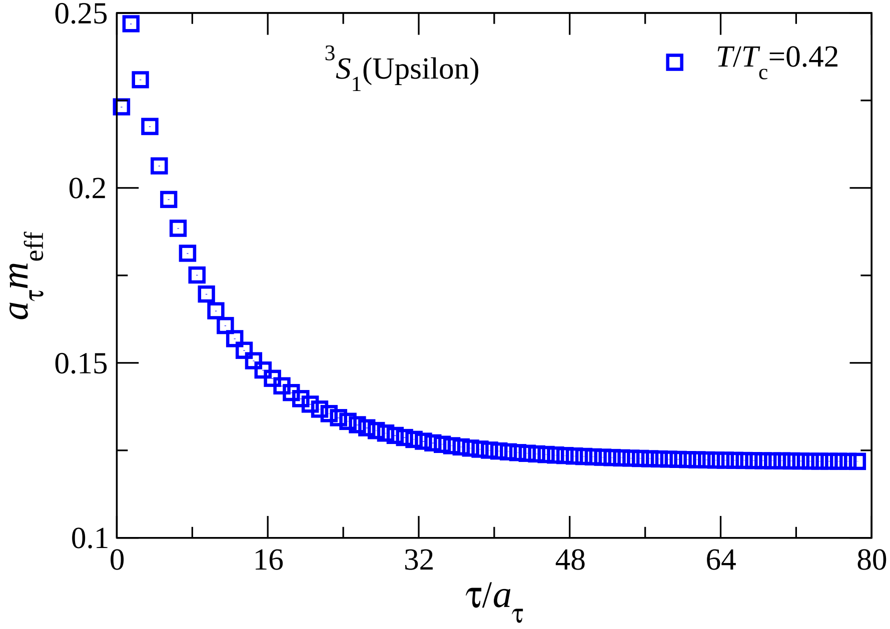
zero temperature: ground and first excited states

state	$a_\tau \Delta E$	Mass (MeV)	Exp. (MeV)
$1^1S_0(\eta_b)$	0.118(1)	9438(8)	9390.9(2.8)
$2^1S_0(\eta_b(2S))$	0.197(2)	10019(15)	-
$1^3S_1(\Upsilon)$	0.121(1)	9460*	9460.30(26)
$2^3S_1(\Upsilon')$	0.198(2)	10026(15)	10023.26(31)
$1^1P_1(h_b)$	0.178(2)	9879(15)	9898.3(1.1)(1.1)
$1^3P_0(\chi_{b0})$	0.175(4)	9857(29)	9859.44(42)(31)
$1^3P_1(\chi_{b1})$	0.176(3)	9864(22)	9892.78(26)(31)
$1^3P_2(\chi_{b2})$	0.182(3)	9908(22)	9912.21(26)(31)

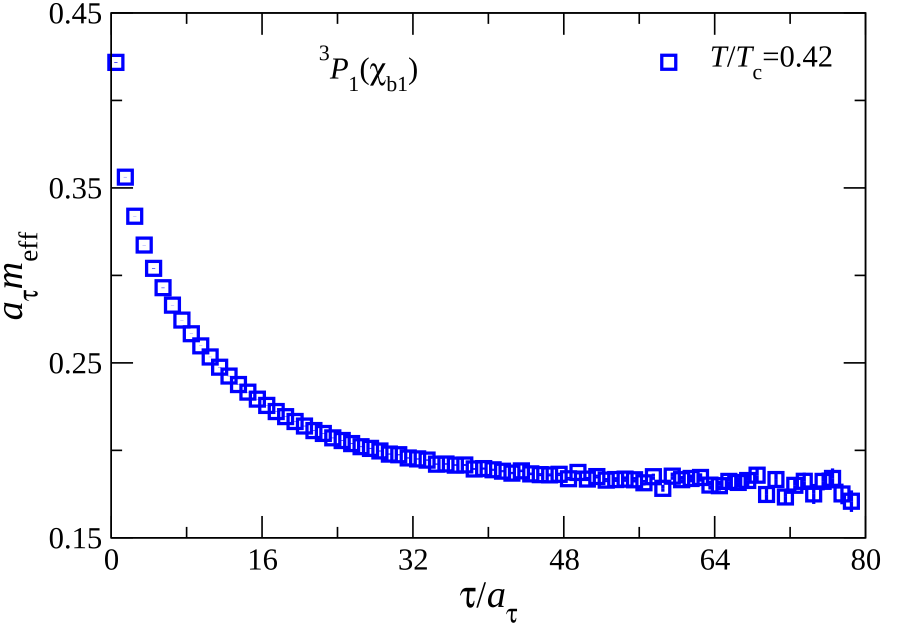
* Υ(1S) used to set the scale

Increasing the temperature

Υ S wave



χ_{b1} P wave

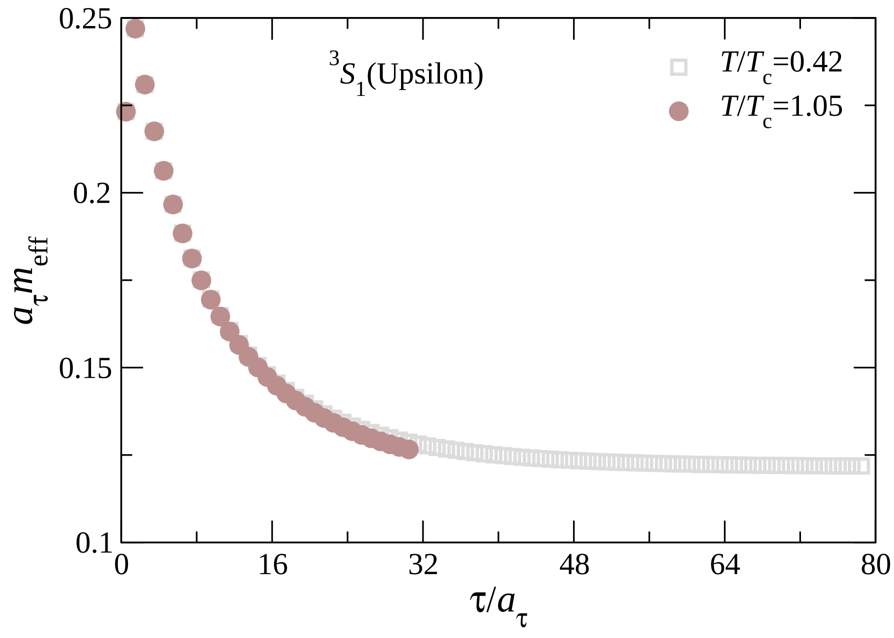


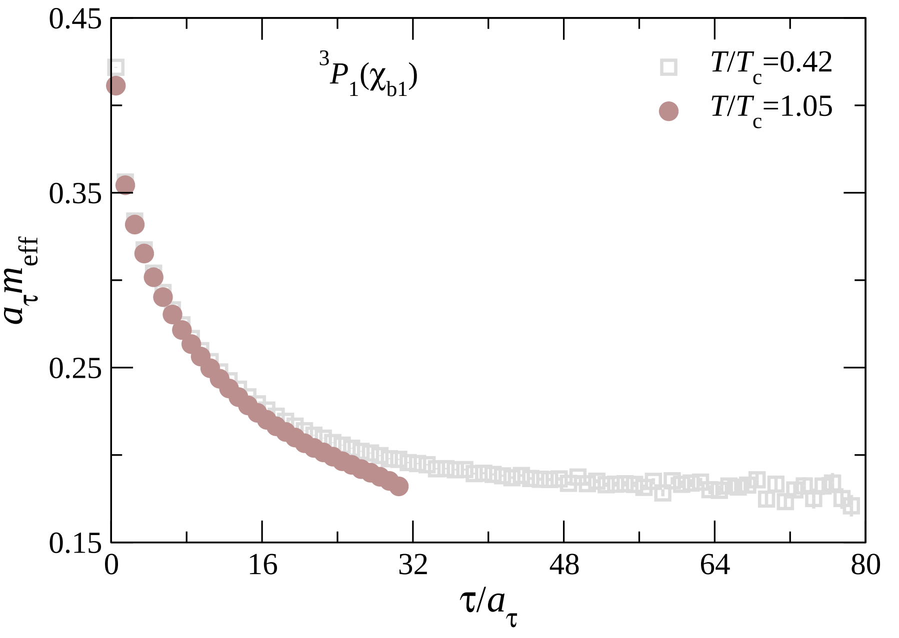
$$T/T_c = 0.42$$

Increasing the temperature

Υ S wave

χ_{b1} P wave

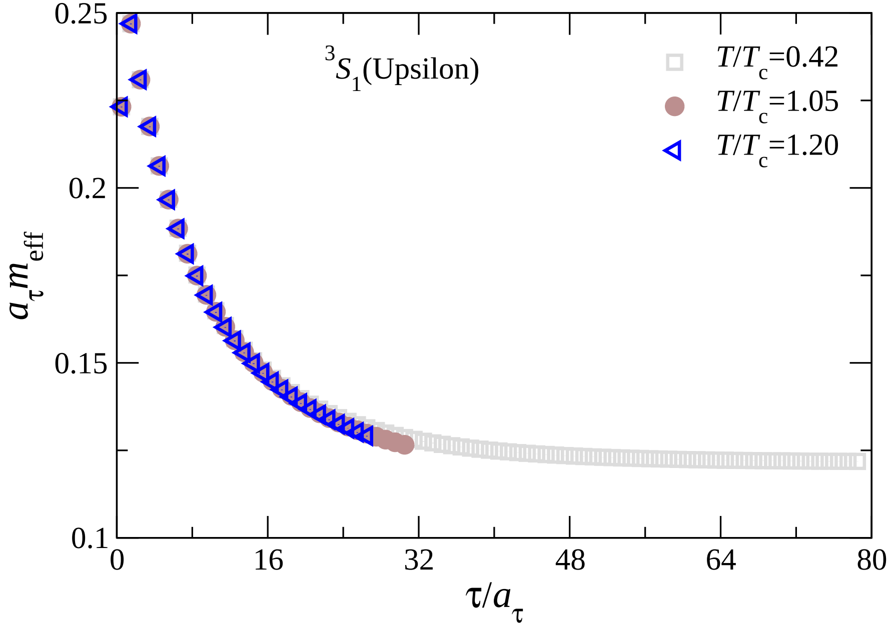




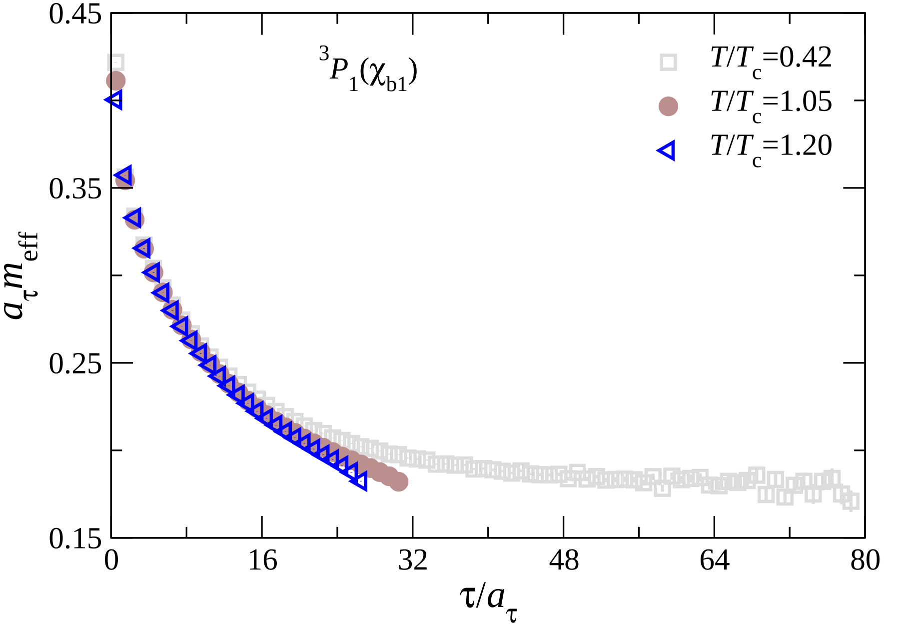
$$T/T_c = 1.05$$

Increasing the temperature

Υ S wave



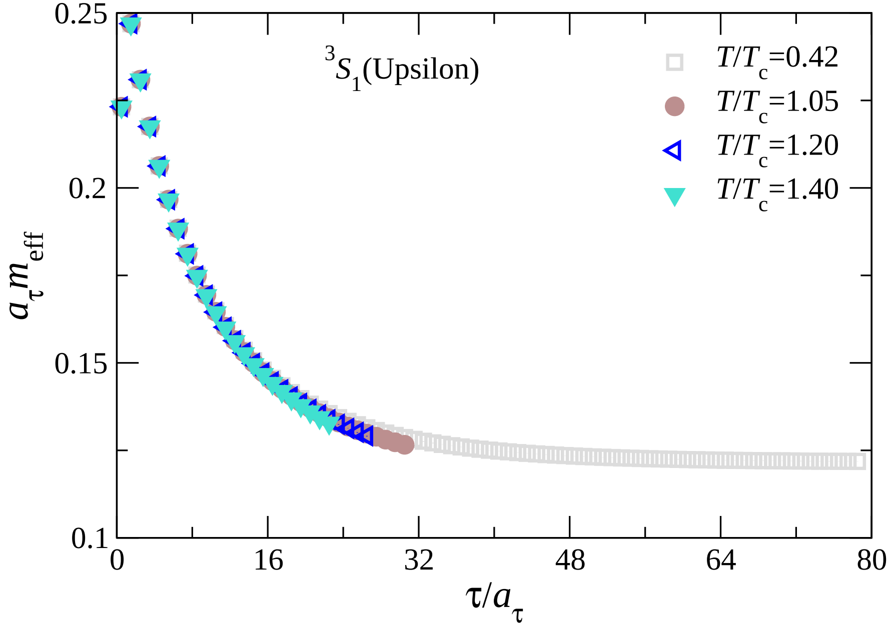
χ_{b1} P wave



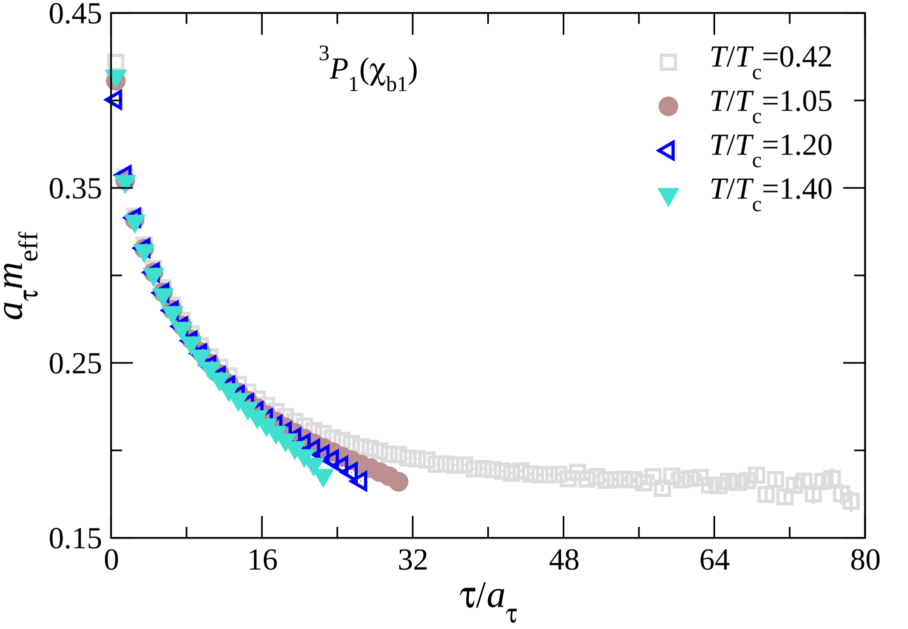
$$T/T_c = 1.20$$

Increasing the temperature

Υ S wave



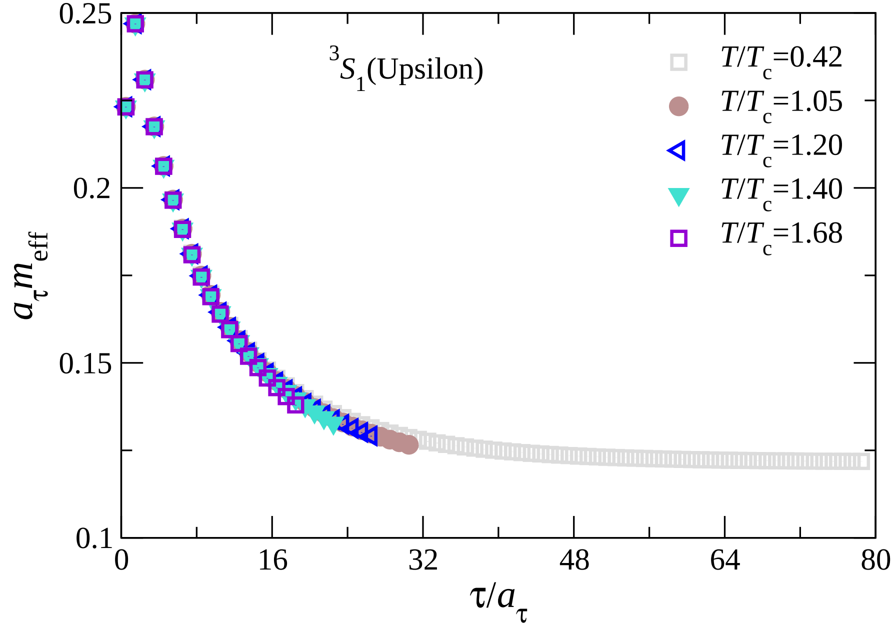
χ_{b1} P wave



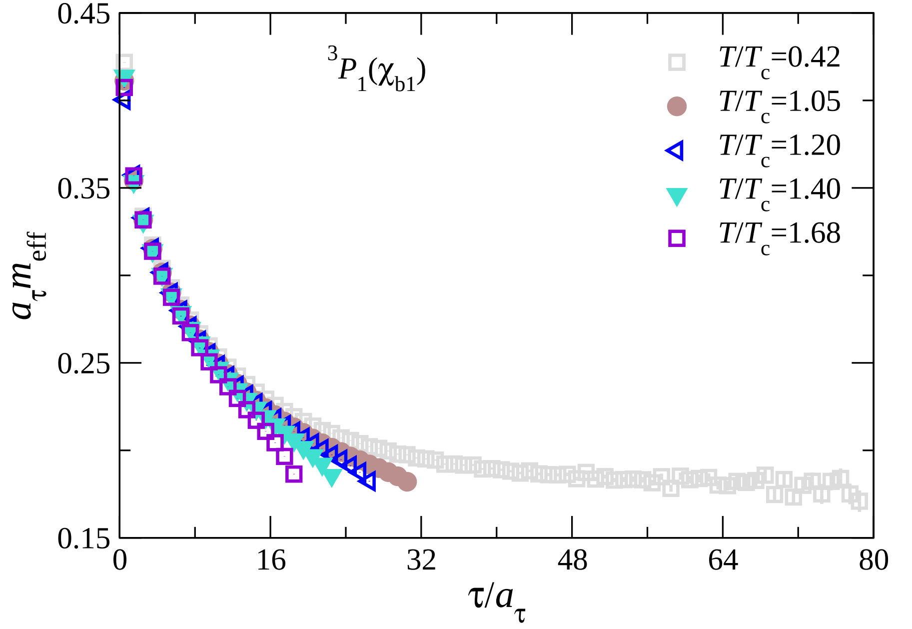
$$T/T_c = 1.40$$

Increasing the temperature

Υ S wave



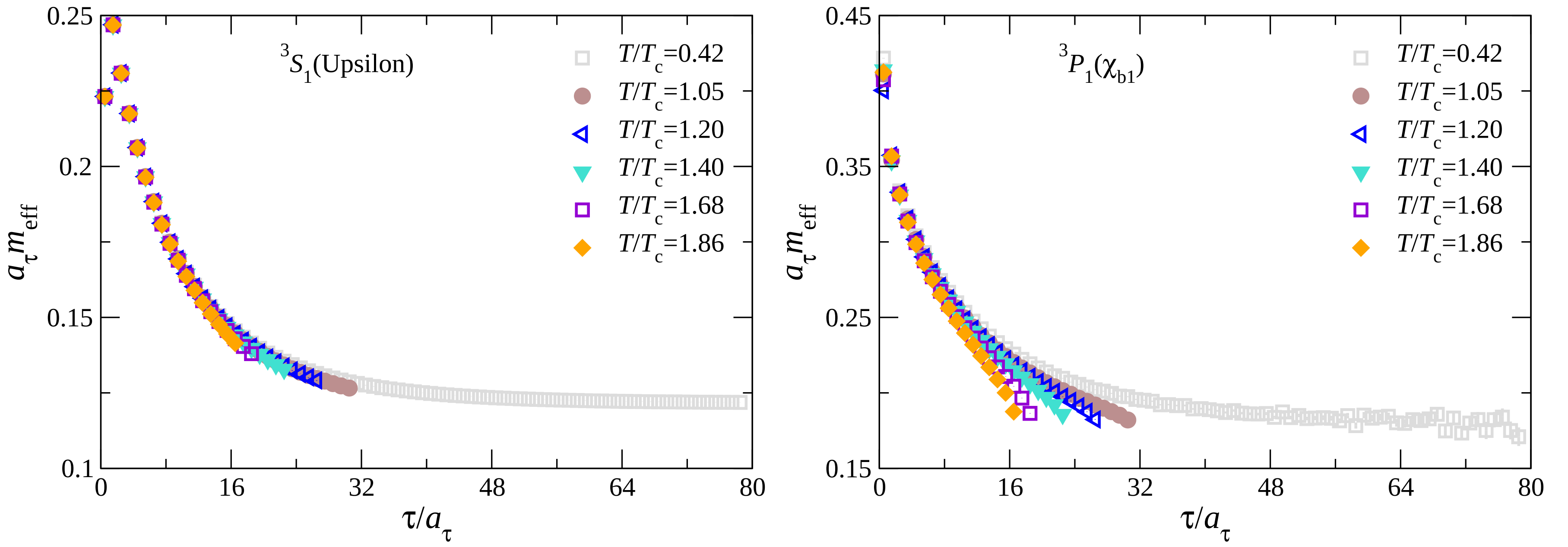
χ_{b1} P wave



$$T/T_c = 1.68$$

Increasing the temperature

Υ S wave χ_{b1} P wave

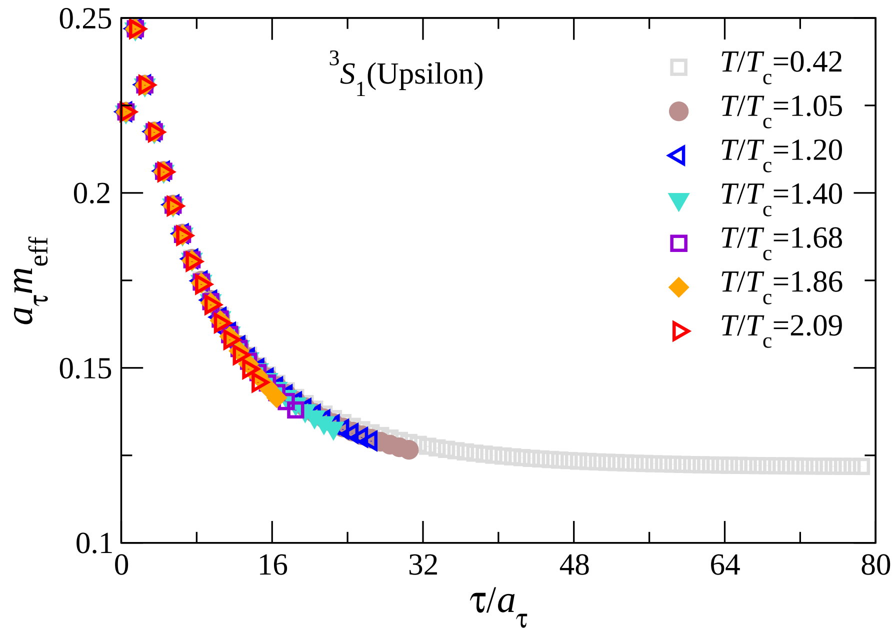


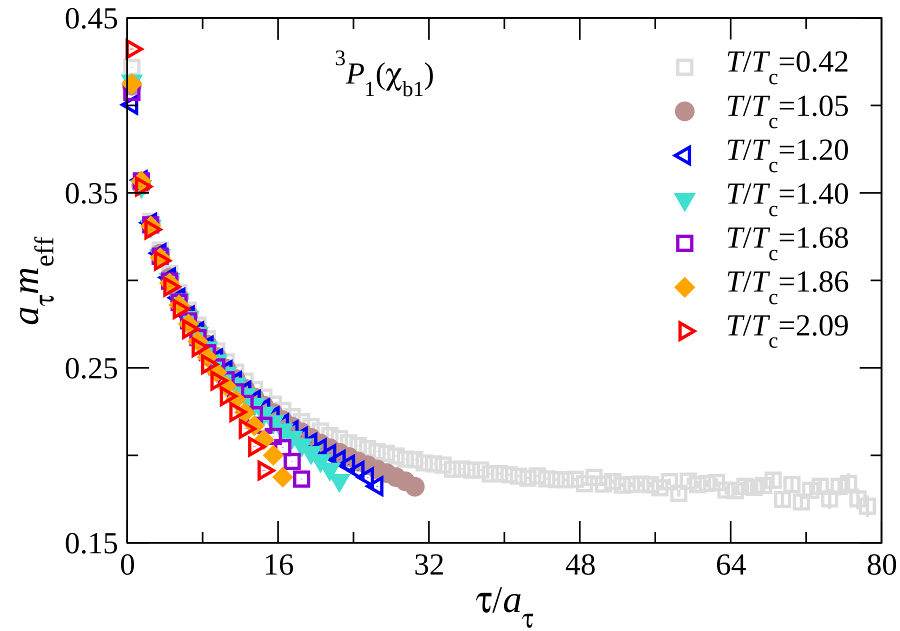
$$T/T_c = 1.86$$

Increasing the temperature

Υ S wave

χ_{b1} P wave





$$T/T_c = 2.09$$

little T dependence

substantial T dependence
no exponential decay
melting?

clear difference in S and P wave correlators

Spectral functions

from euclidean correlators to spectral functions

$$G(\tau, \mathbf{p}) = \int d\omega \, K(\tau, \omega) \rho(\omega, \mathbf{p}) \qquad K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

- inversion: use Maximal Entropy Method (MEM)
- first discussed quite some time ago ...

  Asakawa & Hatsuda 1999, 2001

  Karsch, Petreczky et al 2002

  ...

- ... but full of pitfalls and obstacles

  GA & Martinez Resco 02

  Umeda 07

  Petreczky et al 07-09

Spectral functions

most problems absent in NRQCD:

- effective theory around two-quark threshold
- no transport contribution as $\omega \to 0$
- no thermal boundary conditions
- simple spectral relation

$$G(\tau, \mathbf{p}) = \int d\omega \, e^{-\omega\tau} \rho(\omega, \mathbf{p})$$

why?

- factor out heavy quark mass scale: $\omega = 2M + \omega'$
- $M \gg T$: thermal effects exponentially suppressed

Laine et al 08, GA et al 10

Spectral functions

- no thermal boundary conditions
- simple spectral relation $G(\tau) = \int d\omega\, e^{-\omega\tau} \rho(\omega)$

example at $\mathbf{p} = 0$:

correlators for free quarks with kinetic energy $E_{\mathbf{k}} = \frac{\mathbf{k}^2}{2M}$

$$G_S(\tau) \sim \int d^3k\, \exp\left(-2E_{\mathbf{k}}\tau\right) \qquad \rho_S(\omega) \sim \int d^3k\, \delta(\omega - 2E_{\mathbf{k}})$$

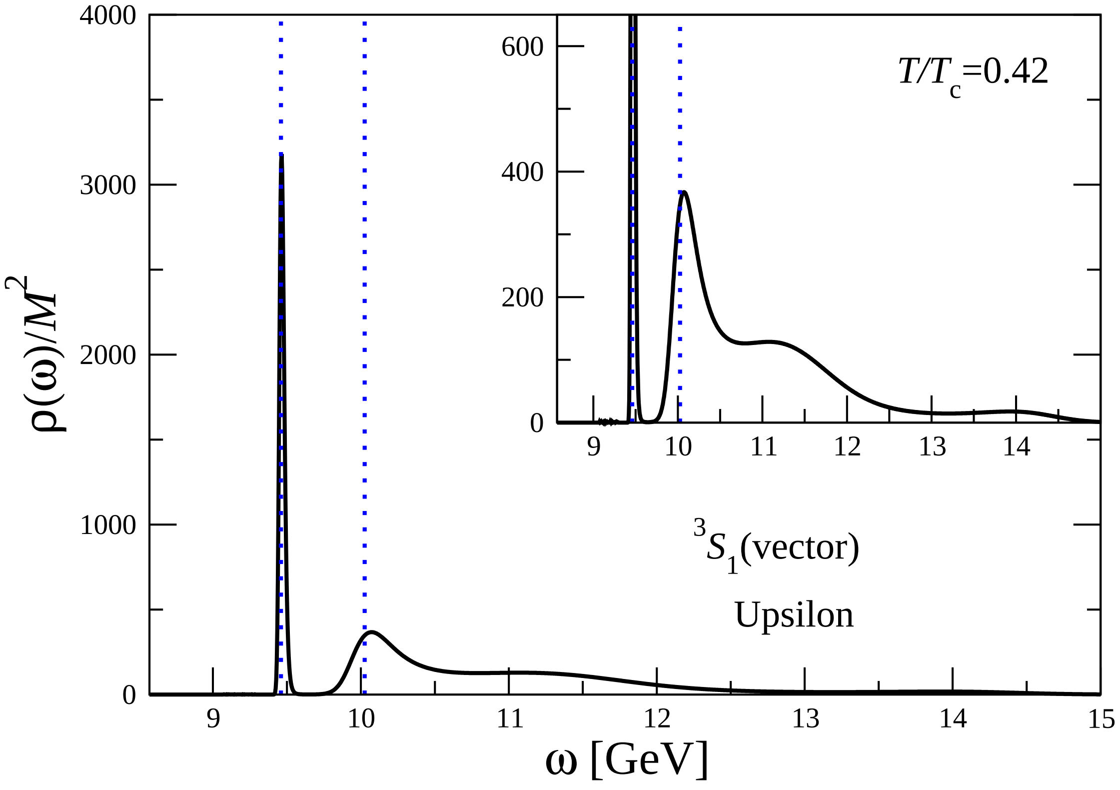
$$G_P(\tau) \sim \int d^3k\, \mathbf{k}^2 \exp\left(-2E_{\mathbf{k}}\tau\right) \qquad \rho_P(\omega) \sim \int d^3k\, \mathbf{k}^2 \delta(\omega - 2E_{\mathbf{k}})$$

Burnier, Laine & Vepsäläinen 08

- temperature dependence only enters via medium !

S wave at finite temperature

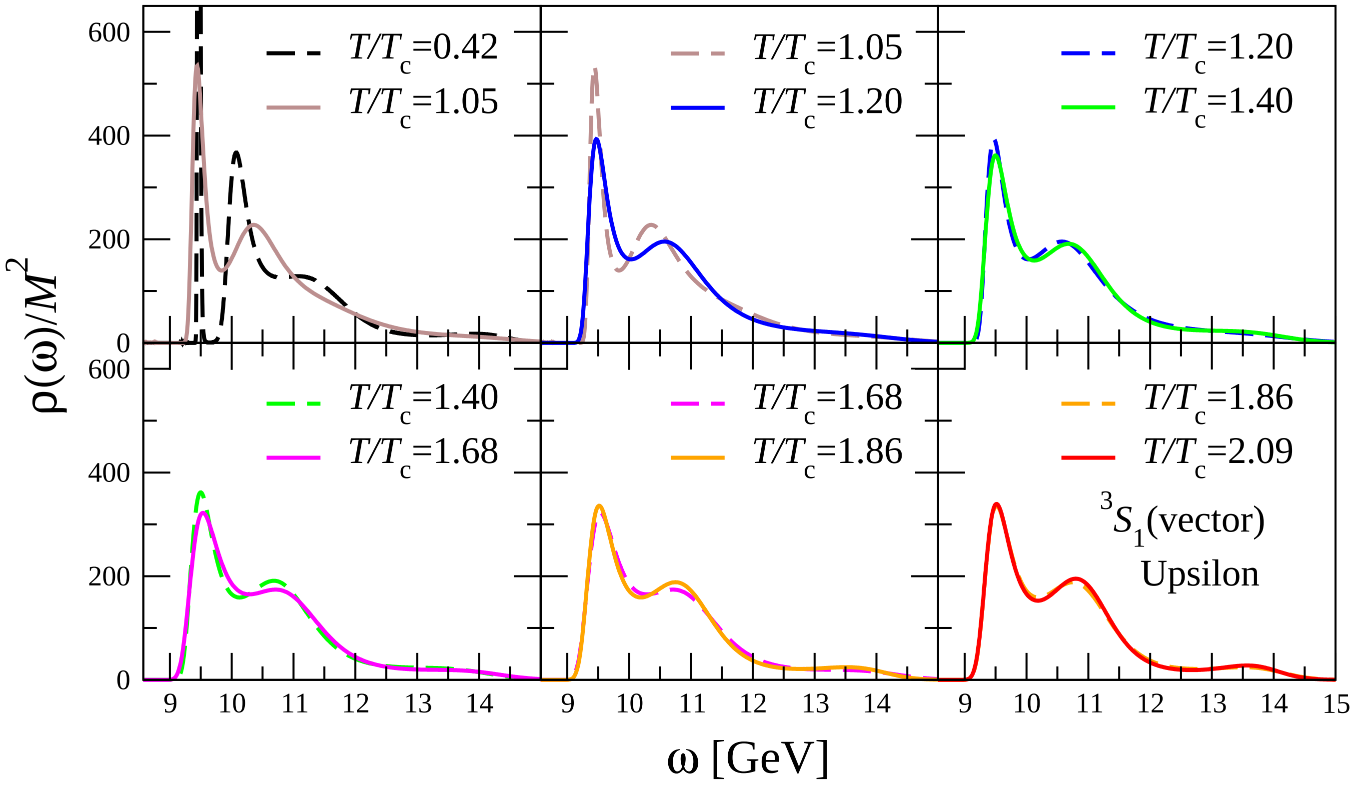
- Υ spectral function: zero temperature



- dotted lines: ground and first excited state Υ(1S, 2S) from exponential fits

S wave at finite temperature

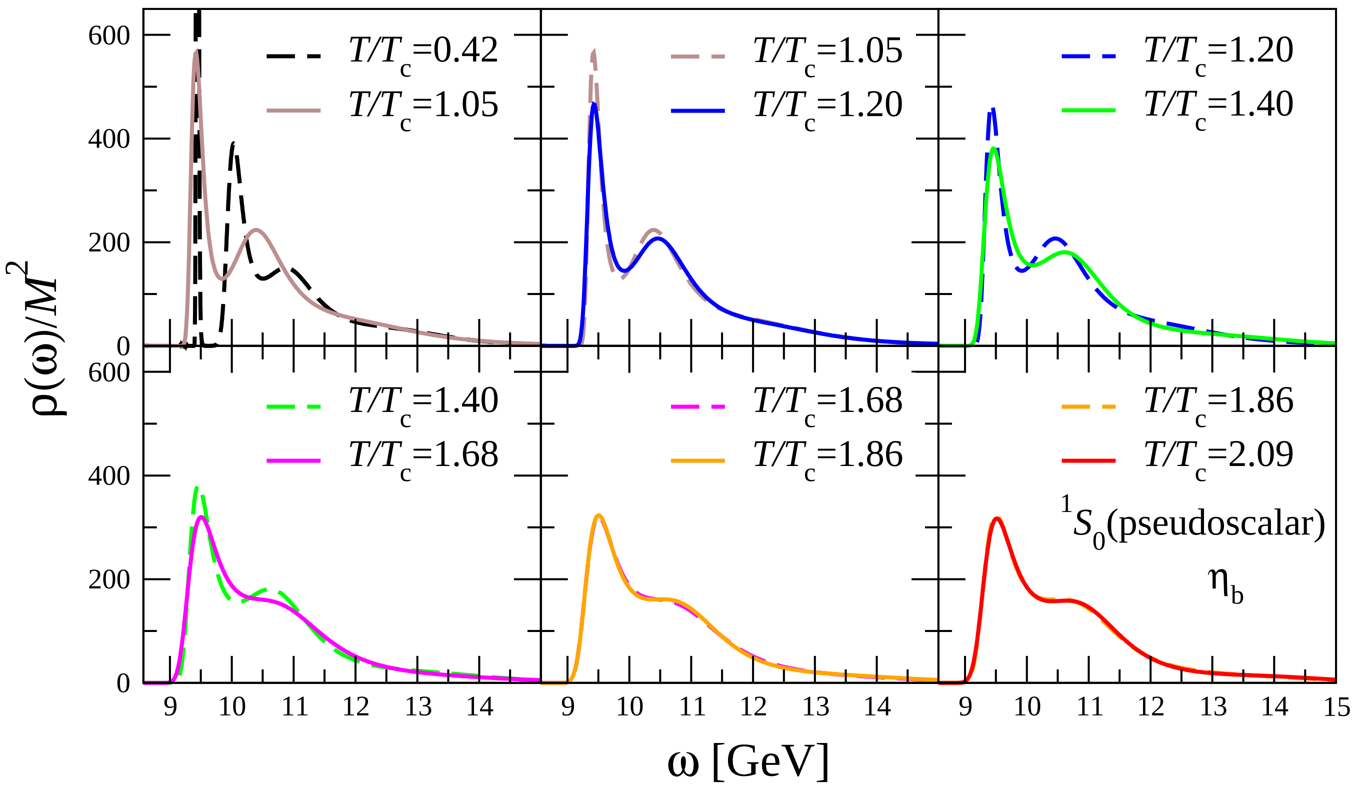
- temperature dependence in Υ channel



- Υ ground state survives – excited states suppressed

S wave at finite temperature

- temperature dependence in η_b channel



- η_b ground state survives – excited states suppressed

moving Υ at finite temperature

non-zero momentum: moving through the QGP

- predictions from EFT, AdS/CFT, potential models, $\ldots$
- no clear picture: e.g. dissociates at lower/higher temperatures

on the lattice

$$a_s p_i = \frac{2\pi n_i}{N_s} \qquad\qquad n_i \lesssim \frac{N_s}{4}$$

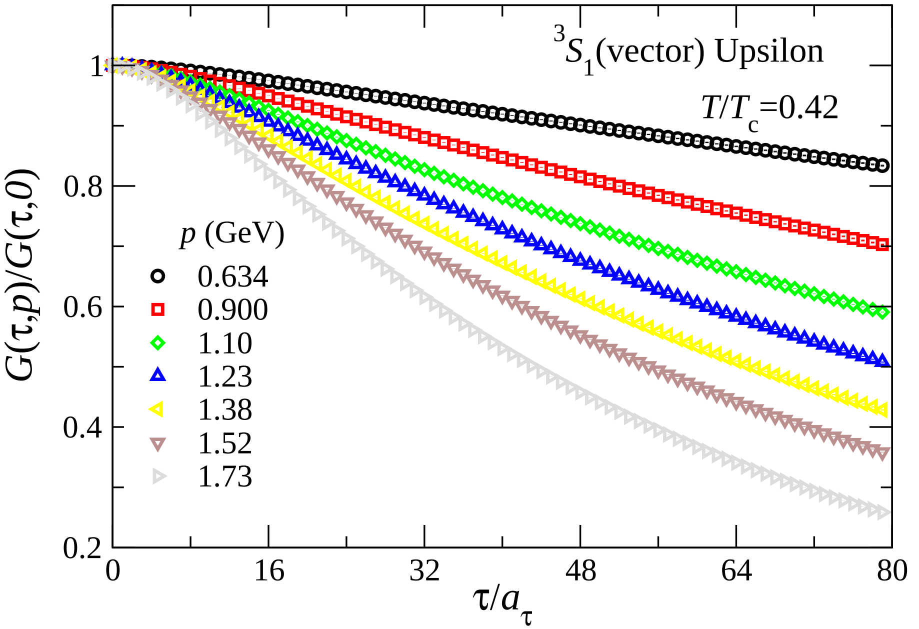
in our case:

- maximal momentum: $p_{\rm max} \sim 1.7$ GeV
- maximal velocity of ground state: $v = p/M_S \lesssim 0.2$

non-relativistic

moving Υ at finite temperature

- non-relativistic speeds: $v/c \lesssim 0.2$

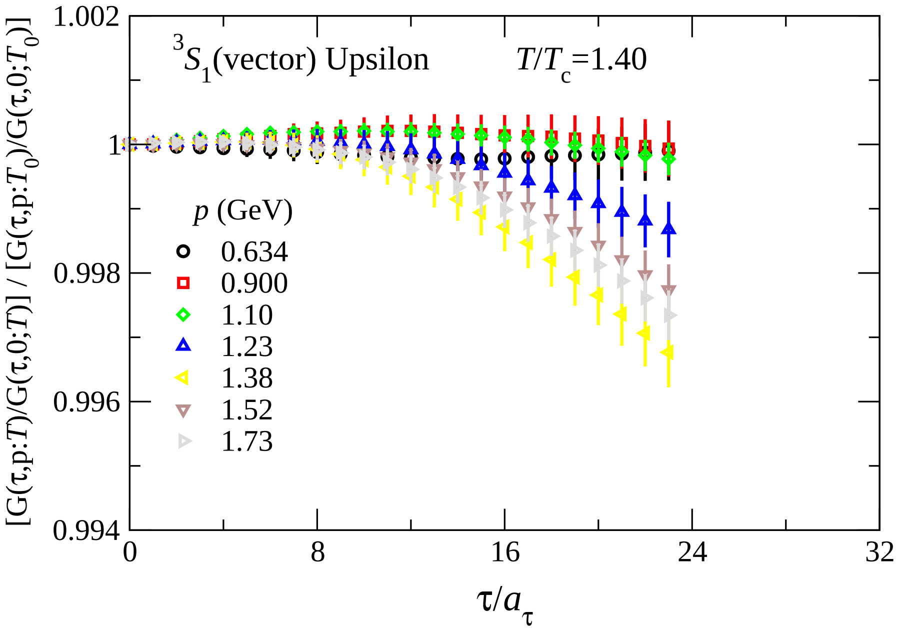


ratio $G(\tau, \mathbf{p})/G(\tau, \mathbf{0})$:

- clear momentum dependence in correlators
- expected from dispersion relation $M(p) = M + p^2/(2M)$

moving Υ at finite temperature

- non-relativistic speeds: $v/c \lesssim 0.2$

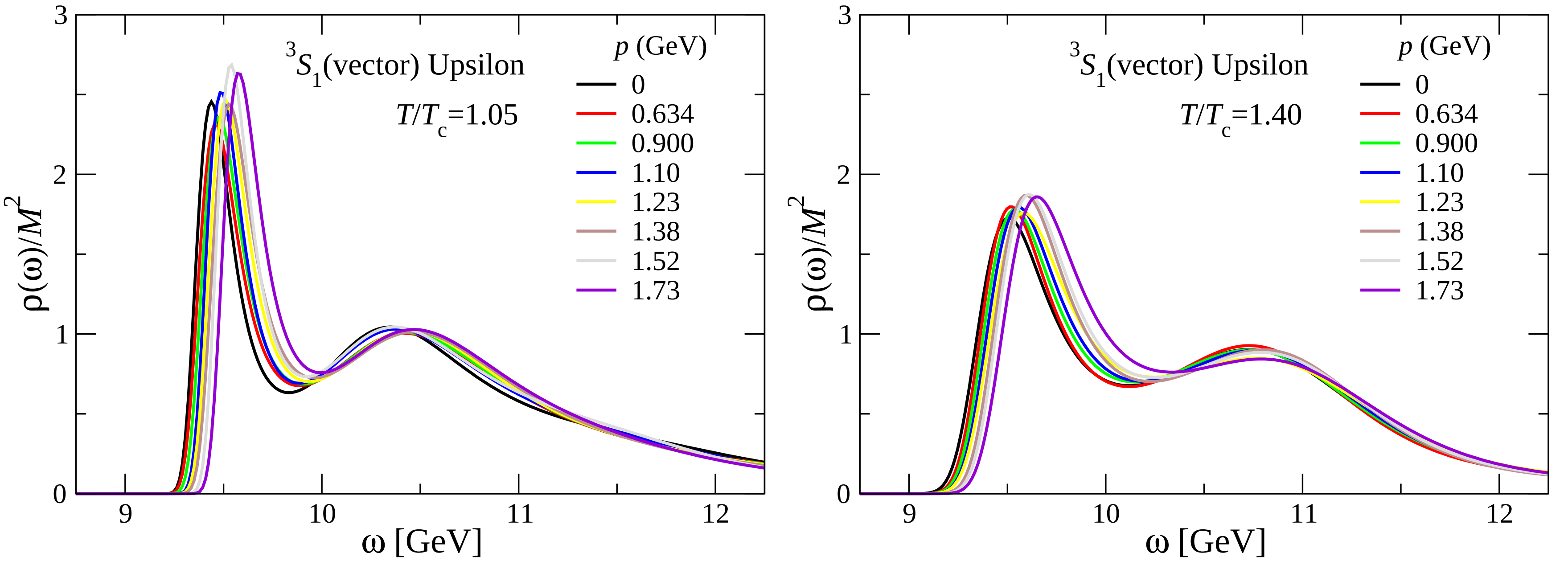


double ratio $[G(\tau,\mathbf{p};T)/G(\tau,\mathbf{0};T)]/[G(\tau,\mathbf{p};T_0)/G(\tau,\mathbf{0};T_0)]$

- very little temperature dependence in the momentum dependence

moving Υ at finite temperature

- non-relativistic speeds: $v/c \lesssim 0.2$



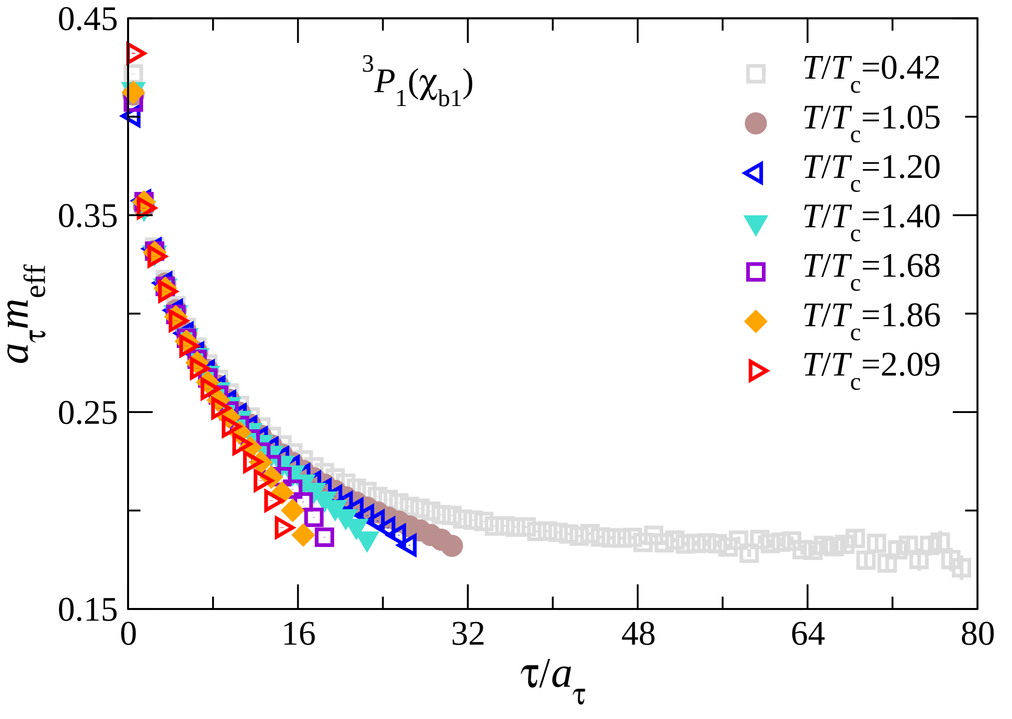
spectral functions:

- survival of moving groundstate

P waves at finite temperature

P waves at finite temperature

- P wave correlators show drastically different behaviour

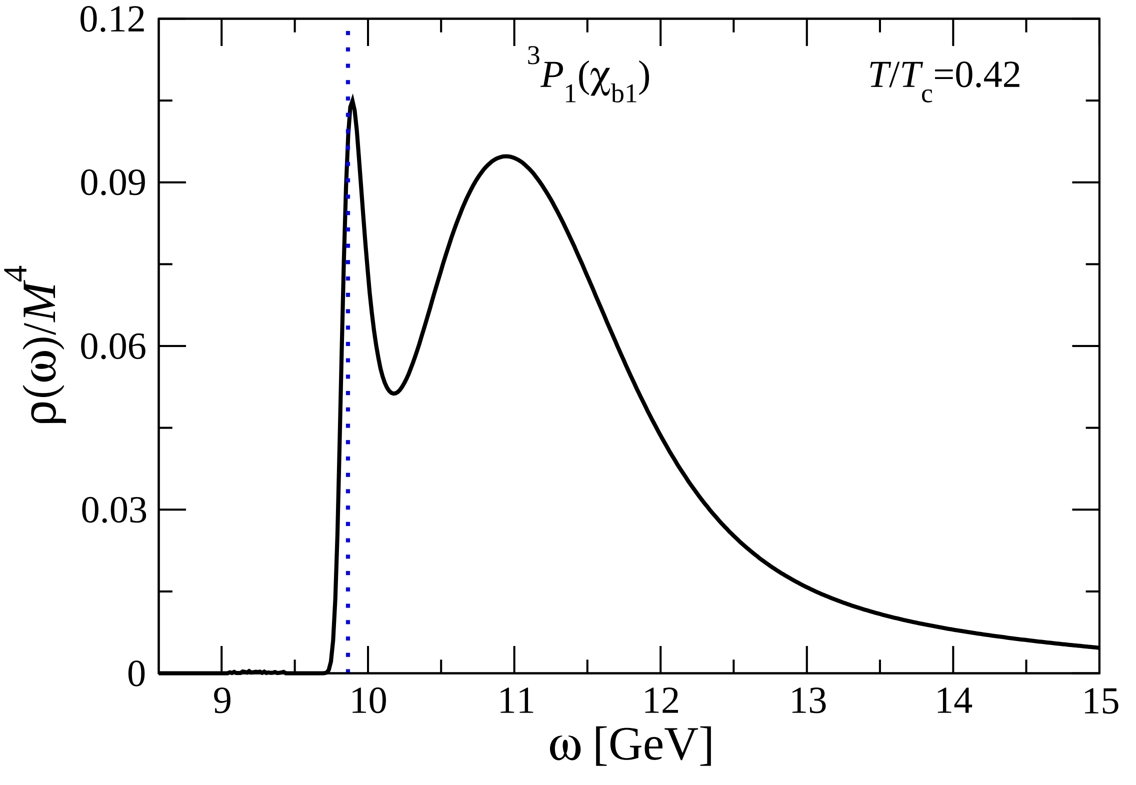


- no exponential decay
- what to expect: no isolated states? melting?

spectral analysis

P waves at finite temperature

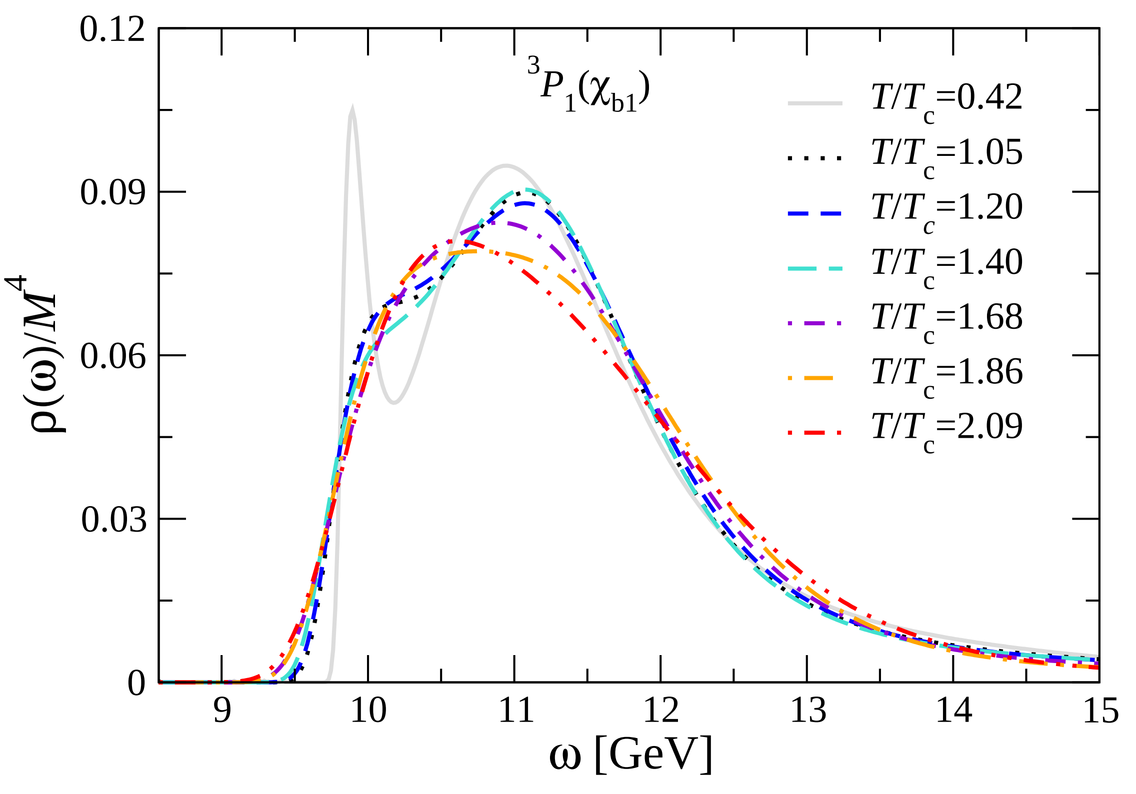
- χ_{b1} axial-vector channel



- groundstate below T_c, agreement with exp. fit

P waves at finite temperature

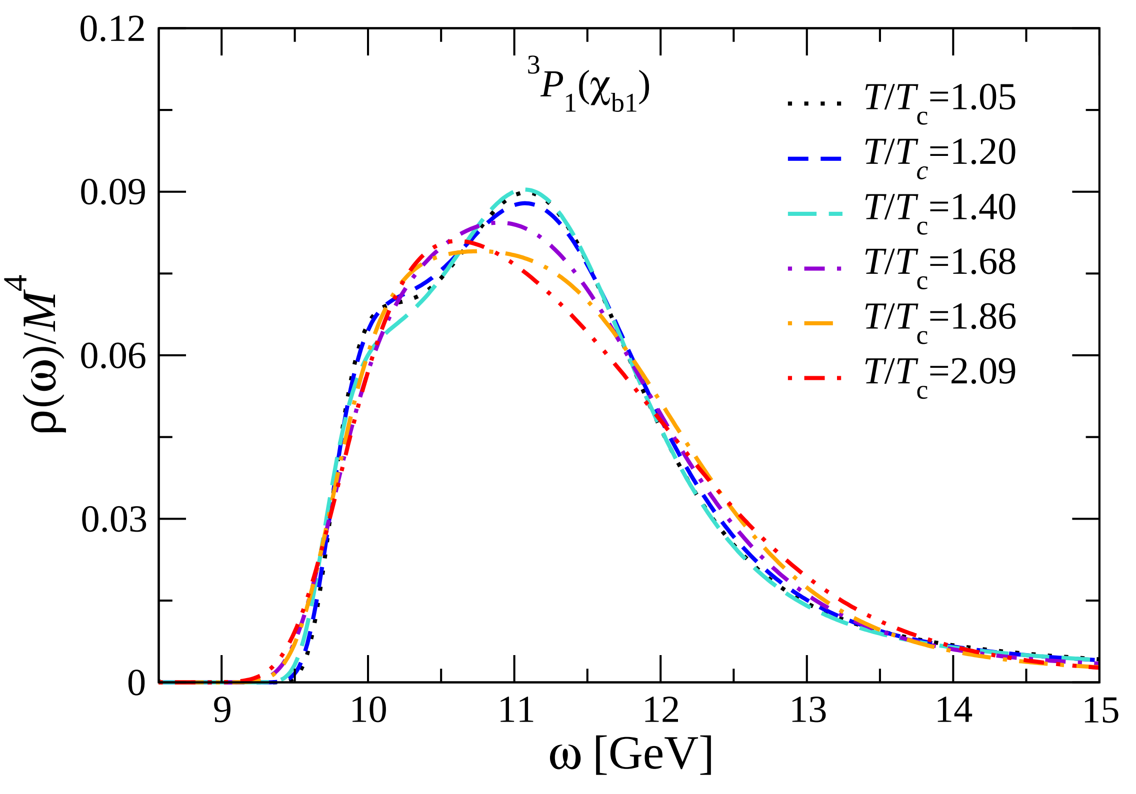
- χ_{b1} axial-vector channel



- melting immediately above T_c
- consistent with correlator decay

P waves at finite temperature

- χ_{b1} axial-vector channel



- no sign of ground state
- immediate melting above T_c

Systematics

systematic checks for MEM:

- default model dependence
- ω range: $\omega_{\rm min} < \omega < \omega_{\rm max}$

  sensitive to $\omega_{\rm min}$, additive constant in NRQCD !
- number of configurations

  high-precision data important, rel. error $\sim 10^{-4}$
- τ range: $\tau = \tau_{\rm min} \ldots \tau_{\rm max} \leq a_\tau (N_\tau - 1)$

see papers for details, in particular 1109.4496, 1310.5467

Transport coefficients

dynamics on long length and timescales:

- effective theory: hydrodynamics
- ideal hydrodynamics: equation of state
- viscous hydro: transport coefficients

shear/bulk viscosity, conductivity, $\ldots$

- depend on underlying microscopic theory
- typically:
  - large in weakly interacting theory
  - small in strongly coupled systems

perfect-fluid paradigm: $\eta/s = 1/4\pi$ (holography)

Transport coefficients

linear response: Kubo relation

- proportional to slope of current-current spectral function at $\omega = 0$
- example: conductivity

$$\sigma = \lim_{\omega \to 0} \frac{1}{6\omega} \rho_{ii}(\omega, \mathbf{0})$$

where

$$\rho_{\mu\nu}(x) = \langle [j_\mu(x), j_\nu(0)] \rangle_{\text{eq}}$$

is current-current spectral function, j_μ is EM current

- real-time correlator in equilibrium
- routinely computed with holography

Transport coefficients from the lattice

- on the lattice: euclidean correlator
- related to spectral function

$$G(\tau) = \int d\omega \, K(\tau,\omega)\rho(\omega) \qquad K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

- inversion/analytical continuation

much harder than previous problem:

- need reliable estimate of $\rho(\omega)/\omega\Big|_{\omega=0}$
  not just $\rho(\omega)$
- in weakly-coupled theories, correlator is remarkably insensitive to details of $\rho(\omega)$ at small ω

GA & Martinez Resco 02

Transport coefficients from the lattice

previous attempts:

- shear and bulk viscosity Nakamura & Sakai 05, Meyer 07
- electrical conductivity

  quenched

  - (TIFR) S. Gupta 04
  - (Swansea) GA, Allton, Foley, Hands & Kim 07
  - (Bielefeld) Ding, Francis, Karsch, Kaczmarek, Laermann & Söldner 11

  dynamical

  - (Mainz) Brandt, Francis, Meyer & Wittig 13
  - (Swansea) GA, Amato, Allton, Giudice, Hands & Skullerud 13

Conductivity from the lattice

several results above T_c:

	T/T_c	$C_{\mathrm{em}}^{-1}\sigma/T$	N_f	
TIFR	1.5, 2, 3	~ 7	0	staggered
Swansea	1.5, 2.25	0.4(1)	0	staggered
Bielefeld	1.45	0.37(1), 0.3-1	0	Wilson, cont. extrapolated
Mainz	1.2	0.40(12)	2	Wilson

note: divide out common EM factor

$$C_{\mathrm{em}} = e^2 \sum_f q_f^2 \qquad q_f = \frac{2}{3}, -\frac{1}{3}$$

all studies used local current $\quad j_\mu = \bar{\psi}\gamma_\mu\psi$

not the exactly conserved lattice current

Conductivity from the lattice

recent work: Amato, GA et al, 1307.6763 [hep-lat]

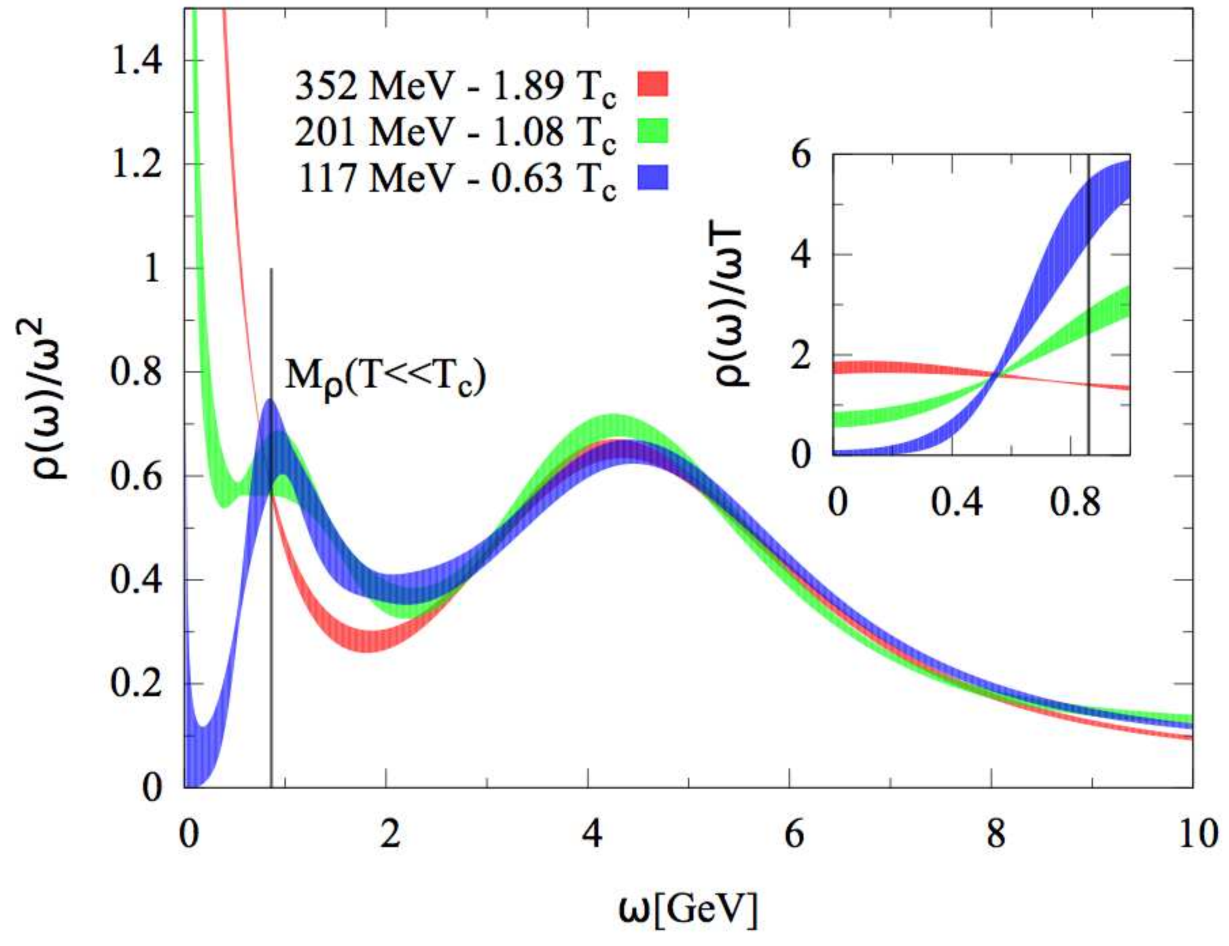
improvements:

- $N_f = 2+1$ dynamical quark flavours
- conserved lattice current (no renormalisation required)
- many temperatures, below and above T_c
- anisotropic lattice, $a_s/a_\tau = 3.5$, many time slices

N_s	32	24	24	32	32	32	24	32
N_τ	48	40	36	32	28	24	20	16
T/T_c	0.63	0.76	0.84	0.95	1.08	1.26	1.52	1.89
$N_{\rm cfg}$	601	523	501	501	502	500	1001	1059

Conductivity from the lattice

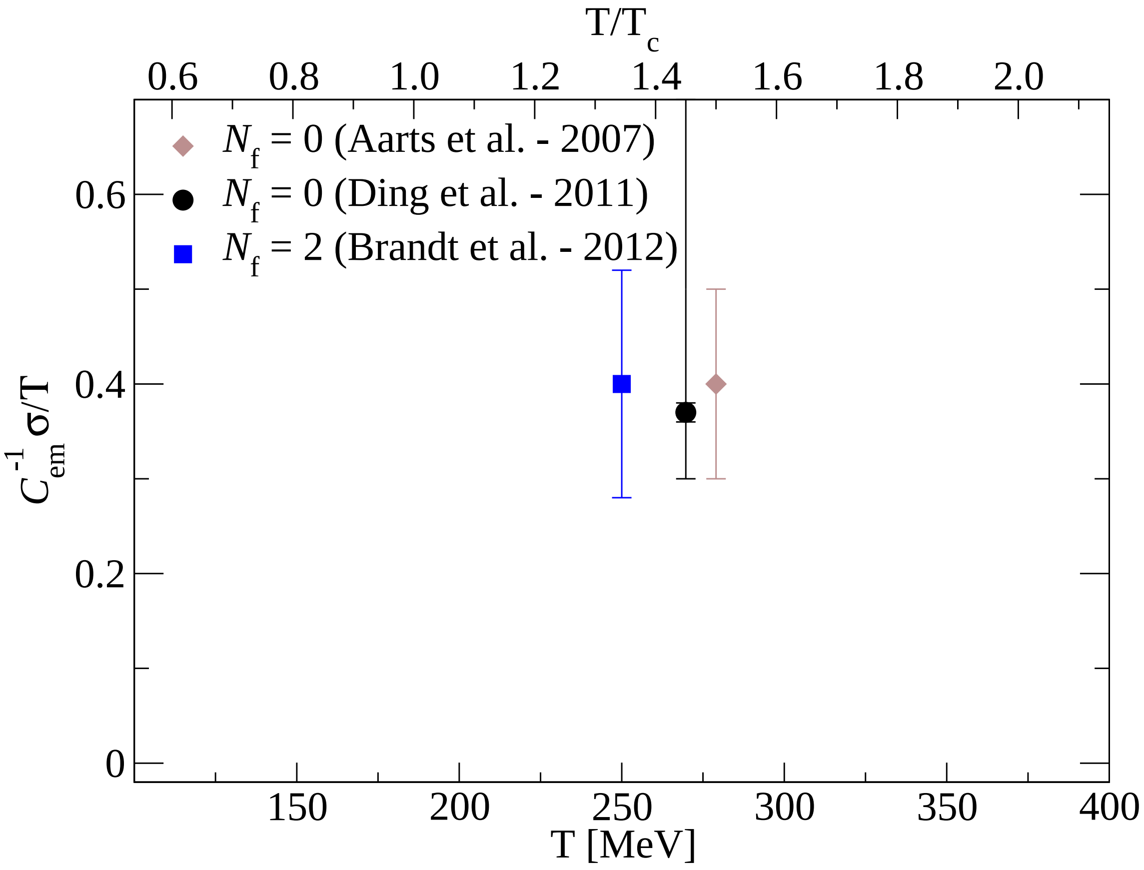
spectral function
$\rho(\omega)/\omega^2$



- rho-particle peak around 800 MeV below T_c
- nonzero conductivity: $\rho(\omega) \sim \sigma\omega + \ldots$
- inset $\rho(\omega)/\omega$: intercept $\sim$ conductivity

Conductivity from the lattice

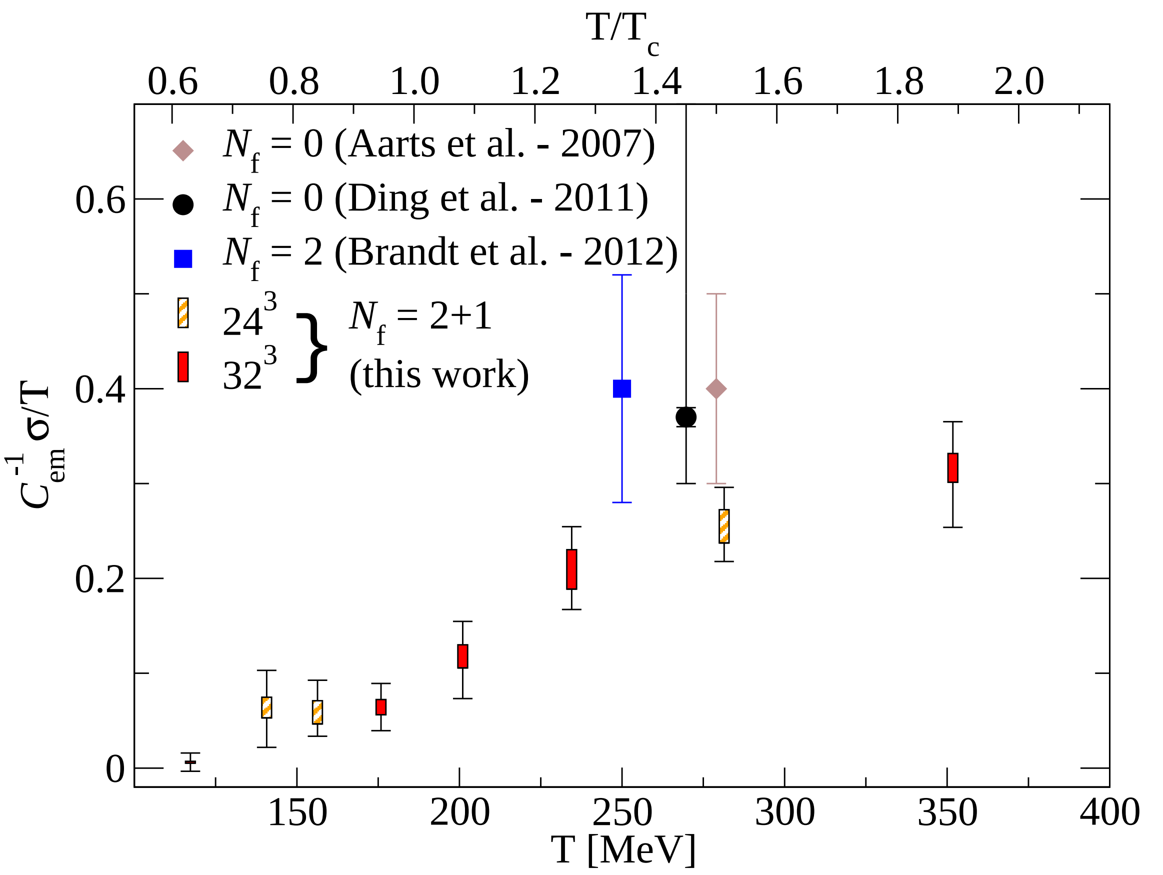
σ/T across the deconfinement transition



- previous results

Conductivity from the lattice

σ/T across the deconfinement transition



- consistent with previous (quenched) results in QGP
- first observation of T dependence

Summary

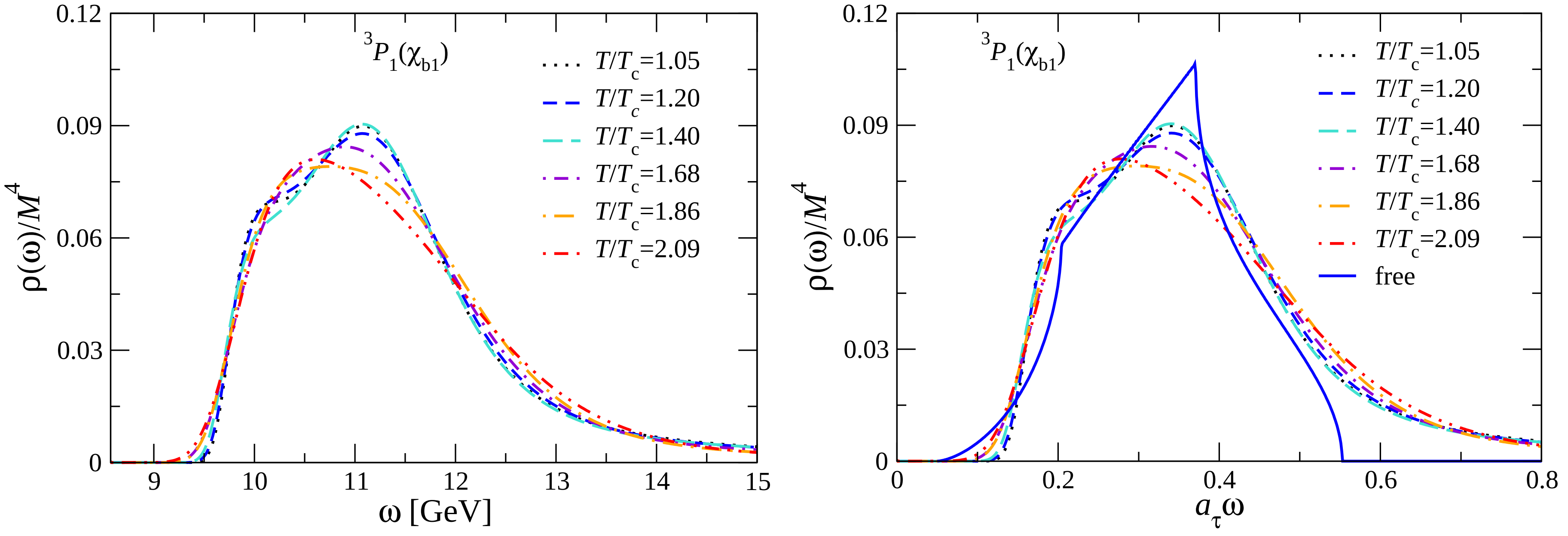
bottomonium: NRQCD on QGP background

- S wave ground states survive, at rest and moving
  excited states appear suppressed
- P wave states melt immediately above T_c

transport from the lattice: electrical conductivity

- first large-scale computation with dynamical quarks
- number of results available above T_c
- mostly consistent, $\sigma/T \sim 0.3 - 0.4 C_{\rm em}$ in QGP
- temperature dependence of σ/T found across T_c

Back-up: melted P waves

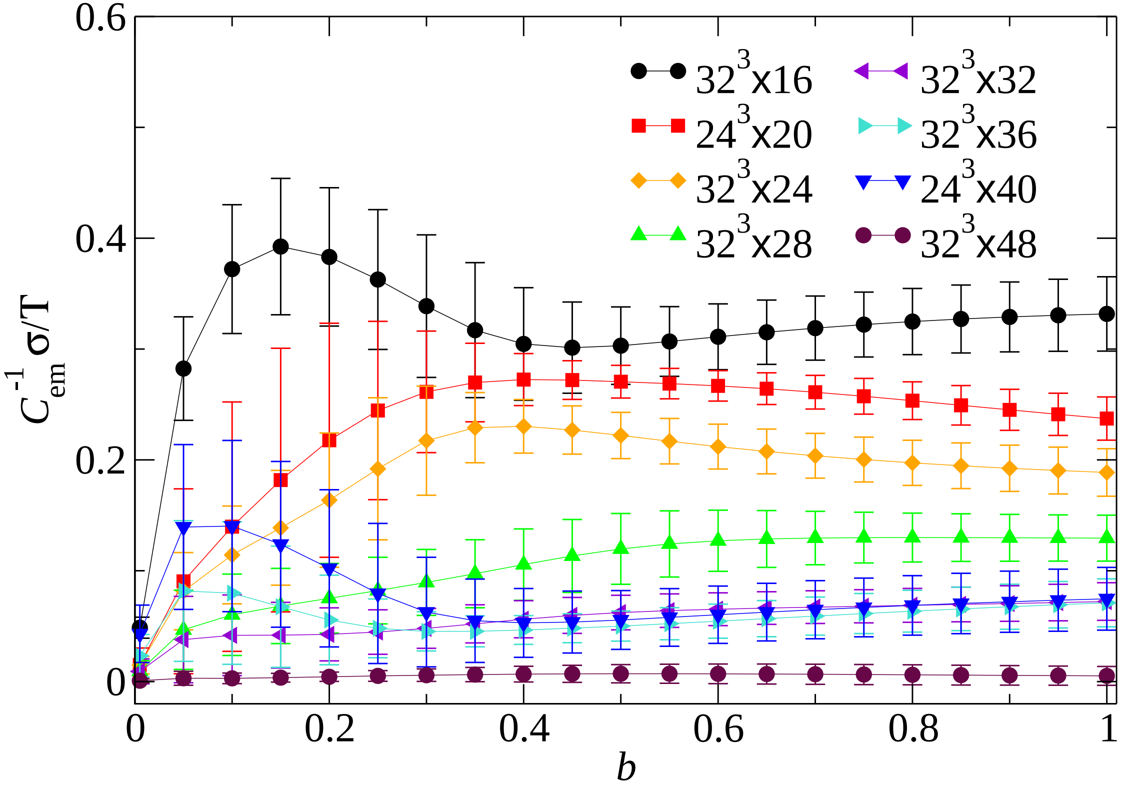


- melting above T_c: a featureless blob ?
- shape is similar to free lattice spectral function

$$\rho_P(\omega) \sim \sum_{\text{Brillouin zone}} \mathbf{p}^2 \delta(\omega - 2E_{\mathbf{p}})$$

Back-up: conductivity

systematics: default model dependence



- input: no structure $\rho_{\rm default}(\omega)/\omega = b + c\omega$
- b needed to allow for a nonzero conductivity
- stable results provided b is not too small (no bias)