QCD phase diagram in the Dyson-Schwinger approach

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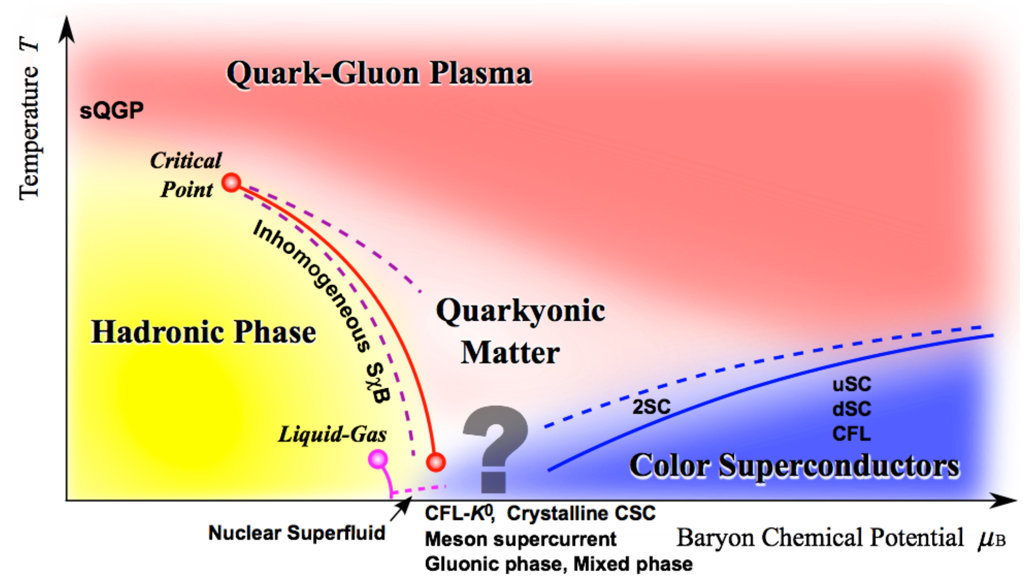
HIC for FAIR
Helmholtz International Center

5th of December 2013

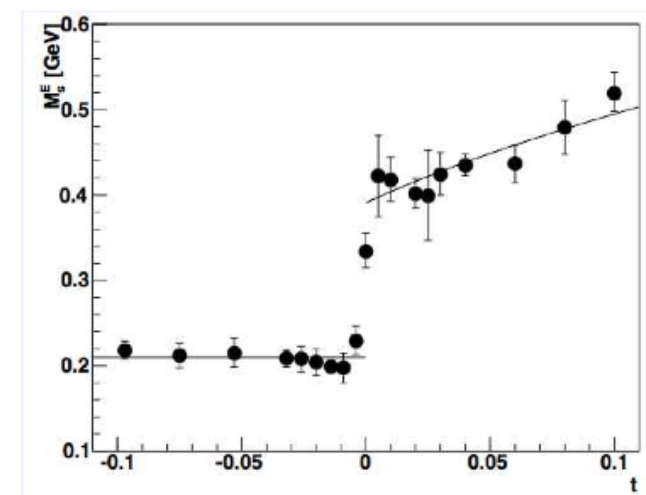
HELMHOLTZ GEMEINSCHAFT

with Jens Mueller, Jan Luecker
Axel Maas, Jan Pawlowski, Leonard Fister

Overview

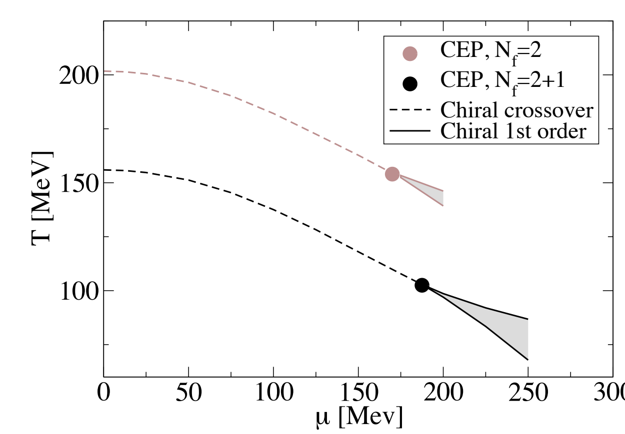


1.Introduction



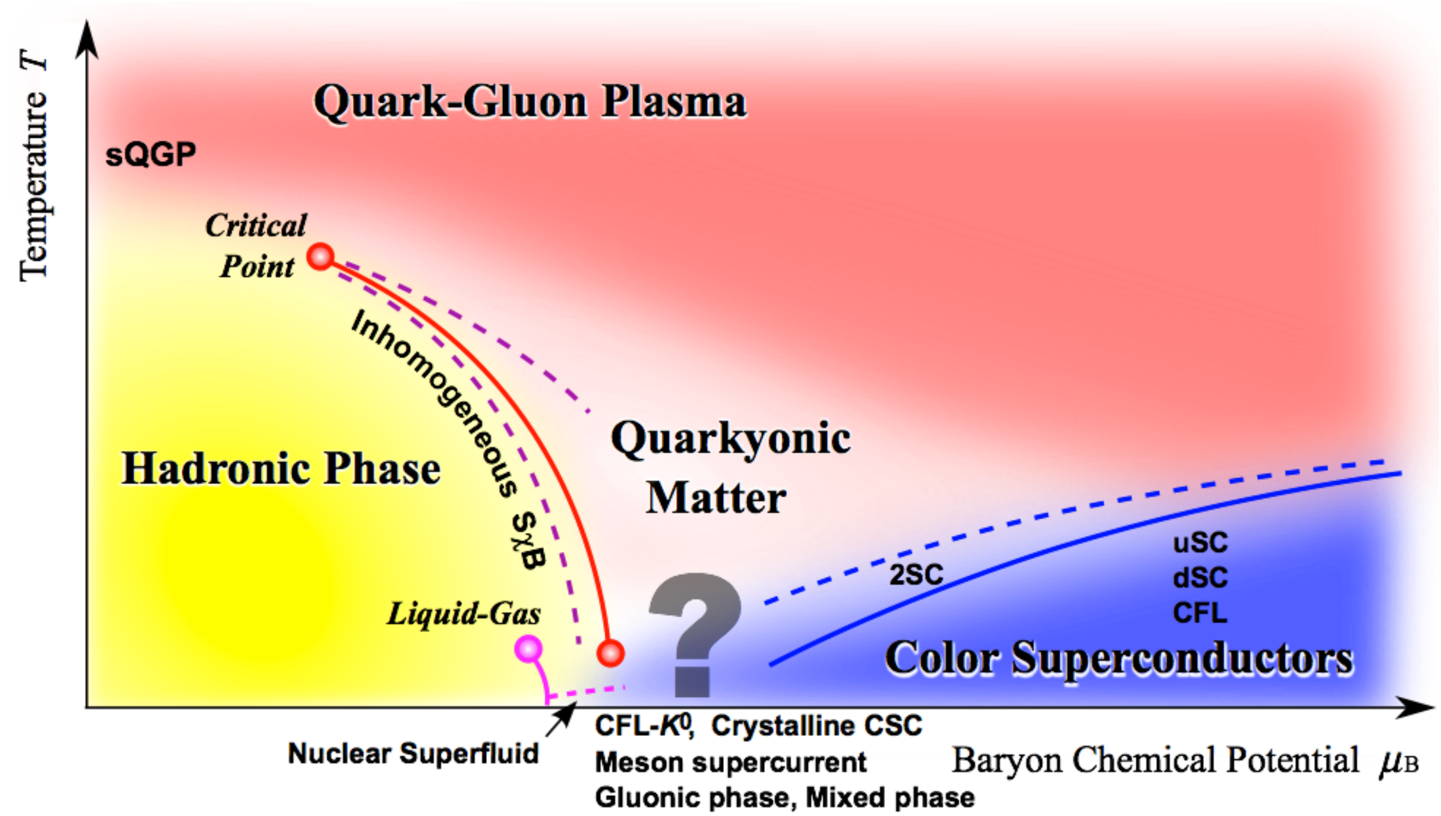
2.Gluons at zero and finite temperature

3.Quarks and the QCD phase diagram



QCD phase transitions I

Fukushima, Hatsuda, Rept. Prog. Phys. 74 (2011) 014001



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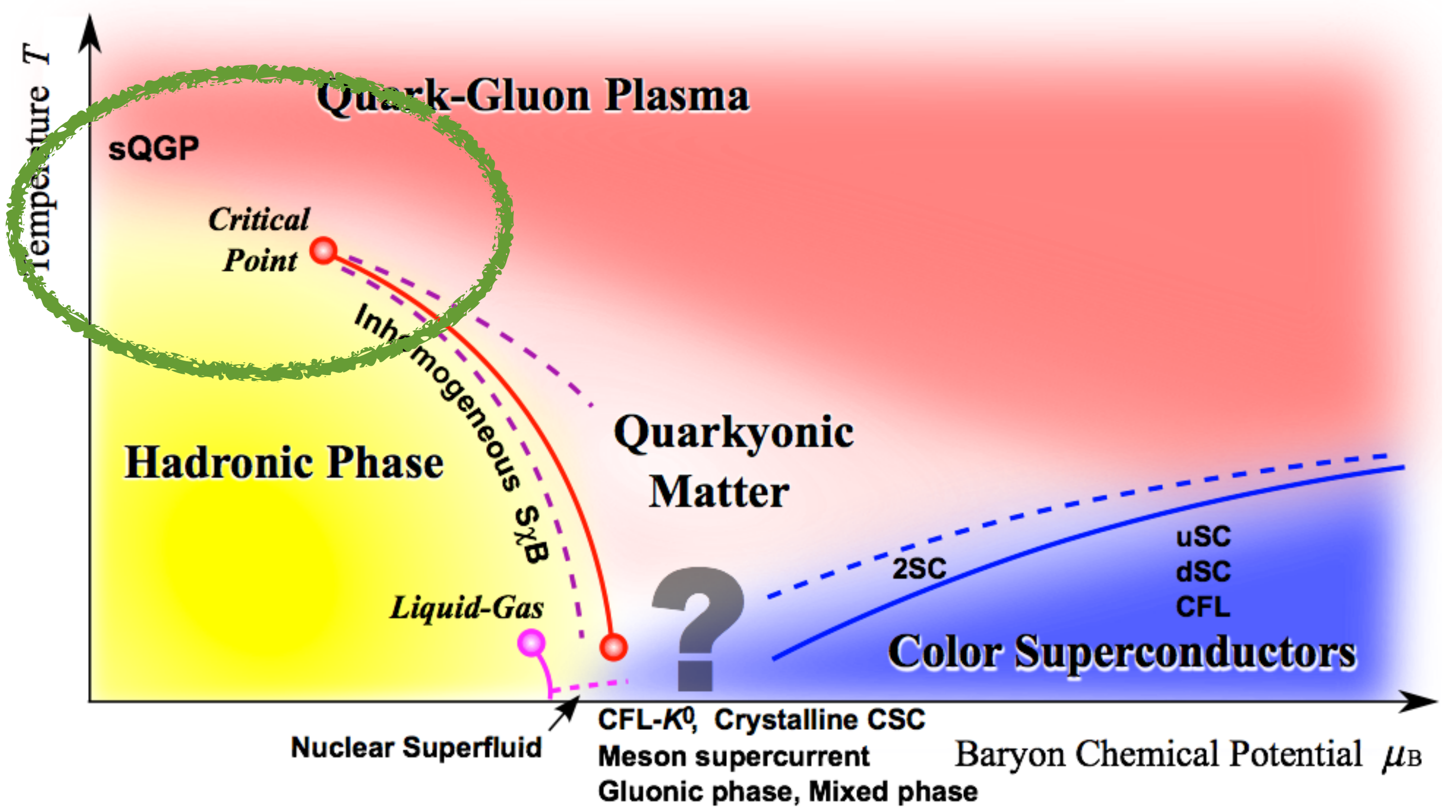

- Chiral limit ($M_{weak} \to 0$): order parameter chiral condensate

$$\langle \bar{\Psi}\Psi \rangle \sim Tr \int S$$

- Static quarks ($M_{weak} \to \infty$): order parameter Polyakov-loop

$$\langle |L| \rangle \sim e^{-F_q/T}$$

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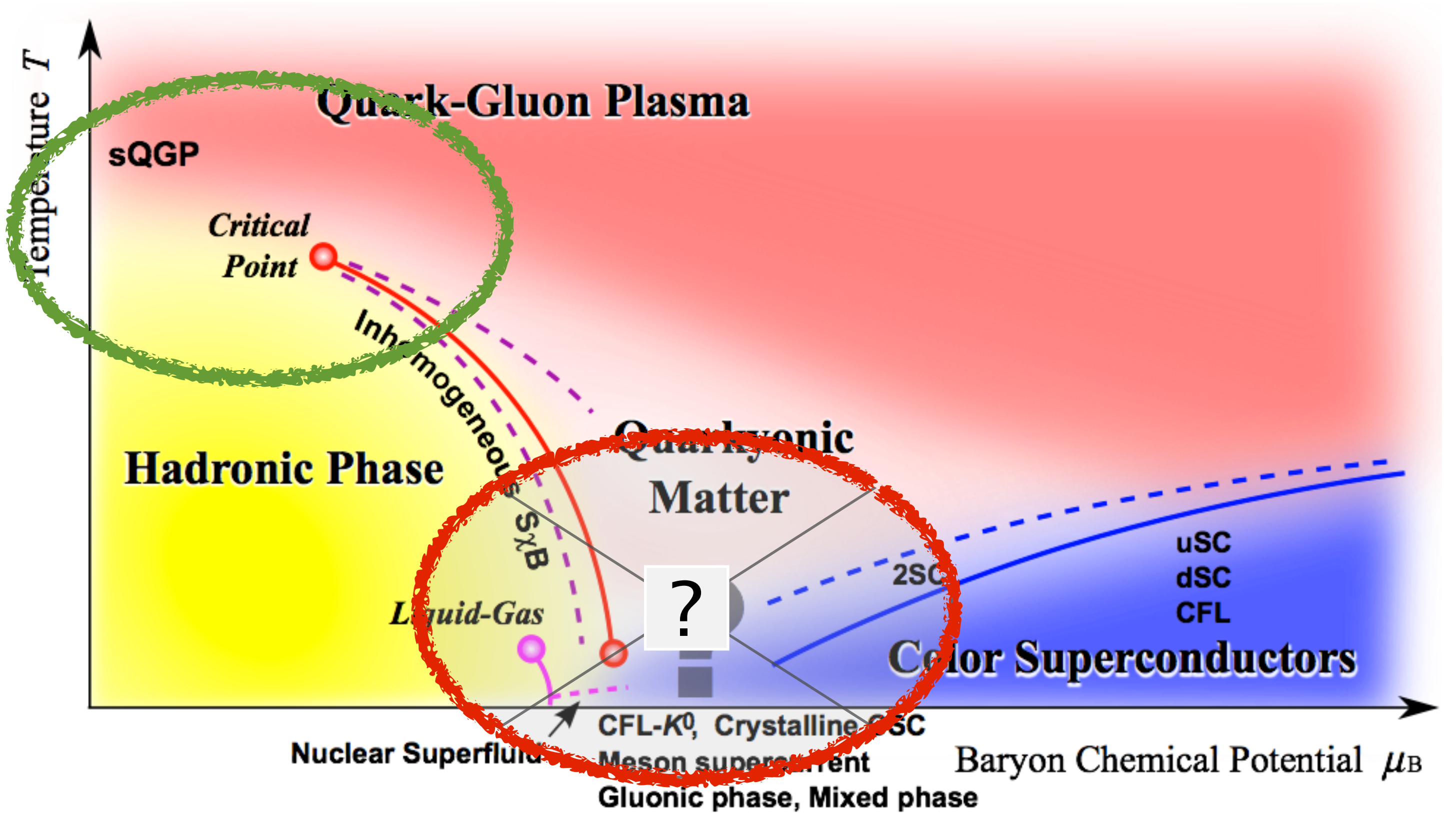

DSEs at low T, large μ:

Müller, Buballa, Wambach, EPJA 49 (2013), PLB 727 (2013) 240

see Talk of M. Buballa, prev. week

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$$\langle \bar{\Psi}\Psi \rangle \sim Tr \int S$$

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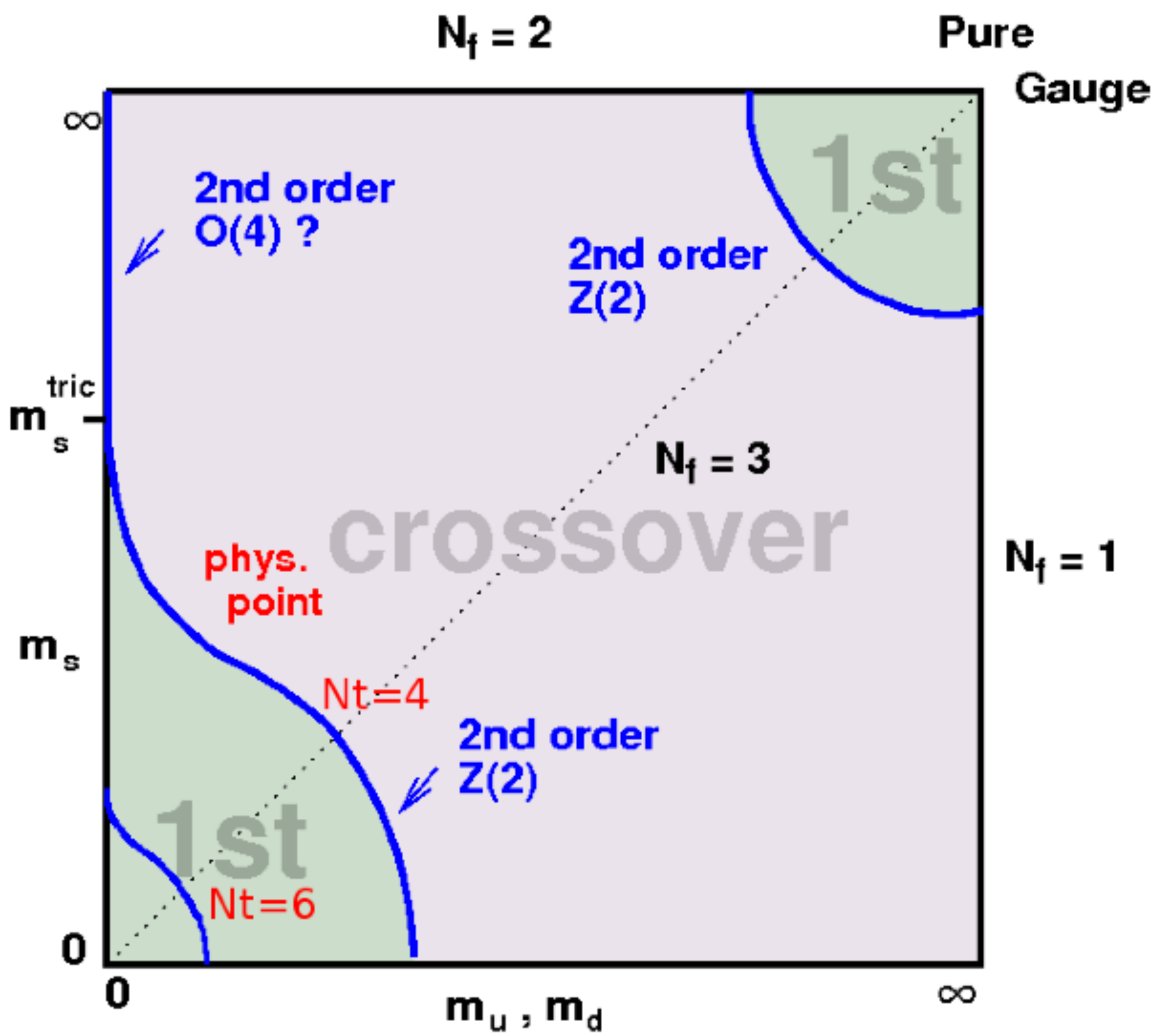
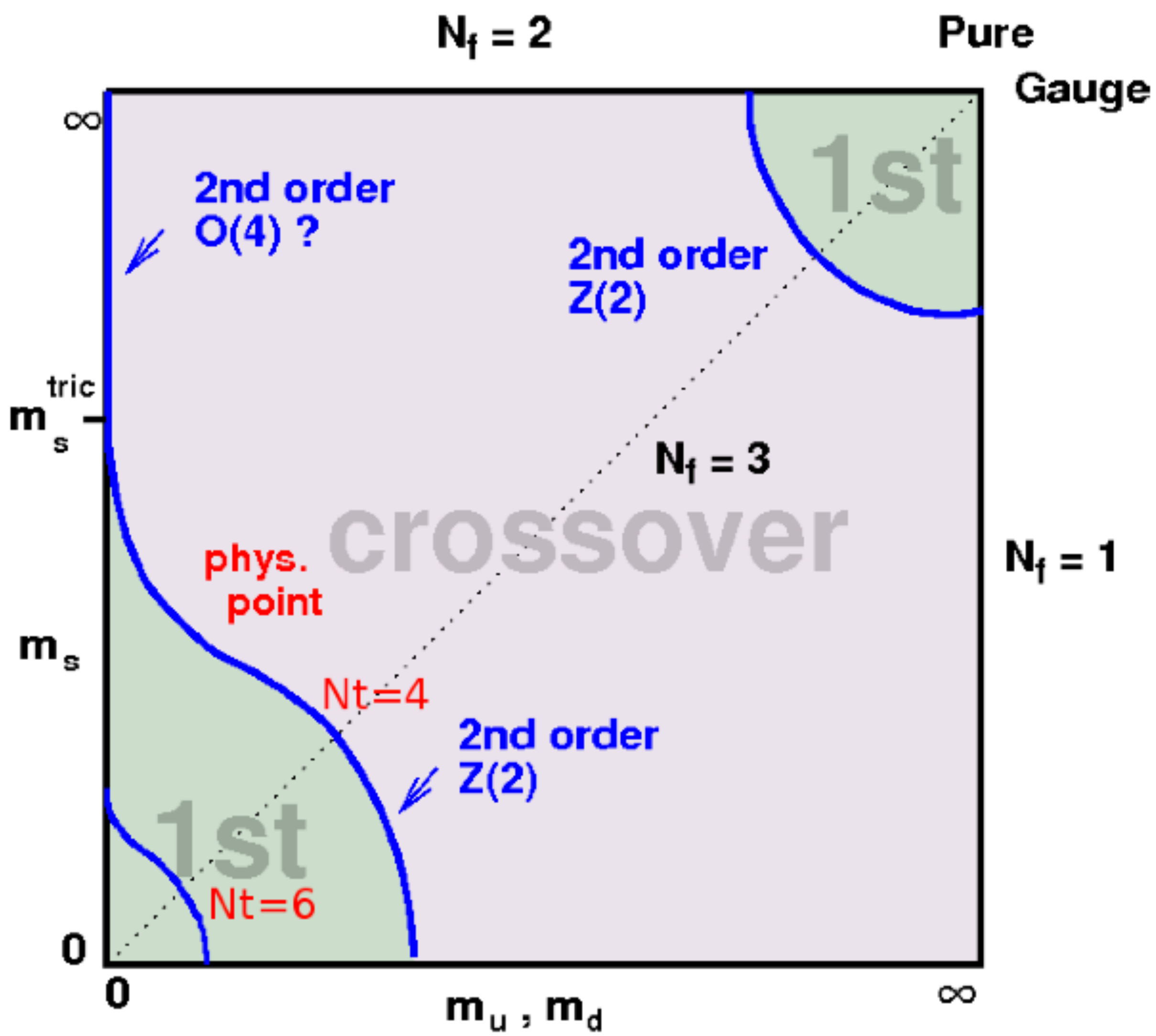
QCD phase transitions II



Plot: O. Philipsen

QCD phase transitions II



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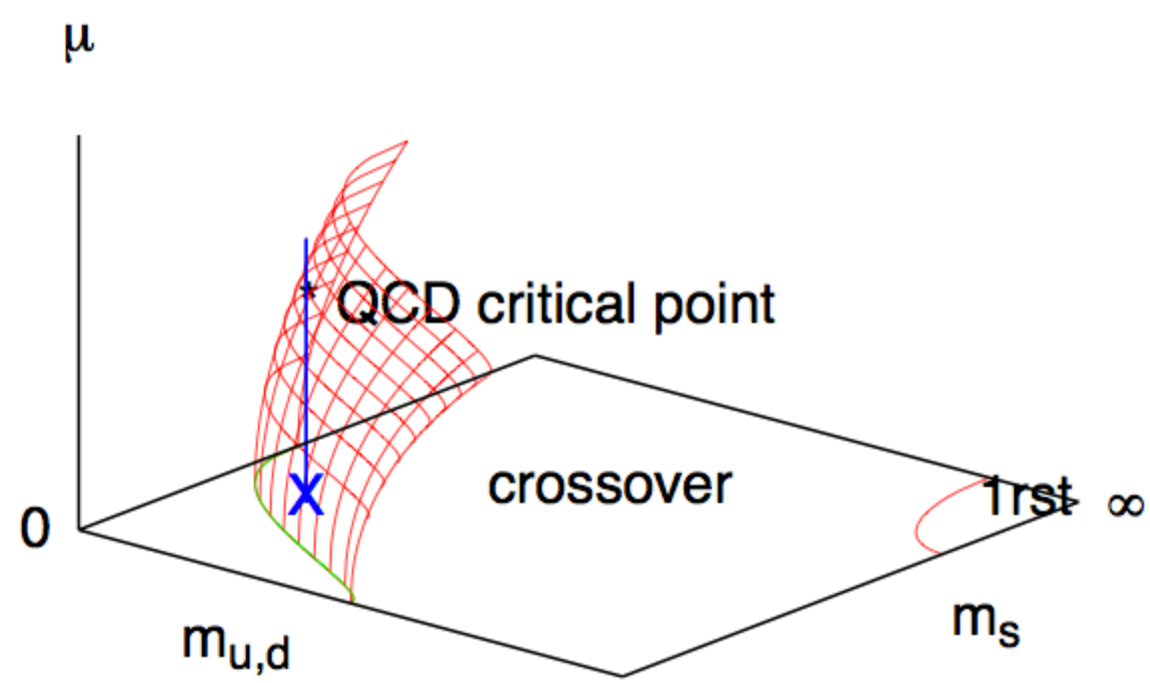
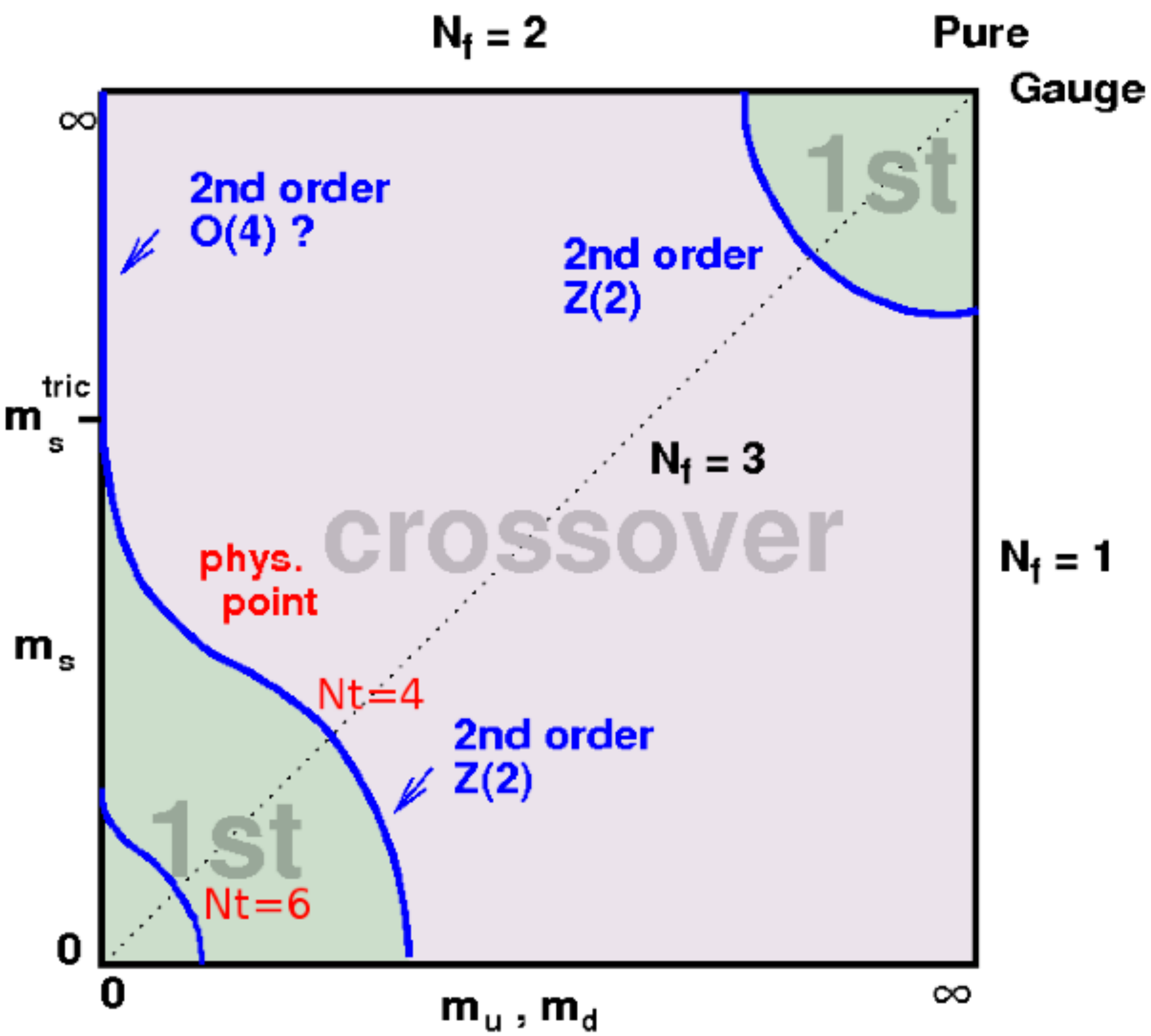


QCD phase transitions II



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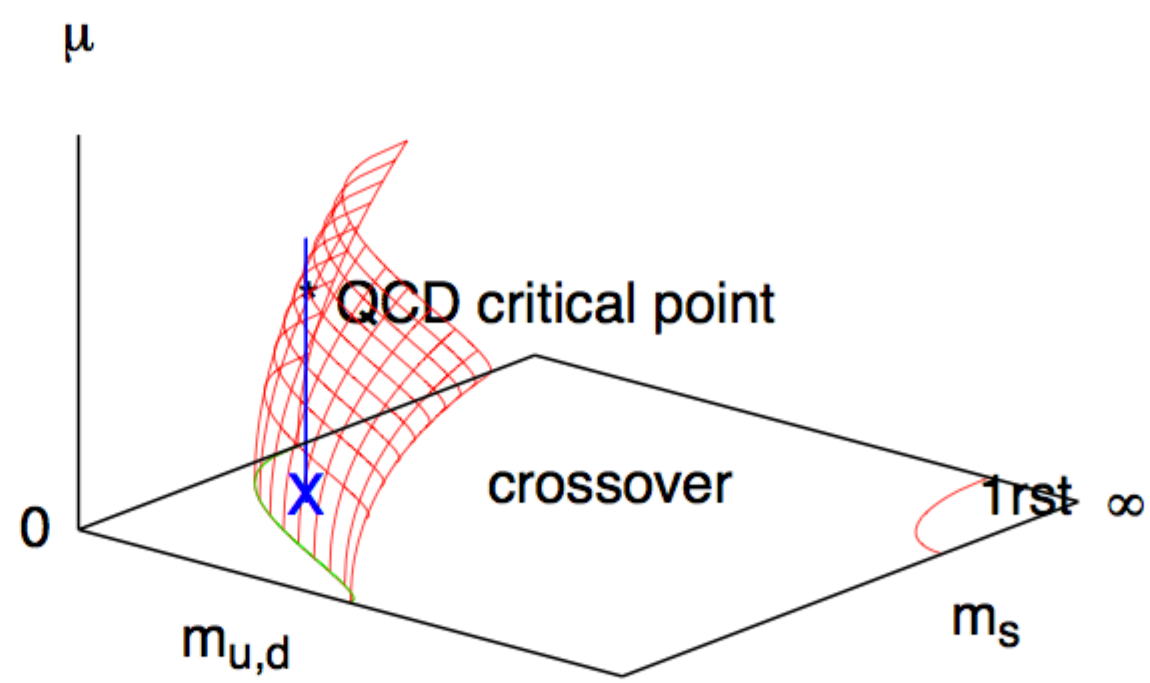
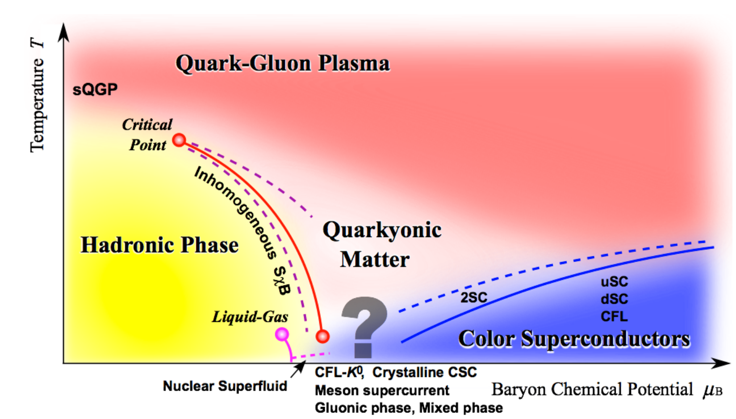


Is this happening ??



Nonperturbative QCD: Complementary approach

Quarks and gluons

- Lattice simulations of QCD
  - Ab initio
  - Gauge invariant

Hadrons

- Effective theories and models (chiral $\mathcal{L}$,PQM,PNJL)
  - Physical degrees of freedom

- Functional approaches to QCD (DSE, FRG,Hamilton):
  - Chiral symmetry: physical quark masses
  - Analytical solutions in IR
  - Infinite volume and continuum limit
  - Multi-scale problems feasible (e.g. $(g-2)_\mu$)
    Goecke, CF, Williams, PRD 87 (2013) 03401
  - Chemical potential: no sign problem

→see also Talk by J. Pawlowski

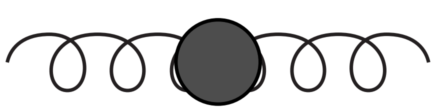
QCD in covariant gauge

Imaginary time formulation:

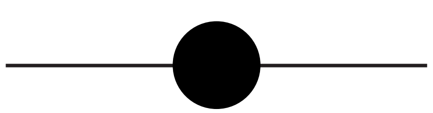
M weak

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \;\exp\left\{ - \int_0^{1/T} dt \int d^3x \left(\overline{\Psi}\,(i\not{D} + \gamma_4 \mu - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \text{gauge fixing} \right) \right\}$$

Landau gauge propagators in momentum space, $p = (\vec{p}, \omega_p)$:

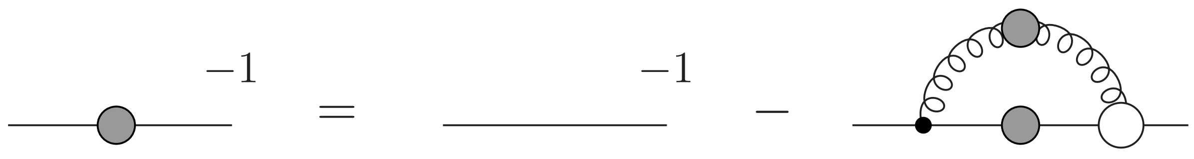


$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{Z_T(p)}{p^2} P^T_{\mu\nu}(p) + \frac{Z_L(p)}{p^2} P^L_{\mu\nu}(p)$$



$$S^{\text{Quark}}(p) = Z_f(p)\left[-i\,\vec{\gamma}\vec{p} - i\,\gamma_4 \tilde{\omega}_n\, Z_c(p) + M(p)\right]^{-1}$$

The Goal: gauge invariant information in a gauge fixed approach.



Chiral order parameter:

$$\langle \bar{\Psi}\Psi \rangle = Z_2 N_c Tr_D \frac{1}{T} \sum_\omega \int \frac{d^3p}{(2\pi)^3} S(\vec{p},\omega)$$

Deconfinement:

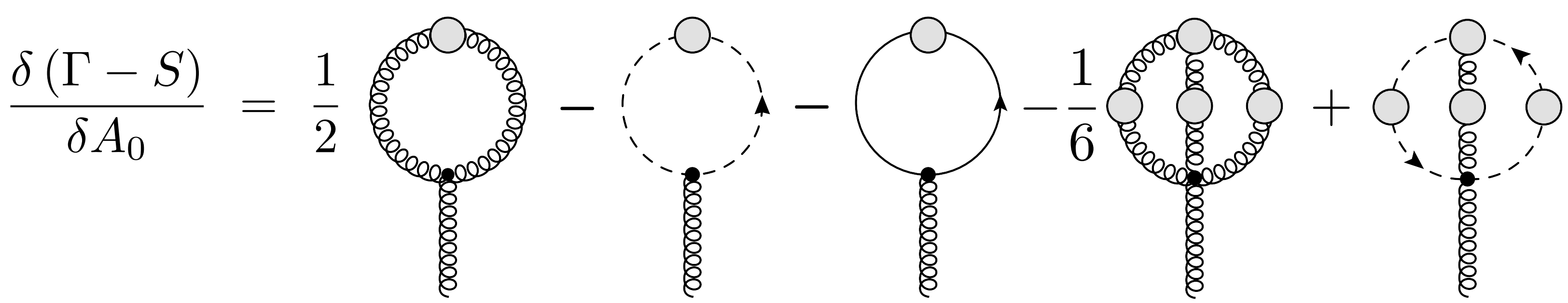
- dressed Polyakov loop

$$\Sigma = -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \langle \bar{\Psi}\Psi \rangle_\varphi$$

Synatschke, Wipf, Wozar, PRD 75, 114003 (2007)
Bilgici, Bruckmann, Gattringer, Hagen, PRD 77 094007 (2008)
CF, PRL 103 052003 (2009)

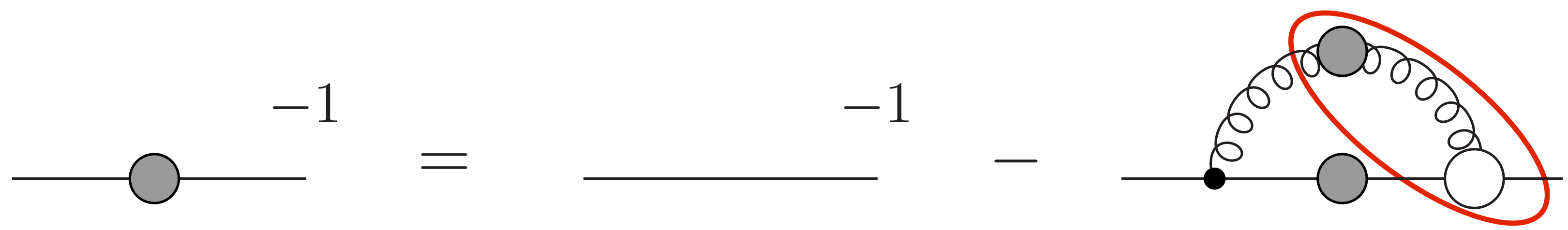
- Polyakov loop potential

$$L = \frac{1}{N_c} Tr\, e^{ig\beta A_0}$$



Braun, Gies, Pawlowski, PLB 684, 262 (2010)
Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011)
Fister, Pawlowski, PRD 88 045010 (2013)
CF, Fister, Luecker, Pawlowski, arXiv:1306.6022

The DSE for the quark propagator



$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$

Input:
- dressed Gluon propagator
- dressed Quark-Gluon-Vertex

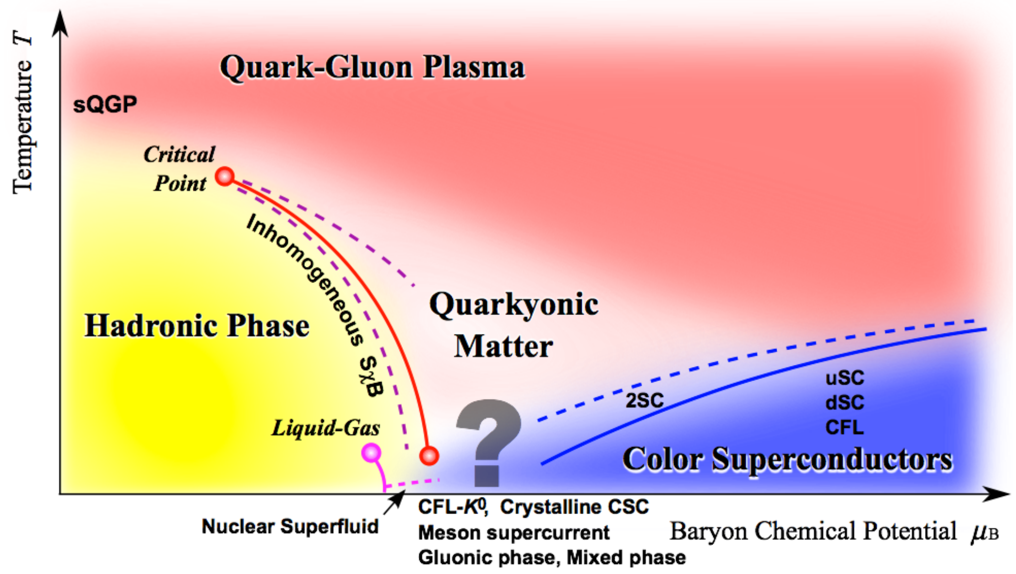
Two strategies: I. use model for gluon and vertex

Qin, Chang, Chen, Liu and Roberts, PRL 106 (2011) 172301
Gutierrez, Ahmad, Ayala, Bashir and Raya, arXiv:1304.8065.

→ ok for first insights
→ not good enough for systematic study

II. determine gluon and vertex explicitly

Overview



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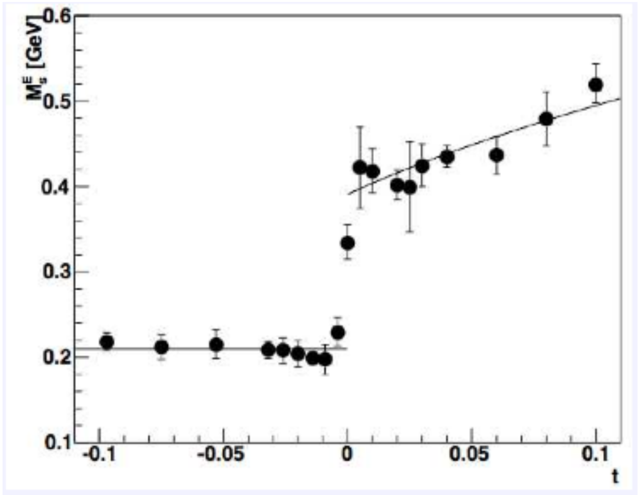

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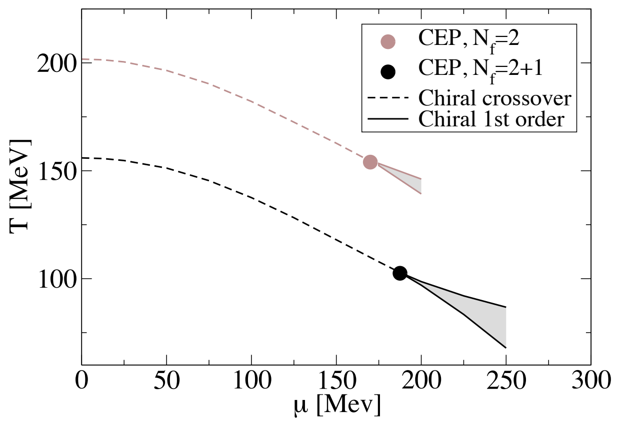
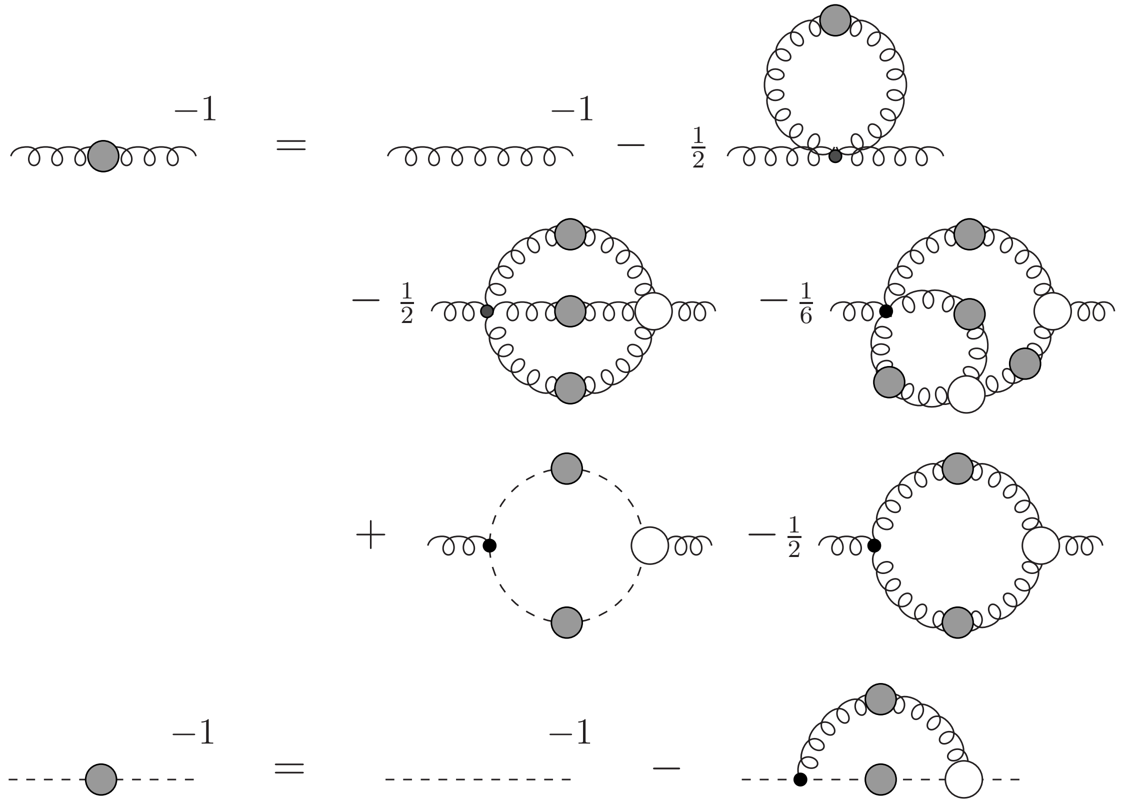
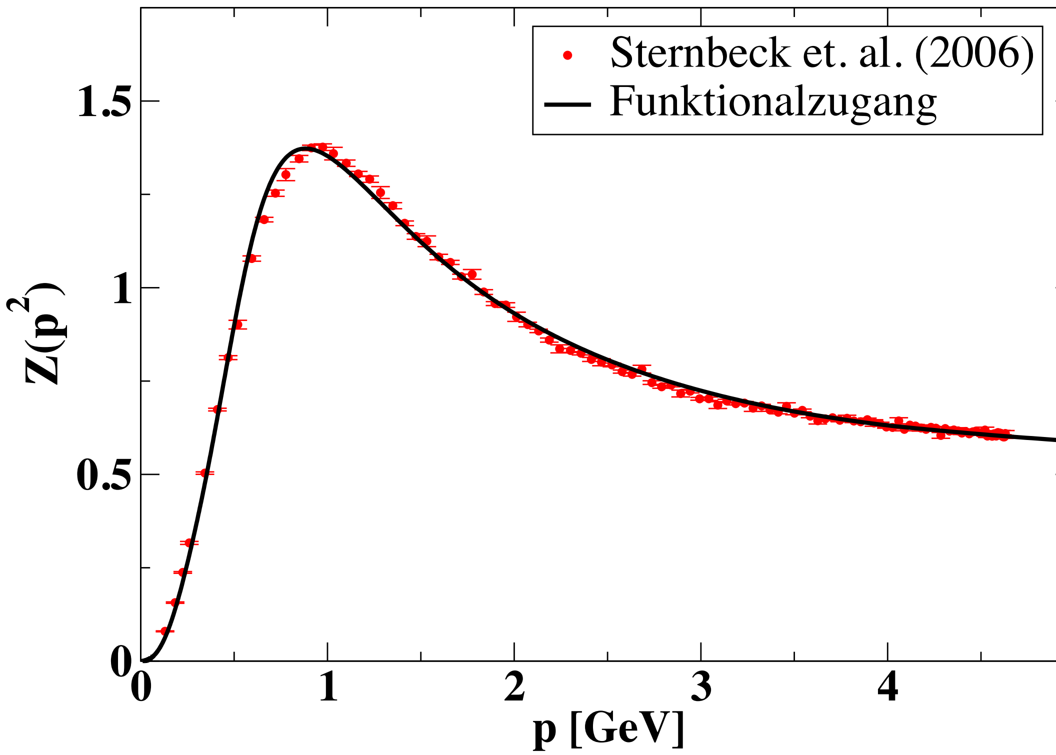


3.Quarks and the QCD phase diagram

Strategy I: Landau gauge gluon propagator

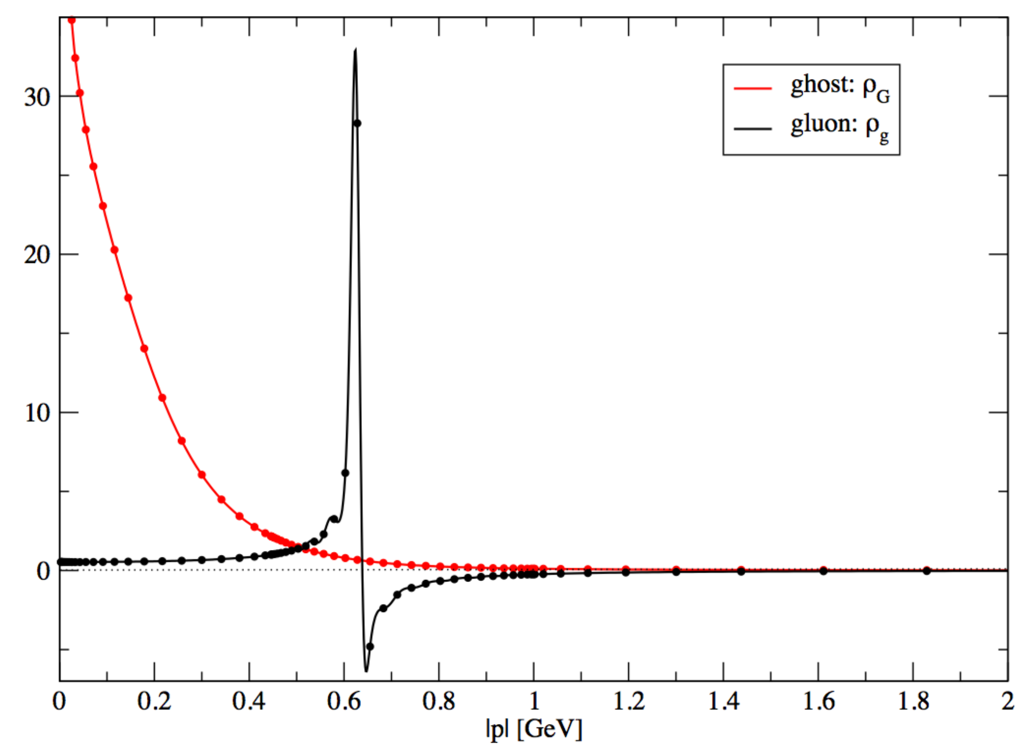




CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408.
Huber and von Smekal, JHEP 1304 (2013) 149

- spacelike momenta: excellent agreement with lattice
- spectral function: positivity violations

Gluon cannot appear in detector!

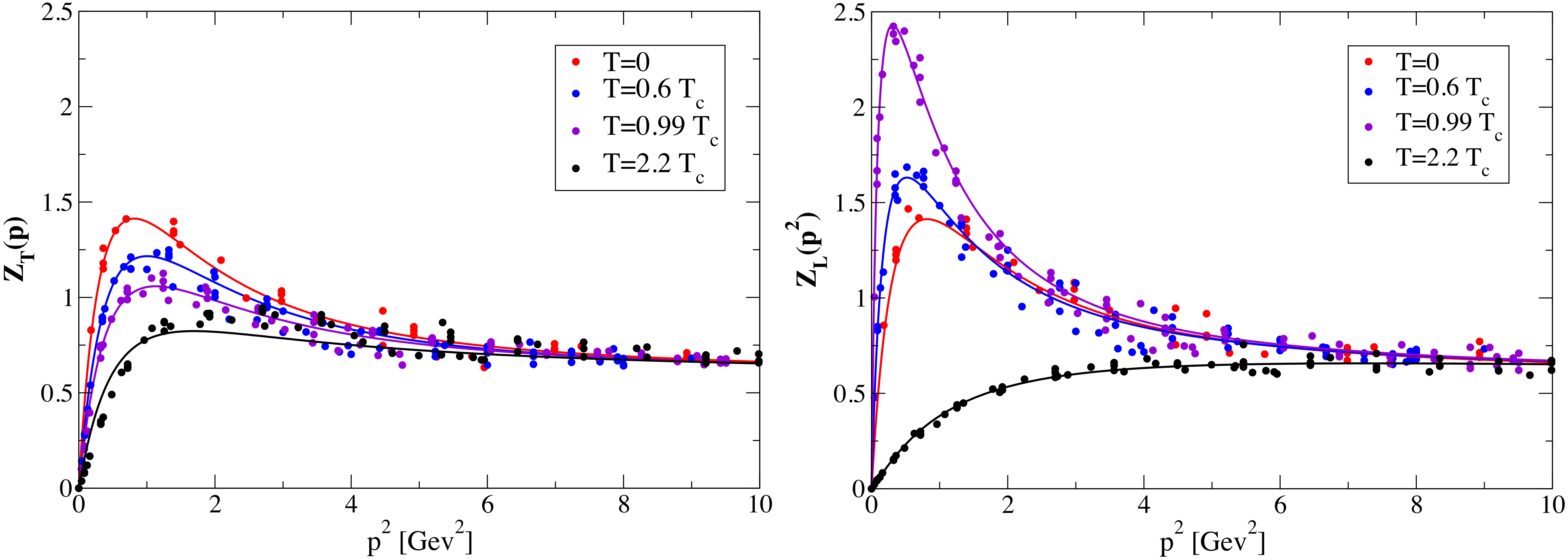


spectral function

Strauss, CF, Kellermann, Phys. Rev. Lett. 109, (2012) 252001

Glue at finite temperature ($T \neq 0$)

T-dependent gluon propagator from quenched lattice simulations:

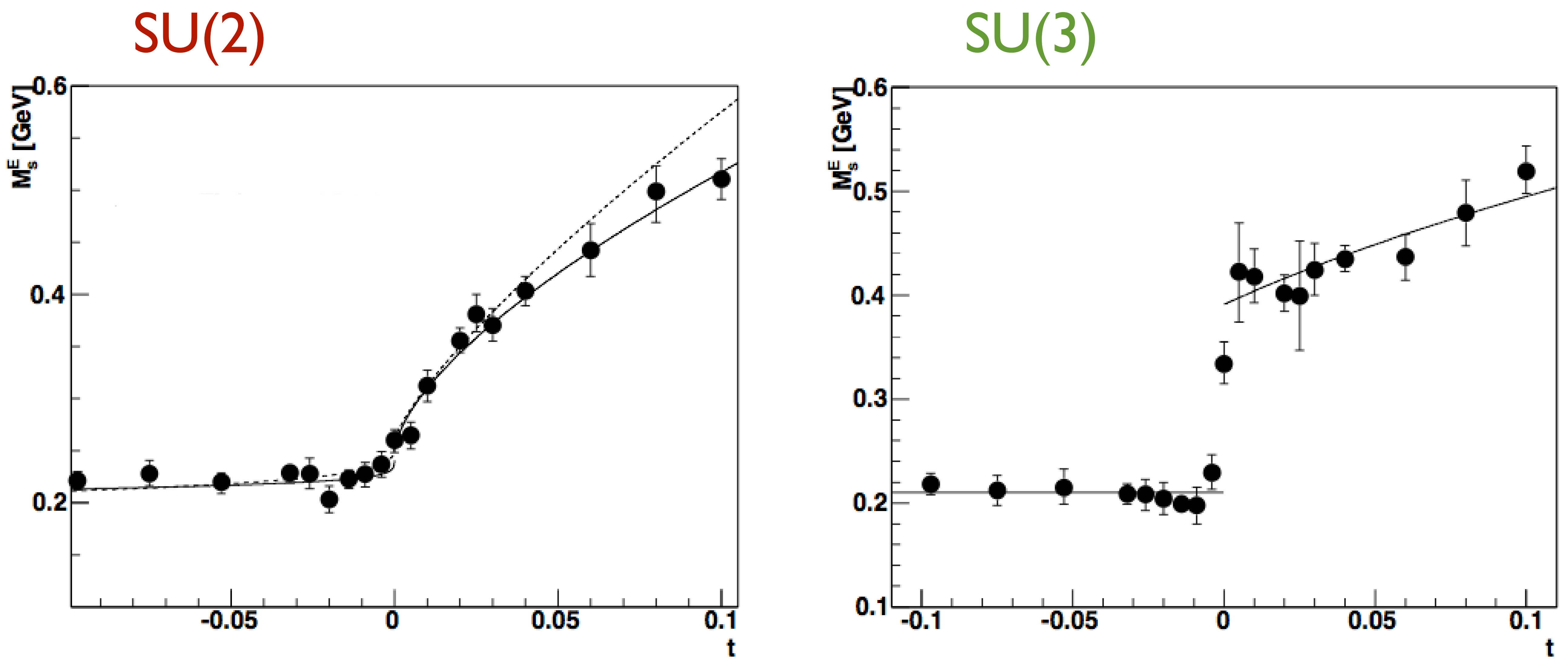


- Crucial difference between magnetic and electric gluon
- Maximum of electric gluon near Tc

Cucchieri, Maas, Mendes, PRD 75 (2007)
CF, Maas, Mueller, EPJC 68 (2010)
Cucchieri, Mendes, PoS FACESQCD 007 (2010)
Aouane, Bornyakov, Ilgenfritz, Mitrjushkin, Muller-Preussker and Sternbeck, PRD 85 (2012) 034501

FRG: Fister, Pawlowski, arXiv:1112.5440

Gluon electric screening mass: SU(2) vs. SU(3)

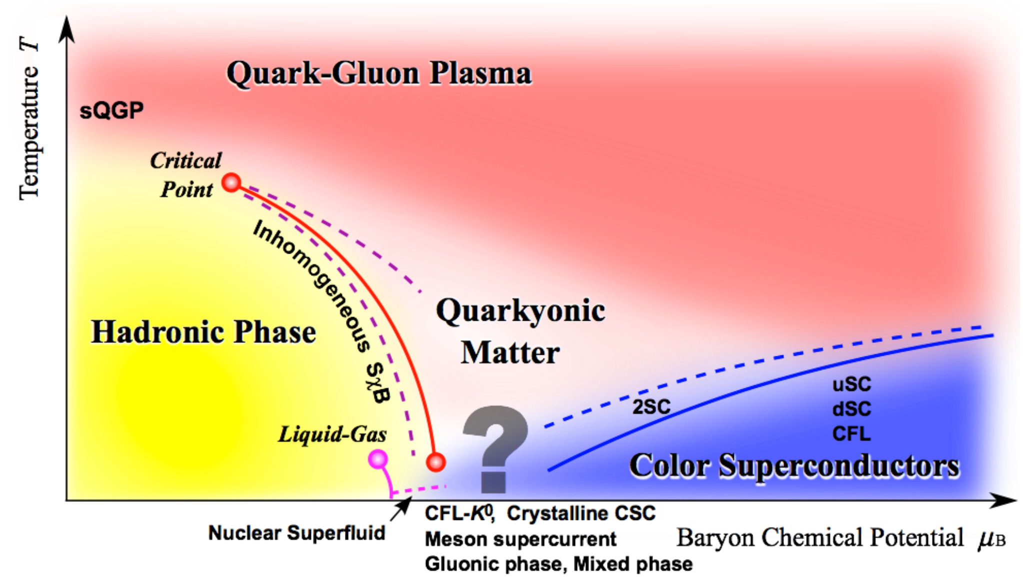


Maas, Pawlowski, Smekal, Spielmann, PRD 85 (2012) 034037
CF, Maas, Mueller, EPJC 68 (2010)

$t = (T-T_c)/T_c$

- phase transition of second and first order visible in electric screening mass

1. Introduction



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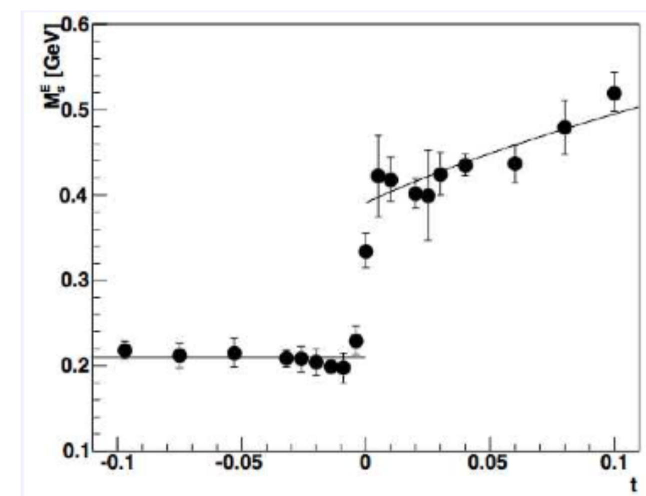

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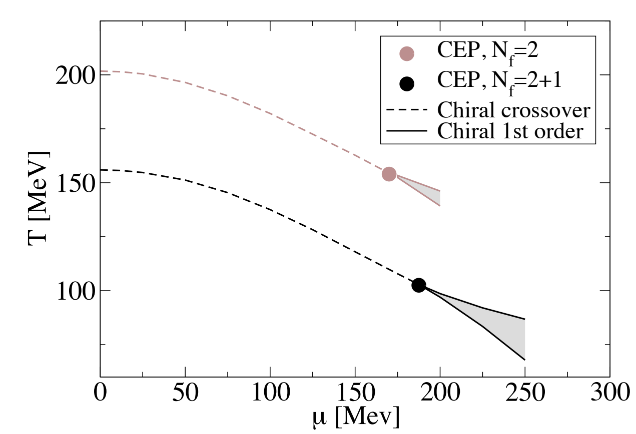
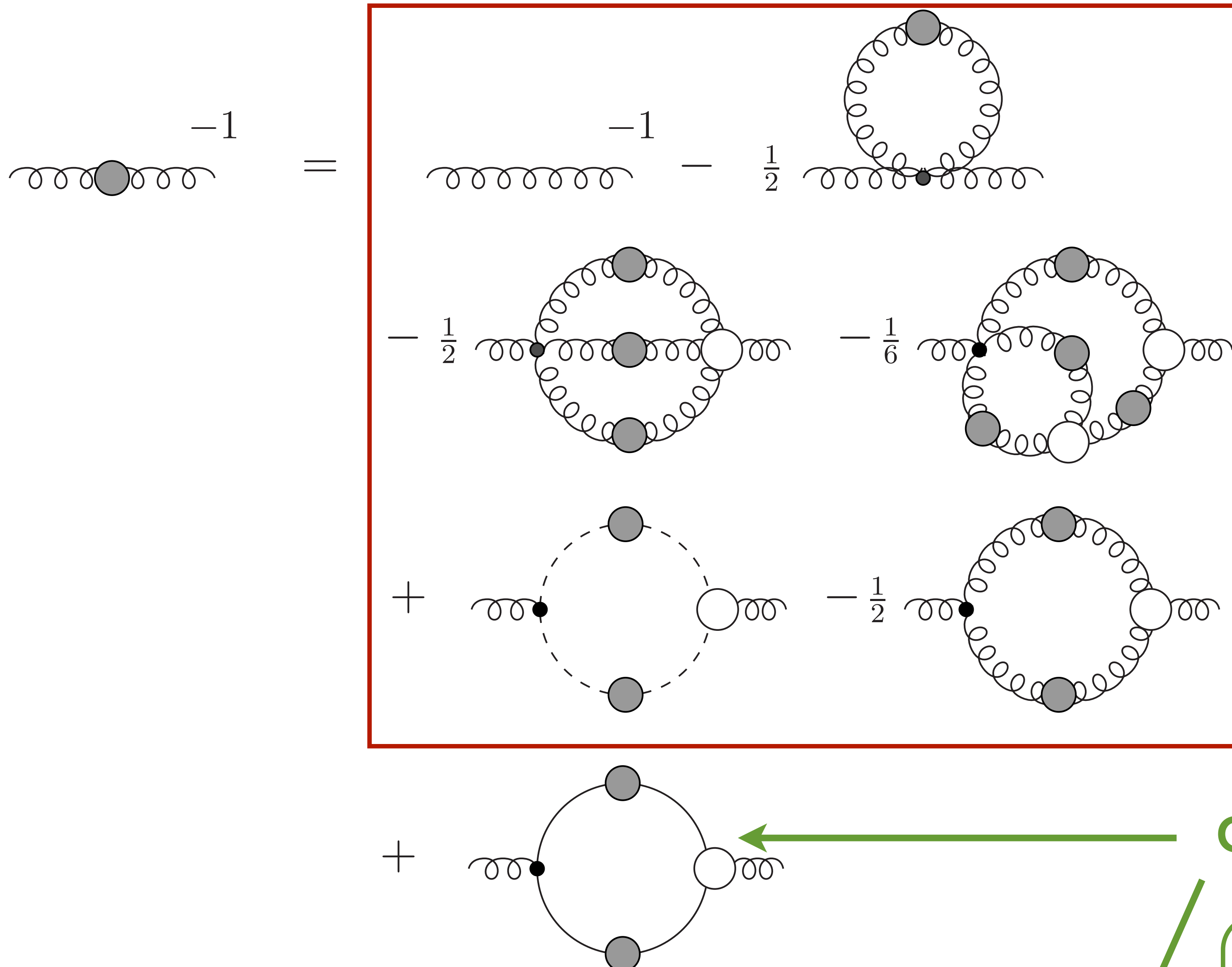
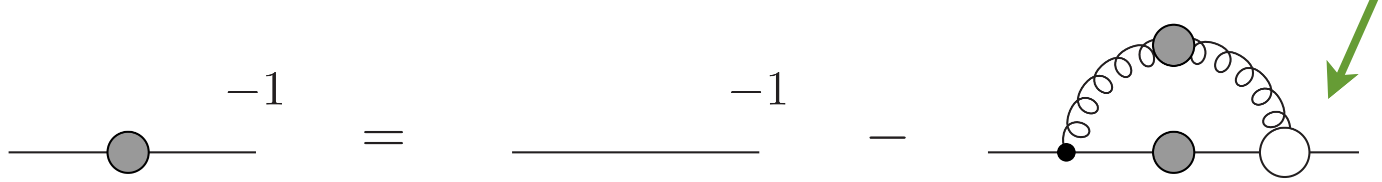


DSEs of QCD



quenched, T-dependent lattice propagator

quark gluon vertex



- under study at T=0
  Skullerud, Kizilersu, JHEP 0209 (2002) 013
  Alkofer, CF, Llanes-Estrada, Schwenzer, Annals Phys. 324 (2009) 106
  CF, Williams PRL 103 (2009) 122001
- T≠0: ansatz, T,m,μ dependent

Approximation for Quark-Gluon interaction

- T,μ,m-dependent vertex:

Abelian WTI

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3 \left(\delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times$$

$$\times \left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right)$$

perturbation theory

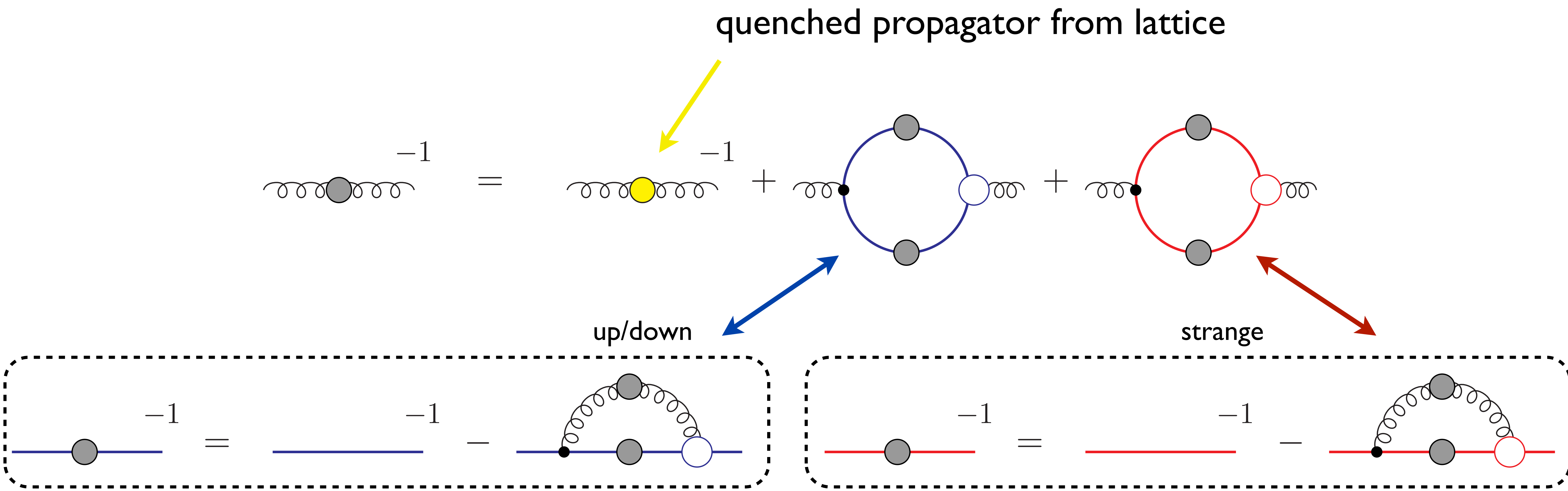
Infrared ansatz:

- d2 fixed to match gluon input
- d1 fixed via quark condensate (see later)
- correct UV and IR-behavior

Alkofer, CF, Llanes-Estrada, Schwenzer, Annals Phys. 324 (2009)
CF, Pawlowski, PRD 80 (2009) 025023

- crosscheck: $f_\pi(T=0) = 88\,\mathrm{MeV}$

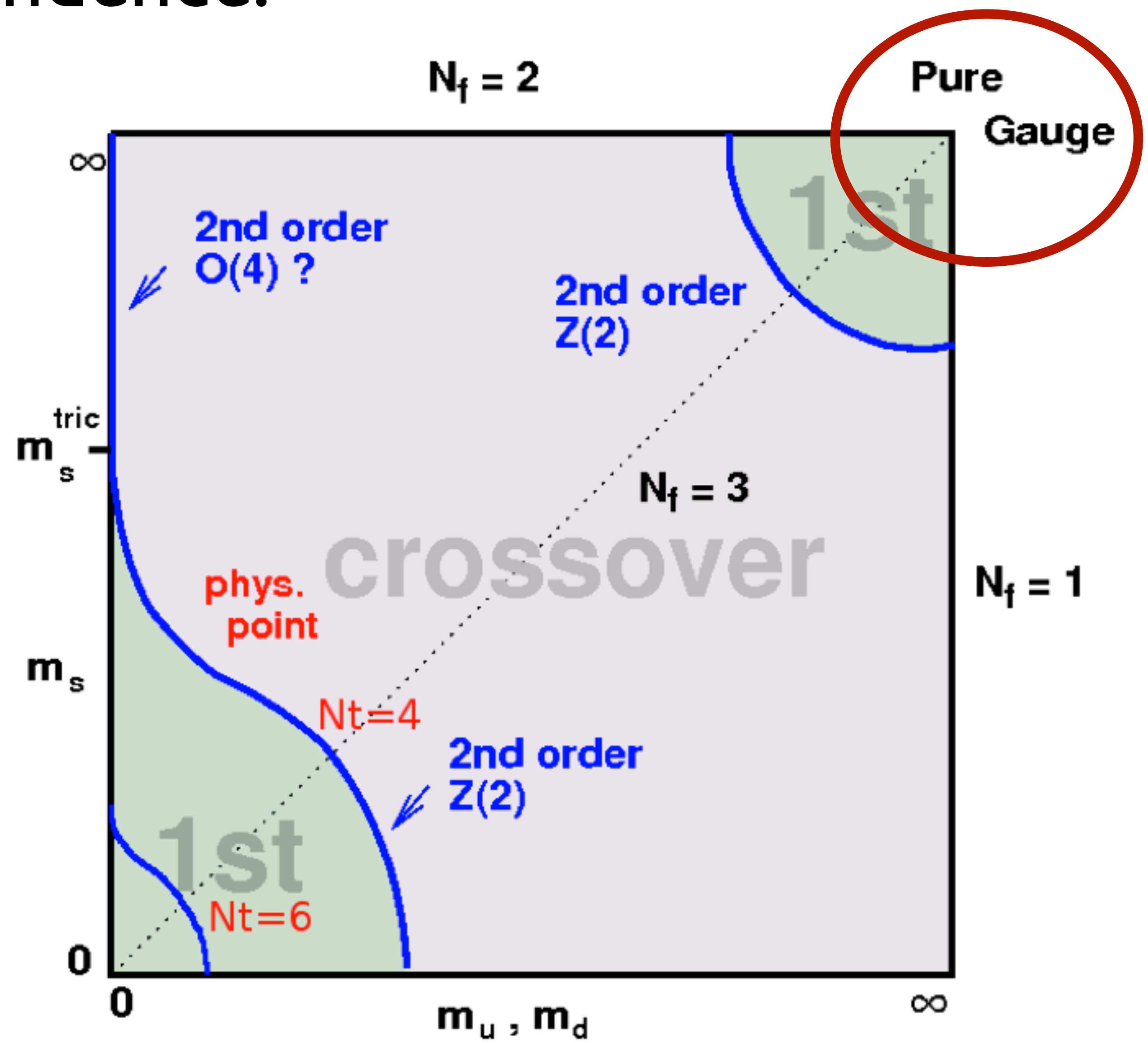
N_f=2+1-QCD with DSEs



- quenched: without quark-loop
- Nf=2: isospin symmetry
- Nf=2+1: solve coupled system of 2+3+3 equations

QCD phase transition: heavy quark limit/quenched

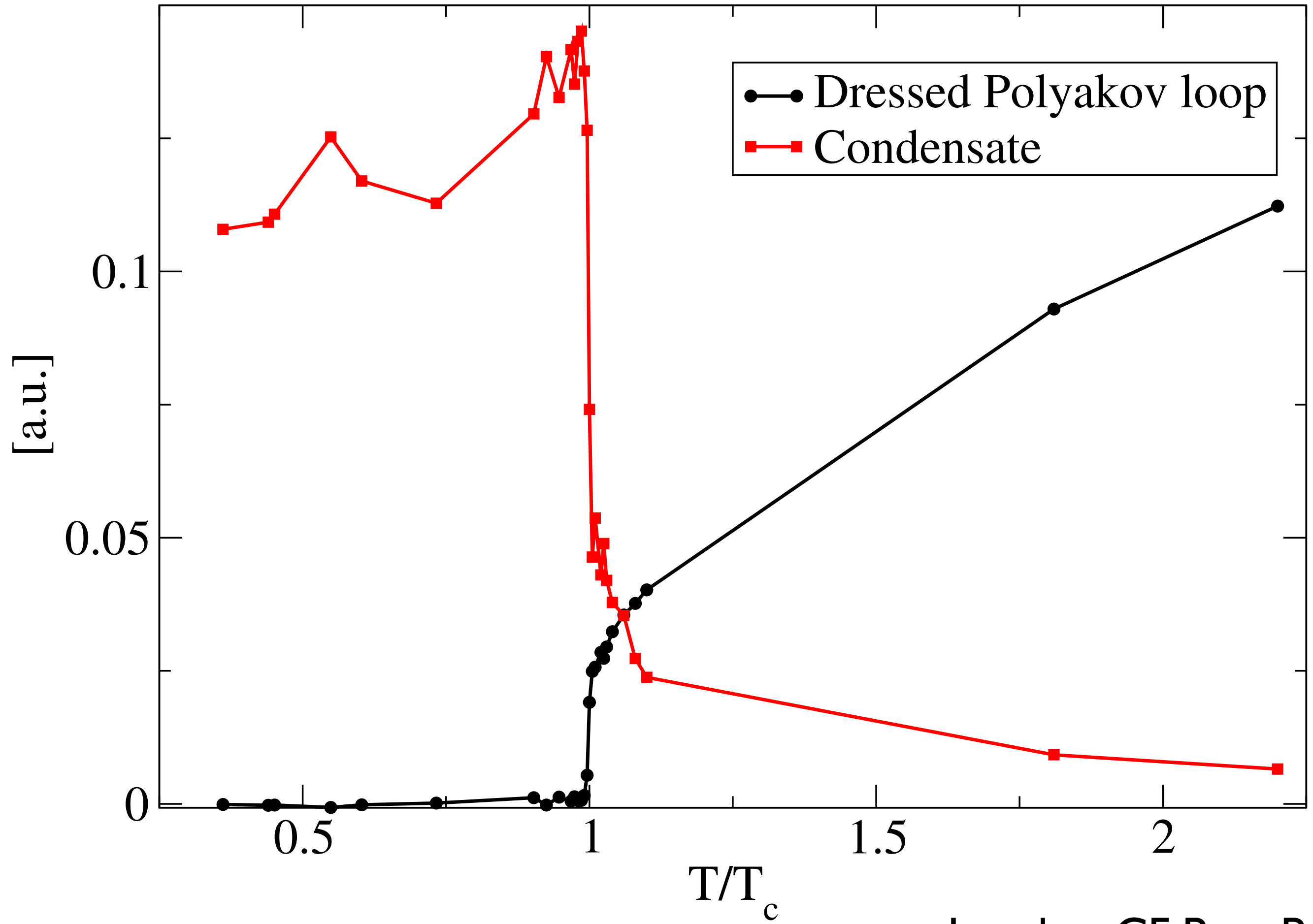
Quark mass dependence:



- Expect: Transitions controlled by deconfinement
- SU(2) second order, SU(3) first order

Transition temperatures, quenched

quenched DSE: SU(3)



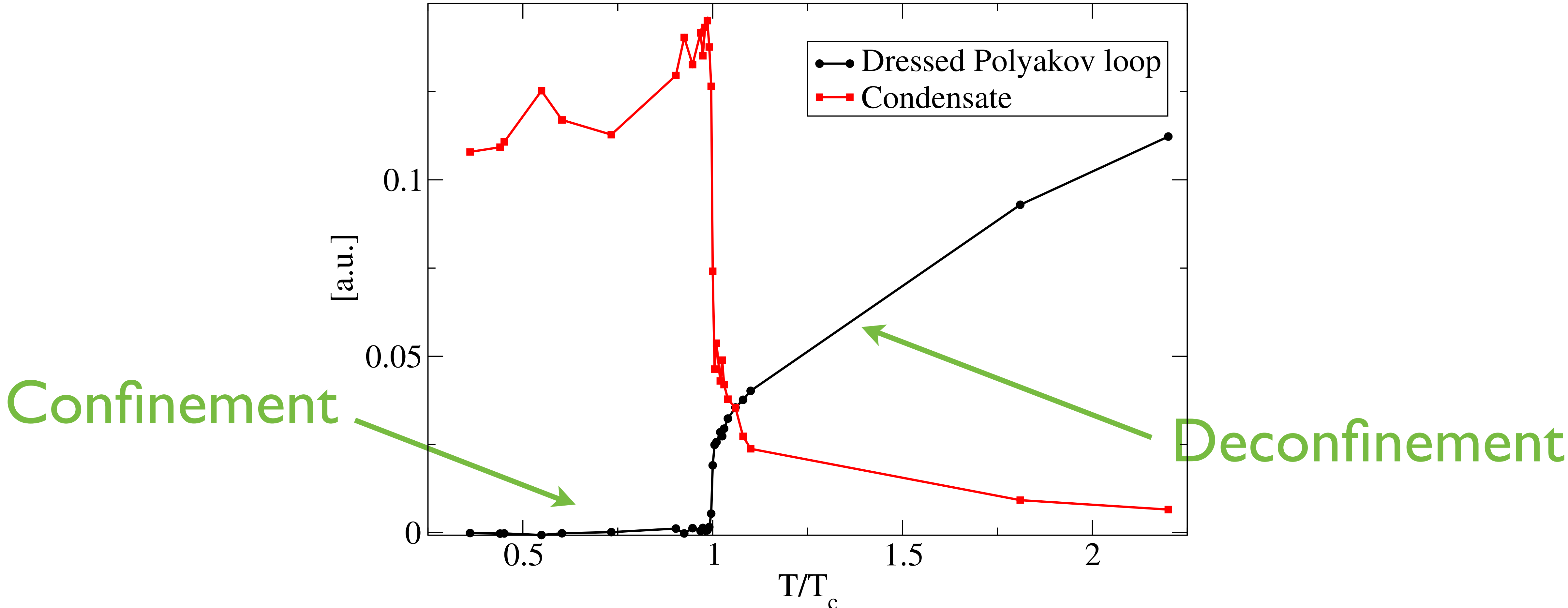
Luecker, CF, Prog.Part.Nucl.Phys. 67 (2012) 200-205
CF, Maas, Mueller EPJC 68 (2010)

- SU(2): $T_c \approx 305$ MeV
- SU(3): $T_c \approx 270$ MeV
- $T \leq T_c$: increasing condensate due to electric part of gluon

cf. Buividovich, Luschevskaya, Polikarpov, PRD 78 (2008) 074505
cf. Braun, Gies, Pawlowski, PLB 684 (2010) 262.

Transition temperatures, quenched

quenched DSE: SU(3)



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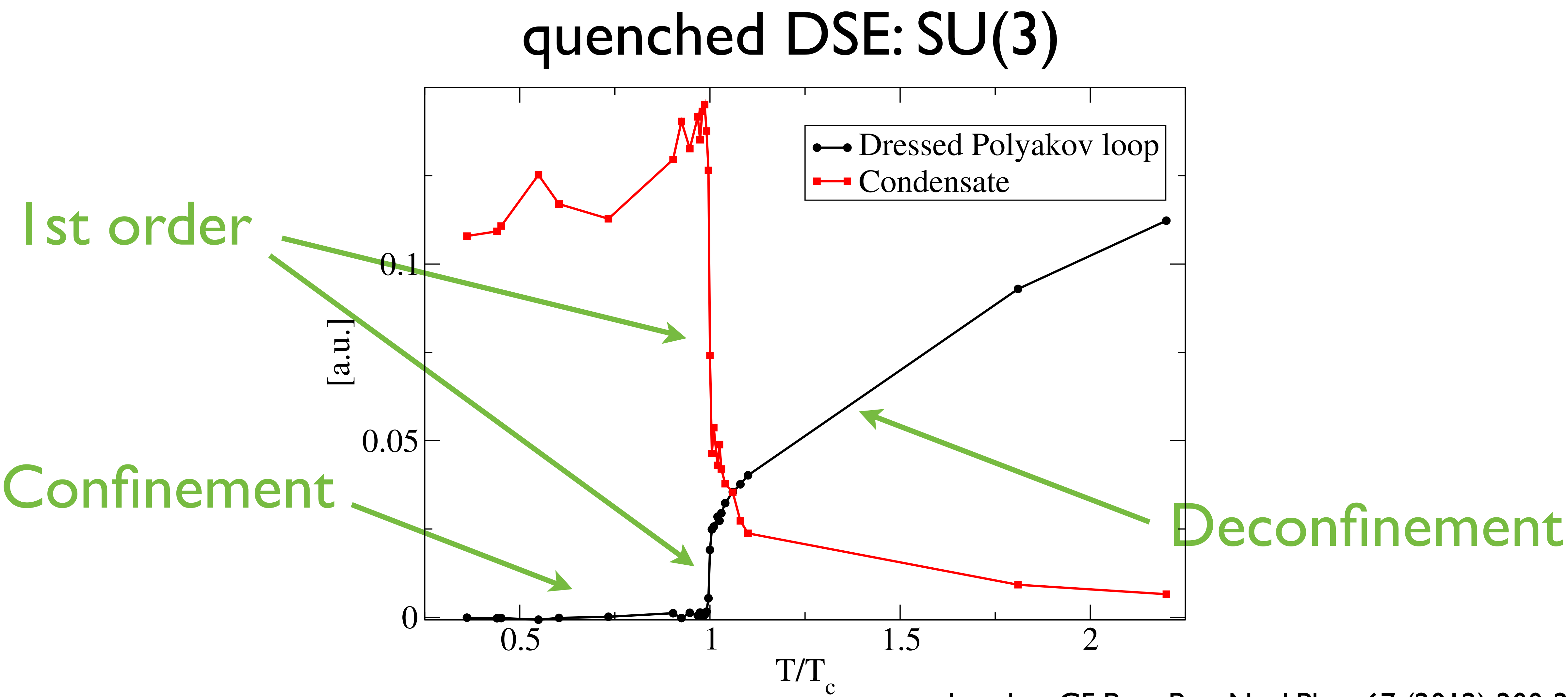

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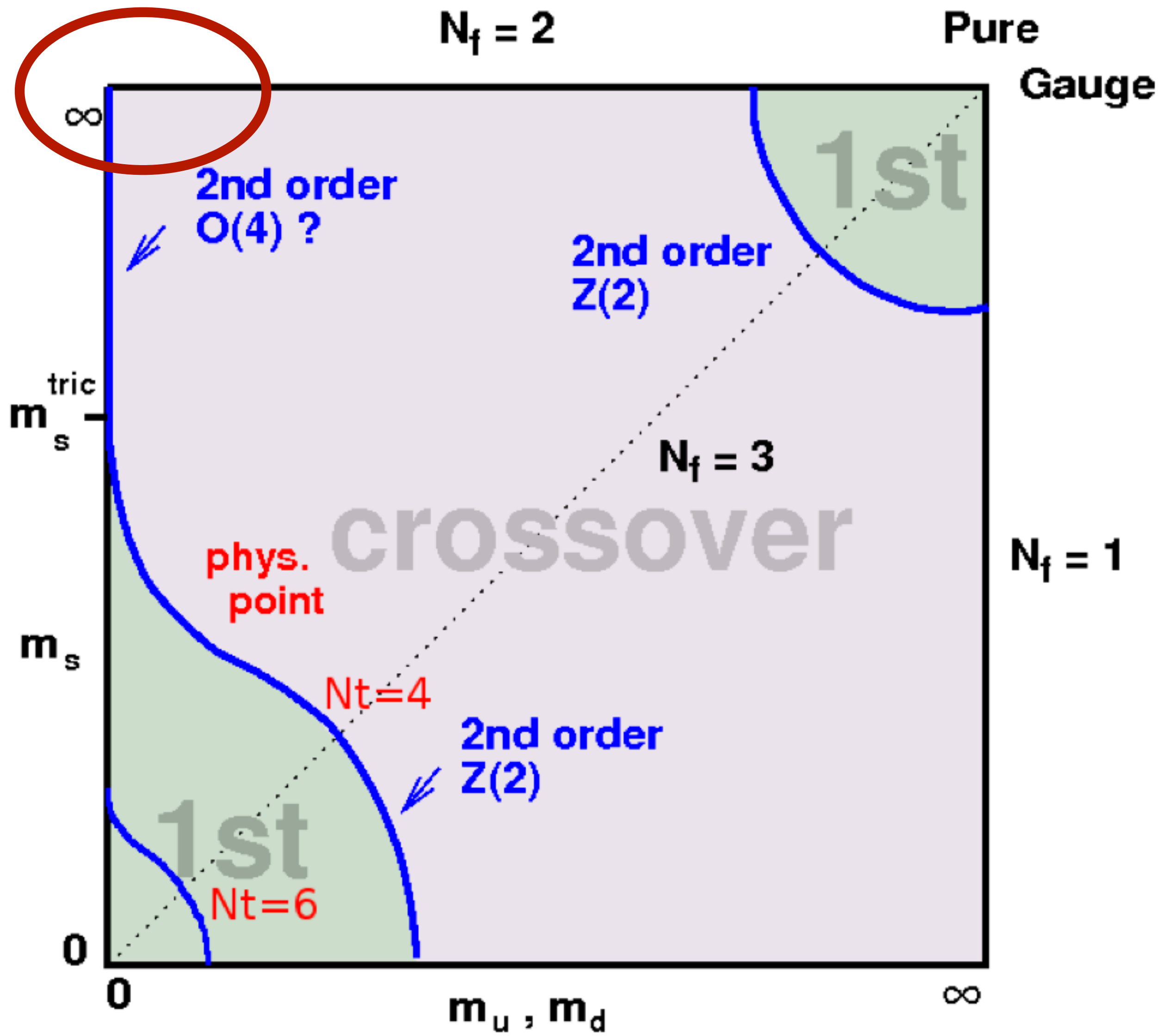
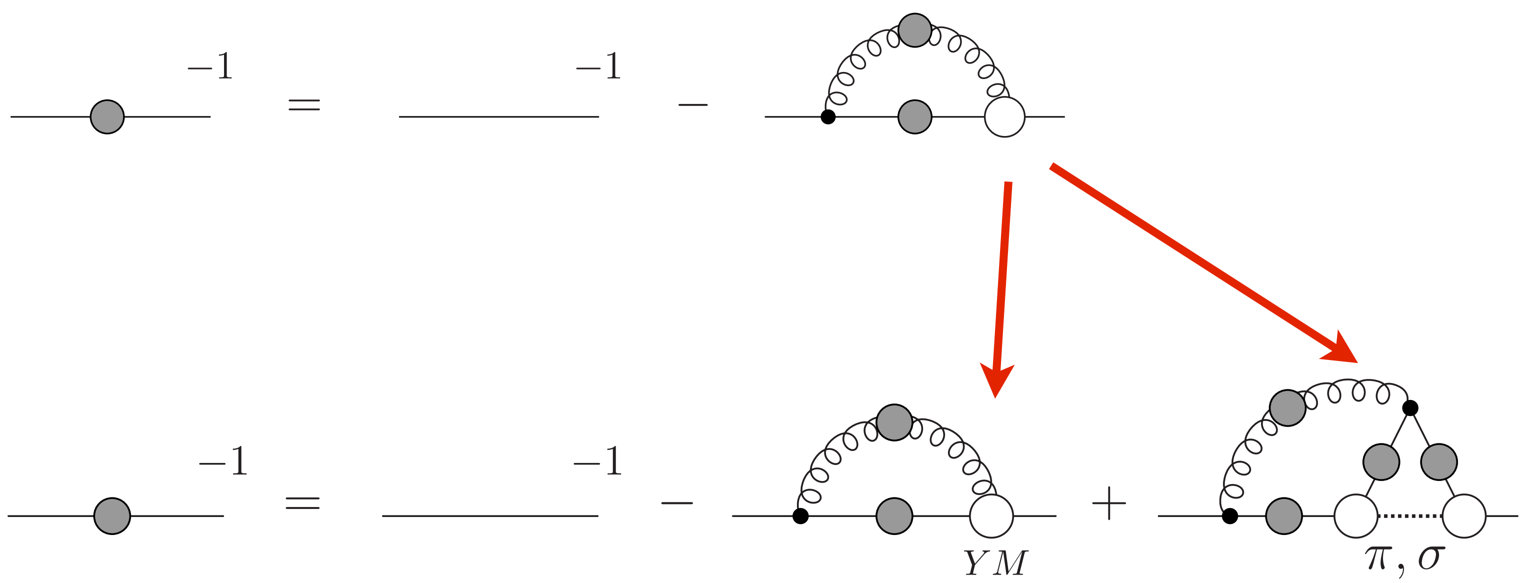
QCD phase transitions: chiral limit

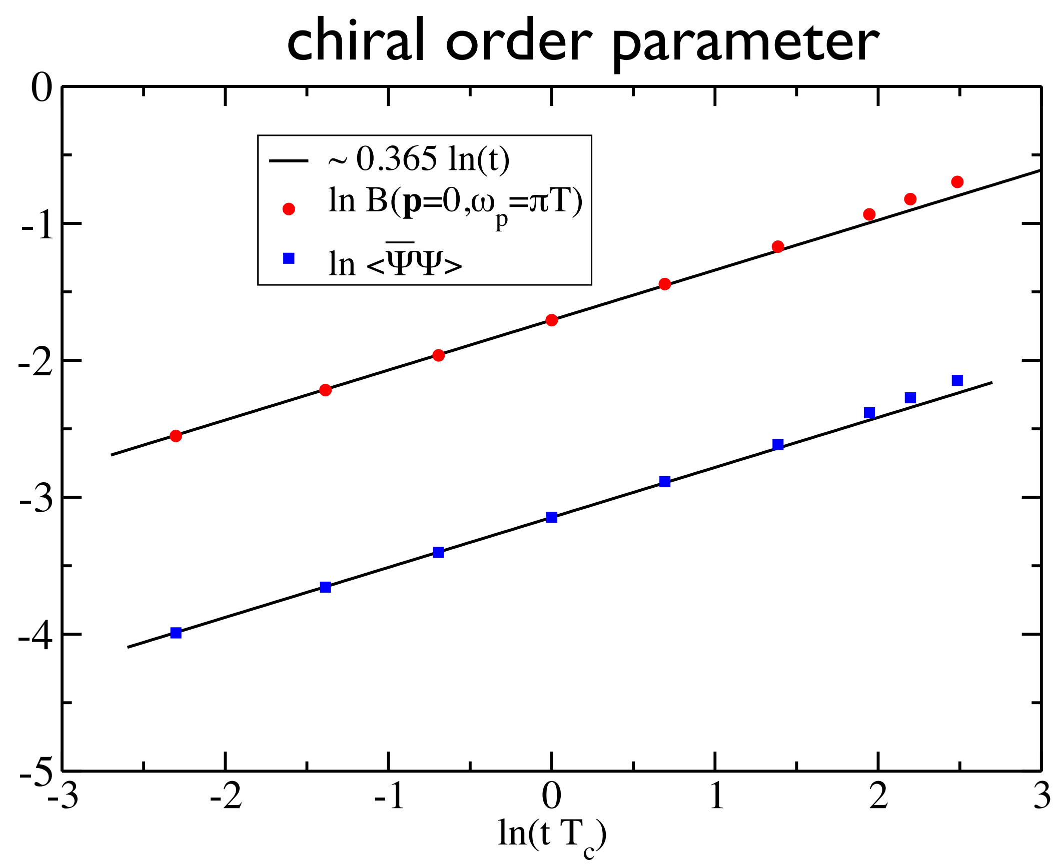


- N_f=2, chiral limit: phase transition dominated by Goldstone boson physics → Quark-Meson (QM) model
- SU(2)×SU(2) ≅ O(4)-second order vs. O(2)×O(4)-first order

Pisarski and Wilczek, PRD 29 (1984) 338

N_f=2, chiral limit: Critical scaling from DSEs



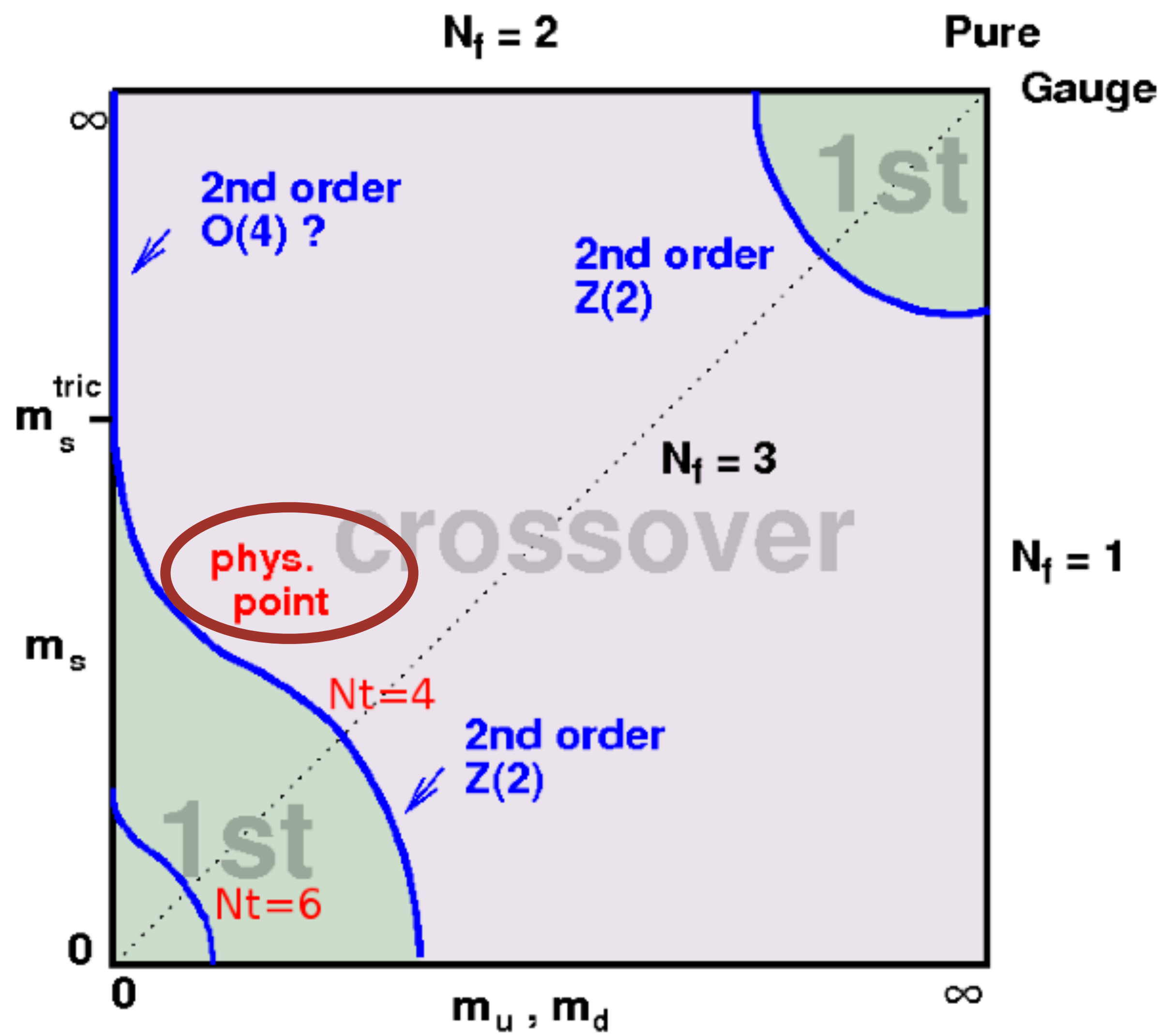


- Crucial: take meson part of vertex explicitly into account
- $T \neq T_c$: meson corrections of order of 10-20 %
  CF, Williams, PRD 78 (2008) 074006
- $T = T_c$: 2nd order phase transition,
  meson conributions are dominant ! (→ universality)
- Critical scaling: $\langle \bar{\Psi}\Psi \rangle(t) \sim B(t) \sim t^{\nu/2}$

$$f^2_{\pi,s} \sim t^{\nu} \qquad (t = (T_c - T)/T_c)$$

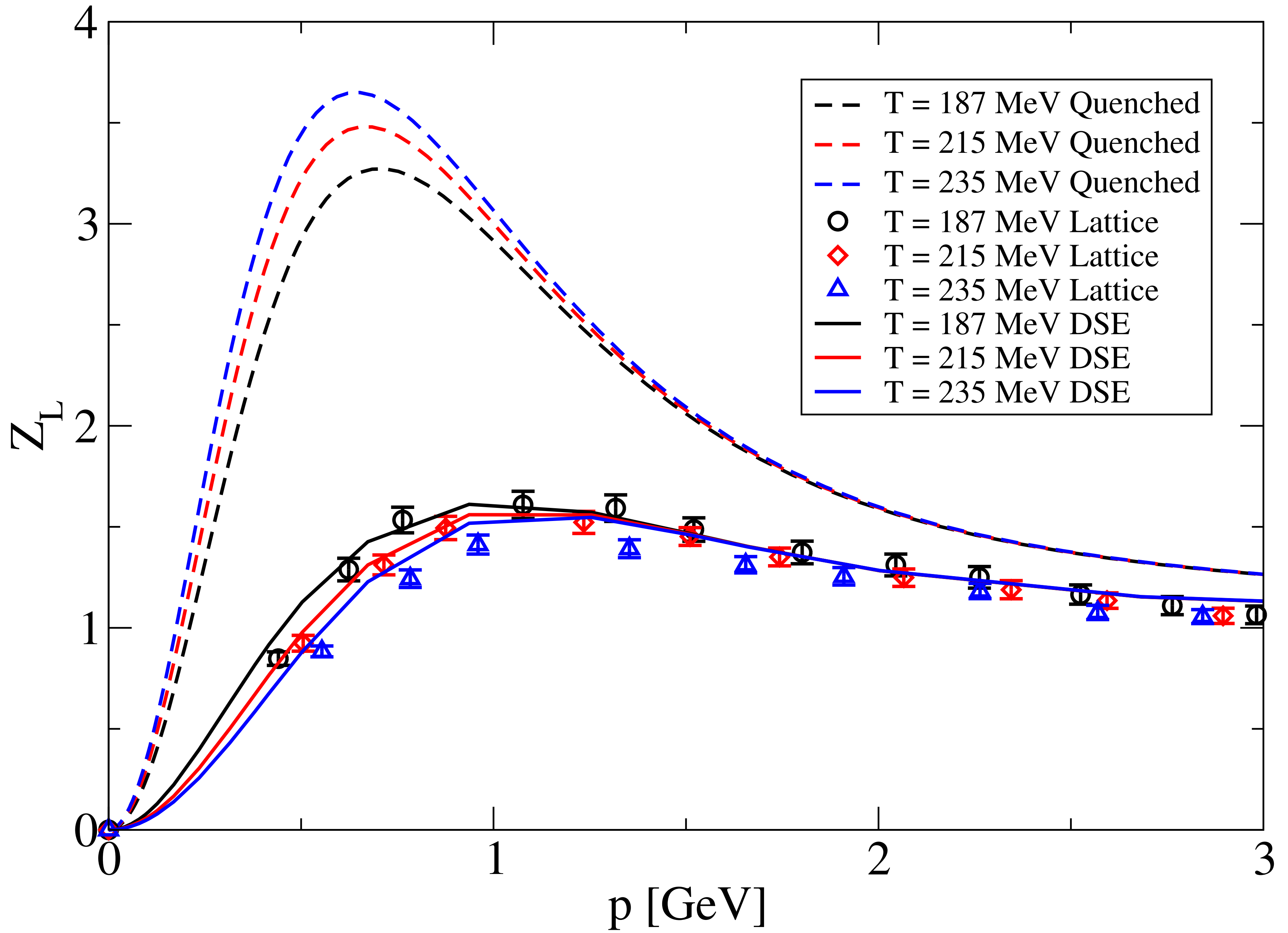
CF, Mueller, PRD 84 (2011) 054013

QCD phase transitions: N_f=2+1



- Physical up/down and strange quark masses
- Transition controlled by chiral dynamics
- at μ=0: compare to available lattice results

Unquenched Gluon DSE vs Lattice

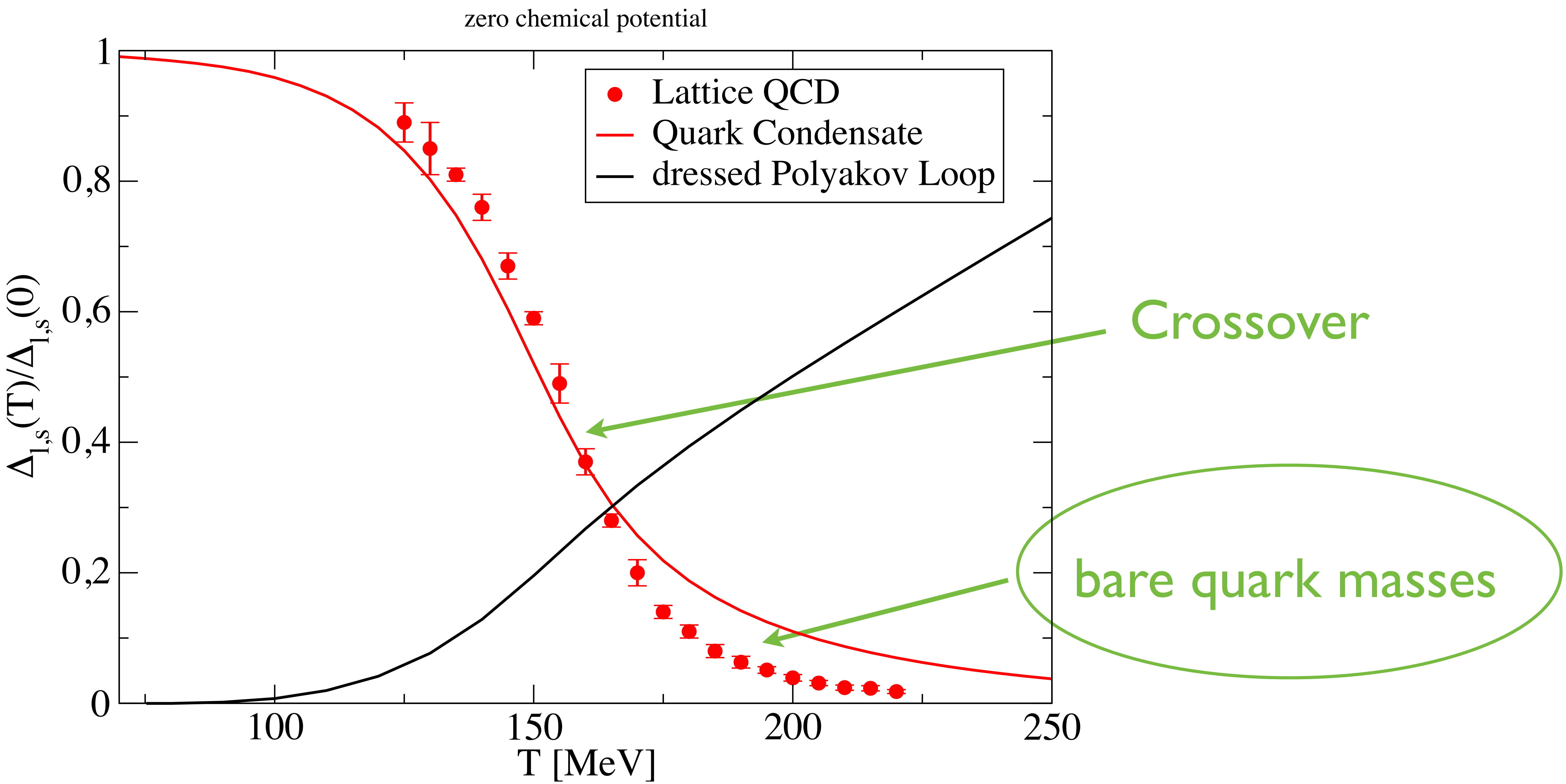


- quantitative agreement: DSE prediction verified by lattice

DSE: CF, Luecker, PLB 718 (2013) 1036 [arXiv:1206.5191]

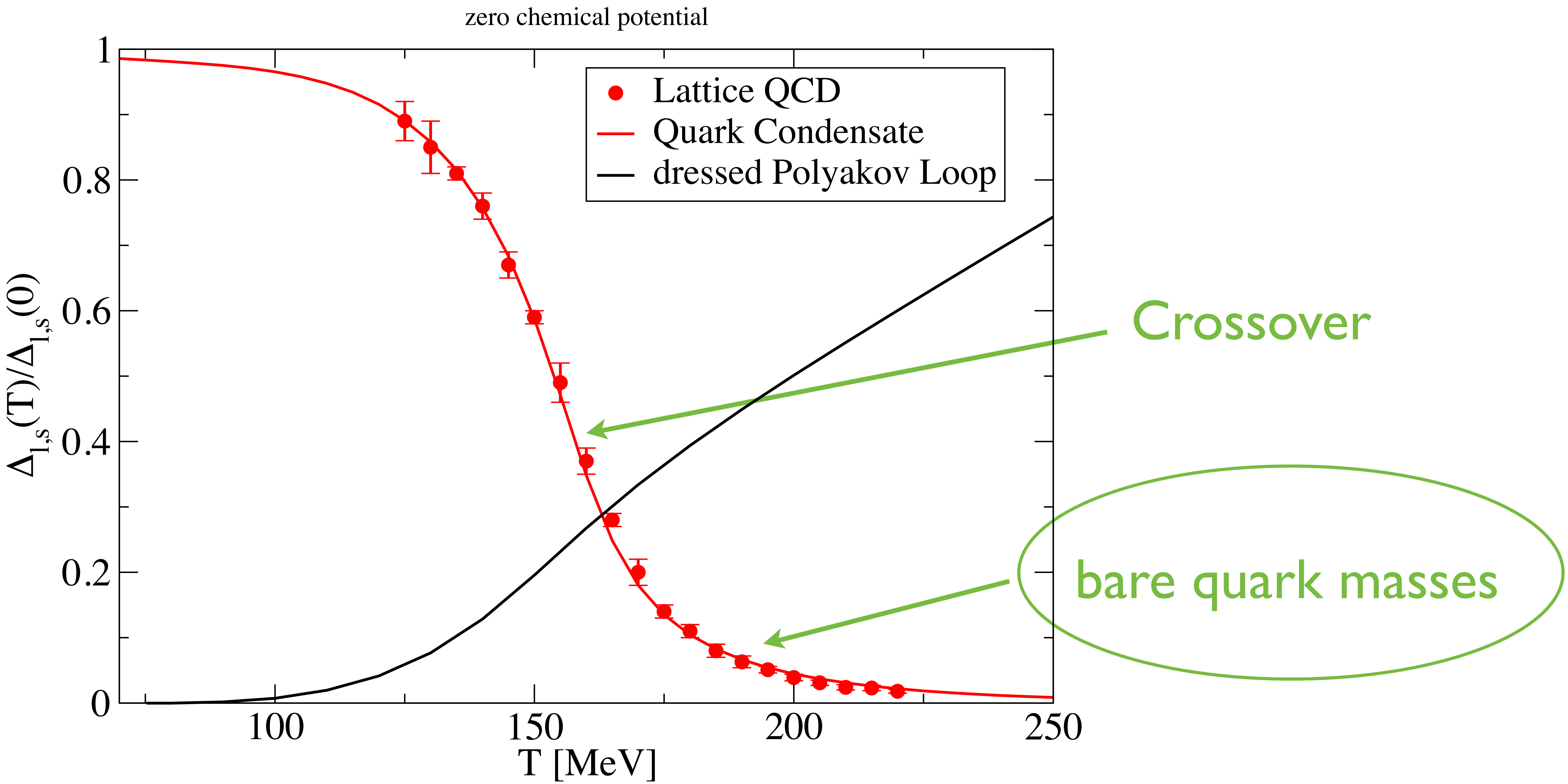
Lattice: Aouane, Burger, Ilgenfritz, Muller-Preussker and Sternbeck, arXiv:1212.1102

N_f=2+1, zero chemical potential



Lattice: Borsanyi *et al.* [Wuppertal-Budapest Collaboration], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036, CF, Luecker, Welzbacher, in prep.

N_f=2+1, zero chemical potential



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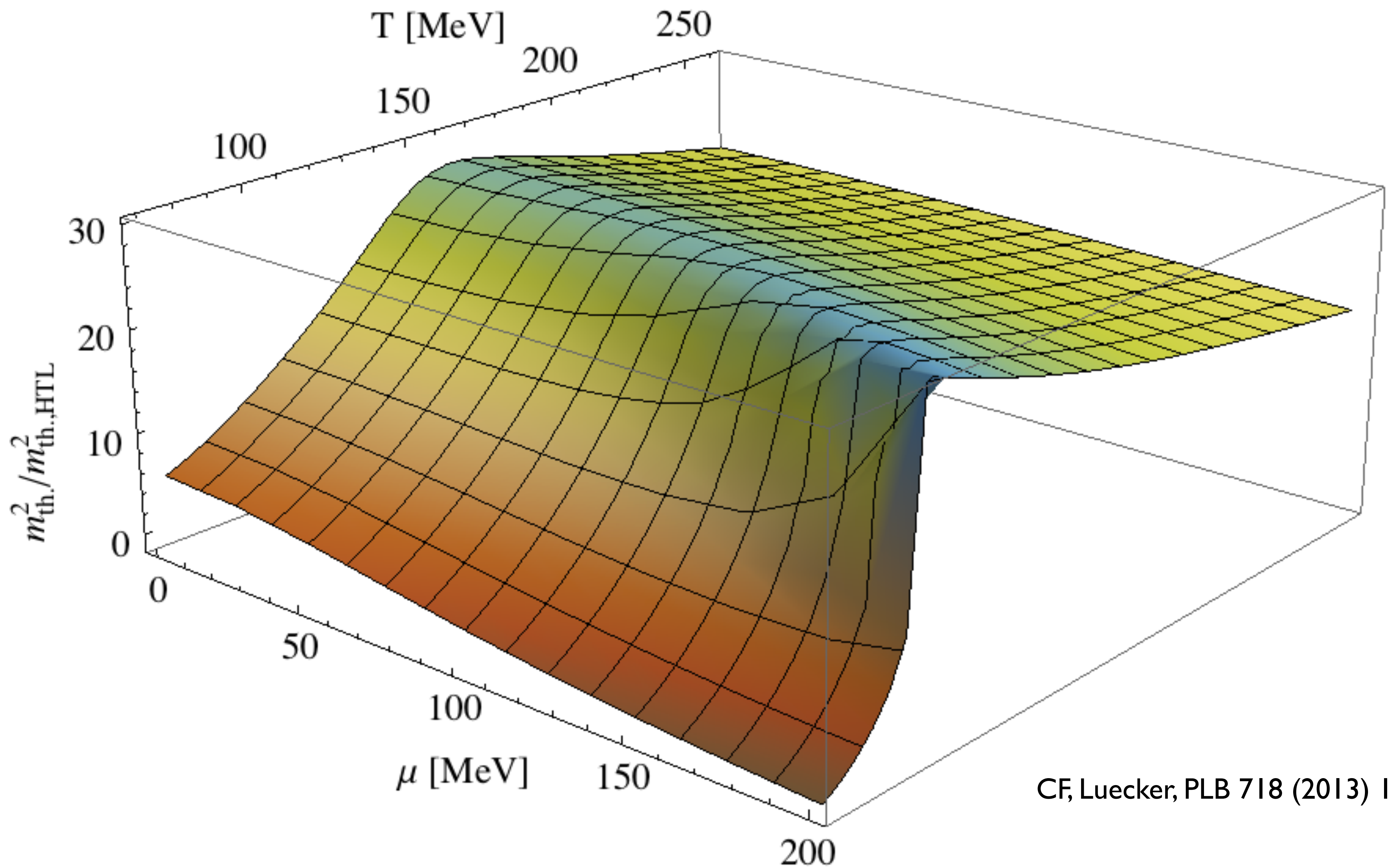
DSE: CF, Luecker, PLB 718 (2013) 1036, CF, Luecker, Welzbacher, in prep.

- quantitative agreement

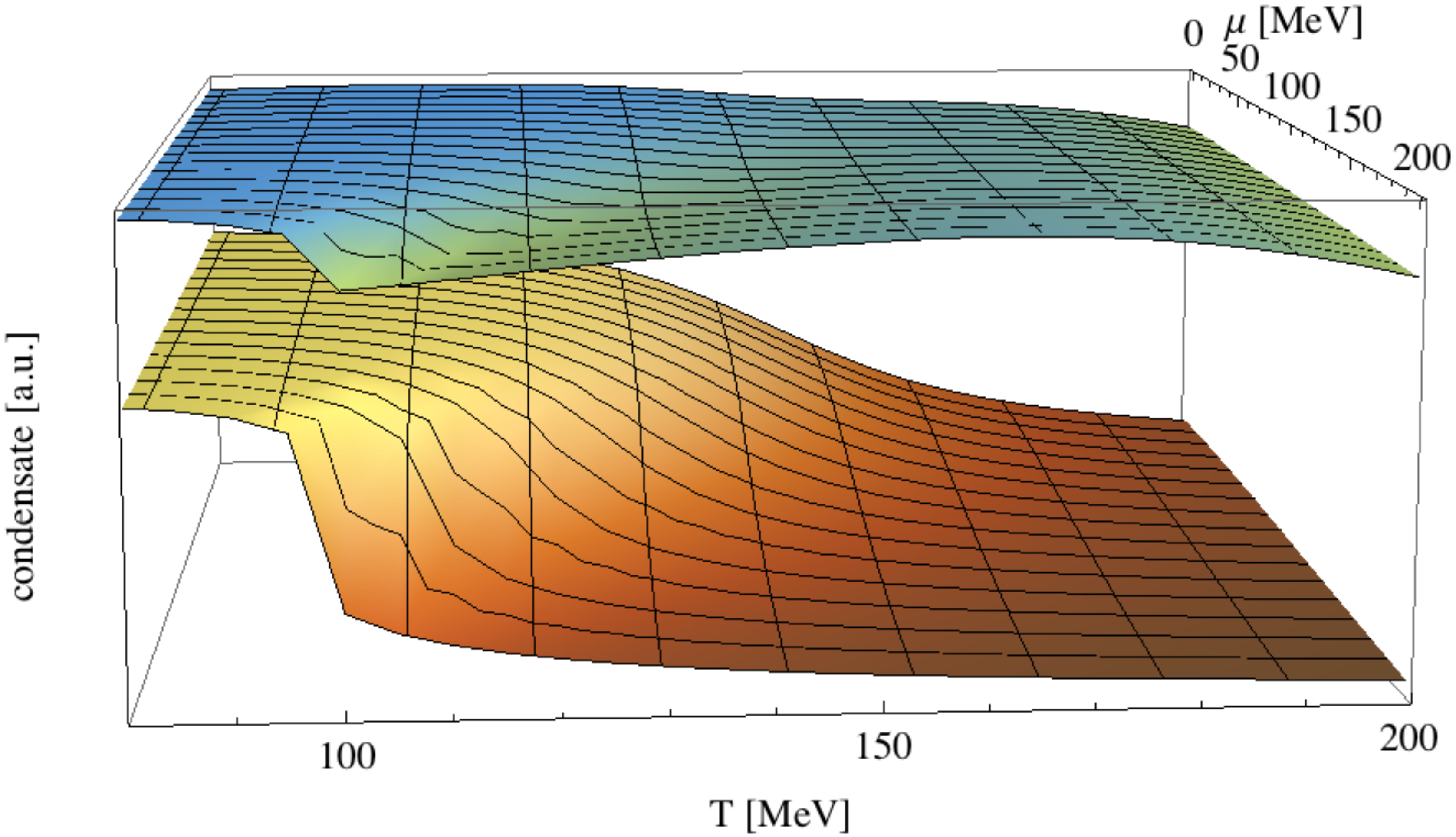
Nf=2+1: thermal electric gluon mass



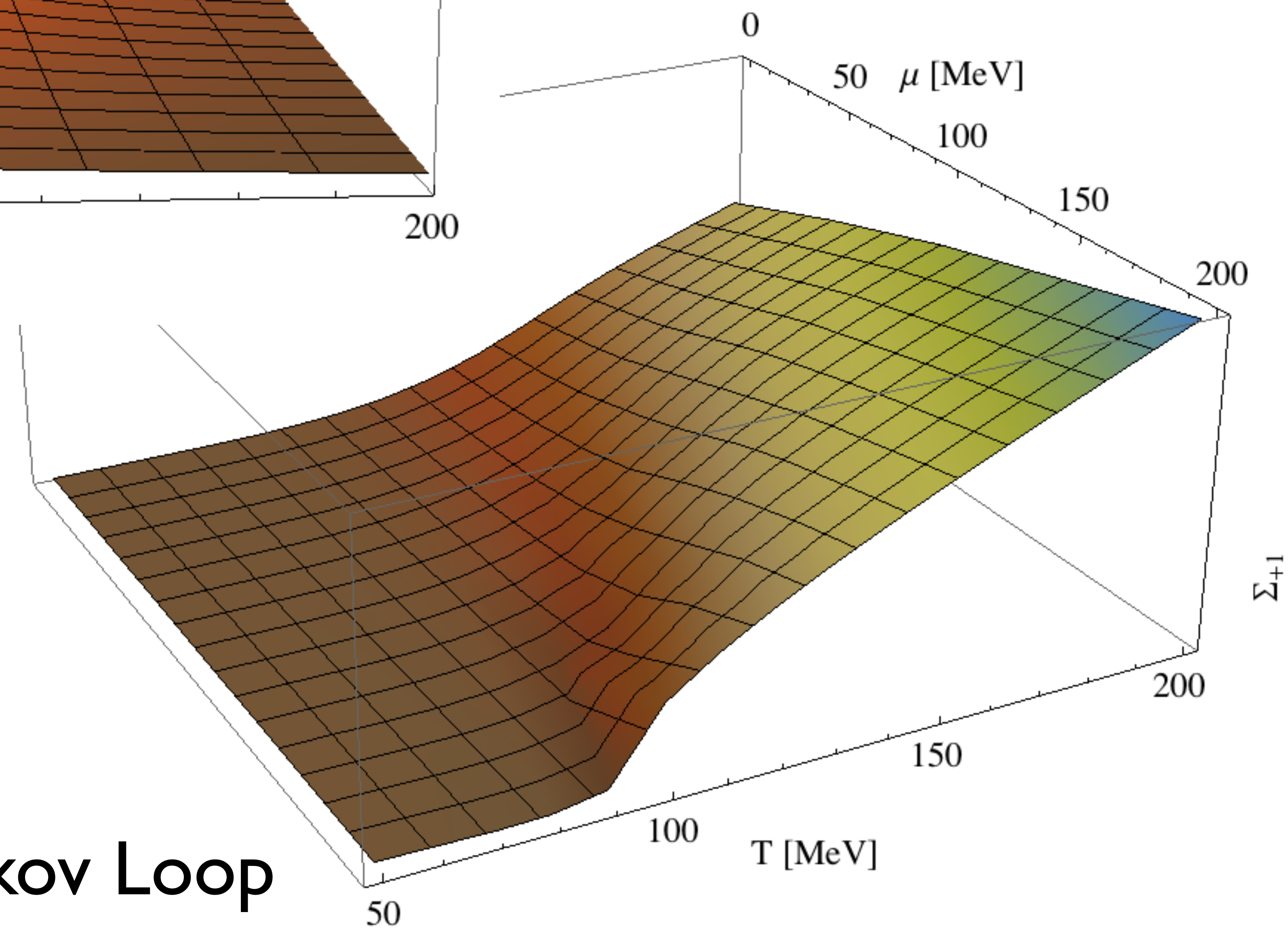
CF, Luecker, PLB 718 (2013) 1036

- large temperatures: behavior as expected from HTL
- first order transition at large chemical potential

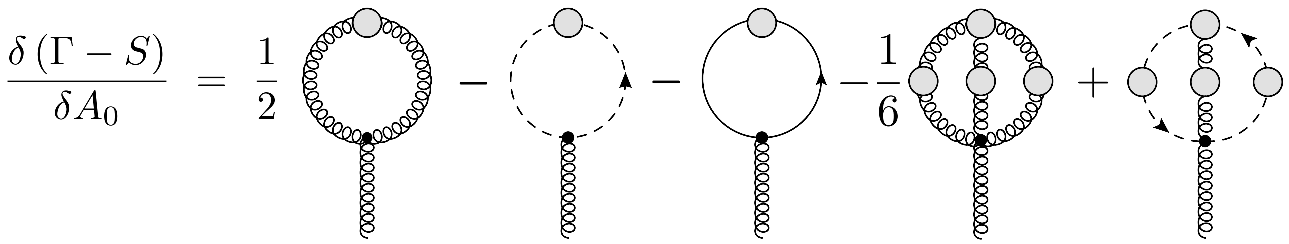
Nf=2+1: Condensate and dressed Polyakov Loop

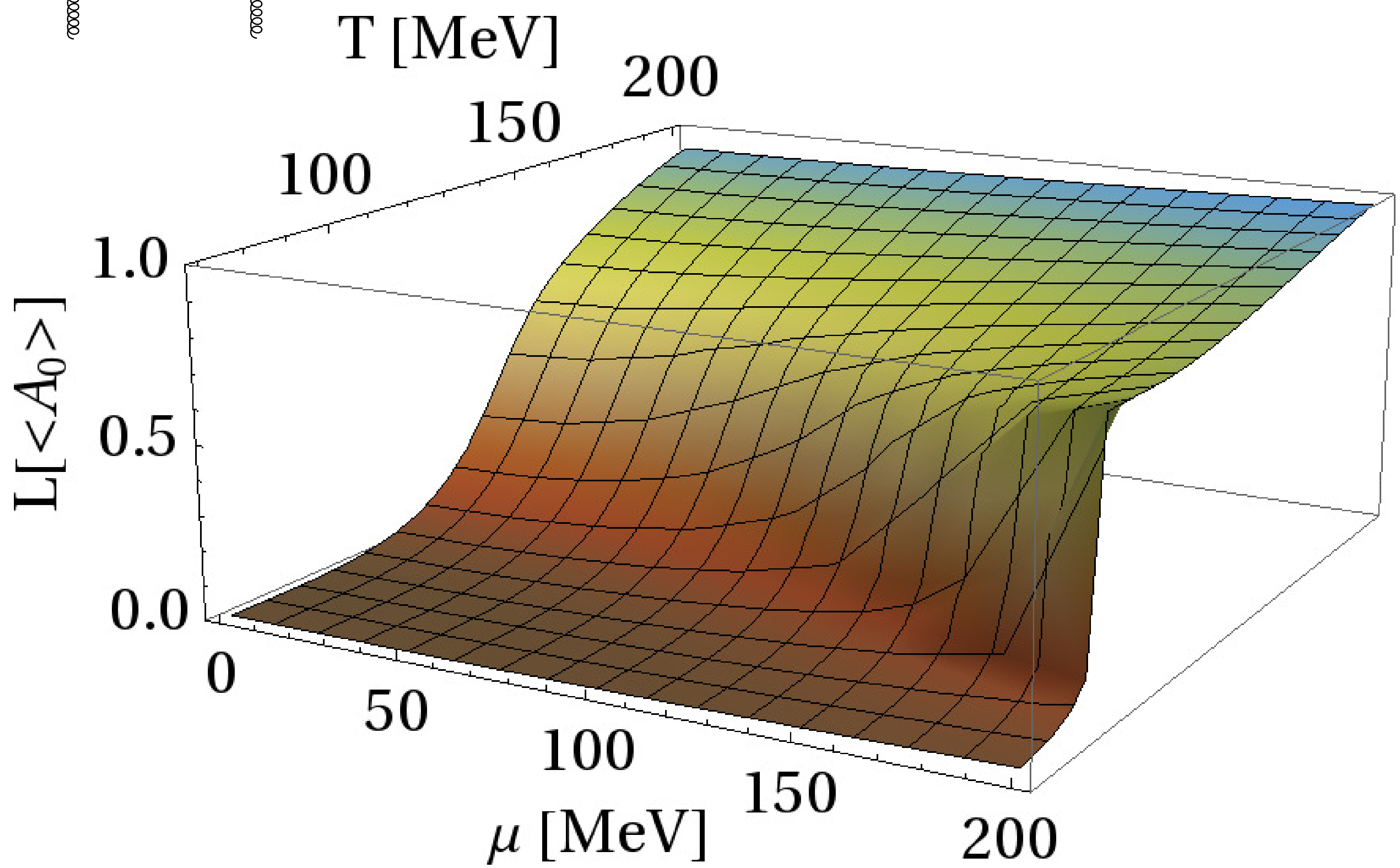


Quark condensate



Dressed Polyakov Loop





CF, Fister, Luecker, Pawlowski, arXiv:1306.6022

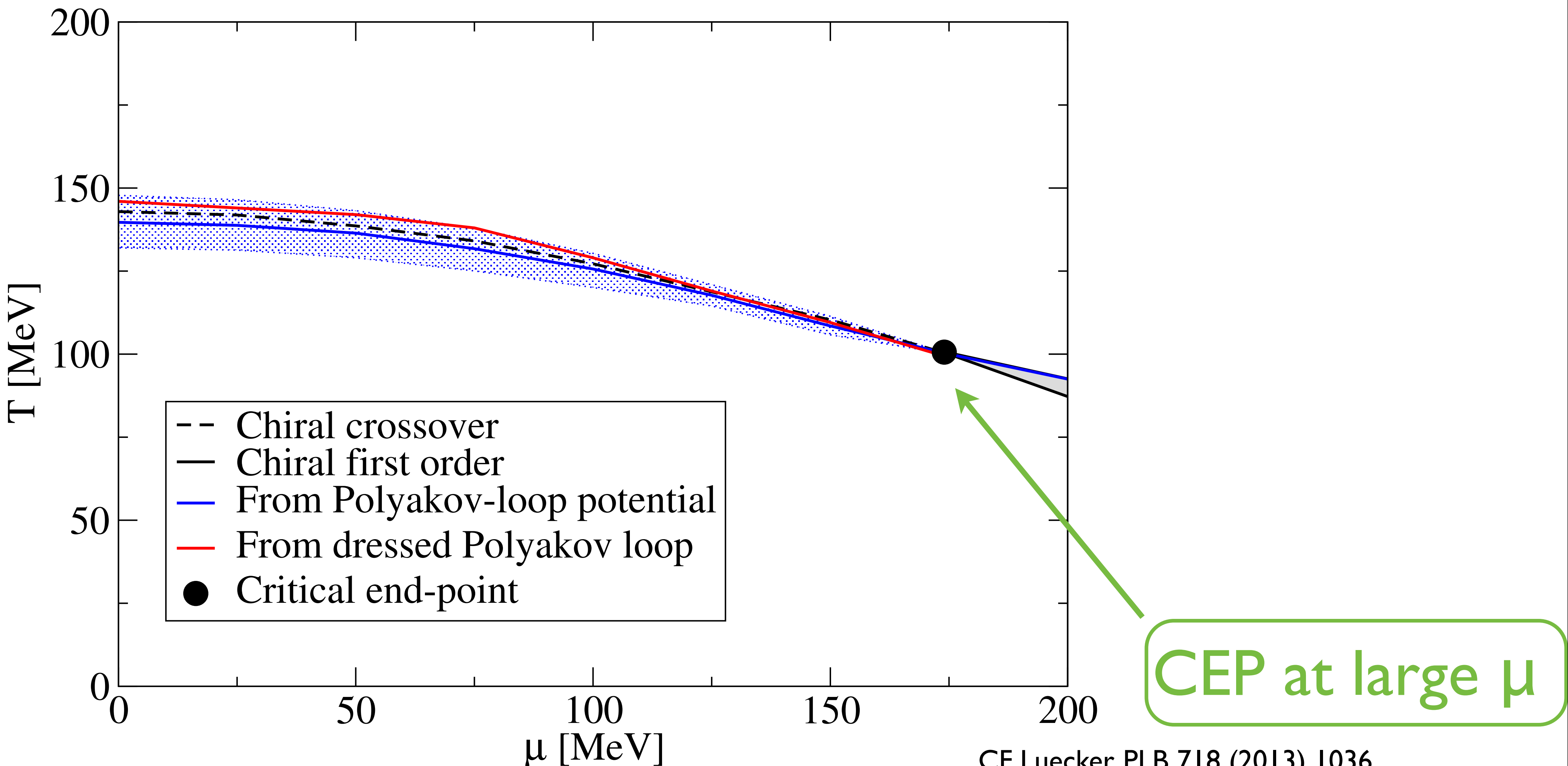
Polyakov-Loop

$$L = \frac{1}{N_c} tr\ e^{ig\int A_0}$$

- evaluated from Polyakov-Loop potential
- important input for P-models: PQM, PNJL !

Herbst, Mitter, Pawlowski, Schaefer, Stiele, arXiv:1308.3621

N_f=2+1: Polyakov loop and phase diagram



CF, Luecker, PLB 718 (2013) 1036,
CF, Fister, Luecker, Pawlowski, arXiv:1306.6022

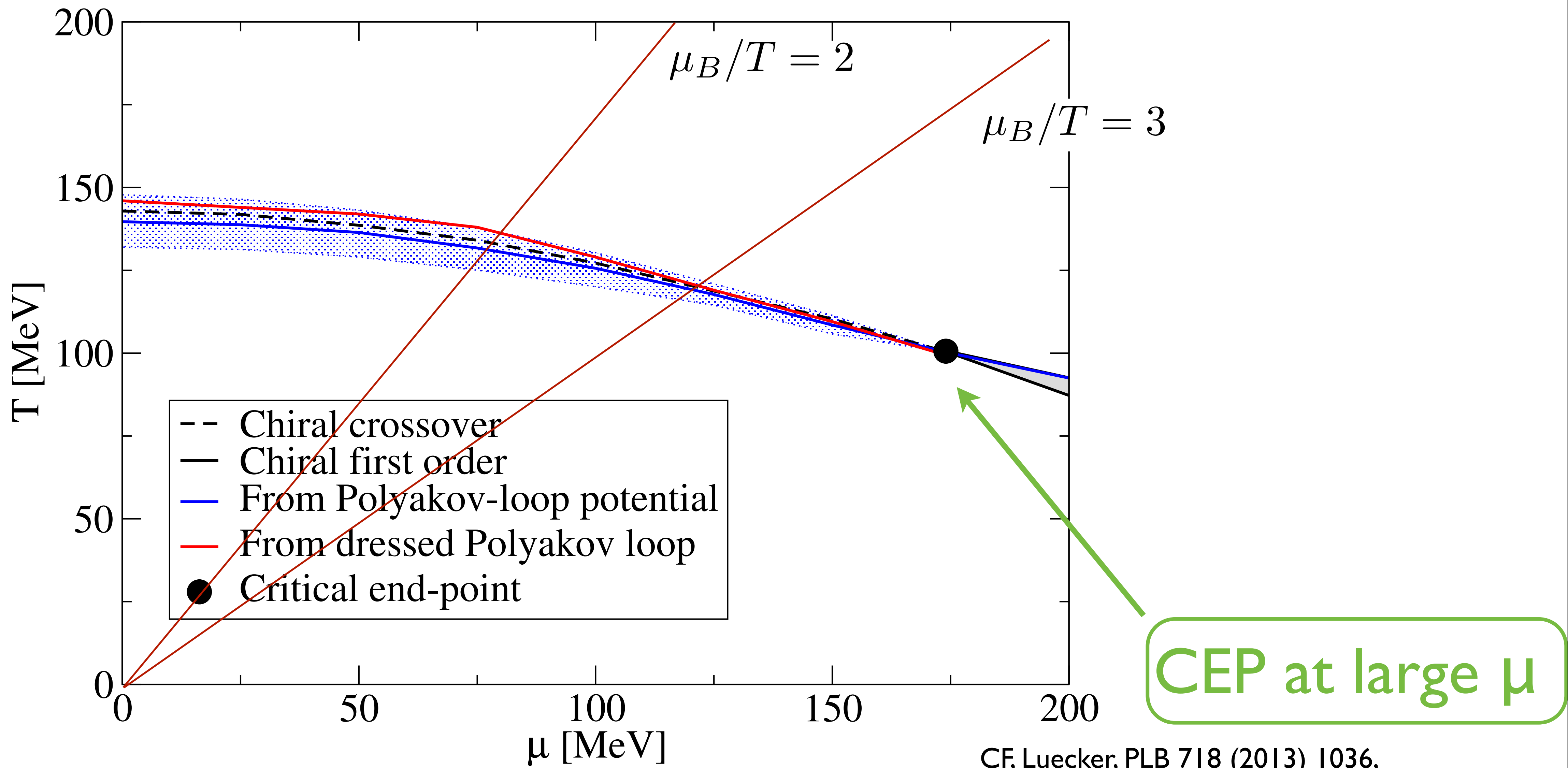
- no CEP at $(\mu_B)_c/T_c < 2$ in agreement with lattice and FRG

de Forcrand, Philipsen, JHEP 0811 (2008) 012; Nucl Phys. B642 (2002) 290-306
Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001
Herbst, Pawlowski, Schaefer, PRD 88 (2013) 014007

Caveat: baryon effects missing...

Nc=2: Brauner, Fukushima and Hidaka, PRD 80 (2009) 74035
Strodthoff, Schaefer and Smekal, PRD 85 (2012) 074007

N_f=2+1: Polyakov loop and phase diagram



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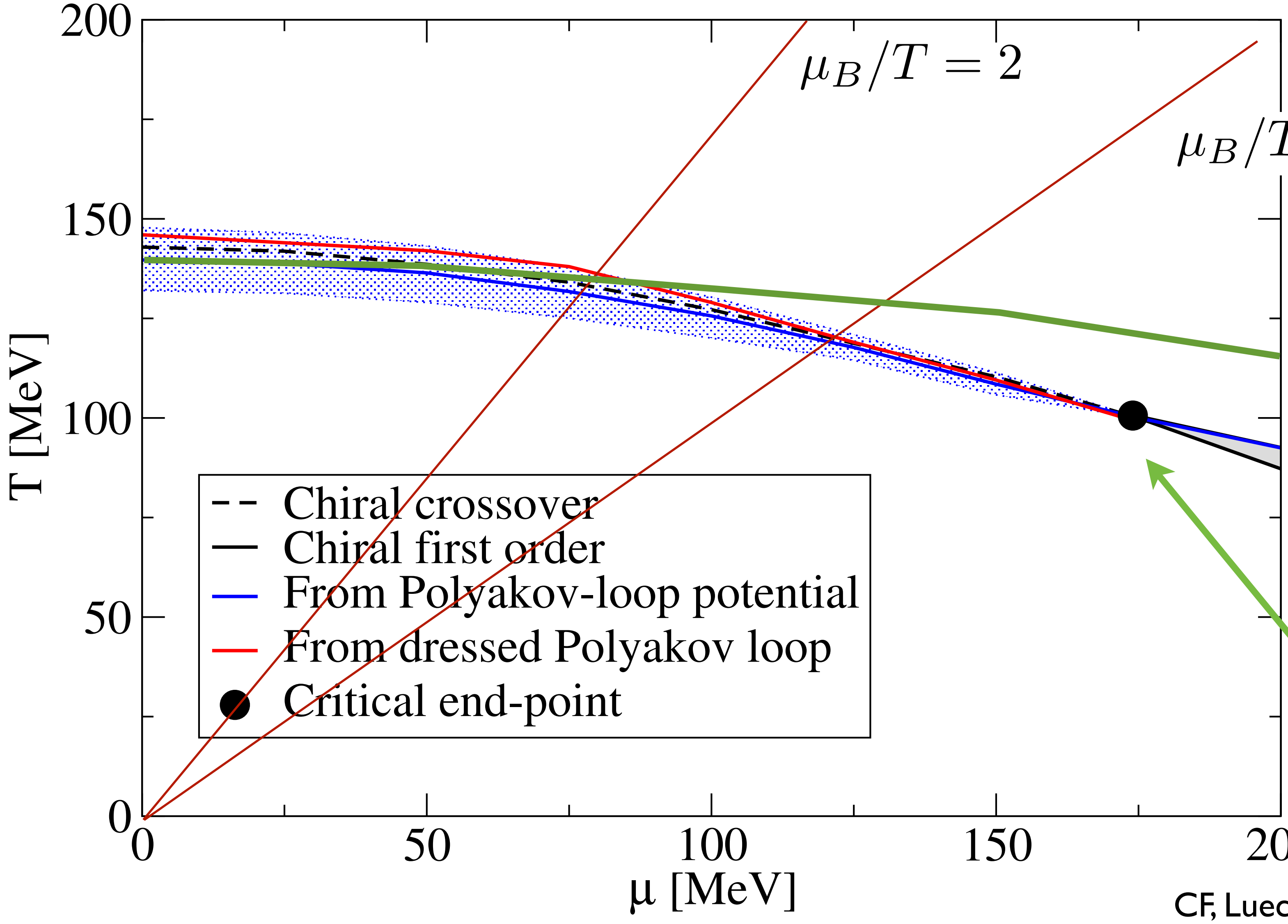


κ = 0.0066

see Talk of F. Karsch

CEP at large μ

CF, Luecker, PLB 718 (2013) 1036,
CF, Fister, Luecker, Pawlowski, arXiv:1306.6022

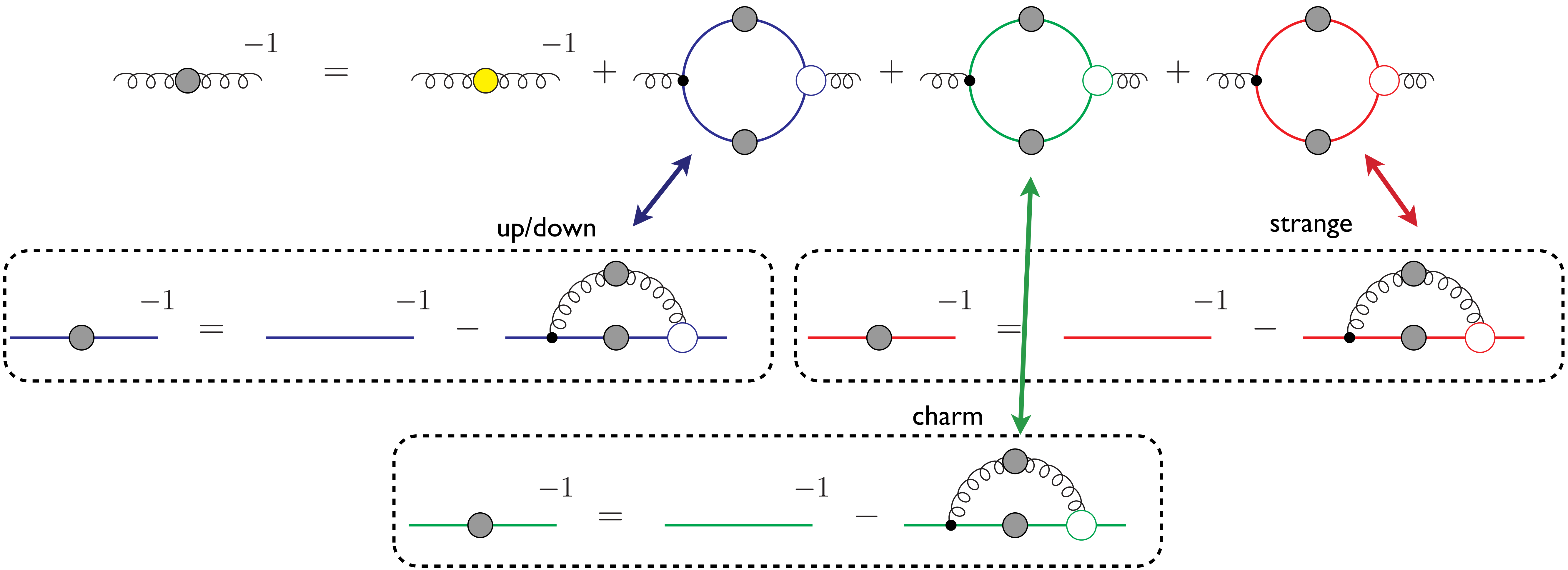
- no CEP at $(\mu_B)_c/T_c < 2$ in agreement with lattice and FRG

de Forcrand, Philipsen, JHEP 0811 (2008) 012; Nucl Phys. B642 (2002) 290-306
Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001
Herbst, Pawlowski, Schaefer, PRD 88 (2013) 014007

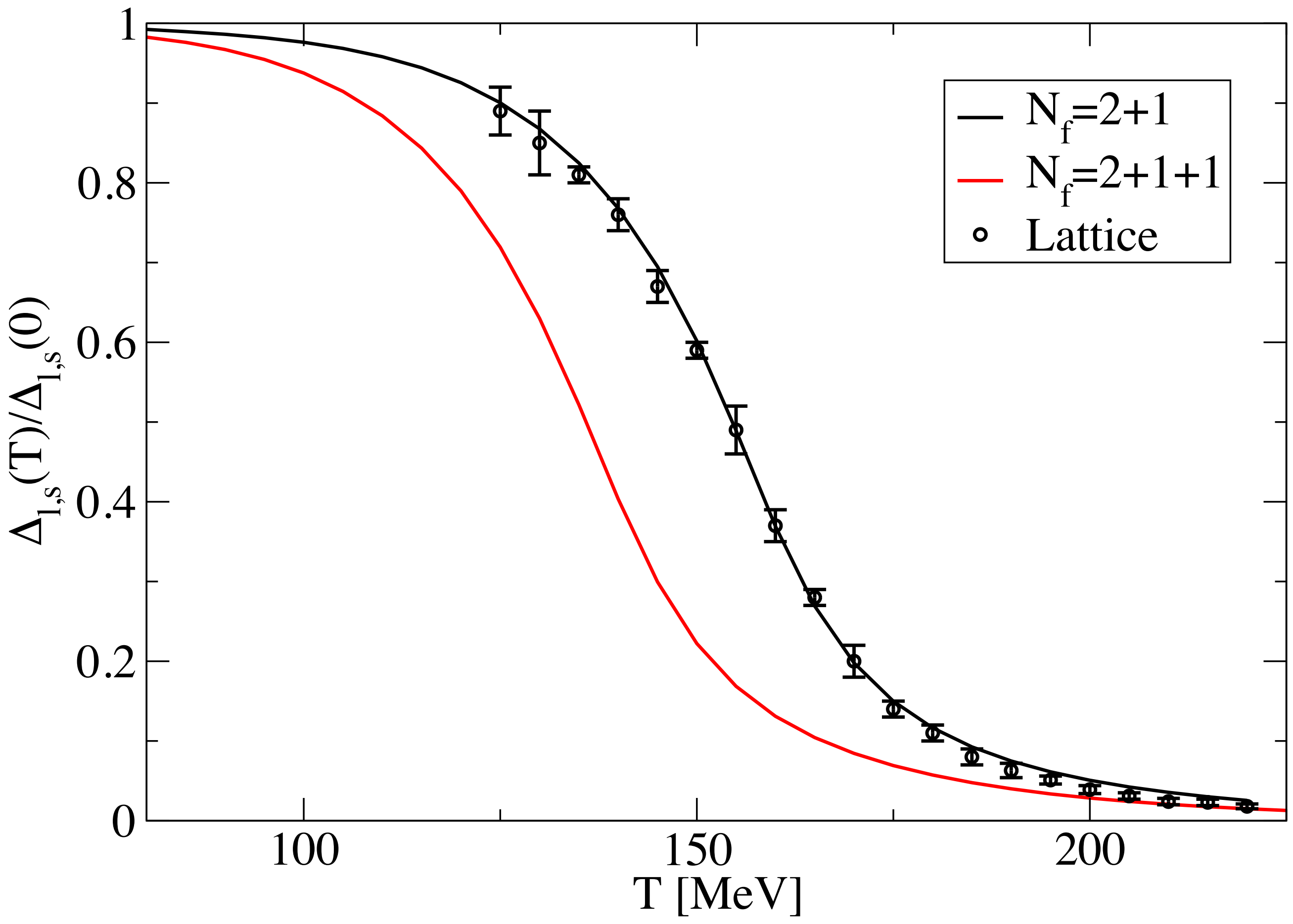
Caveat: baryon effects missing...

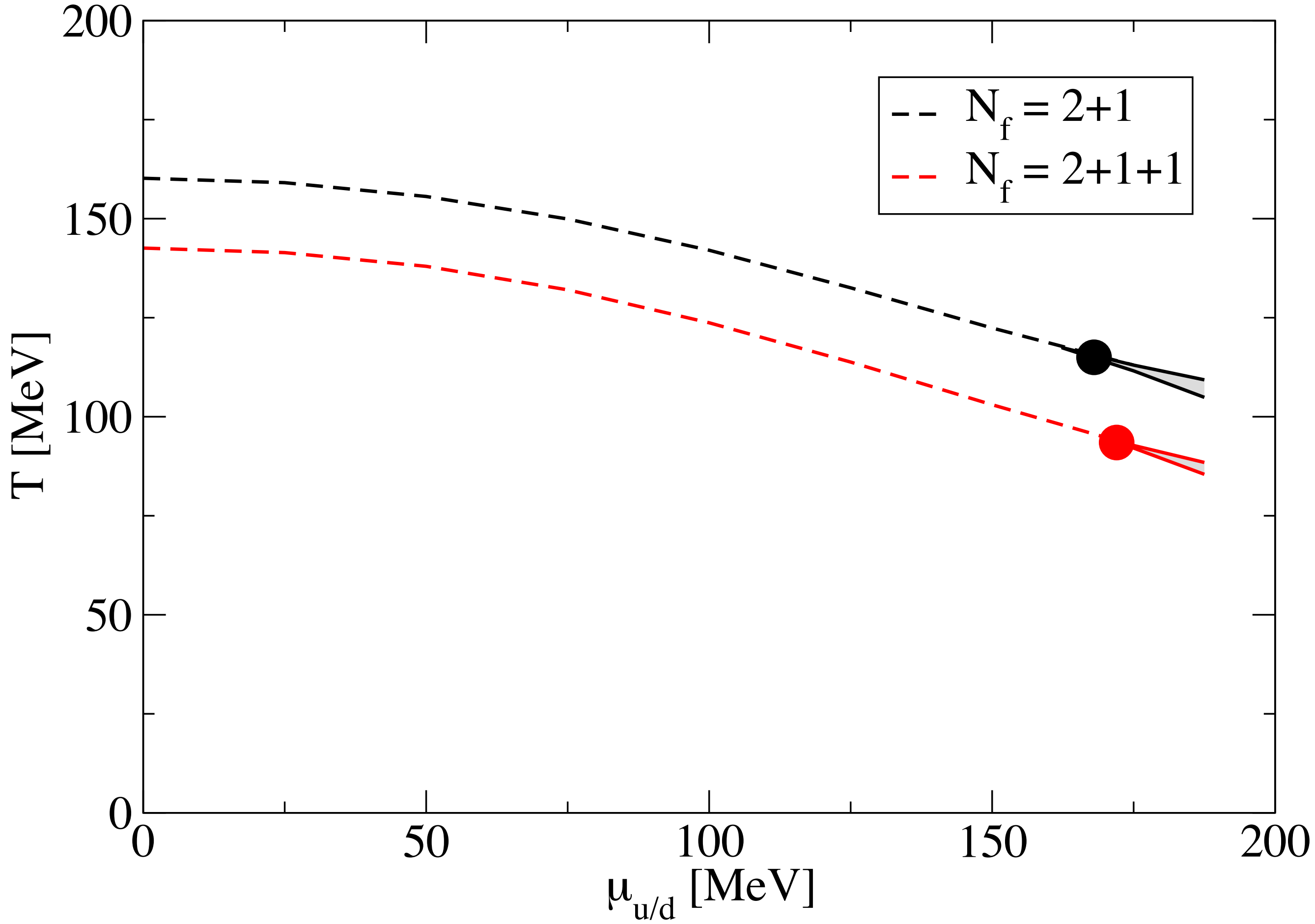
Nc=2: Brauner, Fukushima and Hidaka, PRD 80 (2009) 74035
Strodthoff, Schaefer and Smekal, PRD 85 (2012) 074007

Nf=2+1+1-QCD with DSEs



Nf=2+1+1-QCD with DSEs (preliminary)





CF, Luecker, Welzbacher, in preparation

- Interaction fixed: T_{PC} decreases by O(10 MeV)
- Physics fixed (m_π, f_π): T_{PC} similar (not shown)

→see also Talk by C. Sasaki, 2nd week

Summary

- Gluon spectral functions at T=0: positivity violation
- Temperature dependent gluon propagator
  - characteristic behavior of electric gluon
  - 'melting' of magnetic gluon with temperature
- Deconf. T_{pc} from dressed Polyakov-loop/Polyakov-loop potential
- QCD with finite chemical potential (beyond mean field)
  - backreaction of quarks onto gluons important
  - N_f=2+1 and N_f=2+1+1: CEP at $\mu_c/T_c > 1$

Work in progress: include baryons...
include magnetic field... Mueller, Bonnet, CF in preparation

→see also Talk by T. Kojo, 2nd week