Stony Brook University | The State University of New York

Event-by-event flow at LHC and event-shape-selection technique

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with Peng huo and Soumya Mohapatra arxiv:1311.7091

New Frontiers in QCD 2013

--- Insight into QCD matter from heavy-ion collisions ---

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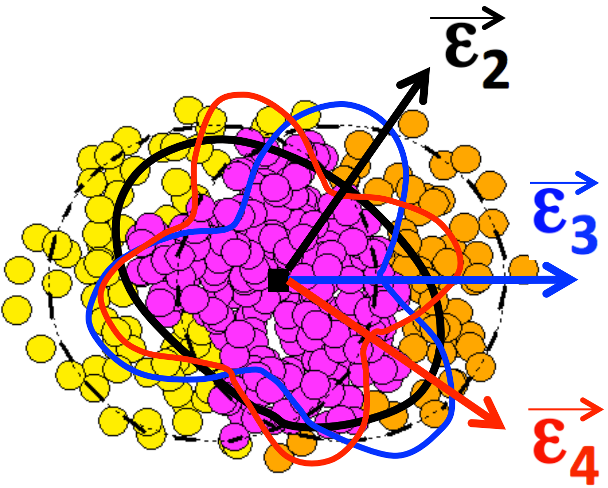
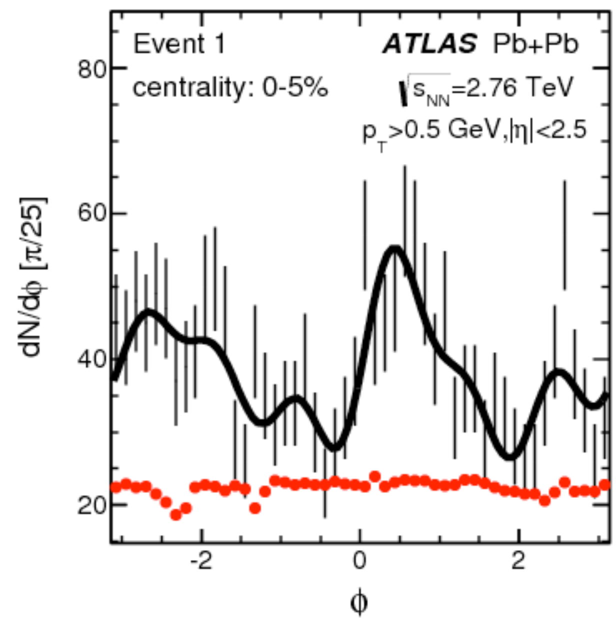
Geometry and harmonic flow



$$\epsilon_n = \frac{\sqrt{\langle r^n \cos n\phi \rangle^2 + \langle r^n \sin n\phi \rangle^2}}{\langle r^n \rangle}$$

$$\tan(n\Phi_n^*) = \frac{\langle r^n \sin(n\phi) \rangle}{\langle r^n \cos(n\phi) \rangle} + \frac{\pi}{n}$$



$$\frac{dN}{d\phi} \propto 1 + 2\sum_n \mathrm{v}_\mathrm{n} \cos n\left(\phi - \Phi_n\right)$$

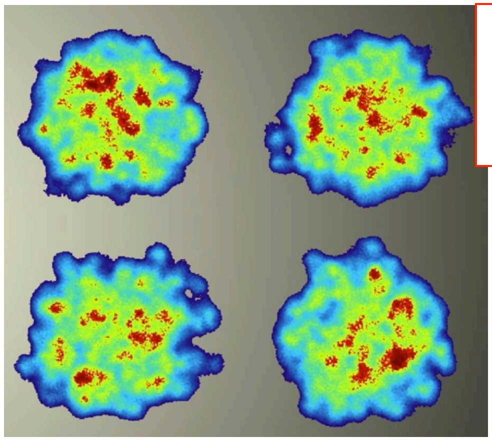
- How $(\varepsilon_n, \Phi_n^*)$ are transferred to (v_n, Φ_n)?
- What is the nature of final state (non-linear) dynamics?
- Are there dynamical p_T or rapidity fluctuations?

Event-by-event flow observables

One little-bang event

$$\frac{dN}{d\phi} \propto 1 + 2\sum_n \mathrm{v}_n \cos n(\phi - \Phi_n)$$

Many little-bang events



$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\mathrm{evts}}} \frac{dN_{\mathrm{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

$$\frac{dN_{\mathrm{evts}}}{d\Phi_1 d\Phi_2 ... d\Phi_l} \propto \sum_{c_n=-\infty}^{\infty} a_{c_1,c_2,...,c_l} \cos(c_1\Phi_1 + c_2\Phi_2 ... + c_l\Phi_l)$$

$$a_{c_1,c_2,...,c_l} = \langle \cos(c_1\Phi_1 + c_2\Phi_2 + ... + c_l\Phi_l)\rangle$$

$$\langle \cos(c_1\Phi_1 + 2c_2\Phi_2 ... + lc_l\Phi_l)\rangle, c_1 + 2c_2 ... + lc_l = 0.$$

So far measured:

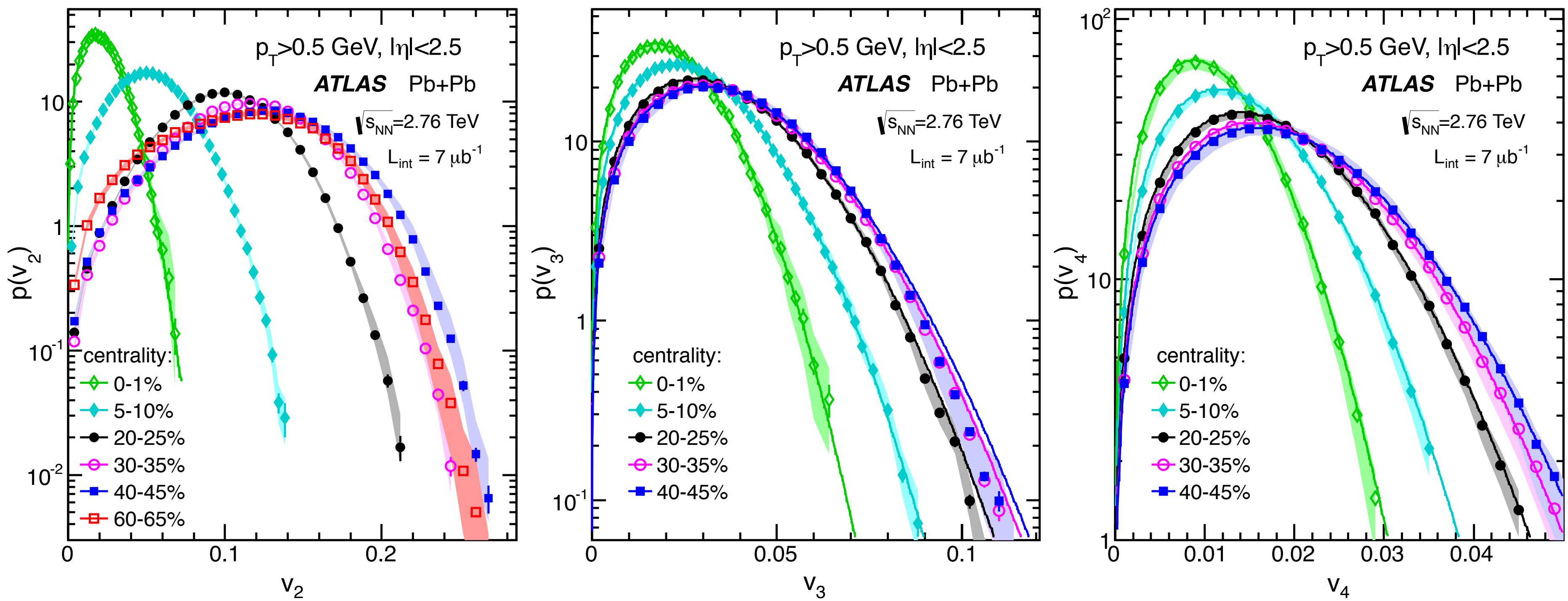
- EbyE v_n: $p(v_2)$, $p(v_3)$ and $p(v_4)$
- Event plane correlation: $p(\Phi_n,\Phi_m)$ and $p(\Phi_n,\Phi_m,\Phi_L)$

Other observables?

- $p(v_n,v_m)$,
- $p(v_n,\Phi_n,\Phi_{m,....})$

with event shape selection

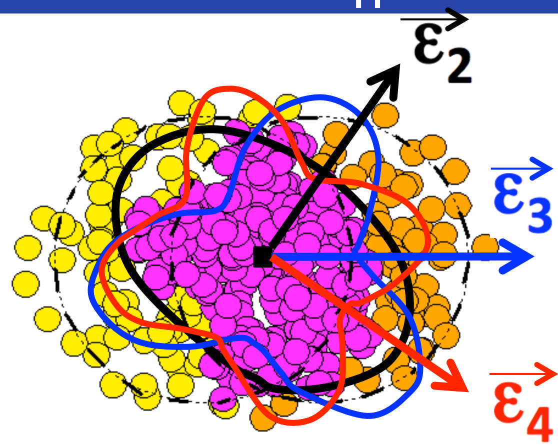
$p(v_n)$ distributions



Probability distribution for v_2, v_3 and v_4 in many centrality ranges

JHEP11(2013)183

Expectation for v_n fluctuations



arXiV: 0708.0800, 0809.2949

$$\vec{\varepsilon}_n = \left(\frac{\langle r^n \cos(n\phi) \rangle}{\langle r^n \rangle}, \frac{\langle r^n \sin(n\phi) \rangle}{\langle r^n \rangle} \right)$$

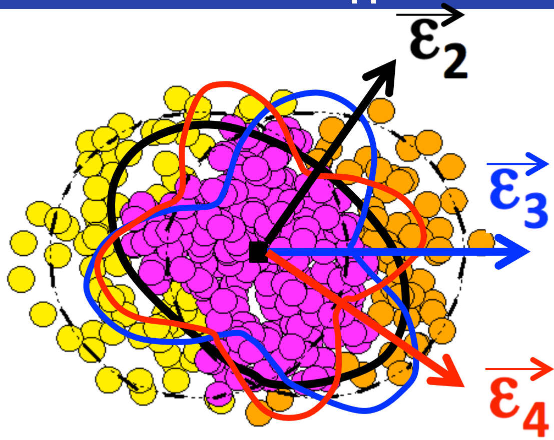
$$\vec{\varepsilon}_n = \vec{\varepsilon}_n^{RP} + \vec{\Delta}_n^{fluc}$$

$$p(\vec{\varepsilon}_n) \propto \exp\left(\frac{-(\vec{\varepsilon}_n - \vec{\varepsilon}_n^{RP})^2}{2\delta_{\varepsilon_n}^2} \right)$$

$$\vec{\varepsilon}_n^{RP} \rightarrow Mean\,Geometry$$

$$\delta_{\varepsilon_n} \rightarrow Fluctuations$$

Expectation for v_n fluctuations



$$\vec{\varepsilon}_n = \left(\frac{\langle r^n \cos(n\phi) \rangle}{\langle r^n \rangle}, \frac{\langle r^n \sin(n\phi) \rangle}{\langle r^n \rangle} \right)$$

arXiV: 0708.0800, 0809.2949

$$\vec{v}_n = (v_n \cos n\Phi_n, v_n \sin n\Phi_n)$$

$$\vec{\varepsilon}_n = \vec{\varepsilon}_n^{RP} + \vec{\Delta}_n^{fluc}$$

$$\vec{v}_n = \vec{v}_n^{RP} + \vec{p}_n^{fluc}$$

$$p(\vec{\varepsilon}_n) \propto \exp\left(\frac{-(\vec{\varepsilon}_n - \vec{\varepsilon}_n^{RP})^2}{2\delta_{\varepsilon_n}^2} \right)$$

$$\vec{v}_n \propto \vec{\varepsilon}_n$$

$$p(\vec{v}_n) \propto \exp\left(\frac{-(\vec{v}_n - \vec{v}_n^{RP})^2}{2\delta_n^2} \right)$$

$\vec{\varepsilon}_n^{RP} \rightarrow$ *Mean Geometry*

$\delta_{\varepsilon_n} \rightarrow$ *Fluctuations*

$\vec{v}_n^{RP} \rightarrow$ *Mean Geometry*

$\delta_n \rightarrow$ *Fluctuations*

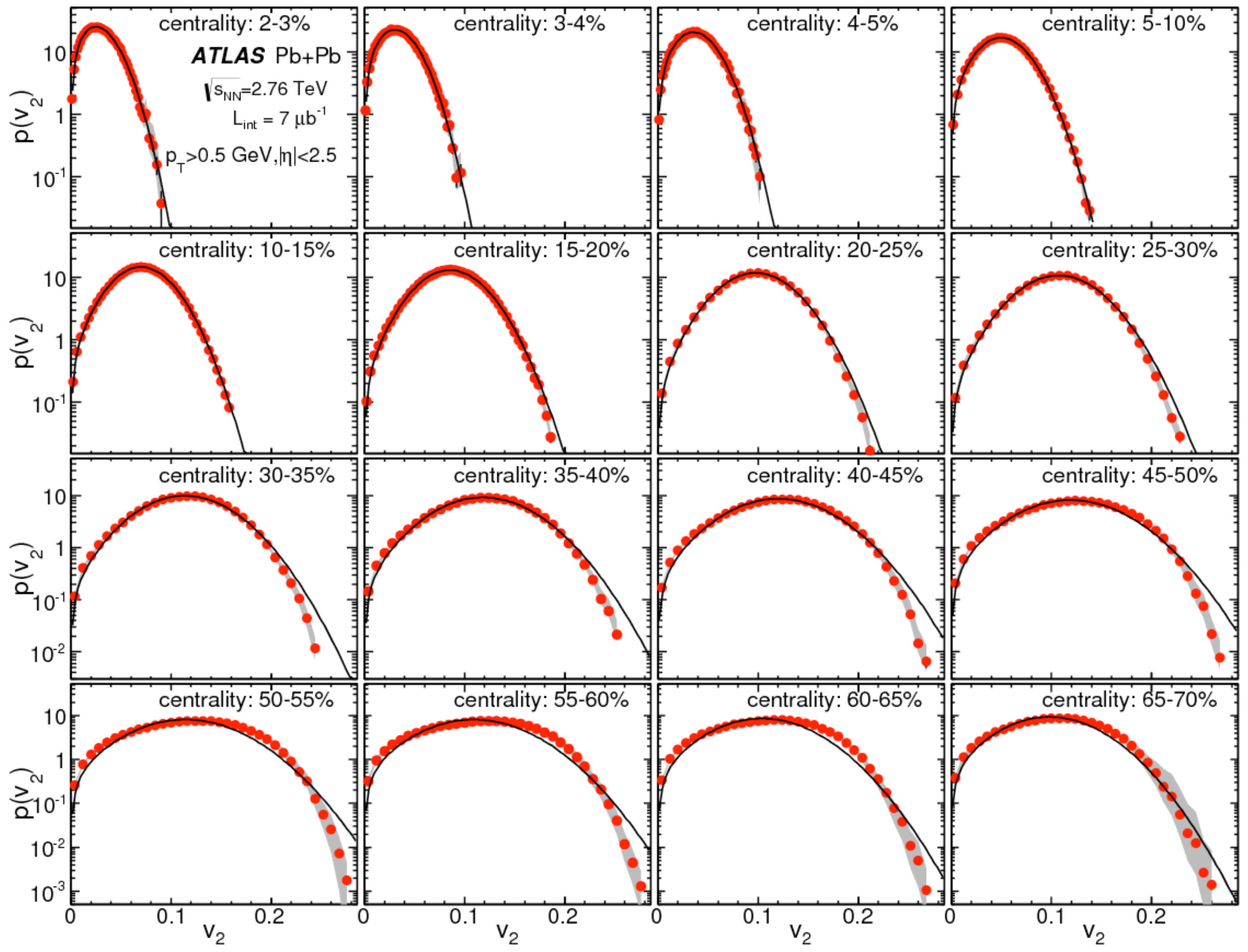
$$p(v_n) \propto v_n \exp\left(\frac{-(v_n^2 + (v_n^{RP})^2)}{2\delta_n^2} \right) I_0\left(\frac{v_n v_n^{RP}}{\delta_n^2}\right)$$

Are flow fluctuations Gaussian?

$$p(v_2) \propto v_2 \exp\left(\frac{-(v_2^2 + (v_2^{RP})^2}{2\delta^2}\right) I_0\left(\frac{v_2 v_2^{RP}}{\delta^2}\right)$$

First indication of non-Gaussian behavior

Onset of non-linear dynamics $v_n \propto \varepsilon_n$?



Flow fluctuations via Cummulants

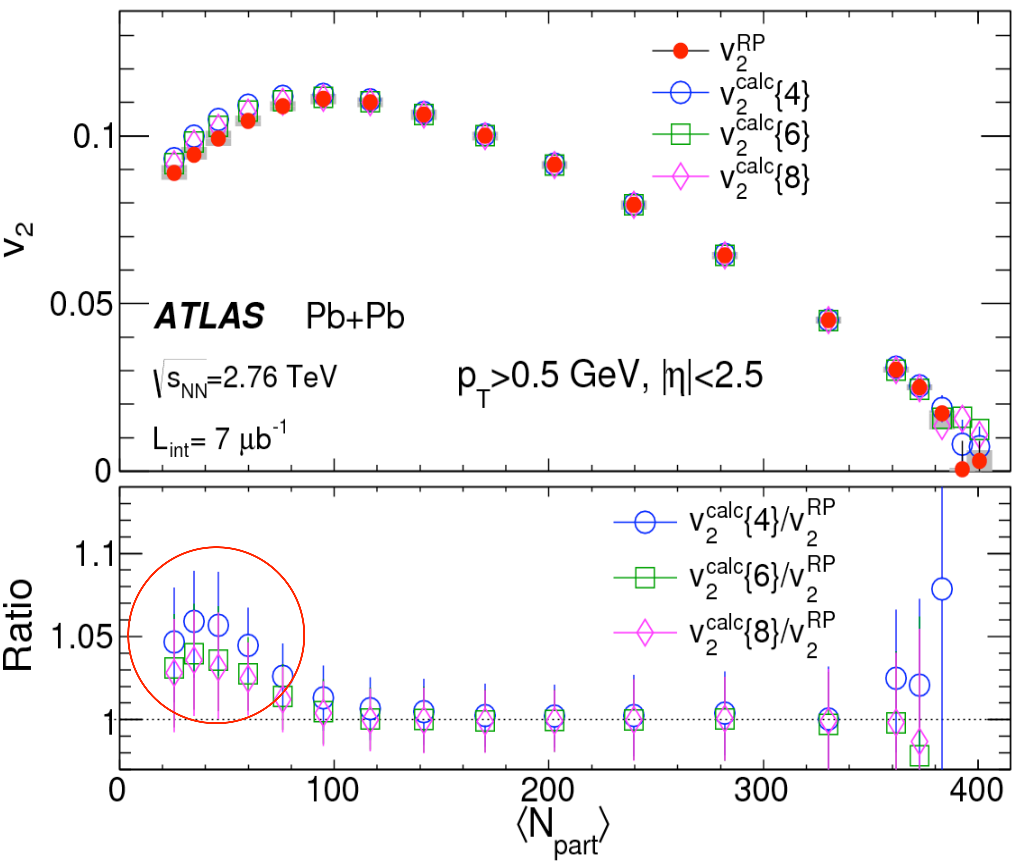
$$v_2^{\rm calc}\{2\}^2 \equiv \langle v_2^2\rangle \approx \left(v_2^{\rm RP}\right)^2 + 2\delta_{v2}^2$$

$$v_2^{\rm calc}\{4\}^4 \equiv -\langle v_2^4\rangle + 2\langle v_2^2\rangle^2 \approx \left(v_2^{\rm RP}\right)^4 ,$$

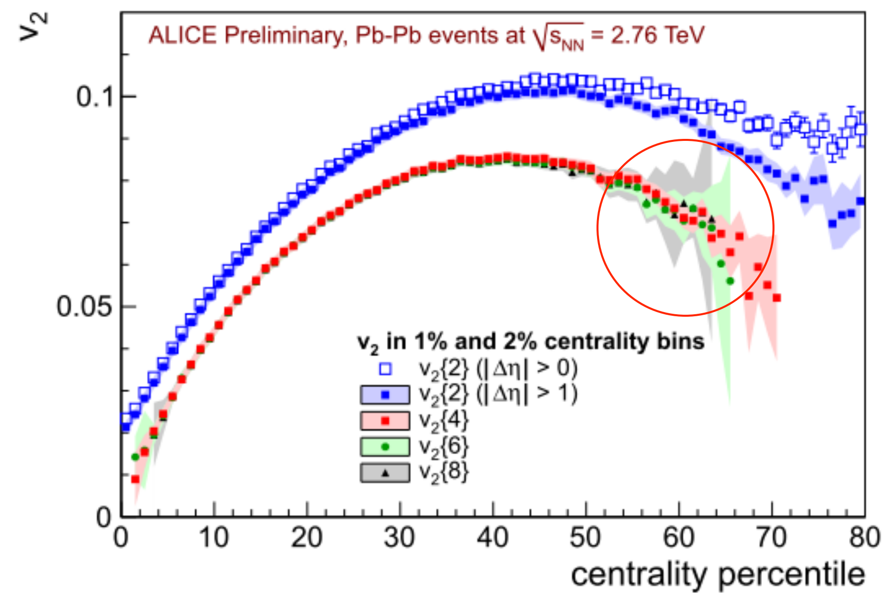
$$v_2^{\rm calc}\{6\}^6 \equiv \left(\langle v_2^6\rangle^2 - 9\langle v_2^4\rangle\langle v_2^2\rangle + 12\langle v_2^2\rangle^3\right)/4 \approx \left(v_2^{\rm RP}\right)^6 ,$$

$$v_2^{\rm calc}\{8\}^8 \equiv -\left(\langle v_2^8\rangle^2 - 16\langle v_2^6\rangle\langle v_2^2\rangle - 18\langle v_2^4\rangle^2 + 144\langle v_2^4\rangle\langle v_2^2\rangle^2 - 144\langle v_2^2\rangle^4\right)/33 \approx \left(v_2^{\rm RP}\right)^8$$

$$p(v_n) \propto v_n \exp\left(\frac{-(v_n^2 + (v_n^{RP})^2}{2\delta_n^2}\right) I_0\left(\frac{v_n v_n^{RP}}{\delta_n^2}\right)$$



Non-Gaussian tail (few % events) leads to few % deviations among $v_2\{4\}$, $v_2\{6\}$ etc



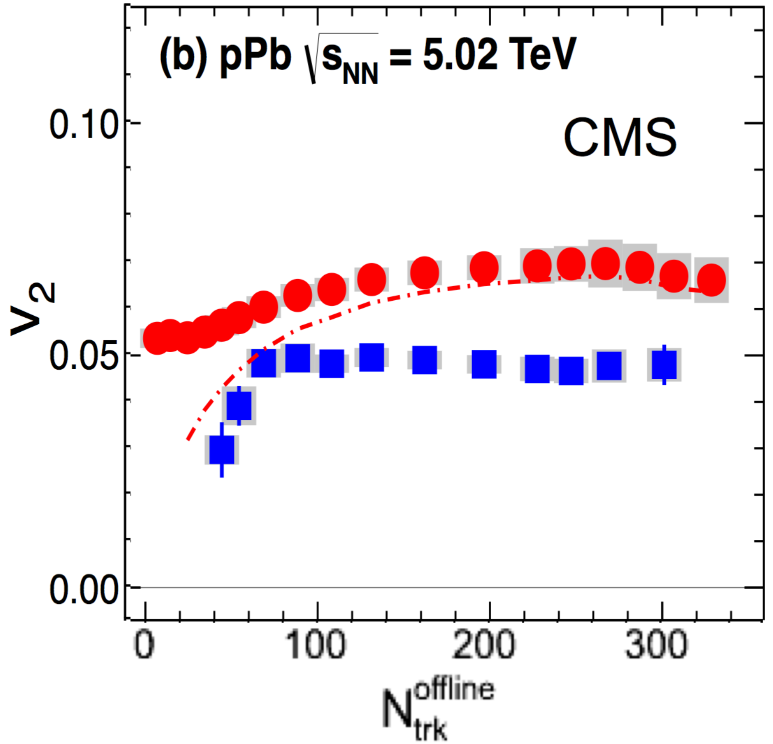
Issues about number of sources

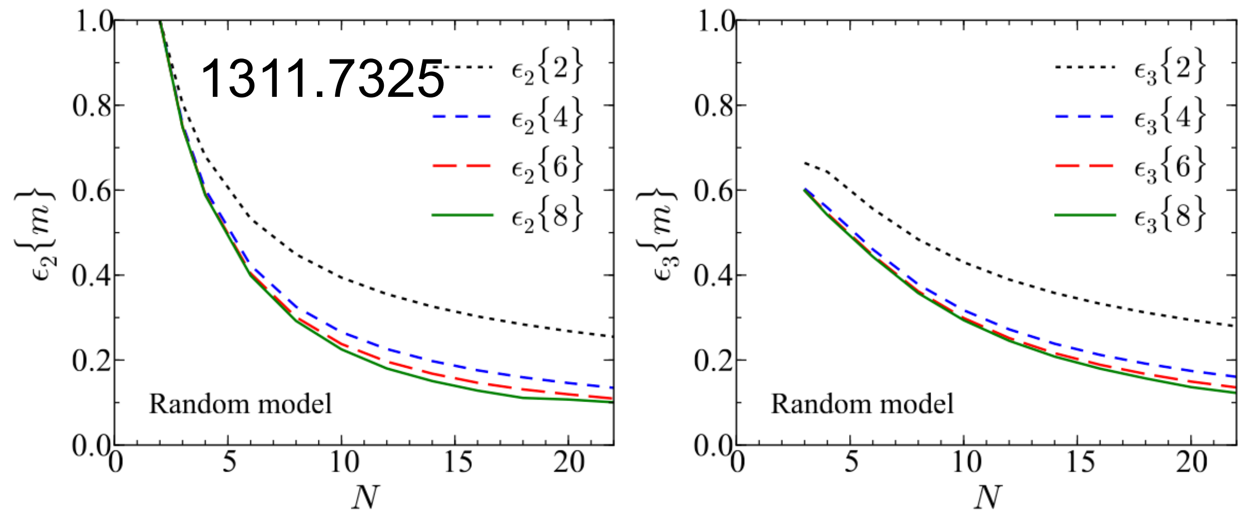
- The non-zero $v_2\{4\}$ in p+Pb and non-zero $\varepsilon_n\{4\}$ could be the effects of finite number of sources

$$p(\vec{\varepsilon}_n) \propto \exp\left(\frac{-(\vec{\varepsilon}_n - \vec{\varepsilon}_n^{RP})^2}{2\delta_{\varepsilon_n}^2}\right) \qquad p(\vec{v}_n) \propto \exp\left(\frac{-(\vec{v}_n - \vec{v}_n^{RP})^2}{2\delta_n^2}\right)$$

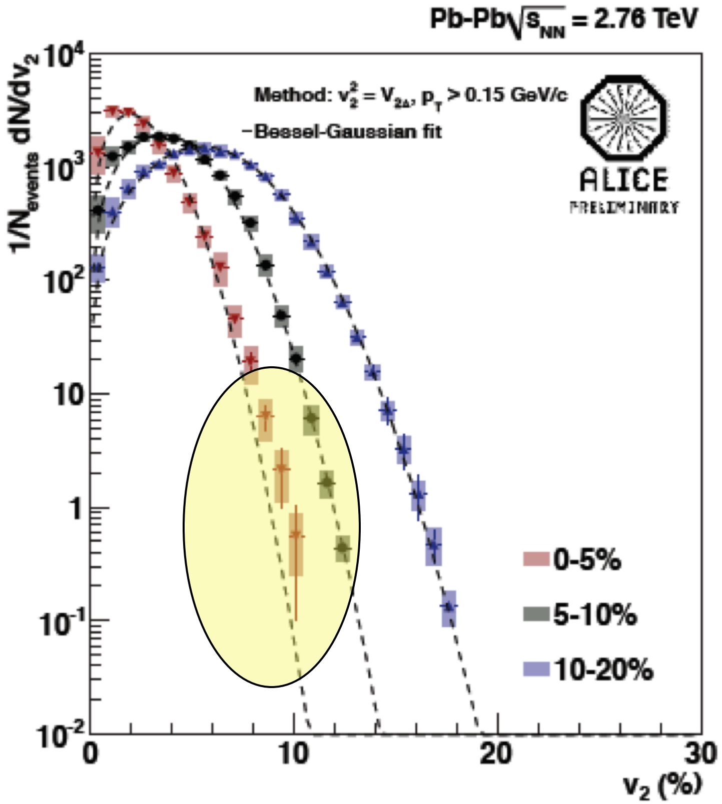
$$v_n\{4\} = v_n\{6\} = \vec{v}_n^{RP}$$

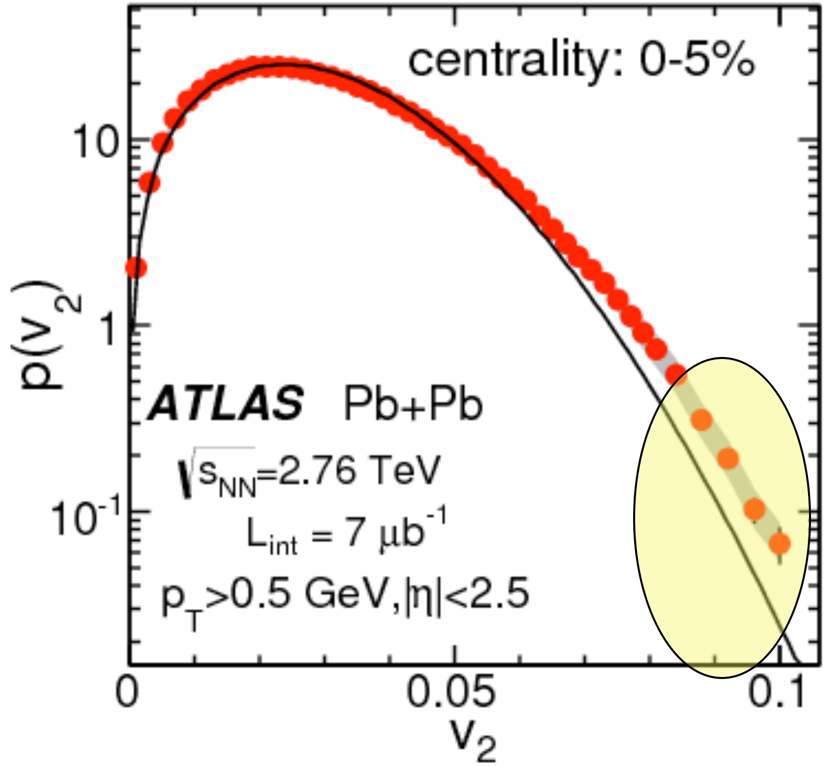
- True only when number of source is large enough,
- negative binomial fluctuation of each source is also important 1304.3044.





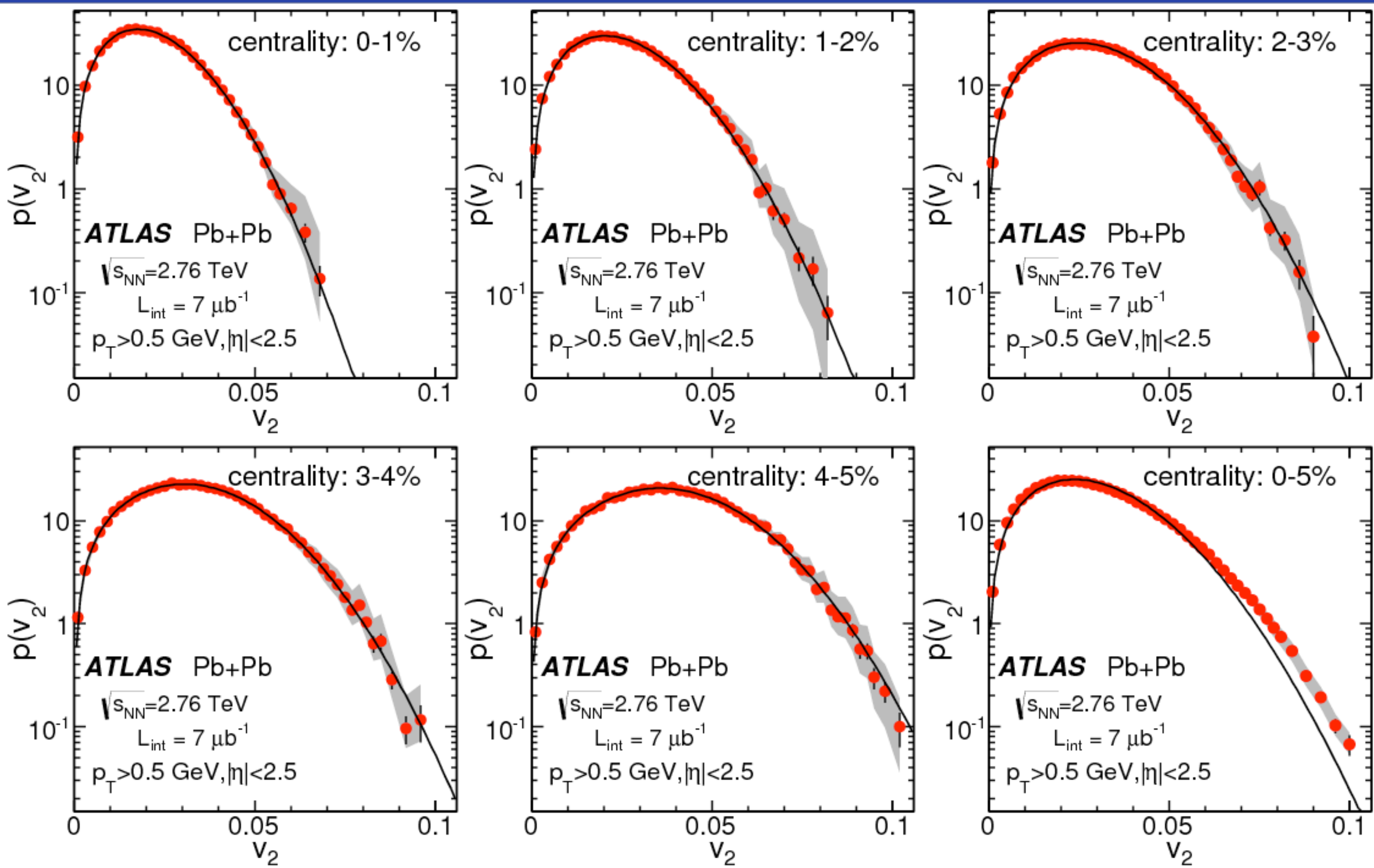
Centrality bin width matters





None BG behavior in centrality collisions?

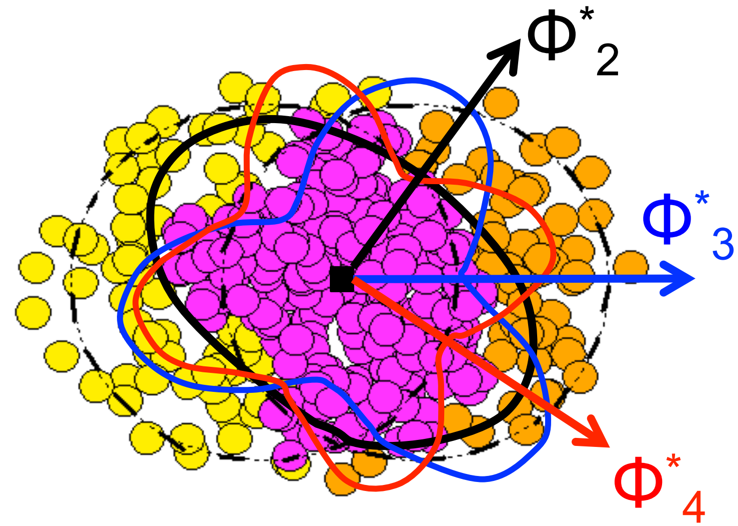
Centrality bin width matters



- 0-1%,1-2%...4-5% bin well described by BG, but the sum does not!!
    - due to rapid increase of v_2^{RP} in 0-5% bin!.
- Crucial to measure $v_2\{4\}$ in narrow centrality bin!

Event plane correlations: $p(\Phi_n, \Phi_m.....)$

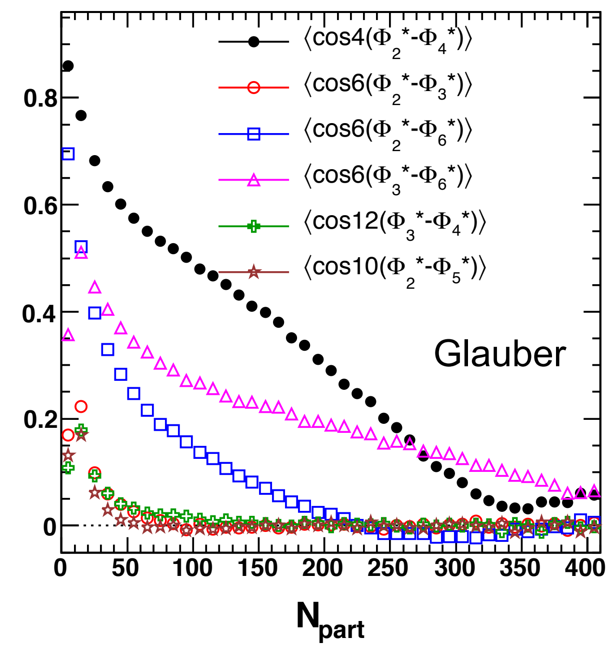
- Correlations exist in the initial geometry and are also generated during hydro evolution: non-linear mixing, e.g.



$$v_4 e^{-i4\Phi_4} \propto \varepsilon_4 e^{-i4\Phi_4^*} + c\, v_2 v_2 e^{-i4\Phi_2} + \ldots$$

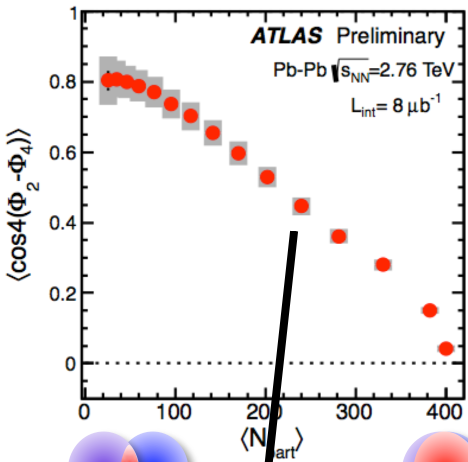
- 8 two-plane and 6 three-plane correlators from ATLAS

$$\langle\cos 4(\Phi_2 - \Phi_4)\rangle \quad \langle\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)\rangle$$
$$\langle\cos 8(\Phi_2 - \Phi_4)\rangle \quad \langle\cos(-8\Phi_2 + 3\Phi_3 + 5\Phi_5)\rangle$$
$$\langle\cos 12(\Phi_2 - \Phi_4)\rangle \quad \langle\cos(2\Phi_2 + 4\Phi_4 - 6\Phi_6)\rangle$$
$$\langle\cos 6(\Phi_2 - \Phi_3)\rangle \quad \langle\cos(-10\Phi_2 + 4\Phi_4 + 6\Phi_6)\rangle$$
$$\langle\cos 6(\Phi_2 - \Phi_6)\rangle \quad \langle\cos(2\Phi_2 - 6\Phi_3 + 4\Phi_4)\rangle$$
$$\langle\cos 6(\Phi_3 - \Phi_6)\rangle \quad \langle\cos(-10\Phi_2 + 6\Phi_3 + 4\Phi_4)\rangle$$

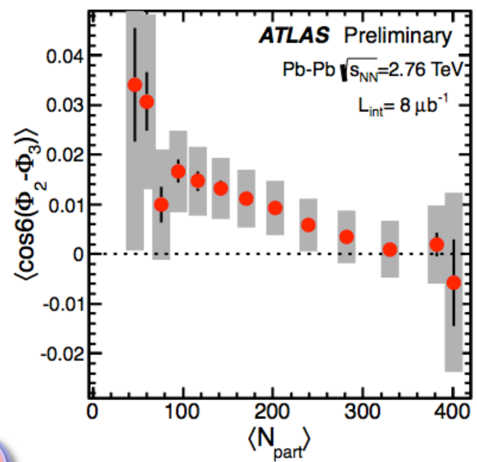


Selected event plane correlation results

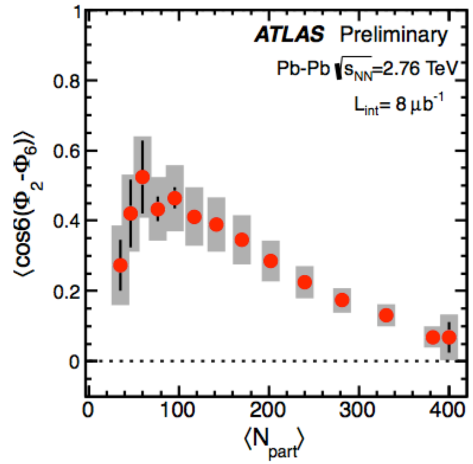
$\left\langle \cos 4(\Phi_2 - \Phi_4) \right\rangle$



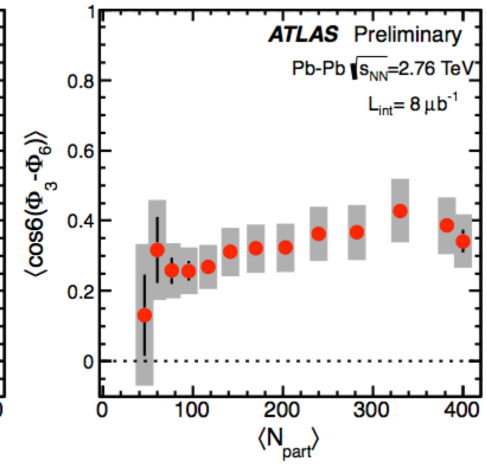
$\left\langle \cos 6(\Phi_2 - \Phi_3) \right\rangle$

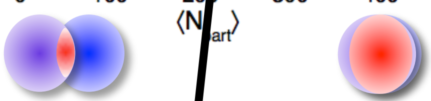


$\left\langle \cos 6(\Phi_2 - \Phi_6) \right\rangle$



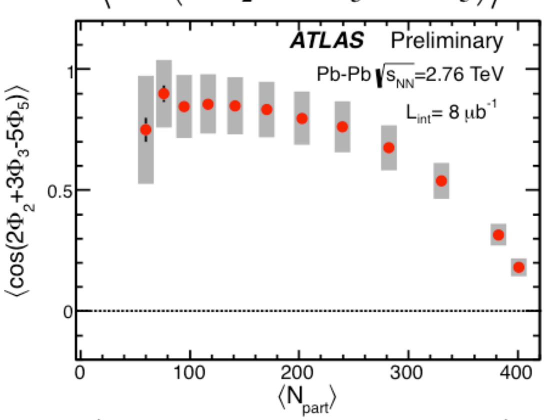
$\left\langle \cos 6(\Phi_3 - \Phi_6) \right\rangle$





$$v_4 e^{-i4\Phi_4} \propto \varepsilon_4 e^{-i4\Phi_4^*} + c v_2 v_2 e^{-i4\Phi_2} + \ldots$$

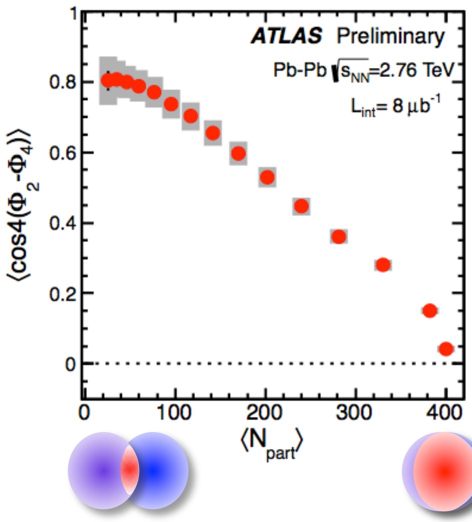
$\left\langle \cos\left(2\Phi_2 + 3\Phi_3 - 5\Phi_5\right) \right\rangle$



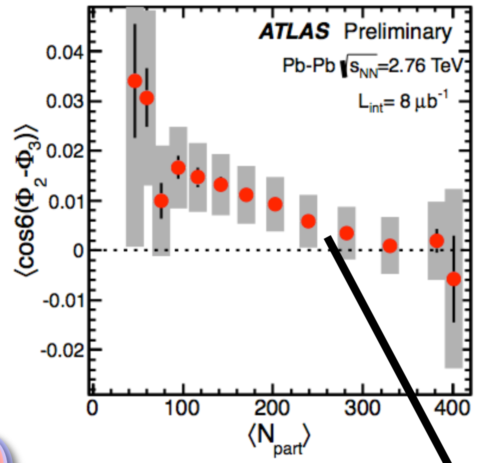
Teaney & Yan

Selected event plane correlation results

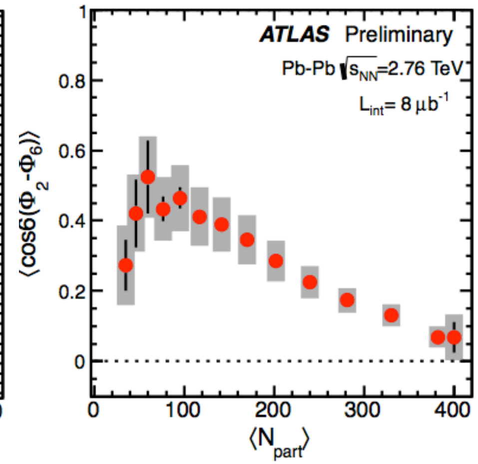
$\langle \cos 4(\Phi_2 - \Phi_4) \rangle$



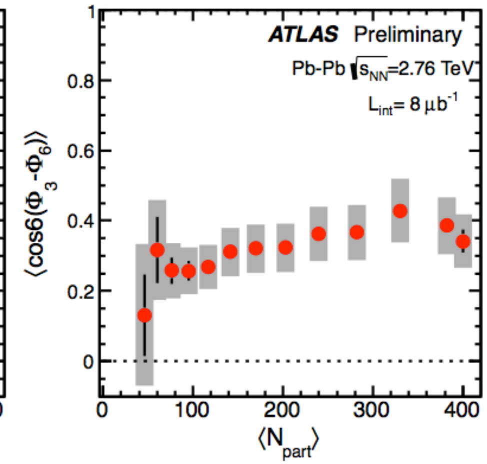
$\langle \cos 6(\Phi_2 - \Phi_3) \rangle$



$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$



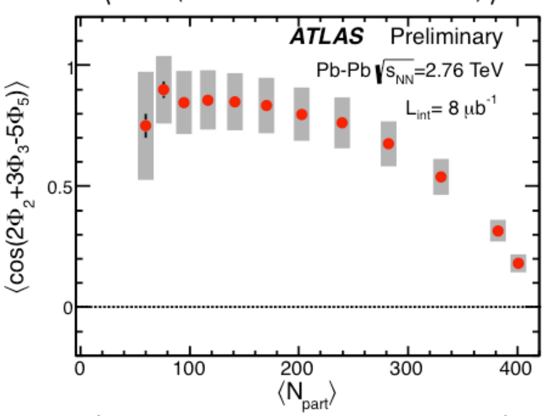
$\langle \cos 6(\Phi_3 - \Phi_6) \rangle$



$$v_2 e^{-i2\Phi_2} \propto \varepsilon_2 e^{-i2\Phi_2^*} + c\, v_1 v_1 e^{-i2\Phi_1} + \ldots$$

$$v_3 e^{-i3\Phi_3} \propto \varepsilon_3 e^{-i3\Phi_3^*} + c\, v_1 v_1 v_1 e^{-i3\Phi_1} + ..$$

$\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$



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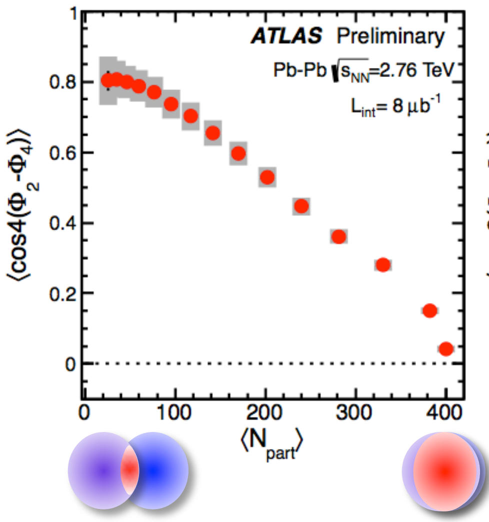
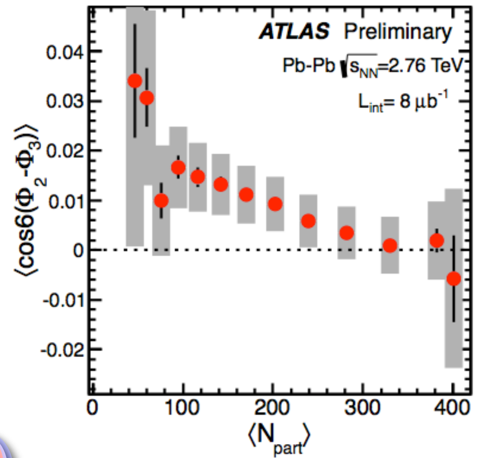
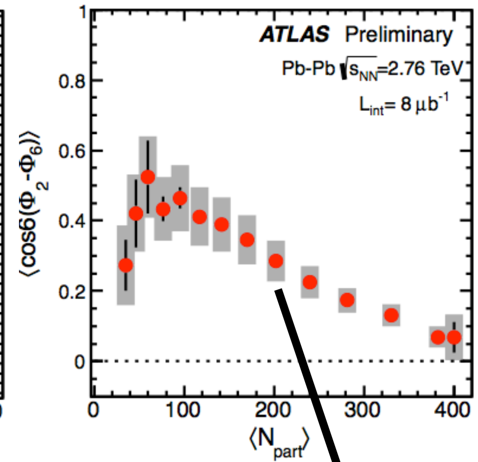


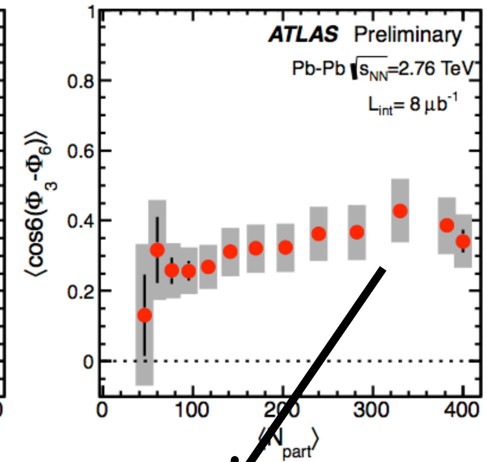
$\langle \cos 6(\Phi_2 - \Phi_3) \rangle$



$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$

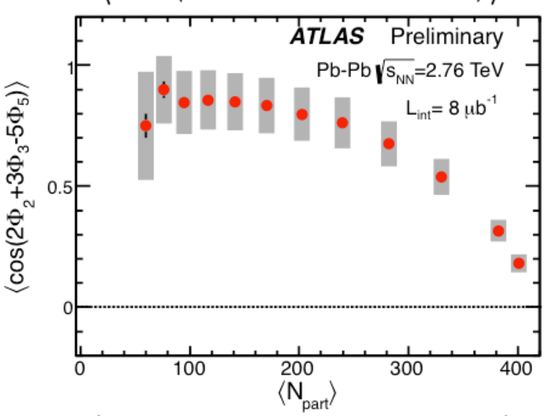


$\langle \cos 6(\Phi_3 - \Phi_6) \rangle$



$$v_6 e^{-i6\Phi_6} \propto \varepsilon_6 e^{-i6\Phi_6^*} + v_2 v_2 v_2 e^{-i6\Phi_2} + v_3 v_3 e^{-i6\Phi_3} + \ldots$$

$\left\langle \cos\left(2\Phi_2 + 3\Phi_3 - 5\Phi_5\right) \right\rangle$



Teaney & Yan

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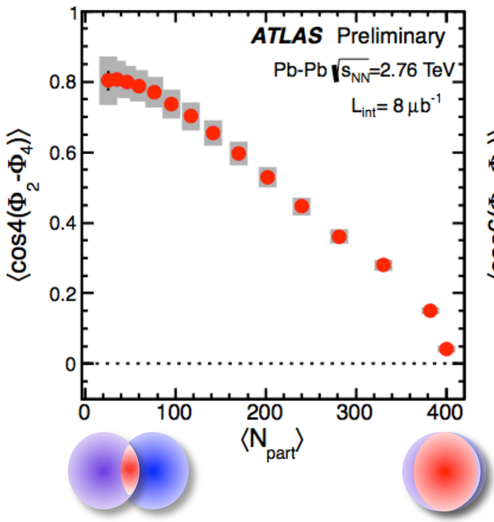
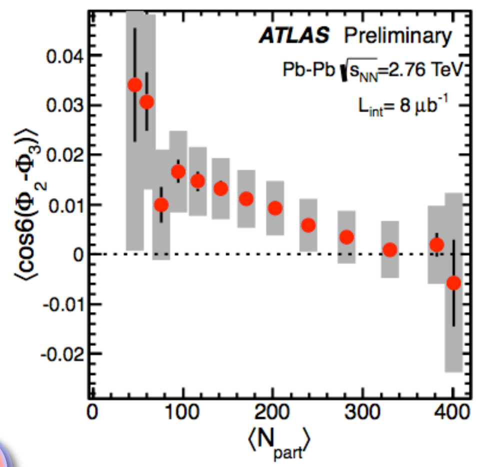
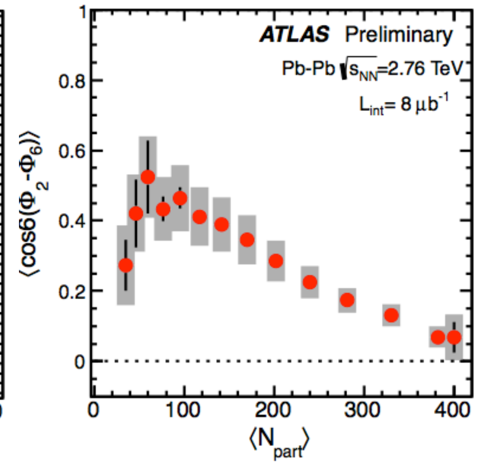


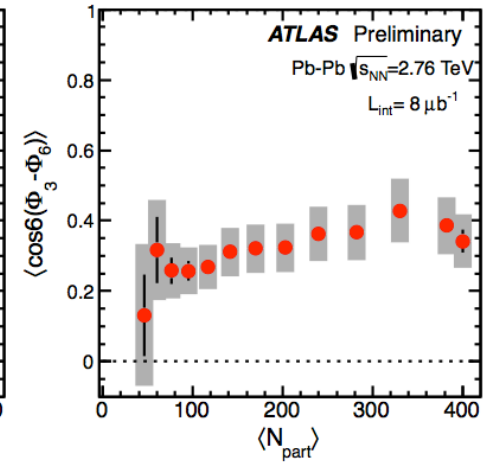
$\langle \cos 6(\Phi_2 - \Phi_3) \rangle$



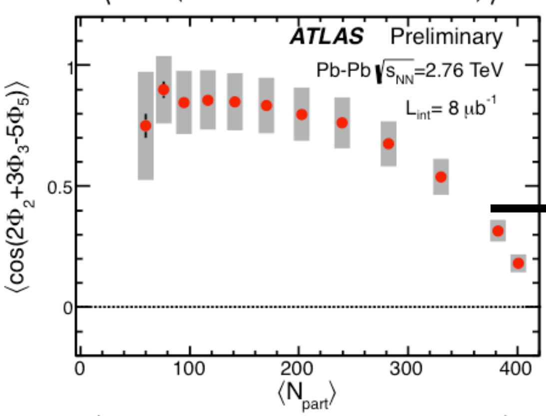
$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$



$\langle \cos 6(\Phi_3 - \Phi_6) \rangle$



$\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$

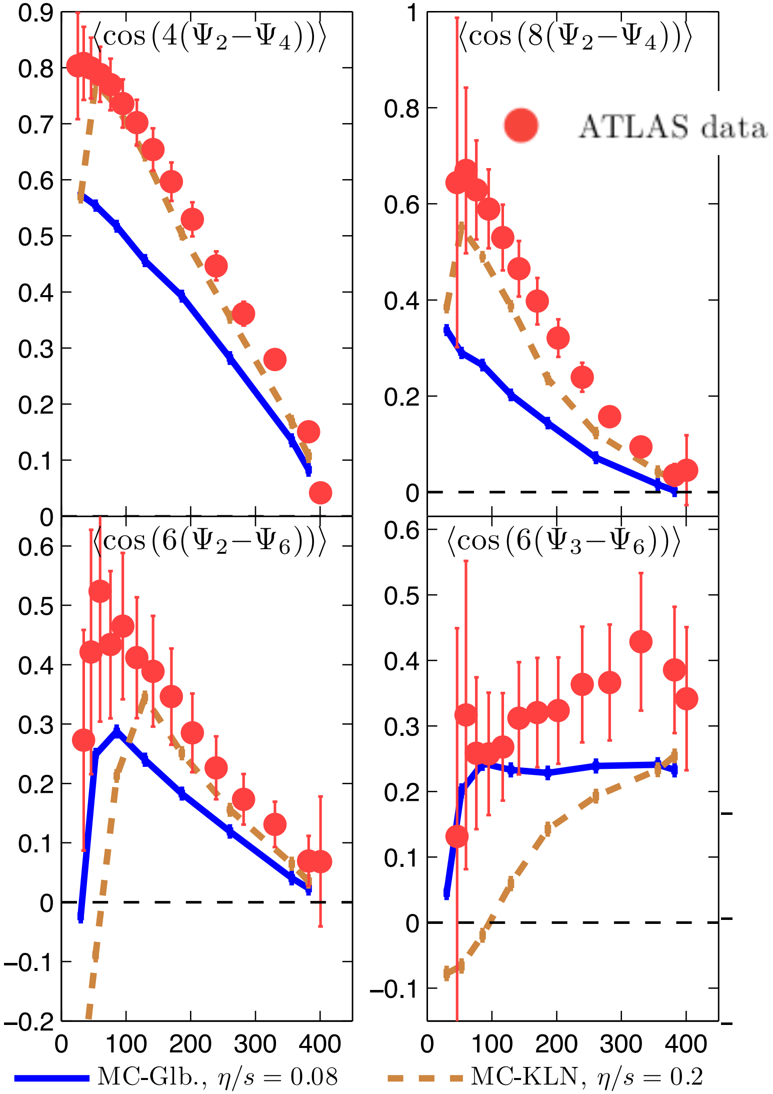


$$v_5 e^{-i5\Phi_5} \propto \varepsilon_5 e^{-i5\Phi_5^*} + v_2 v_3 e^{-i(2\Phi_2+3\Phi_3)} + \ldots$$

Teaney & Yan

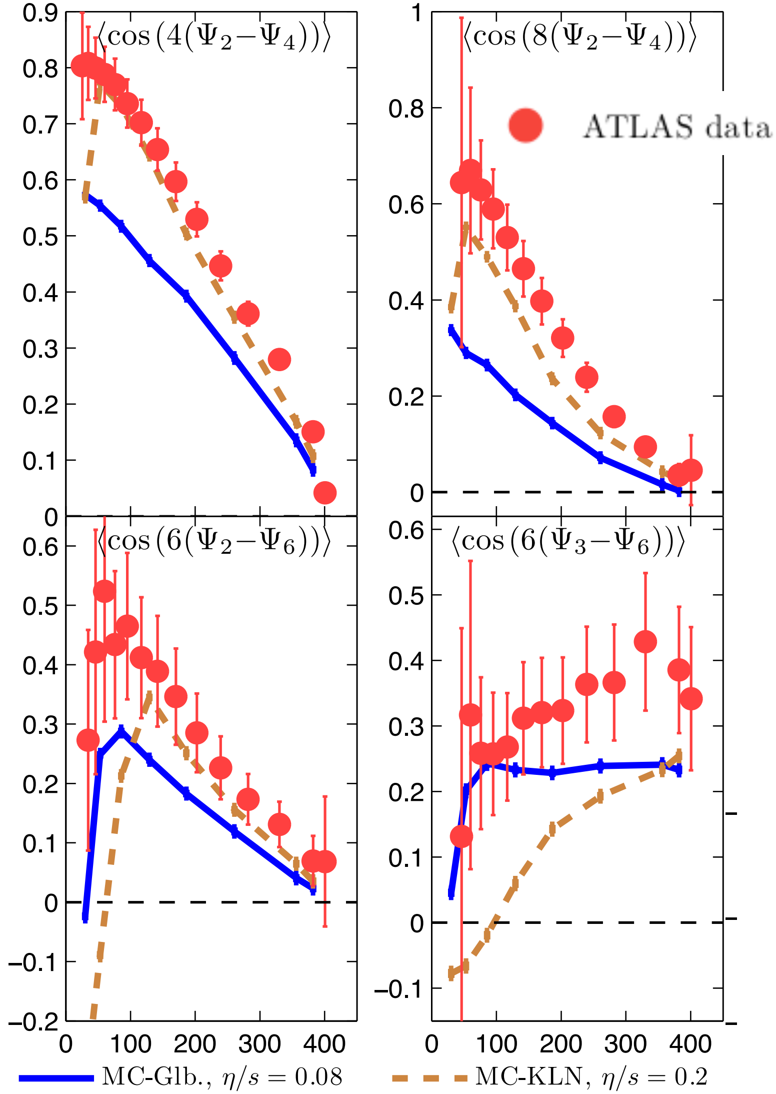
Compare with EbE hydro calculation

Initial geometry + hydrodynamic Zhi & Heinz 1208.1200

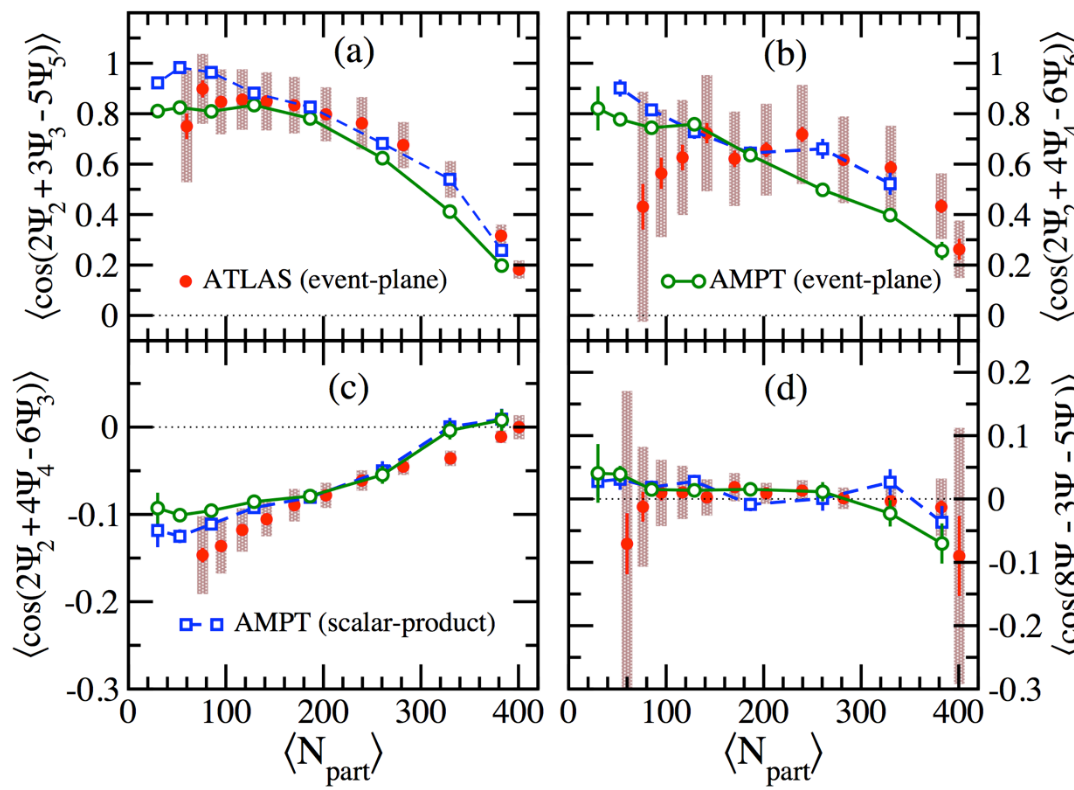


Compare with EbE hydro calculation

Initial geometry + hydrodynamic Zhi & Heinz 1208.1200



Initial geometry + transport 1307.0980 Bhalerao,et.al.



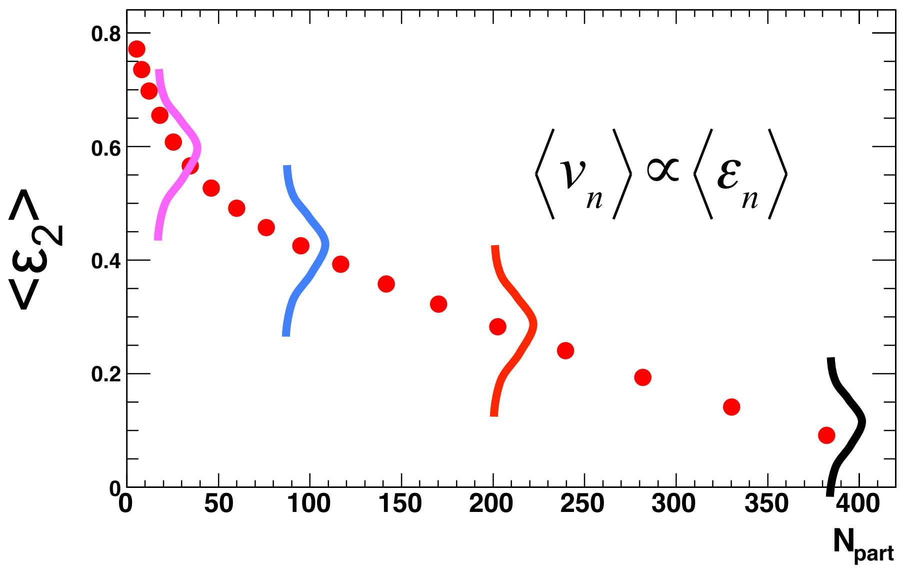
EbyE hydro and transport models reproduce features in the data

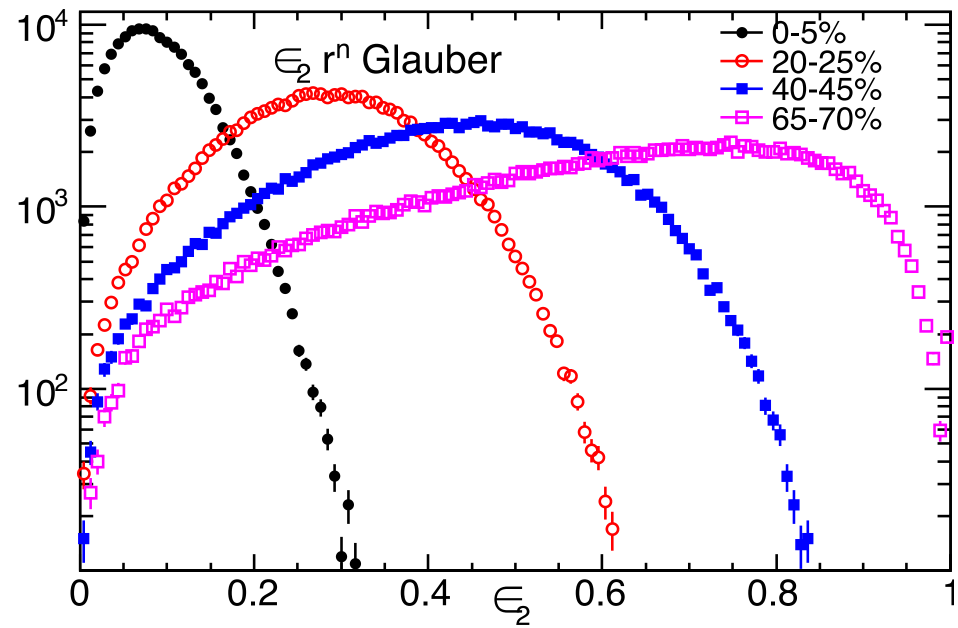
Can we do better?

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\mathrm{evts}}} \frac{dN_{\mathrm{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

Can we do better?

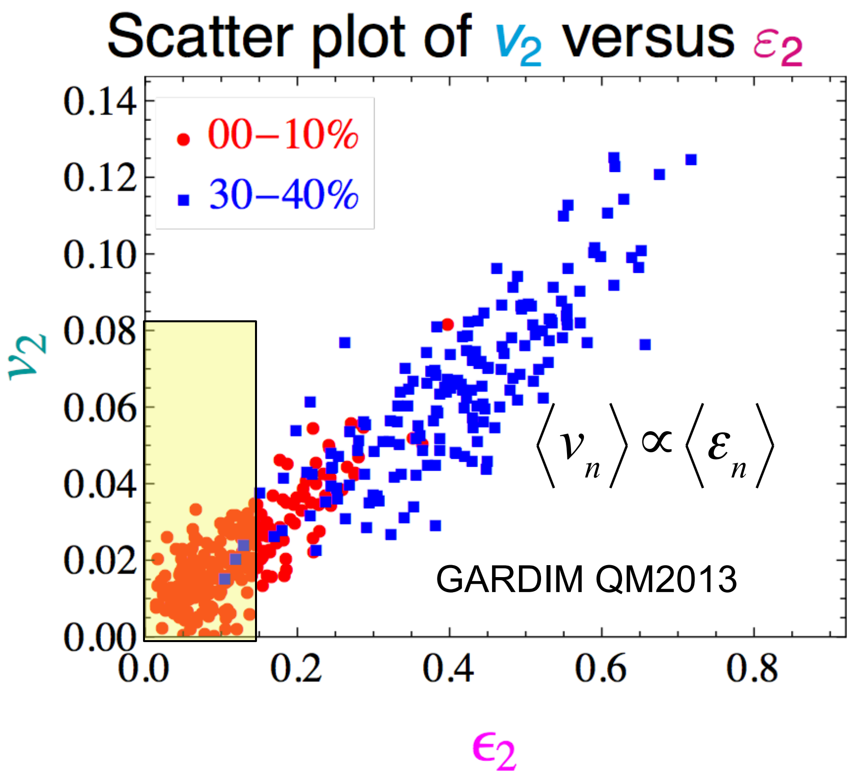
$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{evts}} \frac{dN_{evts}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$



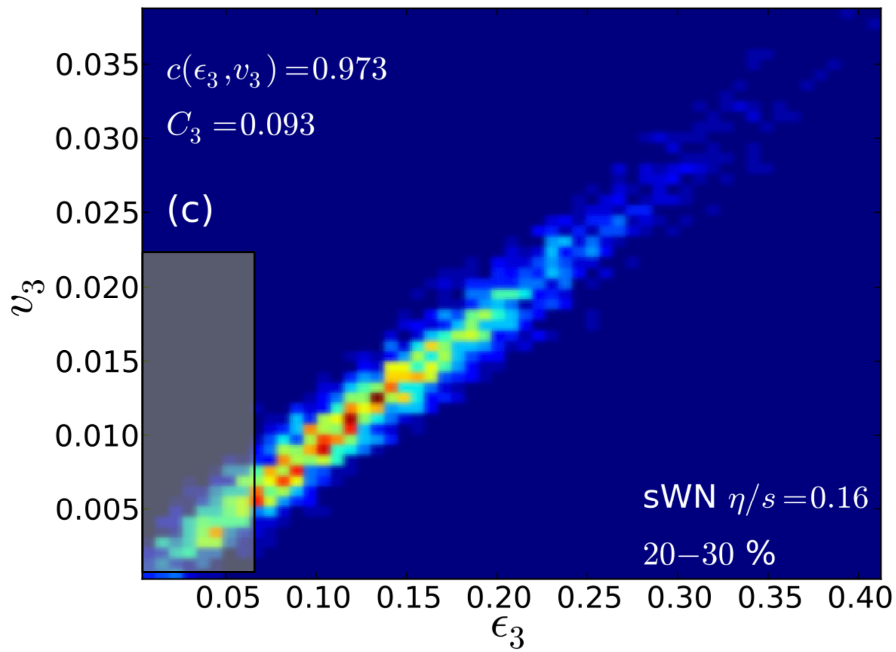


- Variation of event shape at fixed centrality >> variation of mean across the full centrality range!!
  - All of this information has been averaged out!

Dynamics fluctuations in hydro



1212.1008 H. Niemi, G. S. Denicol et.al



- v_2 v_3 responses to ε_2 ε_3 are linear only on average.
  - Spread at fixed ε_n due to dynamic fluctuations (e.g. flow angle flucs).

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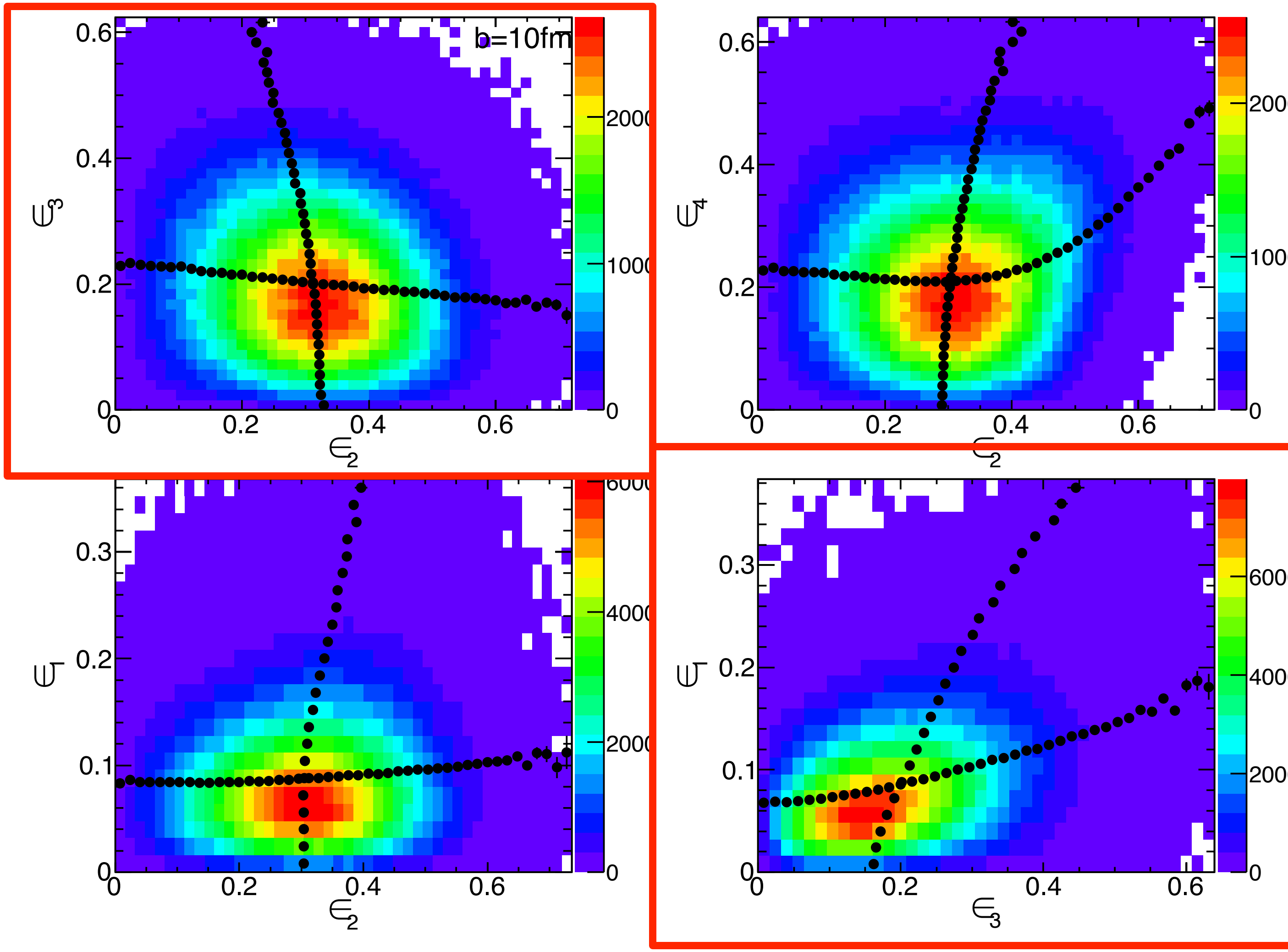
- We can better expose these fluctuations by event-shape selection
- Fireball size is ~ same, so η/s ~ same!

Correlation at initial state: $p(\varepsilon_n,\varepsilon_m)$



- Correlation already exist in the initial state
  - Anti-correlation between ε_2 and ε_3. strong correlation between ε_1 and ε_3.
- Much of them are expected to remain in the final state.

Event shape selection

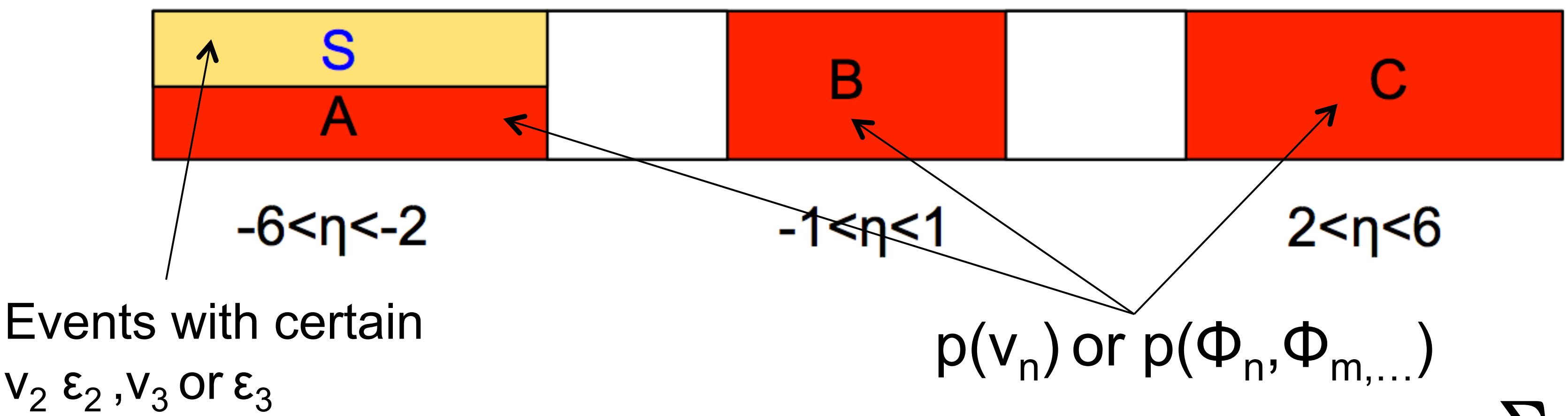
Schukraft, Timmins, and Voloshin, arXiv:1208.4563

- Correlation explored by systematically varying one flow variable

$$p(v_n, v_m) = p(v_n)\,p(v_m | v_n)$$
$$p(v_n, \Phi_n, \Phi_{m,\ldots}) = p(v_n)\,p(\Phi_n, \Phi_{m,\ldots} | v_n)$$

$v_n = v_1, v_2$ or v_3

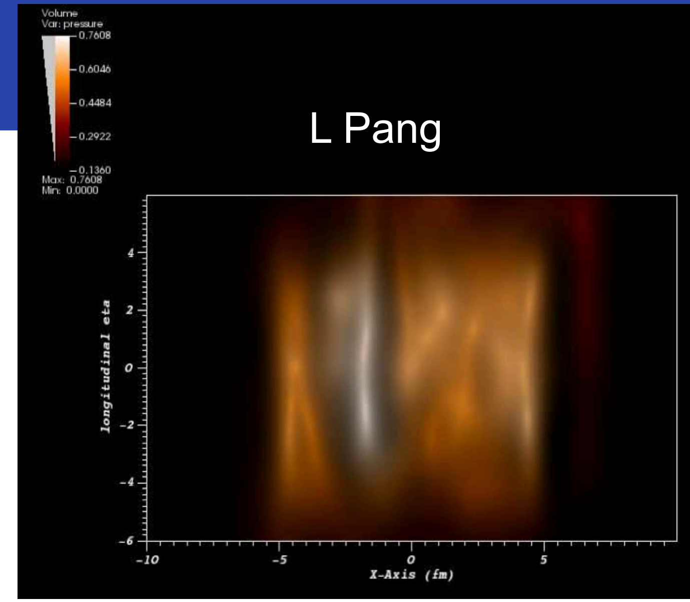
- Shape selection works because event shape is correlated in rapidity.



$$\vec{q}_n = \frac{1}{\Sigma w}\left(\Sigma w \cos n\phi_n, \Sigma w \sin n\phi_n\right),\ w = p_T\ , \qquad q_n = \left|\vec{q}_n\right| \Rightarrow \frac{\sum w v_n}{\sum w}$$

AMPT as a testing model

- AMPT model: Glauber+HIJING+transport
  - Has fluctuating geometry and collective flow
  - Longitudinal fluctuations and initial flow



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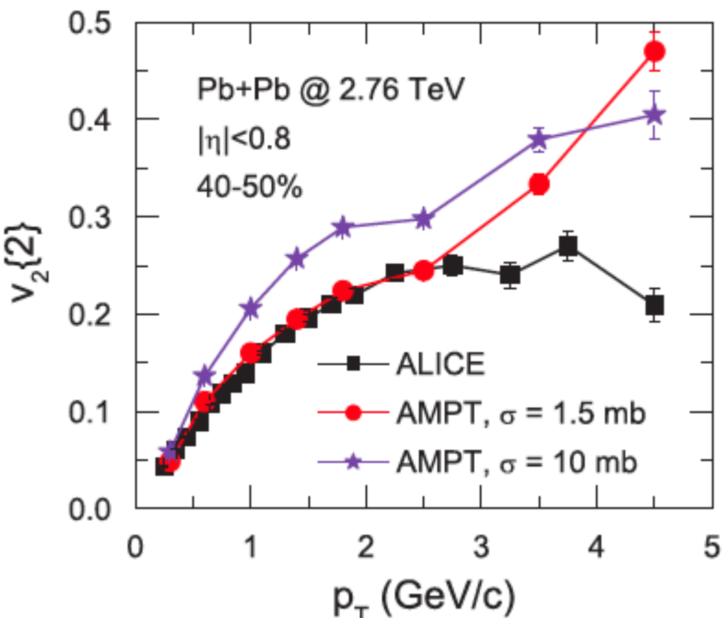
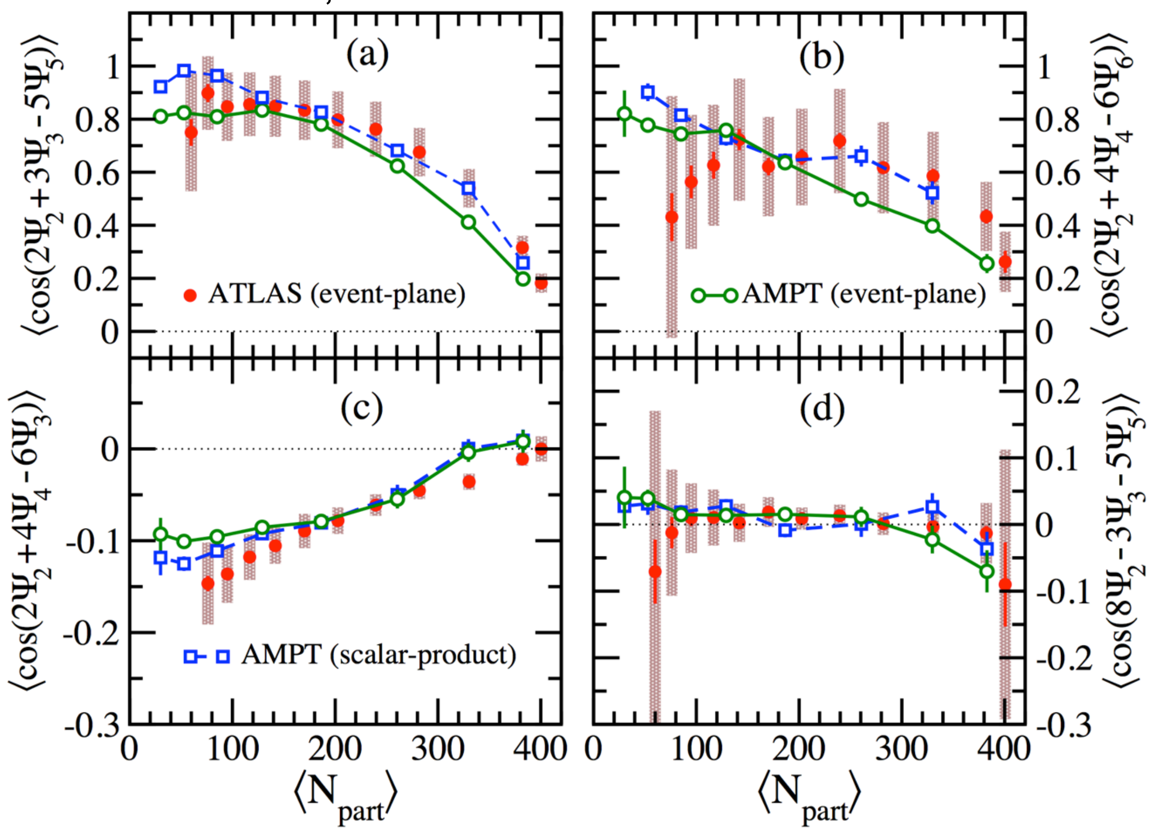
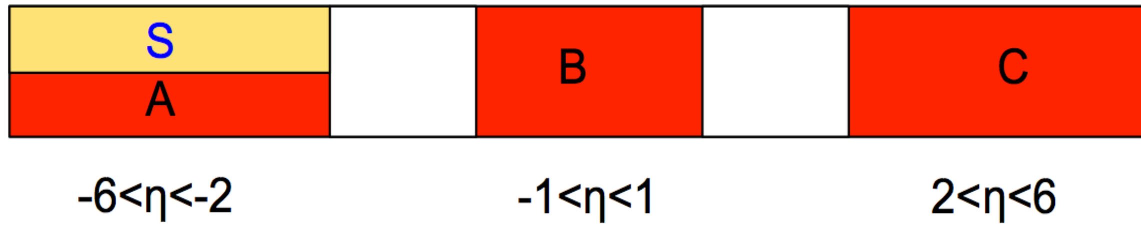


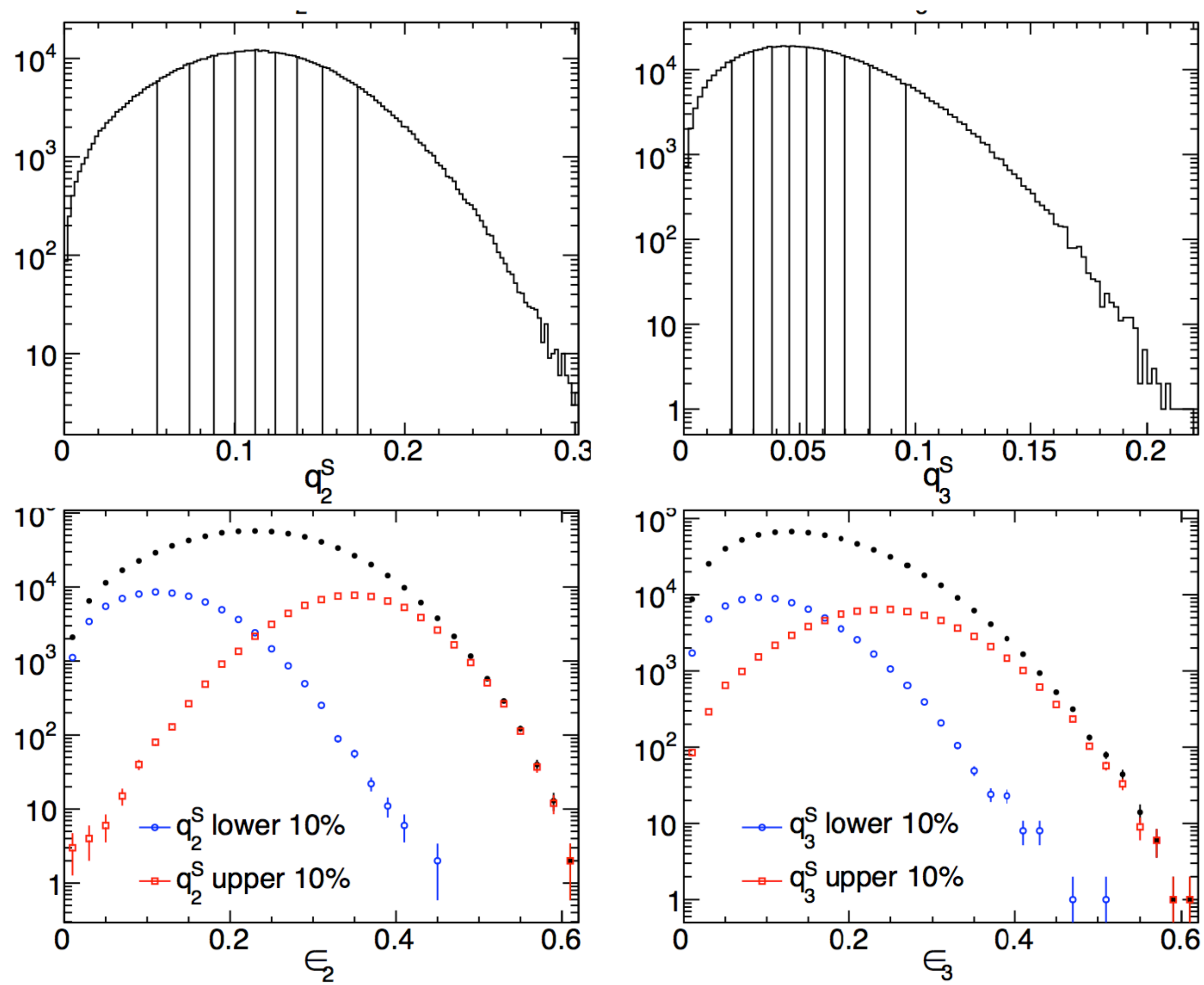
J Xu et al
Phys.Rev.C83,034904(2011)



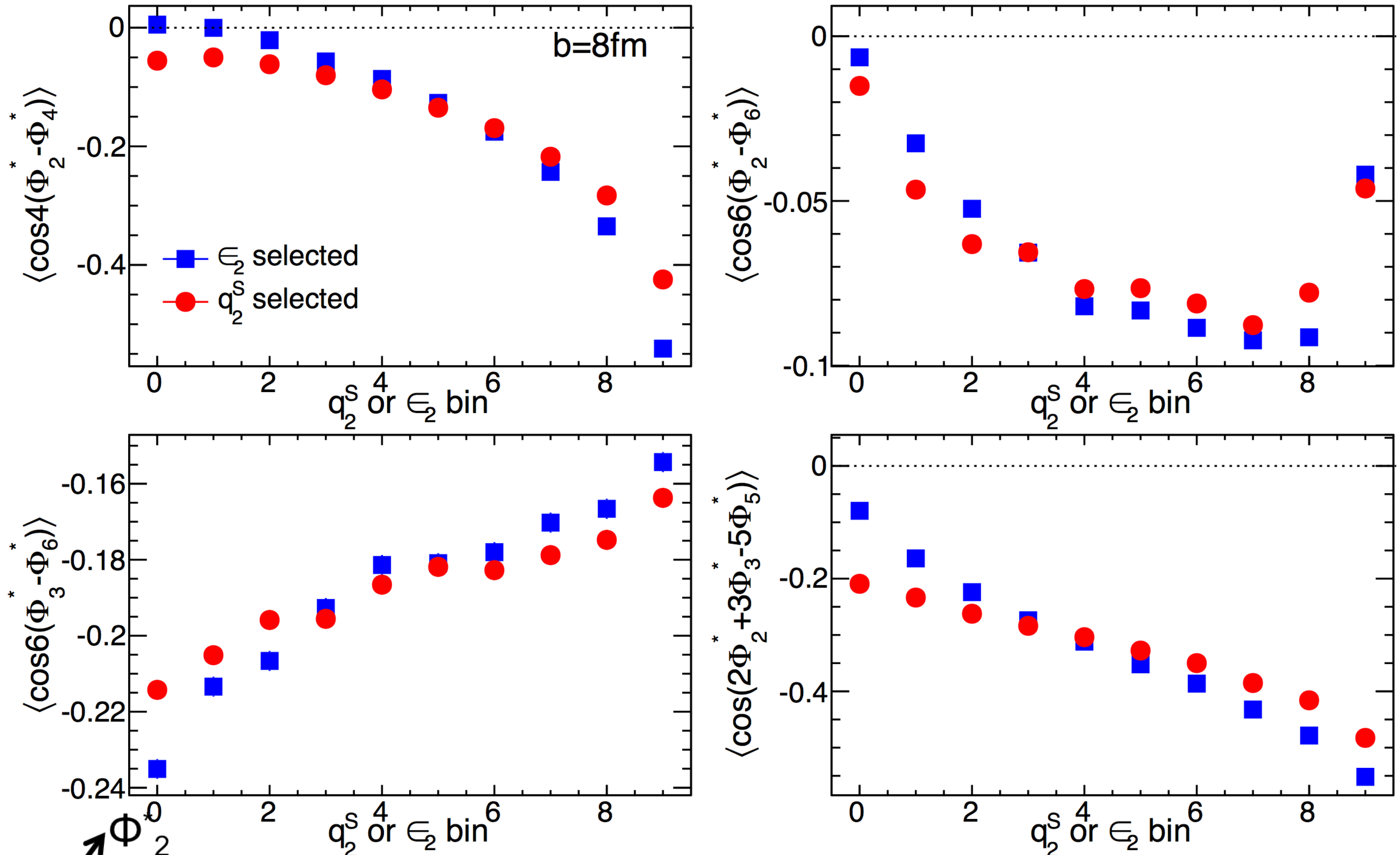
Event shape selection accuracy in AMPT

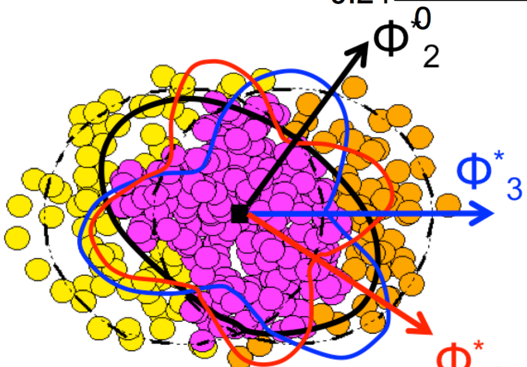
- Fixed impact parameter b=8fm
- Divide events into 10 equal bins
- $\langle\varepsilon_2\rangle$ vary by a factor of 3
- $\langle\varepsilon_3\rangle$ by a factor of 2,





PP correlation $p(\Phi^*_n,\Phi^*_m\ldots|\varepsilon_2)$ or $p(\Phi^*_n,\Phi^*_m\ldots|q_2)$

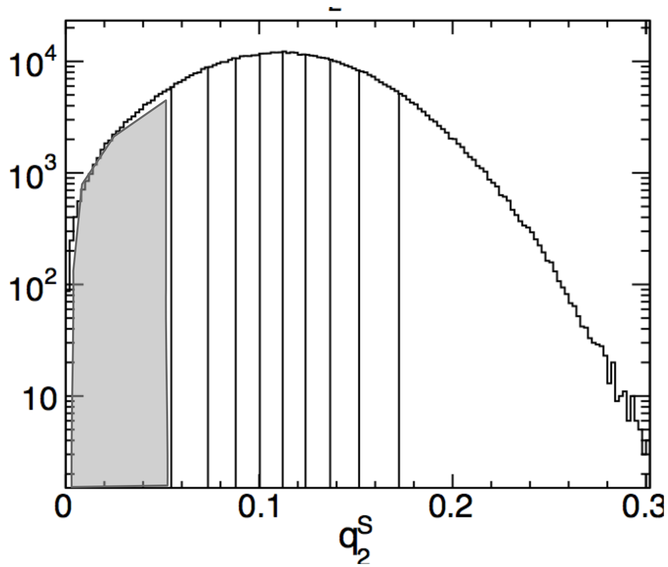


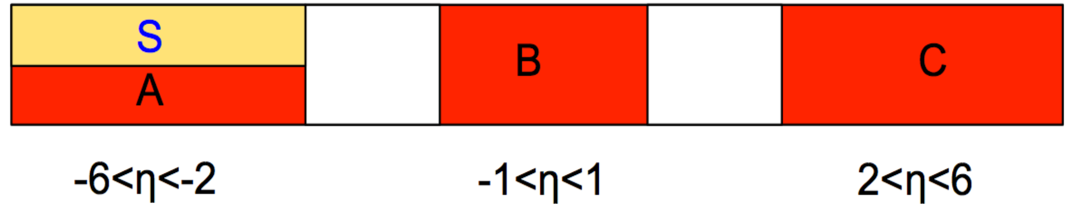


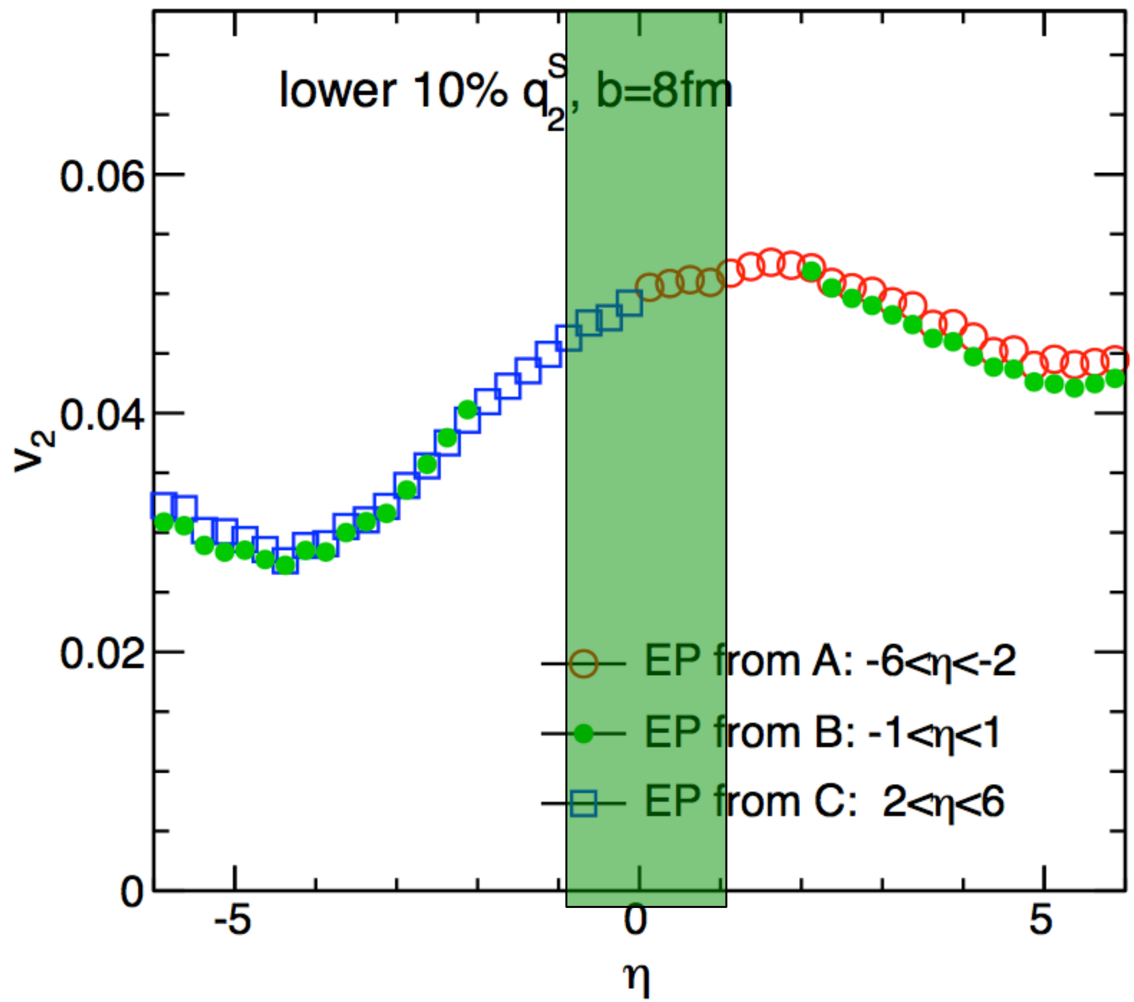
- Events with fixed centrality can have very different participant plane correlation!
- Such dependence is pickup by the q_2 selection!

Final state observables

$v_2(\eta)$: select on q_2







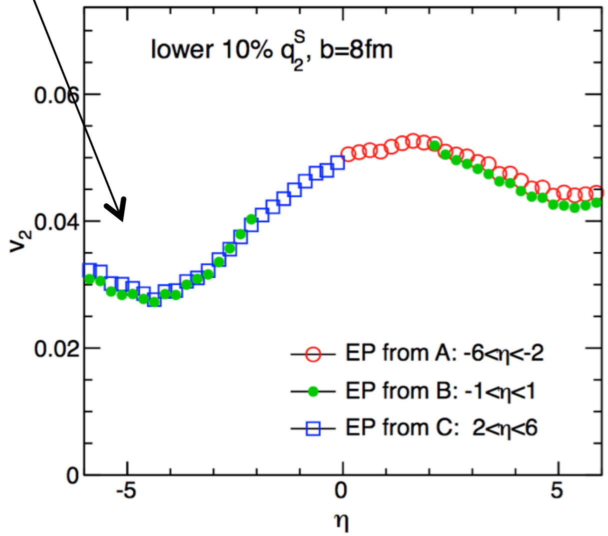
$v_2(\eta)$: select on q_2

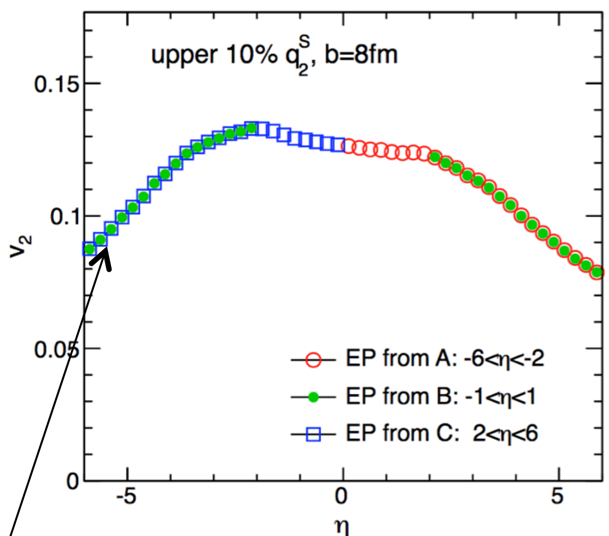
Suppression of flow in the selection window

$v_2(\eta)|_{\eta>0}$ when EP in $-6<\eta<-2$

$v_2(\eta)|_{\eta<0}$ when EP in $2<\eta<6$

$v_2(\eta)|_{|\eta|>2}$ when EP in $|\eta|<1$





enhancement of flow in the selection window

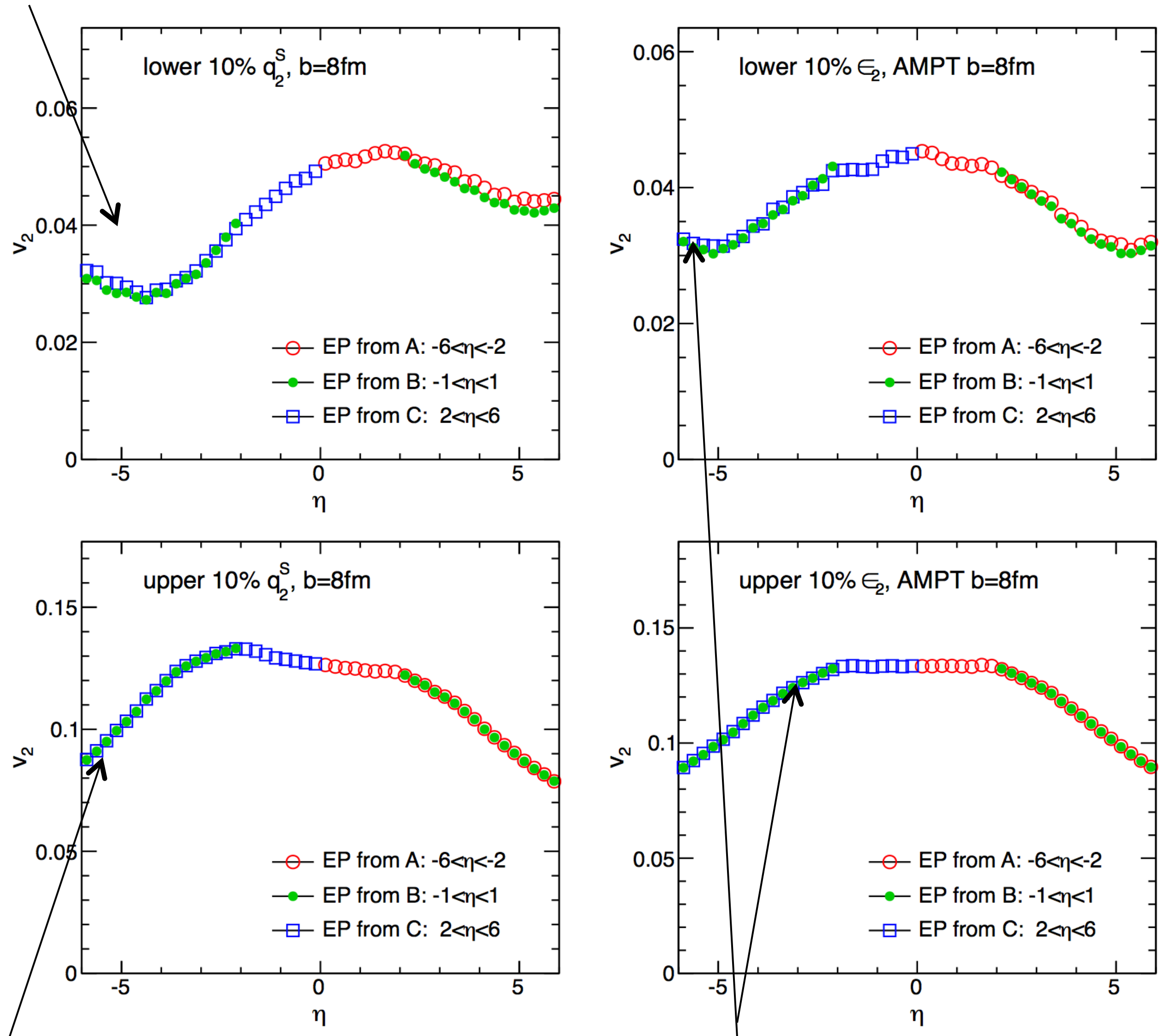
$v_2(\eta)$: compare with selection on ε_2

Suppression of flow in the selection window

$v_2(\eta)|_{\eta>0}$ when EP in $-6<\eta<-2$

$v_2(\eta)|_{\eta<0}$ when EP in $2<\eta<6$

$v_2(\eta)|_{|\eta|>2}$ when EP in $|\eta|<1$

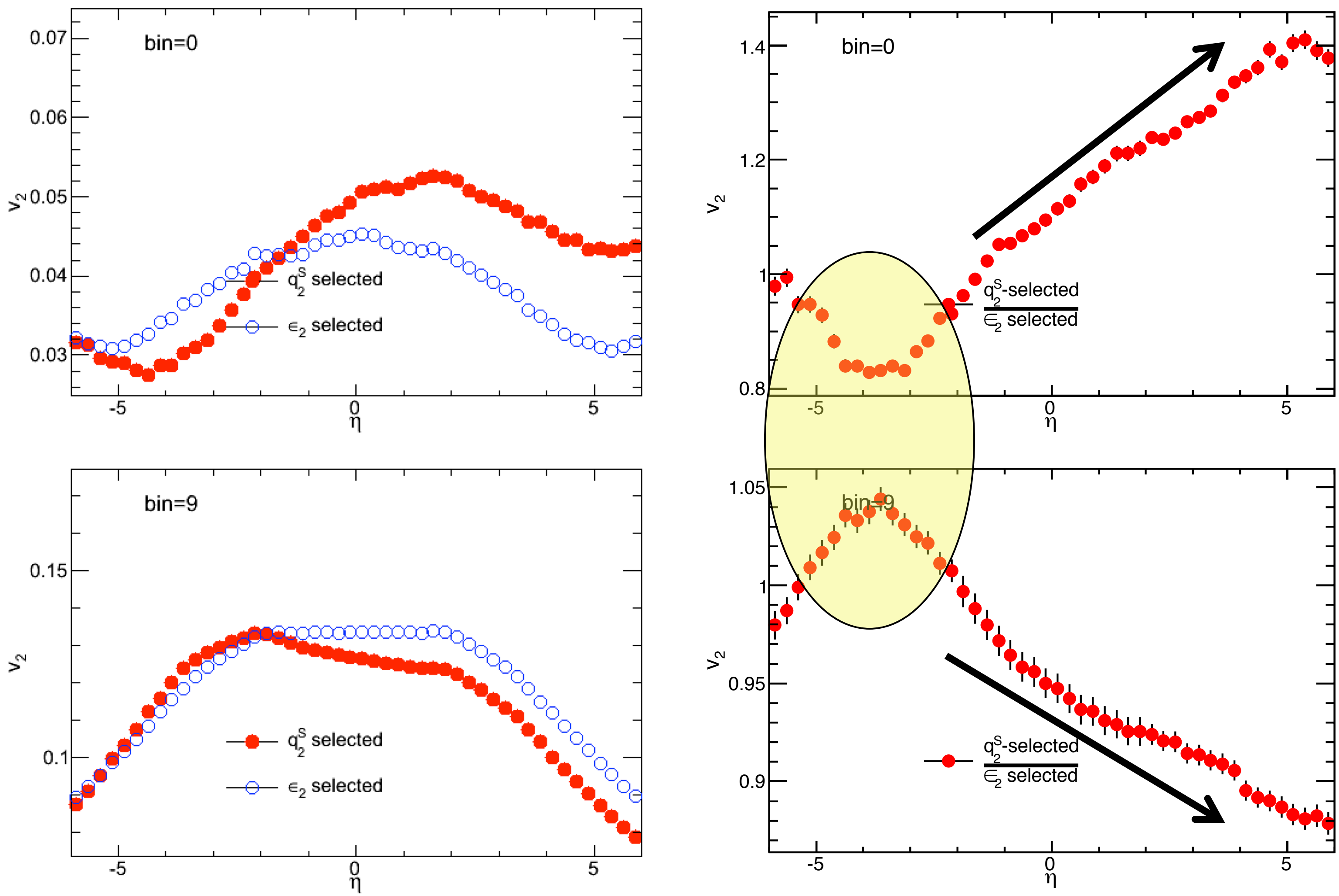


enhancement of flow in the selection window

Symmetric distribution expected

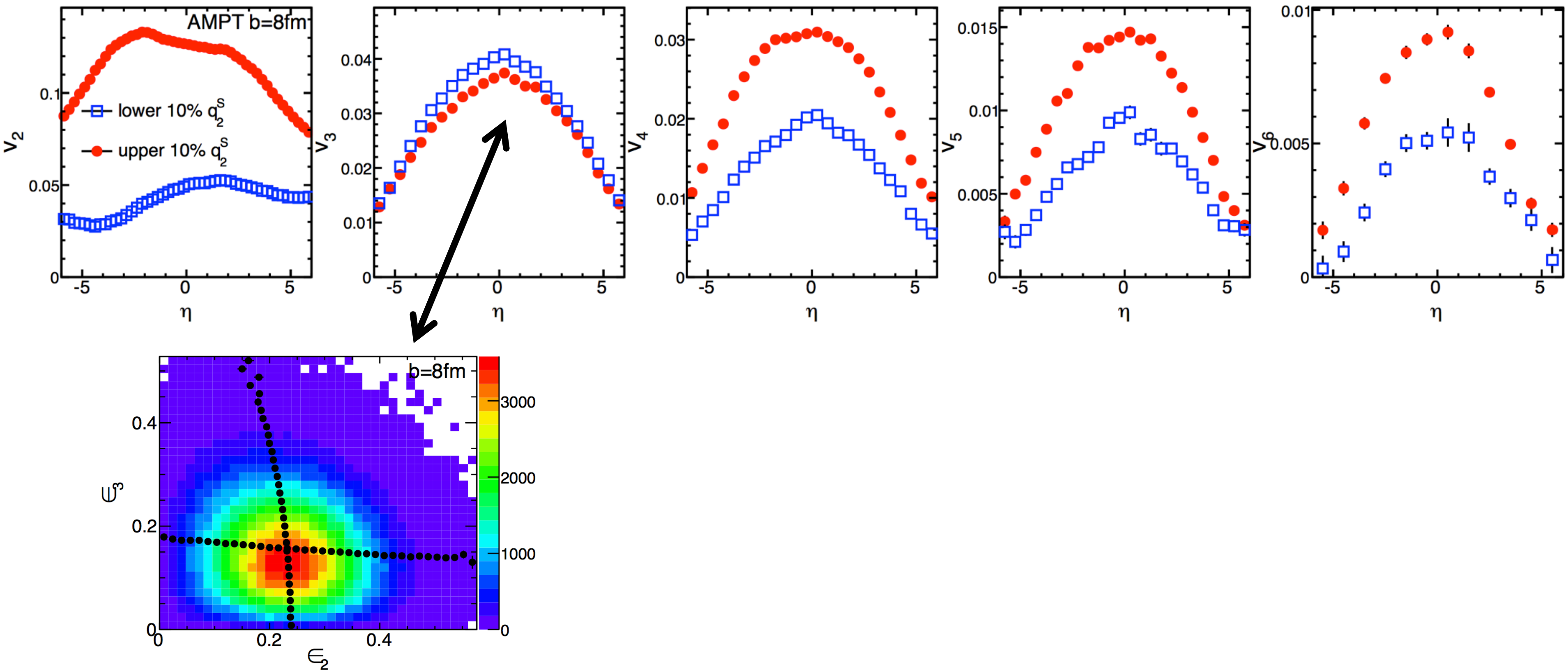
Rapidity asymmetry of v2

- Suppression/enhancement of flow in the selected window
- Decreasing response to flow selection outside the selection window



$v_n(\eta)$ shape for all n: select on q_2

- Smaller asymmetry for other harmonics.
- v_4, v_5, v_6 correlates with v_2 selection, direct measure of non-linear effects!!
- Anti-correlation with v_3 → reflection of $p(\varepsilon_2,\varepsilon_3)$



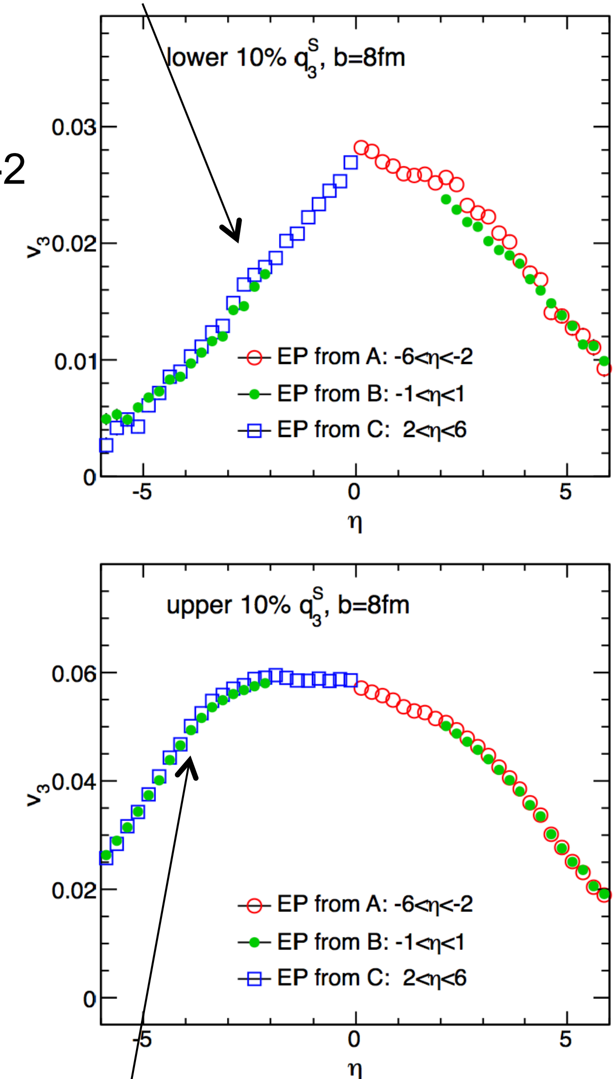
$v_3(\eta)$: select on q_3

Suppression of flow in the selected window

$v_2(\eta)|_{\eta>0}$ when EP in $-6<\eta<-2$

$v_2(\eta)|_{\eta<0}$ when EP in $2<\eta<6$

$v_2(\eta)|_{|\eta|>2}$ when EP in $|\eta|<1$



enhancement of flow in the selected window

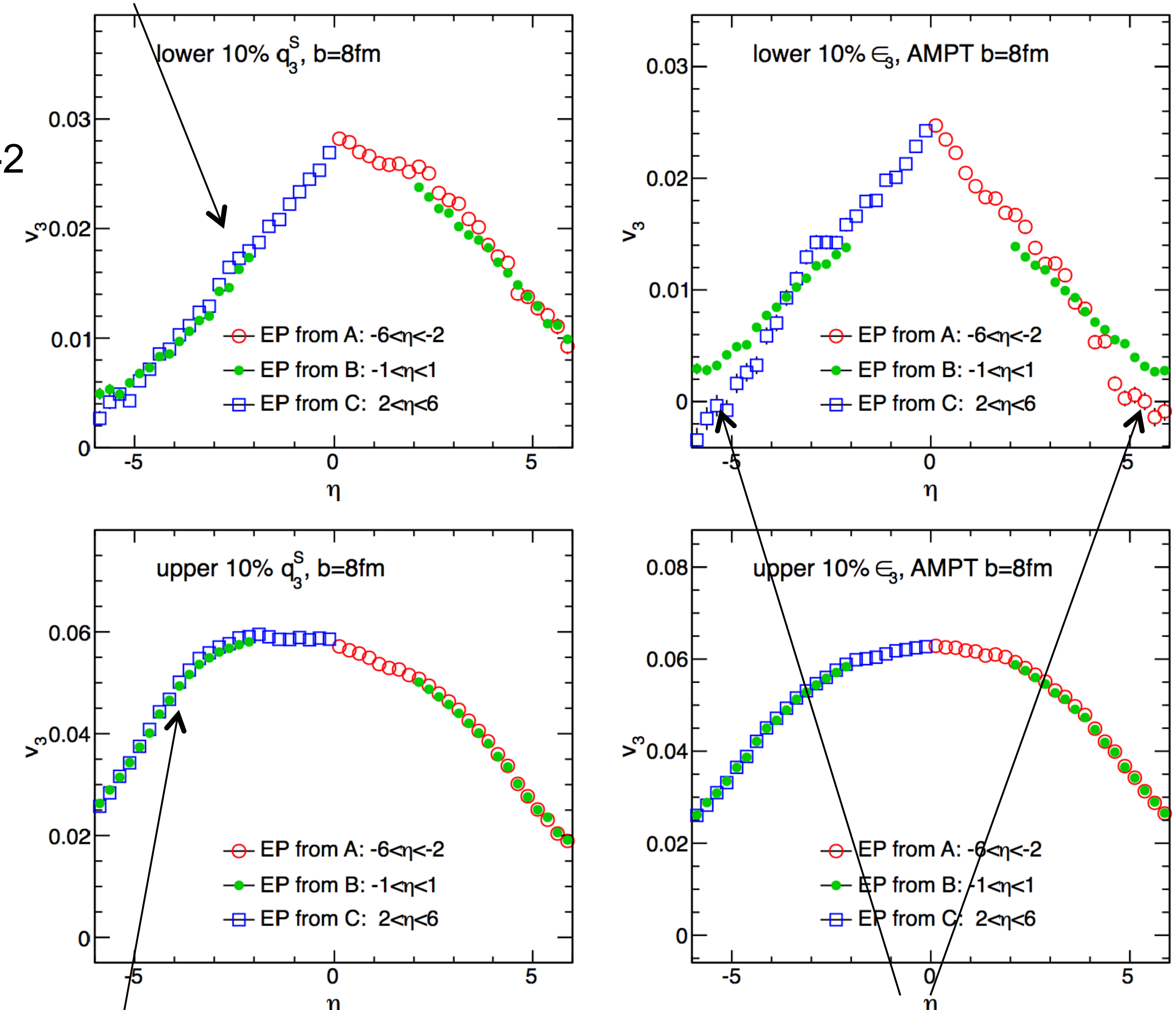
$v_3(\eta)$: select on q_3 or ε_3

Suppression of flow in the selected window

$v_2(\eta)|_{\eta>0}$ when EP in $-6<\eta<-2$

$v_2(\eta)|_{\eta<0}$ when EP in $2<\eta<6$

$v_2(\eta)|_{|\eta|>2}$ when EP in $|\eta|<1$

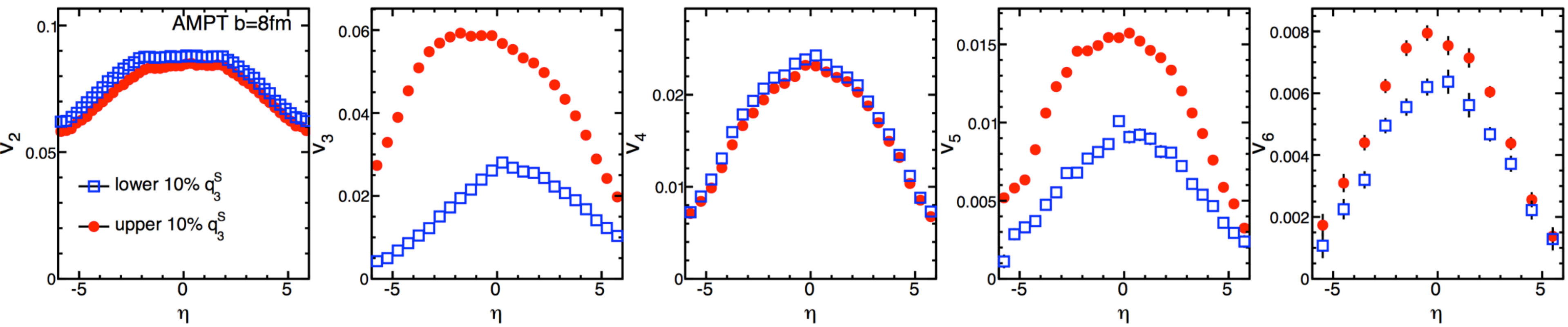


enhancement of flow in the selected window

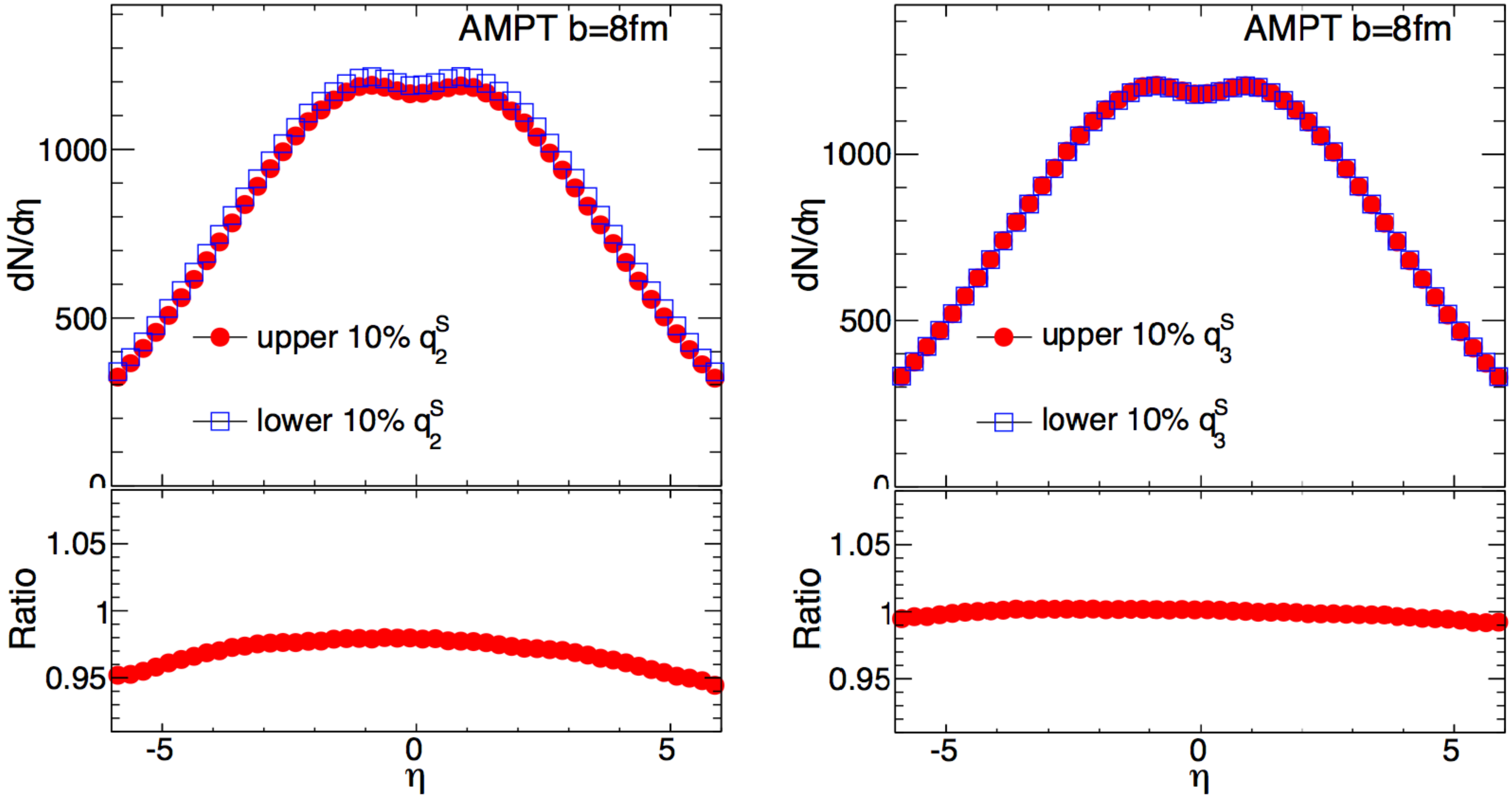
Clear decorrelation effects for events with small ε_3

$v_n(\eta)$ shape for all n: select on q_3

- Smaller asymmetry for other harmonics (but v_5 is noticeable)
- v_5, v_6 correlates with v_3 selection, direct measure of non-linear effects
  - v_5 couples more strongly with v_3 than v_2.
- Anti-correlation of v_3 with v_2 and v_4



There is no asymmetry in particle production



“Engineering” the EP correlations

2Plane correlation selected on q_2

-6<η<-2 Φ^A_2, Φ^A_4

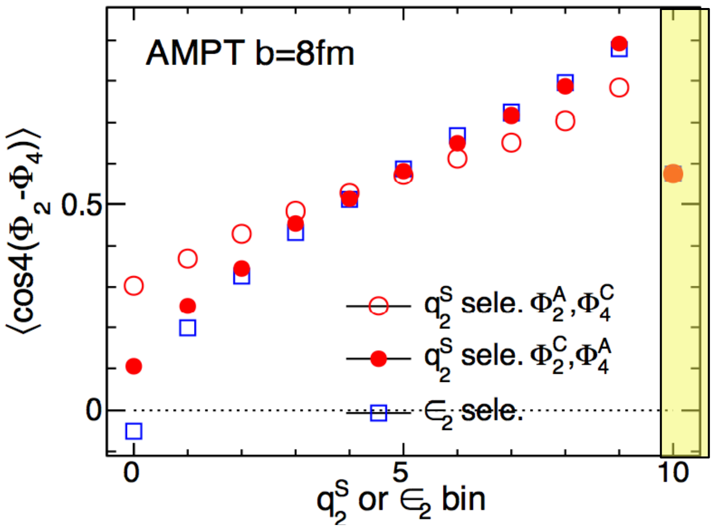
2<η<6 Φ^C_2, Φ^C_4

Shape selection breaks the symmetry of the EP correlations

Two types:

$$4(\Phi_2^A - \Phi_4^C)$$

$$4(\Phi_2^C - \Phi_4^A)$$



2Plane correlation selected on q_2

$-6<\eta<-2$ Φ^A_2, Φ^A_4

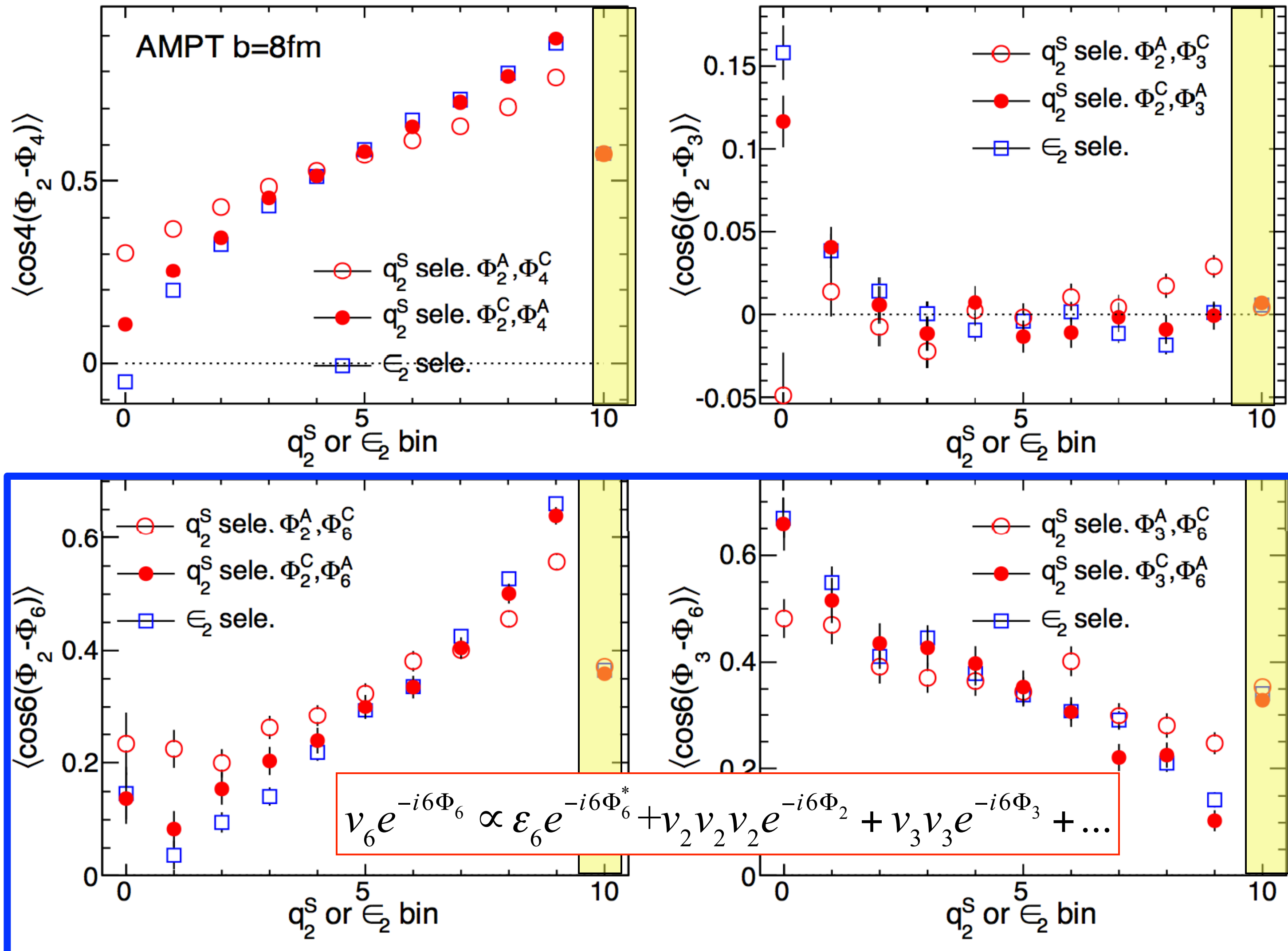
$2<\eta<6$ Φ^C_2, Φ^C_4

Shape selection breaks the symmetry of the EP correlations

Two types:

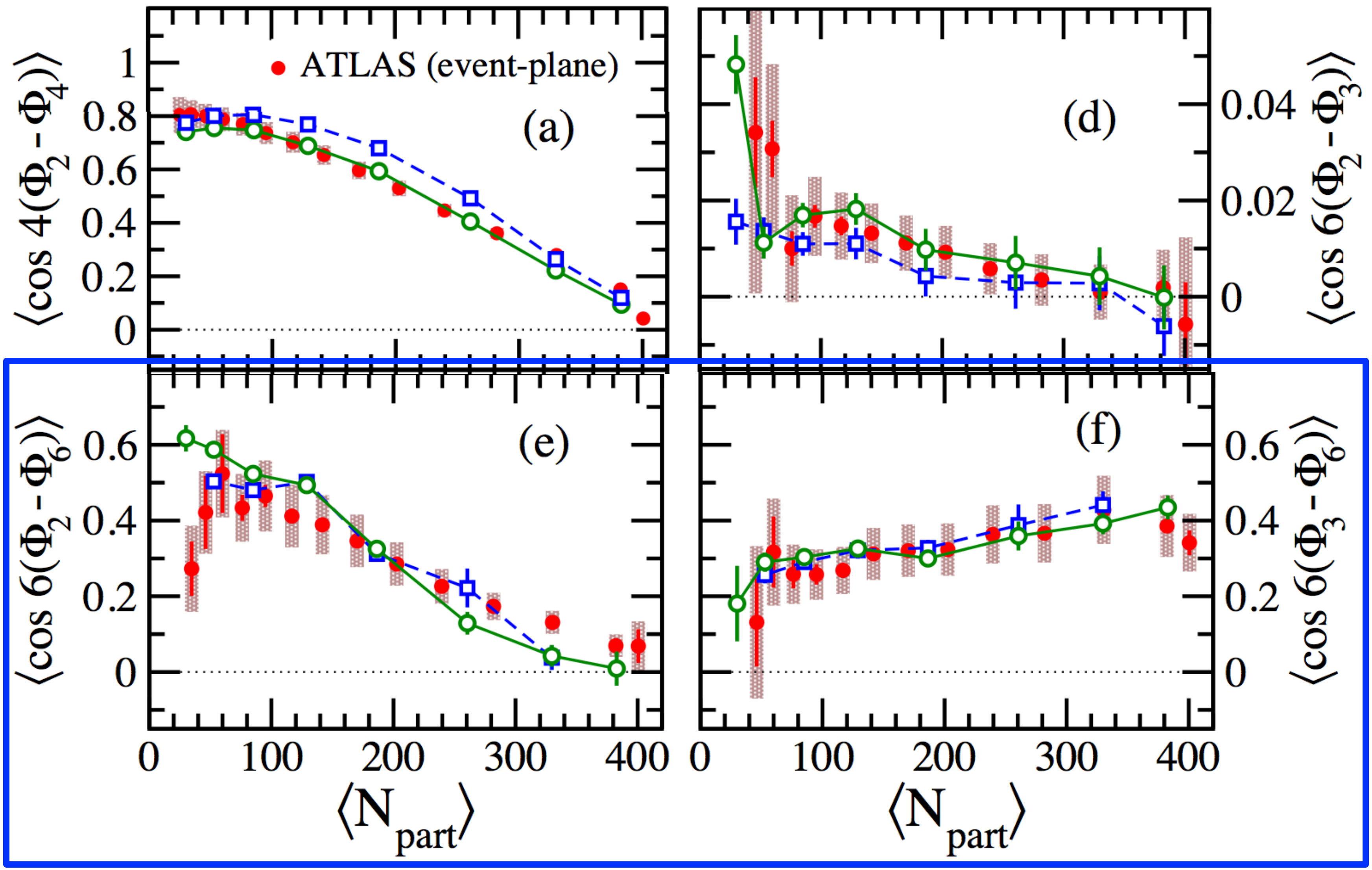
$$4(\Phi_2^A - \Phi_4^C)$$

$$4(\Phi_2^C - \Phi_4^A)$$



$$v_6 e^{-i6\Phi_6} \propto \varepsilon_6 e^{-i6\Phi_6^*} + v_2 v_2 v_2 e^{-i6\Phi_2} + v_3 v_3 e^{-i6\Phi_3} + \ldots$$

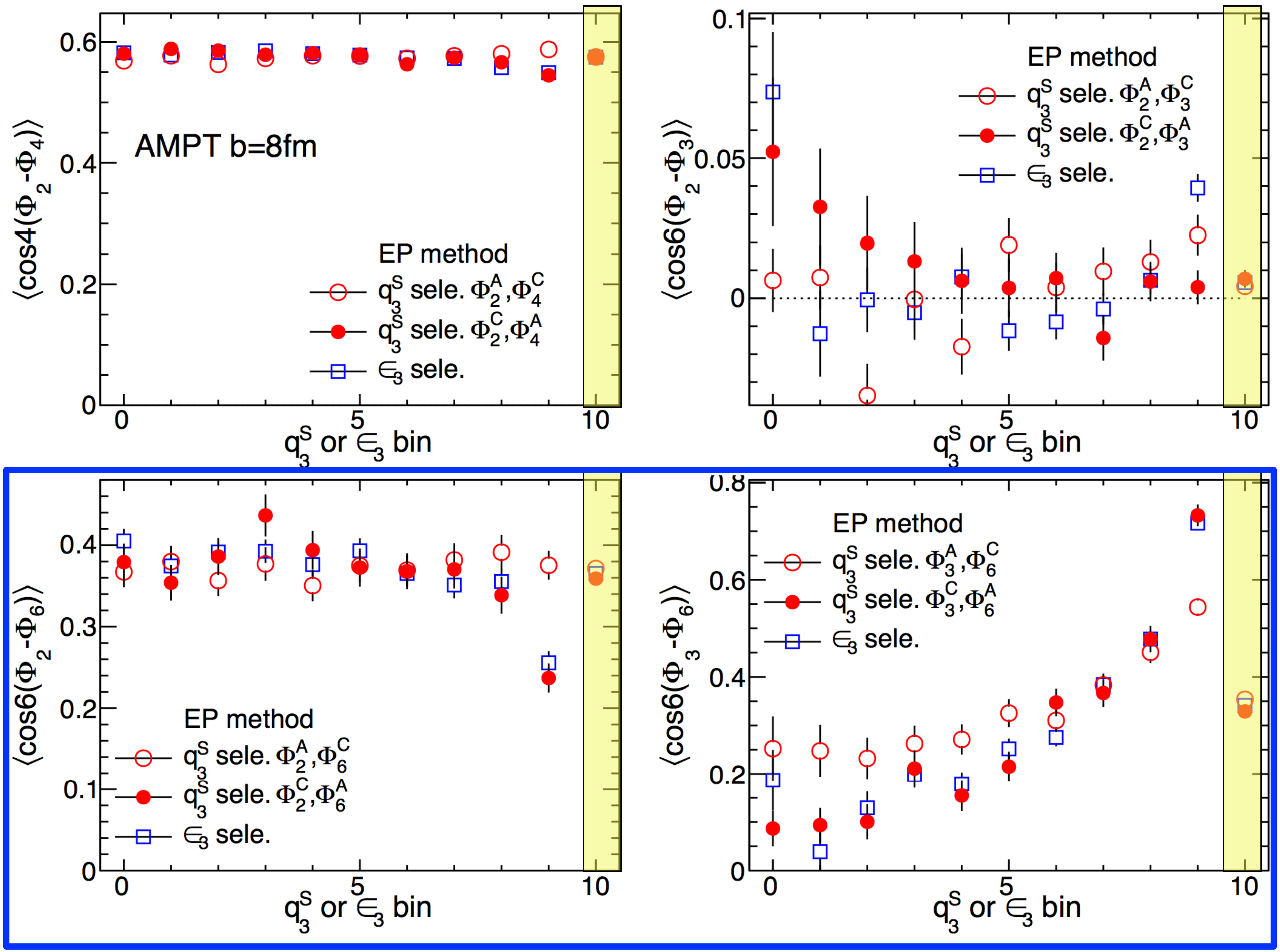
Centrality dependence of 2PC



1307.0980 Bhalerao,et.al.

$$v_6 e^{-i6\Phi_6} \propto \varepsilon_6 e^{-i6\Phi_6^*} + v_2 v_2 v_2 e^{-i6\Phi_2} + v_3 v_3 e^{-i6\Phi_3} + \ldots$$

2Plane correlation selected on q_3 and ε_3



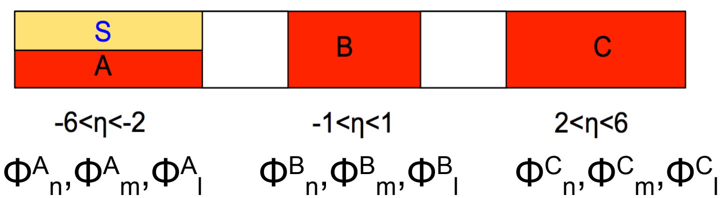
3PC select on q_2 ε_2

$\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$

$\langle \cos(2\Phi_2 - 6\Phi_3 + 4\Phi_4) \rangle$

Six different types:

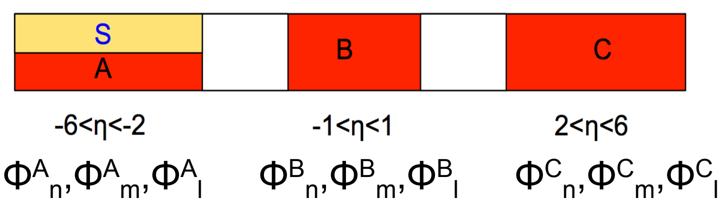
- Type 1a: $c_n n \Phi_n^{\mathrm{B}} + c_m m \Phi_m^{\mathrm{A}} + c_l l \Phi_l^{\mathrm{C}}$
- Type 1b: $c_n n \Phi_n^{\mathrm{B}} + c_m m \Phi_m^{\mathrm{C}} + c_l l \Phi_l^{\mathrm{A}}$
- Type 2a: $c_n n \Phi_n^{\mathrm{A}} + c_m m \Phi_m^{\mathrm{B}} + c_l l \Phi_l^{\mathrm{C}}$
- Type 2b: $c_n n \Phi_n^{\mathrm{C}} + c_m m \Phi_m^{\mathrm{B}} + c_l l \Phi_l^{\mathrm{A}}$
- Type 3a: $c_n n \Phi_n^{\mathrm{A}} + c_m m \Phi_m^{\mathrm{C}} + c_l l \Phi_l^{\mathrm{B}}$
- Type 3b: $c_n n \Phi_n^{\mathrm{C}} + c_m m \Phi_m^{\mathrm{A}} + c_l l \Phi_l^{\mathrm{B}}$



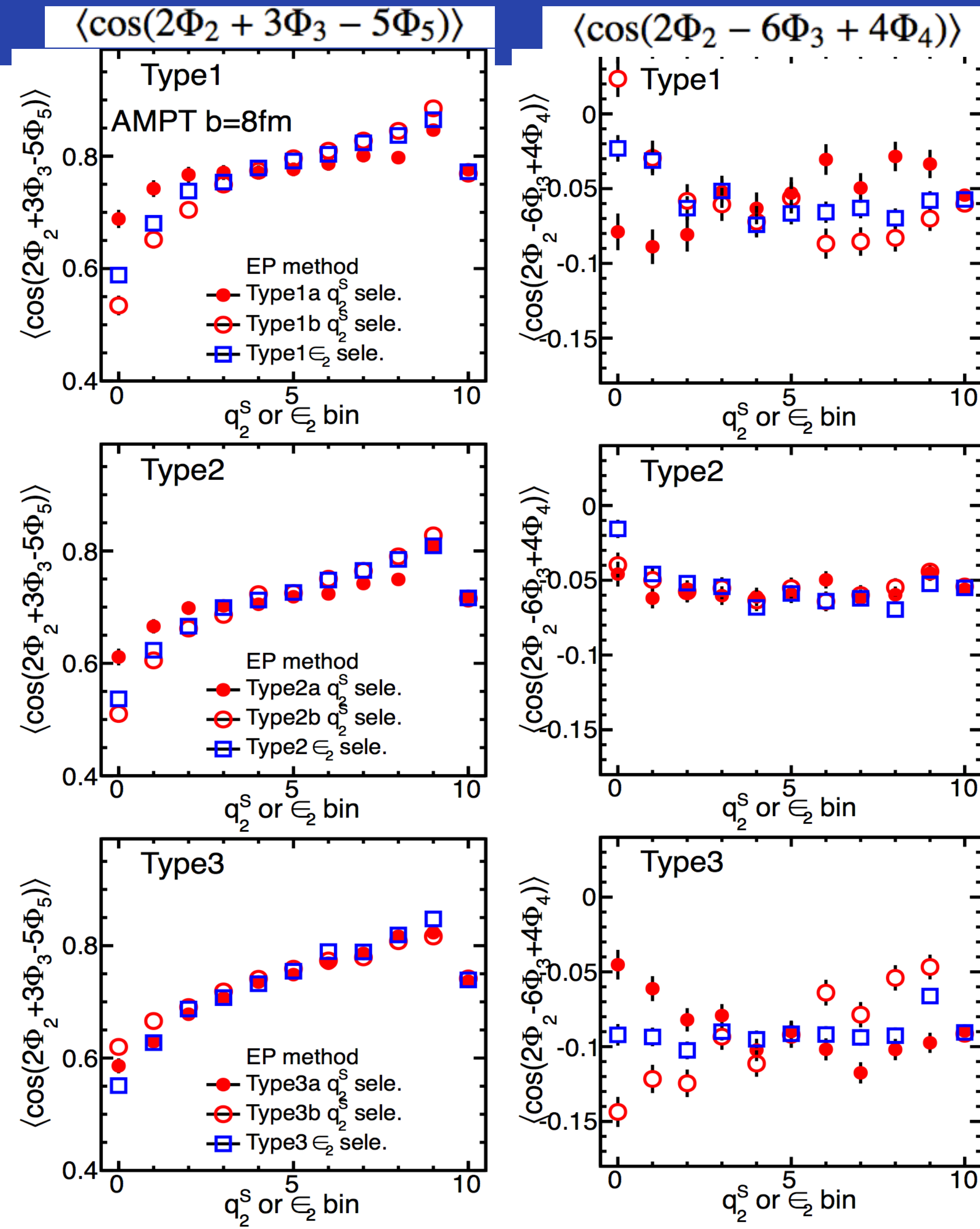
3PC select on q_2 ε_2

Six different types:

- Type 1a: $c_n n\Phi_n^B + c_m m\Phi_m^A + c_l l\Phi_l^C$
- Type 1b: $c_n n\Phi_n^B + c_m m\Phi_m^C + c_l l\Phi_l^A$
- Type 2a: $c_n n\Phi_n^A + c_m m\Phi_m^B + c_l l\Phi_l^C$
- Type 2b: $c_n n\Phi_n^C + c_m m\Phi_m^B + c_l l\Phi_l^A$
- Type 3a: $c_n n\Phi_n^A + c_m m\Phi_m^C + c_l l\Phi_l^B$
- Type 3b: $c_n n\Phi_n^C + c_m m\Phi_m^A + c_l l\Phi_l^B$



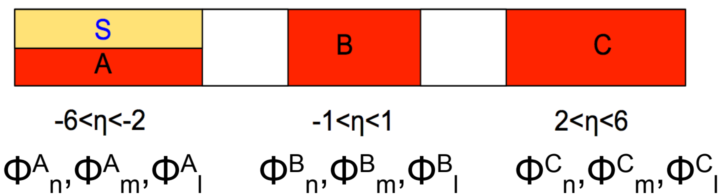
Many interesting patterns



3PC select on q_3 ε_3

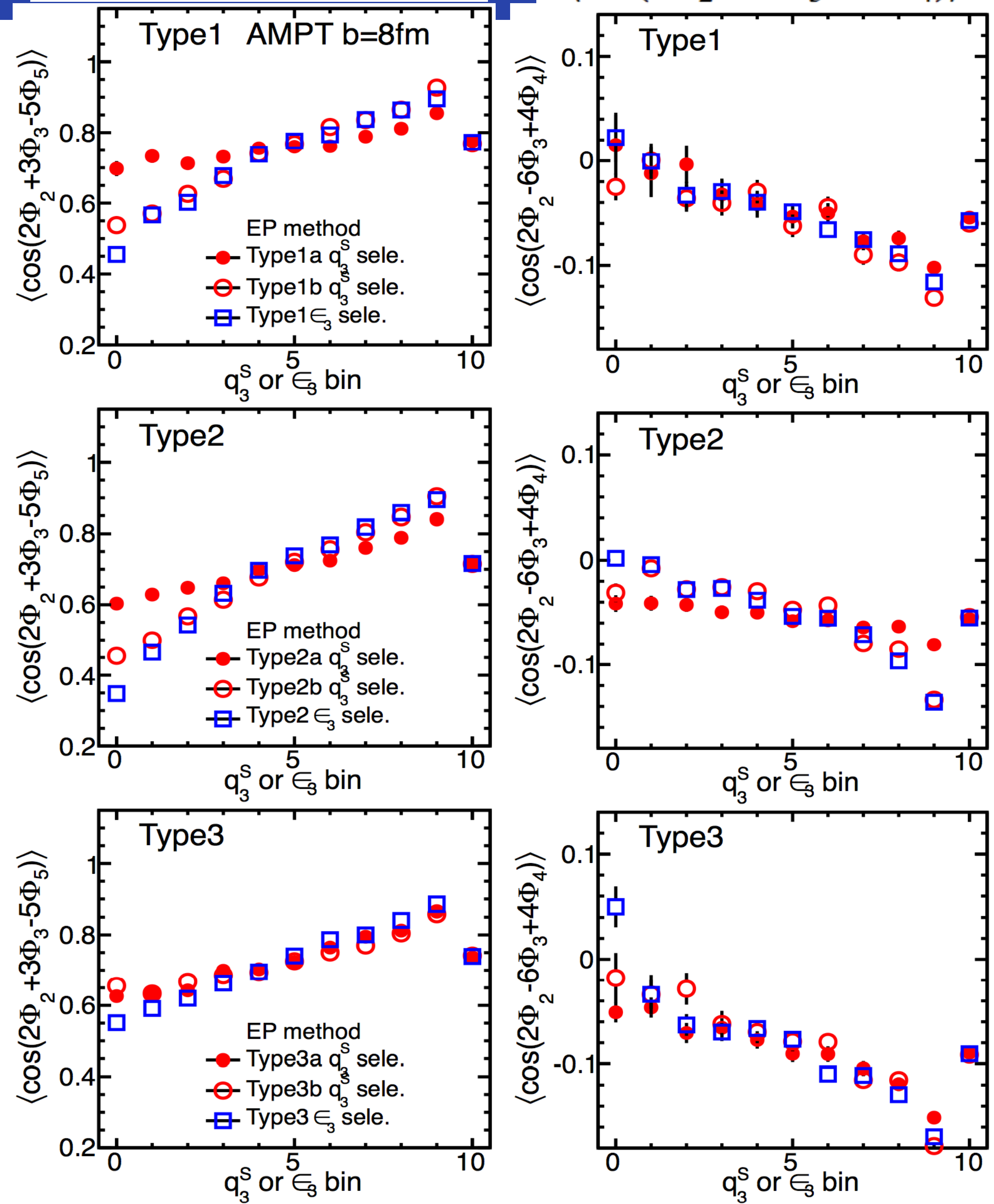
Six different types:

- Type 1a: $c_n n\Phi_n^B + c_m m\Phi_m^A + c_l l\Phi_l^C$
- Type 1b: $c_n n\Phi_n^B + c_m m\Phi_m^C + c_l l\Phi_l^A$
- Type 2a: $c_n n\Phi_n^A + c_m m\Phi_m^B + c_l l\Phi_l^C$
- Type 2b: $c_n n\Phi_n^C + c_m m\Phi_m^B + c_l l\Phi_l^A$
- Type 3a: $c_n n\Phi_n^A + c_m m\Phi_m^C + c_l l\Phi_l^B$
- Type 3b: $c_n n\Phi_n^C + c_m m\Phi_m^A + c_l l\Phi_l^B$

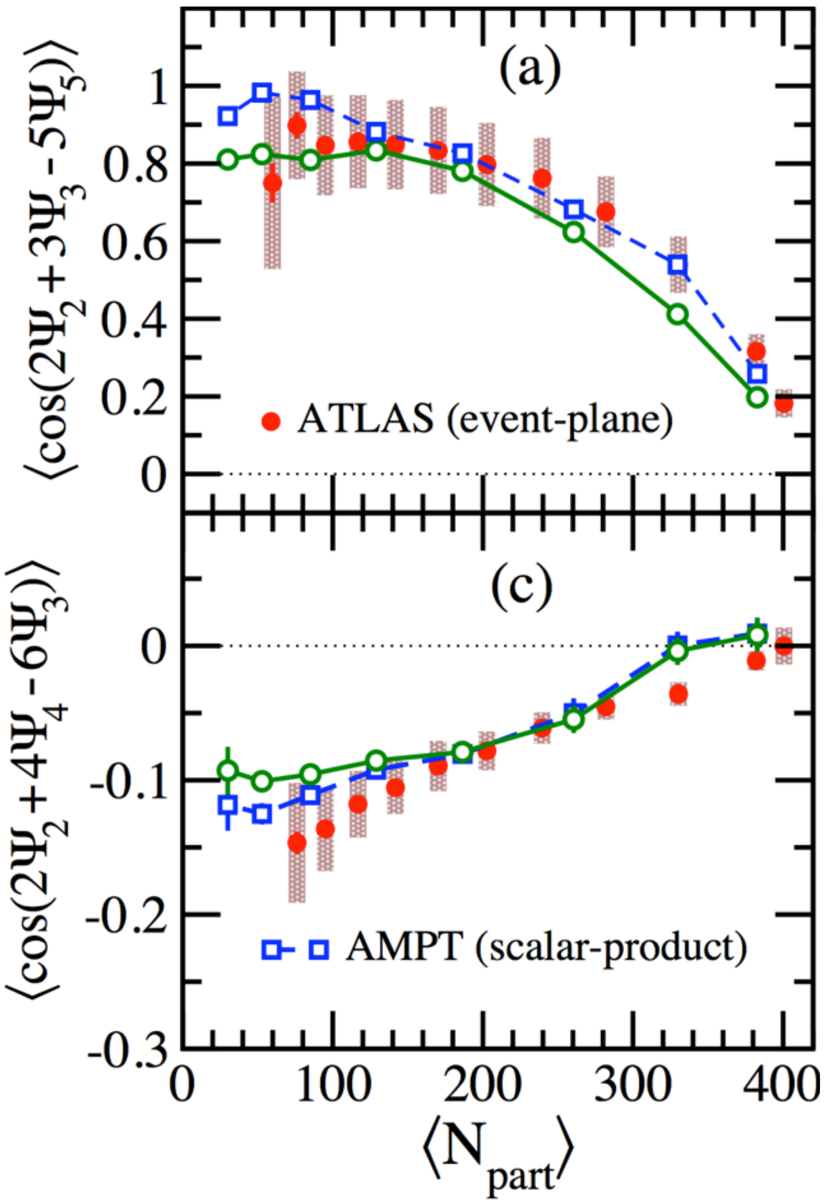


Many interesting patterns

$\langle\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)\rangle$ $\langle\cos(2\Phi_2 - 6\Phi_3 + 4\Phi_4)\rangle$



ESE result helps interpret the centrality dependence



v5 ~ v2v3

Centrality dependence is affected by change in v_2 and v_3

Centrality dependence is controlled mainly by change in v_3!!

Summary

- One key question in EbyE flow study is when/how the fluctuations are generated in the space-time evolution of the system.
  - Need to disentangle dynamical fluctuations from the dominating contribution of the initial condition.

- $p(v_m|v_n)$ and $p(\Phi_n,\Phi_{m,\ldots}|v_n)$ with ESE allow us to systematize the initial condition and better expose the dynamical effects

- First study w/ AMPT show that the flow and EP correlation can be studied much more differentially for fixed centrality. The non-linear dynamics and rapidity fluctuations are directly exposed.

- More detailed study using EbyE hydro are needed.