Exploring the QCD phase diagram with conserved charge fluctuations

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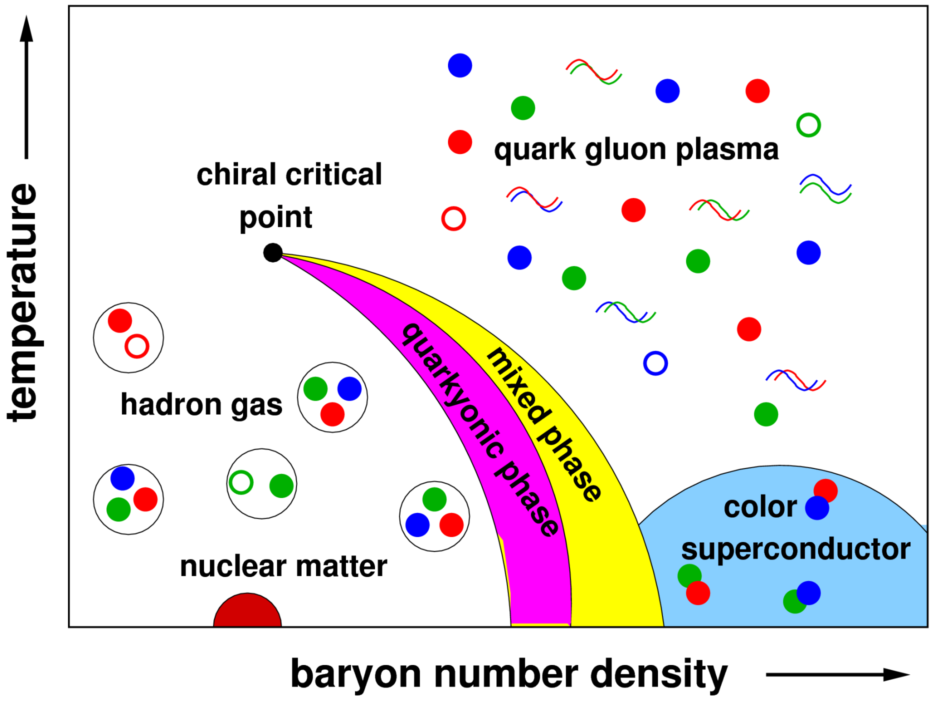
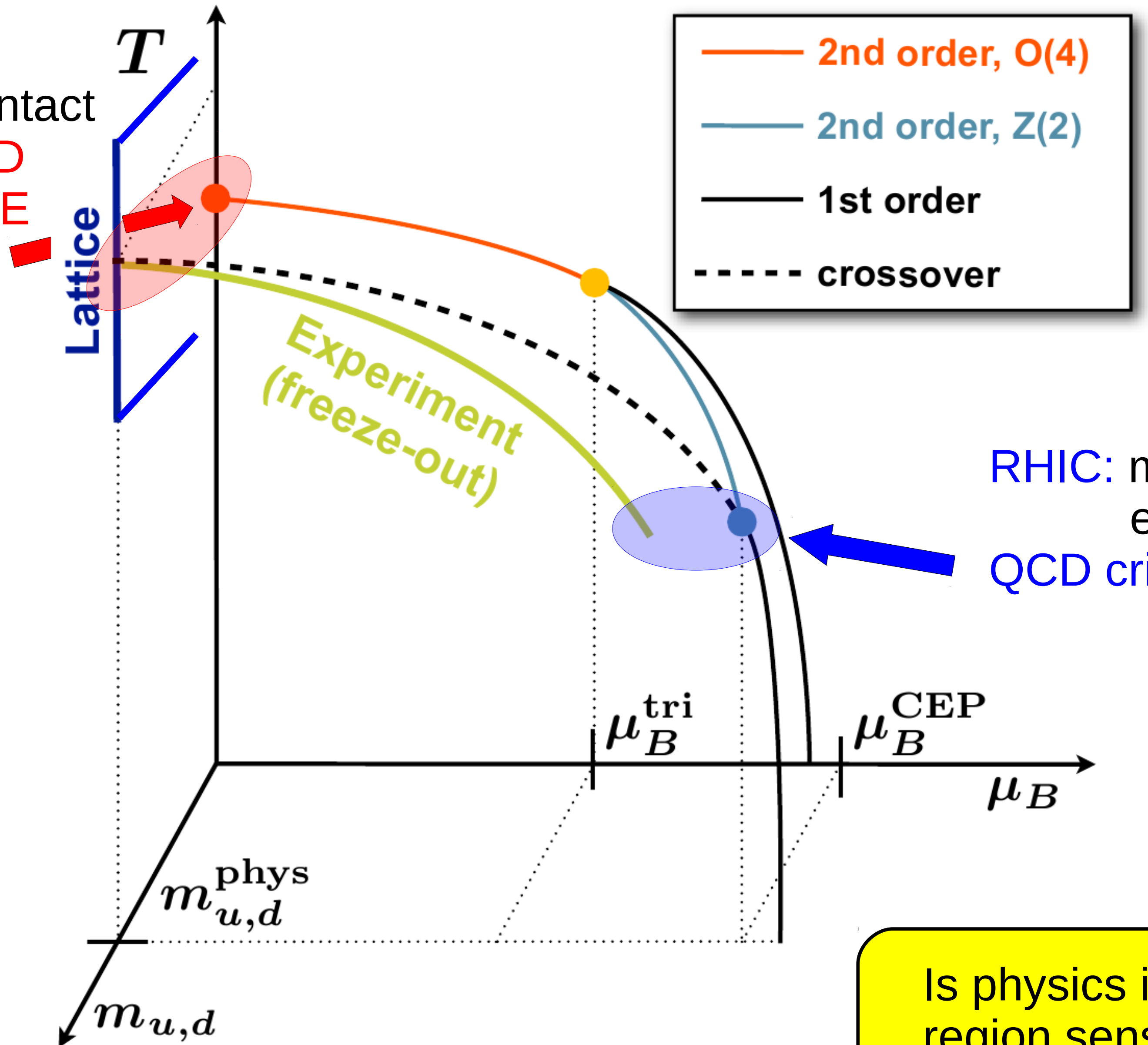


OUTLINE

- conserved charge fluctuations in QCD and HIC
- crossover transition – chiral PHASE transition – HRG
- critical behavior at $\mu_B = 0$ and the critical endpoint at $\mu_B > 0$

Chiral critical point and QCD critical endpoint

LHC: may establish contact with the QCD chiral PHASE transition



RHIC: may establish evidence for the QCD critical endpoint

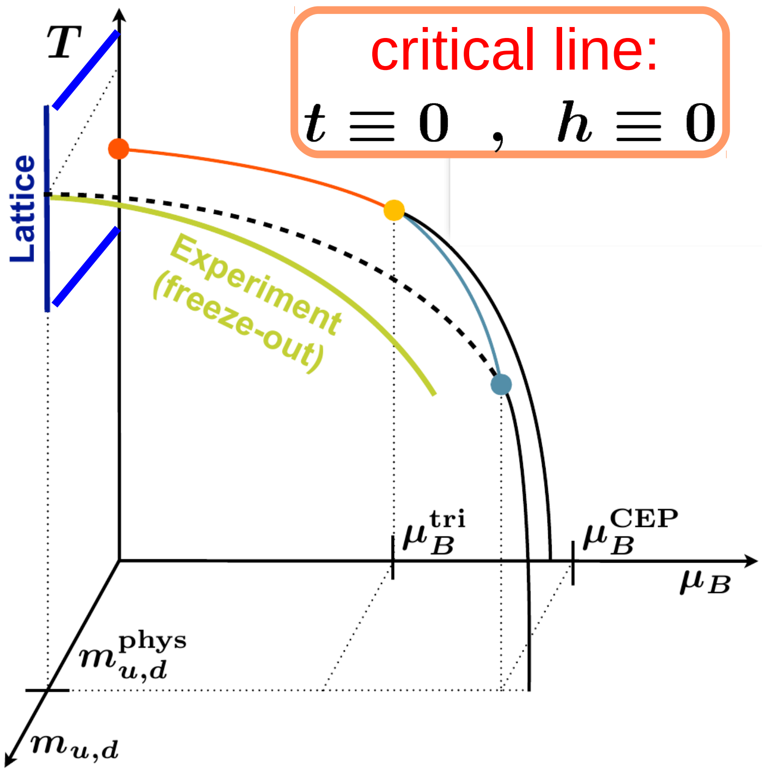
Is physics in the freeze-out region sensitive to critical behavior?

Chiral Transition Temperature at small μ_B/T

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular — regular

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$



critical line: $t \equiv 0 \ , \quad h \equiv 0$

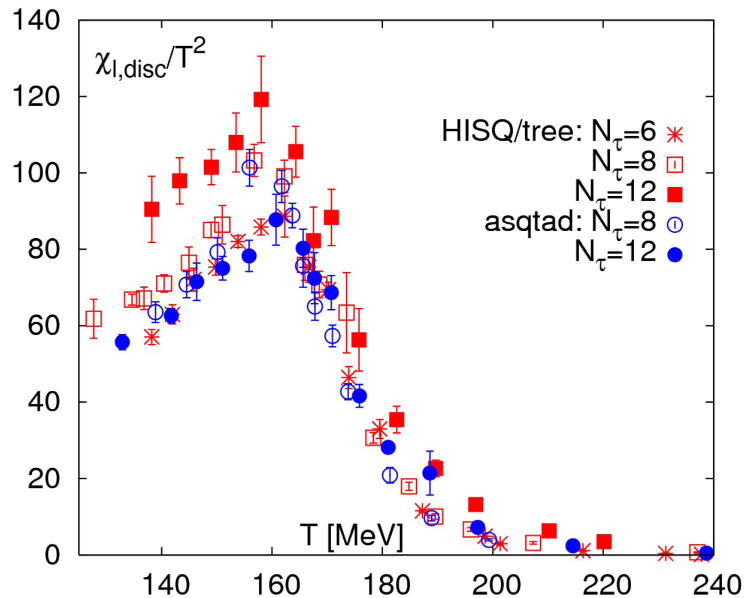
$$t \sim \frac{T-T_c}{T_c} + \kappa_q \left(\frac{\mu_q}{T}\right)^2 \ , \quad h \sim \frac{m_q}{T_c}$$

suppressing dependence on strange quark chemical potential

higher order derivatives==cumulants become sensitive to singular structure

$$\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k} p/T^4}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}$$

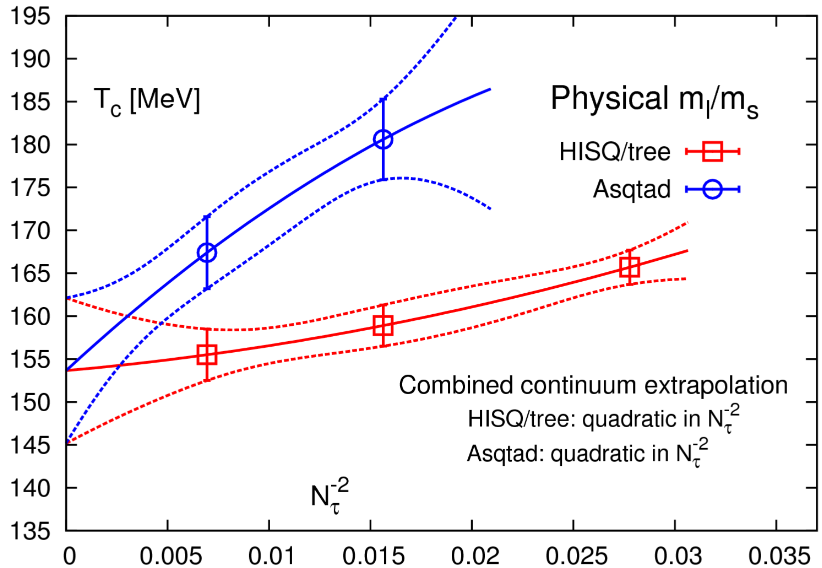
Chiral Transition Temperature



– locate pseudo-critical temperature from chiral susceptibility

$$\chi_{m,l}(T) = \frac{\partial^2 p/T}{\partial m_l^2} = \frac{\partial \langle\bar\psi\psi\rangle_l}{\partial m_l}$$
$$= \chi_{l,disc} + \chi_{l,con}$$

– peak location defines pseudo-critical temperature on $N_\sigma^3 N_\tau$ lattice, $T \equiv 1/N_\tau a$



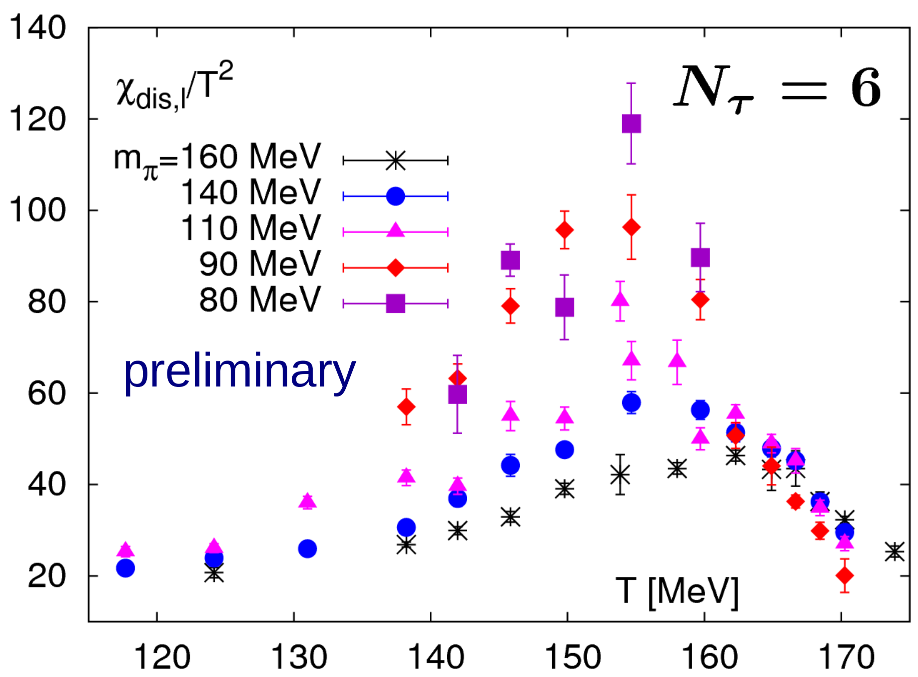
continuum extrapolation of pseudo-critical temperatures at physical light and strange quark masses for two different lattice discretization schemes

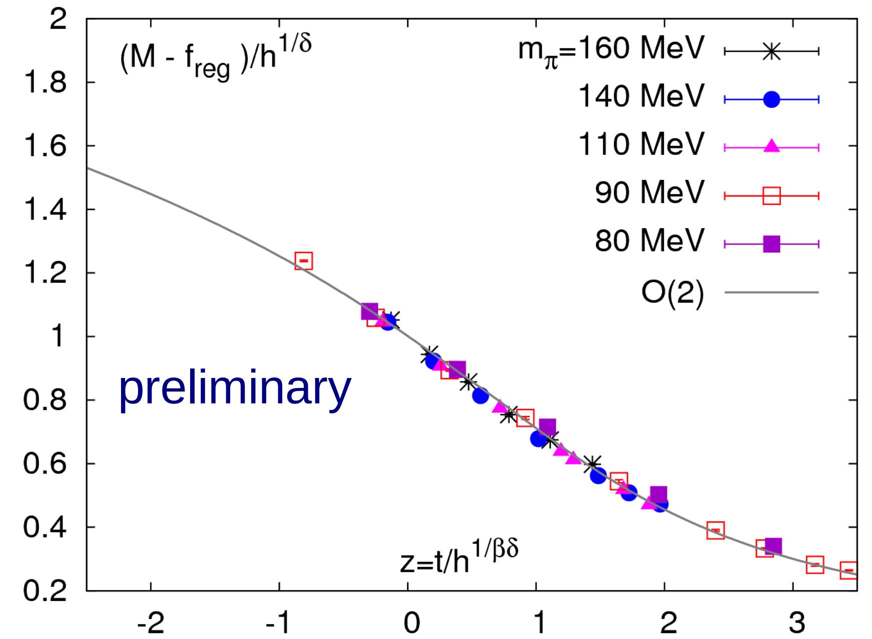
$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

A. Bazavov et al. [hotQCD Collaboration]
Phys. Rev. D 85, 054503 (2012)

consistent with: Y. Aoki et al., JHEP 0906, 088 (2009)

Chiral limit: O(4) scaling





$$\chi_{dis}/T^2 \sim m_\pi^{2(1/\delta-1)}$$

$$M = m_s \langle \bar{\psi}\psi \rangle / T^4$$

magnetic equation of state: $M = h^{1/\delta} f_G(z)$

– scaling analysis in (2+1)-flavor QCD with HISQ fermions

$$m_\pi = 140\text{MeV}$$

small enough to be sensitive to O(4) scaling behavior in the chiral limit

H.-T. Ding et al., Lattice 2013

staggered fermions: O(2) instead of O(4) for non-zero cut-off

O(4) Scaling in QCD: Curvature of the critical line

Bielefeld-BNL, Phys. Rev. D83, 014504 (2011)

p4-action: $N_\tau = 4$

- "thermal" fluctuations of the order parameter

$$t \equiv \frac{1}{t_0}\left(\left(\frac{T}{T_c} - 1\right) + \kappa_q \left(\frac{\mu_q}{T}\right)^2\right) \ , \ z = t/h^{1/\beta\delta}$$

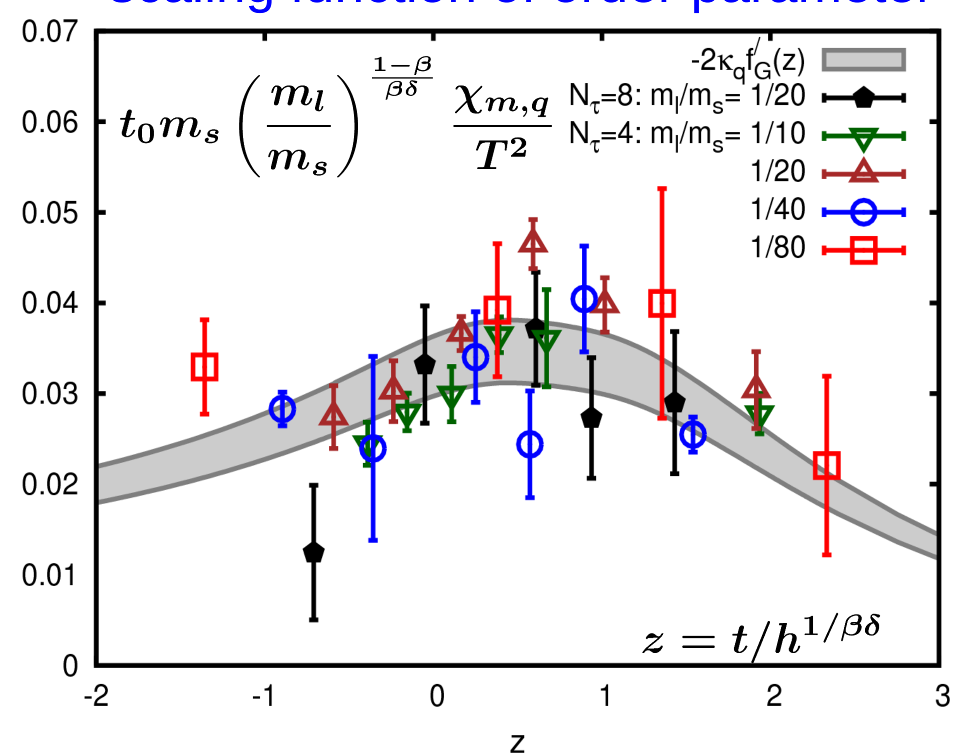
$$M_b \equiv \frac{m_s\langle\bar\psi\psi\rangle}{T^4} = h^{1/\delta} f_G(z)$$

$$\frac{\chi_{m,q}}{T} = \frac{\partial^2\langle\bar\psi\psi\rangle/T^3}{\partial(\mu_q/T)^2}$$

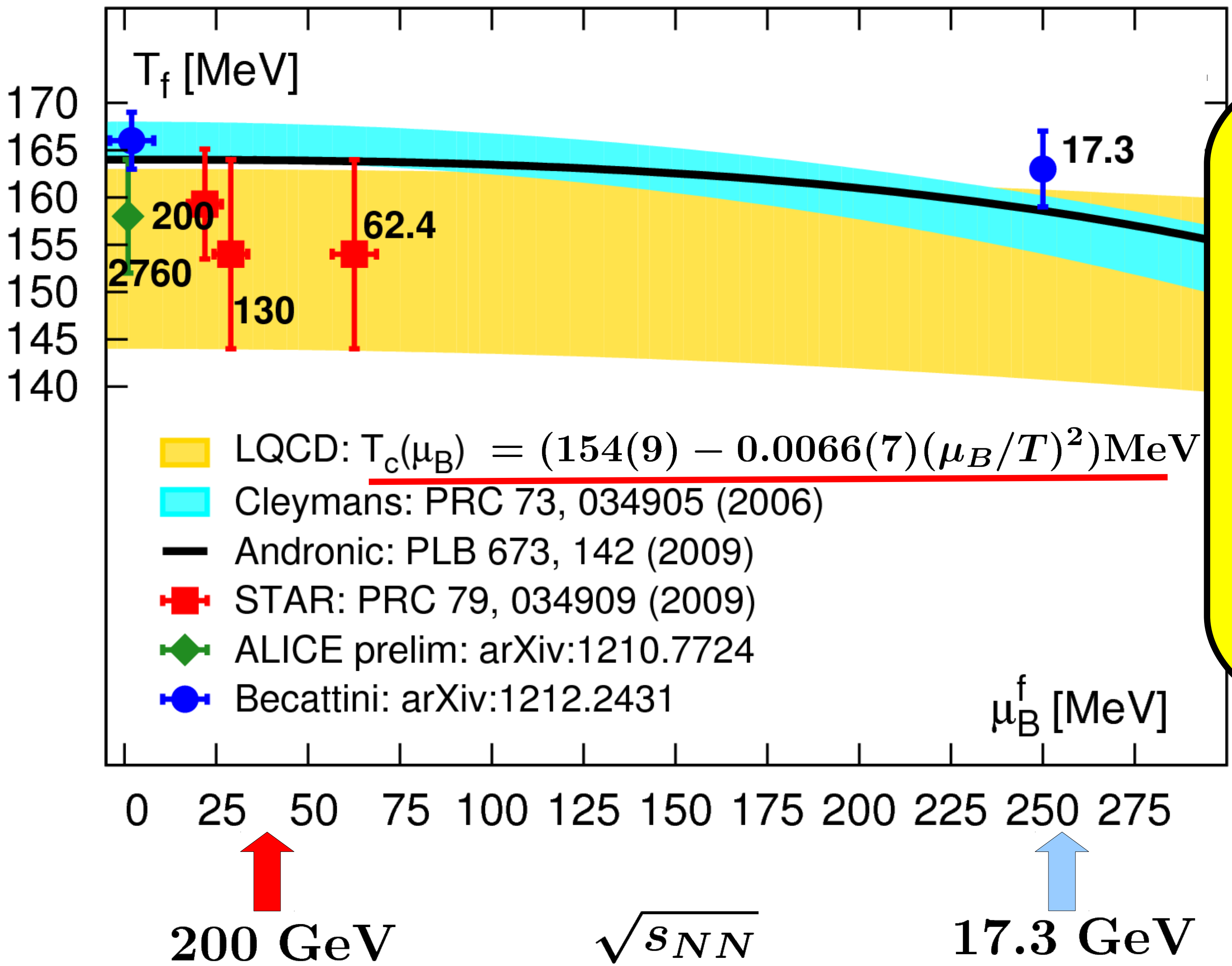
$$= \frac{2\kappa_q T}{t_0 m_s} h^{(\beta-1)/\delta\beta} f_G'(z)$$

$$\kappa_B = \kappa_q/9 = 0.0066(7)$$

scaling function of order parameter



Chiral transition and freeze-out



phenomenological freeze-out / hadronization curve, **QCD transition line** and experimental data (obtained by <u>assuming the validity of the HRG model</u>) are consistent for

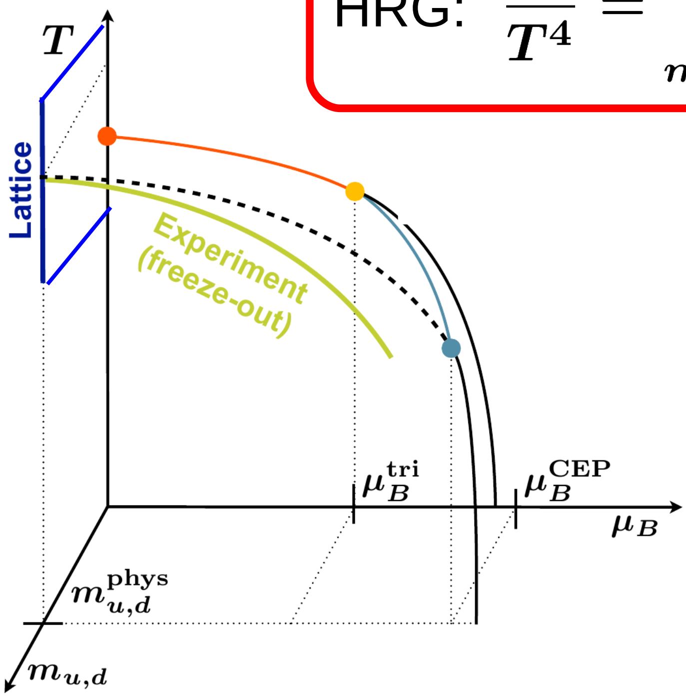
$$\mu_B/T \lesssim 2$$

HRG model, lattice QCD and critical behavior

- **for a wide range of baryon chemical potentials freeze-out happens in or close to the QCD transition region:** predicted ➡ P. Braun-Munzinger et al., Phys. Lett. B596, 61 (2004)

caveat: freeze-out parameter extracted from experimental data by comparing to the HRG model, i.e. not to QCD

HRG: $$\frac{p}{T^4} = \sum_{m \in meson} \ln Z^b_m(T,V,\mu) + \sum_{m \in baryon} \ln Z^f_m(T,V,\mu)$$



goal: describe freeze-out in terms of QCD thermodynamics

➡ freeze-out parameter from comparison of measured moments of charge fluct. with QCD calculation

BNL-Bielefeld, PRL 109, 12302 (2012)

Fluctuations and Correlations: Susceptibilities

– probing the response of a thermal medium to an external field, i.e. variation of one of its external control parameters: $T, \ \mu, \ m_q$

(generalized) response functions == (generalized) susceptibilities

pressure: $$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}, m_{u,d,s})$$

net number density $$\chi_1^q = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial \mu_q / T}$$

(quark) number susceptibility $$\chi_2^q = \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial (\mu_q / T)^2}$$

4th order cumulant $$\chi_4^q = \frac{1}{VT^3} \frac{\partial^4 \ln Z}{\partial (\mu_q / T)^4}$$

mean variance kurtosis

generalized quark number susceptibilities: $$\frac{\partial^{i+j+k} p/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}$$

$$\hat{\mu}_X \equiv \mu_X / T$$

Taylor expansion and baryon number susceptibilities

$$\chi^B_{n,\mu} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi^{B,Q,S}_{i+n,j,k}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

mean: $M_B = VT^3 \chi^B_{1,\mu} = VT^3 \left(\frac{\mu_B}{T} \chi^B_2 + \ldots \right)$

for simplicity: $\mu_s = \mu_Q = 0$

variance: $\sigma^2_B = VT^3 \chi^B_{2,\mu}$

$$= VT^3 \left(\chi^B_2 + \frac{1}{2} \left(\frac{\mu_B}{T}\right)^2 \chi^B_4 + \ldots \right)$$

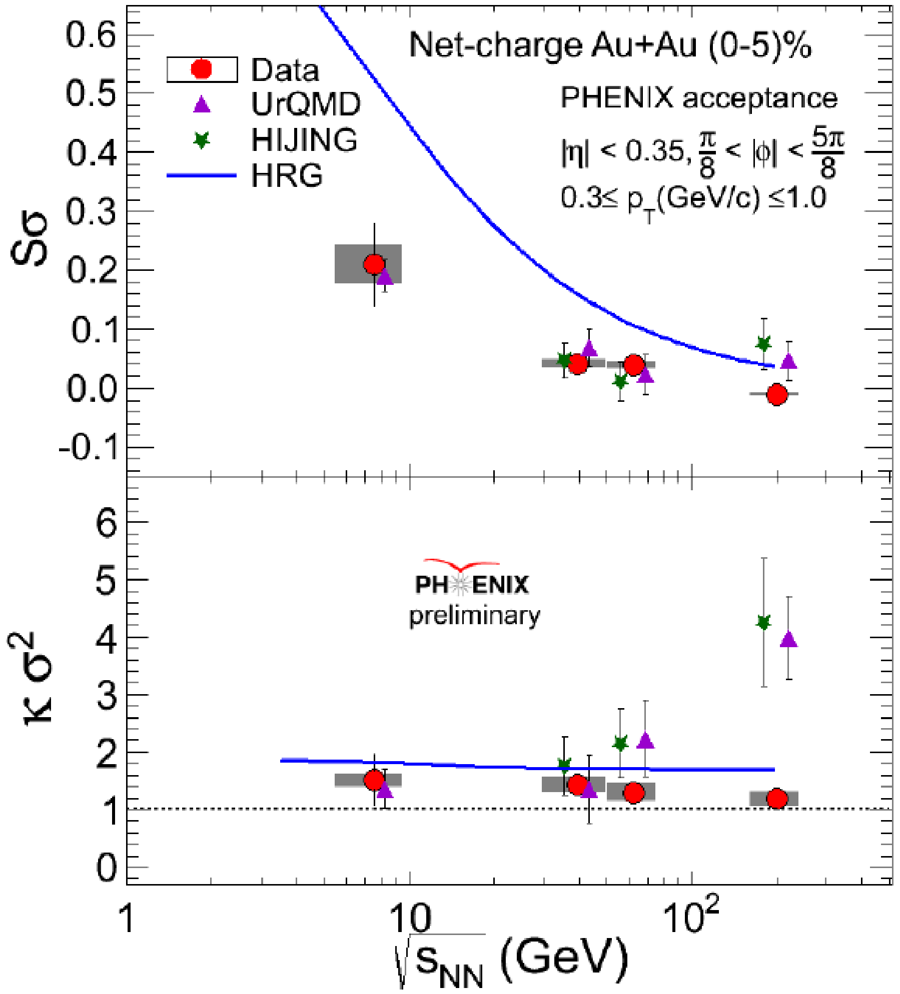
skewness and kurtosis: $S_B \equiv \frac{\langle (\delta N_B)^3 \rangle}{\sigma^3_B}$, $\kappa_B \equiv \frac{\langle (\delta N_B)^4 \rangle}{\sigma^4_B} - 3$

volume independent ratios of susceptibilities

$$\frac{\sigma^2_B}{M_B} = \frac{\chi^B_{2,\mu}}{\chi^B_{1,\mu}}, \qquad S_B \sigma_B = \frac{\chi^B_{3,\mu}}{\chi^B_{2,\mu}}, \qquad \kappa_B \sigma^2_B = \frac{\chi^B_{4,\mu}}{\chi^B_{2,\mu}}$$

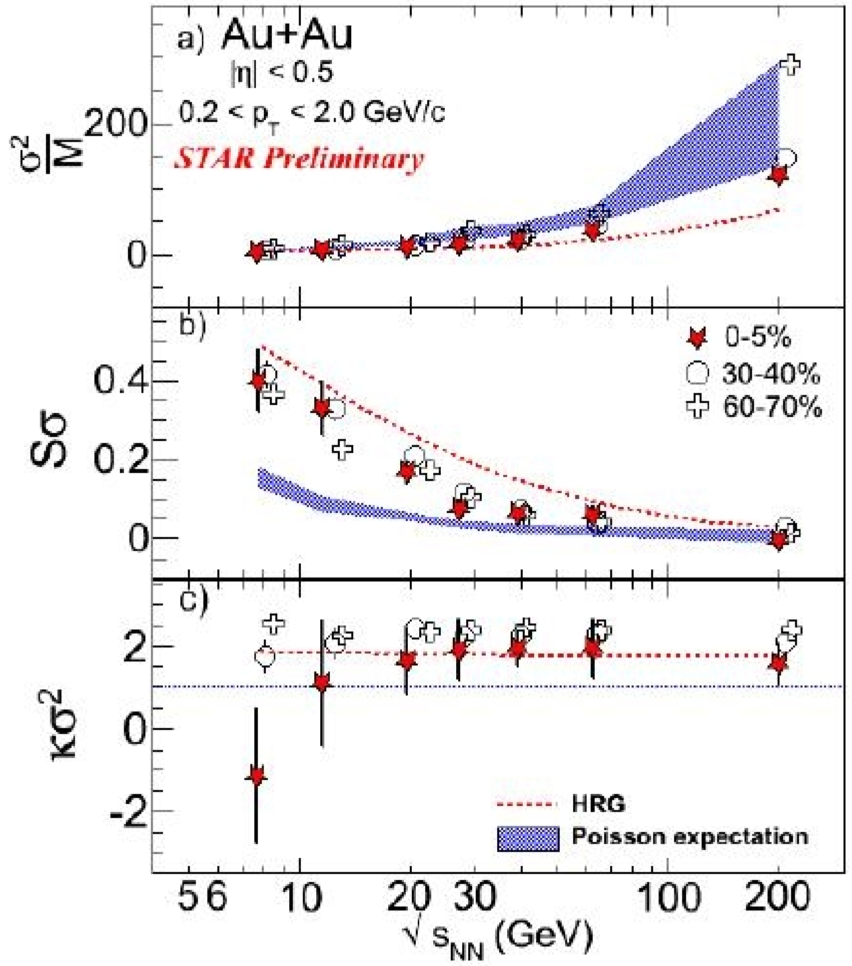
PHENIX and STAR data on electric charge fluctuations

PHENIX



J. Mitchell, CPOD 2013

STAR

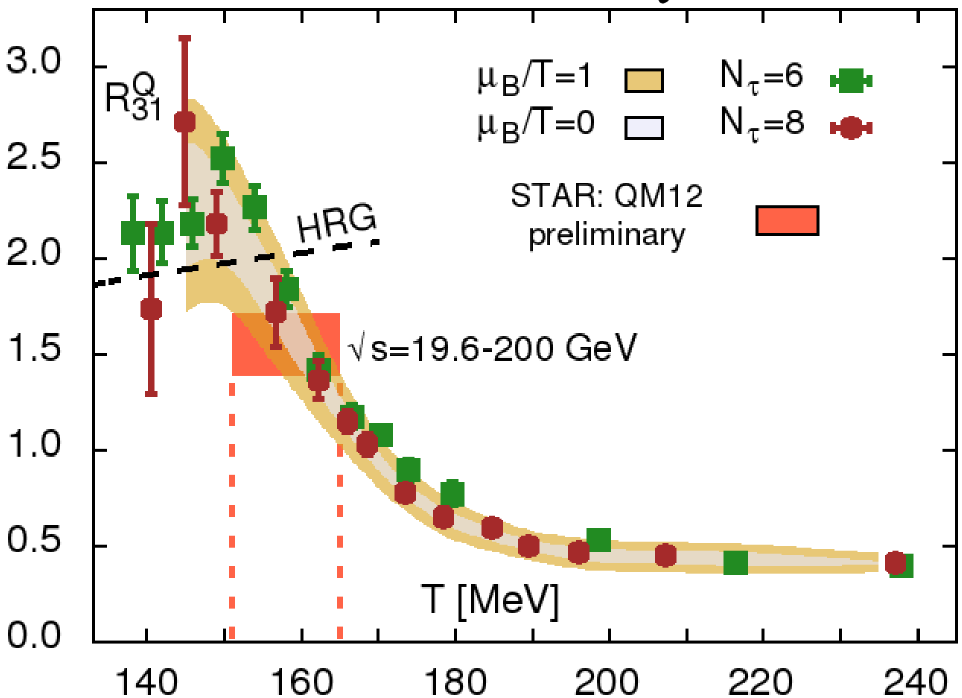


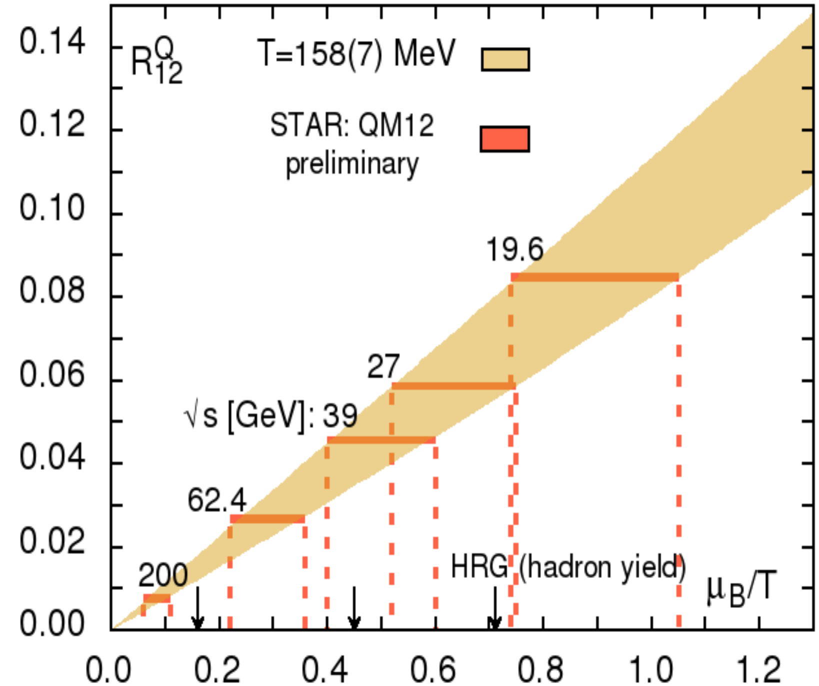
L. Kumar, Quark Matter 2012

Determination of T and μ_B

BNL-Bielefeld, PRL 109, 12302 (2012)

$$R_{31}^Q = \frac{S_Q \sigma_Q^3}{M_Q} \qquad R_{12}^Q = \frac{M_Q}{\sigma_Q^2}$$





T=158(7) MeV

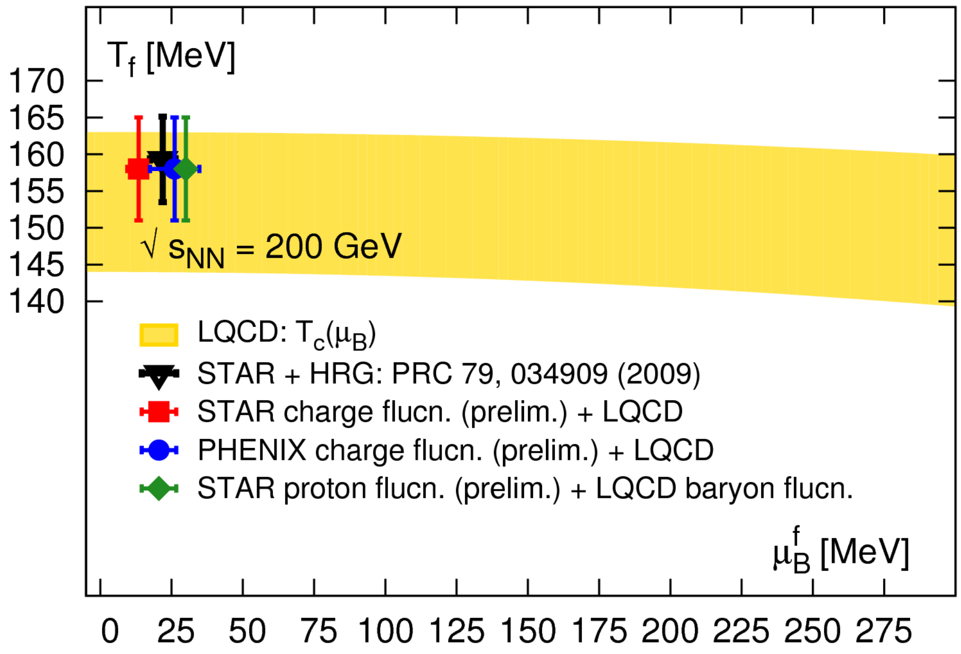
S. Mukherjee and M. Wagner,
PoS CPOD2013, 039 (2013)

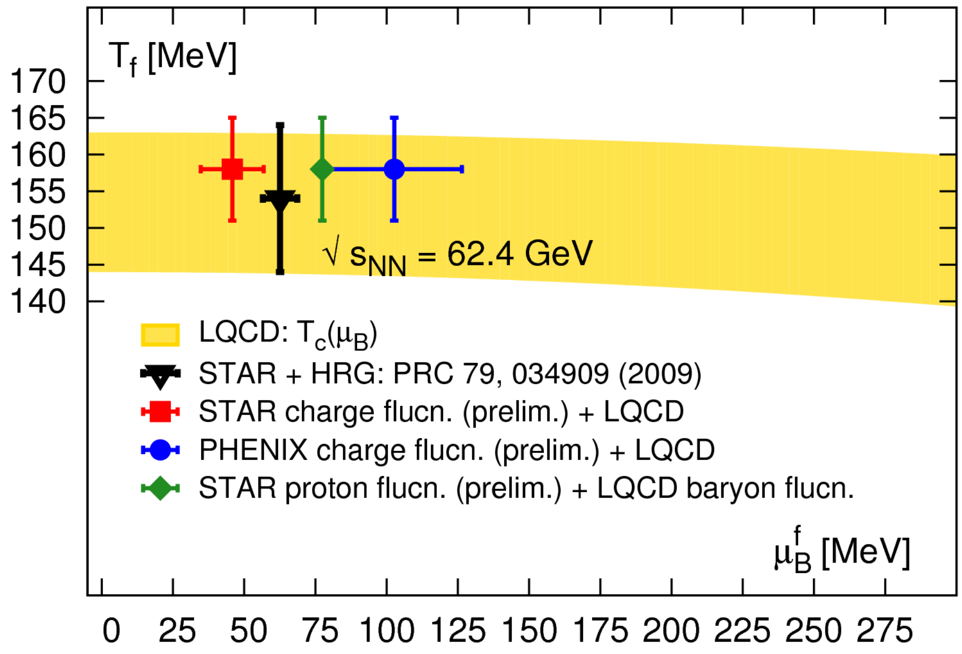
HRG and LGT similar,
however data not yet
efficiency corrected ...etc

from then on all other cumulant ratios probe thermodynamic consistency, i.e.
our basic assumption of a unique freeze-out line, equilibrium thermodyn., etc

Freeze-out parameter

taken at face value the comparison of current experimental data on conserved charge fluctuations with QCD thermodynamics does not yet provide a consistent thermal picture





S. Mukherjee, CPOD2013

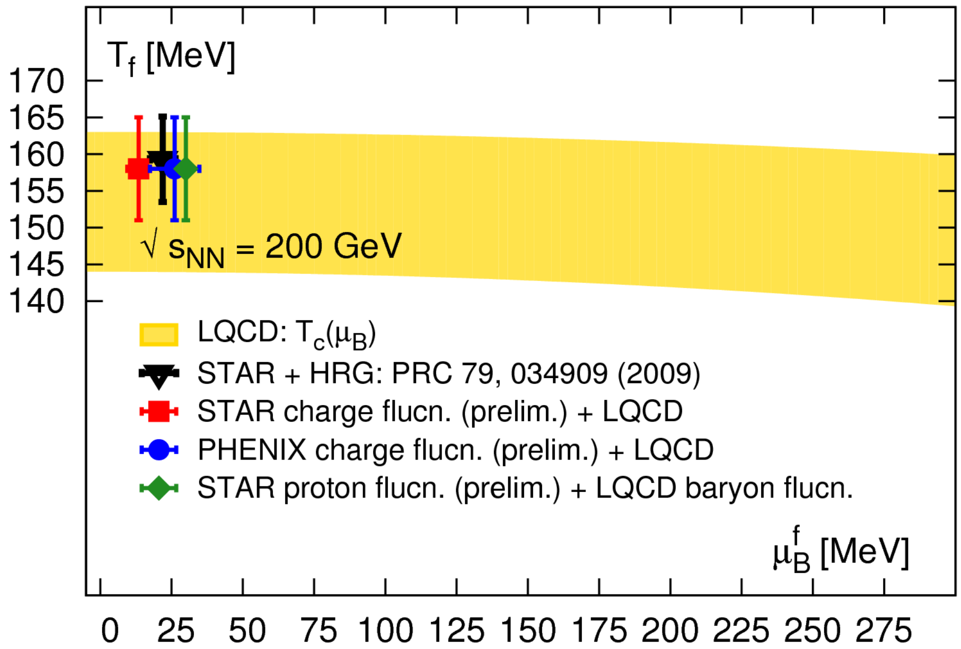
However: this is just the beginning

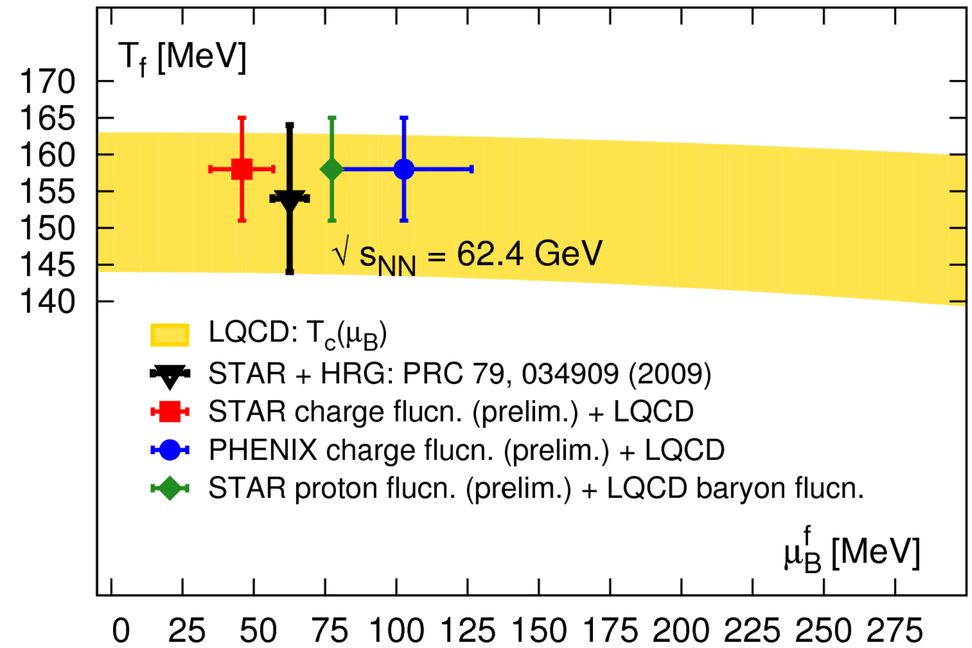
– data are not yet corrected for efficiency
– acceptance cuts and finite volume effects (fluctuations) need to be better understood...

see: M. Kitazawa, last week

Freeze-out parameter

taken at face value the comparison of current experimental data on conserved charge fluctuations with QCD thermodynamics does not yet provide a consistent thermal picture





S. Mukherjee, CPOD2013

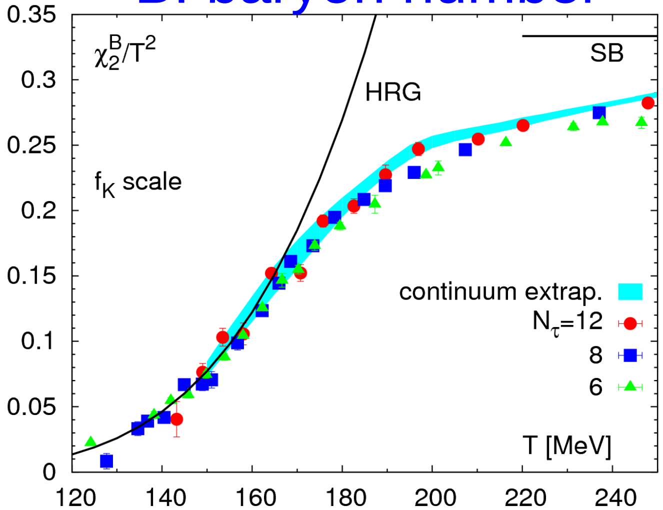
NOTE: This discrepancy also exists, when one would use for comparison cumulants calculated within the HRG model rather than QCD.

In fact, the agreement between QCD and HRG on moments up to 4th order is pretty good for temperatures below T=160 MeV.

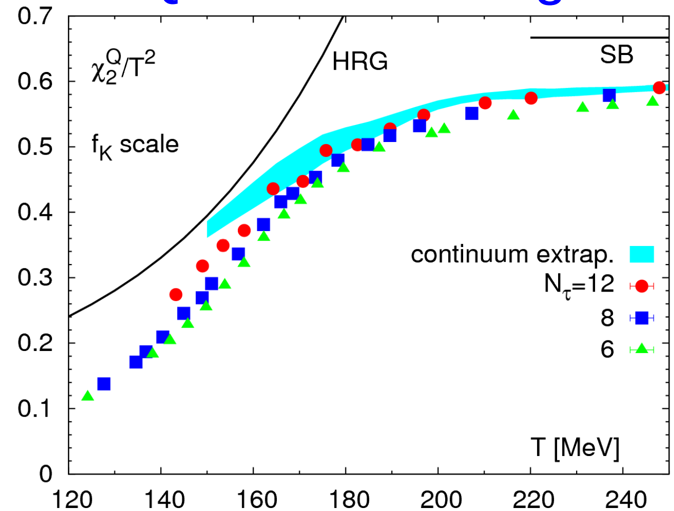
Quadratic charge fluctuations

continuum extrapolated results: A. Bazavov et al (hotQCD), PRD86, 034509 (2012)

B: baryon number



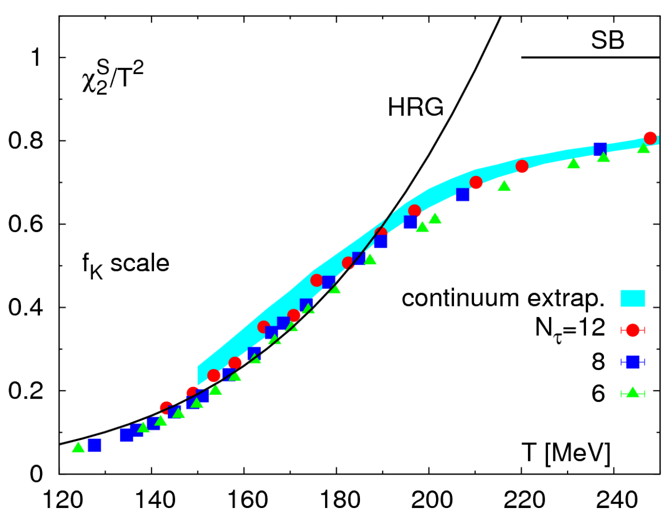
Q: electric charge



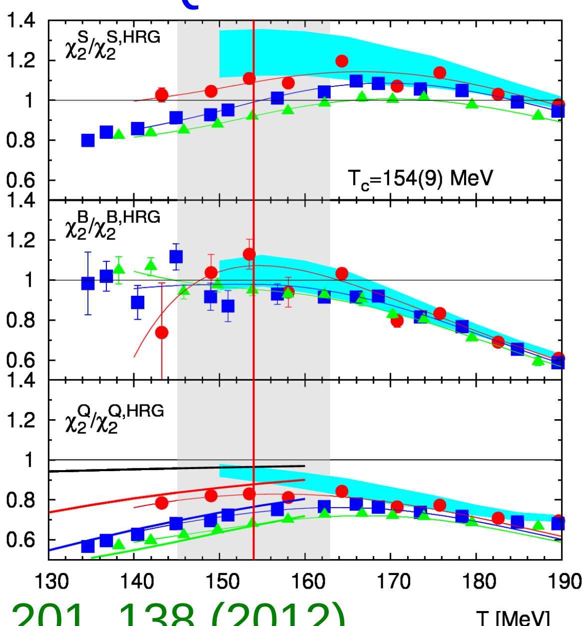
HISQ-action on $(4N_\tau)^3 \times N_\tau$ lattices;

$$\mu_B = 0$$

S: strangeness



BQS vs. HRG



(2+1)-flavor QCD physical strange quark sector;

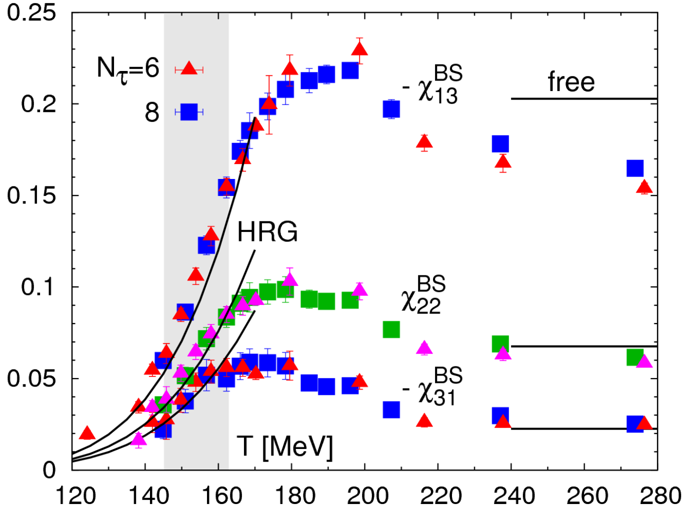
$$m_l/m_s = 1/20$$

$$m_{ps} \simeq 160\ \mathrm{MeV}$$

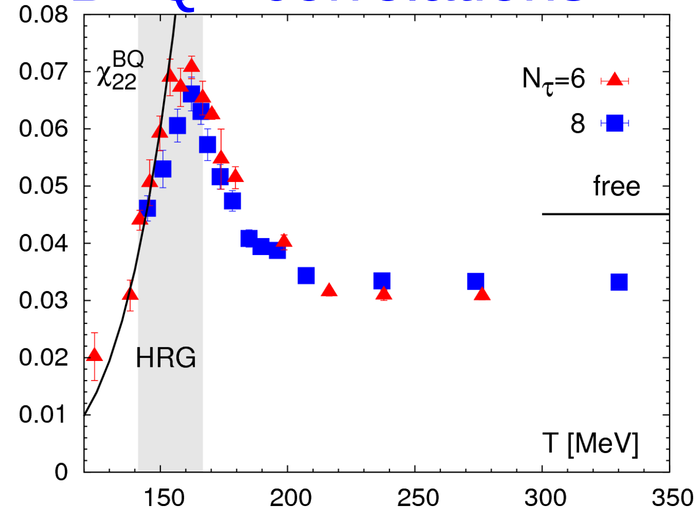
consistent with S. Borsanyi et al., JHEP 1201, 138 (2012)

Some 4th order charge fluctuations

B^x-S^y correlations



B^2-Q^2 correlations



HISQ-action on

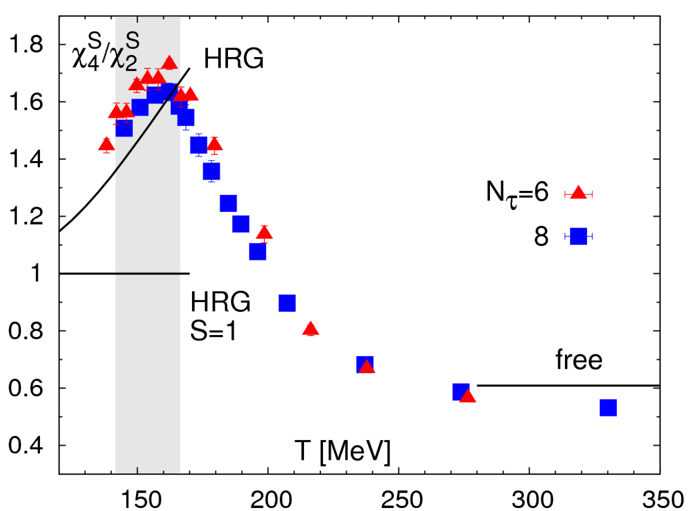
$(4N_\tau)^3 \times N_\tau$

lattices;

$m_{ps} \simeq 160$ MeV

$\mu_B = 0$

S^4/S^2 fluctuations: $\kappa_S \sigma_S^2$



in the chiral limit the singular part reduces to: $\sim A_\pm |t|^{-\alpha}$

$\alpha < 0!!$

generates cusps in 4th order cumulants at $\mu_B = 0$

band:
$T_c = (154 \pm 9)$MeV

Bielefeld-BNL, preliminary similar: S. Borsanyi, Quark Matter 2012

Critical behavior and higher order cumulants

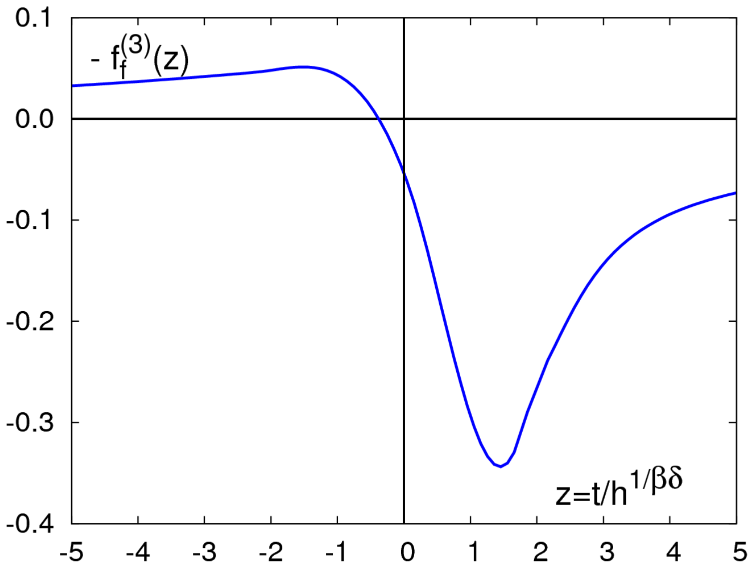
– the breakdown of the HRG model description in the "vicinity of Tc" becomes obvious in properties of higher order cumulants,

pressure: $$\frac{p}{T^4} = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

	alpha
O(4)	-0.213
Z(2)	+0.107

cumulants:

$$\chi_{B,\mu_B}^{(n)} \sim \begin{cases} m_q^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(t/h^{1/\beta\delta}) \ , & \mu_B = 0 \\ m_q^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(t/h^{1/\beta\delta}) \ , & \mu_B > 0 \end{cases}$$



FK, J. Engels, PRD 85, 094506 (2012)

higher order cumulants:

- become singular at Tc in the chiral limit
- are no longer strictly positive

controlled by → universal, O(4) scaling function

Universal properties of the 6th order cumulant at $\mu_B = 0$

$$\mu_B = 0: \quad \chi_6^B = -(2\kappa_B t_0^{-1})^3 h^{-(1+\alpha)/\Delta} f_s^{(3)}(z) + regular$$

in the chiral limit the singular part diverges: $\sim A_\pm |t|^{-(1+\alpha)}$

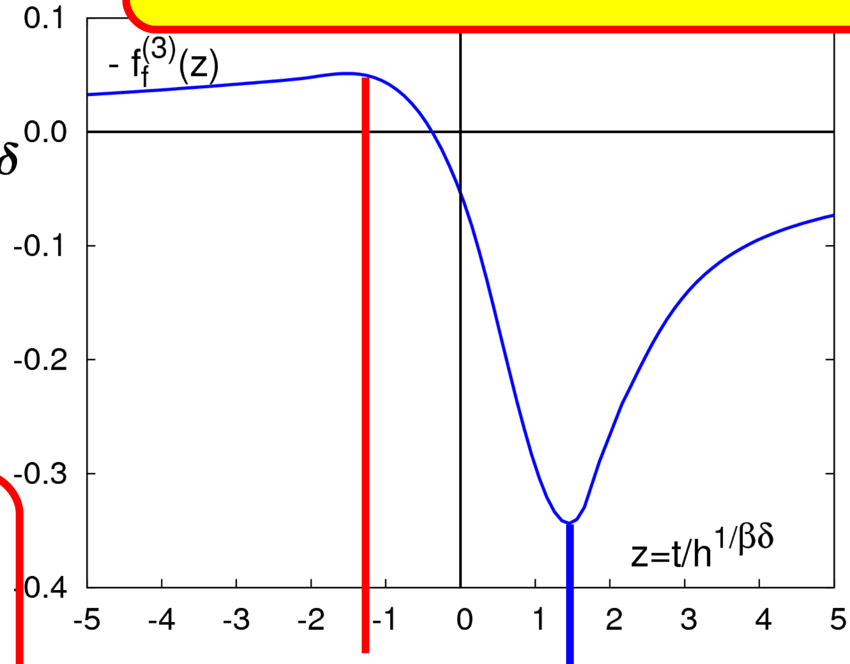
the width of the transition region

(as seen by χ_6^B)

$$\Delta z = z_+ - z_- = (t_+ - t_-)/h^{1/\beta\delta}$$

universal numbers

$$\Delta z = z_+ - z_- \simeq 3$$



$z_- \simeq -1.50$ $z_+ \simeq 1.48$

$$T_+ - T_- = \frac{1}{\Delta z} \frac{t_0 T_c}{h_0^{1/\beta\delta}} \left(\frac{m_l}{m_s}\right)^{1/\beta\delta}$$

$T_+ - T_- \simeq 0.20(5) T_c$

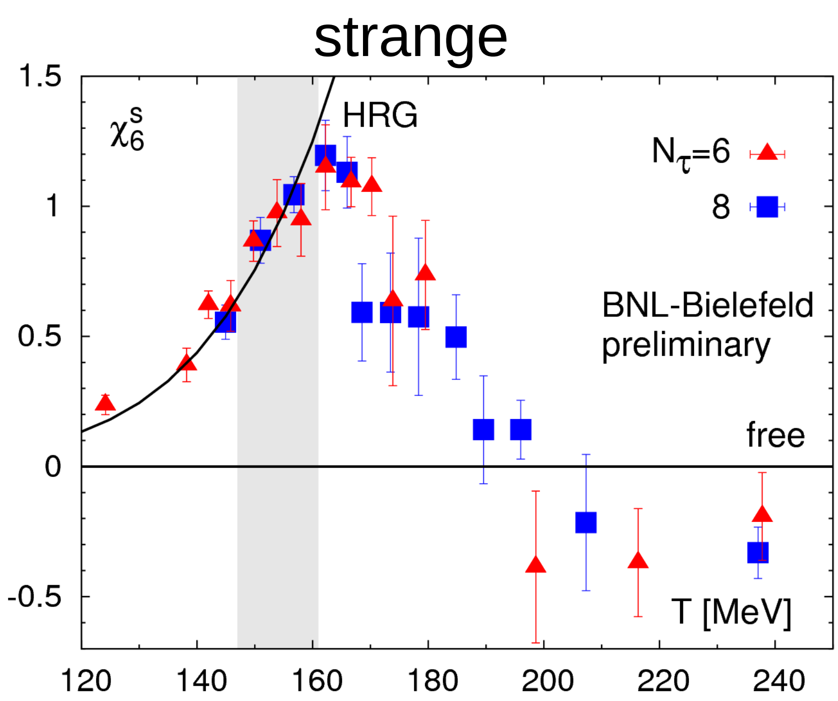
for $m_l/m_s = 1/27$

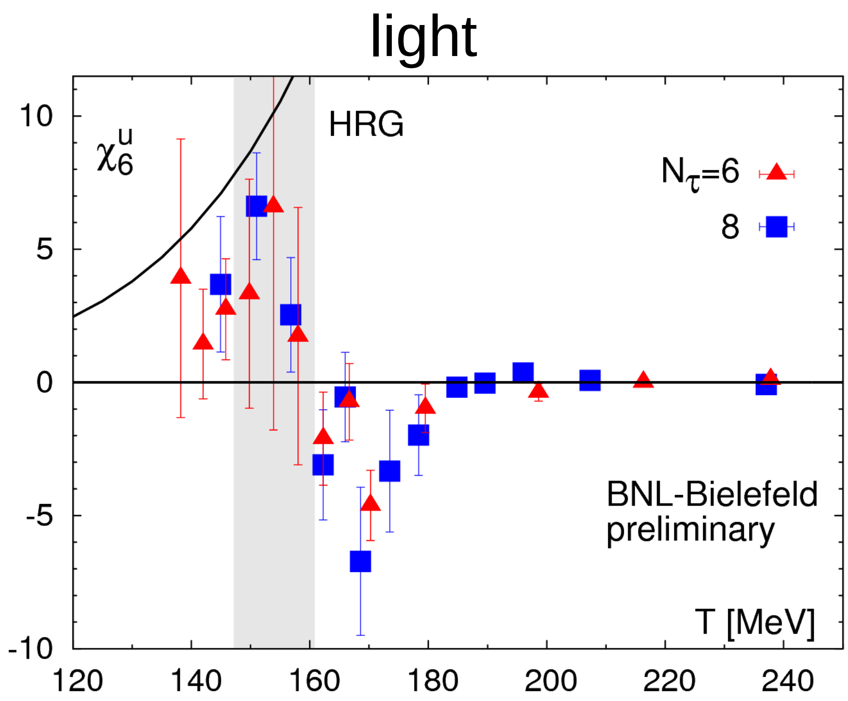
$$\frac{\chi_6^{B,min}}{\chi_6^{B,max}} = -6.7$$

chiral limit

universal number

6^{th} order light and strange quark number cumulants



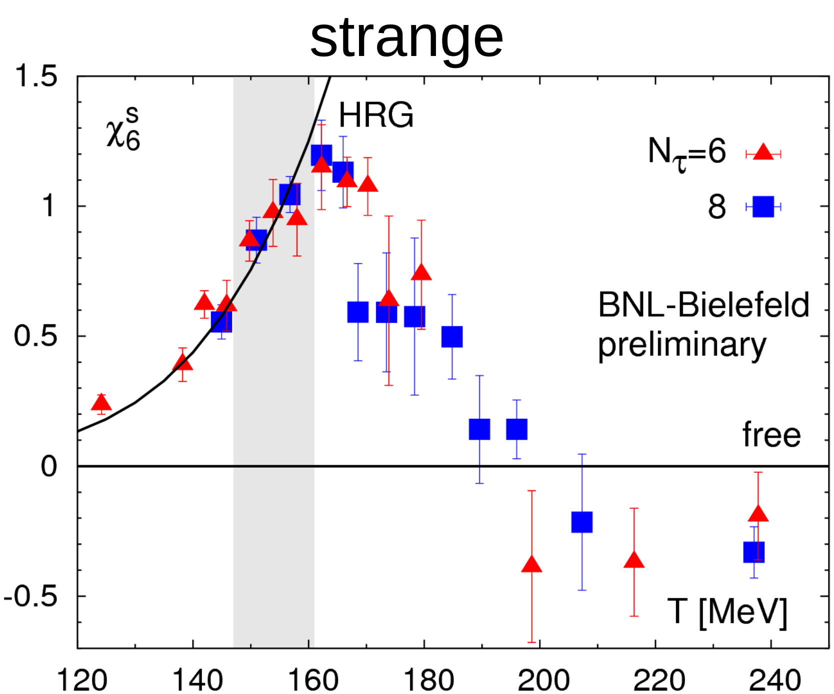


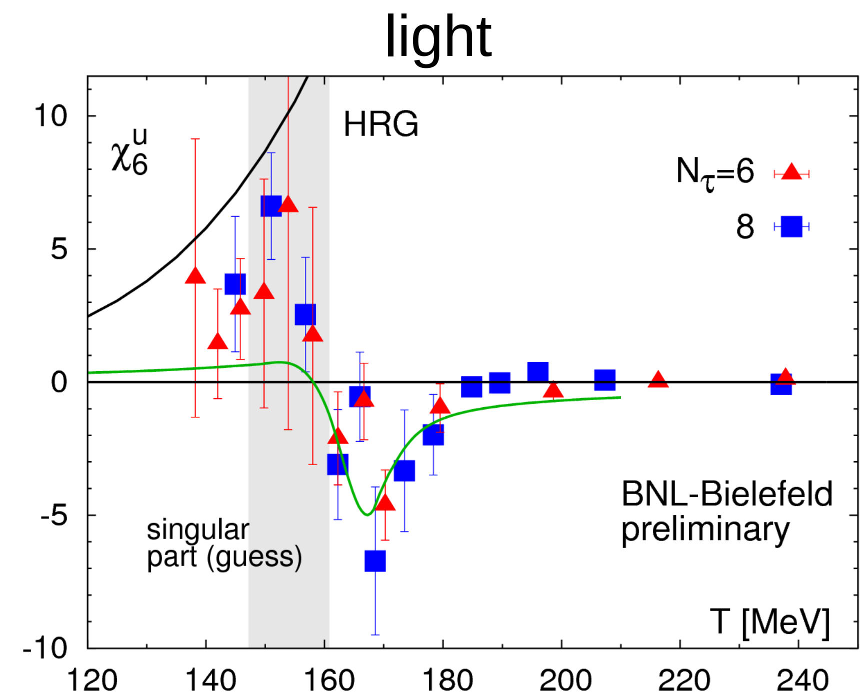
– no evidence for
'typical' O(4) singular structure
– regular contribution dominates

– clear evidence for
'typical O(4) singular structure
– regular and singular contributions

still need to take continuum limit!!

6^{th} order light and strange quark number cumulants





– no evidence for 'typical' O(4) singular structure
– regular contribution dominates

– clear evidence for 'typical O(4) singular structure
– regular and singular contributions

this is important for following the discussion on estimates of the critical point location !!

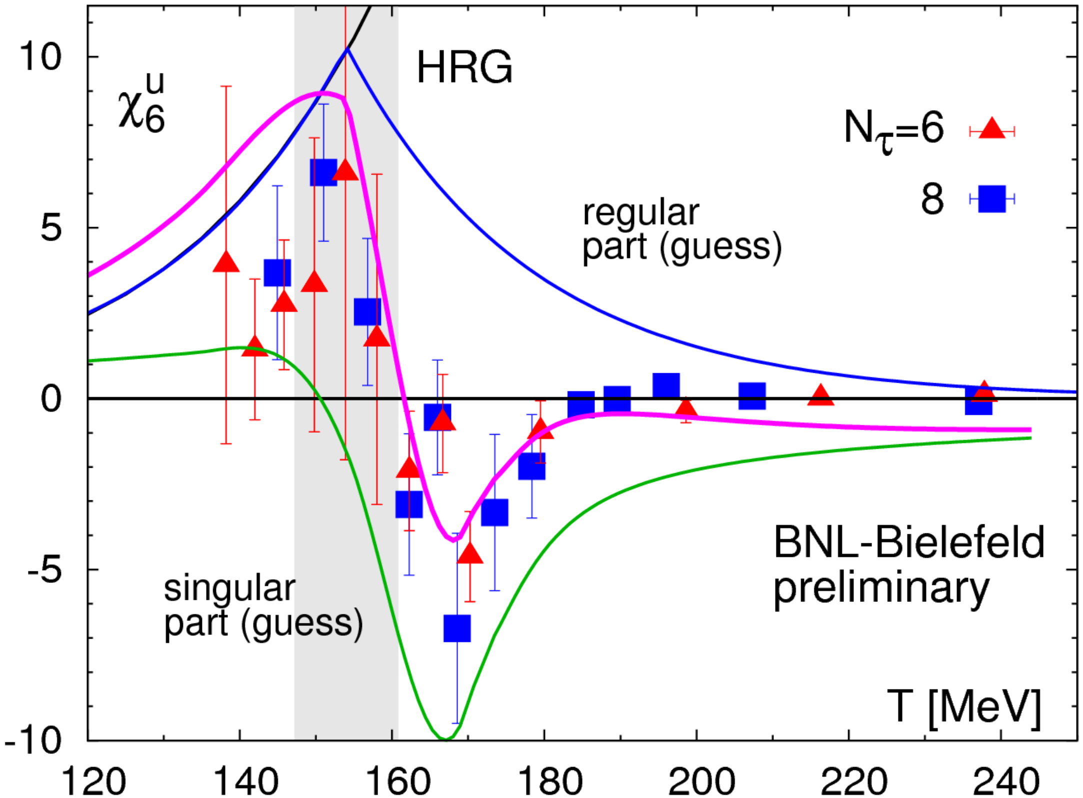
depth of the minimum at high T fixes maximal sinular contribution at low T !!!

$$\frac{\chi_6^{u,min}}{\chi_6^{u,max}} = -6.7$$

6th order light quark number cumulants

On the interplay of regular and singular contributions – a guess, not a fit

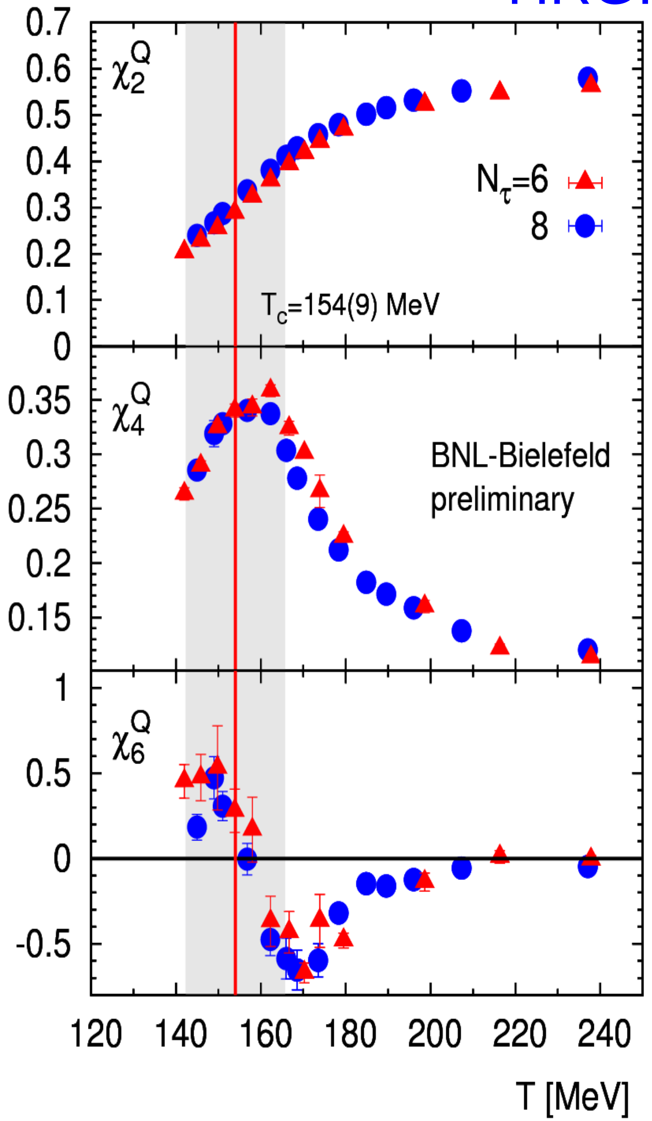
regular part
dominated by
HRG at low-T



singular part
generates dip
at high-T

total=singular+regular:
one may expect an overshooting of HRG
by (at most) a factor 2 ??

Electric charge fluctuations in QCD and the HRG



- HRG: (I) all cumulants are positive
  (II) ratios of even cumulants are larger than unity;
  e.g. $\chi_4^Q/\chi_2^Q \simeq 1.6$ in the crossover region

QCD

if $T_{freeze} \simeq 160$ MeV

$$\chi_6^Q \lesssim 0$$
$$\chi_4^Q/\chi_2^Q \simeq 1$$

HRG

$$\chi_6^Q > 0$$
$$\chi_4^Q/\chi_2^Q \simeq 1.6$$

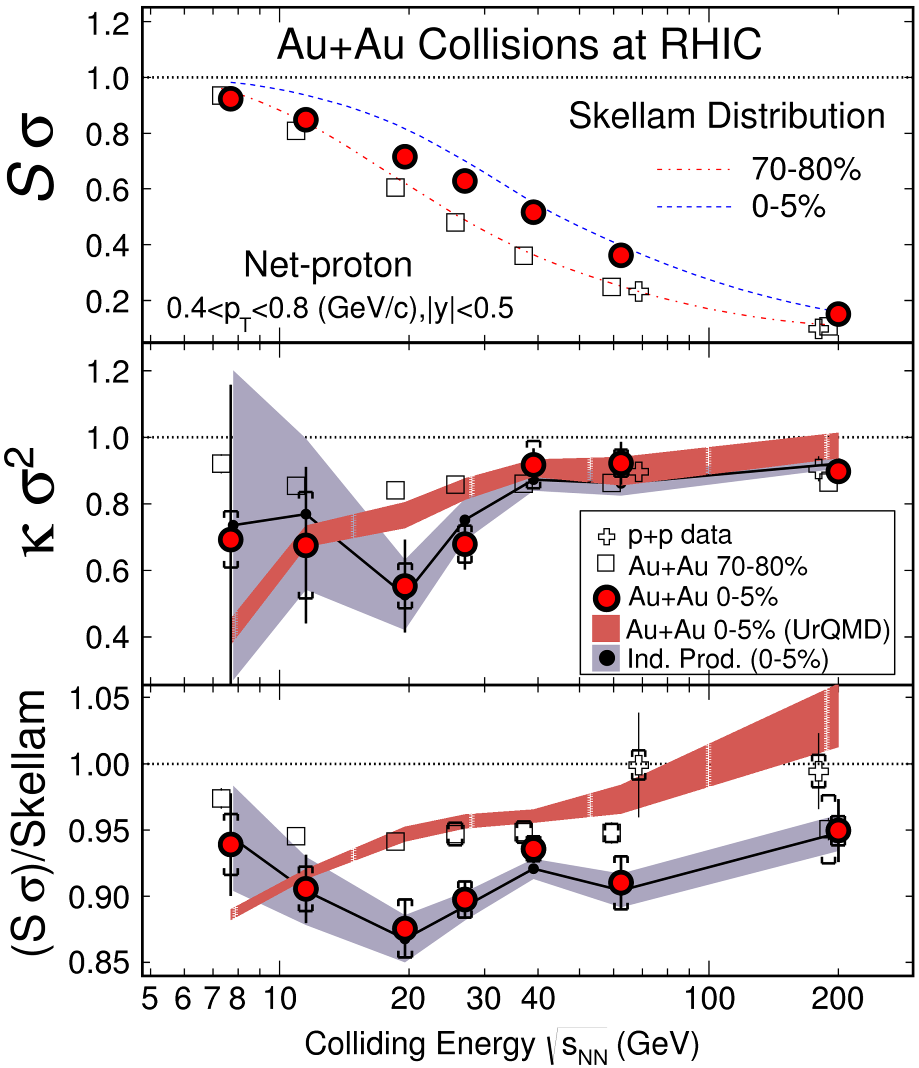
cut-off effects;
net electric charge fluctuations are difficult to calculate in lattice QCD;
light pions induce large cut-off effects

still need to take continuum limit!!

C. Schmidt (Bielefeld-BNL), Quark Matter 2012,
Nucl. Phys. A904-905, 865c (2013)

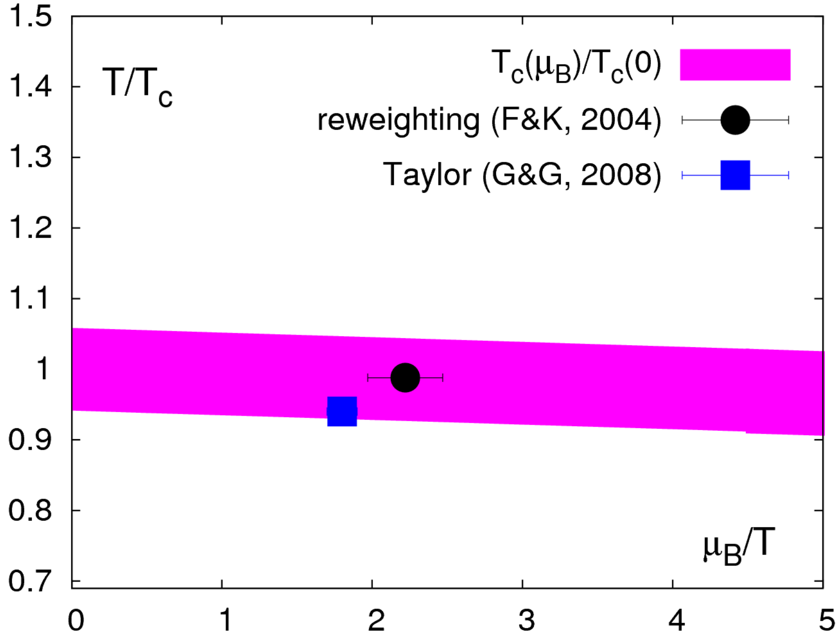
Critical Point searches

cumulants of net proton number fluctuations



STAR Collaboration, arXiv:1309.5681

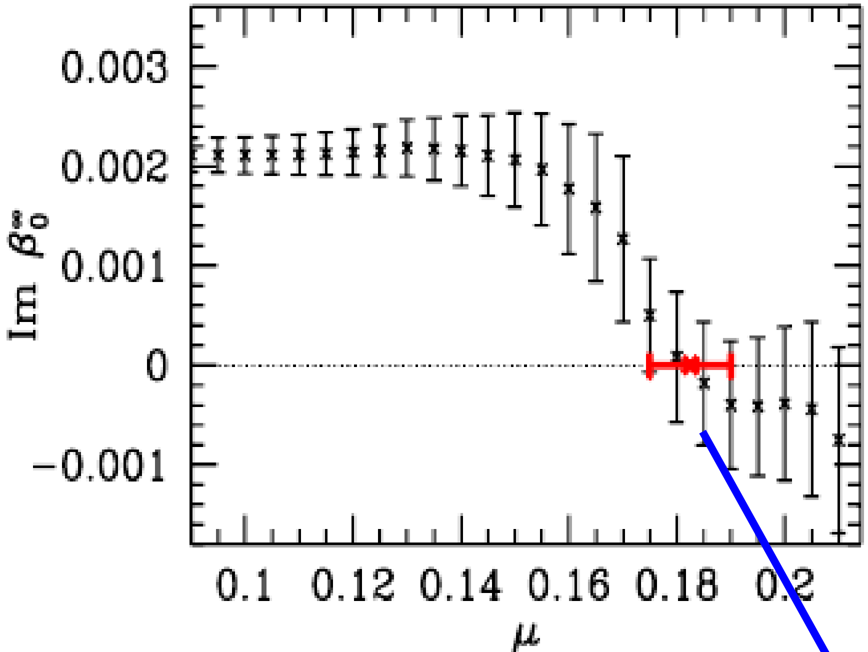
lattice QCD



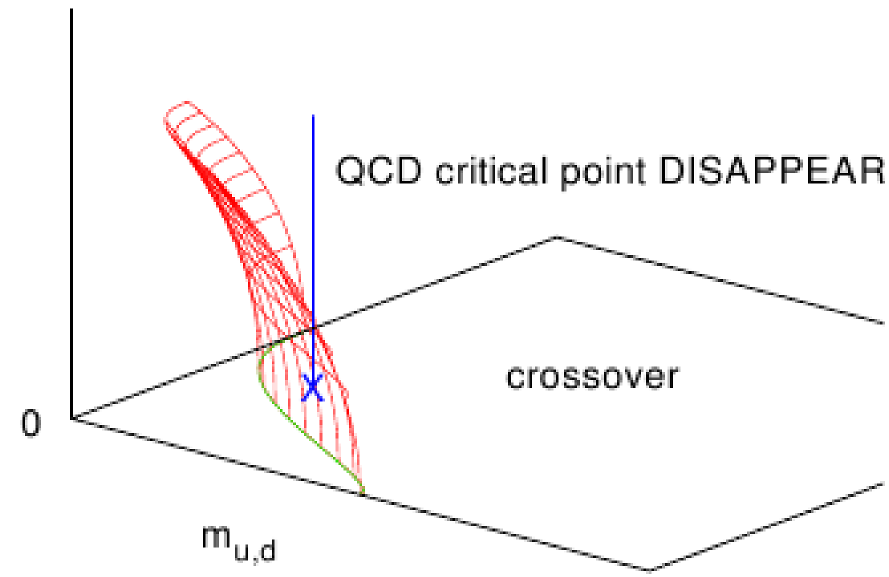
reweighting:
Z. Fodor, S. Katz,
JHEP 04, 204 (2004)

Taylor expansion:
R.V. Gavai, S. Gupta,
PRD78, 114503 (2008)

LGT attempts to find the critical point



Z. Fodor, S. Katz. 2001, 2004



P. deForcrand, O. Philipsen, 2002

these calculations were possible because
(I) the lattices were coarse,
(II) the discretization schemes were crude

critical point or breakdown of the reweighting approach (loosing the overlap) ?

S. Ejiri, PRD69, 094506 (2004)

since 10 years no progress along this line

Taylor expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_B,\mu_S,\mu_Q)$$

$$= \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

generalized susceptibilities: $\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k} \right|_{\mu=0}$

– can be evaluated using standard MC simulation algorithms;
- valid up to radius of convergence: μ_c (critical point?)

– radius of convergence corresponds to a critical point only, iff

$$\chi_n > 0 \text{ for all } n \geq n_0$$

provides also a criterium to locate $T_c(\mu_c)$

Taylor expansion of the pressure

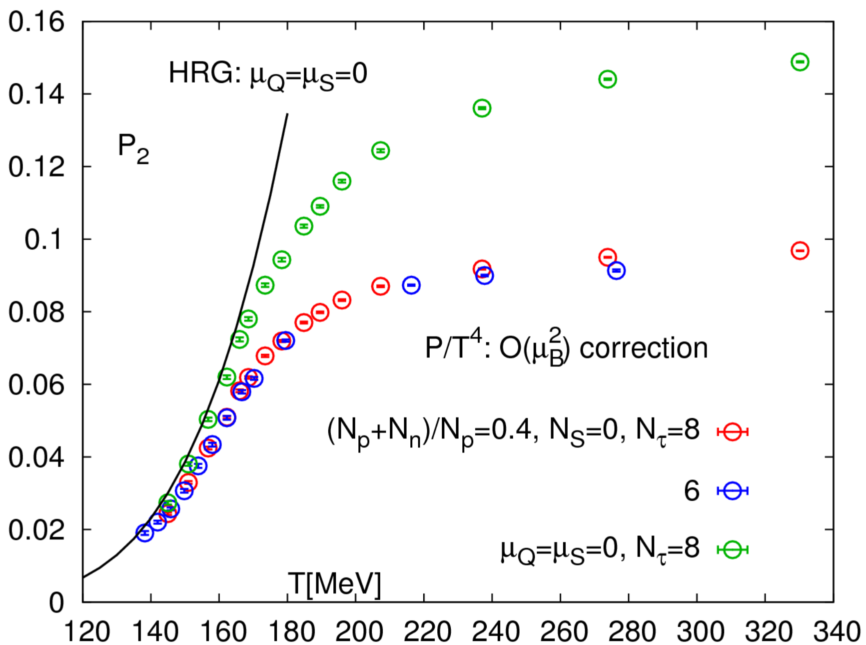
e.g., assume vanishing electric charge and strangeness chemical potential:

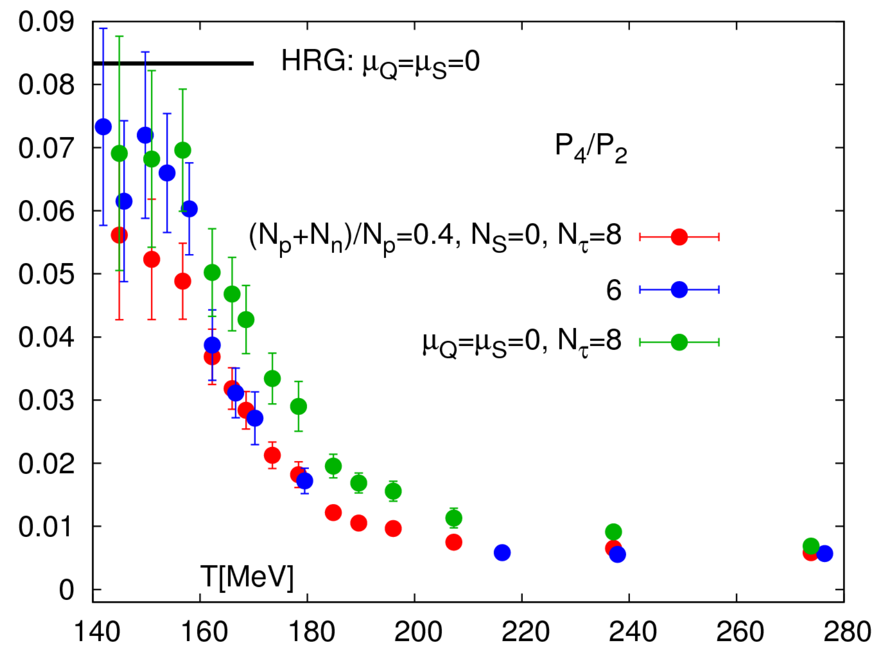
$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_B) = \sum_{n,even} \frac{1}{n!} \chi_n^B \left(\frac{\mu_B}{T}\right)^n$$

$$\Delta p/T^4 = (p(\mu_B) - p(0))/T^4 = P_2\ \hat{\mu}_B^2 + P_4\ \hat{\mu}_B^4 + \ldots$$

BNL-Bielefeld-CCNU, preliminary

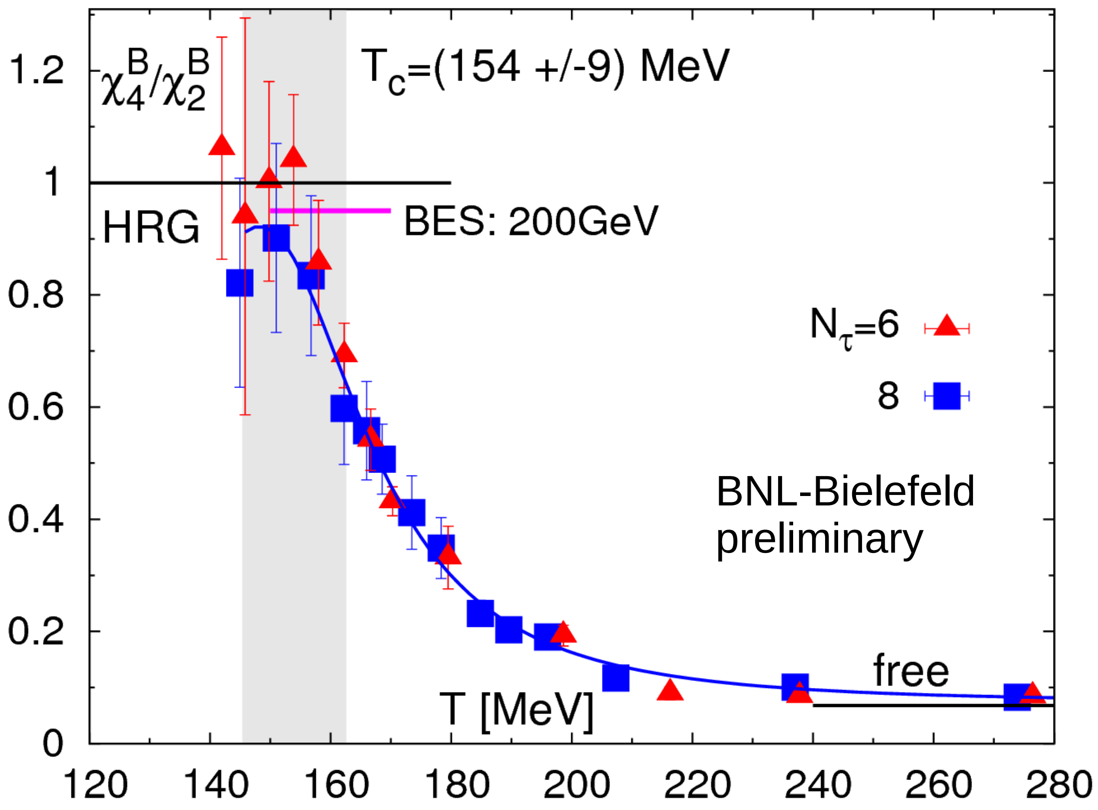
$P_4/P_2 < 1$ for $\mu_B \lesssim 3.5$





Taylor expansion of the pressure

for $\mu_Q = \mu_S = 0$ the relative strength of the next-to-leading order correction to the pressure is controlled by χ_4^B/χ_2^B



$T < T_c$:

$0.8 \leq \chi_4^B/\chi_2^B \leq 1.0$

⇓

$P_4/P_2 < 1$ for $\mu_B \lesssim 3.5$

similar: S. Borsanyi et al, PRL 111, 062005 (2013)

Radius of convergence and the critical point

e.g., assume vanishing electric charge and strangeness chemical potential:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_B) = \sum_{n,even} \frac{1}{n!} \chi_n^B \left(\frac{\mu_B}{T} \right)^n$$

– radius of convergence: $$\left(\frac{\mu_B}{T} \right)_{crit} = \lim_{n\to\infty} \sqrt{\left| \frac{(n+2)! \chi_n^B}{n! \chi_{n+2}^B} \right|}$$

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– may use expansion of another quantity (Gavai&Gupta), e.g. baryon number susceptibility:

$$\chi_{2,\mu}^B = \sum_{n,even} \frac{1}{n!} \chi_{n+2}^B \left(\frac{\mu_B}{T}\right)^n$$

– radius of convergence: $\left(\frac{\mu_B}{T}\right)_{crit}^{\chi} = \lim_{n\to\infty} \sqrt{\left|\frac{n!\chi_n^B}{(n-2)!\chi_{n+2}^B}\right|}$

Radius of convergence and the critical point

may use another estimator:

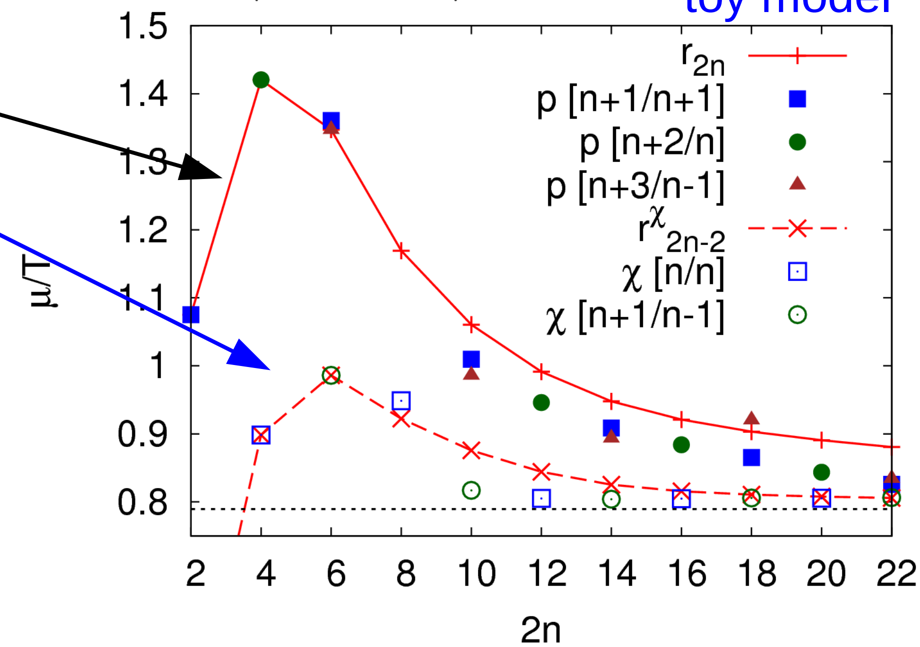
$$\left(\frac{\mu_B}{T}\right)_{crit} = \lim_{n\to\infty} \sqrt{\left|\frac{(n+2)!\chi_n^B}{n!\chi_{n+2}^B}\right|} \equiv \lim_{n\to\infty} \left|\frac{(n+2)!\chi_2^B}{2!\chi_{n+2}^B}\right|^{1/n}$$

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit} = \lim_{n\to\infty} \sqrt{\left|\frac{n!\chi_n^B}{(n-2)!\chi_{n+2}^B}\right|} \equiv \lim_{n\to\infty} \left|\frac{n!\chi_2^B}{\chi_{n+2}^B}\right|^{1/n}$$

PQM toy model

pressure series

suscept. series



prefactors are not unique for $n < \infty$

⇨ inherent systematic errors

$\sim 1/n^2$ (~30% for n=6)

$$\frac{\chi_B^{n+2}}{\chi_B^n}$$

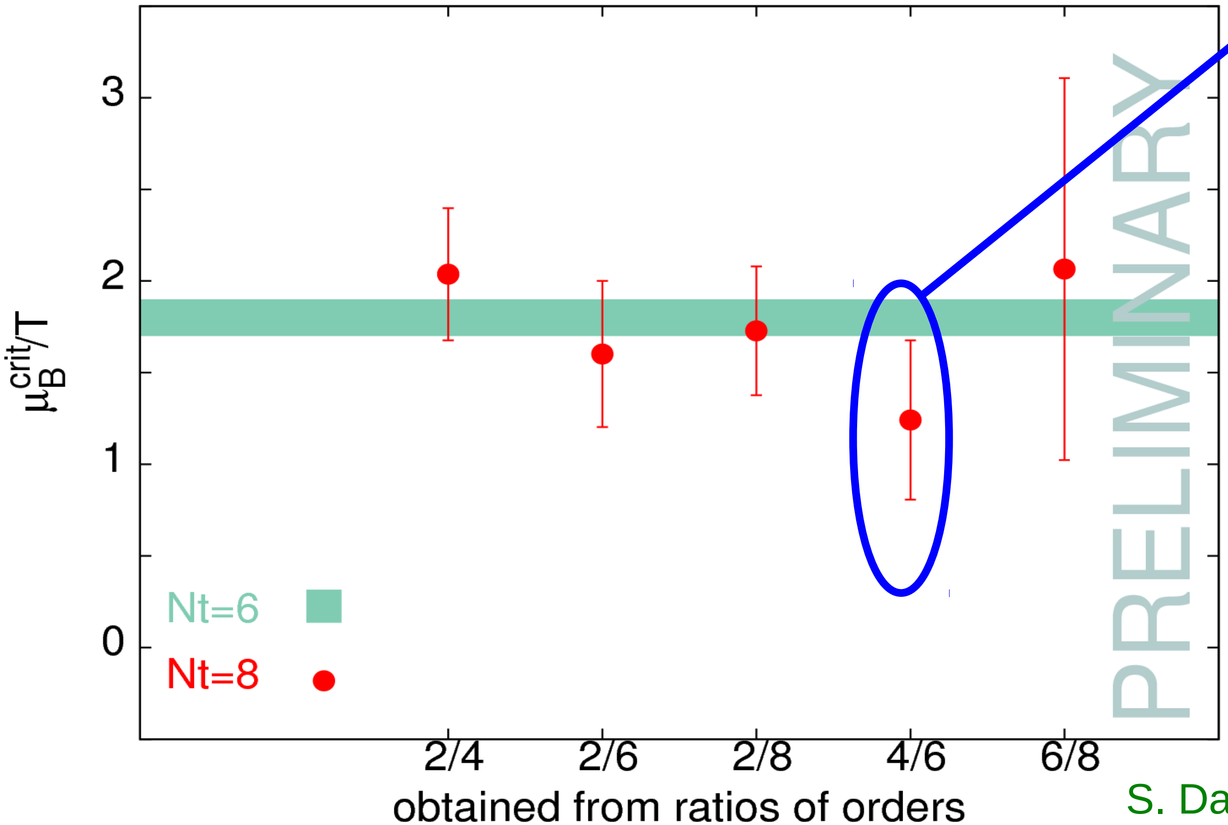
basic observables need to deviate from HRG like n^2

FK et al., Phys.Lett. B698, 256 (2011)

Estimates of the radius of convergence

a challenging prediction from susceptibility series: $\left(\frac{\mu_B}{T}\right)^{\chi}_{crit} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$

suggests large deviations from HRG in the hadronic phase



huge deviations from HRG in 6^{th} order cumulants?

$$\frac{\chi_6^B}{\chi_4^B} = 5.5(1.5)$$

S. Datta et al., Nucl. Phys. A904 (2013), 883c

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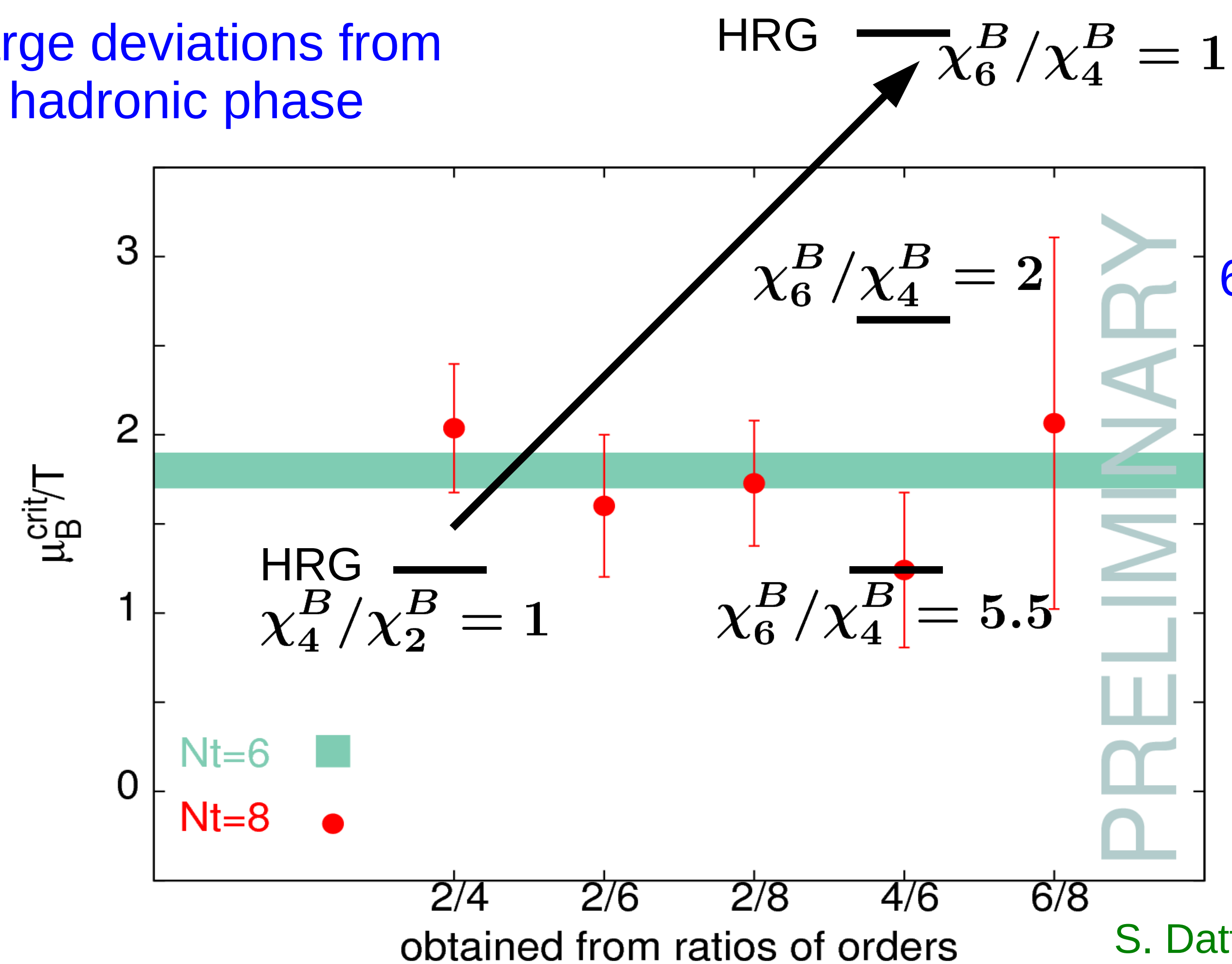


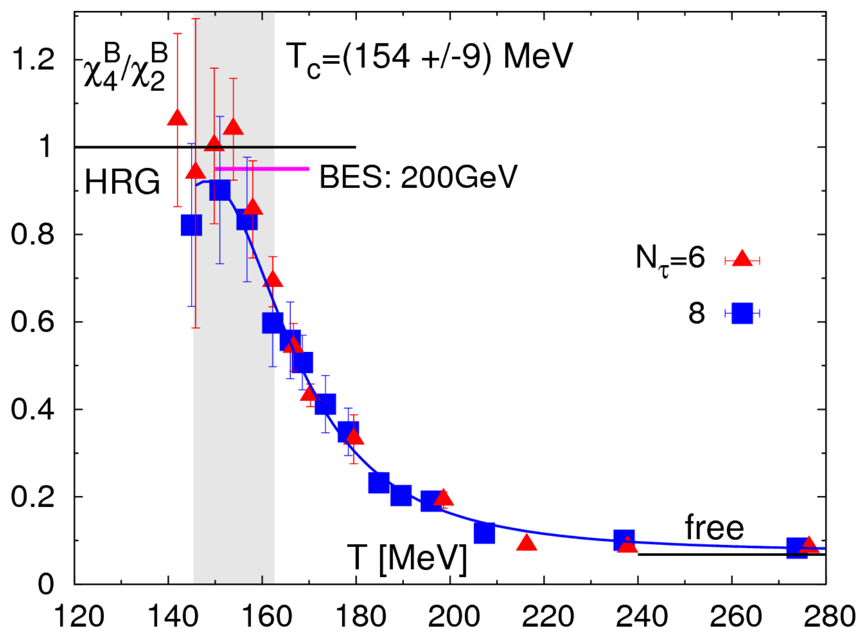
S. Datta et al., Nucl. Phys. A904 (2013), 883c

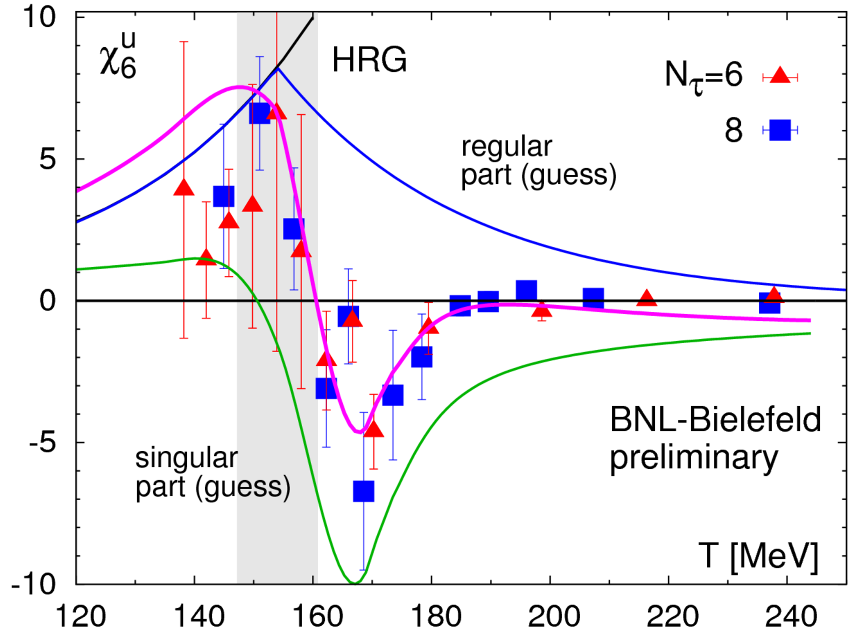
Radius of convergence and the critical point

basic observables $\dfrac{\chi_B^{n+2}}{\chi_B^n}$ need to deviate from HRG like n^2

However, so far no evidence for large enhancement over HRG for $T < T_c$

...remember:



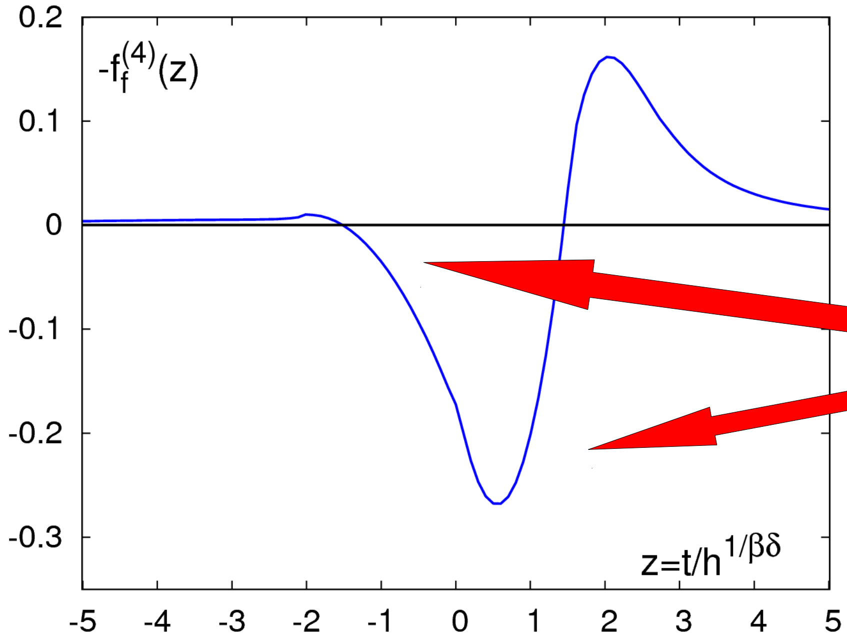


this suggests a large μ_B^{crit}

Conserved charge fluctuations in QCD and HIC

4th order cumulant: A dip in the kurtosis ?

$$\mu_B > 0: \quad \chi_{4,\mu}^B = -3(2\kappa_B t_0^{-1})^2 h^{-\alpha/\Delta} f_f^{(2)}(z)$$
$$-6(2\kappa_B t_0^{-1})^3 (\hat{\mu}_B^c)^2 h^{-(1+\alpha)/\Delta} f_f^{(3)}(z)$$
$$-(2\kappa_B t_0^{-1} \hat{\mu}_B^c)^4 h^{-(2+\alpha)/\Delta} f_f^{(4)}(z) \quad \text{+ regular}$$



dominates in the chiral limit, or if

$$\hat{\mu}_B^c = \mu_B^c / T \gtrsim 1$$

$$\frac{\chi_{4,\mu}^{B,min}}{\chi_{4,\mu}^{max-}} \simeq -25$$

$\Rightarrow \chi_4^B(\mu_B) < 0$

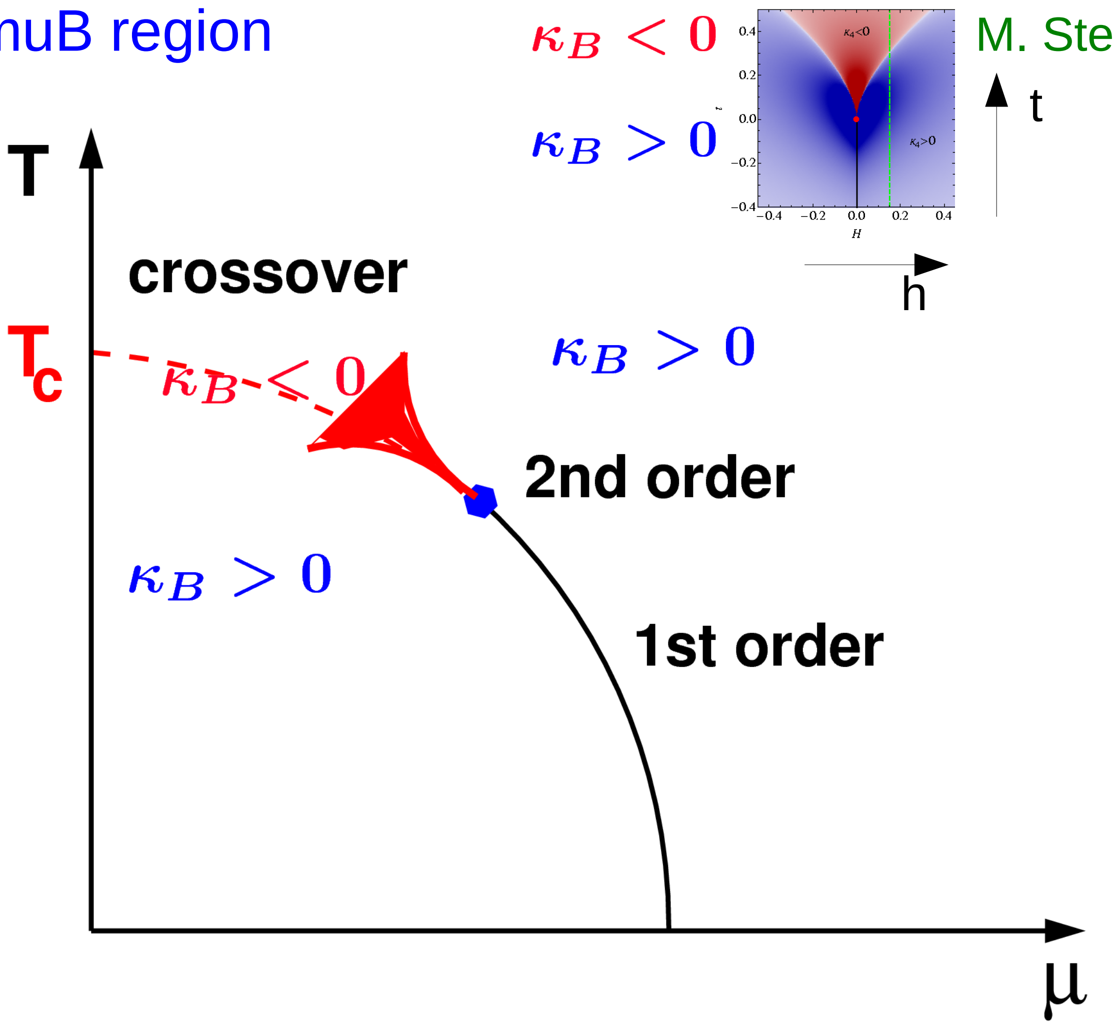
B.Friman, FK, K.Redlich, V.Skokov, Eur. Phys. J. C71, 1694 (2011)

4^{th} order cumulant (kurtosis) and the critical point

in the vicinity of the critical point the kurtosis will be negative in a certain T, muB region

M. Stephanov, PRL 107, 052301 (2011)

mapping of the Ising variables t, h on the T, μ_B plane is non-trivial



4^{th} order cumulant (kurtosis) and the critical point

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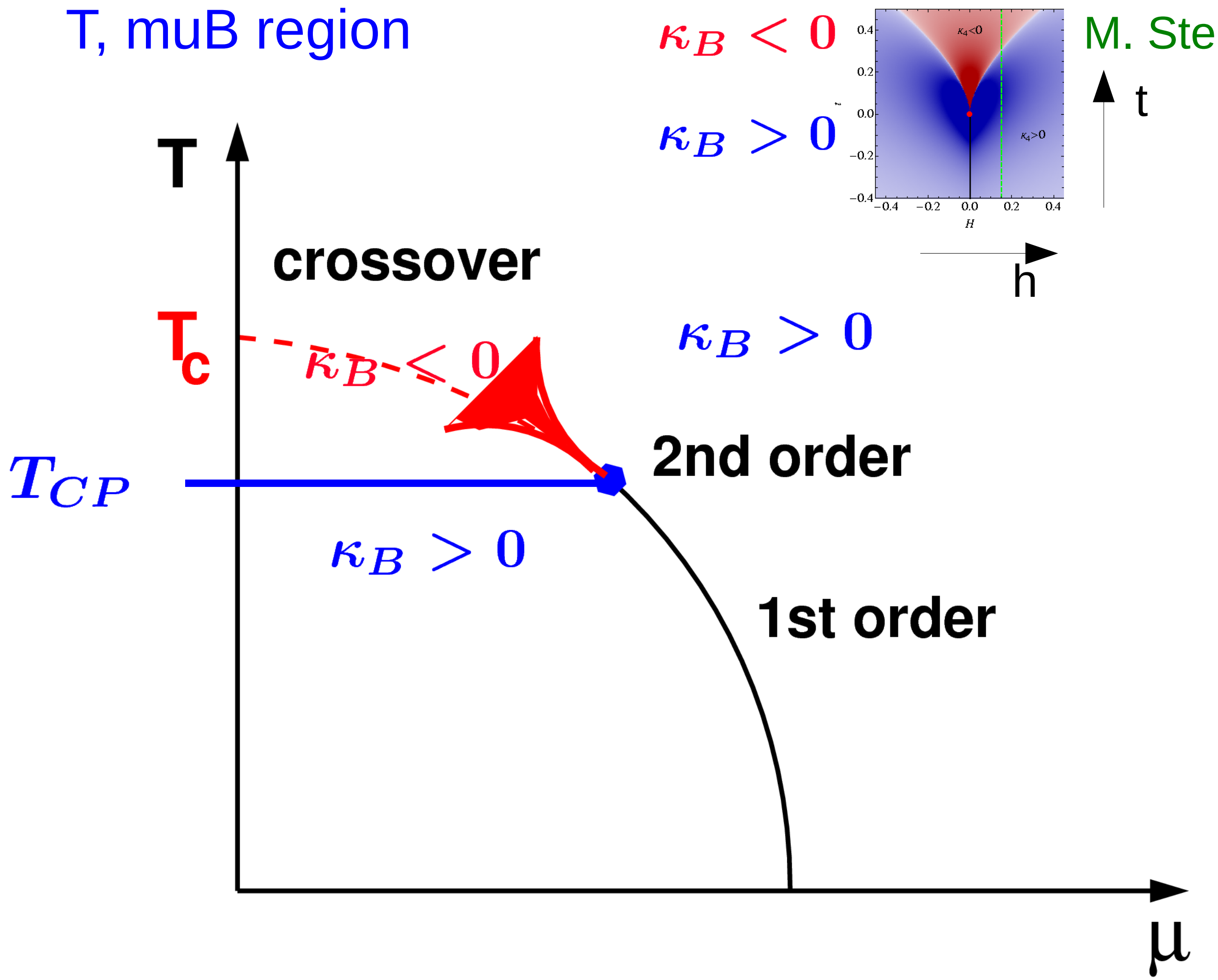
M. Stephanov, PRL 107, 052301 (2011)



mapping of the Ising variables t, h on the T, μ_B plane is non-trivial

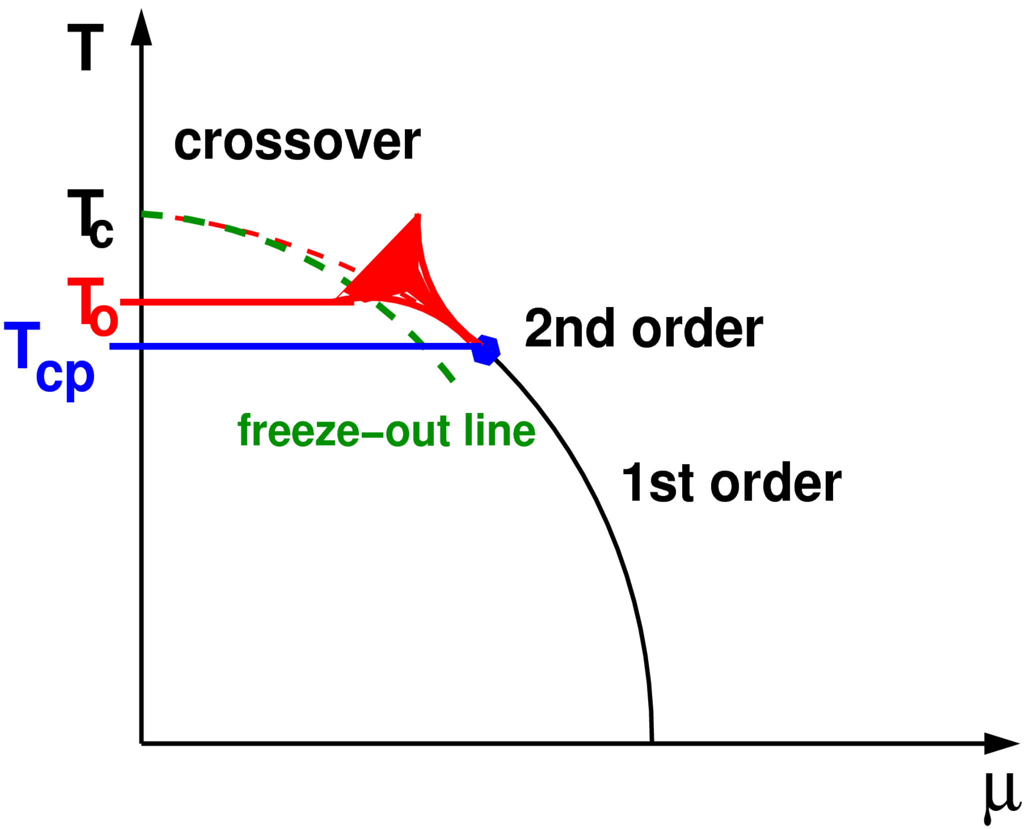
generically, expect:

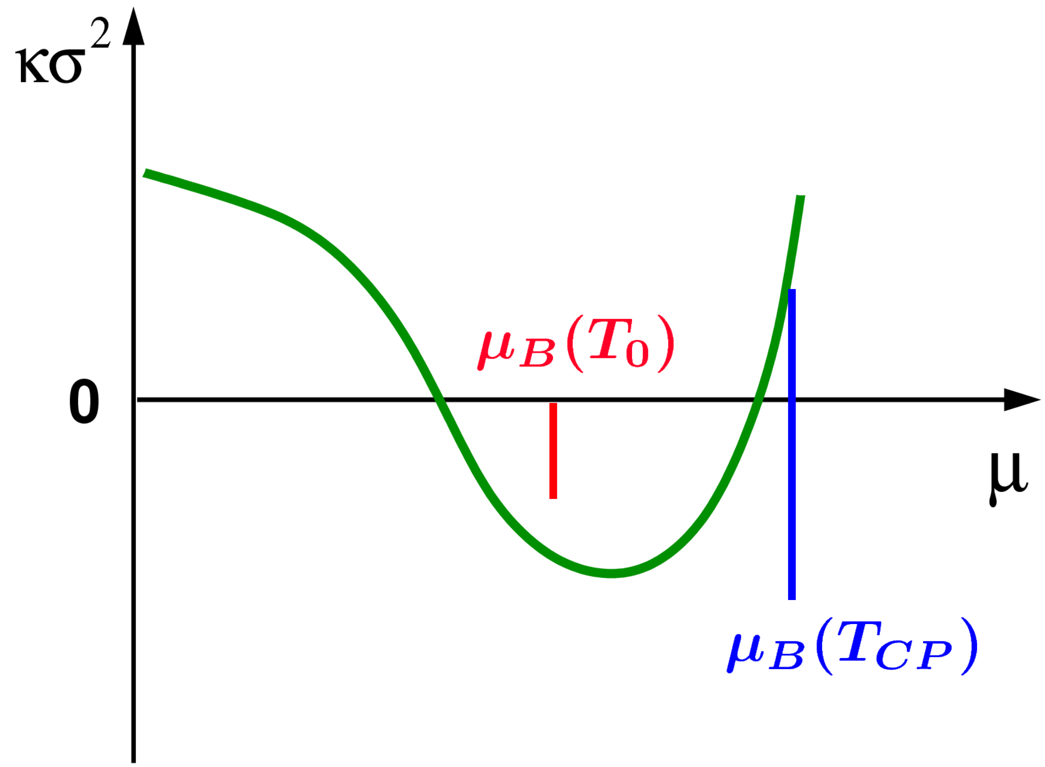
$\kappa_B > 0$ for $T \le T_{CP}$

expect all cumulants to be positive on the line of fixed $T \equiv T_{CP}$

prerequisite for well-behaved estimates of the location of the critical point based on the radius of convergence of the Taylor series for $\chi_{B,\mu}$

Kurtosis on the freeze-out curve





to determine the importance of regular terms and the non-universal scales requires lattice QCD

a dip in the kurtosis seems to be generic: whether or not it becomes negative depends on the magnitude of regular terms in the QCD partition function (pressure)

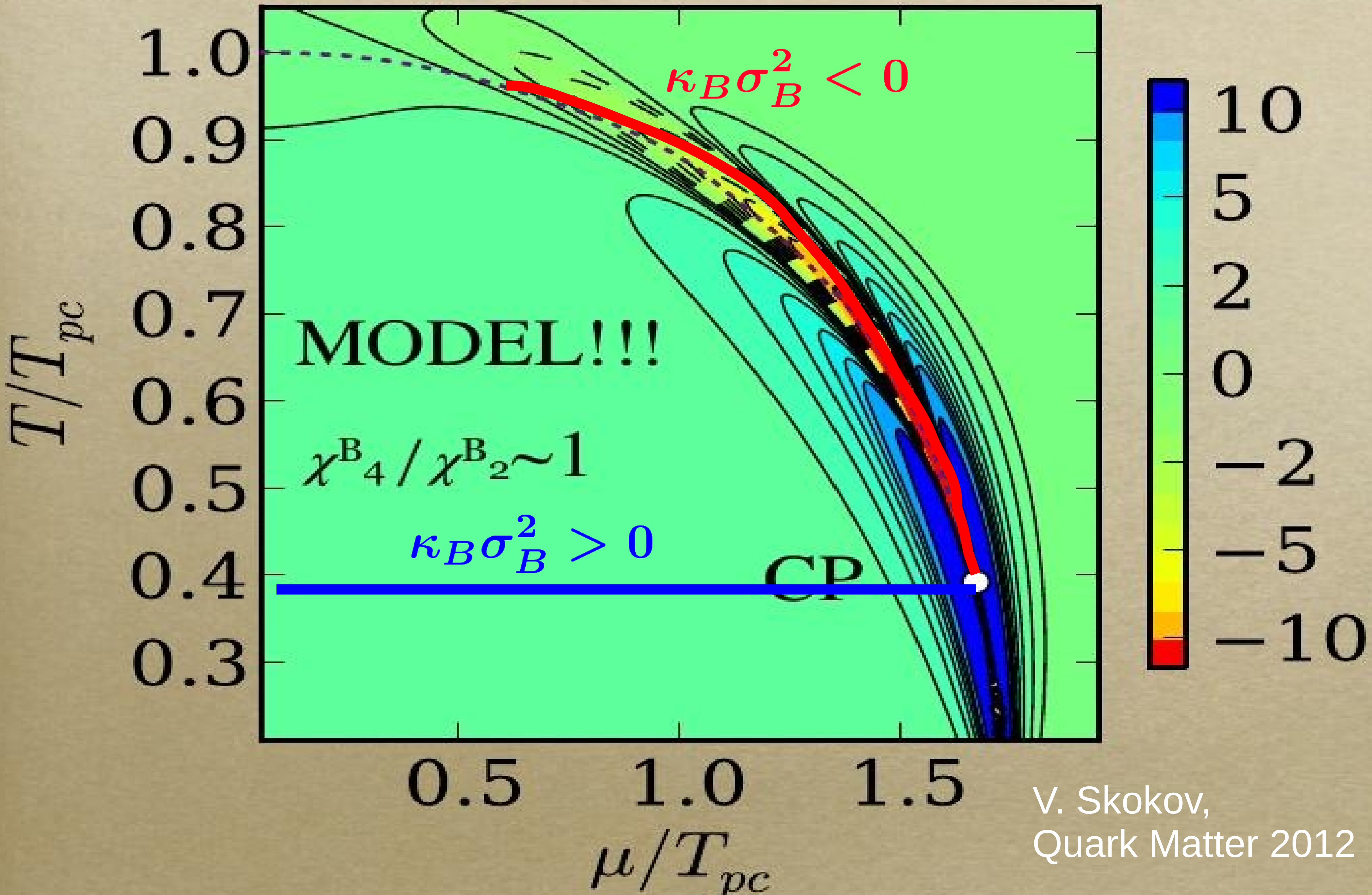
Conclusions

- approximate agreement with HRG model calculations at freeze-out and sensitivity to O(4) criticality in the crossover region are not inconsistent with each other

- 6th order cumulants are sensitive to O(4) scaling but will pick up only a small singular contribution below Tc. This favors estimates for the location of a critical endpoint at large μ_B/T

- a dip in the kurtosis*variance is likely to show up on the freeze-out line in the vicinity of a critical endpoint

Chiral model and negative $\chi^B{}_4/\chi^B{}_2$:



V. Skokov,
Quark Matter 2012