Probing Saturation Physics in pA Collisions

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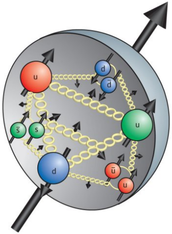
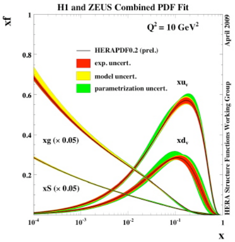
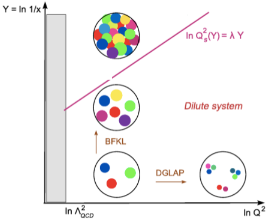
Deep into low-x region



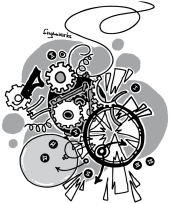


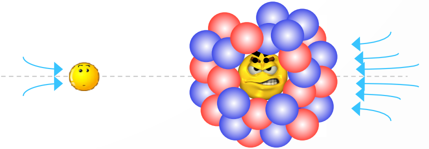


- Partons in the low-x region is dominated by gluons. See HERA data.
- BFKL equation ⇒ Resummation of the $\alpha_s \ln \frac{1}{x}$.
- When too many gluons squeezed in a confined hadron, gluons start to overlap and recombine ⇒ Non-linear dynamics ⇒ BK (JIMWLK) equation
- Use $Q_s(x)$ to separate the saturated dense regime from the dilute regime.
- Core ingredients: Multiple interactions + Small-x (high energy) evolution

Saturation physics (Color Glass Condensate)

Saturation physics describes high density parton distributions at high energy limit.

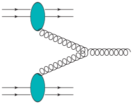




- Saturation is an inevitable consequence of QCD dynamics at high energy.
- R. Feynman used to say: Scattering protons on protons is like banging two fine Swiss watches to find out how they are built.
- Using *AA* collisions to search for saturation is too hard due to factorization issues: Finding a needle in a haystack
- The search for parton saturation is much easier in dilute-dense scatterings.
  - 1. single hadron (*pA* and *eA*);
  - 2. dijet (dihadron) correlation (*pA* and *eA*).

k_t factorization vs Dilute-Dense factorizations

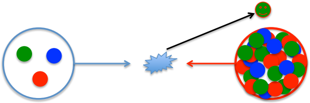
k_t factorization for single inclusive gluon productions in hadron-hadron collision:



$$\frac{d\sigma}{d^2p_T dy} = \frac{2\alpha_s}{C_F\, p_T^2} \times$$
$$\times \int d^2k_{A,T}\, f_A(x_A, k_{A,T})\, f_B(x_B, p_T - k_{A,T}).$$

- Factorization and NLO correction? Only proved for DY and Higgs !
- For dijet processes in pp, AA collisions, no k_t factorization[Collins, Qiu, 08],[Rogers, Mulders; 10].

Dilute-Dense factorizations



projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim 1$ valence

target: $x_2 \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$ gluon

- Protons and virtual photons are dilute probes of the dense target hadrons.
- For dijet productions in forward *pA* collisions, effective k_t factorization:

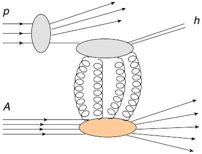
$$\boxed{\frac{d\sigma^{pA\to ggX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p g(x_p, \mu) x_A g(x_A, q_\perp) \frac{1}{\pi} \frac{d\hat{\sigma}}{d\hat{t}}.}$$

Forward hadron production in *pA* collisions

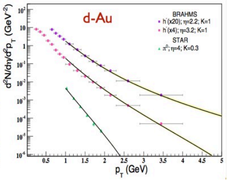
[Dumitru, Jalilian-Marian, 02] Inclusive forward hadron production in *pA* collisions

$$\frac{d\sigma_{\text{LO}}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[\sum_f x_p q_f(x_p,\mu)\mathcal{F}(k_\perp)D_{h/q}(z,\mu) + x_p g(x_p,\mu)\tilde{\mathcal{F}}(k_\perp)D_{h/g}(z,\mu)\right].$$

$$p + A \to h(y, p_\perp) + X$$



Albaete-Marquet, 2010



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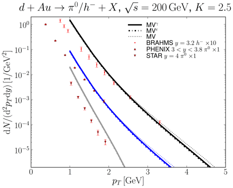
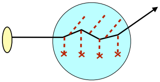


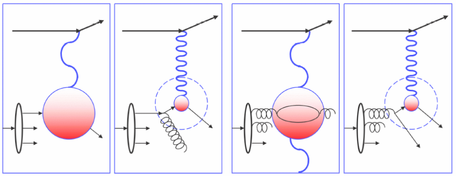
- Caveats: arbitrary choice of the renormalization scale μ and K factor.
- NLO correction? [Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Chirilli, Xiao and Yuan, 12]

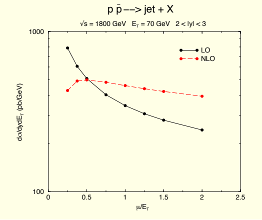


Why do we need NLO calculations?

Large x: valence quarks

Small x: Gluons, sea quarks





- Due to quantum evolution, PDF and FF changes with scale. This introduces large theoretical uncertainties in $xf(x)$ and $D(z)$. Choice of the scale at LO requires information at NLO.
- LO cross section is always a monotonic function of μ, thus it is just order of magnitude estimate.
- NLO calculation significantly reduces the scale dependence. More reliable.
- $K = \frac{\sigma_{\mathrm{LO}}+\sigma_{\mathrm{NLO}}}{\sigma_{\mathrm{LO}}}$ is not a good approximation.
- NLO is vital in establishing the QCD factorization in saturation physics.

NLO Calculation and Factorization

- Factorization is about separation of short distant physics (perturbatively calculable hard factor) from large distant physics (Non perturbative).

$$\sigma \sim xf(x) \otimes \mathcal{H} \otimes D_h(z) \otimes \mathcal{F}(k_\perp)$$

- NLO (1-loop) calculation always contains various kinds of divergences.
  - Some divergences can be absorbed into the corresponding evolution equations.
  - The rest of divergences should be cancelled.
- Hard factor

$$\mathcal{H} = \mathcal{H}^{(0)}_{\text{LO}} + \frac{\alpha_s}{2\pi}\mathcal{H}^{(1)}_{\text{NLO}} + \cdots$$

should always be finite and free of divergence of any kind.

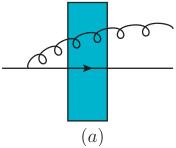
- NLO vs NLL   Naive α_s expansion sometimes is not sufficient!

	LO	NLO	NNLO	$\cdots$
LL	1	$\alpha_s L$	$(\alpha_s L)^2$	$\cdots$
NLL		α_s	$\alpha_s (\alpha_s L)$	$\cdots$
$\cdots$			$\cdots$	$\cdots$

- Evolution → Resummation of large logs.
  LO evolution resums LL; NLO ⇒ NLL.

Approximation and Relavent Diagrams

- Large N_c approximation.
- In the high energy limit $s \to \infty$, one can assume that medium size L is much less than the typical coherence time.
  Three possible real diagrams:



- In the shock wave approximation, only (a) and (b) are taken into account.
- As to jet quenching related calculation, if one assumes $L \to \infty$, (c) diagram must be included.
- Also Virtual diagrams must be included.
- By integrating over the gluon phase space, we encounter several types of divergences.

Factorization for single inclusive hadron productions

Systematic factorization for the $p + A \to H + X$ process
[G. Chirilli, BX and F. Yuan, Phys. Rev. Lett. 108, 122301 (2012)]

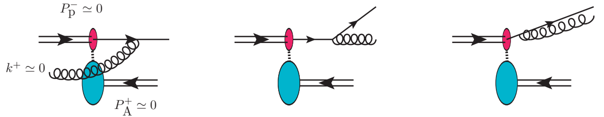
$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\xi x f_a(x,\mu) D_{h/c}(z,\mu) \int [dx_\perp] S^Y_{a,c}([x_\perp]) \mathcal{H}_{a\to c}(\alpha_s,\xi,[x_\perp]\mu)$$

Collinear divergence: pdfs

Collinear divergence: fragmentation functs

Rapidity divergence: BK evolution

Finite hard factor



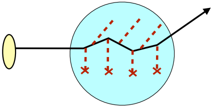
Rapidity Divergence   Collinear Divergence (P)   Collinear Divergence (F)

- Subtracting off divergence ⇒ the NLO correction.
- By invoking the NLO evolutions, we promote the NLO calculation up to NLL.
- All 4 different channels are included. $q \to q$, $q \to g$, $g \to q(\bar{q})$ and $g \to g$.

Numerical implementation of the NLO result

Single inclusive hadron production up to NLO

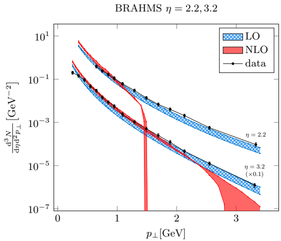
$$d\sigma = \int xf_a(x) \otimes D_a(z) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} + \frac{\alpha_s}{2\pi} \int xf_a(x) \otimes D_b(z) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)}.$$

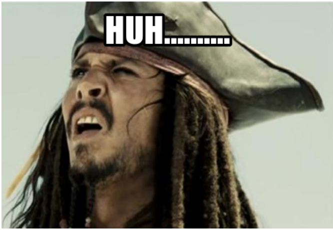


Consistent implementation should include all the NLO α_s corrections.

- NLO parton distributions. (MSTW or CTEQ)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors. Partially by [Albacete, Dumitru, Fujii, Nara, 12]
- Use the one-loop approximation for the running coupling
- rcBK evolution equation for the dipole gluon distribution [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. Full NLO BK evolution not available.
- Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]

Surprise

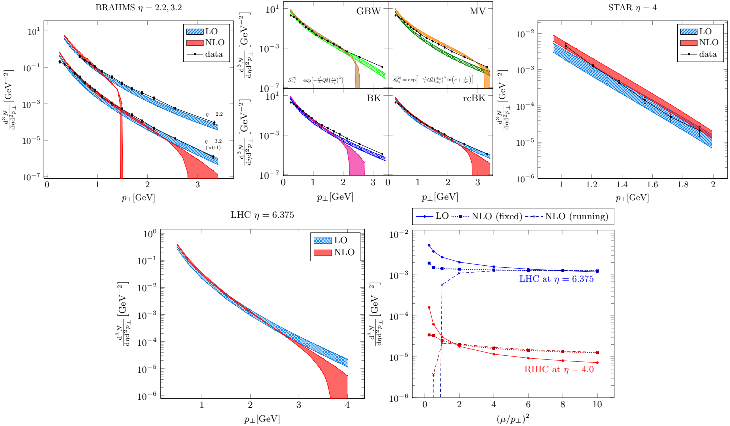




- The abrupt drop of the NLO correction when $p_\perp > Q_s$ was really a surprise!
- What is going wrong?
  - Saturation formalism? Dilute-dense factorization? Not necessarily positive definite! Does this indicate that we need NNLO correction? $\cdots$
  - Some hidden large correction in $\frac{\alpha_s}{2\pi}\mathcal{H}^{(1)}_{\mathrm{NLO}}$?

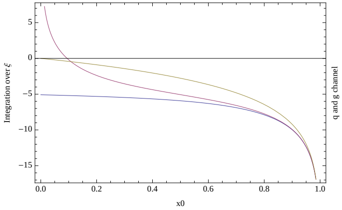
Numerical implementation of the NLO result

[Stasto, Xiao, Zaslavsky, 13, accepted for publication in PRL]



- Agree with data for $p_\perp < Q_s(y)$, and reduced scale dependence, no K factor.
- For more forward rapidity, the agreement gets better and better.
- Additional $+$-function resummation ?

Looking into the high $p_\perp$ region



What is going on exactly? Perform large $p_\perp$ expansion

- Both the LO and NLO calculation manifest themselves in single hard scattering. $\sigma \sim \frac{1}{p_\perp^4}$.
- The coefficient of the NLO correction for $q \to q$ and $g \to g$ channels are

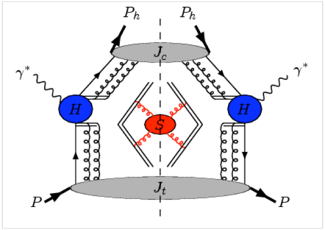
$$\mathcal{I}_q(x_0) = \int_{x_0}^{1} d\xi \frac{(1+\xi^2)^2}{(1-\xi)_+} \to 4\ln(1-x_0)$$

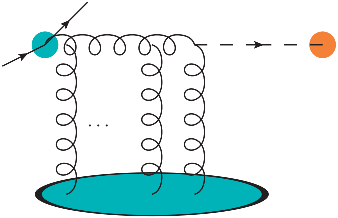
$$\mathcal{I}_g(x_0) = 2\int_{x_0}^{1} d\xi [1+\xi^2+(1-\xi)^2]\left[\frac{\xi}{(1-\xi)_+}+\frac{(1-\xi)}{\xi}+\xi(1-\xi)\right].$$

- Threshold resummation, since x_0 is related to the x fraction of produced hadron w.r.t. the proton projectile. Still preliminary!

k_t Factorization in Small-x VS TMD Factorization

[Collins-Soper-Sterman,85], [Ji-Ma-Yuan, 04]





- (Mueller's dipole model) k_t factorization is widely used in small-x physics. TMD factorization is widely used in spin physics, etc (large x).
- The same gauge invariant operator definition! What about evolution ? [F. Dominguez, BX and F. Yuan, Phys.Rev.Lett. 106, 022301 (2011).]
- k_t factorization $\simeq$ TMD factorization ? Yes, but more complicated.
- TMD evolution (CSS), however, UGD evolution (small-x).

Lesson from the one-loop calculation for Higgs productions

[A. Mueller, BX, F. Yuan, Phys. Rev. Lett. 110, 082301 (2013)]

$$\frac{d\sigma^{(\text{resum})}}{dyd^2k_\perp}|_{k_\perp\ll M} = \sigma_0\int\frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2}e^{ik_\perp\cdot R_\perp}e^{-\mathcal{S}_{\text{sud}}(M^2,R^2_\perp)}S^{WW}_{Y=\ln 1/x_g}(x_\perp,x'_\perp)$$

$$\times x_pg_p(x_p,\mu^2=\frac{c_0^2}{R_\perp^2})\left[1+\frac{\alpha_s}{\pi}\frac{\pi^2}{2}N_c\right],$$

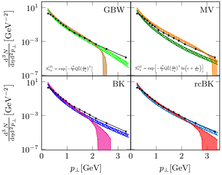
- Four independent (at LL level) renormalization equation. No divergences left.
- Unified description of the *CSS* and small-*x* evolution? TMD and UGD share the same operator definition.

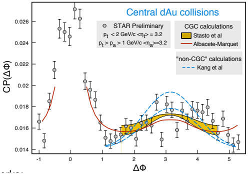
$$\text{Define}\quad \mathcal{S}^{WW}(R_\perp,M;x_g)=\mathcal{C}e^{-\mathcal{S}_{\text{sud}}(M^2,R^2_\perp)}S^{WW}_{Y=\ln 1/x_g}(x_\perp-x'_\perp)\quad\Rightarrow$$

$$\frac{d\sigma^{(\text{resum})}}{dyd^2k_\perp}|_{k_\perp\ll M}=\sigma_0\int\frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2}e^{ik_\perp\cdot R_\perp}x_pg_p(x_p,\mu^2=\frac{c_0^2}{R_\perp^2})\mathcal{S}^{WW}(R_\perp,M;x_g)$$

- Straight forward generalization for dijet processes.
- Competition between Sudakov and Saturation suppressions.

Conclusion





- Inclusive forward hadron productions in *pA* collisions in the small-x saturation formalism at one-loop order. (More interesting).
- Towards the test of saturation physics beyond LL (More precise).
- Dijet (dihadron) correlation in *pA* collisions. (More striking evidence !)
- One-loop calculation for hard processes in *pA* collisions, Sudakov factor. (More complete understanding of TMD or UGD).
- Gluon saturation could be the next interesting discovery at the LHC.
- Of course, this needs a lot of hard work and we are hiring postdocs!