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# Extracting the bulk viscosity of the quark-gluon plasma


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with:

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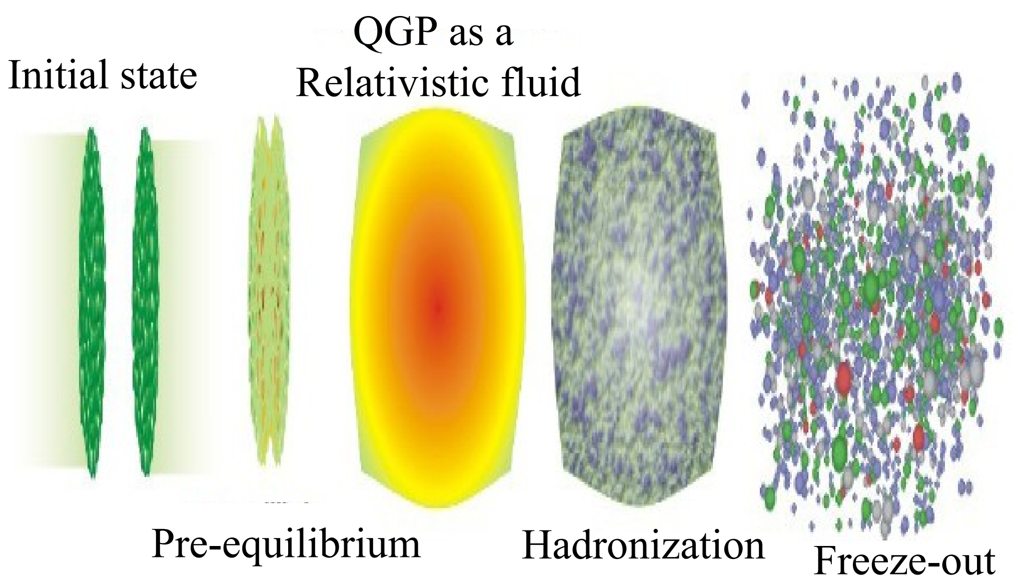
**New Frontiers in QCD 2013**

--- *Insight into QCD matter from heavy-ion collisions* ---

# Main Motivation

• Relativistic **fluid dynamics** has played a key role in our current understanding of the novel "**near perfect**" fluid behavior displayed by the Quark-Gluon Plasma (QGP)




**Is there any point in improving the current fluid-dynamical modeling?**

# Basics of fluid dynamics

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

$$\partial_\mu N^\mu = 0$$

$$N^\mu = nu^\mu + n^\mu,$$
$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - \Delta^{\mu\nu}(P_0 + \Pi) + \pi^{\mu\nu}$$

Particle diffusion current

Bulk viscous pressure

Shear stress tensor

**Need to be closed!**

Spatial projector

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$u_\mu u^\mu = 1$$

# What everyone does

- Most simulations neglect nonlinear terms
- Most simulations neglect bulk viscous pressure
- All simulations neglect heat flow

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta$$

**Majority of conclusions of our field are based on these equations**
**(e.g., MUSIC 1.0, Ohio Group)**

**Is there any point in improving this?**

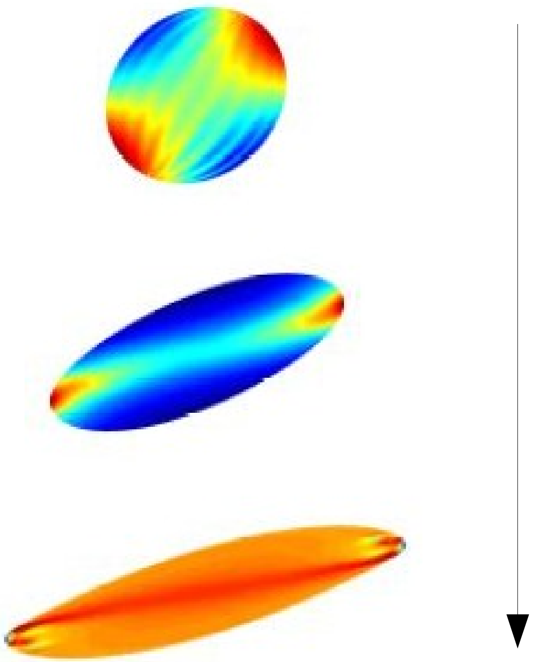
# Sources of dissipation

**Bulk**

Resistance to expansion

**Shear**

Resistance to deformation



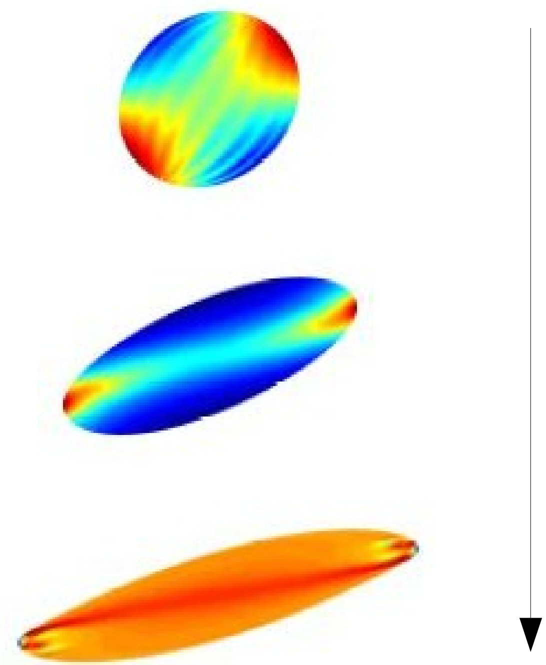
usually ignored

**Bulk**

**Shear**

Resistance to expansion

Resistance to deformation

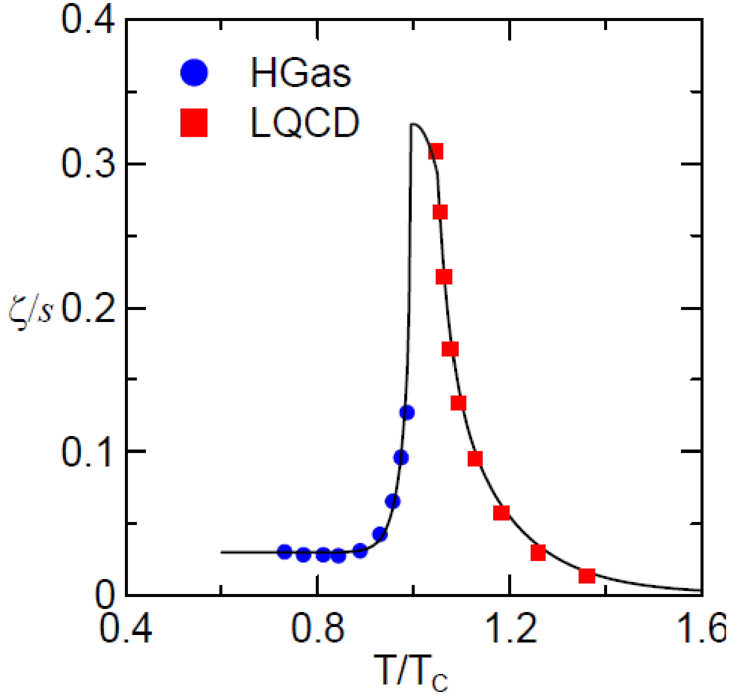


**Why?**

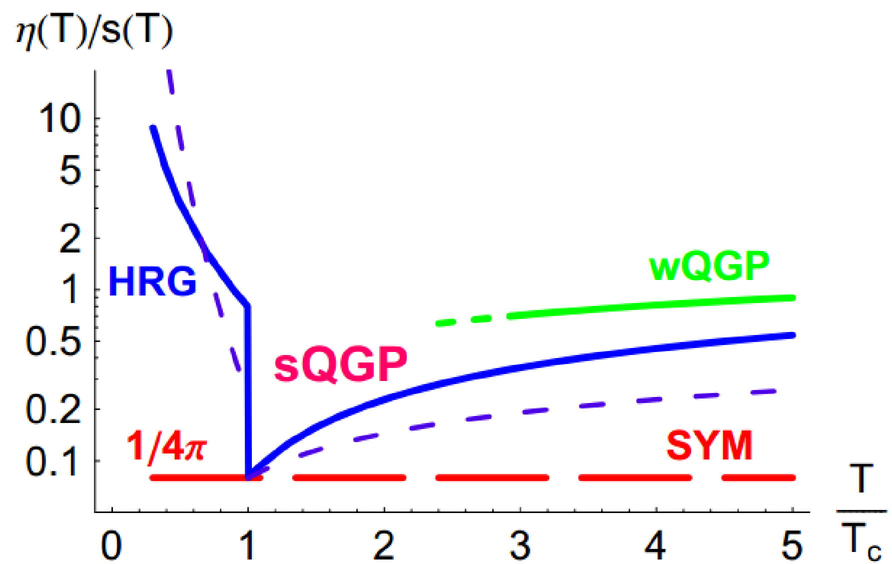
because it's small? some estimates ...

## Bulk viscosity

## Shear viscosity



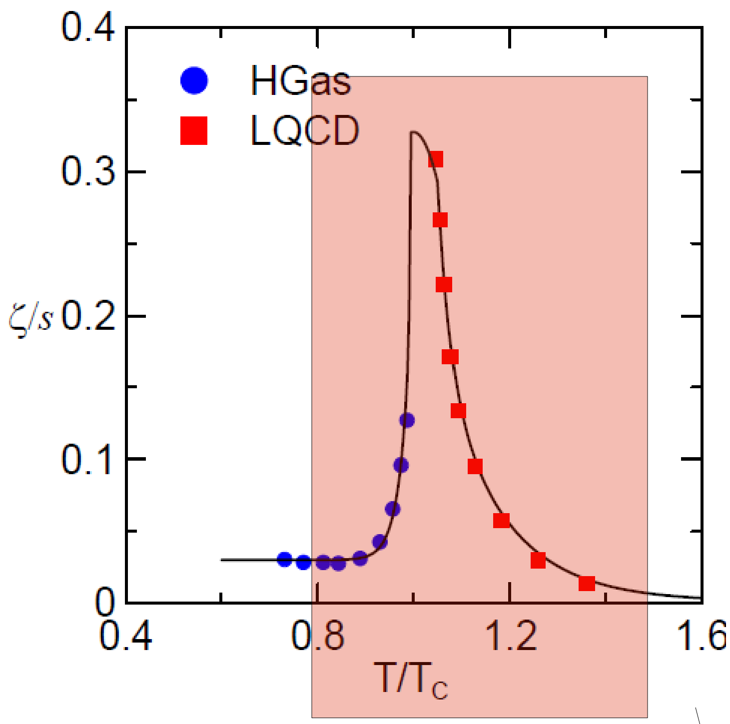

Karsh&Kharzeev&Tuchin
Noronha&Noronha&Greiner



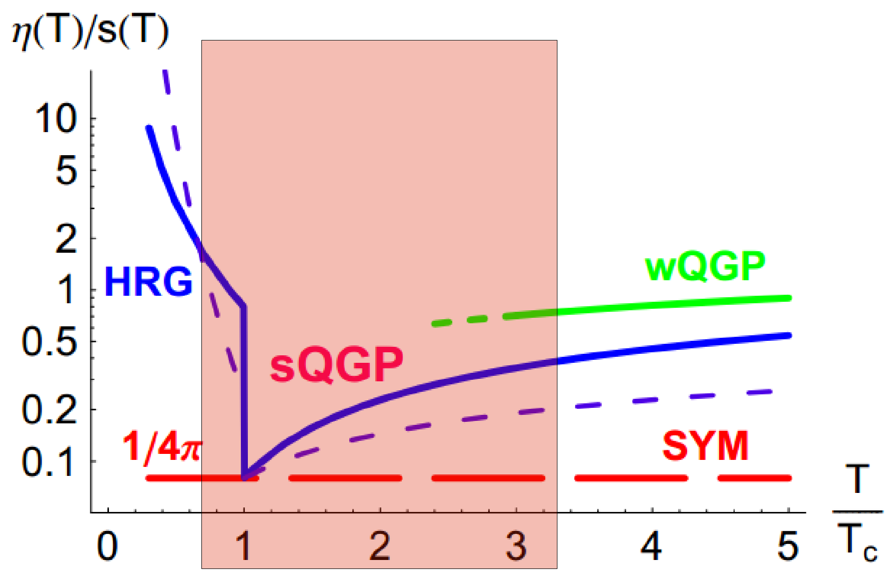

Hirano&Gyulassy

because it's small? some estimates ...

# Bulk viscosity




# Shear viscosity




**But in the region of interest, we don't really know ...**

# Questions addressed

✔ Can bulk viscosity have an effect on flow?

✔ Is the effect the same as the one from shear viscosity?

✔ Can bulk viscosity change the previous conclusions obtained for shear viscosity?

**everything in ultracentral collisions; 0-1% centrality; LHC lowest energy**

# What you will see in this talk

**Instead of the traditional equation**

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta$$

$$\Downarrow$$

**Inclusion of bulk viscous pressure, shear-stress tensor, and all couplings**

**MUSIC 2.0**

$$\begin{aligned}
\dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \varphi_1 \Pi^2 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu}\pi_{\mu\nu} \,, \\
\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} \\
&\quad + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \varphi_6 \Pi\pi^{\mu\nu}.
\end{aligned}$$

**Equations of motion obtained from kinetic theory**

# Transport equations

**MUSIC 2.0**

**Inclusion of bulk viscous pressure, shear-stress tensor, and all couplings**

$$\boxed{\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \varphi_1 \Pi^2} + \boxed{\lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu}\pi_{\mu\nu}\,,}$$

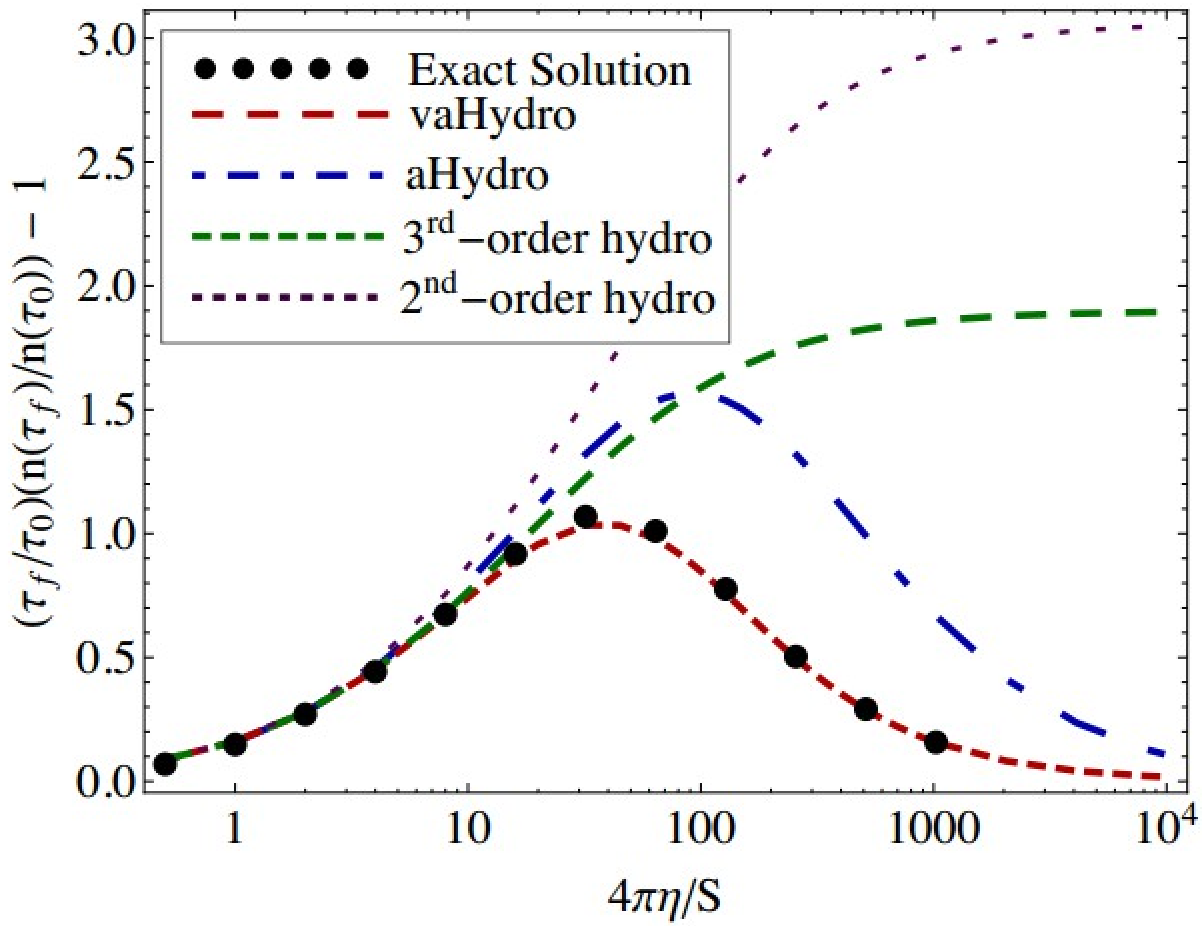
$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + \boxed{2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha}}$$
$$\boxed{+\lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \varphi_6 \Pi\pi^{\mu\nu}\,.}$$

**Second-Order Nonlinear source terms**

**Bulk viscous pressure**

**Coupling between bulk viscous pressure and shear-stress tensor**

# Viscous hydro works very well




D. Bazow, U. Heinz, M. Strickland, arxiv:1311.6720

**In contrast to the naive expectation:**
**even at η/s~10, second order viscous hydro seems to work**

# Coefficients employed MUSIC 2.0

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \varphi_1 \Pi^2 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu}\pi_{\mu\nu} \,,$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \varphi_6 \Pi\pi^{\mu\nu} .$$

Transport coefficients computed within the 14-moment approximation

$$\beta_\pi = \frac{\varepsilon_0 + P_0}{5},\ \delta_{\pi\pi} = \frac{4}{3}\tau_\pi \,, \quad \tau_{\pi\pi} = \frac{10}{7}\tau_\pi \,, \quad \varphi_7 = \frac{9}{70 P_0}\tau_\pi \,.$$

$$\beta_\Pi = \frac{\zeta}{\tau_\Pi} = 14.55 \times \left(\frac{1}{3} - c_s^2\right)^2 (\varepsilon_0 + P_0) + \mathcal{O}\left(z^5\right),$$

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = 1 - c_s^2 + \mathcal{O}\left(z^2 \ln z\right) \,,$$

$$\frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right) + \mathcal{O}\left(z^4\right) \,,$$

$$z \equiv m/T,$$

# Viscosity Ansatz

## Shear viscosity

$$\frac{\eta}{s} = \mathrm{const}$$

**"effective" shear viscosity**

## Bulk viscosity

$$\frac{\zeta}{s} = \mathrm{const}$$

**"effective" bulk viscosity**

**QGP?**

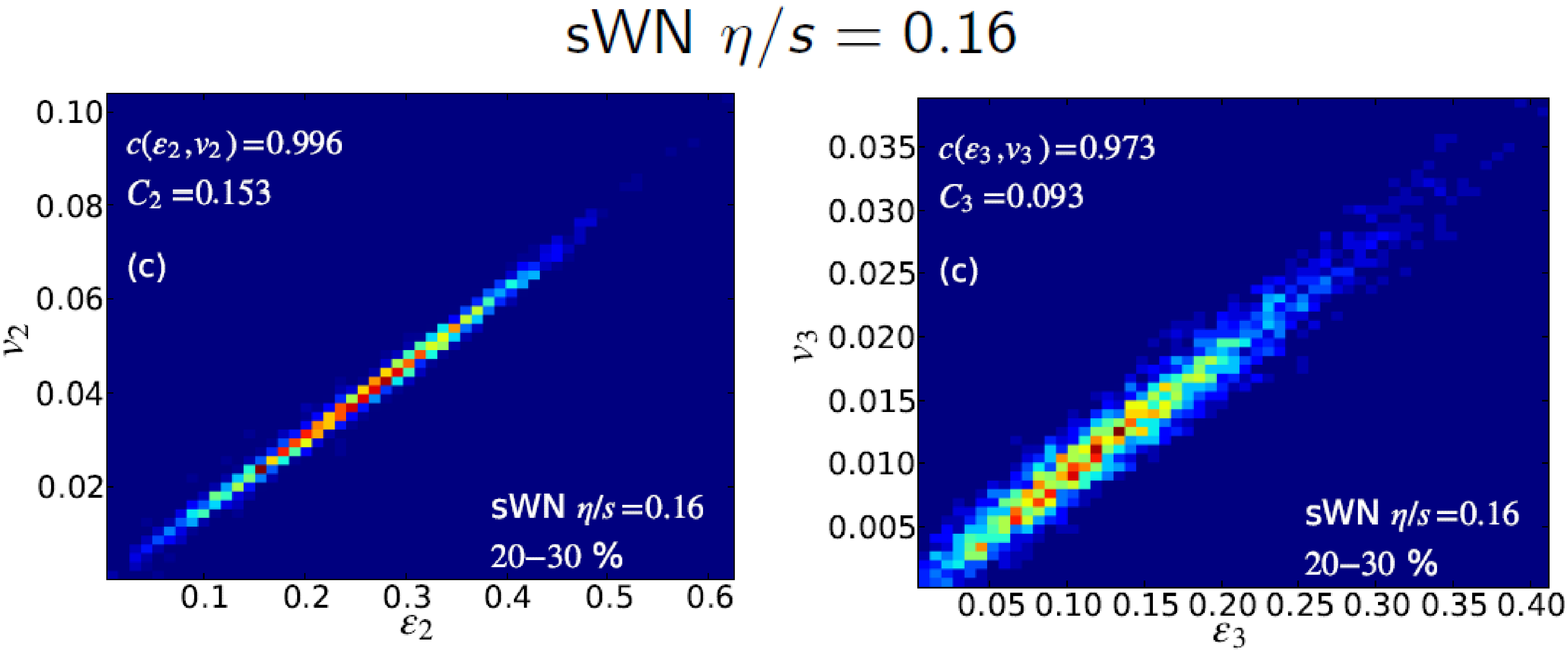
**Possibly other functional forms**

**or**

$$\frac{\zeta}{s} = \underbrace{\mathrm{const}}_{b} \times \frac{\eta}{s}\left(\frac{1}{3} - c_s^2\right)^2$$

**inspired in the weakly coupled limit**

# Why ultracentral? we don't need to do EbE




Niemi&GSD&Huovinen&Holopainen

$$\boxed{v_n = C_n \epsilon_n} \Longleftrightarrow \boxed{C_n = \langle v_n \rangle_{\rm ev} / \langle \epsilon_n \rangle_{\rm ev}}$$

**Works for n=2 and n=3**

# In ultracentral collisions: **Works for all of them**

Gardim&Grassi&Luzum&Ollitrault, arxiv:1111.6538

$$\boxed{v_n = C_n \epsilon_n} \Longleftrightarrow \boxed{C_n = \langle v_n \rangle_{\rm ev} / \langle \epsilon_n \rangle_{\rm ev}}$$

**very small dependence on initial state**

Same as Luzum&Ollitrault, arxiv:1210.6010

Similar philosophy to Retinskaya&Luzum&Ollitrault, arxiv:1311.5339

## What we do:

We compute this coefficient for an arbitrary IC, but with the **correct multiplicity** and **average pT**

# Simulation

- ✔ We solve the fluid-dynamical equations using a relativistic version of the KT algorithm – **MUSIC**

  Schenke&Jeon&Gale

  Phys.Rev. C82 (2010) 014903

- ✔ Freeze-out via Cooper-Frye, T=140 MeV

- ✔ $\delta$f from Monnai&Hirano, Phys. Rev. C80 (2009) 054906

  Detailed study of $\delta$f → J.Noronha-Hostler *et al*

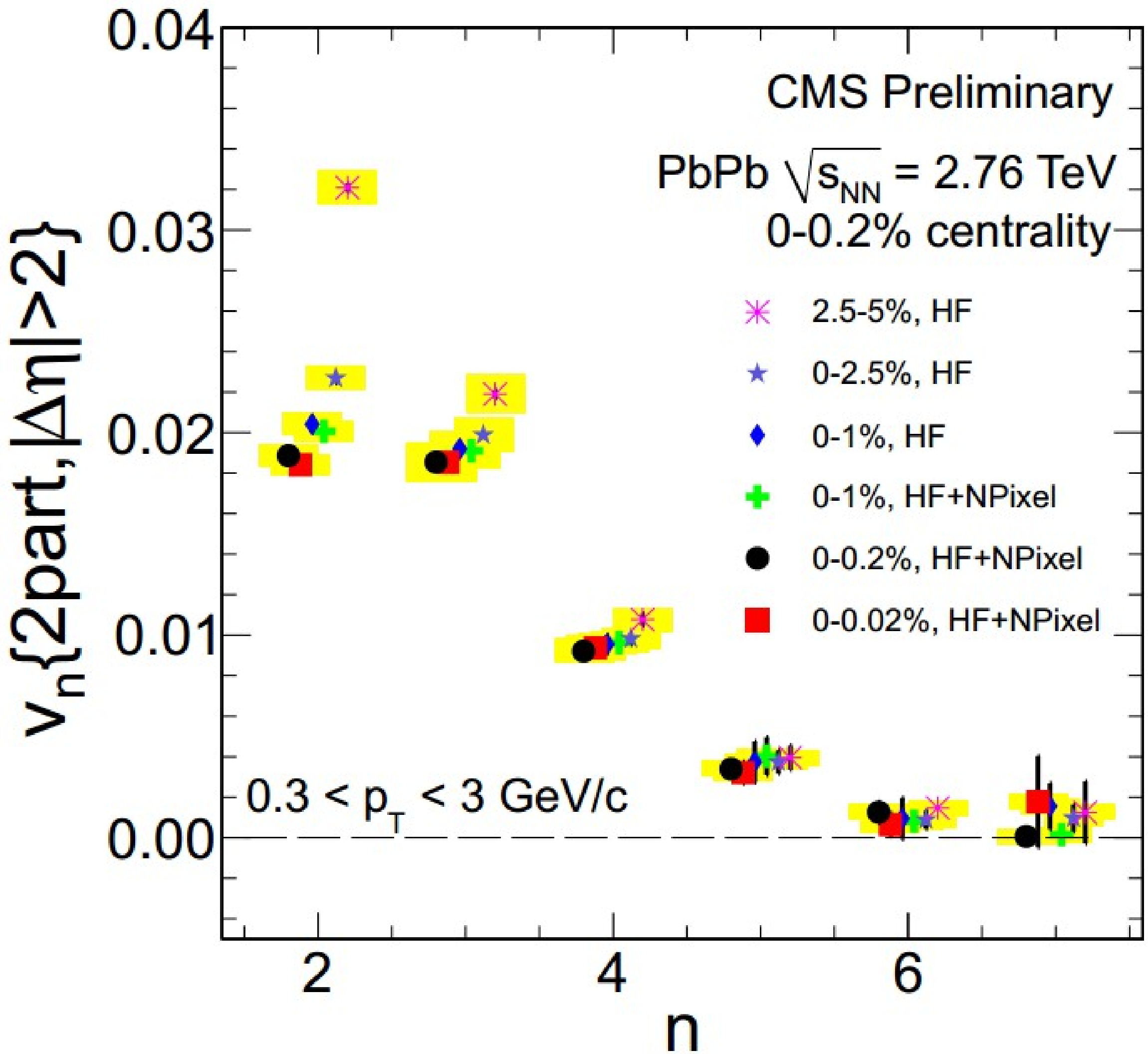
  Phys. Rev. C88 (2013) 044916

- ✔ lQCD + HRG EoS by Huovinen&Petrescky

  Nucl.Phys. A837 (2010) 26-53

- ✔ $\tau_0$=1 fm, equilibrium

# **Data from CMS:** ultracentral collisions




**$v_2 \sim v_3$**

hard to get with hydro

# Effect of bulk viscous pressure (1)

**Bulk Only**

$$\boxed{\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \left(1 - c_s^2\right) \tau_\Pi \Pi \theta}$$

**Shear Only**

$$\boxed{\begin{aligned} \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta \\ &\quad + \frac{18}{35}\tau_\pi \frac{\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}}{\varepsilon_0 + P_0} - \frac{10}{7}\tau_\pi \pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha}. \end{aligned}}$$

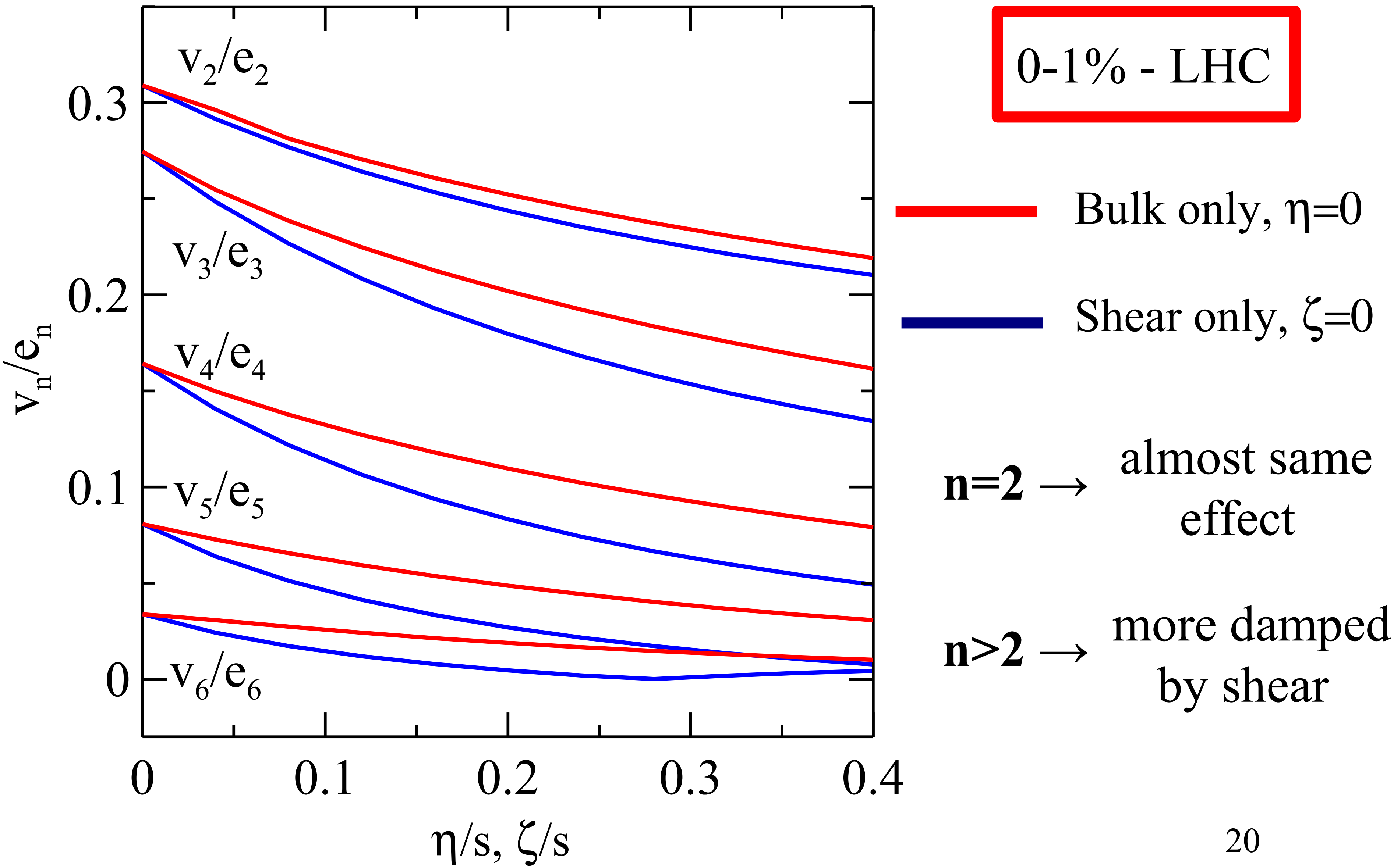
**assume effective viscosities:**

$$\frac{\eta}{s} = \text{const}$$

$$\frac{\zeta}{s} = \text{const}$$

# Effect of bulk viscous pressure

**MUSIC 2.0**

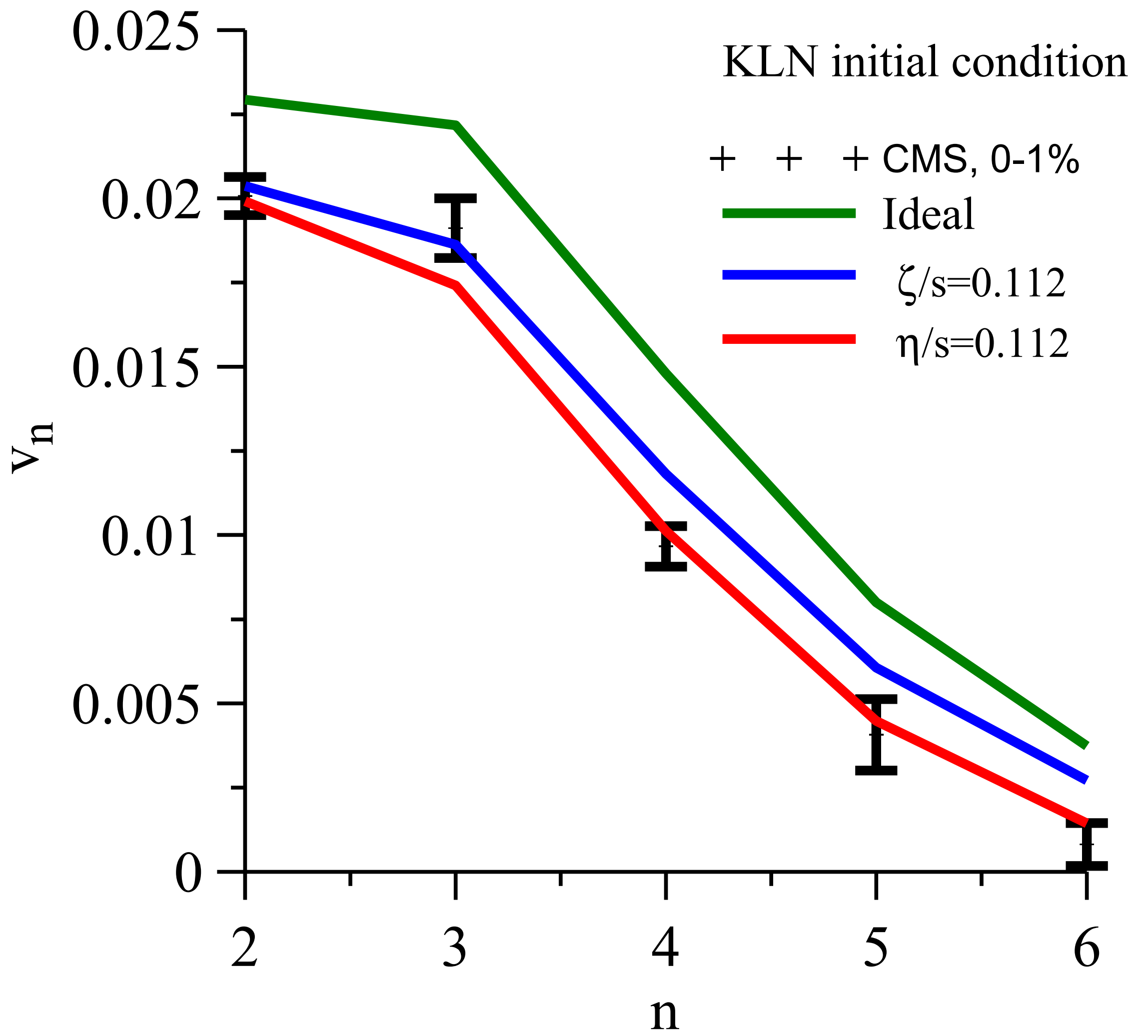



**n=2** → almost same effect

**n>2** → more damped by shear

# Effect of bulk viscous pressure

**MUSIC 2.0**




0-1% - LHC

**bulk** → $v_3$ less damped
misses n=4,5,6

**shear** → $v_3$ too damped
describes n=4,5,6

# Effect of bulk viscous pressure (2)

## Complete equations

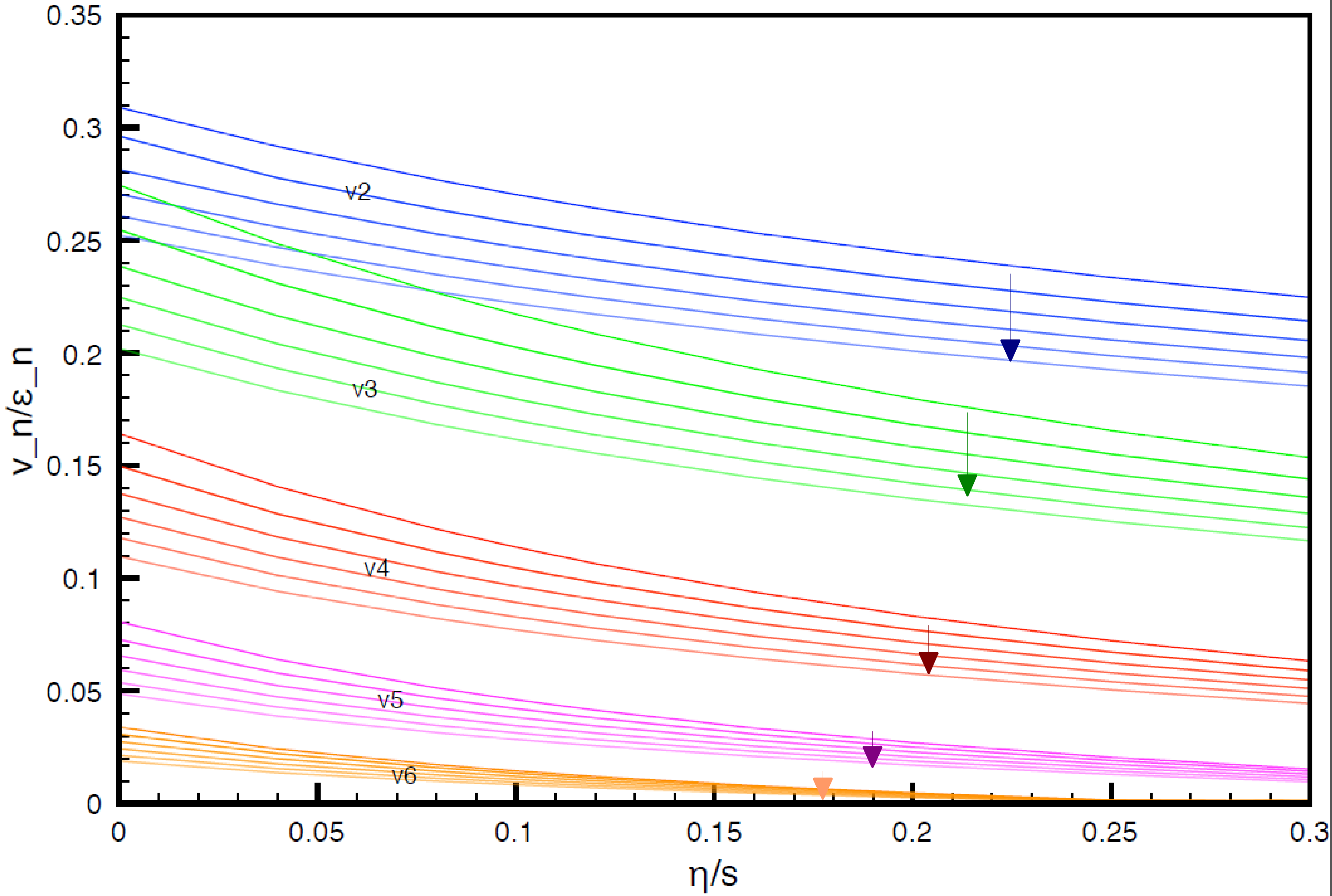
$$
\begin{aligned}
\tau_\Pi \dot{\Pi} + \Pi &= -\zeta\theta - \left(1 - c_s^2\right)\tau_\Pi \Pi\theta + \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)\tau_\Pi \pi^{\mu\nu}\sigma_{\mu\nu} \; , \\
\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta - \frac{10}{7}\tau_\pi \pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} \\
&\quad + \frac{18}{35}\tau_\pi \frac{\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}}{\varepsilon_0 + P_0} + \frac{6}{5}\tau_\pi \Pi\sigma^{\mu\nu} .
\end{aligned}
$$

# Effect of bulk viscous pressure

**MUSIC 2.0**

$\zeta/s$=0, 0.04, 0.08, 0.12, 0.16, 0.20

0-1% - LHC



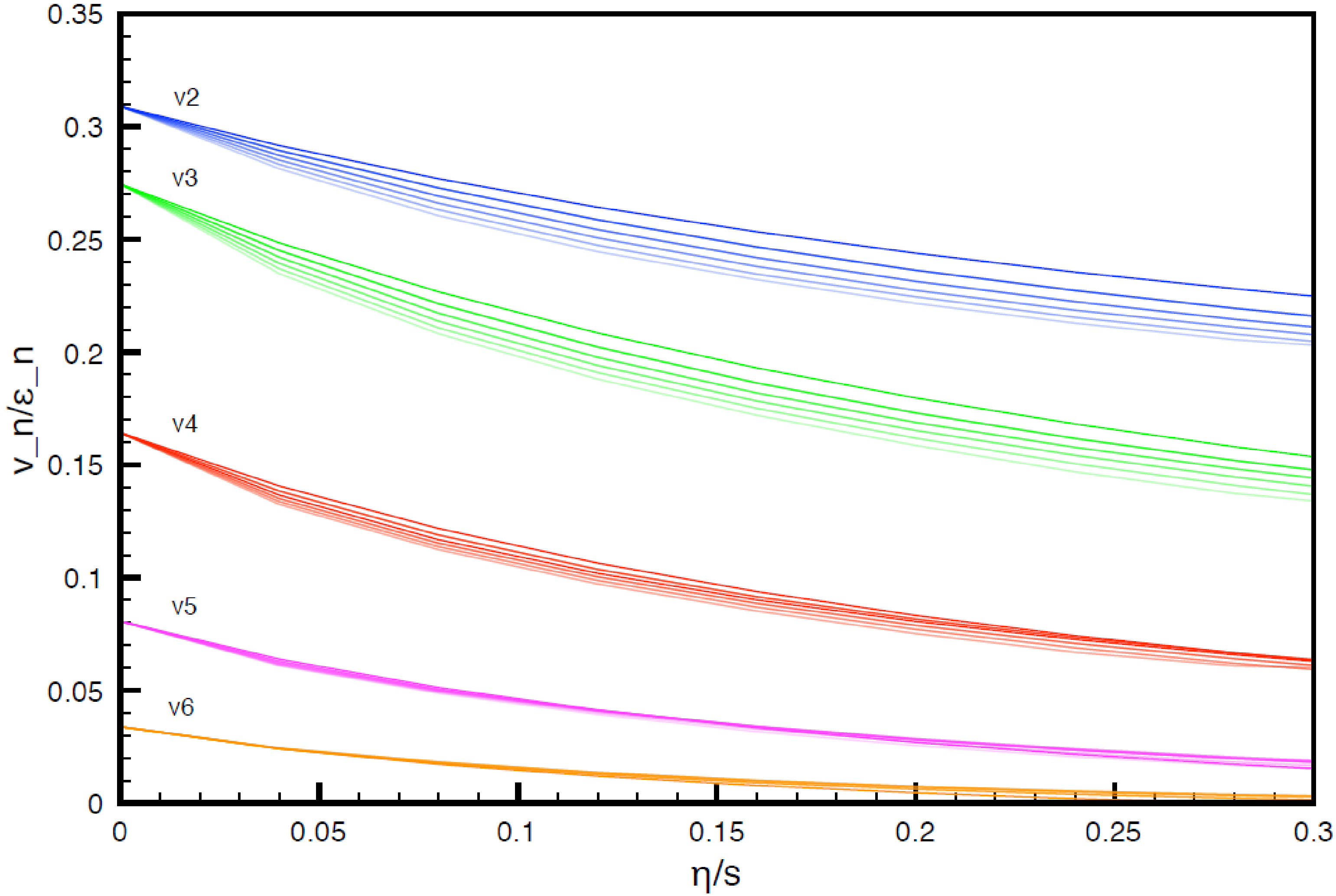

**not a small effect!**

# Effect of bulk viscous pressure

**MUSIC 2.0**

0-1% - LHC

$$\frac{\zeta}{s} = \mathrm{b} \times \frac{\eta}{s}\left(\frac{1}{3} - c_s^2\right)^2$$ b=0, 15, 30, 45, 60, 75

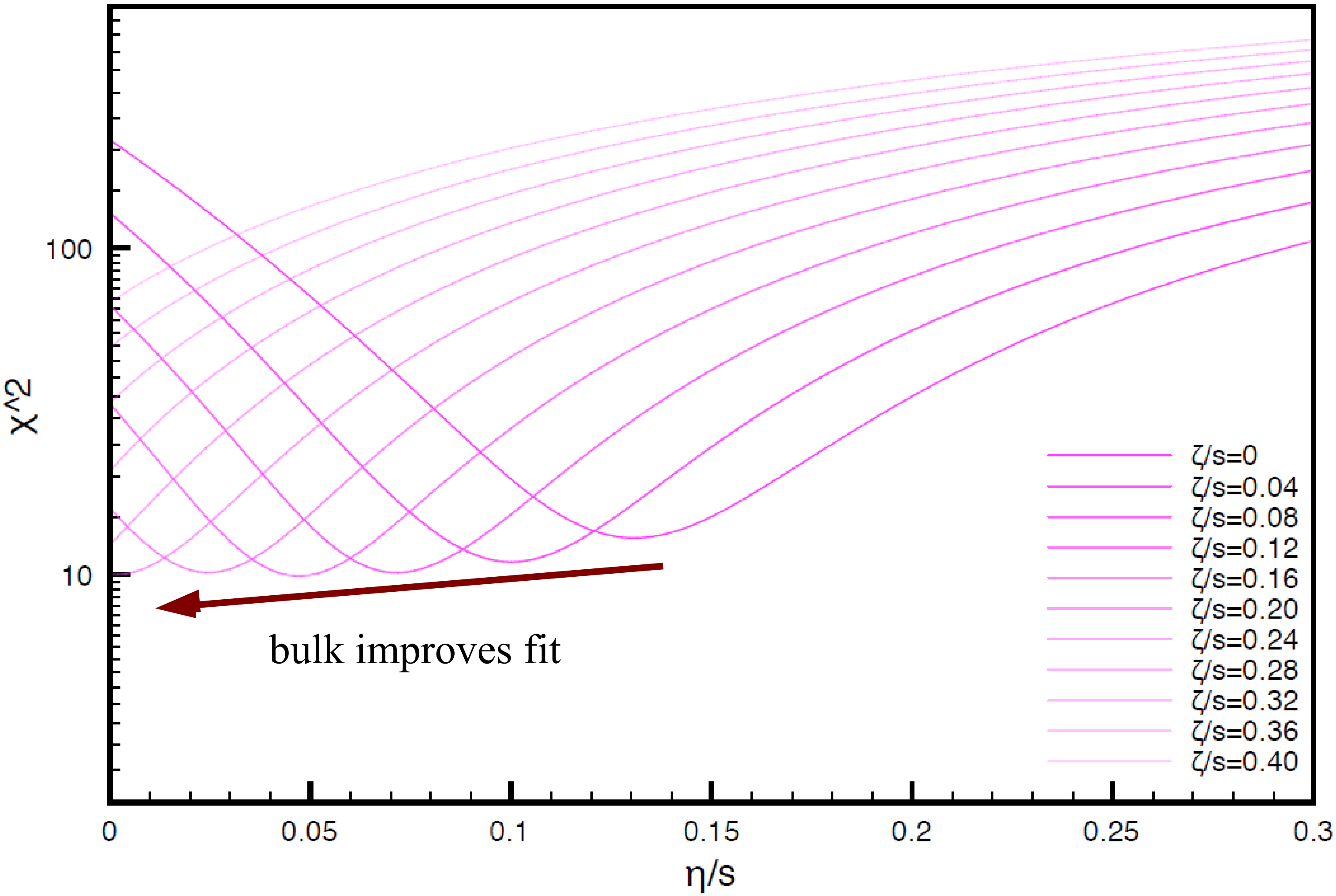


# Comparison with data (Glauber)

**MUSIC 2.0**

0-1% - LHC

$\zeta/s$ = constant

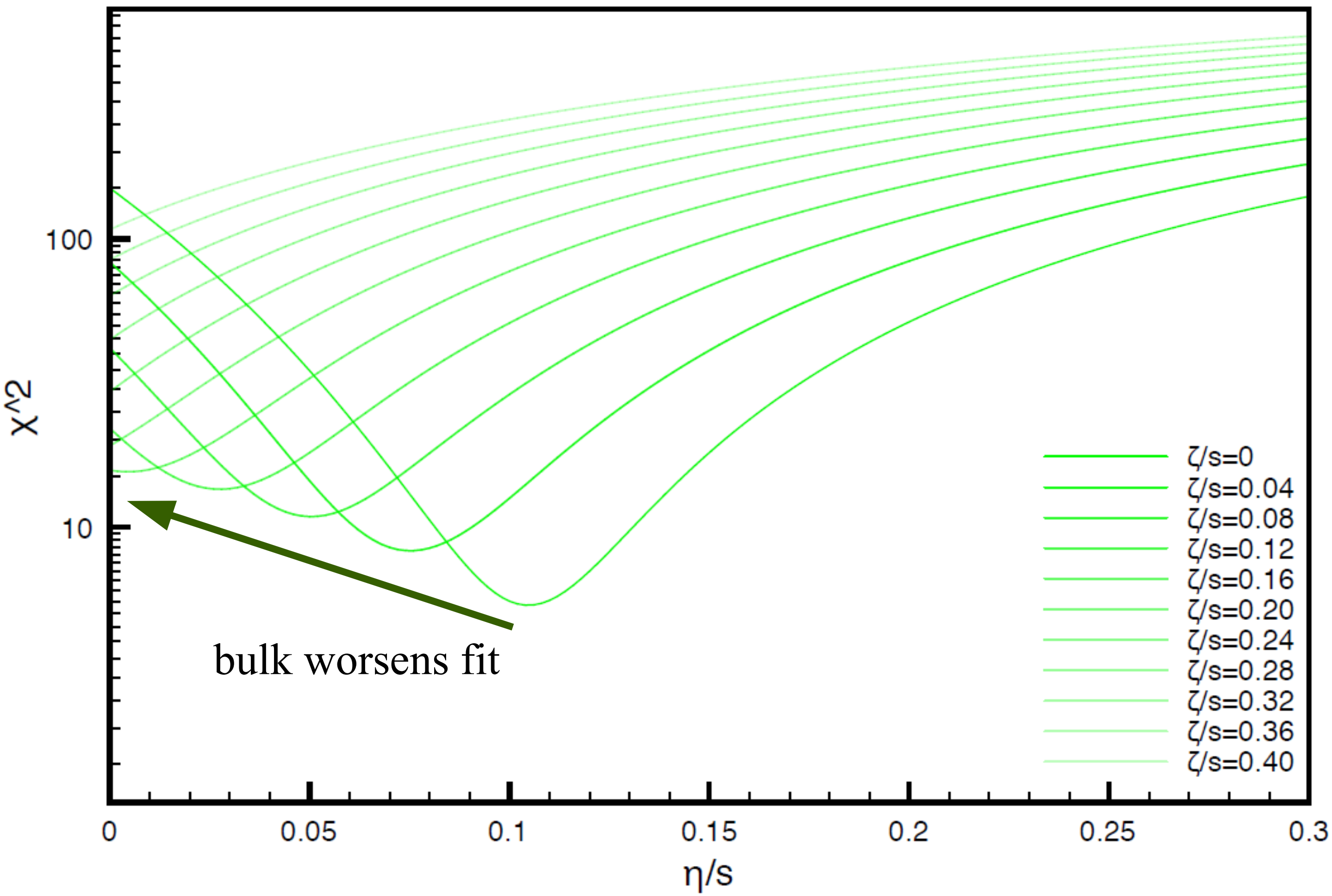


# Comparison with data (KLN)

**MUSIC 2.0**

0-1% - LHC

$\zeta/s$ = constant

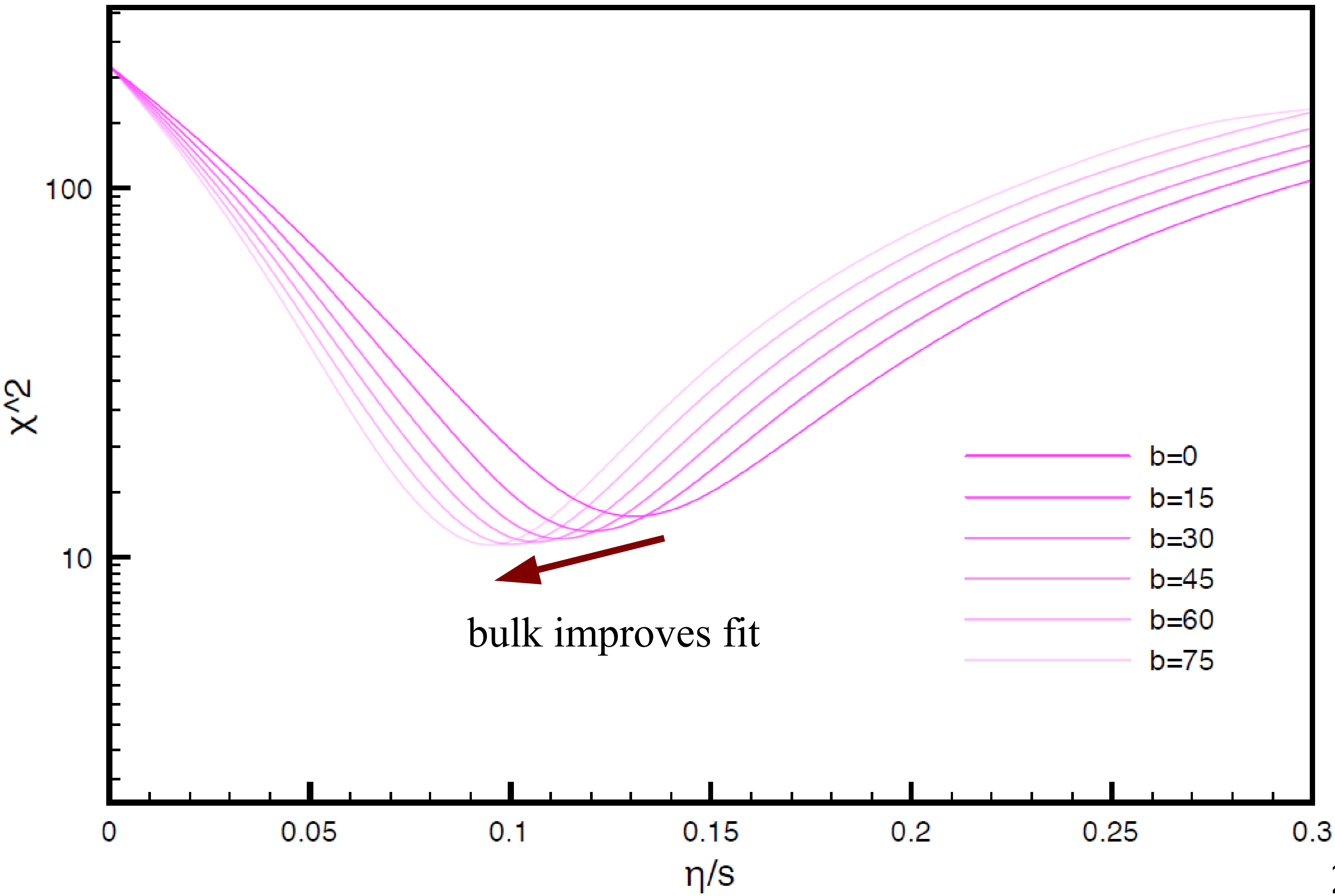


# Comparison with data (Glauber)

**MUSIC 2.0**

0-1% - LHC

$$\frac{\zeta}{s} = \mathrm{b} \times \frac{\eta}{s}\left(\frac{1}{3} - c_s^2\right)^2$$

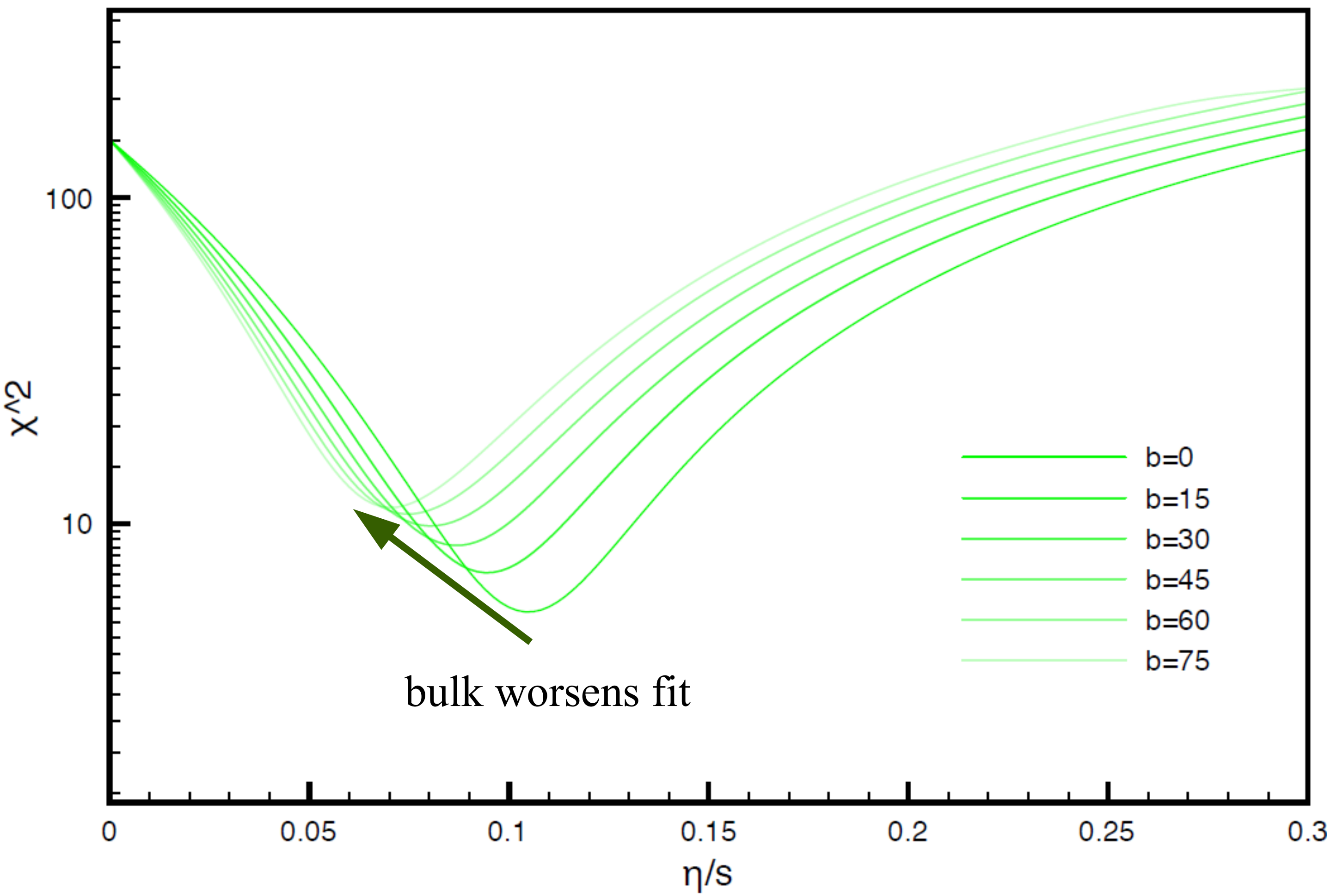


# Comparison with data (KLN)

**MUSIC 2.0**

0-1% - LHC

$$\frac{\zeta}{s} = \mathrm{b} \times \frac{\eta}{s}\left(\frac{1}{3} - c_s^2\right)^2$$



# Summary/conclusions

**We studied the effect of bulk viscosity and nonlinear terms on the azimuthal momentum anisotropies**

✔ We see a clear effect of bulk viscosity on flow; specially on v2 and v3

✔ Ultracentral collisions are a challenge to hydro models

**Ongoing:**

✗ Actual extraction of bulk viscosity from data

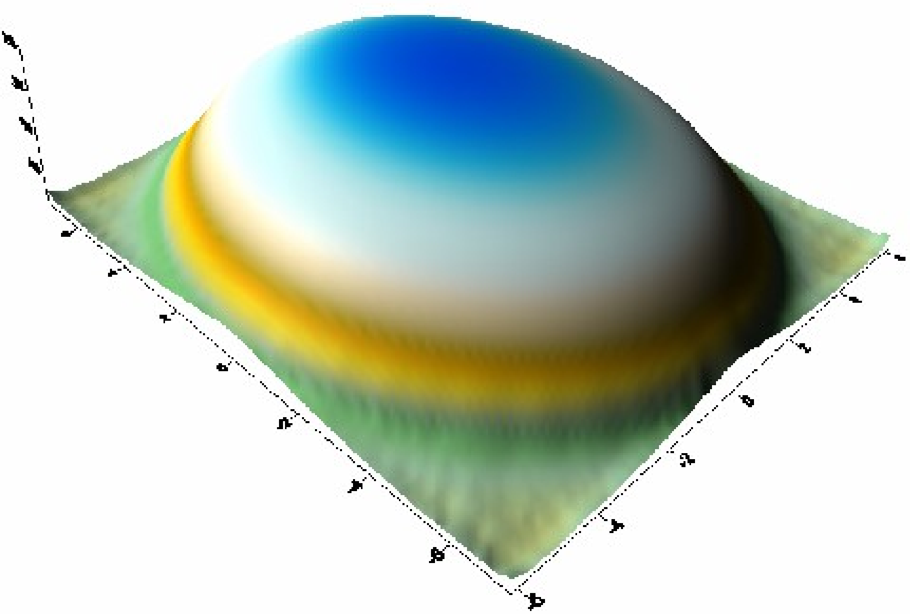
✗ Inclusion of heat flow

✗ Effects on photons and dileptons; and pPb

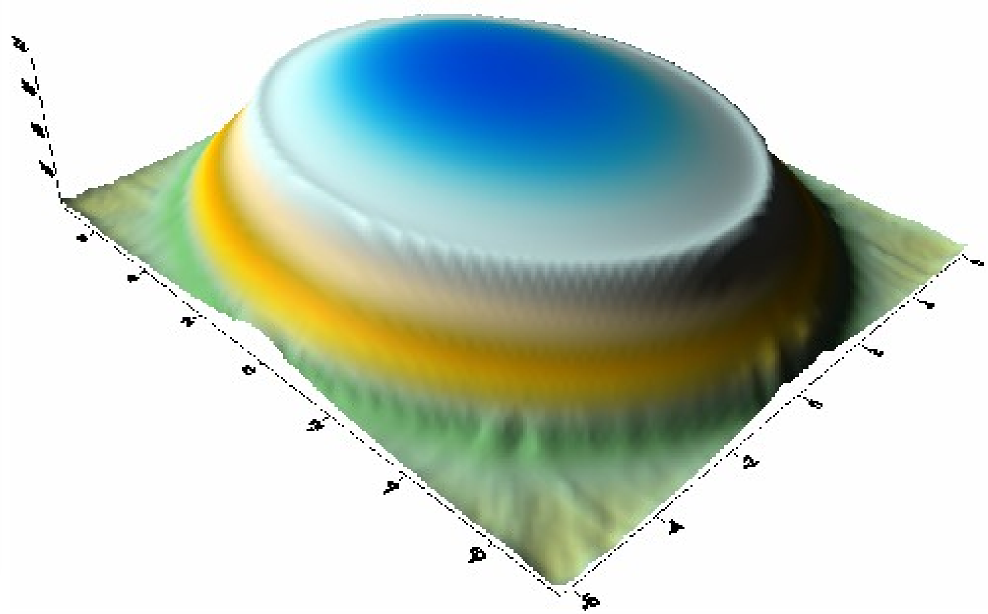
# My old calculation (with Kodama)

**Temperature profile** (Glauber IC, $\tau_0$=0.6fm)

**Ideal** - 2.1 fm

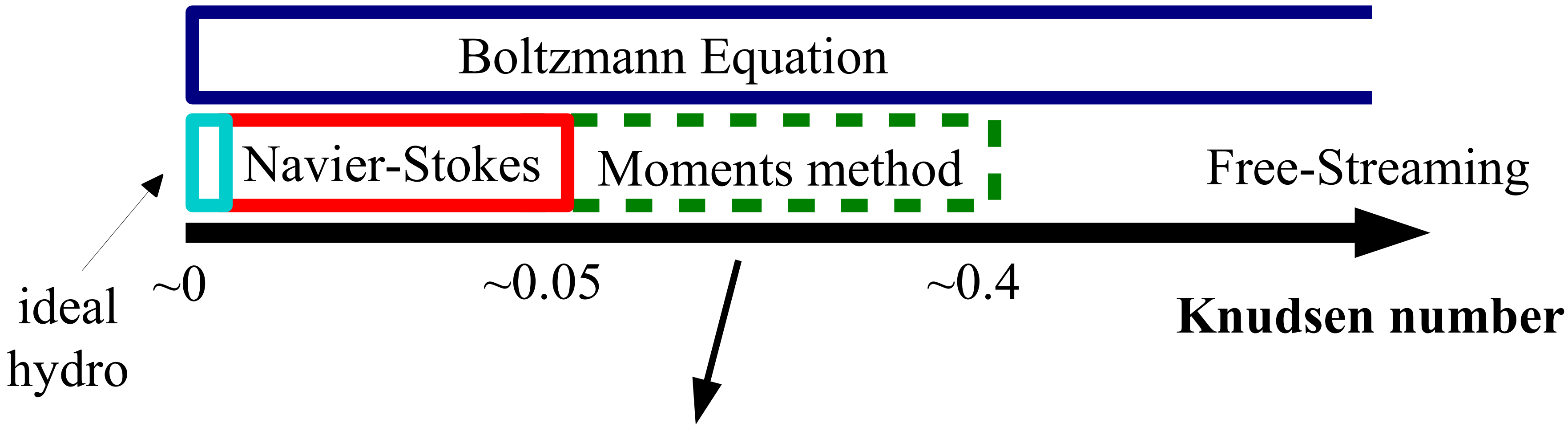


**Viscous** - 2.1 fm



# For a dilute gas

In terms of Knudsen number $\mathrm{Kn} = \dfrac{\ell_{\mathrm{micro}}}{L_{\mathrm{macro}}}$



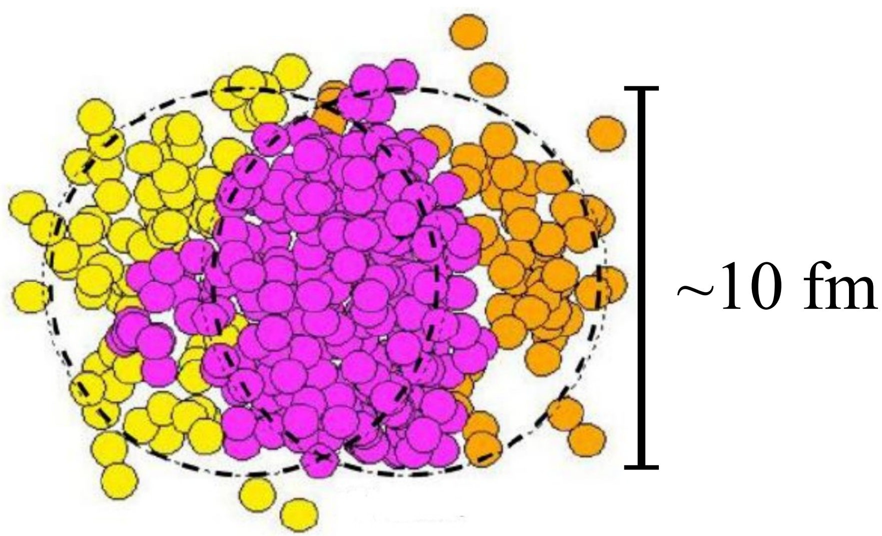

**heavy ion collisions (?)**

In a heavy ion collision ... $\mathrm{Kn} \sim \tau_\pi \nabla_\mu u^\mu \sim \dfrac{\eta}{s}\dfrac{1}{T\tau} \sim 0.2 - 1$

**Higher order terms can be important …**

# Heavy ion Collisions

- **smallest** fluid ever created
- **largest** gradients ever seen
- **expansion rate of the order of the relaxation time**




- From the fluid-dynamical point of view, challenging to describe

# Effect of nonlinear terms

**usual**

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta$$

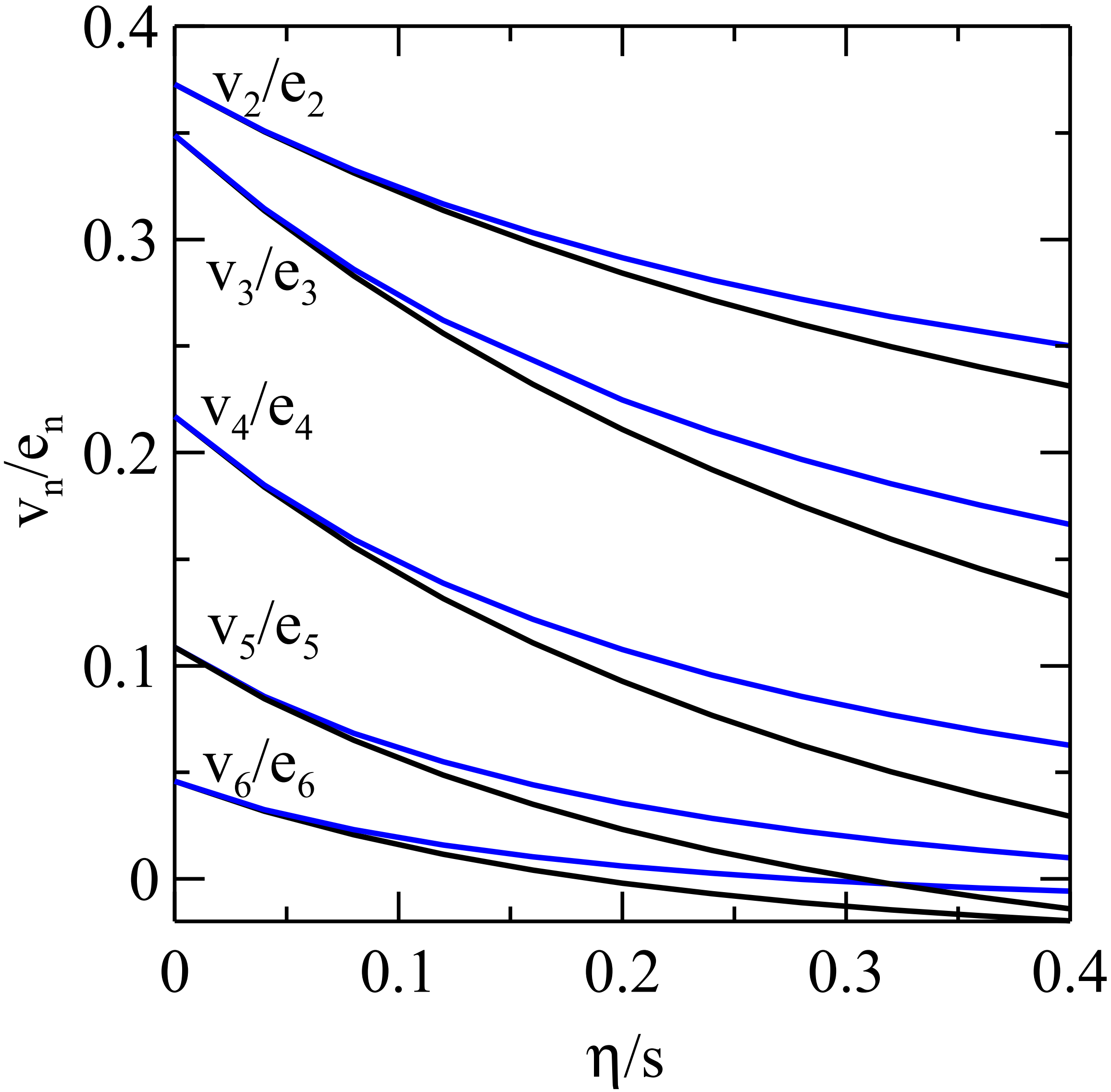
**with nonlinear terms**

$$\begin{aligned}\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} \;=\; & 2\eta\sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta \\ & + \frac{18}{35}\tau_\pi \frac{\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}}{\varepsilon_0 + P_0} - \frac{10}{7}\tau_\pi \pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha}.\end{aligned}$$

**can we see a difference?**

# Effect of nonlinear terms

**MUSIC 2.0**




0-1% - LHC

wo/ nonlinear terms

w/ nonlinear terms

**nonlinear terms reduce viscous effects**