Dissipative corrections to anisotropic flow

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Dissipative corrections to anisotropic flow

- Dissipative effects in heavy ion collisions
- Principle and ideas of the calculations
- Results

Christian Lang & N.B., arXiv:1312.????

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Dissipative effects
in heavy ion collisions

When going from ideal fluid dynamics to dissipative fluid dynamics, the corrections (viscosity, (heat conductivity), ...) are twofold:

- modification of the fluid four-velocity $\mathrm{u}(\mathrm{x})$
  - ☛ solution of $\partial_\mu T^{\mu\nu}(\mathrm{x}) = 0$ with

$$T^{\mu\nu}(\mathrm{x}) = \epsilon(\mathrm{x}) u^\mu(\mathrm{x}) u^\nu(\mathrm{x}) - \mathcal{P}(\mathrm{x}) \Delta^{\mu\nu}(\mathrm{x}) + \pi^{\mu\nu}(\mathrm{x})$$

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- at freeze-out
  - ☛ within the (naive?) Cooper–Frye prescription

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

f receives corrections $f = f_{\rm id.} + \delta f^{(1)} + \delta f^{(2)} + \cdots$ so that $T^{\mu\nu}$ remain continuous in the transition from a fluid to a collection of particles.

Dissipative effects
in heavy ion collisions

When going from ideal fluid dynamics to dissipative fluid dynamics, the corrections (viscosity, (heat conductivity), ...) are twofold:

- modification of the fluid four-velocity $u(x)$

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$$T^{\mu\nu}(x) = \epsilon(x) u^\mu(x) u^\nu(x) - \mathcal{P}(x) \Delta^{\mu\nu}(x) + \pi^{\mu\nu}(x)$$

The dissipative part $\pi^{\mu\nu}$ of the stress tensor involves the various transport coefficients (η, ζ, κ...).

Here, the temperature dependences of the coefficients over the whole history of the hydrodynamical evolution affect the particle spectra.

☛ (as yet) unknown functional dependences $\eta(T)$, $\zeta(T)$, $\kappa(T)$...

Dissipative effects
in heavy ion collisions

Dissipative correction at freeze-out: in the Cooper–Frye prescription

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

there come corrections $f = f_{\rm id.} + \delta f^{(1)} + \delta f^{(2)} + \cdots$ to the phase space occupation factor.

- The functional form of the corrections has been computed

  Teaney 2003 (shear); Dusling & Teaney, Denicol et al., Monnai & Hirano 2008- (bulk); Teaney & Yan 2013 (conformal 2nd order terms)

  mostly assuming freeze-out to a simple-component kinetic gas in the relaxation time approximation.

Dissipative effects

in heavy ion collisions

Dissipative correction at freeze-out: in the Cooper–Frye prescription

there … space

occup…

The …

Te… (bulk);

… terms)

mo… as in

the…

Dissipative corrections to the occupation factor:

$$\delta f^{(1)}_{\rm shear} = C'_{\rm shear}\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\underbrace{\pi^{\mu\nu}}(\mathsf{x})\,p_\mu p_\nu\, f_{\rm id.}\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)$$

→ involves η

$$\delta f^{(1)}_{\rm bulk} = C'_{\rm bulk}\left(\mathsf{p}\cdot\mathsf{u}(\mathsf{x}), \mathsf{p}^2\right)\underbrace{\Pi(\mathsf{x})}\, f_{\rm id.}\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)$$

→ involves ζ

...

Dissipative effects
in heavy ion collisions

Dissipative correction at freeze-out: in the Cooper–Frye prescription

$$E_{\vec{p}} \frac{\mathrm{d}^3 N}{\mathrm{d}^3 \vec{p}} = \frac{g}{(2\pi)^3} \int_{\Sigma} f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right) \mathsf{p}\cdot \mathrm{d}^3\sigma(\mathsf{x})$$

there come corrections $f = f_{\mathrm{id.}} + \delta f^{(1)} + \delta f^{(2)} + \cdots$ to the phase space occupation factor.

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  mostly assuming freeze-out to a simple-component kinetic gas in the relaxation time approximation.

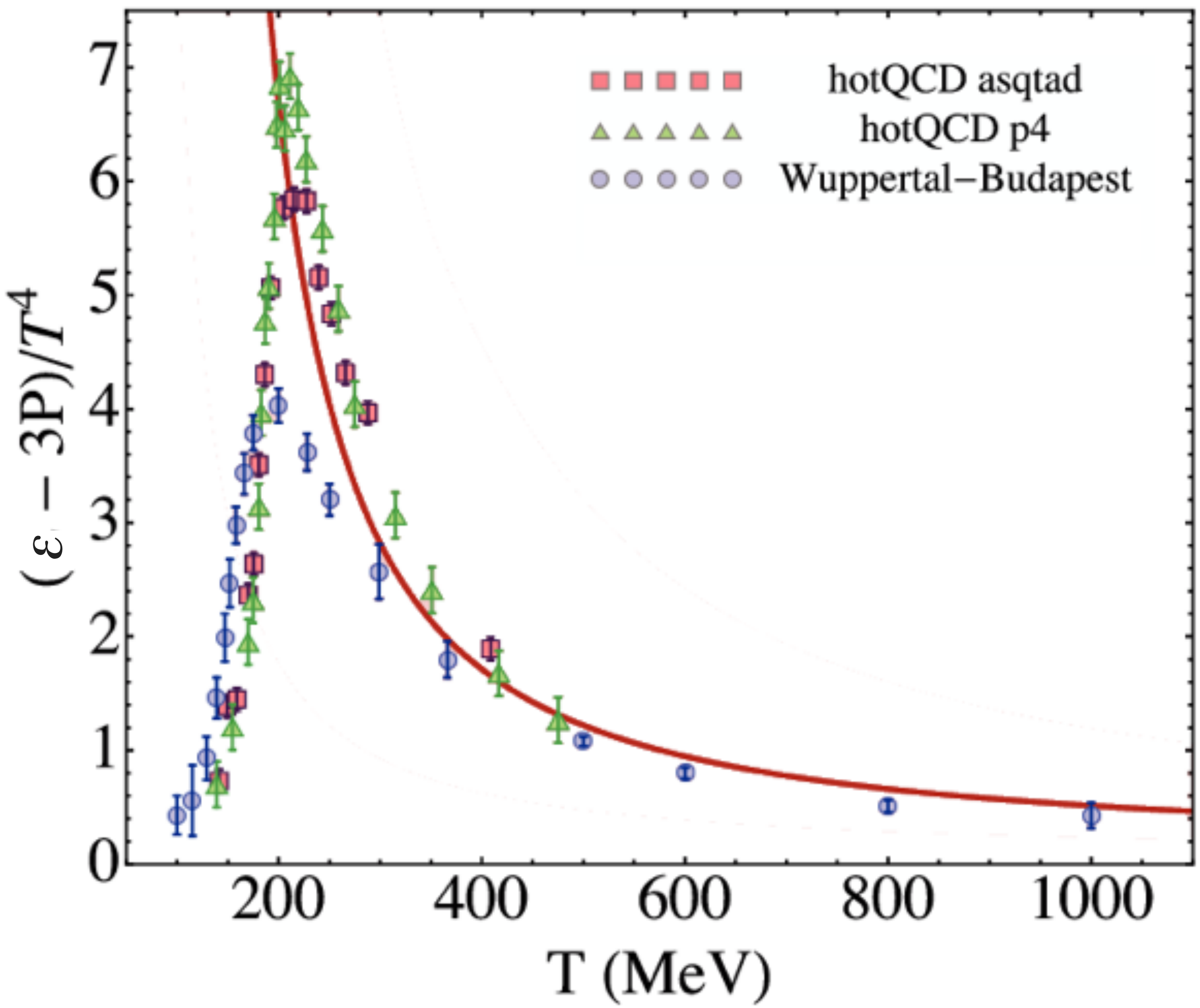
  ☛ actual functional forms not fully known.

- Only the values of the transport coefficients at freeze-out matter.

  ☛ if freeze-out at some temperature $T_{\text{f.o.}}$, only $\eta(T_{\text{f.o.}})$, $\zeta(T_{\text{f.o.}})$...

Dissipative corrections at freeze-out

If $T_{\rm f.o} < T_{\rm c}$, the decoupling fluid is to a good approximation conformal:



☛ one may first forget ζ and the non-conformal 2nd order terms.

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Particle emission at freeze-out

Consider the Cooper–Frye formula (T is the freeze-out temperature)

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

The phase space occupation factor f is proportional to $f_{\text{id.}}$, which will be approximated by a Maxwell–Boltzmann distribution.

The integral can be computed with the saddle-point approximation, without needing any detail on the freeze-out surface Σ.

(☞ the ensuing results are thus to a large extent irrespective of the velocity profile, i.e. of whether $\mathsf{u}(\mathsf{x})$ is a solution to ideal or dissipative fluid dynamics)

☛ one needs to determine the saddle point, which is* the minimum of $\mathsf{p}\cdot\mathsf{u}(\mathsf{x})/T$.

* at least approximately

Particle emission at freeze-out

Taking the Cooper–Frye formula seriously

$$E_{\vec{p}}\frac{d^3N}{d^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot d^3\sigma(\mathsf{x})$$

one may approximate the integral using the steepest-descent method.

N.B. & J.-Y.Ollitrault 2005

Two categories of particles:

- "slow particles": velocity $\frac{\mathsf{p}}{m}$ coincides with that of the fluid $\mathsf{u}(\mathsf{x})$ at some point on Σ.
  - ☛ at a given rapidity y, $|\mathbf{p}_t| < m u_{\max}(y)$.

   → maximum fluid velocity in the direction of p^μ

The minimum value of $\mathsf{p}\cdot\mathsf{u}(\mathsf{x})/T$ simply equals m/T, independent of the particle momentum.

Particle emission at freeze-out

Taking the Cooper–Frye formula seriously

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

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  - ☛ at a given rapidity y, $|\mathbf{p}_t| < m u_{\max}(y)$.
    - $u_{\max}(y)$: maximum fluid velocity in the direction of p^μ
- "fast particles": $|\mathbf{p}_t| > m u_{\max}(y)$.

  The minimum value of $\mathsf{p}\cdot\mathsf{u}(\mathsf{x})/T$ is larger than m/T.

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Results for slow particles

"Slow particles": emitted by fluid cells w.r.t. which they are at rest.

Velocity $\frac{\mathsf{p}}{m}$ coincides with that of the fluid $\mathsf{u}(\mathsf{x})$ at the saddle point.

- Freezing out from an ideal fluid:

  The Cooper–Frye integral $E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f_{\mathrm{id.}}\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$

  yields a function of the particle velocity only, with an m-dependent prefactor:

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = C(m)F\left(\frac{\mathbf{p}_t}{m}, y\right)$$

  Fourier-expanding, the flow coefficients v_n for all particles coincide when considered at the same transverse velocity and rapidity.

$\Rightarrow$ "mass-scaling" of anisotropic flow

N.B. & J.-Y.Ollitrault 2005

Results for slow particles

Particles with velocity $\frac{\mathrm{p}}{m}$ are all emitted from the same saddle point.

- Freezing out from a dissipative fluid:

  Considering first order only, the Cooper–Frye integral again gives an m-dependent pre factor times a function of the particle velocity.

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = C'(m)\,F'\!\left(\frac{\mathbf{p}_t}{m}, y\right)$$

$\Rightarrow$ mass-scaling of anisotropic flow persists

Results for slow particles

Particles with velocity $\frac{\mathrm{p}}{m}$ are all emitted from the same saddle point.

- Freezing out from a dissipative fluid:

  Considering first order only, the Cooper–Frye integral again gives an m-dependent pre factor times a function of the particle velocity.

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = C'(m)\,F'\!\left(\frac{\mathbf{p}_t}{m}, y\right)$$

⇒ mass-scaling of anisotropic flow persists

Proof:

- Since the saddle point obeys $u^\mu(\mathrm{x}) = p^\mu/m$, the shear term vanishes

$$\delta f^{(1)}_{\mathrm{shear}} = C'_{\mathrm{shear}}\left(\frac{\mathrm{p}\cdot\mathrm{u(x)}}{T}\right)\pi^{\mu\nu}(\mathrm{x})\,p_\mu p_\nu\, f_{\mathrm{id.}}\left(\frac{\mathrm{p}\cdot\mathrm{u(x)}}{T}\right) \quad \text{with } p^\mu p^\nu \pi_{\mu\nu} = 0$$

- For the bulk viscosity term, use the "universality" of the saddle point

$$\delta f^{(1)}_{\mathrm{bulk}} = C'_{\mathrm{bulk}}(\mathrm{p}\cdot\mathrm{u(x)}, \mathrm{p}^2)\,\Pi(\mathrm{x})\, f_{\mathrm{id.}}\left(\frac{\mathrm{p}\cdot\mathrm{u(x)}}{T}\right)$$

$= m \quad m^2$ → universal

Fast particles

Fast particles emitted in a given direction come from the same saddle point, where the fluid has velocity $u_{\rm max}(y,\varphi)$.

At the saddle point, $\mathsf{p}\cdot\mathsf{u}(\mathsf{x}) = m_t u^0_{\rm max}(y,\varphi) - p_t u_{\rm max}(y,\varphi)$, with here $u^0_{\rm max} \equiv \sqrt{1+u^2_{\rm max}}$.

☛ governs the particle spectrum.

As a second step, write

$$u_{\rm max}(y,\varphi) = \bar{u}_{\rm max}(y)\left[1 + 2\sum_{n\geq 1} V_n(y)\cos n(\varphi - \Psi_n)\right]$$

and Taylor-expand (with respect to the small coefficients V_n) to find the flow coefficients $v_n(p_t,y)$.

Fast particles from an ideal fluid

Reasonable(?) assumption: the velocity of the freezing-out fluid mostly has elliptic and triangular anisotropies: $V_2, V_3 \gg V_1, V_4, V_5, V_6$

One then finds

- $v_2(p_t) = I(p_t)V_2$
- $v_3(p_t) = I(p_t)V_3$
- $v_4(p_t) = \frac{I(p_t)^2}{2}V_2^2 + I(p_t)V_4$
- $v_5(p_t) = I(p_t)^2 V_2 V_3 + I(p_t)V_5$
- $v_6(p_t) = \frac{I(p_t)^3}{6}V_2^3 + \frac{I(p_t)^2}{2}V_3^2 + I(p_t)^2 V_2 V_4 + I(p_t)V_6$

where $I(p_t) \equiv \frac{\bar{u}_{\max}}{T}(p_t - m_t \bar{\mathrm{v}}_{\max})$ with $\bar{\mathrm{v}}_{\max} \equiv \frac{\bar{u}_{\max}}{\sqrt{1+\bar{u}^2_{\max}}}$

N.B. & J.-Y.Ollitrault 2005; D.Teaney & L.Yan 2012

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One then finds at high momentum (not too high, hydro should hold!)

- $v_2(p_t) = I(p_t)V_2$
- $v_3(p_t) = I(p_t)V_3$
- $v_4(p_t) = \frac{I(p_t)^2}{2}V_2^2 + I(p_t)V_4 \quad \sim \frac{1}{2}v_2(p_t)^2$
- $v_5(p_t) = I(p_t)^2 V_2 V_3 + I(p_t)V_5 \sim v_2(p_t)v_3(p_t)$
- $v_6(p_t) = \frac{I(p_t)^3}{6}V_2^3 + \frac{I(p_t)^2}{2}V_3^2 + I(p_t)^2 V_2 V_4 + I(p_t)V_6 \sim \frac{1}{6}v_2(p_t)^3 + \frac{1}{2}v_3(p_t)^2$

where $I(p_t) \equiv \frac{\bar{u}_{\max}}{T}(p_t - m_t \bar{\mathrm{v}}_{\max})$ with $\bar{\mathrm{v}}_{\max} \equiv \frac{\bar{u}_{\max}}{\sqrt{1+\bar{u}_{\max}^2}}$

N.B. & J.-Y.Ollitrault 2005; D.Teaney & L.Yan 2012

Fast particles from a viscous fluid

Assumptions: V_2, $V_3 \gg V_1$, V_4, V_5, V_6 & azimuthal dependence of the shear part of the stress tensor is neglected (for the sake of simplicity only).

Remarks:

- Strictly speaking, the values of these Fourier coefficients, as well as that of $\bar{u}_{\max}$ (which enters the function $I(p_t)$) are different from those of the ideal case.

- Hereafter I only mention the correction due to shear viscosity. That arising from bulk viscosity and those from second order terms lead to similar results.

Fast particles from a viscous fluid

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One then finds

- $v_2(p_t) = [I(p_t) - D(p_t)]V_2$
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where $D(p_t)$ is a positive function proportional to η, whose functional dependence reflects that of the viscous correction $\delta f^{(1)}_{\rm shear}$.

Fast particles from a viscous fluid

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- $v_5(p_t) = [I(\ldots D(p_t)]V_2V_3 + [I(p_t) - D(p_t)]V_5$
- $v_6(p_t \ldots \left[\frac{\ldots}{6} - \frac{I(p_t)^2 D(p_t)}{2}\right]V_2^3 + \left[\frac{I(p_t)^2}{2} - I(p_t)D(p_t)\right]V_3^2 + \cdots$

What do we do with that stuff??

where $D(p_t)$ is a positive function proportional to η, whose functional dependence reflects that of the viscous correction $\delta f^{(1)}_{\rm shear}$.

Fast particles from a viscous fluid: relations between flow harmonics

Inspecting these results more carefully...

- $v_2(p_t) = [I(p_t) - D(p_t)]V_2$
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- $v_3(p_t) = [I(p_t) - D(p_t)]V_3$

same momentum dependence, i.e. the ratio $\dfrac{v_3(p_t)}{v_2(p_t)}$ should be constant

- $v_4(p_t) = \left[\dfrac{I(p_t)^2}{2} - I(p_t)D(p_t)\right]V_2^2 + [I(p_t) - D(p_t)]V_4$
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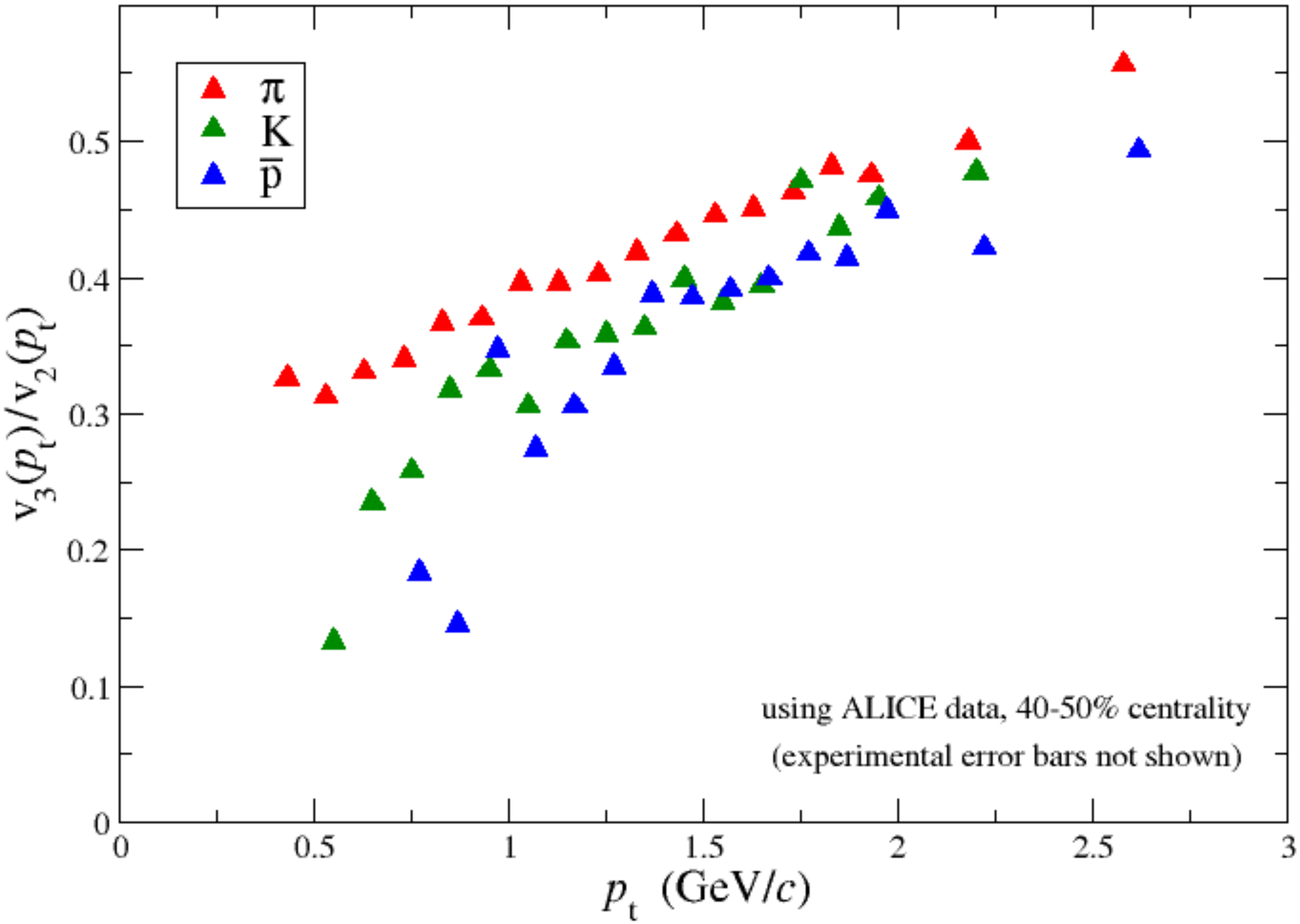
Fast particles from a viscous fluid: relations between flow harmonics

The ratio $\frac{v_3(p_t)}{v_2(p_t)}$ should be constant



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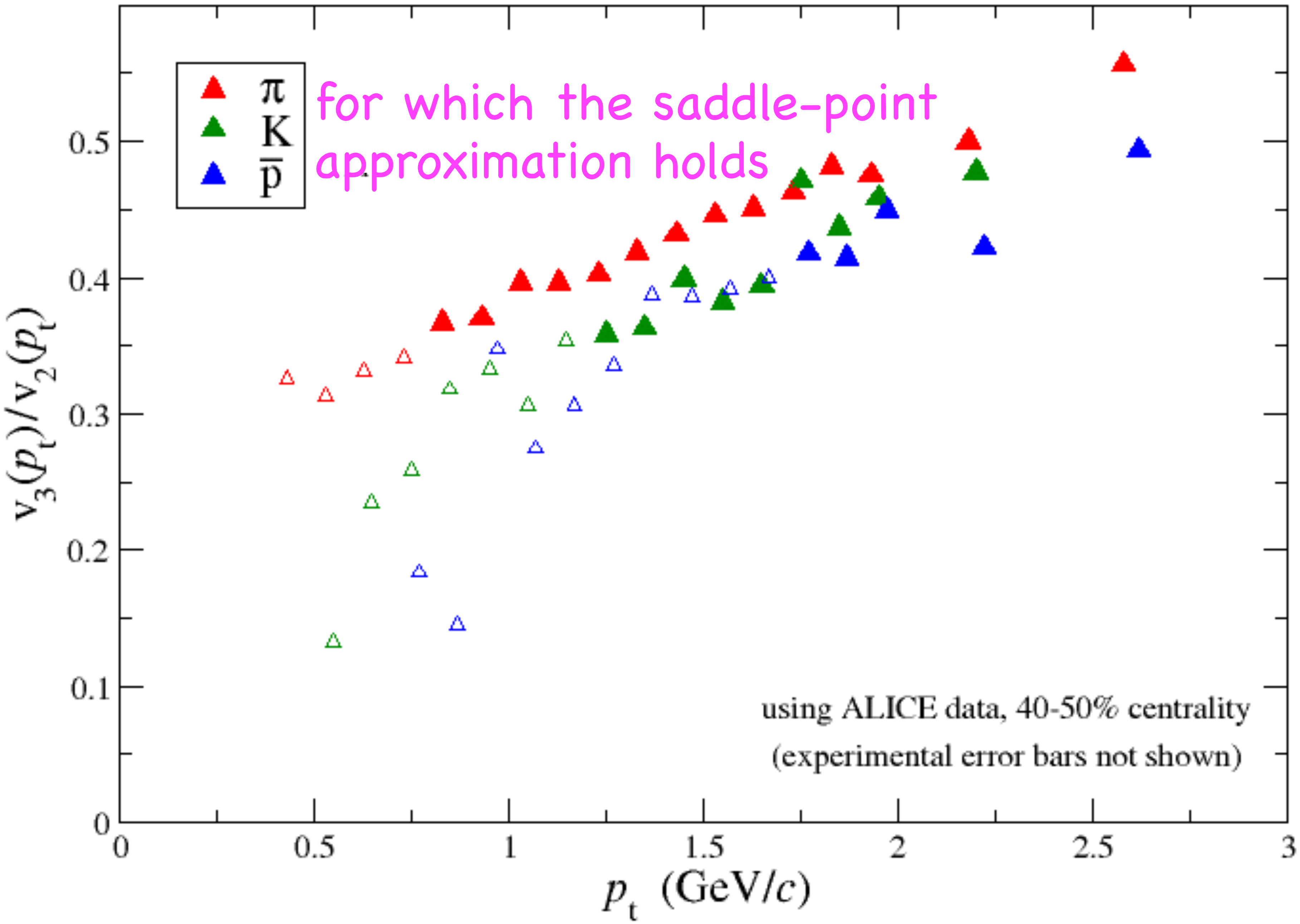

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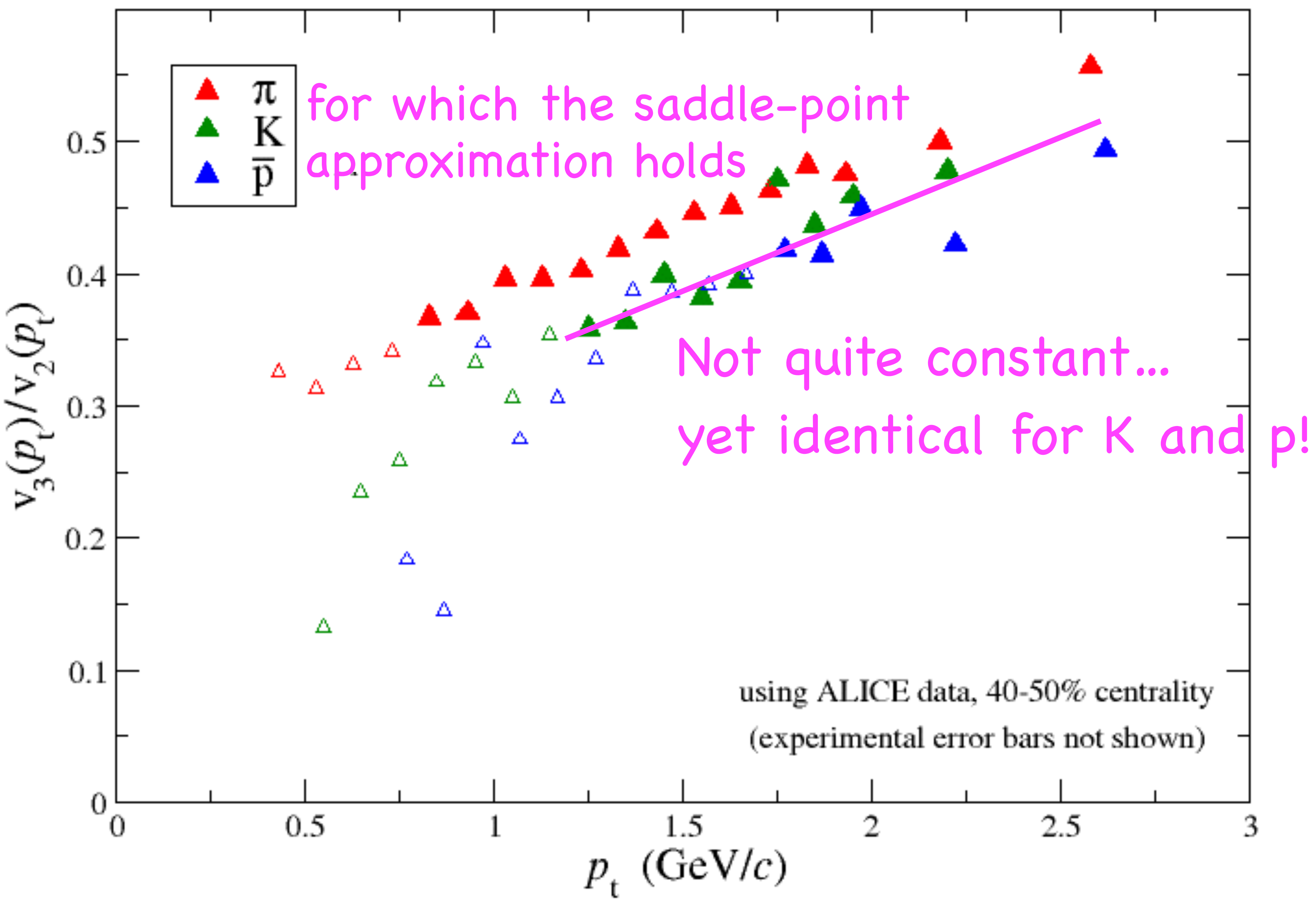

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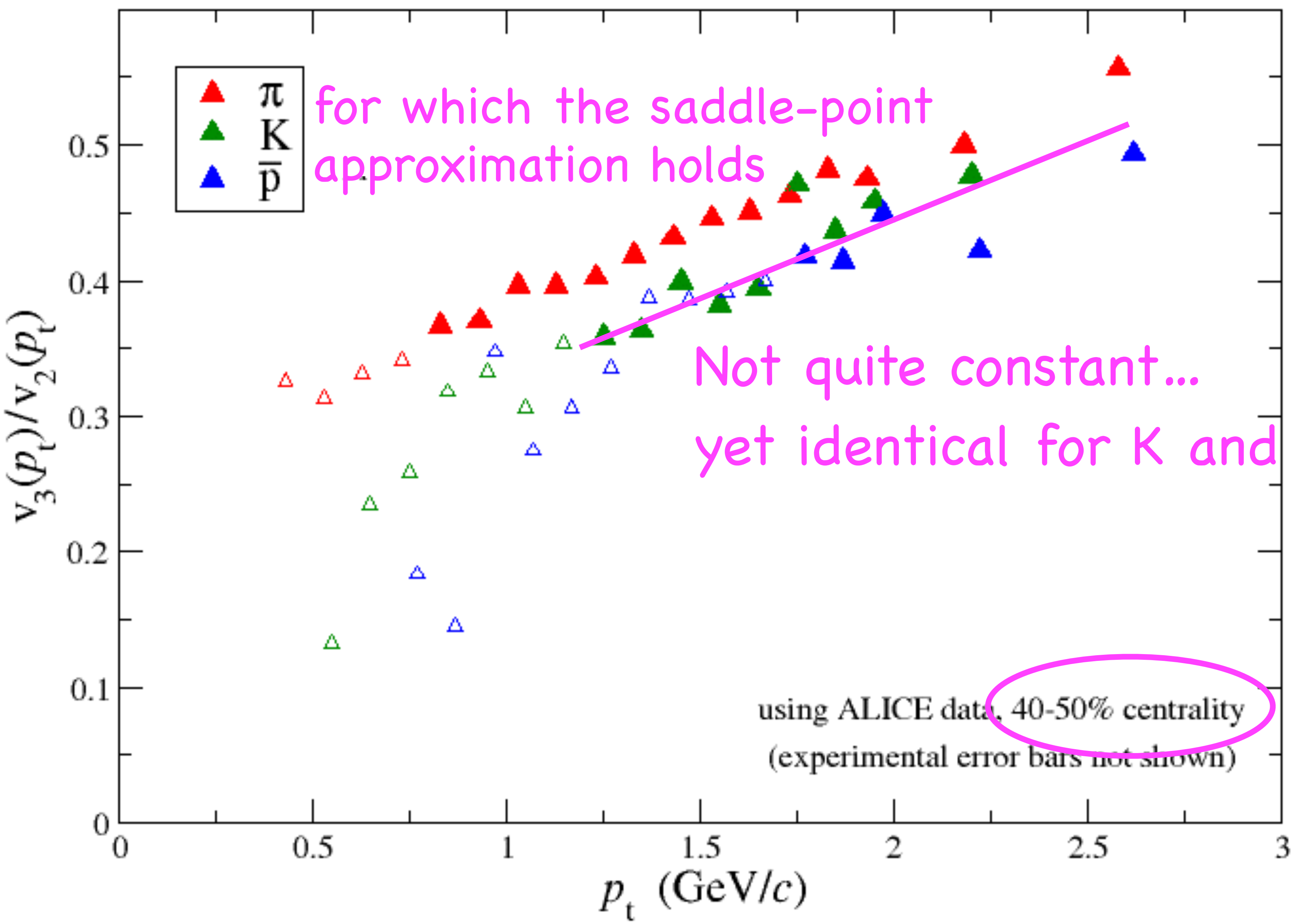

Does $V_2, V_3 \gg V_1$ hold?

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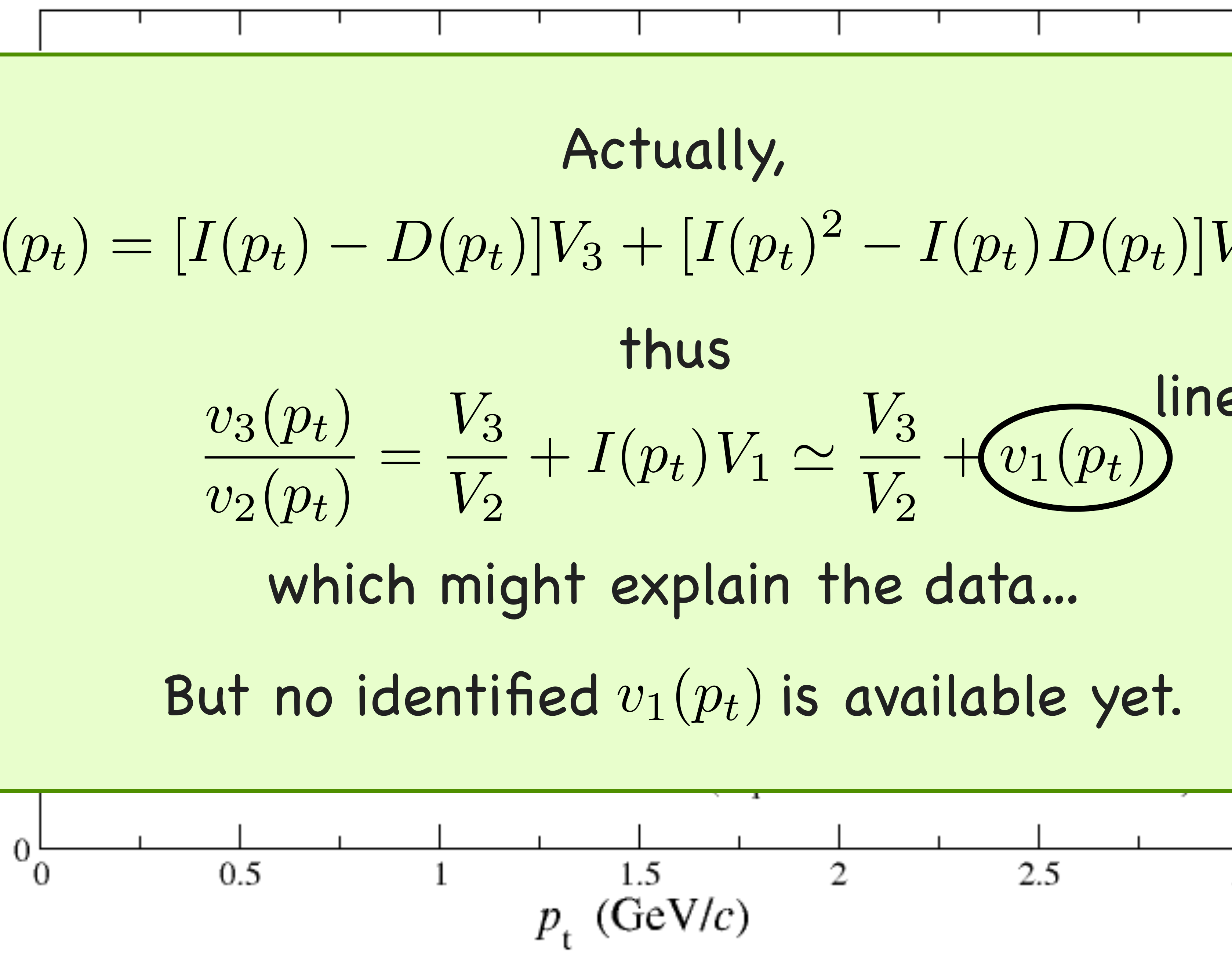
The ratio $\dfrac{v_3(p_t)}{v_2(p_t)}$ should be constant



Actually,

$$v_3(p_t) = [I(p_t) - D(p_t)]V_3 + [I(p_t)^2 - I(p_t)D(p_t)]V_1V_2$$

thus

$$\frac{v_3(p_t)}{v_2(p_t)} = \frac{V_3}{V_2} + I(p_t)V_1 \simeq \frac{V_3}{V_2} + v_1(p_t)$$

linear(?)

which might explain the data...

But no identified $v_1(p_t)$ is available yet.

oes
$V_3 \gg V_1$
hold?

Fast particles from a viscous fluid: relations between flow harmonics

Inspecting the results more carefully...

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- $v_3(p_t) = [I(p_t) - D(p_t)]V_3$
- $v_4(p_t) = \left[\frac{I(p_t)^2}{2} - I(p_t)D(p_t)\right]V_2^2 \; < \frac{1}{2}v_2(p_t)^2$

  as seen in transport and in (real) hydro simulations
- $v_5(p_t) = [I(p_t)^2 - I(p_t)D(p_t)]V_2V_3 + [I(p_t) - D(p_t)]V_5$
- $v_6(p_t) = \left[\frac{I(p_t)^3}{6} - \frac{I(p_t)^2 D(p_t)}{2}\right]V_2^3 + \left[\frac{I(p_t)^2}{2} - I(p_t)D(p_t)\right]V_3^2 + \cdots$

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- $v_5(p_t) = [I(p_t)^2 - I(p_t)D(p_t)]V_2V_3$ $\quad > v_2(p_t)v_3(p_t)$

as seen in (real) hydro simulations

(I use $I(p_t) > D(p_t)$)

Fast particles from a viscous fluid: relations between flow harmonics

Inspecting the results more carefully…

- $v_2(p_t) = [I(p_t) - D(p_t)]V_2$
- $v_3(p_t) = [I(p_t) - D(p_t)]V_3$

together with

- $v_5(p_t) = [I(p_t)^2 - I(p_t)D(p_t)]V_2V_3$

yield
$$\frac{v_5(p_t) - v_2(p_t)v_3(p_t)}{v_2(p_t)} = D(p_t)V_3$$

i.e. isolate the dissipative contribution to $v_3(p_t)$.

Fast particles from a viscous fluid: relations between flow harmonics

Inspecting the results more carefully...

- $v_2(p_t) = [I(p_t) - D(p_t)]V_2$
- $v_3(p_t) = [I(p_t) - D(p_t)]V_3$

and

- $v_4(p_t) = \left[\frac{I(p_t)^2}{2} - I(p_t)D(p_t)\right]V_2^2 + [I(p_t) - D(p_t)]V_4$
- $v_5(p_t) = [I(p_t)^2 - I(p_t)D(p_t)]V_2V_3 + [I(p_t) - D(p_t)]V_5$

yield $2v_4(p_t) - v_2(p_t)^2 = -D(p_t)^2V_2^2$

i.e. again isolate the dissipative contribution (here to $v_2(p_t)$).

Fast particles from a viscous fluid: relations between flow harmonics

More relations can be found...

For instance, combining

$$\frac{v_5(p_t) - v_2(p_t)v_3(p_t)}{v_3(p_t)} = D(p_t)V_2$$

(analogous to the relation on slide 20) and

$$2v_4(p_t) - v_2(p_t)^2 = -D(p_t)^2V_2^2$$

one at once comes up with a relation between four harmonics...

... up to the caveats on my summary slide!

Dissipative corrections to anisotropic flow

- Viscosity and other dissipative phenomena strike twice:
  - ☛ throughout the evolution and at freeze-out
- Their contributions at freeze-out might be isolated
  - ☛ relations between various flow harmonics (for a given particle species)
  - ☛ (similar relations with multiparticle correlations; not shown here)
  - ☛ hope (naïve?): using particles which decouple earlier / later, one may access the temperature dependence of the transport coefs.
- Caveats:
  - Throughout this talk, fluctuations were neglected (not a big deal)
  - How much of these ideas survives: 1. real hydro; 2. rescatterings
    - ☛ realistic studies needed!