Numerical studies of JIMWLK (and glasma) evolution

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Yukawa Institute, Kyoto, December 2013

Outline

- CGC, Glasma, JIMWLK evolution
- JIMWLK equation in Langevin form
- Running coupling in JIMWLK T.L., H. Mäntysaari EPJC 2013
- JIMWLK as initial condition for CYM T.L., PLB 2011
- Wilson loop in glasma with JIMWLK (or MV) initial conditions
  Dumitru, T.L., Nara, work in progress: more in Adrian's talk

JIMWLK ["gym-walk"] Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

Gluon saturation, Glass and Glasma

Small x: the hadron/nucleus wavefunction is characterized by **saturation scale** $Q_s \gg \Lambda_{QCD}$.

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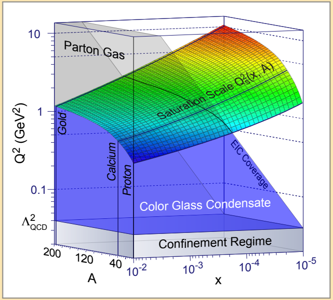
$$\Downarrow$$

p $\sim Q_s$: strong fields $A_\mu \sim 1/g$

- occupation numbers $\sim 1/\alpha_s$
- classical field approximation.
- small α_s, but nonperturbative



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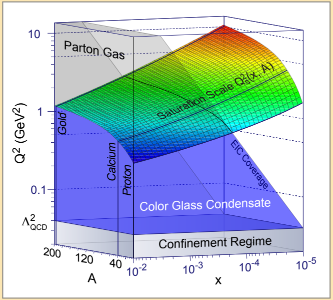

CGC: Effective theory for wavefunction of nucleus

- Large x = color charge ρ, **probability** distribution $W_y[\rho]$
- Small x = classical gluon field A_μ + quantum flucts.

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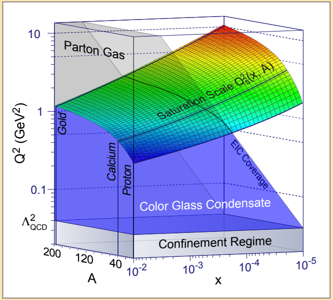

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Glasma field configuration of two colliding sheets of CGC.

Wilson line

Classical color field described as Wilson line

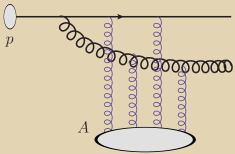
$$U(\mathbf{x}) = P \exp\left\{ ig \int \mathrm{d}x^- A^+_{\mathrm{cov}}(\mathbf{x}, x^-) \right\} \quad \in \quad \mathrm{SU}(3)$$

Relation to color charge

$$\nabla^2 A^+_{\mathrm{cov}}(\mathbf{x}, x^-) = -g\rho(\mathbf{x}, x^-)$$

Example of usage: forward pA

- Quark from p (large x pdf) , radiate gluon(s)
- Eikonal propagation $\Longrightarrow$ Wilson lines $U(\mathbf{x})$



Need target expectation values of operators:

$$\mathrm{Tr}\, U(\mathbf{x})U^\dagger(\mathbf{y}) \quad \mathrm{Tr}\, U(\mathbf{x})U^\dagger(\mathbf{y})U(\mathbf{u})U^\dagger(\mathbf{v}) \quad \ldots$$

See Bowen's talk

JIMWLK evolution

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$$U(\mathbf{x}) = P\exp\left\{ig \int \mathrm{d}x^- A^+(\mathbf{x}, x^-)\right\} \in \mathrm{SU}(3)$$

- Energy dependent **probability** distribution $W_y[U]$ $(y \sim \ln\sqrt{s})$
- Energy/rapidity dependence of $W_y[U]$ given by JIMWLK renormalization group equation

$$\partial_y W_y[U(\mathbf{x})] = \mathcal{H} W_y[U(\mathbf{x})]$$

JIMWLK Hamiltonian: (fixed coupling)

$$\mathcal{H} \equiv \frac{1}{2}\alpha_s \int_{\mathbf{xyz}} \frac{\delta}{\delta A_c^+(\mathbf{y})} \mathbf{e}^{ba}(\mathbf{x},\mathbf{z}) \cdot \mathbf{e}^{ca}(\mathbf{y},\mathbf{z}) \frac{\delta}{\delta A_b^+(\mathbf{x})},$$

$$\mathbf{e}^{ba}(\mathbf{x},\mathbf{z}) = \frac{1}{\sqrt{4\pi^3}} \frac{\mathbf{x}-\mathbf{z}}{(\mathbf{x}-\mathbf{z})^2} \left(1 - U^\dagger(\mathbf{x})U(\mathbf{z})\right)^{ba}$$

Fokker-Planck and Langevin

Textbook example: two descriptions of Brownian motion

- 1-d diffusion eq. ($\supset$ F.-P. eq.)

$$\partial_t P(x,t) = D\partial_x^2 P(x,t)$$

- $P(x,t)$=probability for particle to be at location x at time t.
- For $x=0$ at $t=0$ solution:

$$P(x,t) = \frac{1}{\sqrt{4\pi D t}} \exp\left\{-\frac{x^2}{4Dt}\right\}$$

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- Langevin equation:

$$\begin{aligned} x(t) &= \sqrt{2D}\eta(t) \\ \langle \eta(t)\eta(t')\rangle &= \delta(t-t') \end{aligned}$$

- $$\begin{aligned} \langle x(t)\rangle &= 0 \\ \langle x^2(t)\rangle &= 2Dt \\ \langle x(t)x(t')\rangle &= 2D\min(t,t') \end{aligned}$$

  $\Longrightarrow$ same as F.-P.

1d Brownian motion to JIMWLK

- Replace $x \Longrightarrow U(\mathbf{x})$ and $t \Longrightarrow y$.
- Constant $D \Longrightarrow$ nonlinearity (U-dependence) in kernel
- $({N_c}^2-1)N_\perp^2$-dimensional nonlinear diffusion equation.
  ($N_\perp^2$ = number of lattice points in transverse plane.)

Langevin formulation

Fokker-Planck $\Longrightarrow$ Langevin in JIMWLK Blaizot, Iancu, Weigert 2002

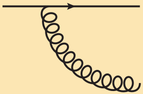
Original Langevin form: only right derivative ($\xi_{\mathbf{z}}^{b,i}$ is noise)

$$U_{\mathbf{x}}(y+\mathrm{d}y) = U_{\mathbf{x}}(y)\exp\left\{it^a\int_{\mathbf{z}}\varepsilon_{\mathbf{x},\mathbf{z}}^{ab,i}\xi_{\mathbf{z}}^{b,i}\sqrt{\mathrm{d}y}+\sigma_{\mathbf{x}}^a\,\mathrm{d}y\right\}.$$

Simpler, equivalent (for $\mathrm{d}y\to 0$) form T.L., H.M.

$$U_{\mathbf{x}}(y+\mathrm{d}y) = \exp\left\{-i\frac{\sqrt{\alpha_{\mathrm{s}}\,\mathrm{d}y}}{\pi}\int_{\mathbf{z}}\mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot(U_{\mathbf{z}}\xi_{\mathbf{z}}U_{\mathbf{z}}^\dagger)\right\}$$
$$\times\, U_{\mathbf{x}}(y)\exp\left\{i\frac{\sqrt{\alpha_{\mathrm{s}}\,\mathrm{d}y}}{\pi}\int_{\mathbf{z}}\mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot\xi_{\mathbf{z}}\right\},$$

$$K_{\mathbf{x}-\mathbf{z}}^i = \frac{(\mathbf{x}-\mathbf{z})^i}{(\mathbf{x}-\mathbf{z})^2}$$



$i = x, y$

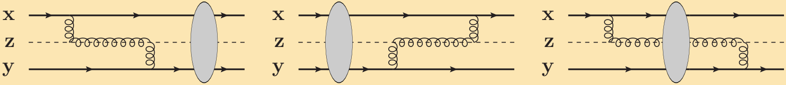
Fixed α_{s} noise: $\langle\xi_{\mathbf{x}}(y_m)_i^a\xi_{\mathbf{y}}(y_n)_j^b\rangle = \alpha_{\mathrm{s}}\delta^{ab}\delta^{ij}\delta_{\mathbf{xy}}^{(2)}\delta_{mn}, \quad \xi = \xi^a t^a$

Multiply from left **and** right $\Longrightarrow$ remove deterministic term

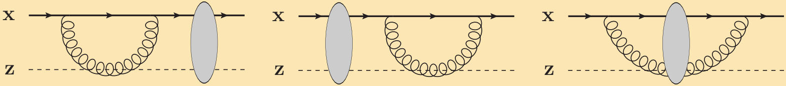
Interpreting JIMWLK: derive BK

$$U_{\mathbf{x}}(y+\mathrm{d}y) = e^{-i\frac{\sqrt{\alpha_s\,\mathrm{d}y}}{\pi}\int_{\mathbf{z}}\mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot(U_{\mathbf{z}}\boldsymbol{\xi}_{\mathbf{z}}U_{\mathbf{z}}^\dagger)}\,U_{\mathbf{x}}\,e^{i\frac{\sqrt{\alpha_s\,\mathrm{d}y}}{\pi}\int_{\mathbf{z}}\mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot\boldsymbol{\xi}_{\mathbf{z}}},$$

- At $\mathrm{d}y \to 0$ develop to $\mathcal{O}(\xi^2)$ and take expectation values.
- BK Balitsky-Kovchegov is equation for **dipole** $\hat{D}_{\mathbf{x},\mathbf{y}} = \mathrm{Tr}\,U^\dagger(\mathbf{x})U(\mathbf{y})/N_c$
- Contract ξ's from timestep of $U^\dagger(\mathbf{x})$ with one from $U(\mathbf{y})$: real terms



- Contract two ξ's from timestep of $U^\dagger(\mathbf{x})$ or $U(\mathbf{y})$: virtual terms



- Result

$$\partial_y \hat{D}_{\mathbf{x},\mathbf{y}}(y) = \frac{\alpha_s N_c}{2\pi^2}\int_{\mathbf{z}}\left(\mathbf{K}_{\mathbf{x}-\mathbf{z}}^2 + \mathbf{K}_{\mathbf{y}-\mathbf{z}}^2 - 2\mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot\mathbf{K}_{\mathbf{y}-\mathbf{z}}\right)\left[\hat{D}_{\mathbf{x},\mathbf{z}}\hat{D}_{\mathbf{z},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}}\right].$$

Scale of running α_s in JIMWLK ?

BK for $\hat{D}_{\mathbf{x},\mathbf{y}}(y)$ describes dipole splitting $\mathbf{x}-\mathbf{y} \quad \longrightarrow \quad \mathbf{x}-\mathbf{z}\ ;\ \mathbf{z}-\mathbf{y}$

- α_s given by parent $\mathbf{x}-\mathbf{y}$: easy in BK, but funny in JIMWLK: Langevin is only for one Wilson line
- Daughter (scale in $\mathbf{K}$): easy to implement as $\sqrt{\alpha_s}$, but why?

$$\sqrt{\alpha_s}\mathbf{K}_{\mathbf{x}-\mathbf{z}} \to \sqrt{\alpha_s(\mathbf{x}-\mathbf{z})}\mathbf{K}_{\mathbf{x}-\mathbf{z}}$$

- Used in BK: combinations of these two Balitsky, Kovchegov, Weigert
- Suggestion T.L., H.Mäntysaari 2012 : α_s at k_T of radiated gluon.
- Easily implemented by new momentum space noise correlator

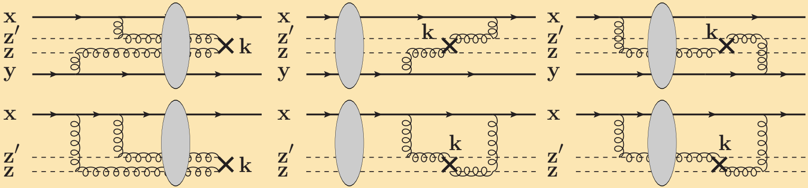
$$\langle \xi_{\mathbf{x}}(m)_i^a \xi_{\mathbf{y}}(n)_j^b \rangle \sim \alpha_s \delta^{(2)}_{\mathbf{xy}} = \alpha_s \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}$$

$$\Longrightarrow \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \alpha_s(\mathbf{k})$$

Reinterpreting JIMWLK

$$U_{\mathbf{x}}(y + \mathrm{d}y) = \exp\left\{ -i \frac{\sqrt{\mathrm{d}y}}{\pi} \int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}} \cdot (U_{\mathbf{z}} \xi_{\mathbf{z}} U_{\mathbf{z}}^{\dagger}) \right\}$$
$$\times\, U_{\mathbf{x}}(y)\, \exp\left\{ i \frac{\sqrt{\mathrm{d}y}}{\pi} \int_{\mathbf{z}'} \mathbf{K}_{\mathbf{x}-\mathbf{z}'} \cdot \xi_{\mathbf{z}'} \right\},$$

$$\langle \xi_{\mathbf{x}}(m)_i^a \xi_{\mathbf{y}}(n)_j^b \rangle \sim \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \alpha_{\mathrm{s}}(\mathbf{k}) \equiv \widetilde{\alpha}_{\mathbf{x}-\mathbf{y}}$$



- Breaks time-reversal-symmetry: choose scale as momentum of gluon either before or after the target
- Two gluon coordinates instead of one

Recovering BK

- Equation for dipole now involves higher point functions:

$$\partial_y \hat{D} = \frac{N_c}{2\pi^2} \int_{\mathbf{u},\mathbf{v}} \tilde{\alpha}_{\mathbf{u}-\mathbf{v}} \Big(\mathbf{K}_{\mathbf{x}-\mathbf{u}} \cdot \mathbf{K}_{\mathbf{x}-\mathbf{v}} + \mathbf{K}_{\mathbf{y}-\mathbf{u}} \cdot \mathbf{K}_{\mathbf{y}-\mathbf{v}} - 2\mathbf{K}_{\mathbf{x}-\mathbf{u}} \cdot \mathbf{K}_{\mathbf{y}-\mathbf{v}} \Big) \times \frac{1}{2} \Big[\hat{D}_{\mathbf{x},\mathbf{u}} \hat{D}_{\mathbf{u},\mathbf{y}} + \hat{D}_{\mathbf{x},\mathbf{v}} \hat{D}_{\mathbf{v},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}} - \hat{D}_{\mathbf{v},\mathbf{u}} \hat{Q}_{\mathbf{x},\mathbf{v},\mathbf{u},\mathbf{y}} \Big],$$

- But recall that α_s is a slowly varying function of the scale:

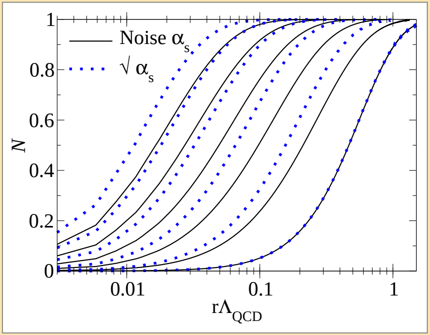
$$\tilde{\alpha}_{\mathbf{x}-\mathbf{y}} \equiv \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \alpha_s(\mathbf{k}) \sim \alpha_s \delta^2(\mathbf{x}-\mathbf{y})$$

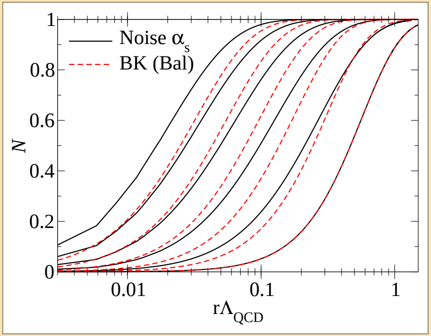
$\Longrightarrow$ $\mathbf{u} \approx \mathbf{v}$ and structure simplifies to BK:

$$\frac{1}{2} \Big[\hat{D}_{\mathbf{x},\mathbf{u}} \hat{D}_{\mathbf{u},\mathbf{y}} + \hat{D}_{\mathbf{x},\mathbf{v}} \hat{D}_{\mathbf{v},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}} - \hat{D}_{\mathbf{v},\mathbf{u}} \hat{Q}_{\mathbf{x},\mathbf{v},\mathbf{u},\mathbf{y}} \Big] \approx \hat{D}_{\mathbf{x},\mathbf{u}} \hat{D}_{\mathbf{u},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}}$$

- Parametrically dominant length scale in coupling is “smallest dipole”, just like in Balitsky prescription for BK.

Comparison BK/JIMWLK





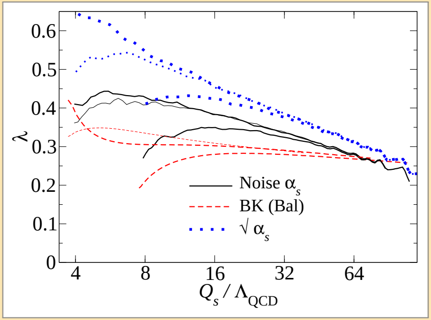
Evolution with our prescription is slower than with $\sqrt{\alpha_s}$.
This is good, data favors slower evolution

But this is still faster than with Balitsky prescription in BK
(Although parametricallly dominant scales are the same.)

Note: rcBK fits to HERA data need to take $\Lambda_{QCD} \approx 50$MeV to make evolution slow enough.

Evolution speed

$$\lambda \equiv \frac{\mathrm{d} \ln Q_s^2}{\mathrm{d}y}$$

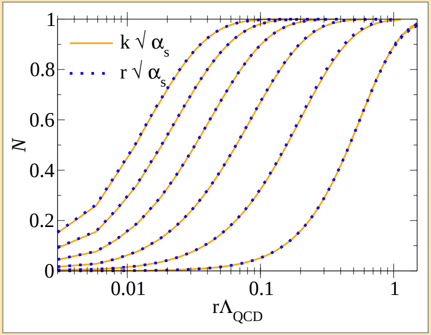


At very IR scales also dependence on how the Landau pole is regulated (different line shapes)

Side note: scale in coordinate vs momentum space

If running coupling depends only on scale in **K** ($\sqrt{\alpha_s}$-prescription), can use either coordinate or momentum space:

$$\sqrt{\alpha_s(\mathbf{x})}\frac{\mathbf{x}}{\mathbf{x}^2} \quad \text{vs.} \quad \sqrt{\alpha_s(\mathbf{k})}\frac{\mathbf{k}}{\mathbf{k}^2}$$



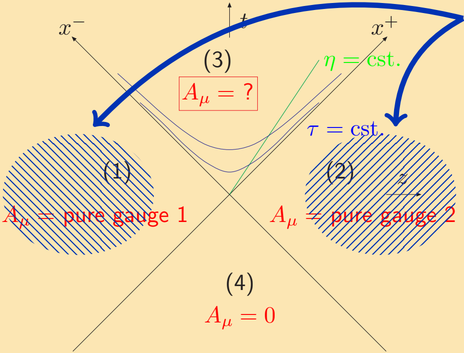
Numerically verified identification Kovchegov, Weigert (for this kernel!)

$$\ln\frac{\mathbf{k}^2}{\Lambda_{QCD}^2} \sim \ln\frac{4e^{-2\gamma_E}}{r^2\Lambda_{QCD}^2}$$

Gluon fields in AA collision

2 pure gauges

Classical **Y**ang-**M**ills



$$A^i_{(1,2)} = \frac{i}{g} U_{(1,2)}(\mathbf{x})\partial_i U^\dagger_{(1,2)}(\mathbf{x})$$

$$U_{(1,2)}(\mathbf{x}) = P e^{ig \int dx^- \frac{\rho(\mathbf{x},x^-)}{\nabla^2}}$$

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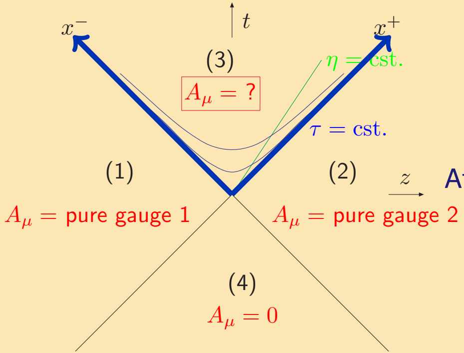

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At $\boldsymbol{\tau = 0}$:

$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

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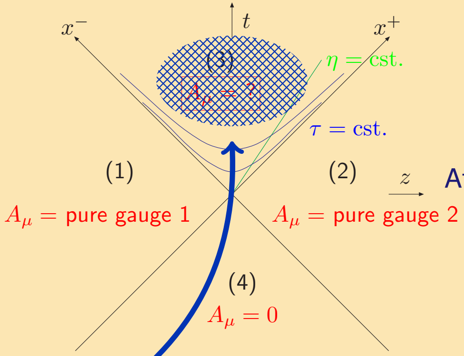

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$\boldsymbol{\tau > 0}$ Solve numerically Classical Yang-Mills **CYM** equations.
This is the **glasma** field $\Longrightarrow$ Then average over ρ.

Gluons with $p \sim Q_s$ — strings of size $R \sim 1/Q_s$

Gluon spectrum in the glasma

T.L., *Phys.Lett.* **B703** (2011) 325 ; 1st direct use of JIMWLK in CYM calculation

Q_s is only dominant scale

Parametrically gluon spectrum $$\frac{\mathrm{d}N_g}{\mathrm{d}y\,\mathrm{d}^2\mathbf{x}\,\mathrm{d}^2\mathbf{p}} = \frac{1}{\alpha_s} f\left(\frac{p}{Q_s}\right)$$

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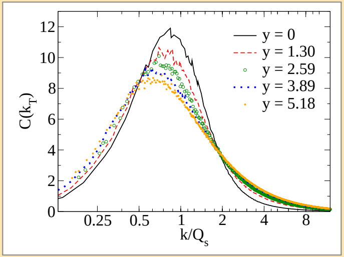

ugd for 1 nucleus

$$C(\mathbf{k}) = \frac{k^2}{N_c} \mathrm{Tr}\left\langle U(\mathbf{k}) U^\dagger(\mathbf{k}) \right\rangle$$

becomes **harder** with evolution.

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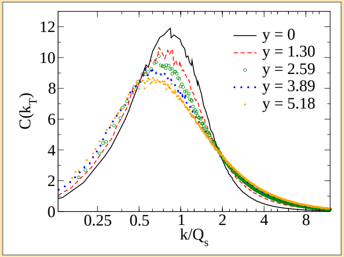

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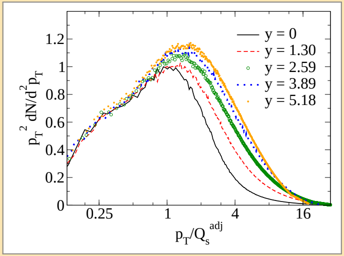

Produced gluon spectrum: harder at higher $\sqrt{s}$

(Here: midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

Gluon multiplicity and mean p_T

Q_s is only dominant scale

Parametrically $$\frac{\mathrm{d}N_g}{\mathrm{d}y\,\mathrm{d}^2\mathbf{x}} = c_N \frac{C_F}{2\pi^2\alpha_s} Q_s^2 \qquad \langle p \rangle \sim Q_s$$

Note: in full CYM total gluon multiplicity is IR finite, no cutoff.

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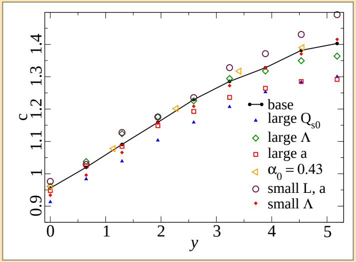

Scaled multiplicity grows with $\sqrt{s}$

(Midrapidity, $y \equiv \ln\sqrt{s/s_0}$)

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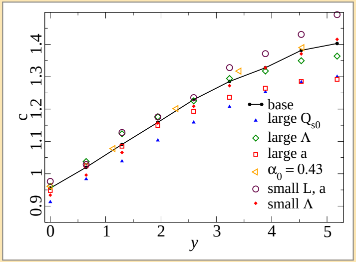
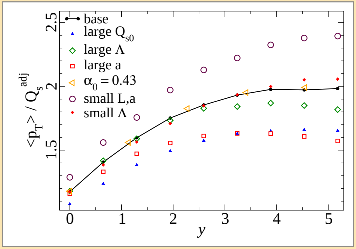




Scaled multiplicity grows with $\sqrt{s}$

(Midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

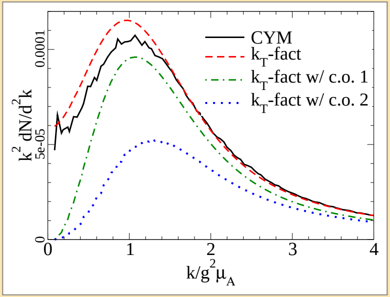
Harder gluon spectrum $\Longrightarrow$ higher $\langle p\rangle/Q_s$ as scaling regime sets in.

(Large lattice cutoff effects.)

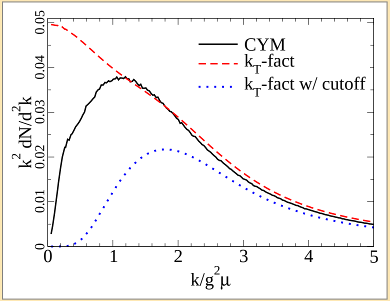
Side note: CYM vs. k_T-factorization

Blaizot, T.L., Mehtar-Tani 2010

$$\frac{dN}{d^2\mathbf{p}\,dy} = \frac{\#}{\alpha_s}\frac{1}{\mathbf{p}^2}\int_{\mathbf{k}}\left[\theta(p-k)\right]\phi_y(\mathbf{k})\phi_y(\mathbf{p}-\mathbf{k})$$



pA



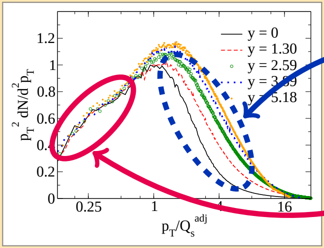
AA

k_T-factorization works only for $p_T \gtrsim Q_s$

- OK for high-p_T spectra
- Not for total gluon multiplicity

(Suggested interpretation Levin, 2010 Sudakov suppression factor.)

Universality in the IR spectrum?



- Scaled gluon spectrum in the UV depends on anomalous dimension $\Longrightarrow$ different for MV, JIMWLK
- IR seems to scale

Gauge inv. probe for $p \lesssim Q_s$?

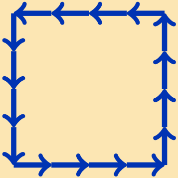
Spatial Wilson loop

$$W(A) = \frac{1}{N_c} \operatorname{Tr} \mathbb{P} \exp \left\{ ig \oint_A \mathrm{d}\mathbf{x} \cdot \mathbf{A} \right\}$$

2d lattice: transverse links:

$$\uparrow = U_i(\mathbf{x}) = \exp\{igaA_i\}$$

$$W(A) = \frac{1}{N_c} \operatorname{Tr}$$

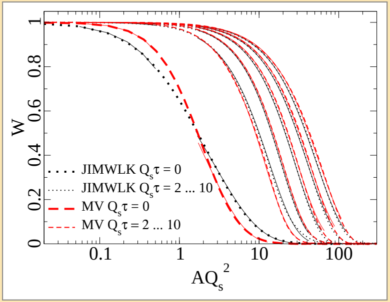


Measure Wilson loops

Calculation is simple:

- Construct initial glasma fields at $\tau = 0$ using e.g.
  - MV model
  - rcJIMWLK
  - fcJIMWLK

  (Try to have same $Q_s a$ to minimize lattice effects)
- Evolve forward in τ
- Measure $W(A)$



Behavior in both UV ($AQ_s^2 \lesssim 1$) and IR ($AQ_s^2 \gtrsim 1$) parametrized as

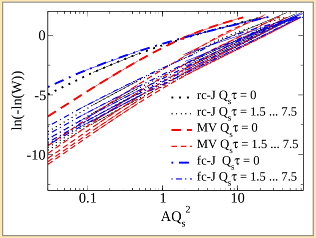
$$W = \exp\left\{-C(AQ_s)^\gamma\right\}$$

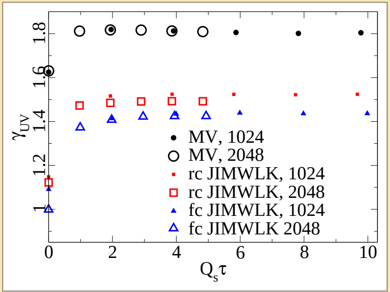
(Fits restricted to $e^{-2.5} < AQ_s^2 < e^{-0.5}$ (UV) and $e^{0.5} < AQ_s^2 < e^5$ (IR))

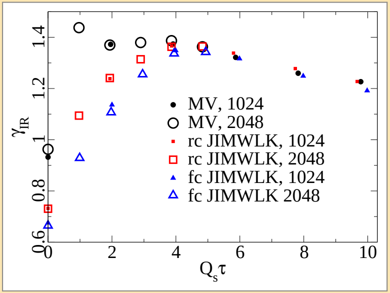
Wilson loop scaling exponents

In practice fit logarithm

$$W = \exp\{-C(AQ_s)^\gamma\}$$
$$\iff \ln(-\ln W) = \gamma \ln(AQ_s) + \ln C$$







UV: remembers initial condition

IR: **nontrivial universal behavior**

Magnetic field correlator

Wilson loop measures magnetic flux:

$$W(A) = \frac{1}{N_\mathrm{c}} \operatorname{Tr} \mathbb{P} \exp\left\{ ig \oint_A \mathrm{d}\mathbf{x} \cdot \mathbf{A} \right\} = \frac{1}{N_\mathrm{c}} \operatorname{Tr} \exp\left\{ ig \int \mathrm{d}^2\mathbf{x} B_z(\mathbf{x}) \right\}$$

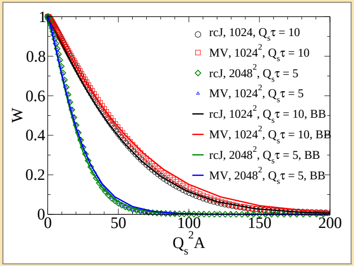
If magnetic field consists of uncorrelated Gaussian domains:

$$\langle W(A) \rangle = \exp\left\{ -\frac{1}{2} \frac{1}{N_\mathrm{c}} \operatorname{Tr} \left\langle \left[\int \mathrm{d}^2\mathbf{x} g B_z(\mathbf{x}) \right]^2 \right\rangle \right\}$$

(Connect B:s at different locations with gauge link for gauge invariance)

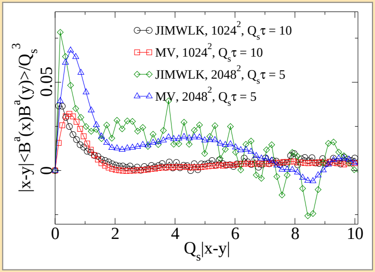
$\Longrightarrow$ $W(A)$ related to $\langle B(\mathbf{x}) B(\mathbf{y}) \rangle$

Check: compare $W(A)$ with reconstruction from BB-correlator



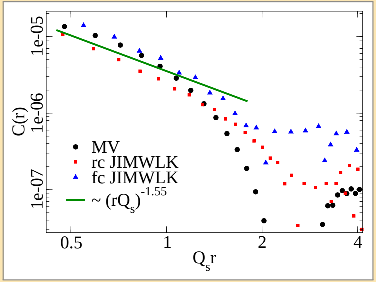
Magnetic field correlator

However: no obvious scaling seen in *BB*-correlator



Same on log plot

$$C(|\mathbf{x}-\mathbf{y}|) \equiv \text{Tr}\left\langle [B(\mathbf{x})B(\mathbf{y})]_{\text{gauge link}}\right\rangle$$



Straight line: $\sim (rQ_s)^{-1.55}$.

(For $C(r) \sim (rQ_s)^{-\alpha}$ one would get $\gamma = 2 - \alpha/2 \iff \alpha = 4 - 2\gamma$;

from $W(A)$ measured $\gamma = 1.22$)

Conclusions

- JIMWLK can actually be applied; for some observables BK is not enough
- Running coupling, going towards NLO . . . necessary for realistic phenomenology
- Provides initial state for AA collision in CYM: glasma
- Universal behavior in the for $p_T \ll Q_s$ seen both in gluon spectrum and spatial Wilson loop.