Turbulent Thermalization

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arXiv:1303.5650, arXiv:1311.3005

Content

Initial state:
Far from equilibrium

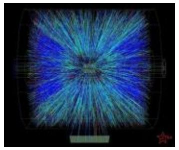
Non-equilibrium dynamics

Final state:
Thermal equilibrium

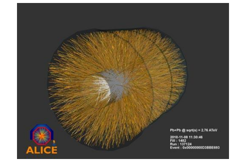
I. Universal attractor in longitudinal expanding plasmas: Turbulence and \`bottom up´ thermalization

II. Limitations of classical-statistical lattice simulations: A well understood quantum example

Heavy Ion Collisions



- fluid-like behavior from very early time on
- very special transport properties, such as small η/s



How is local isotropization/thermalization achieved?

Strong correlations:

a) strong coupling? Gauge-string duality: Heller, Janik, Witaszczyk; Chesler, Yaffe …

Sizeable anisotropy even at transition to hydrodynamic regime

b) weak coupling but highly occupied? CGC: McLerran, Venugopalan, …

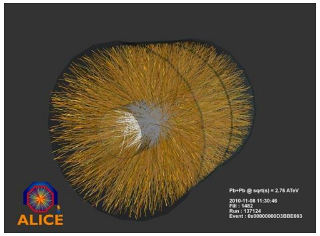
Energy density of gluons with typical momentum Q (at time $\sim 1/Q$)

$$\epsilon \sim \frac{Q^4}{\alpha_s} \qquad \text{i.e. ‘occupation numbers‘} \qquad f\,(p \lesssim Q\,) \sim \frac{1}{\alpha_s}$$

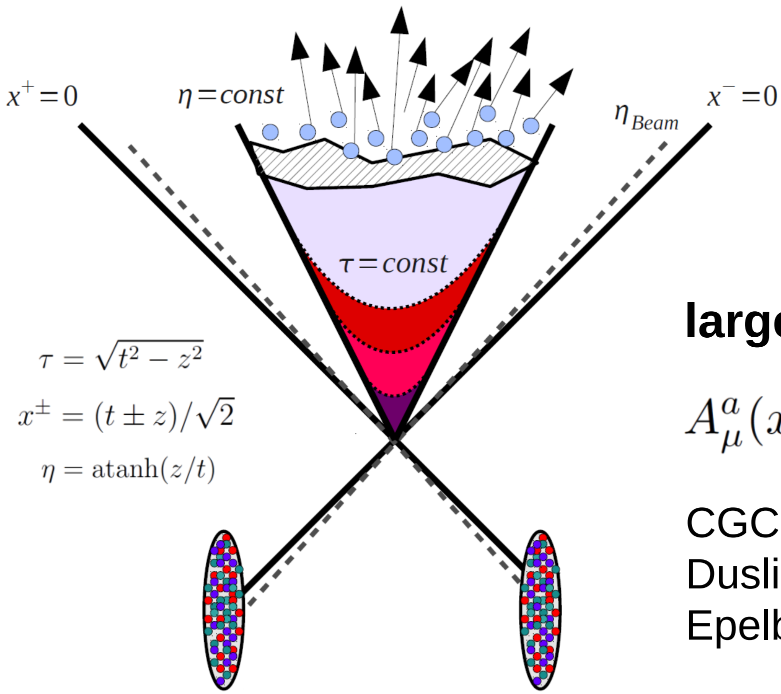
Strongly correlated/nonperturbative even for weak coupling $\alpha_s \ll 1$

High-energy/weak-coupling limit

Particle production in the presence of large fields



$$g \ll 1$$



large initial fields:

$$A^a_\mu(x) = \langle \hat{A}^a_\mu(x) \rangle \ \sim \ \mathcal{O}(1/g)$$

CGC: Lappi, McLerran, Dusling, Gelis, Venugopalan, Epelbaum…

small initial (vacuum) fluctuations:

$$F^{ab}_{\mu\nu}(x,y) = \frac{1}{2}\left\langle \left\{ \hat{A}^a_\mu(x), \hat{A}^b_\nu(y) \right\} \right\rangle - A^a_\mu(x) A^b_\nu(y)$$

$$\sim \ \mathcal{O}(1)$$

→ ***plasma instabilities!***

Mrowczynski; Rebhan, Romatschke, Strickland; Arnold, Moore, Yaffe …

Different weak-coupling thermalization scenarios

Kinetic theory (parametric):

- Bottum-up isotropization

  BMSS: Baier, Mueller, Schiff, Son (2001)
- Rescattering due to plasma (Weibel) instabilities

  BD: Boedeker (2005); KM: Kurkela, Moore (2011)
- Transient Bose condensation+fixed anisotropy

  BGLMV: Blaizot, Gelis, Liao, McLerran, Venugopalan (2012)

How can we decide from first principles?

→ for typical occupancies $1 < f < 1/g^2$ a dual description is feasible:

$f(p) \sim 1/g^2$	$1/g^2 \gg f(p) \gg 1$	$f(p) \lesssim 1$
('over-occupied')	(classical particle)	(quantum)

$\underbrace{\text{('over-occupied')} \quad \text{(classical particle)}}_{\text{classical-statistical field theory}}$

$\underbrace{\text{(classical particle)} \quad \text{(quantum)}}_{\text{kinetic theory}}$

Non-linear evolution: Classical-statistical lattice gauge theory

Wilson action:

$$S[U] = -\beta_0 \sum_x \sum_i \left\{ \frac{1}{2\mathrm{Tr}\,\mathbf{1}} \left(\mathrm{Tr}\, U_{x,0i} + \mathrm{Tr}\, U_{x,0i}^{-1} \right) - 1 \right\} + \beta_s \sum_x \sum_{\substack{i,j \\ i<j}} \left\{ \frac{1}{2\mathrm{Tr}\,\mathbf{1}} \left(\mathrm{Tr}\, U_{x,ij} + \mathrm{Tr}\, U_{x,ij}^{-1} \right) - 1 \right\}$$

Plaquette variables $U_{x,\mu\nu} \equiv U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu} \approx \exp\left[-\mathrm{i} g a^2 F_{\mu\nu}(x) \right]$

Sampling introduces classical-statistical fluctuations (‘loops to all orders‘)
→ accurate for characteristic ’large fields/high occupation numbers’:

anti-commutator $\langle \{A, A\} \rangle \gg \langle [A, A] \rangle$ commutator

ab initio description of non-Abelian real-time dynamics!

→ instability dynamics, turbulence:

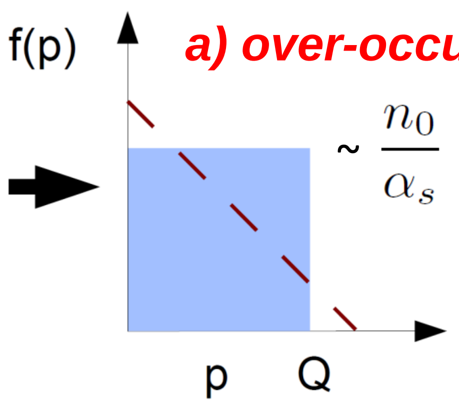
Romatschke, Venugopalan; Berges, Boguslavski, Gelfand, Scheffler, Schlichting, Sexty; Kunihiro, Müller, Ohnishi, Schäfer, Takahashi, Yamamoto; Fukushima, Gelis, Eppelbaum; …

Plasma instabilities at early times

- quasi-exponential growth of initial fluctuations
- clear separation between instability and scaling (turbulent) regime
- characteristic distributions after instabilities saturate; schematic:

a) over-occupation:

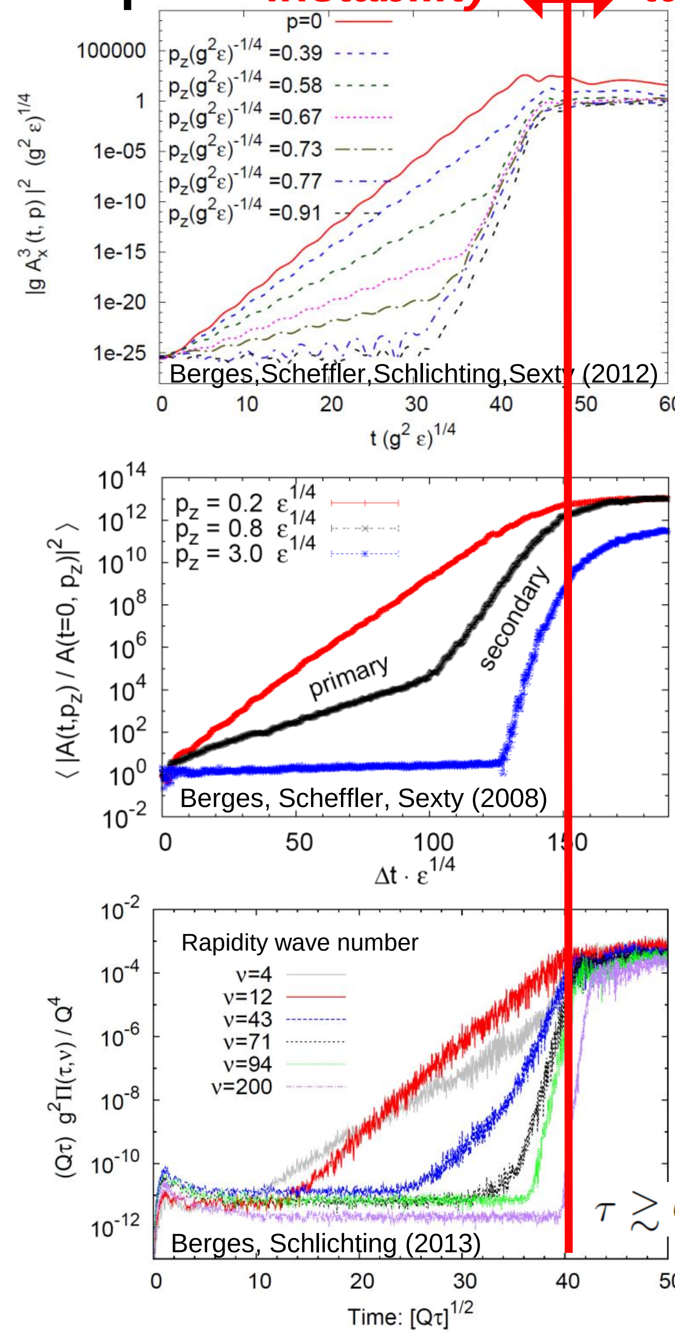
Insta-
bilities



$\sim \dfrac{n_0}{\alpha_s}$

b) anisotropy: $p \approx \sqrt{p_T^2 + (\xi_0 p_z)^2}$

Examples: ***instability*** ↔ ***turbulence***



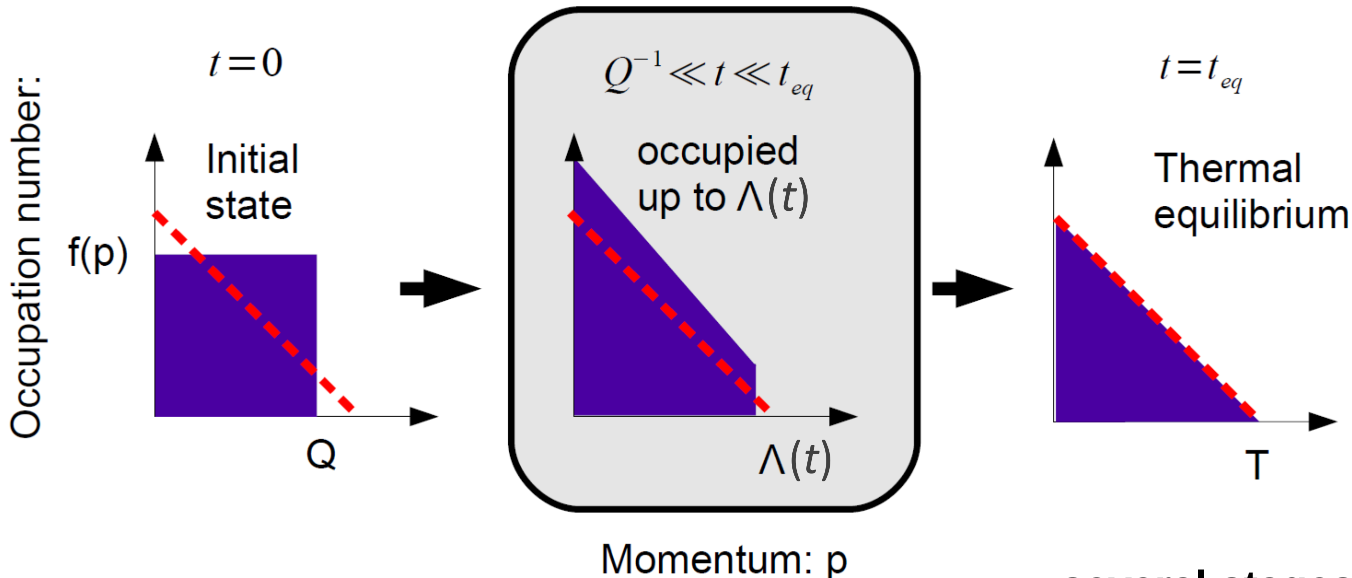
Large initial fields in fixed box

Large initial fluctuations, fixed box

CGC longitudinal expansion

$\tau \gtrsim Q^{-1} \ln^2(\alpha_s^{-1})$

Thermalization process – schematic



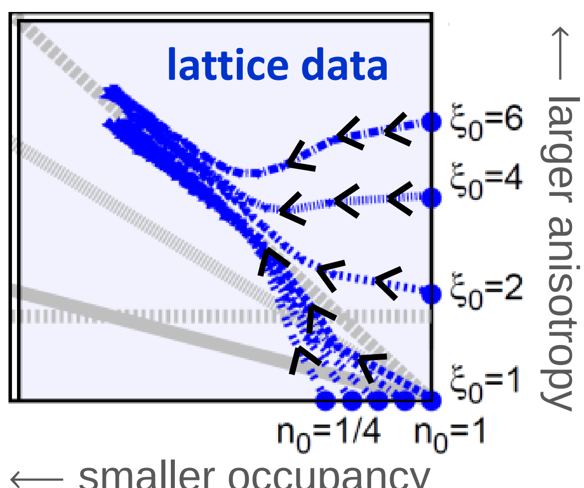
$Q\tau_0 \sim \ln^2(\alpha_s^{-1})$ → ***Universal attractor*** →

several stages (BMSS):

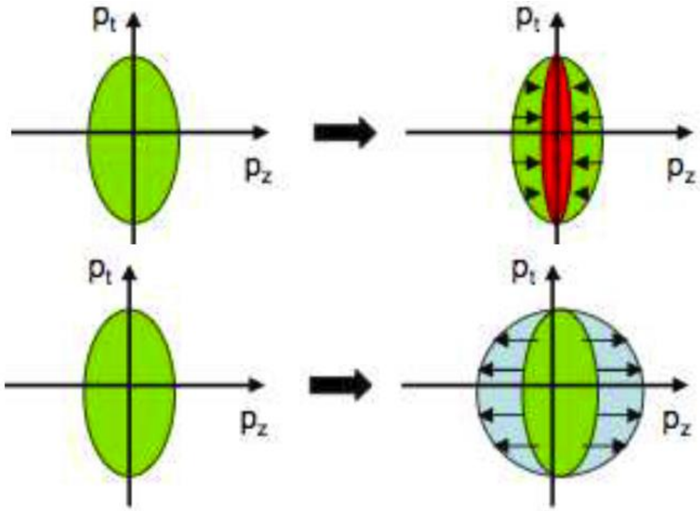
$Q\,\tau \sim \alpha^{-3/2},\ \alpha^{-5/2},\ \alpha^{-13/5}$

attractor (classical), isotropization (quantum), thermalization (quantum)

different initial occupancies & anisotropies lead to same universal properties!



Thermalization of expanding systems



Longitudinal Expansion:

- Red-shift of longitudinal momenta p_z
  $\rightarrow$ increase of anisotropy
- Dilution of the system

Interactions:

- Isotropize the system

- Redshift and dilution require extremely large lattices
- Will report here on real-time classical-statistical simulations on

$256^2 \times 4096$ (!) spatial lattices

-- fully capture important infrared dynamics

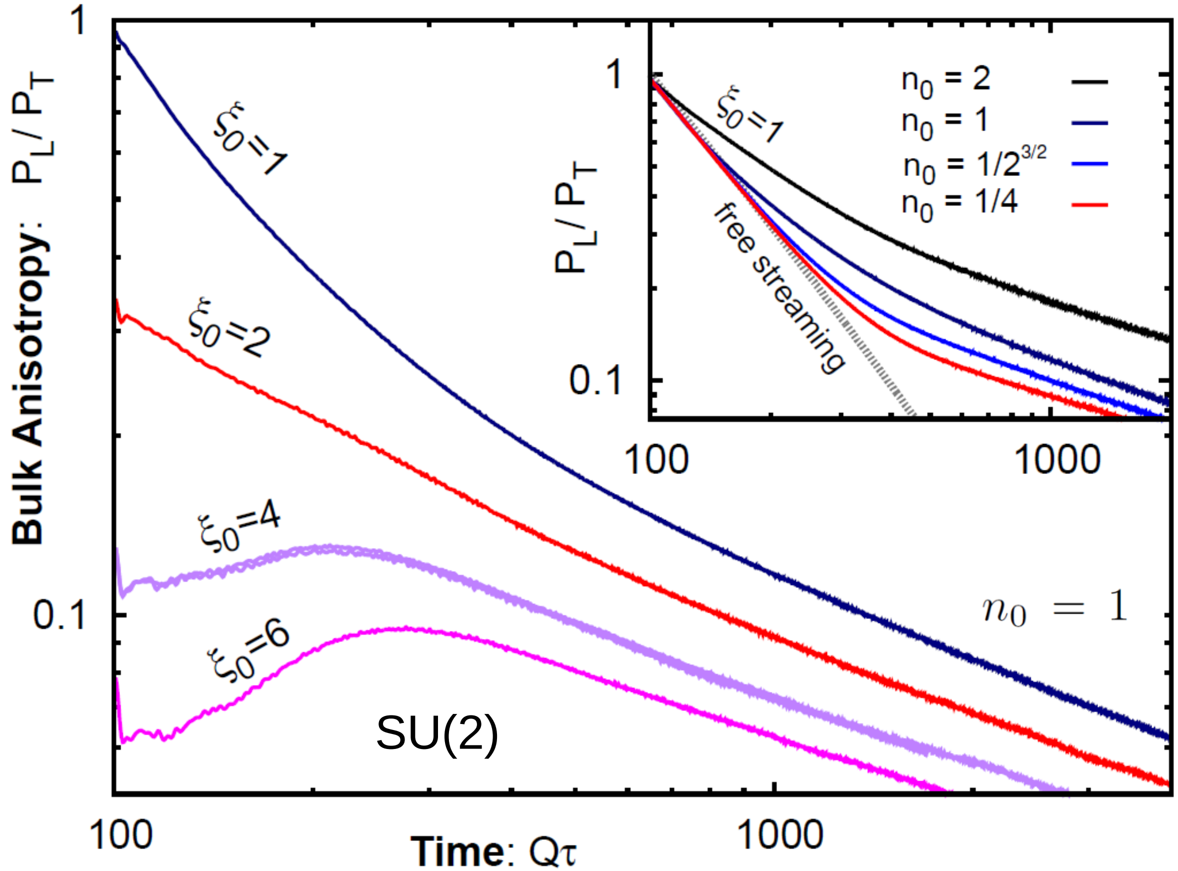
Berges, Boguslavski, Schlichting, Venugopalan:1303.5650; 1311.3005

Longitudinal expanding non-Abelian plasma: Anisotropy

occupancy parameter

Initial gluon distributions: $$f(\mathrm{p_T}, \mathrm{p_z}, \tau_0) = \frac{n_0}{2g^2}\,\Theta\left(Q - \sqrt{\mathrm{p_T^2} + (\xi_0 \mathrm{p_z})^2}\right)$$

anisotropy parameter

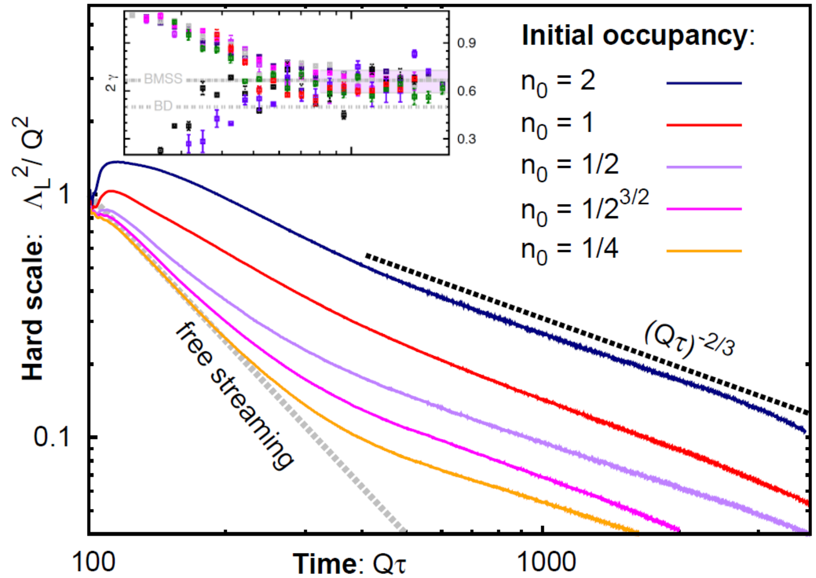


Large initial anisotropy leads to transient isotropy increase (→ instabilities).

Smaller initial occupancy leads to transient period of free streaming.

Continues to be ***strongly correlated*** throughout the entire turbulent stage!

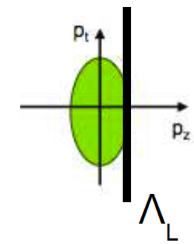
Gauge invariant hard scales: Universal power laws

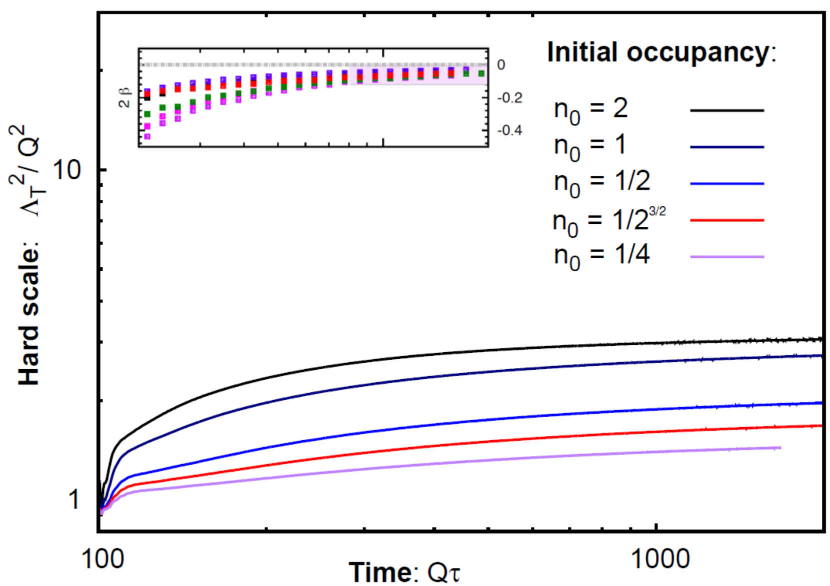


- The typical ***longitudinal momentum*** of hard excitations exhibits a ***universal scaling*** behavior

$$\Lambda_L^2(\tau) \propto Q^2\ (Q\tau)^{-2\gamma}$$

$$\boxed{2\gamma \;=\; 0.67 \pm 0.07}$$

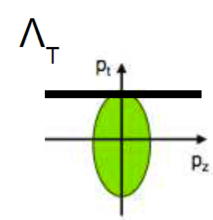




- The typical ***transverse momentum*** of hard excitations remains approximately ***constant***

$$\Lambda_T^2(\tau) \propto Q^2\ (Q\tau)^{-2\beta}$$

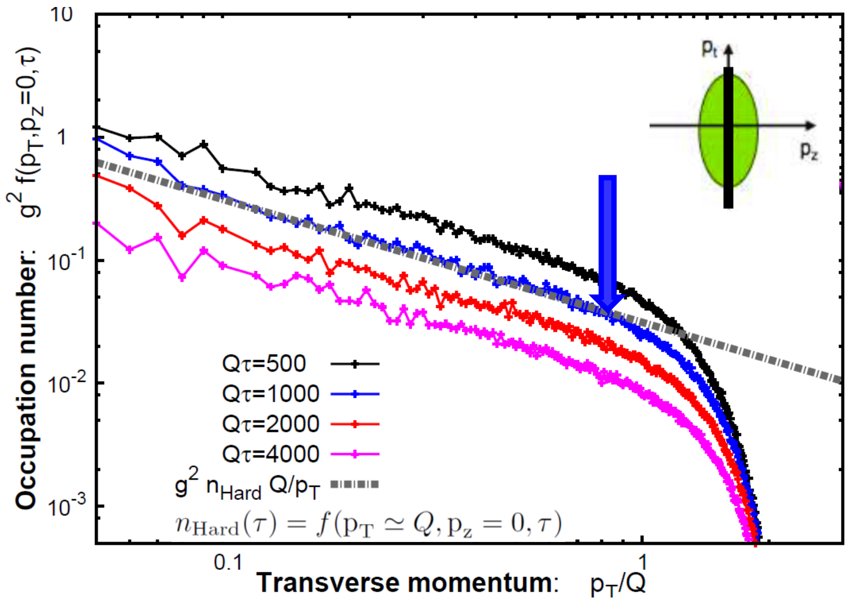
$$\boxed{2\beta \simeq 0}$$



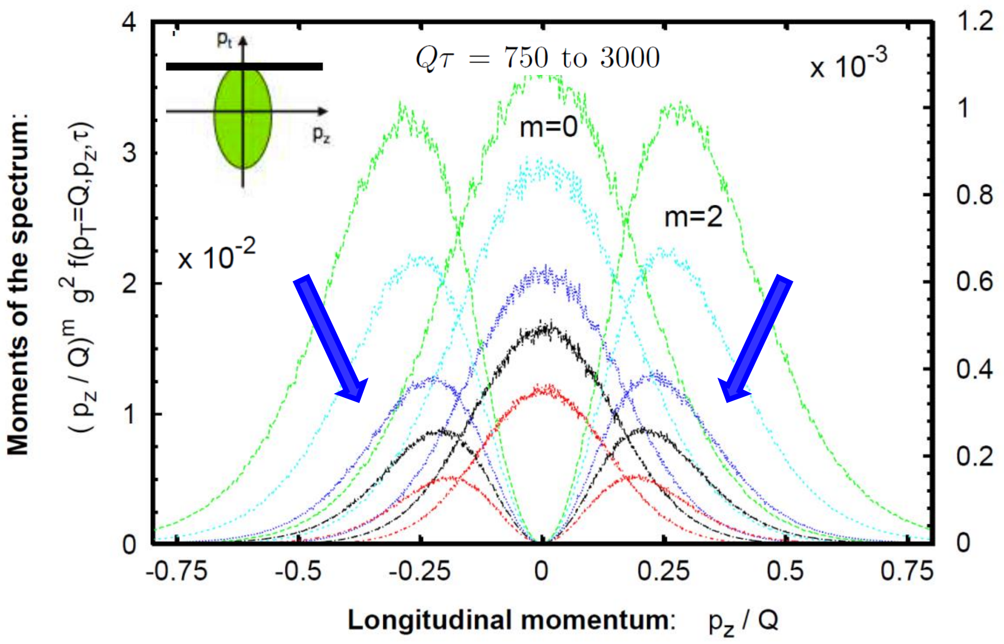
Transverse and longitudinal spectra

(Coulomb type gauge)

Transverse spectrum

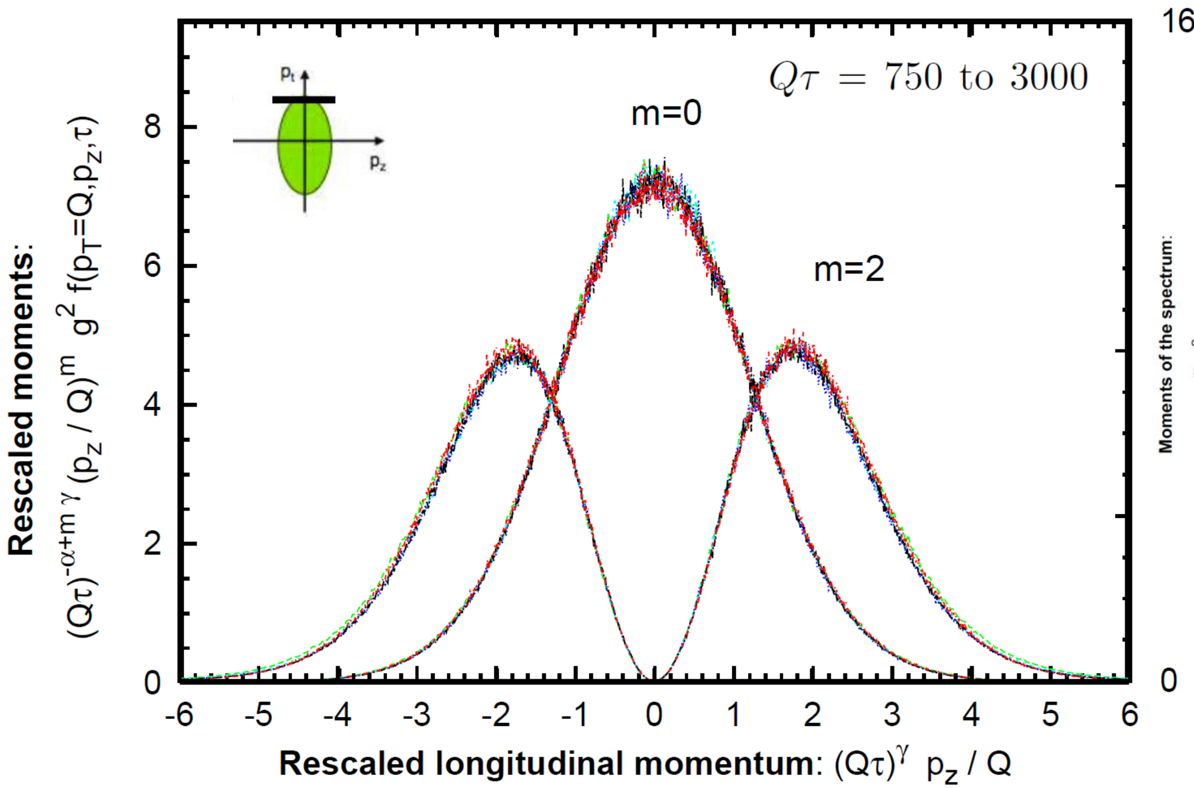


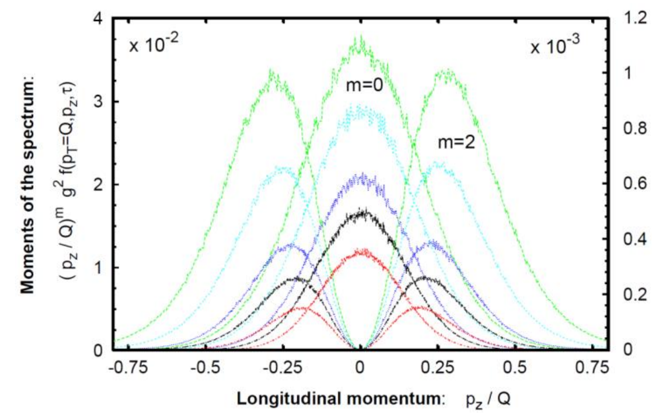
Longitudinal spectrum



Thermal-like transverse shape ~1/p_T even as longitudinal distribution is beeing `squeezed´

Nonthermal fixed point: Self-similar evolution!





The spectrum shows a self-similar evolution with universal scaling exponents α, β, γ and scaling function f_S:

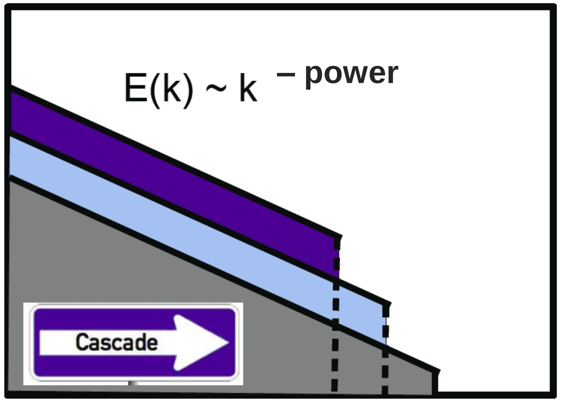
$$f(\mathrm{p_T},\mathrm{p_z},\tau) = (Q\tau)^{\alpha} \underbrace{f_S\Big((Q\tau)^{\beta}\mathrm{p_T},(Q\tau)^{\gamma}\mathrm{p_z}\Big)}_{\text{stationary fixed-point distribution}}$$

Nature of nonthermal fixed point: wave turbulence

Boltzmann equation with generic collision term for longitudinal expansion:

$$\left[\partial_t - \frac{p_z}{t}\partial_{p_z}\right] f(p_T, p_z, t) = C[p_T, p_z, t; f]$$

Turbulent Thermalization



Self-similar evolution:

$$f(p_T, p_z, t) = t^{\alpha} f_S(t^{\beta} p_T, t^{\gamma} p_z)$$

$$C[p_T, p_z, t; f] = t^{\mu}\, C[t^{\beta} p_T, t^{\gamma} p_z; f_S]$$

→ *a)* ***fixed point equation for stationary distribution****:*

$$\alpha f_S(p_T, p_z) + \beta p_T \partial_{p_T} f_S(p_T, p_z + (\gamma - 1)\, p_z \partial_{p_z} f_S(p_T, p_z) = C[p_T, p_z; f_S]$$

→ *b)* ***scaling condition****:*

$$\boxed{\alpha - 1 = \mu(\alpha, \beta, \gamma)}$$

Nonthermal fixed point

Interpret scaling condition with **energy/number conserving*** Fokker-Planck-type dynamics for the collisional broadening of longitudinal momentum distribution:

$$C^{(\mathrm{elast})}[p_T, p_z; f] = \hat{q}\; \partial^2_{p_z} f(p_T, p_z, t)$$

with **momentum diffusion parameter**: $\hat{q} \sim \alpha_S^2 \int \frac{d^2 p_T}{(2\pi)^2} \int \frac{dp_z}{2\pi}\; f^2(p_T, p_z, t)$

→ 1) $\mu = 3\alpha - 2\beta + \gamma \xrightarrow{\alpha - 1 = \mu(\alpha,\beta,\gamma)} 2\alpha - 2\beta + \gamma + 1 = 0$

2) number conservation $\longrightarrow$ $\alpha - 2\beta - \gamma + 1 = 0$

3) energy conservation $\longrightarrow$ $\alpha - 3\beta - \gamma + 1 = 0$

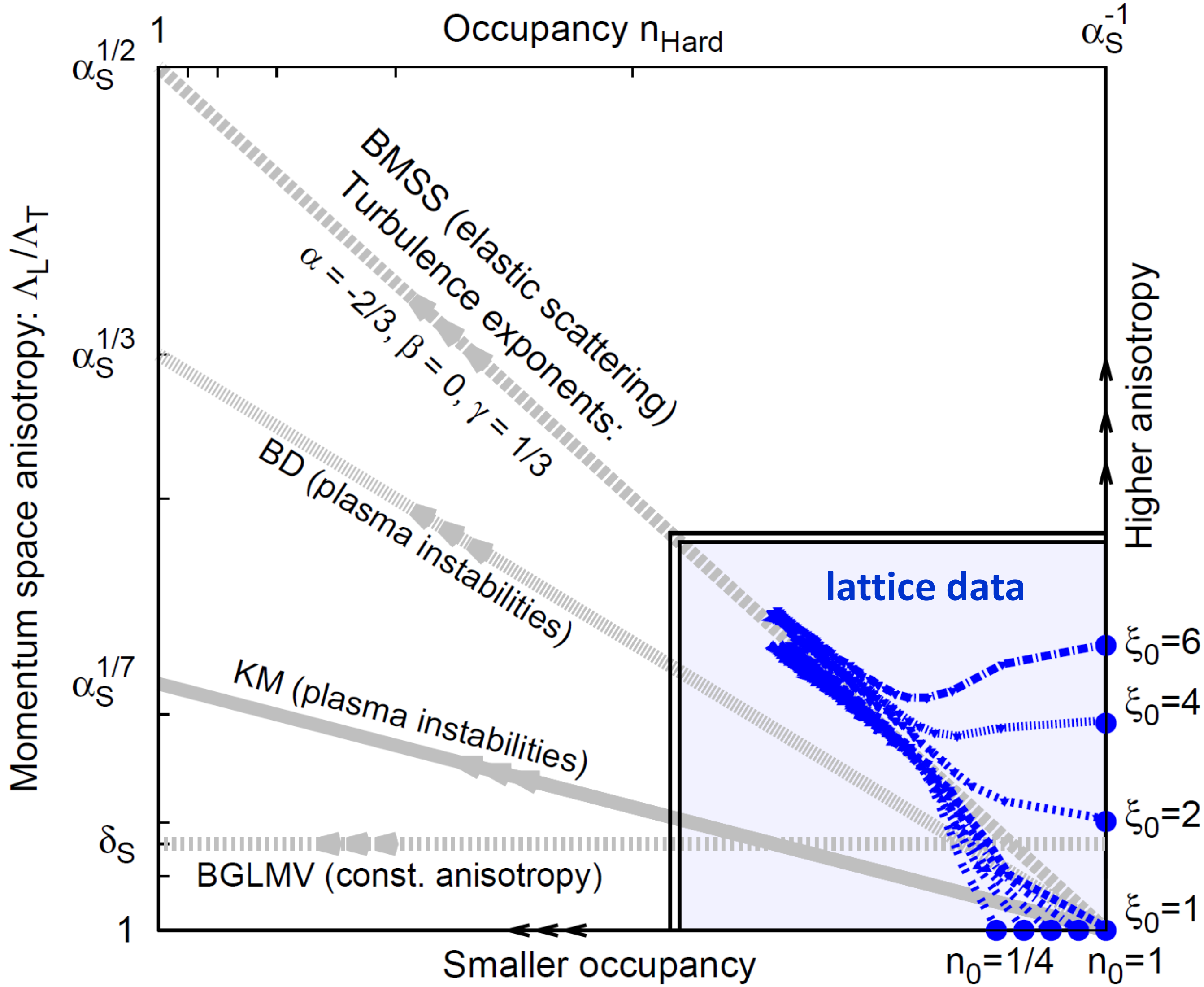
$$\boxed{\alpha = -2/3\,, \qquad \beta = 0\,, \qquad \gamma = 1/3}$$

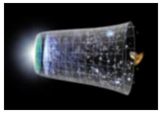
remarkable agreement with lattice data!

*cf. early stages of Baier, Mueller, Schiff, Son, PLB 502 (2001) 51 ('BMSS')

Universal attractor

Evolution in the `anisotropy-occupancy plane´

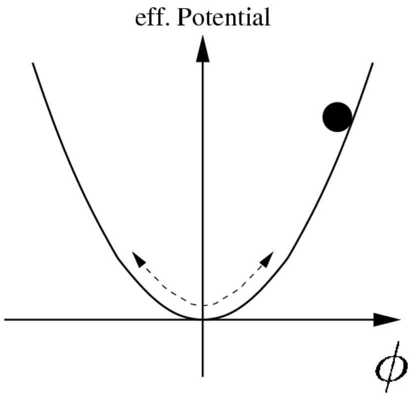




A well understood *quantum* example

Early universe preheating:

Kofman, Linde, Starobinsky, PRL 73 (1994) 3195



Scalar $\lambda\Phi^4$ inflaton: $\lambda \ll 1$

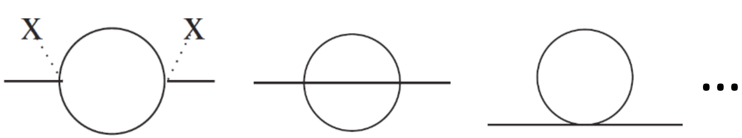
- **large** initial field $\phi = \langle\Phi\rangle \sim 1/\lambda^{1/2}$
- **small** fluctuation $F \sim \langle\{\Phi,\Phi\}\rangle - \phi\phi \sim 1$

Instability: $F(t) \sim e^{\gamma t}$ $(\gamma > 0)$

Quantum field theory:

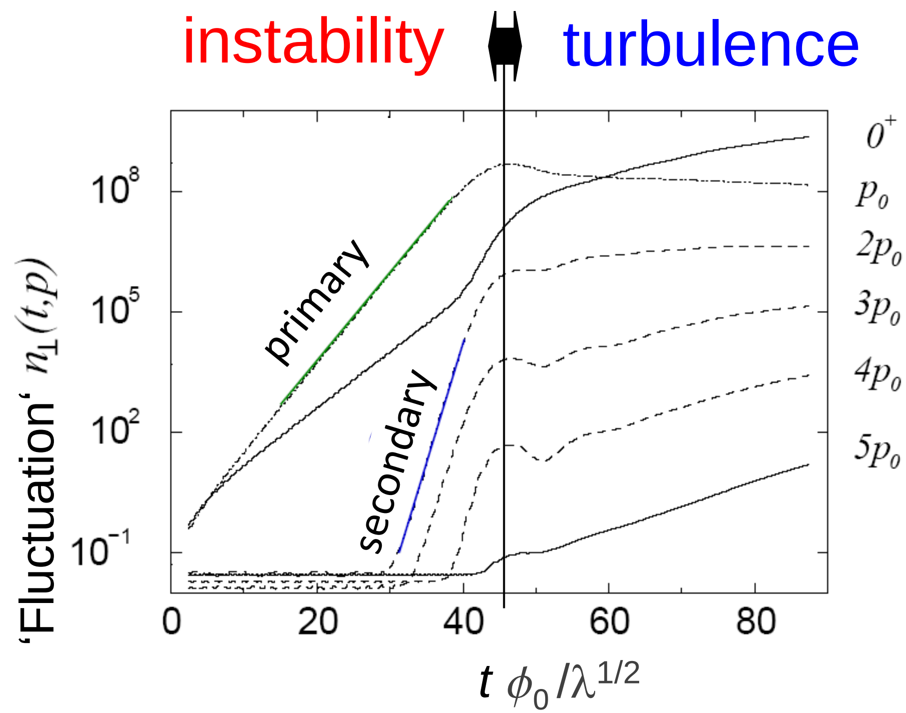
Berges, Serreau, PRL 91 (2003) 111601

Dynamical power counting (2PI):



$F \sim 1/\lambda^{1/2}$ $F \sim 1/\lambda^{3/2}$ $F \sim 1/\lambda$

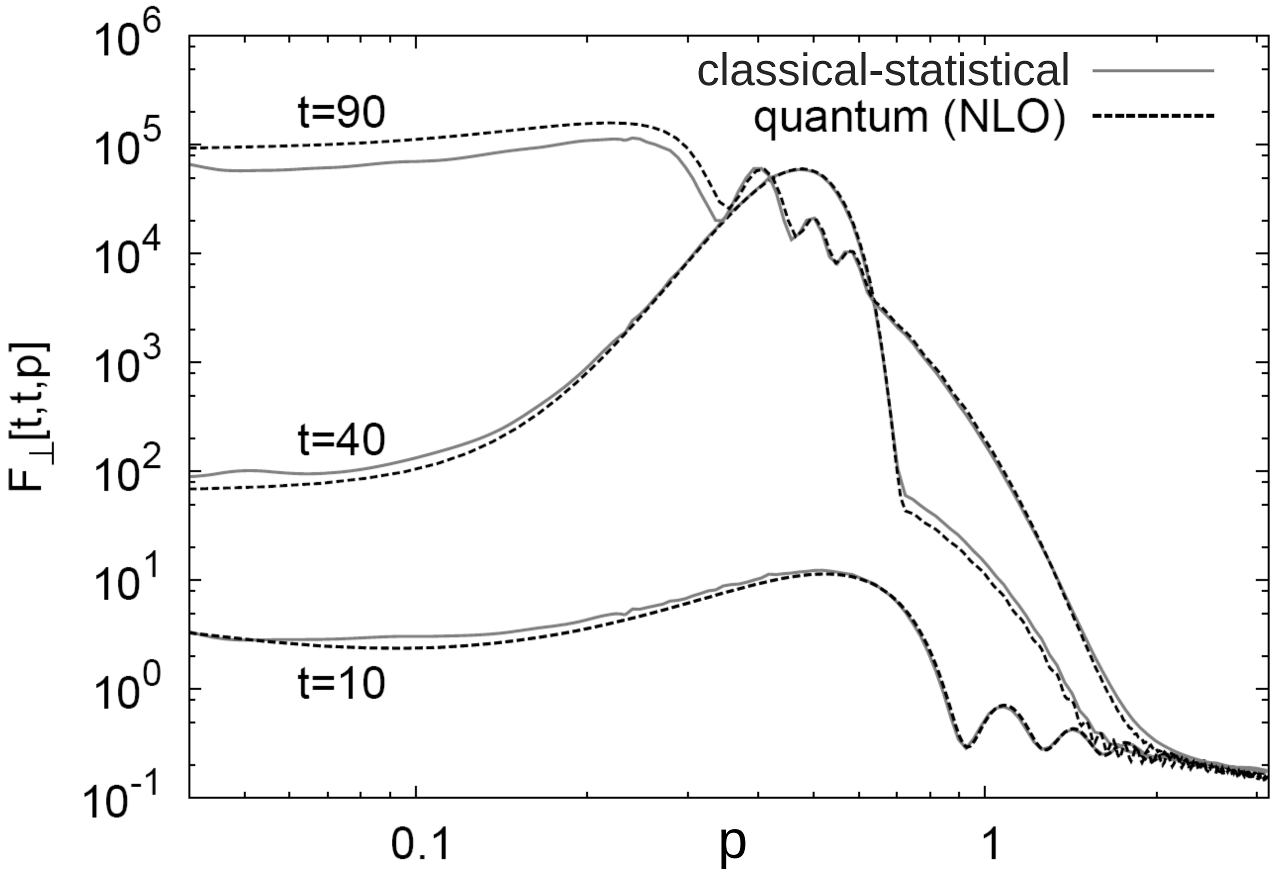
time



Build up of fluctuations

Build up of fluctuations during instability regime:

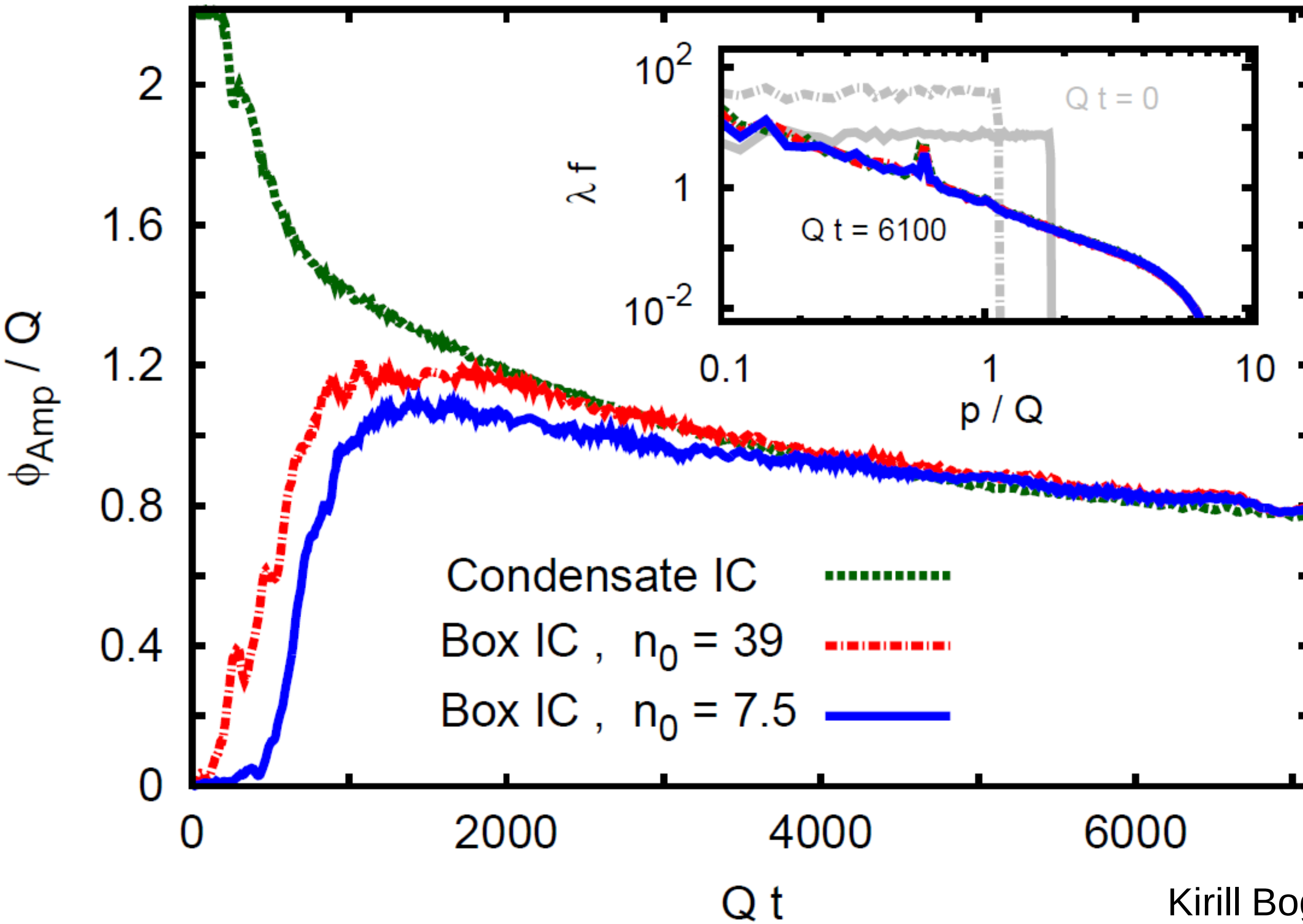
large initial field $\phi \sim 1/\lambda^{1/2}$ $\xrightarrow{t \sim \log 1/\lambda}$ **large** fluctuations $F \sim 1/\lambda$



in units of $\phi_0/\lambda^{1/2}$

Good agreement of classical-statistical and quantum for large ϕ, large F

Insensitivity to initial conditions



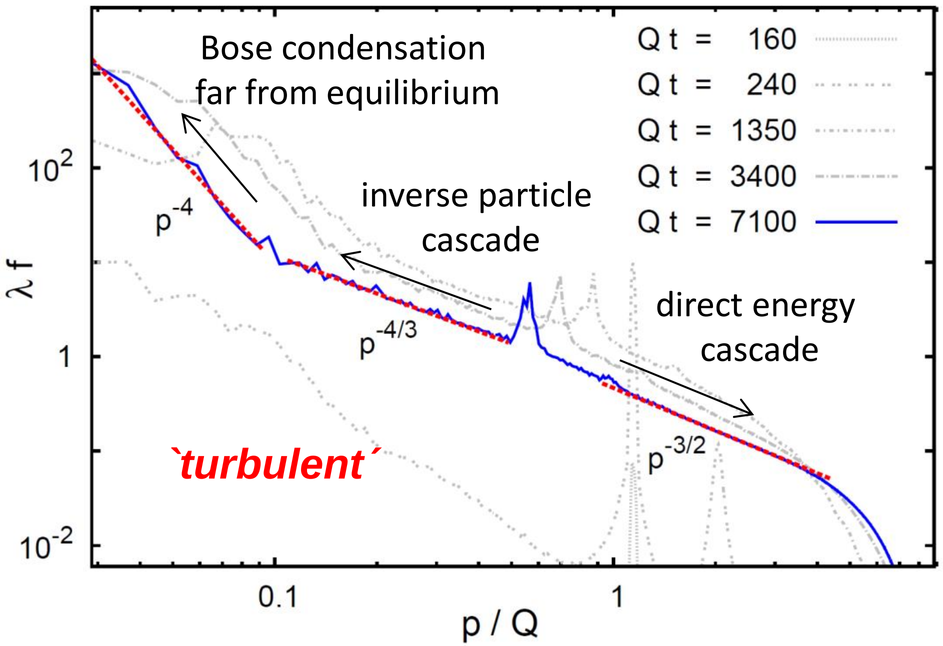
Kirill Boguslavski

Details about initial conditions are lost after instability period

Limitations of classical-statistical simulations

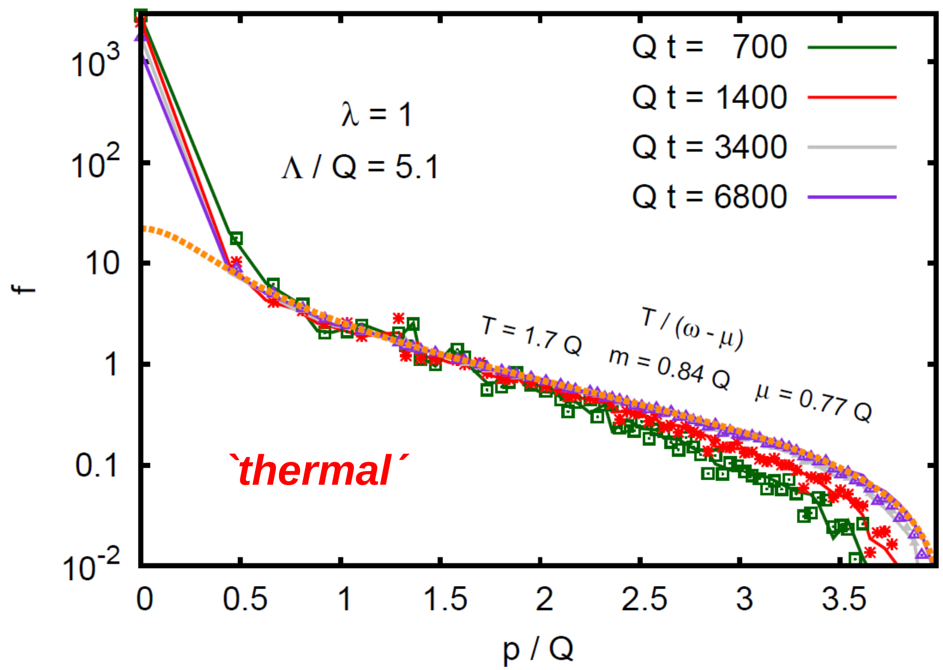
Comparing classical-statistical results at different couplings:

Turbulent thermalization
at weak coupling $\lambda \ll 1$



(logarithmic)

λ = 1 thermalization scenario in
Epelbaum, Gelis, NPA 872 (2011) 210



(linear)

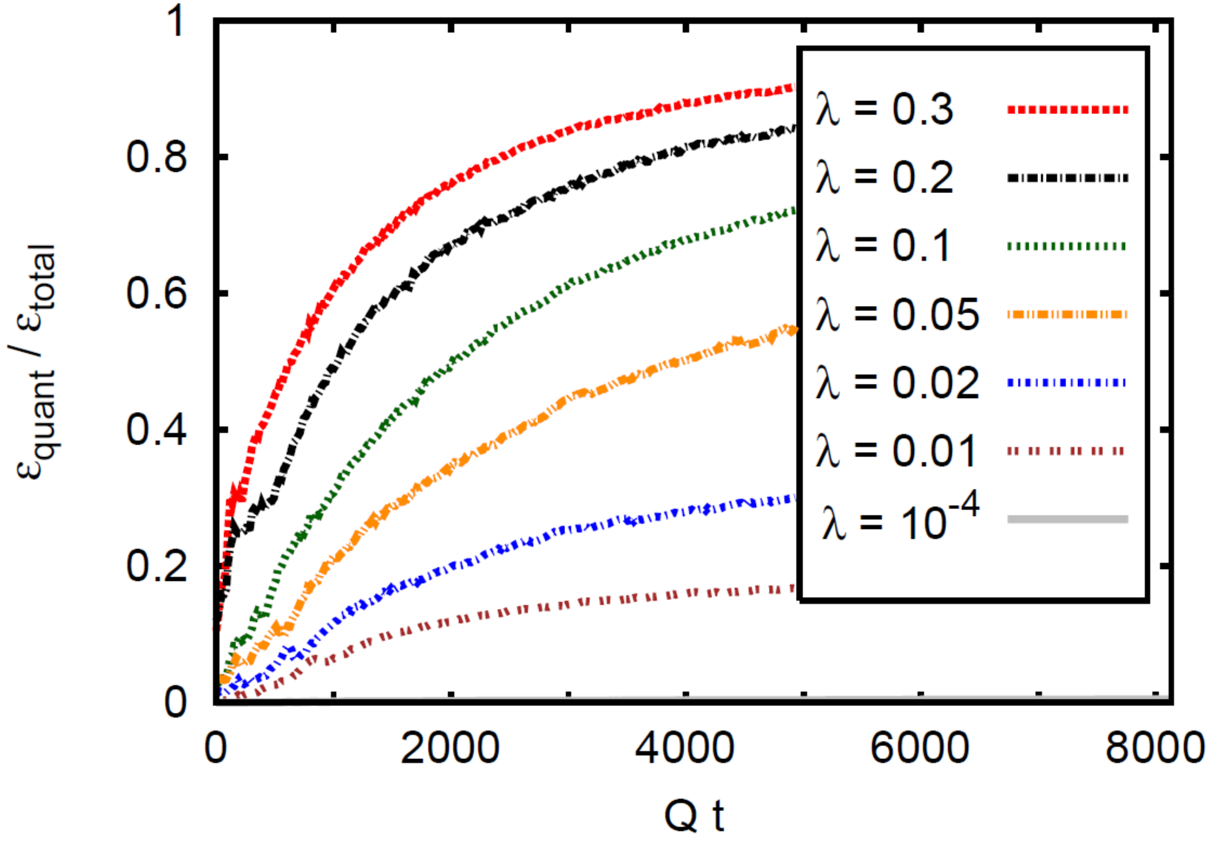
Kirill Boguslavski

data provided by Thomas Epelbaum

See also Micha,Tkachev (2003): UV; Berges, Rothkopf, Schmidt (2008): IR

Some diagnostics

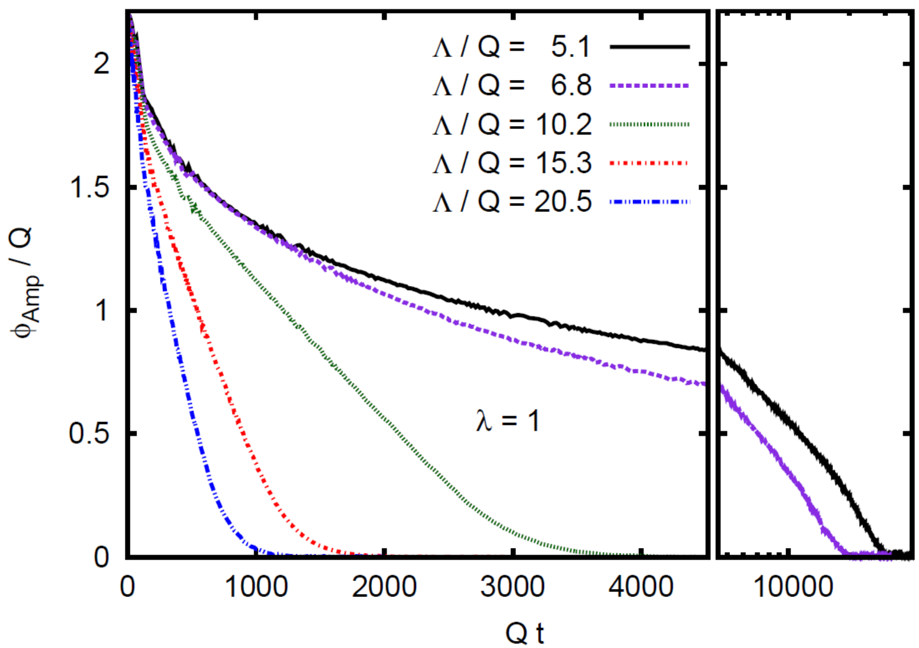
$$\epsilon_{\rm quant} = \int \frac{d^3p}{(2\pi)^3}\, p\, f(p)\, \Theta(1-f)$$

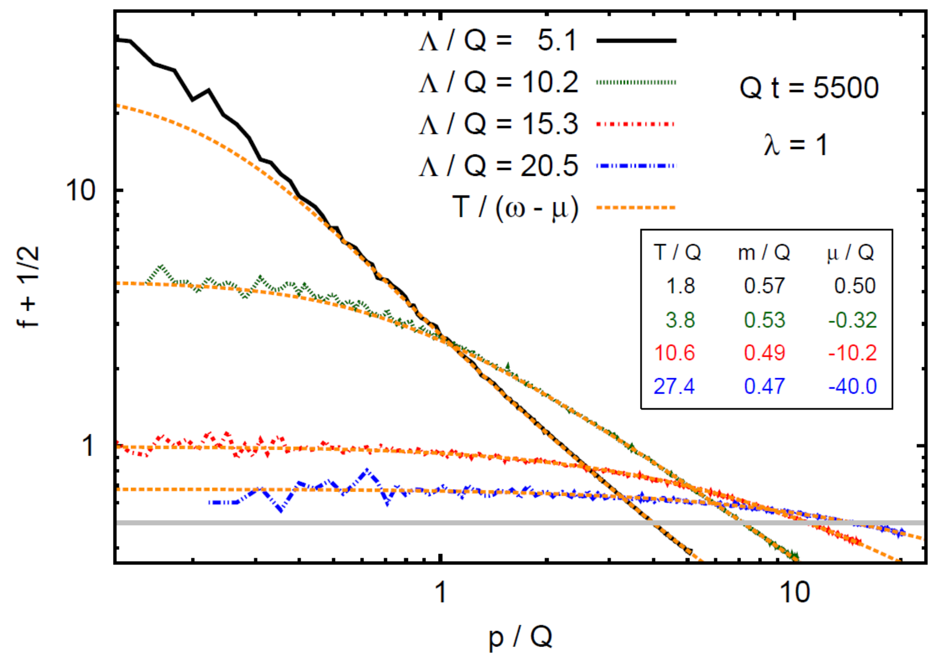


Already for rather small couplings a sizeable fraction of modes does not fulfill classicality condition f(p) > 1

Cutoff dependence

- $\lambda \ll 1$ results are insensitive to variation of UV cutoff
- Beyond their range of validity classical-statistical simulations show strong cutoff dependencies:





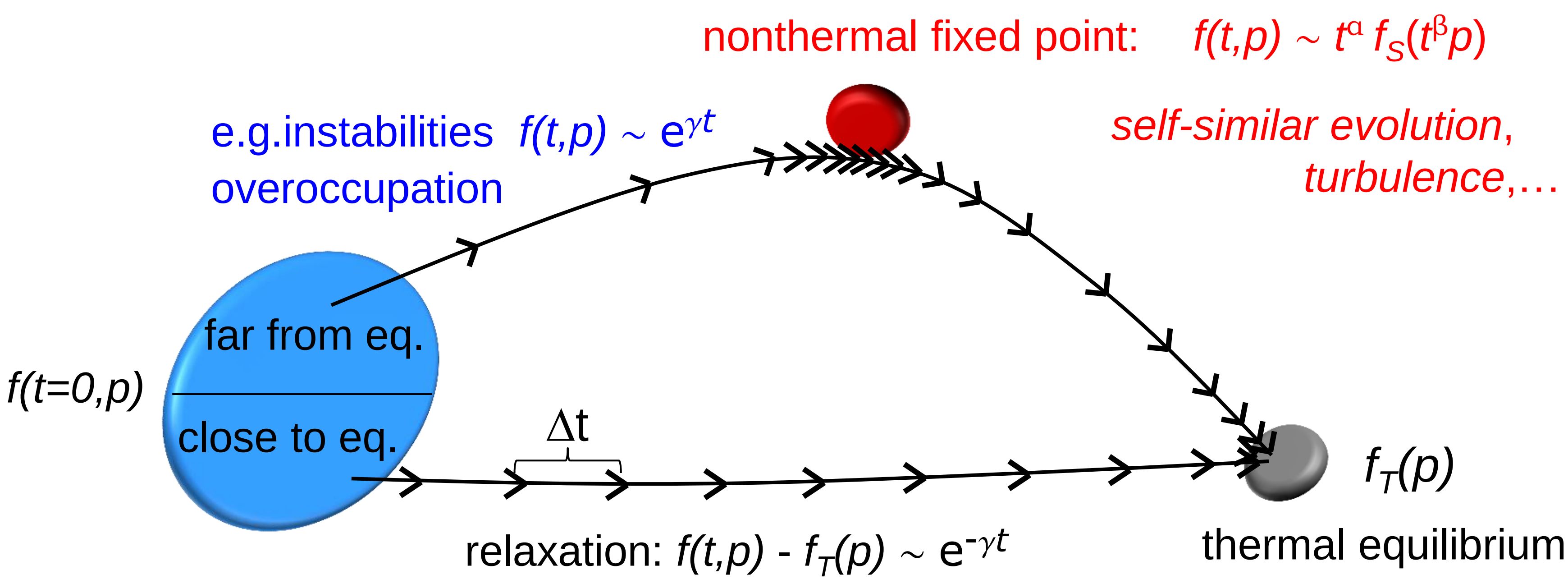
See also Aarts, Berges (2001); Smit, Arrizabalaga, Tranberg (2004)…

Conclusions

- ***Universal attractor*** ➡ \`bottom up´ thermalization

  BMSS: $\tau_{\text{iso.}} \sim Q^{-1}\alpha_S^{-5/2}$; $T_{\text{therm}} \sim \alpha_S^{2/5} Q$

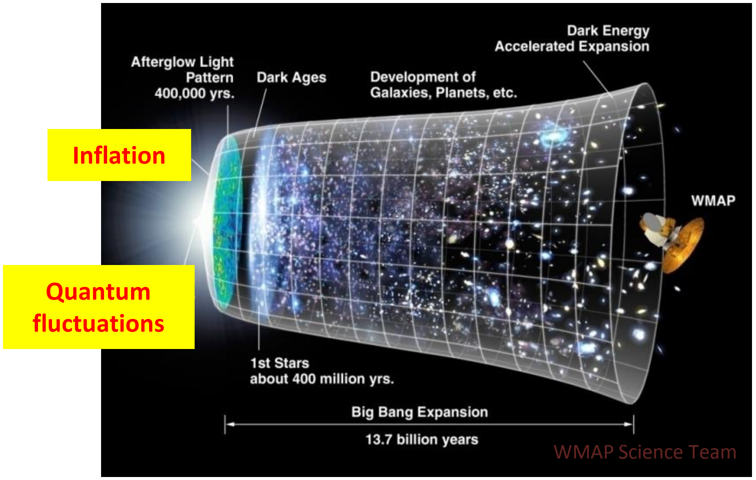
- ***Strongly correlated dynamics*** throughout the entire turbulent regime, despite very weak coupling (universal scaling, \`self-tuned criticality´)!

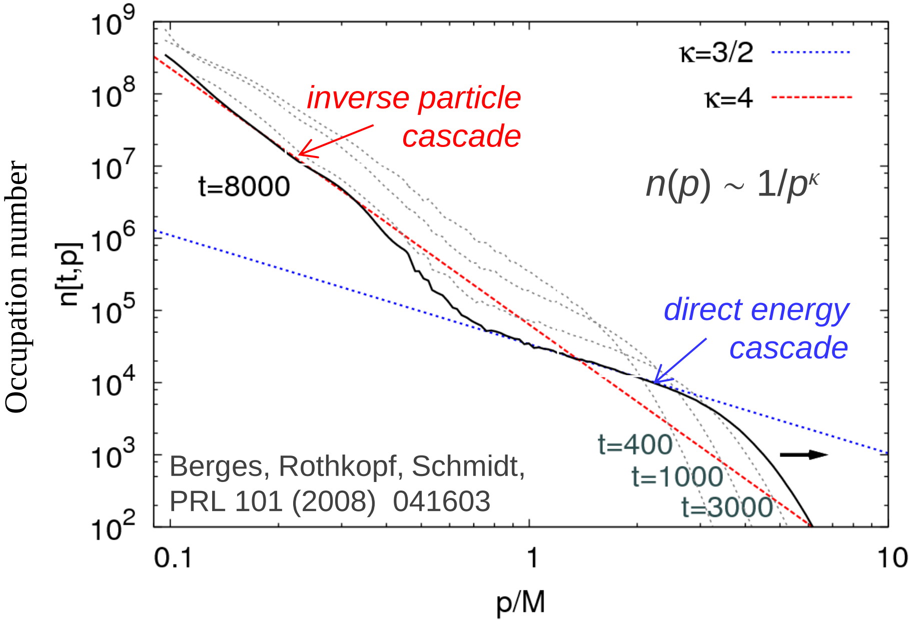


DFG

Universality far from equilibrium

- Reheating dynamics after chaotic inflation





- Superfluid turbulence in a cold Bose gas