Isotropization in Heavy Ion Collisions at High Energy

Yukawa Institute of Theoretical Physics, Kyoto, December 2013

T. Epelbaum, FG :
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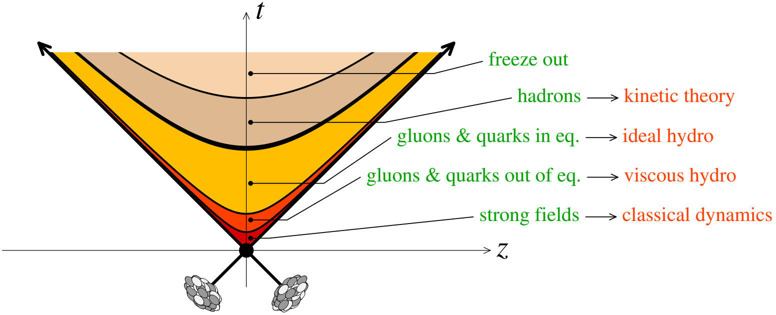
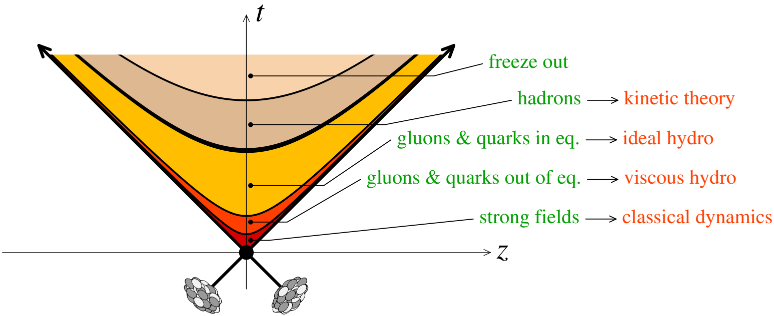
Outline

1. CGC description of heavy ion collisions
2. Isotropization in Heavy Ion Collisions

Stages of a nucleus-nucleus collision

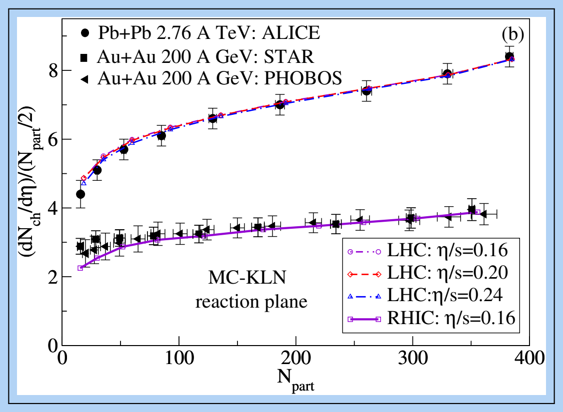


Stages of a nucleus-nucleus collision



- Well described as a fluid expanding into vacuum according to relativistic hydrodynamics

Evidence for hydrodynamical behavior



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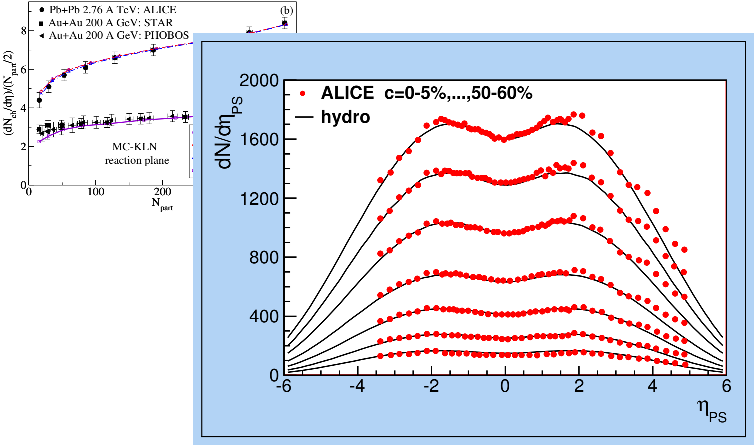

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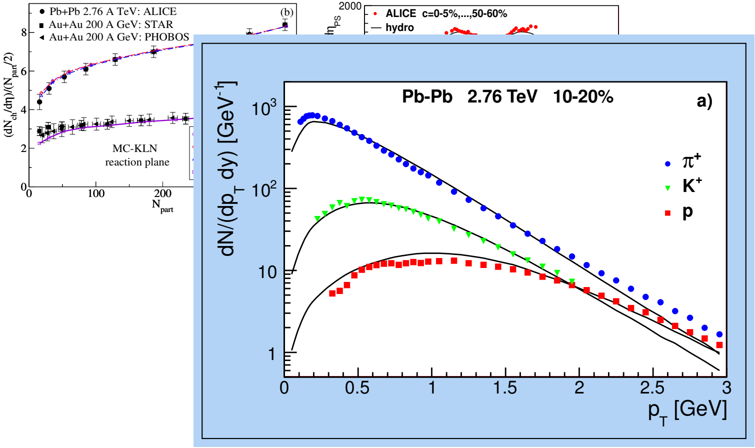

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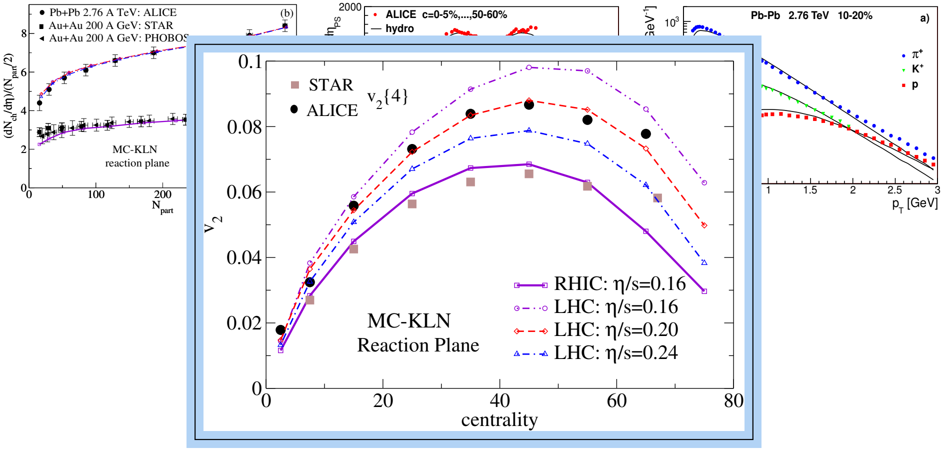

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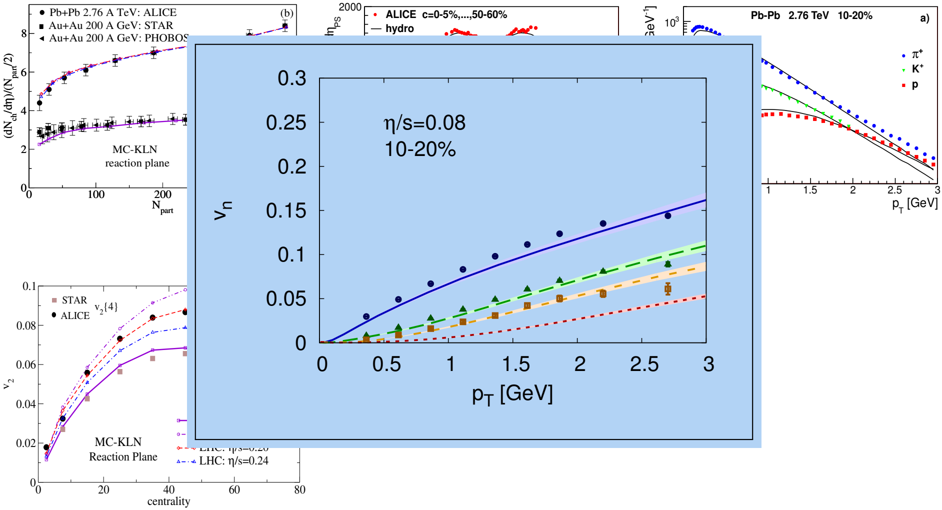

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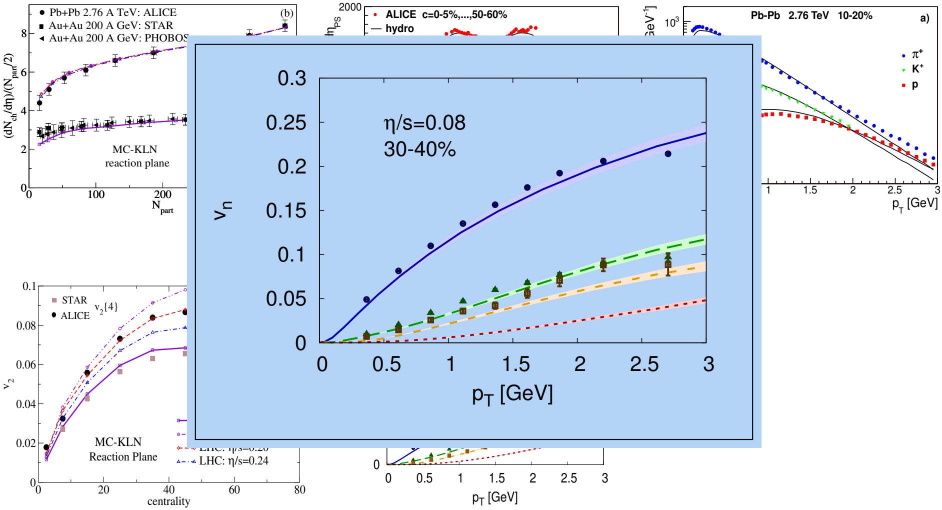

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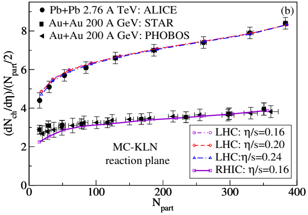

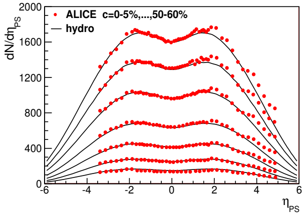

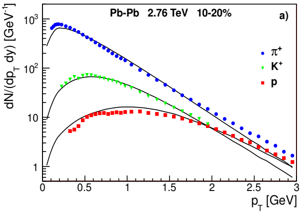

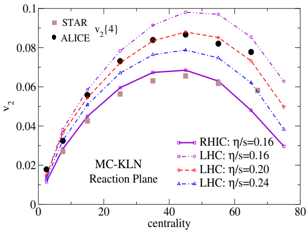

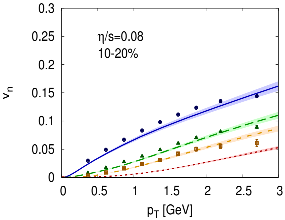

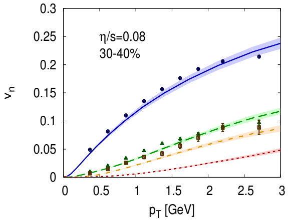

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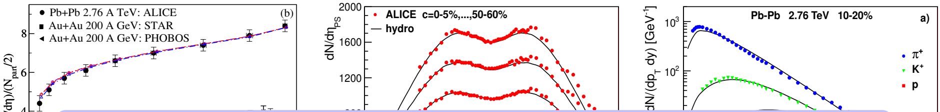
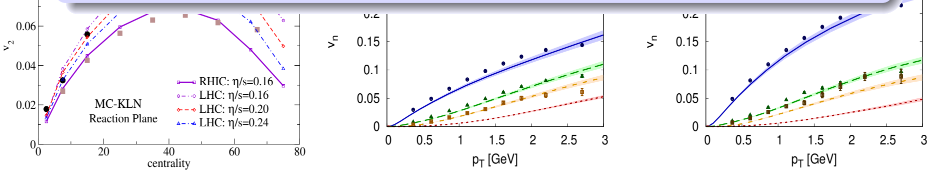


Impressive agreement, but: What makes hydrodynamics work so well?

- Near isotropic pressure tensor (in the local rest frame)
- Not too far from equilibrium
- Low viscosity

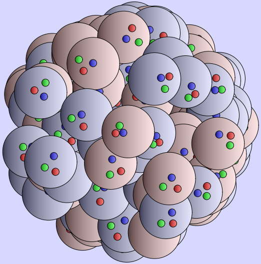
Where does this come from in pQCD...?



CGC Description of Heavy Ion Collisions

What do we need to know about nuclei?

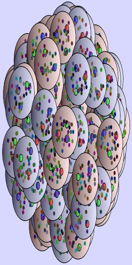
Nucleus at rest



- At low energy : valence quarks

What do we need to know about nuclei?

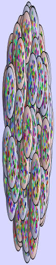
Slightly boosted nucleus



- At low energy : valence quarks
- At higher energy :
  - Lorenz contraction of longitudinal sizes
  - Time dilation ▷ slowing down of the internal dynamics
  - Gluons start becoming important

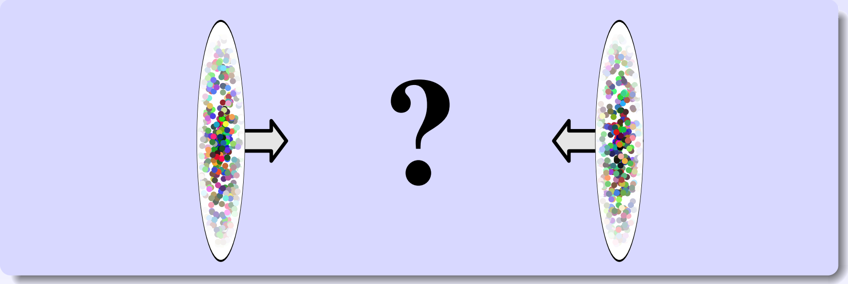
What do we need to know about nuclei?

High energy nucleus



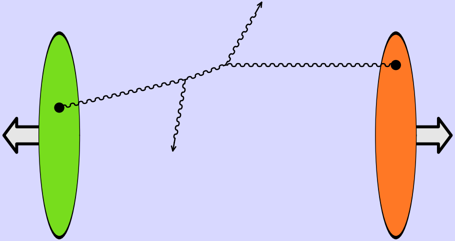
- At low energy : valence quarks
- At higher energy :
  - Lorenz contraction of longitudinal sizes
  - Time dilation ▷ slowing down of the internal dynamics
  - Gluons start becoming important
- At very high energy : gluons dominate

Multiple scatterings and gluon recombination



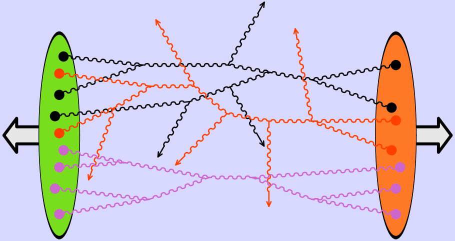
- Main difficulty: How to treat collisions involving a large number of partons?

Multiple scatterings and gluon recombination



- Dilute regime : one parton in each projectile interact
  - ▷ single parton distributions, standard perturbation theory

Multiple scatterings and gluon recombination



- Dense regime : multiparton processes become crucial
  - ▷ gluon recombinations are important (saturation)
  - ▷ multi-parton distributions
  - ▷ alternative approach : treat the gluons in the projectiles as external currents

$$\mathcal{L} = -\frac{1}{4}F^2 + A \cdot (J_1 + J_2)$$

(gluons only, field A for $k^+ < \Lambda$, classical source J for $k^+ > \Lambda$)

Color Glass Condensate

CGC = effective theory of small x gluons

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu +}\rho(x^-, \vec{x}_\perp) \qquad (0 < x^- < 1/\Lambda^+)$$

- The color sources ρ are random, and described by a probability distribution $W_{\Lambda^+}[\rho]$

- Slow partons ($k^+ < \Lambda^+$) cannot be considered static over the time-scales of the collision process
  ▷ must be treated as standard gauge fields
  ▷ eikonal coupling to the current J^μ : $A_\mu J^\mu$

Semantics

- Weakly coupled : $g \ll 1$

- Weakly interacting : $g\mathcal{A} \ll 1 \qquad g^2 f(\mathbf{p}) \ll 1$

  $$(2 \to 2) \gg (2 \to 3), (3 \to 2), \cdots$$

- Strongly interacting : $g\mathcal{A} \sim 1 \qquad g^2 f(\mathbf{p}) \sim 1$

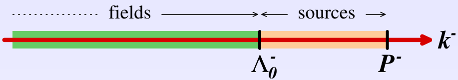
  $$(2 \to 2) \sim (2 \to 3) \sim (3 \to 2) \sim \cdots$$

  No well defined quasi-particles

CGC = weakly coupled, but strongly interacting effective theory

Power counting

- CGC effective theory with cutoff at the scale Λ_0 :

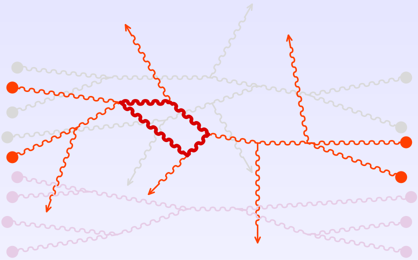


$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\mathcal{S}_{\rm YM}} + \int \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}} \; A_\mu$$

- Expansion in g^2 in the saturated regime:

$$T^{\mu\nu} \sim \frac{1}{g^2}\Big[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\Big]$$

Power counting



In the saturated regime: $J^\mu \sim g^{-1}$

$$g^{-2}\; g^{\#\text{ of external legs}}\; g^{2\times(\#\text{ of loops})}$$

- No dependence on the number of sources J^μ
  - ▷ infinite number of graphs at each order

Leading Order in g^2 : tree diagrams

- The Leading Order is the sum of all the tree diagrams

  Observables can be expressed in terms of classical solutions of Yang-Mills equations :

$$\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = J_1^\nu + J_2^\nu$$

- Boundary conditions for inclusive observables :

$$\lim_{x^0 \to -\infty} \mathcal{A}^\mu(x) = 0$$

Example : 00 component of the energy-momentum tensor

$$T_{\text{LO}}^{00} = \frac{1}{2}\Big[\underbrace{\mathcal{E}^2 + \mathcal{B}^2}_{\text{class. fields}}\Big]$$

Next to Leading Order in g^2 : 1-loop diagrams

Getting the NLO from tree graphs...

$$\mathcal{O}_{\rm NLO} = \left[\frac{1}{2}\int_{u,v} \Gamma_2(u,v)\,\mathbb{T}_u\mathbb{T}_v + \int_u \alpha(u)\,\mathbb{T}_u\right]\mathcal{O}_{\rm LO}$$

- $\mathbb{T}$ is the generator of the shifts of the initial value of the field :

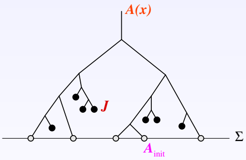
$$\mathbb{T}_u \sim \frac{\partial}{\partial \mathcal{A}_{\rm init}}$$

$$\exp\left[\int_u \alpha_u\,\mathbb{T}_u\right]\;\mathcal{O}\Big[\overbrace{\mathcal{A}_\tau(\underbrace{\mathcal{A}_{\rm init}}_{\text{init. value}})}^{\text{class. field at }\tau}\Big] = \mathcal{O}\Big[\mathcal{A}_\tau(\underbrace{\mathcal{A}_{\rm init}+\alpha}_{\text{shifted init. value}})\Big]$$

Shift operator $\mathbb{T}$ – Definition

Equations of motion for a field $\mathcal{A}$ and a small perturbation α

$$
\begin{aligned}
\square\mathcal{A} + V'(\mathcal{A}) &= J \\
[\square + V''(\mathcal{A})]\ \alpha &= 0
\end{aligned}
$$



- Getting the perturbation by shifting the initial condition of $\mathcal{A}$ at one point :

$$
\alpha(x) = \int_u \alpha_u\, \mathbb{T}_u\ \mathcal{A}(x)
$$

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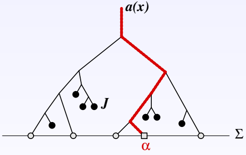

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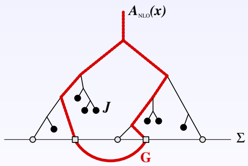


- Getting the perturbation by shifting the initial condition of $\mathcal{A}$ at one point :

$$\alpha(x) = \int_u \alpha_u\, \mathbb{T}_u\ \mathcal{A}(x)$$

- A loop is obtained by shifting the initial condition of $\mathcal{A}$ at two points

Initial state logarithms

- In the CGC, upper cutoff on the loop momentum : $k^\pm < \Lambda$, to avoid double counting with the sources $J^\nu_{1,2}$
  $\triangleright$ logarithms of the cutoff

Central result for factorization at Leading Log

$$\frac{1}{2}\int_{u,v} \Gamma_2(u,v)\,\mathbb{T}_u\mathbb{T}_v + \int_u \alpha(u)\,\mathbb{T}_u =$$

$$= \log\left(\Lambda^+\right)\,\mathcal{H}_1 + \log\left(\Lambda^-\right)\,\mathcal{H}_2 + \text{terms w/o logs}$$

$\mathcal{H}_{1,2}$ = JIMWLK Hamiltonians of the two nuclei

- No mixing between the logs of the two nuclei
- Since the LO $\leftrightarrow$ NLO relationship is the same for all inclusive observables, these logs have a universal structure

Factorization of the logarithms

Inclusive observables at Leading Log accuracy

$$\left\langle \mathcal{O} \right\rangle_{\text{Leading Log}} = \int \left[D\rho_1 \; D\rho_2 \right] \; W_1\left[\rho_1\right] \; W_2\left[\rho_2\right] \; \underbrace{\mathcal{O}_{\text{LO}}[\rho_1, \rho_2]}_{\text{fixed } \rho_{1,2}}$$

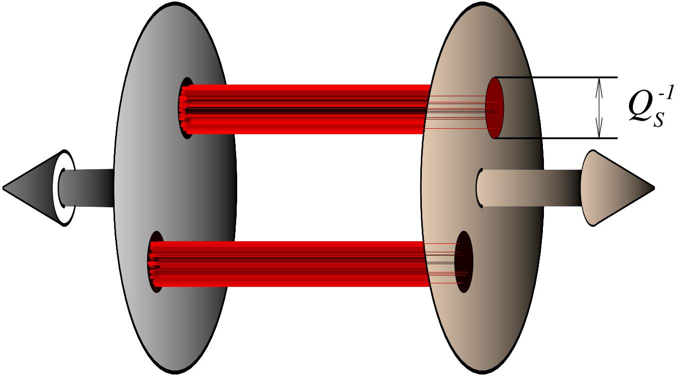
- Logs absorbed into the scale evolution of $W_{1,2}$

$$\Lambda \frac{\partial W}{\partial \Lambda} = \mathcal{H}\, W \qquad \text{(JIMWLK equation)}$$

- Universality : the same W's for all inclusive observables

Isotropization in Heavy Ion Collisions

Energy momentum tensor of the initial classical field



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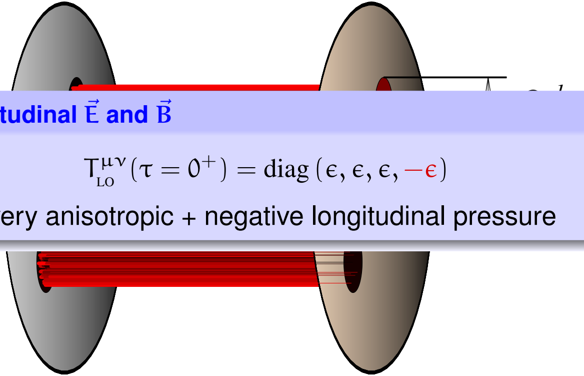
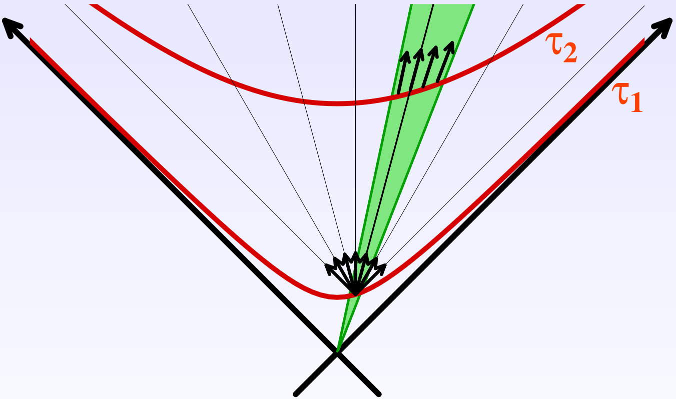


$T^{\mu\nu}$ for longitudinal $\vec{E}$ and $\vec{B}$

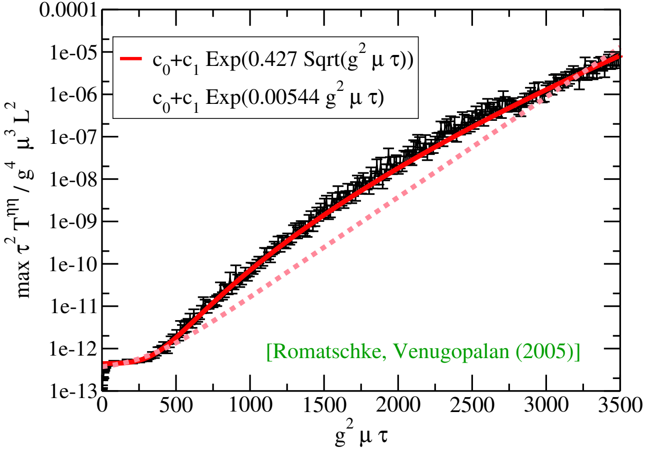
$$T^{\mu\nu}_{\text{LO}}(\tau = 0^+) = \text{diag}\,(\epsilon, \epsilon, \epsilon, -\epsilon)$$

▷ very anisotropic + negative longitudinal pressure

Competition between Expansion and Isotropization



Weibel instabilities for small perturbations



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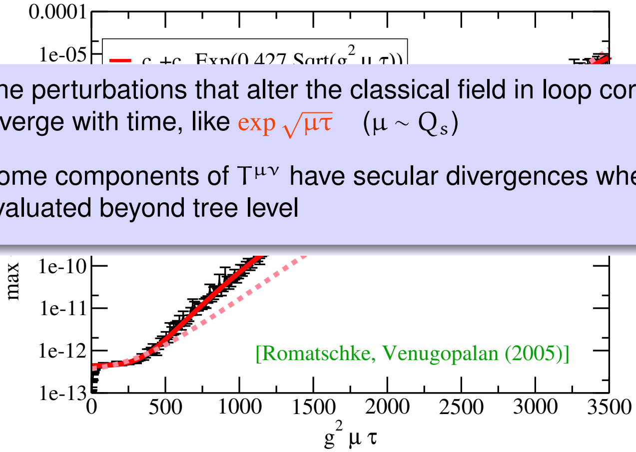
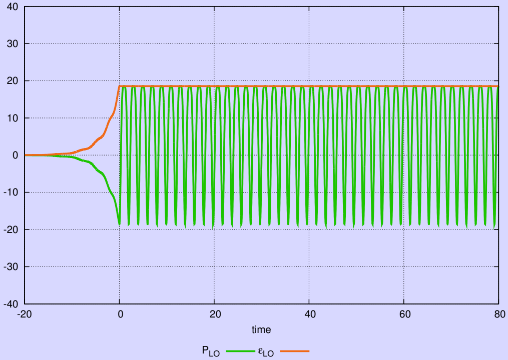


- The perturbations that alter the classical field in loop corrections diverge with time, like $\exp\sqrt{\mu\tau}$ $\quad(\mu \sim Q_s)$
- Some components of $T^{\mu\nu}$ have secular divergences when evaluated beyond tree level

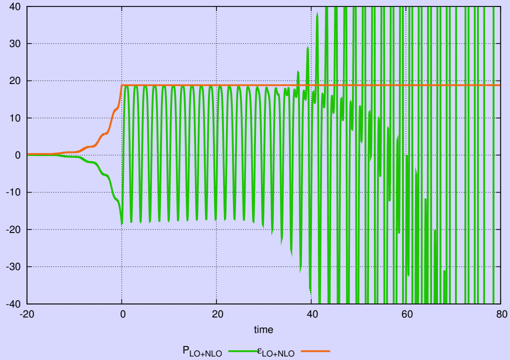
Example of pathologies in fixed order calculations (scalar theory)

LO



Example of pathologies in fixed order calculations (scalar theory)

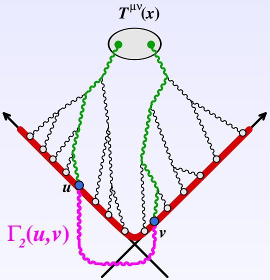
LO + NLO



- Small correction to the energy density (protected by energy conservation)
- Secular divergence in the pressure

Improved power counting and resummation

$$\text{Loop} \sim g^2 \qquad , \qquad \mathbb{T} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $(g e^{\sqrt{\mu\tau}})^2$

Improved power counting and resummation

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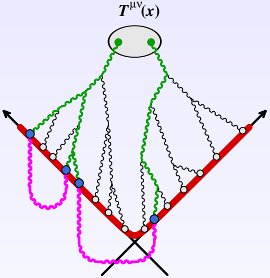
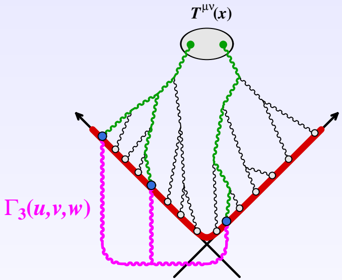


- 1 loop : $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops : $(ge^{\sqrt{\mu\tau}})^4$

Improved power counting and resummation

$$\text{Loop} \sim g^2 \qquad , \qquad \mathbb{T} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $\left(g e^{\sqrt{\mu\tau}}\right)^2$
- 2 disconnected loops : $\left(g e^{\sqrt{\mu\tau}}\right)^4$
- 2 entangled loops : $g\left(g e^{\sqrt{\mu\tau}}\right)^3$ ▷ subleading

Leading terms

- All disconnected loops to all orders
  - ▷ exponentiation of the 1-loop result

Resummation of the leading secular terms

$$T^{\mu\nu}_{\text{resummed}} = \exp\left[\frac{1}{2}\int_{u,v} \Gamma_2(u,v)\,\mathbb{T}_u\mathbb{T}_v\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}]$$

$$= \underbrace{T^{\mu\nu}_{\text{LO}} + T^{\mu\nu}_{\text{NLO}}}_{\text{in full}} + \underbrace{T^{\mu\nu}_{\text{NNLO}} + \cdots}_{\text{partially}}$$

- The exponentiation of the 1-loop result collects all the terms with the worst time behavior

Resummation of the leading secular terms

$$
\begin{aligned}
T^{\mu\nu}_{\text{resummed}} &= \exp\left[\frac{1}{2}\int\limits_{u,v} \Gamma_2(u,v)\mathbb{T}_u\mathbb{T}_v\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}] \\
&= \int[Da]\,\exp\left[-\frac{1}{2}\int\limits_{u,v} a(u)\Gamma_2^{-1}(u,v)a(v)\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}+a]
\end{aligned}
$$

- The exponentiation of the 1-loop result collects all the terms with the worst time behavior
- Equivalent to Gaussian fluctuations of the initial field + classical time evolution
- At $Q_s\tau_0 \ll 1$: $\mathcal{A}_{\text{init}} \sim Q_s/g \quad , \quad a \sim Q_s$

Note : Classical field + Fluctuations = Coherent state

- This Gaussian distribution of initial fields is the Wigner distribution of a **coherent state** $\big|\mathcal{A}\big\rangle$

  Coherent states are the "most classical quantum states"

  Their Wigner distribution has the minimal support permitted by the uncertainty principle ($\mathcal{O}(\hbar)$ for each mode)

- $\big|\mathcal{A}\big\rangle$ is not an eigenstate of the full Hamiltonian
  $\triangleright$ decoherence via interactions

What needs to be done?

Main steps

1. Determine the 2-point function $\Gamma_2(u, v)$ that defines the Gaussian fluctuations, for the initial time $Q_s\tau_0$ of interest

   Note : this is an initial value problem, whose outcome is uniquely determined by the state of the system at $x^0 = -\infty$, and depends on the history of the system from $x^0 = -\infty$ to $\tau = \tau_0$

   Problem solvable only if the fluctuations are weak, $a^\mu \ll Q_s/g$

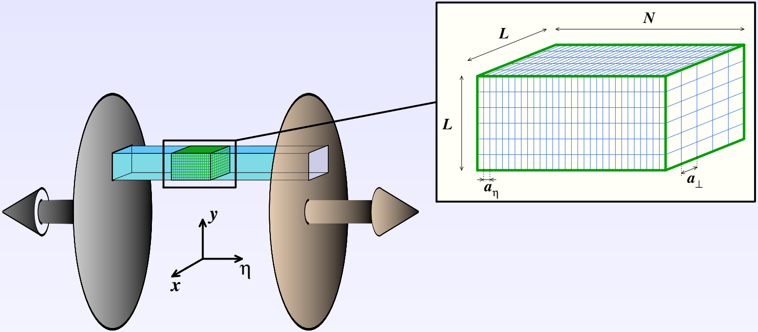
   $Q_s\tau_0 \ll 1$ necessary for the fluctuations to be Gaussian

2. Solve the classical Yang-Mills equations from τ_0 to τ_f

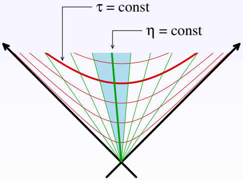
   Note : the problem as a whole is boost invariant, but individual field configurations are not $\implies$ 3+1 dimensions necessary

3. Do a Monte-Carlo sampling of the fluctuating initial conditions

Discretization of the expanding volume



- Comoving coordinates : $\tau, \eta, x_\perp$
- Only a sub-volume is simulated + periodic boundary conditions
- $L^2 \times N$ lattice

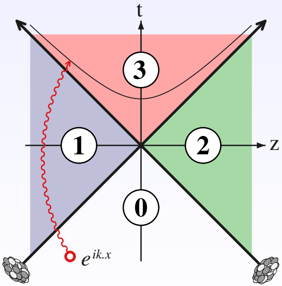


Gaussian spectrum of fluctuations

Expression of the variance (from 1-loop considerations)

$$\Gamma_2(u,v) = \int_{\text{modes } \mathbf{k}} a_{\mathbf{k}}(u)\, a^*_{\mathbf{k}}(v)$$

$$\Big[\mathcal{D}_\rho \mathcal{D}^\rho \delta_\mu^\nu - \mathcal{D}_\mu \mathcal{D}^\nu + ig\,\mathcal{F}_\mu{}^\nu\Big] a_{\mathbf{k}}^\mu = 0 \quad , \quad \lim_{x^0 \to -\infty} a_{\mathbf{k}}(x) \sim e^{ik\cdot x}$$



0. $\mathcal{A}^\mu = 0$, trivial

1,2. $\mathcal{A}^\mu =$ pure gauge, analytical solution

3. $\mathcal{A}^\mu$ non-perturbative
$\Rightarrow$ expansion in $Q_s\tau$

- We need the fluctuations in Fock-Schwinger gauge $x^+a^- + x^-a^+ = 0$
- Delicate light-cone crossings, since $\mathcal{F}^{\mu\nu} = \infty$ there

Mode functions for given quantum numbers : $\nu, k_\perp, \lambda, c$

$$a^i = \beta^{+i} + \beta^{-i} \qquad a^\eta = \mathcal{D}^i\left(\frac{\beta^{+i}}{2+i\nu} - \frac{\beta^{-i}}{2-i\nu}\right)$$

$$e^i = -i\nu\left(\beta^{+i} - \beta^{-i}\right) \qquad e^\eta = -\mathcal{D}^i\left(\beta^{+i} - \beta^{-i}\right)$$

$$\beta^{+i} \equiv e^{\frac{\pi\nu}{2}}\Gamma(-i\nu)e^{i\nu\eta}\,\mathcal{U}_1^\dagger(\mathbf{x}_\perp)\int_{\mathbf{p}_\perp} e^{i\mathbf{p}_\perp\cdot\mathbf{x}_\perp}\,\widetilde{\mathcal{U}}_1(\mathbf{p}_\perp+\mathbf{k}_\perp)\left(\frac{p_\perp^2\tau}{2k_\perp}\right)^{i\nu}\left(\delta^{ij}-2\frac{p_\perp^i p_\perp^j}{p_\perp^2}\right)\epsilon_\lambda^j$$

$$\beta^{-i} \equiv e^{-\frac{\pi\nu}{2}}\Gamma(i\nu)e^{i\nu\eta}\,\mathcal{U}_2^\dagger(\mathbf{x}_\perp)\int_{\mathbf{p}_\perp} e^{i\mathbf{p}_\perp\cdot\mathbf{x}_\perp}\,\widetilde{\mathcal{U}}_2(\mathbf{p}_\perp+\mathbf{k}_\perp)\left(\frac{p_\perp^2\tau}{2k_\perp}\right)^{-i\nu}\left(\delta^{ij}-2\frac{p_\perp^i p_\perp^j}{p_\perp^2}\right)\epsilon_\lambda^j$$

- Linearized EOM and Gauss' law satisfied up to terms of order $(Q_s\tau)^2$
- Fock-Schwinger gauge condition ($a^\tau = e^\tau = 0$)
- Evolved from plane waves in the remote past

Computational cost

Initial Conditions

- Naive :

$$N \log(N) \times L^4 \log(L) \times N_{\text{confs}}$$

- Better algorithm :

$$N \log(N) \times L^4 \times \left(\log(L) + N_{\text{confs}}\right)$$

Time evolution

$$N \times L^2 \times N_{\text{confs}} \times N_{\text{time steps}}$$

Useful statistics (at fixed volume)

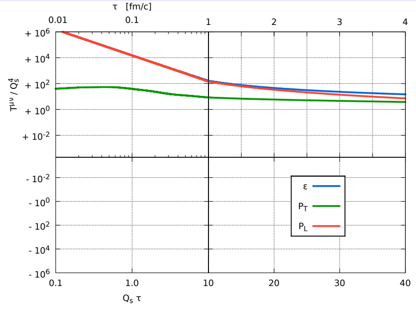
$$\sqrt{N_{\text{confs}}} \sim \frac{g^2}{(a_\perp a_\eta)^2}$$

Ultraviolet subtractions

- Fixed spacing in η

  $\Longleftrightarrow \quad \Lambda_z \sim \tau^{-1}$

  Bare ϵ and P_L diverge as τ^{-2} when $\tau \to 0^+$



Ultraviolet subtractions

- Fixed spacing in η
  $\Longleftrightarrow \quad \Lambda_z \sim \tau^{-1}$

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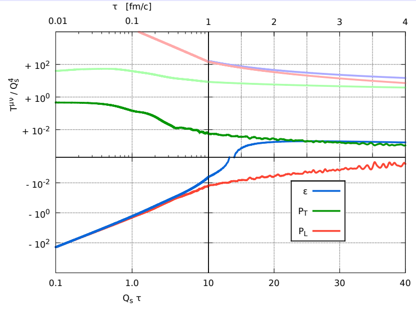
- Zero point energy $\sim \Lambda_\perp^2 \Lambda_z^2$:

  Subtracted by redoing the calculation with the sources turned off



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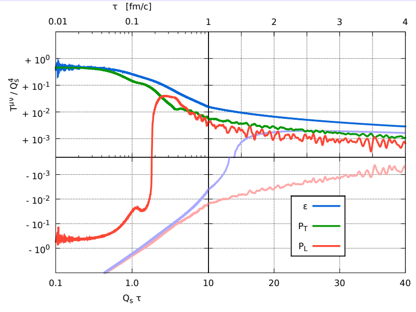


- Subleading divergences $\sim \Lambda_z^2$ in ϵ and P_L :

  Exist only at finite $\perp$ lattice spacing (not in the continuum)

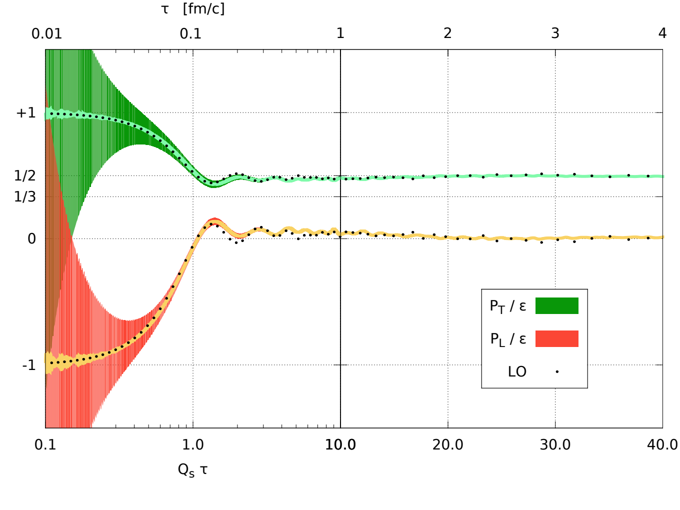
  Same counterterm in ϵ and P_L to preserve $T^\mu{}_\mu = 0$

  Must be of the form $A \times \tau^{-2}$ to preserve Bjorken's law

  At the moment, not calculated from first principles $\Rightarrow$ A fitted

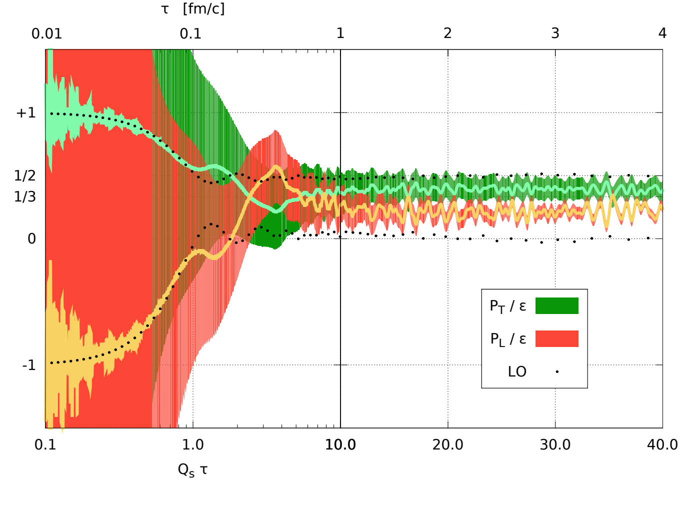
Time evolution of P_T/ϵ and P_L/ϵ $(64 \times 64 \times 128$ lattice)

$g = 0.1$ $(N_{\text{confs}} = 200)$



Time evolution of P_T/ϵ and P_L/ϵ $(64 \times 64 \times 128$ lattice)

$g = 0.5$ ($N_{\text{confs}} = 2000$)



Summary

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- CGC calculation of the energy momentum tensor in a nucleus-nucleus collision, up to $Q_s\tau \lesssim 20$
- Method :
  - Classical statistical method
  - Initial Gaussian fluctuations : analytical, from a 1-loop calculation
  - Time evolution : numerical, 3+1d Yang-Mills equations on a lattice
- Accuracy :

  $\big\langle 0_{\rm in}\big|T^{\mu\nu}(\tau,x)\big|0_{\rm in}\big\rangle$ at LO + NLO + leading secular terms
- Results :
  - Sizeable longitudinal pressure ($P_L/P_T \sim 60\%$ for $g = 0.5$)
  - Typical timescale : $Q_s\tau \sim 2-3$