New Results for the Transport Coeff cients of the QGP from Holography

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New Frontiers in QCD, Dec. 2013, Kyoto, Japan

- Interpretation of anisotropic flow data + hydrodynamical calculations indicate that the QGP is a strongly interacting plasma.

- Phenomenological estimates suggest that the minimum of $\eta/s(T)$ in the QGP is close to the "perfect fluid" value $\sim 1/(4\pi)$.

- Weak coupling (g << 1) calculations (AMY) obtain $\eta/s \sim 1/(g^4 \ln 1/g)$

- I will take this as an indication that the perfect fluid character of the QGP near deconfinement involves some unknown mechanism that is beyond weak coupling methods. Perhaps the plasma is truly strongly coupled.

Once you buy the idea of strong coupling, a few questions come up:

1) What are the equations of motion that describe how this strongly coupled fluid relaxes towards local thermal equilibrium?

2) How does one determine the several microscopic coefficients that may appear in this coarse grained description?

3) Is it possible to extrapolate this knowledge to learn something about the QGP?

Answers: 1) Not known. 2) More or less known. 3) Hopefully :)

What are the equations of motion that describe how this strongly interacting fluid relaxes towards local thermal equilibrium?

Basically, given the conserved energy-momentum tensor: $\nabla_\mu \langle T^{\mu\nu} \rangle = 0$

$$\langle T^{\mu\nu} \rangle = \varepsilon\, u^\mu u^\nu + p\Delta^{\mu\nu} + \Pi^{\mu\nu}$$

Ideal fluid (inviscid) part

Dissipative part

Spatial projector

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

Flow velocity

$$u_\mu u^\mu = -1$$

General decomposition: $\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu}\Pi$

How does one determine $\pi^{\mu\nu}$ and Π ???

Remember that at strong coupling, our intuition built up by studying the relaxation to equilibrium via the Boltzmann equation cannot be trusted.

Yet, we can make the following simplifications:

- The system is close to equilibrium.

- No other degrees of freedom besides energy density, flow and $\Pi^{\mu\nu}$ are relevant for the low frequency, long wavelength phenomena.

- Within an effective theory approach to this problem, it makes sense to try an expansion in spacetime gradients.

The strategy: Consider ε , u^{μ} as fundamental quantities and

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu}\Pi = \Pi^{\mu\nu}(\nabla\varepsilon, \nabla u^{\lambda})$$

Expansion to second order in gradients (Romatschke, 2009):

$$\begin{aligned}\pi^{\mu\nu} =& -\eta\sigma^{\mu\nu} + \eta\tau_{\pi}\left[\langle D\sigma^{\mu\nu}\rangle + \frac{\nabla\cdot u}{3}\sigma^{\mu\nu}\right] + \kappa\left[R^{<\mu\nu>} - 2u_{\alpha}u_{\beta}R^{\alpha<\mu\nu>\beta}\right]\\ &+\lambda_1\sigma^{<\mu}{}_{\lambda}\sigma^{\nu>\lambda} + \lambda_2\sigma^{<\mu}{}_{\lambda}\Omega^{\nu>\lambda} + \lambda_3\Omega^{<\mu}{}_{\lambda}\Omega^{\nu>\lambda}\\ &+\kappa^{*}2u_{\alpha}u_{\beta}R^{\alpha<\mu\nu>\beta} + \eta\tau_{\pi}^{*}\frac{\nabla\cdot u}{3}\sigma^{\mu\nu} + \lambda_4\nabla^{<\mu}\ln s\nabla^{\nu>}\ln s\,.\end{aligned}$$

Shear channel

$$\begin{aligned}\Pi =& -\zeta\left(\nabla\cdot u\right) + \zeta\tau_{\Pi}D\left(\nabla\cdot u\right) + \xi_1\sigma^{\mu\nu}\sigma_{\mu\nu} + \xi_2\left(\nabla\cdot u\right)^2\\ &+\xi_3\Omega^{\mu\nu}\Omega_{\mu\nu} + \xi_4\nabla^{\perp}_{\mu}\ln s\nabla^{\mu}_{\perp}\ln s + \xi_5 R + \xi_6 u^{\alpha}u^{\beta}R_{\alpha\beta}\end{aligned}$$

Bulk channel

17 transport coefficients – 5 constraints = 12 unknown functions of T.

The 12 coefficients are determined by the underlying microscopic theory.

This is interesting because the equations of motion are is not sensitive to the strength of the coupling.

However, this approach the way it is presented is not very useful in a relativistic theory.

It has been known since the work of Israel and Stewart (1978) that causality (and stability) requires the dissipative tensor to have its own differential equation.

Dissipative currents must be treated as independent dynamical variables. For instance, for shear

$$\tau_1\,\dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} = 2\,\eta\,\sigma^{\mu\nu} + \ldots$$

This makes sense because flow and energy density disturbances cannot propagate instantaneously through the system (this is also well known in non-relativistic systems).

What was done before (Baier, Romatschke, Son, Starinets, Stephanov, 2007):

Use the asymptotic formulas derived from the gradient expansion to obtain the equation of motion obeyed by the dissipative tensor.

Basically: From an asymptotic solution $x(t) = F(t) - c\,\dot{F}(t) + \ldots$

$$c\,\dot{F}(t) + x(t) = F(t) + \cdots \to c\,\dot{x}(t) + x(t) = F(t) + \ldots$$

find a differential equation

- It is clear, however, that this process in general does not lead to the correct equation of motion for the dynamical variable (from the asymptotic solution one cannot know about the previous transient phenomena).

- However, this is fine if the timescales associated with transient phenonema are too short and the actual differential equation does not matter (in this case it is OK to have the wrong diff. equation since you only care about asymptotics).

- In the QGP, since these timescales are not known, I see no reason to believe that this asymptotic "triviality" has been reached.

- Also, given that the initial conditions fluctuate event by event, it does not seem to be reasonable to neglect this intrinsic transient phenomena.

- Moreover, there are observables that are sensitive to short time dynamics (jets, photons) and they could be affected by all of this.

- In fact, it should be clear that the only way (in terms of dynamics) in which strongly coupled fluids can be different than their weakly coupled counterparts is in the transient regime where the gradients are still not entirely negligible.

- Within the Boltzmann equation (hence weak coupling), the leading order equations of motion for the dissipative currents have been derived (see Denicol et al, 2012).

- Their analysis cannot be applied in the case of a strongly coupled QGP.

- It should be clear after this discussion that all the phenomenological studies involving the hydrodynamic behavior of the QGP relied on the unproven hypothesis that the transient hydrodynamic equations of motion found at weak coupling are also valid for a strongly coupled plasma. This hypothesis must be checked.

- Lattice gauge theory is not the best tool to investigate this issue.

- One needs a method that is better suited to deal with out-of-equilibrium phenomena in strongly coupled gauge theories.

- The gauge/gravity correspondence (holography) has provided important insights in the last years into the dynamical properties of strongly coupled non-Abelian fluids.

- It makes sense to see what this approach has to say about this problem.

What is Holography ???

Overview

Holography is a duality between QFT and gravity

Maps quantum many body physics to classical dynamics of black hole horizons in one higher dimension

Replaces quasiparticles with geometry as the effective d.o.f.

When QFT is strongly coupled, new weakly coupled d.o.f. in the gravity theory emerge.

Emergent fields in the theory of gravity live in a dynamical spacetime with an extra dimension.

This extra dimension plays the role of an energy scale in the QFT with the motion along the extra dimension providing a geometric representation of the QFT's renormalization group (RG) flow.

Quantum many body physics problems, such as thermodynamics and transport phenomena, become equivalent to problems in classical gravity.

Universal black hole phenomena are mapped into universal behavior in QFT's

HOLOGRAPHY

When is Holography applicable ????

Holography is under control when:

I) The coupling of the QFT, say, λ , is $\lambda \gg 1$

II) The number of d.o.f. /volume, ~ N, is very large, i.e., N >> 1.

$\lambda \gg 1$
d-dimensional QFT
with d.of./vol $\rightarrow \infty$

$\Rightarrow$ **HOLOGRAPHY** $\Rightarrow$

$\sim 1/\lambda \ll 1$
d+1-dimensional
classical gravity

- A well defined gauge theory/gravity dictionary exists that allows one to determine any gauge invariant quantity at strong coupling from gravity.

The Holographic Dictionary

The “Rosetta” stone

(Gubser, Klebanov, Polyakov, Witten, 1998)

Mathematical Definition of the Holographic Duality

$$\boxed{Z_{\text{QFT}}[J_i] \;=\; Z_{\text{QG}}[\Phi[J_i]]}$$

$Z_{\text{QFT}}[J_i]$ Partition function of the QFT as a function of the sources

$Z_{\text{QG}}[\Phi[J_i]]$ Partition function of the gravitational theory in AdS

The bulk fields play the role of the coupling constants of the QFT that are now promoted to dynamical fields on the higher dimensional spacetime where the extra dimension plays the role of the RG scale.

Therefore, by classifying the different operators in the d-dimensional theory according to their Lorentz structure we see that

d-dimensional theory			d+1-dimensional theory	
Scalar operator	$\mathcal{S}(x)$	$\Rightarrow$	$\Phi(x, r)$	Bulk scalar field
Current operator	$J^{\mu}(x)$	$\Rightarrow$	$A_M(x, r)$	Bulk spin 1 field
Tensor operator	$T^{\mu\nu}$	$\Rightarrow$	$g_{MN}(x, r)$	Bulk spin 2 field

Diffeomorphism in the bulk → Energy-momentum conservation in the gauge theory.

Gauge invariance in the bulk → Global charge conservation in the gauge theory.

So, what do we need then?

- List of local operators in the CFT $\mathcal{O}_i$ labeled by their Lorentz structure, their charges, and UV scaling dimensions Δ_i defined at the fixed point.

- To generate the RG flow of interest, we perturb the system away from this fixed point by turning on appropriate sources J_i with which we construct the generating functional

$$Z_{\text{QFT}}[J_i(x)] = \langle e^{\int dx^d J_i(x)\mathcal{O}_i(x)} \rangle \Longrightarrow \langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \frac{\delta^n \ln Z_{\text{QFT}}[J(x)]}{\delta J_1(x_1)\ldots\delta J_n(x_n)}\Big|_{J_i=0}$$

- All the information needed to determine how energy-momentum disturbances in the strongly coupled gauge theory evolve towards equilibrium must be contained in the metric (on-shell solution of Einstein's equations with a black hole).

- Therefore, it must be possible to use this quantity to determine what the equations of motion of the fluid are.

- It is now known how to obtain the gradient expansion from gravity. The general idea is called Fluid/Gravity correspondence and has been developed by several people in the last 5 years.

- Basically, they perform an expansion in gradients in the 5d metric, which is mapped into the gradient expansion of the dissipative tensor in the gauge theory. The nice thing about this method is that it gives you all the transport coefficients of the gradient expansion directly from gravity.

- However, as I mentioned before, the asymptotic equations given by the gradient expansion are not very useful in relativistic hydrodynamics simulations ...

- It is necessary to generalize the analysis of Minwalla et al. (fluid/gravity duality) to find a way to determine, directly from gravity, what are the equations of motion obeyed by the dissipative tensor in strongly coupled fluids.

- This is an open problem. This means that we do not know, for instance, what is the relationship between relaxation time coefficients and shear and bulk viscosities in a strongly coupled fluid.

- Are these equations the same as the relaxation time equations currently used in hydrodynamical calculations?

- I will give now a simple example that suggests that this is not the case.

- The shear viscosity transport coefficient can be computed via linear response using the Kubo formula

Retarded correlator

$$G_R^{\mu\nu\alpha\beta}(x-x') = i\,\theta(x-x')\langle[\hat{T}^{\mu\nu}(x), \hat{T}^{\alpha\beta}(x')]\rangle_T$$

$$\boxed{\eta = -\frac{\mathrm{Im}G_R^{xyxy}(\omega, \mathbf{0})}{\omega}\Big|_{\omega=0}}$$

- Note that this correlator is defined in Minkowski space.

- Lattice can reliably compute the Euclidean version of this correlator.

- Analytical continuation for this is tricky (also w->0 limit required is hard).

Let us now focus on the spectral density associated with the (xy, xy) channel of the energy-momentum correlator.

$$\rho(\omega) = -\mathrm{Im}\, G_R^{xy,xy}(\omega)$$

I will show you that gravity calculations give very different results for this quantity in comparison to what is expected from relaxation equations defined at weak coupling.

Linear Response via Metric Disturbances

G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011

Metric disturbances around Minkowski

$$g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$$

Gravitational wave

Fluid in equilibrium

Variation of the energy-momentum tensor

$$\delta T^{\mu\nu} = T^{\mu\nu}(\eta^{\alpha\beta} + h^{\alpha\beta}) - T^{\mu\nu}(\eta^{\alpha\beta})$$

Linear response theory (a la Kubo)

$$\delta T^{\mu\nu}(x) = \frac{1}{2} \int d^4x' \, G_R^{\mu\nu\alpha\beta}(x - x') \, h_{\alpha\beta}(x')$$

Retarded 2-point function

$$G_R^{\mu\nu\alpha\beta}(x - x') = i\,\theta(x - x') \langle [\hat{T}^{\mu\nu}(x), \hat{T}^{\alpha\beta}(x')] \rangle_T$$

Application: Linear Response via Metric Disturbances

G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011

A very specific disturbance is chosen where only $h_{xy}(t,z) \neq 0$ so then the equations decouple ...

$$\delta T^{xy}(t,z) = \int dt'\, dz'\, G_R^{xyxy}(t-t', z-z')\, h_{xy}(t',z')$$

The traditional form for the energy-momentum tensor is used

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}$$

To linear order in the metric disturbances we obtain …

$$\delta\pi^{xy}(t,z) = P_0\, h^{xy}(t,z) + \int dt'\, dz'\, G_R^{xyxy}(t-t', z-z')\, h_{xy}(t',z')$$

If the Green's function first pole is **purely imaginary** then the equation of motion for the dissipative current is

$$\tau_\pi \partial_t \delta\pi^{xy} + \delta\pi^{xy} = D_0 h_{xy} + D_1 \partial_t h_{xy} + D_2 \partial_t^2 h_{xy} + \mathcal{O}\left(\partial_t^3 h_{xy}, \partial_z^2 h_{xy}\right)$$

G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011

And the coefficients are

Relaxation time given by the first pole !!!

$$\tau_\pi = \frac{1}{i\omega_1(\mathbf{0})},$$

$$D_0 = \tilde{G}_R(\omega, \mathbf{0})\Big|_{\omega=0} = -P_0 + \tilde{G}_R^{xyxy}(\omega, \mathbf{0})\Big|_{\omega=0} \equiv -P_0 + P_0 = 0\,,$$

$$D_1 = i\partial_\omega \tilde{G}_R(\omega, \mathbf{0})\Big|_{\omega=0} + \tau_\pi D_0 = i\partial_\omega \tilde{G}_R(\omega, \mathbf{0})\Big|_{\omega=0} \equiv \eta\,,$$

Kubo formula

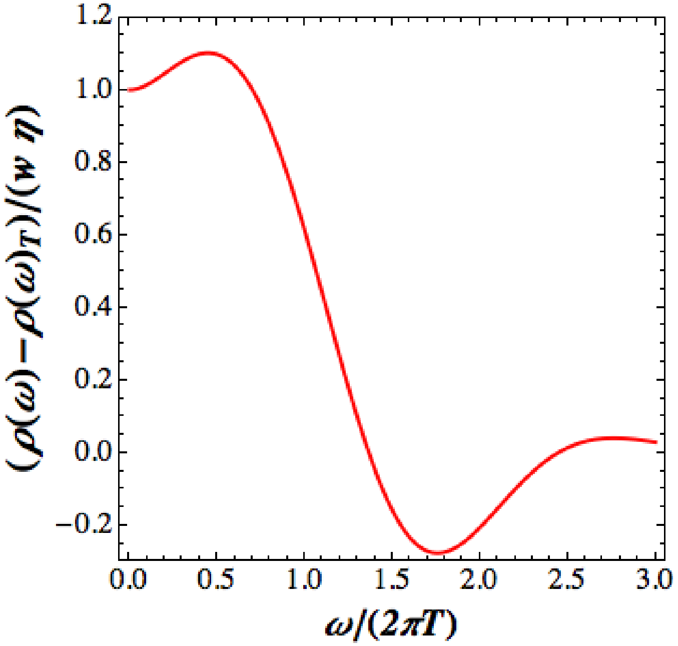
$$D_2 = -\frac{1}{2}\partial_\omega^2 \tilde{G}_R(\omega, \mathbf{0})\Big|_{\omega=0} + D_1\tau_\pi - D_0\tau_\pi^2 \equiv -\frac{1}{2}\partial_\omega^2 \tilde{G}_R(\omega, \mathbf{0})\Big|_{\omega=0} + \eta\tau_\pi$$

One finds for a relaxation time hydro theory: $$\frac{\rho(\omega)}{\omega\eta} = \frac{1}{1+\omega^2\tau_\pi^2}$$

This is the case for the low frequency limit of the Boltzmann equation.

At strong coupling, it is well known that

Zero temperature subtracted spectral function in strongly coupled N=4 SYM



Non-conformal theories display similar behavior (see T. Springer et al. 2010)

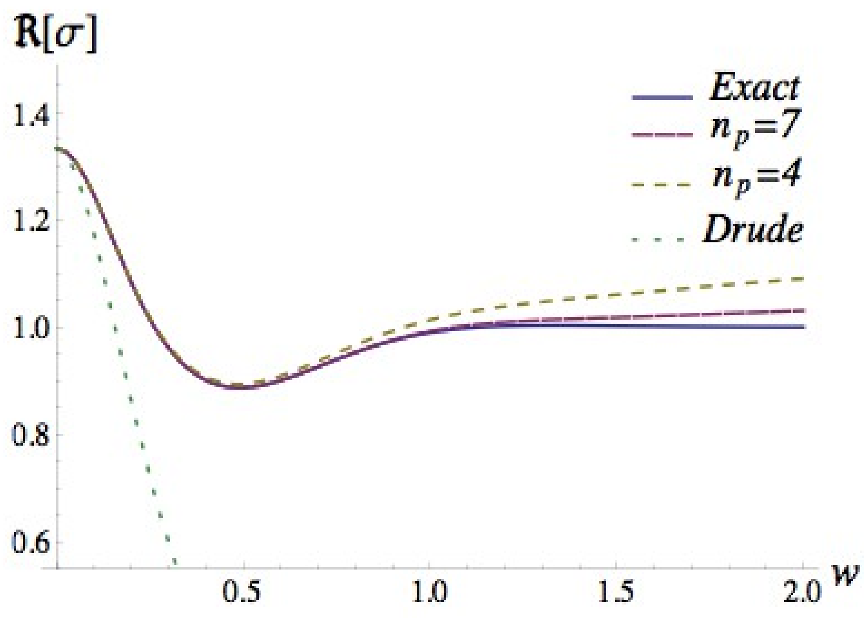
The fact that the effective theory at low frequencies at strong coupling is different from the relaxation time-like equations in kinetic theory is well known in other areas – condensed matter physics!

Conductivity of a 2+1 CFT

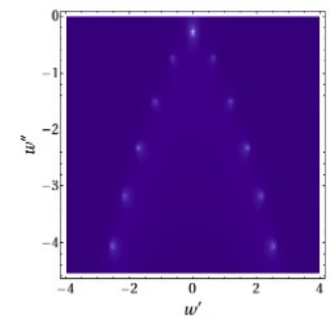
$$S_{\rm bulk} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right) - \frac{1}{4g_4^2}F_{ab}F^{ab} + \gamma\frac{L^2}{g_4^2}C_{abcd}F^{ab}F^{cd}\right]$$

Drude

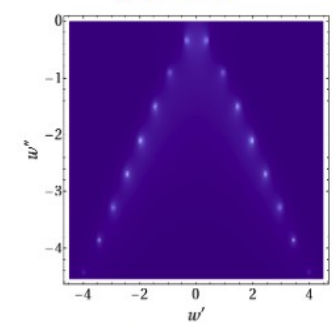
$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$



(a)



(a)$\sigma(\gamma = 1/12)$



(c)$\sigma(\gamma = -1/12)$

Witczak-Krempa, Sachdev, arxiv:1210.4166

- Thus, these oscillations in the spectral density found at strong coupling indicate that the equations of motion for the dissipative shear stress tensor does not follow relaxation time equations (Denicol, JN, 2011).

- So, there are two options:

a) The perfect fluidity of the QGP involves some yet unknown non-perturbative mechanism at weak coupling and, thus, relaxation time equations are OK and the current hydrodynamic models are theoretically justified.

OR

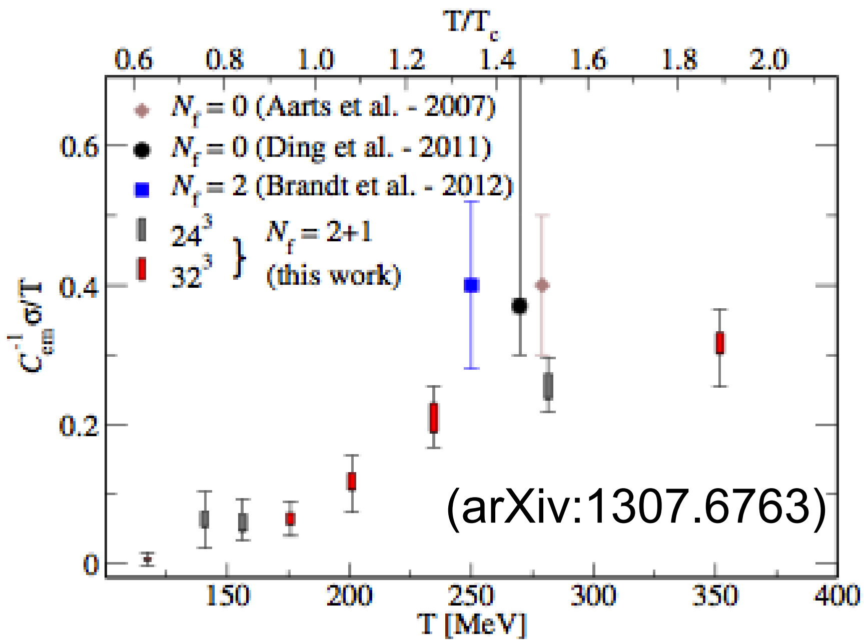
2) Perfect fluidity implies truly strong coupling physics and, thus, at least according to holography we are not even solving the correct equations of motion. Non-asymptotical phenomena are not described by Israel-Stewart like equations.

Now let us talk about a less controversial topic:

$$\boxed{\text{Electric conductivity of the QGP near deconfinement from holography}}$$

Based on Stefano Finazzo, JN, arxiv:1311.6675

We have seen this week the nice recent work from Gert and collaborators on the electric conductivity of the QGP

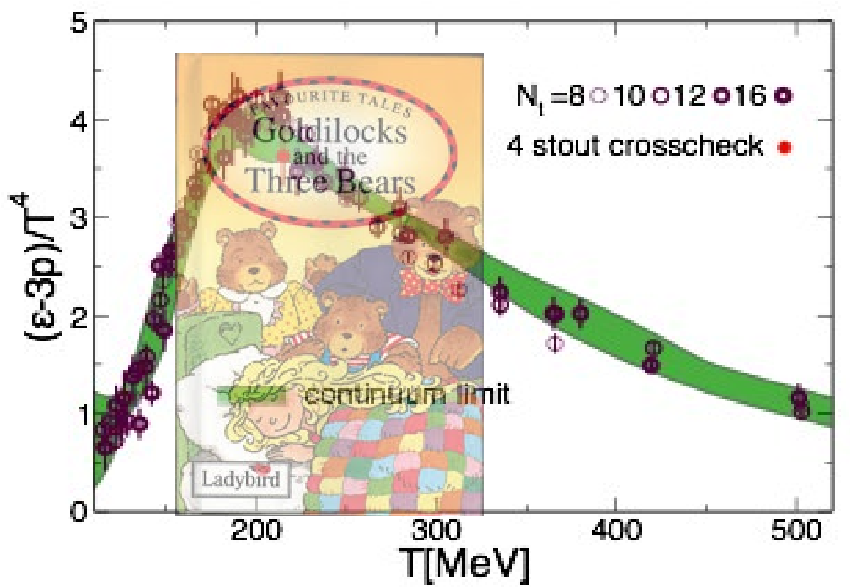


It is interesting to see if holographic models of the QGP display similar temperature behavior (model must be non-conformal).

The interesting part here is the region near “Tc” ~ 150-400 MeV.

The Goldilocks zone for holographic models: Not too hot (where pQCD dominates), not too cold (hadron dominance).

Fodor et al., arxiv:1309.5258



Obviously, the QGP is strongly non-conformal in the near Tc region.

- We need a holographic model for the QGP near Tc.

- Non-conformal models involving a dynamical scalar field are ideal for this task since it is possible to engineer a black brane that has the same thermodynamic properties of the QGP near Tc.

- Such models have been used in the past to compute several different quantities ranging from hydrodynamic spectral functions to jet energy loss.

- In the rest of this talk I will show you how to construct a holographic model that is tuned to reproduce lattice data for thermodynamics and the electric charge susceptibility and use this model to estimate the behavior of the electric charge transport properties of the QGP.

Gubser et al., 2008
Kiritsis et al, 2008
Noronha, 2009

Minimal extension of the good and old gravity setup (bottom-up approach)

Scalar potential

$$V(\phi)$$

$$S_{ES} = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(\mathcal{R} - \frac{(\partial\phi)^2}{2} - V(\phi)\right)$$

Nontrivial fields in the 5d bulk: g_{MN}, ϕ

Dual to a relevant deformation of a 4d CFT $\mathcal{L}_{CFT} + \Lambda_\phi^{4-\Delta}\mathcal{O}_\phi$

Here Λ_ϕ is the energy scale of the deformation and Δ is the dimension of $\mathcal{O}_\phi$ in the boundary, which is dual to ϕ in the bulk.

General assumptions:

Gubser et al. 2008
Noronha, 2009.

- Relevant deformation (important in the IR) $\Delta < 4$

- Spacetime is asymptotically AdS_5 with radius R

$$\lim_{\phi\to 0} V(\phi) = -\frac{12}{R^2} + \frac{1}{2R^2}\Delta(\Delta - 4)\phi^2 + \mathcal{O}(\phi^4)$$

- Breitenlohner-Freedman bound $1 < \Delta < 4$ $m_\phi^2 < 0$

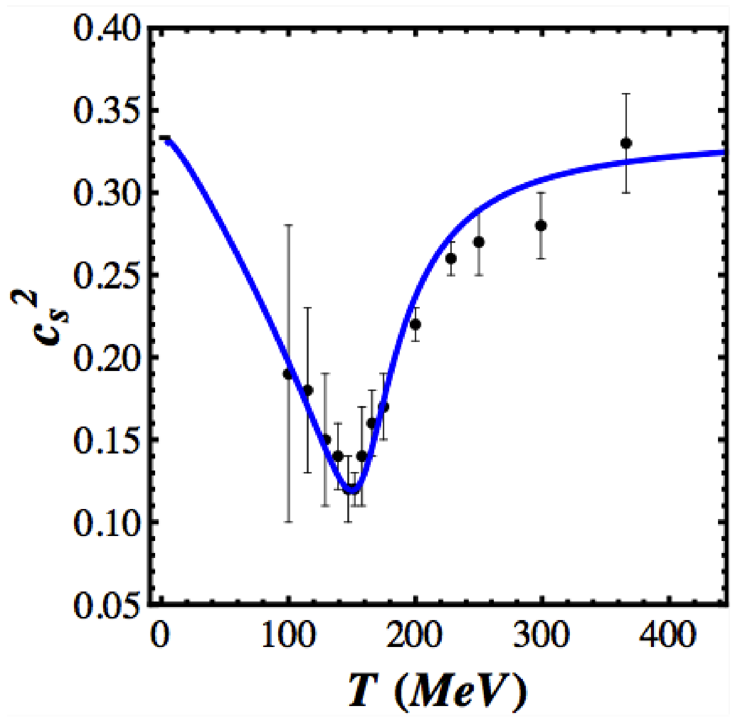
- Gauge theory is conformal in the UV $E \gg \Lambda_\phi$ (not asymptotically free)

The theory has a nontrivial UV fixed point.

A simple choice of parameters in the potential leads to a reasonable fit to lattice data from Fodor et al, 2010.

$$V(\phi) = -12\cosh\gamma\phi + b_2\phi^2 + b_4\phi^4 + b_6\phi^6$$

where $\gamma = 0.606,\ b_2 = 0.703,\ b_4 = -0.12,\ b_6 = 0.0044$



The metric and the scalar field define the background, i.e., the thermodynamical properties of the plasma at zero chemical potential.

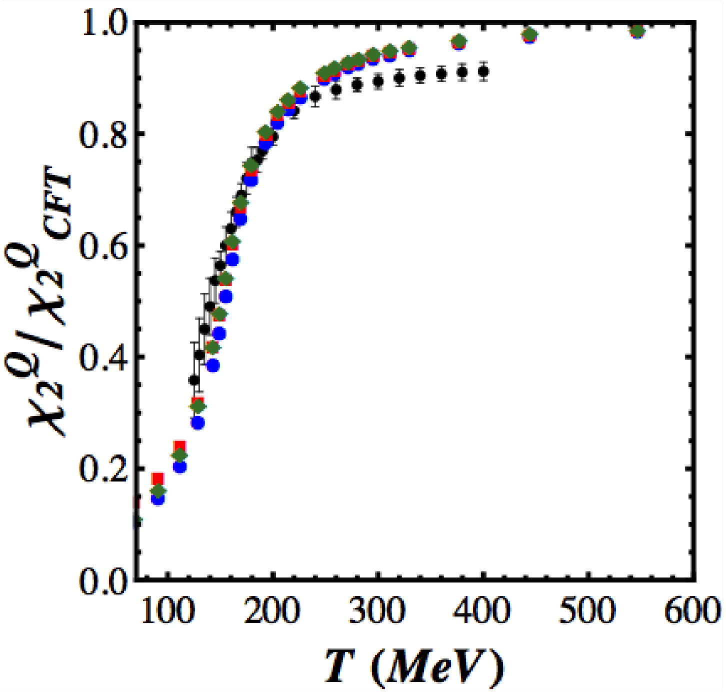
Processes involving electric charge conservation require, according to the duality dictionary, the presence of an Abelian gauge field in the bulk.

$$S_M = -\frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\,\frac{f(\phi)}{4}F_{MN}F^{MN}$$

Function $f(\phi)$, a priori unknown in this effective model, can be fixed

by matching the lattice data for the charge susceptibility.

Charge susceptibility



Lattice data from Fodor et. al. 2012

One-parameter model choices

$$f_1(\phi) = \frac{\operatorname{sech}(a_1\,\phi)}{g_{5,1}^2},$$

$$f_2(\phi) = \frac{1}{g_{5,2}^2}\,\frac{1}{(\phi^2 + a_2^2)}$$

$$f_3(\phi) = \frac{e^{-a_3^2\phi^2}}{g_{5,3}^2},$$

Decent description of temperature behavior of this equilibrium quantity near Tc for all model choices.

Now, everything is fixed. The model can be used to compute the charge transport properties.

Retarded Green's function of the current

$$G_R^{ij}(k) = -i \int d^4x \, e^{-ik\cdot x} \theta(t) \left\langle \left[\hat{J}^i(t, \mathbf{x}), \hat{J}^j(0, \mathbf{0}) \right] \right\rangle_T$$

DC conductivity

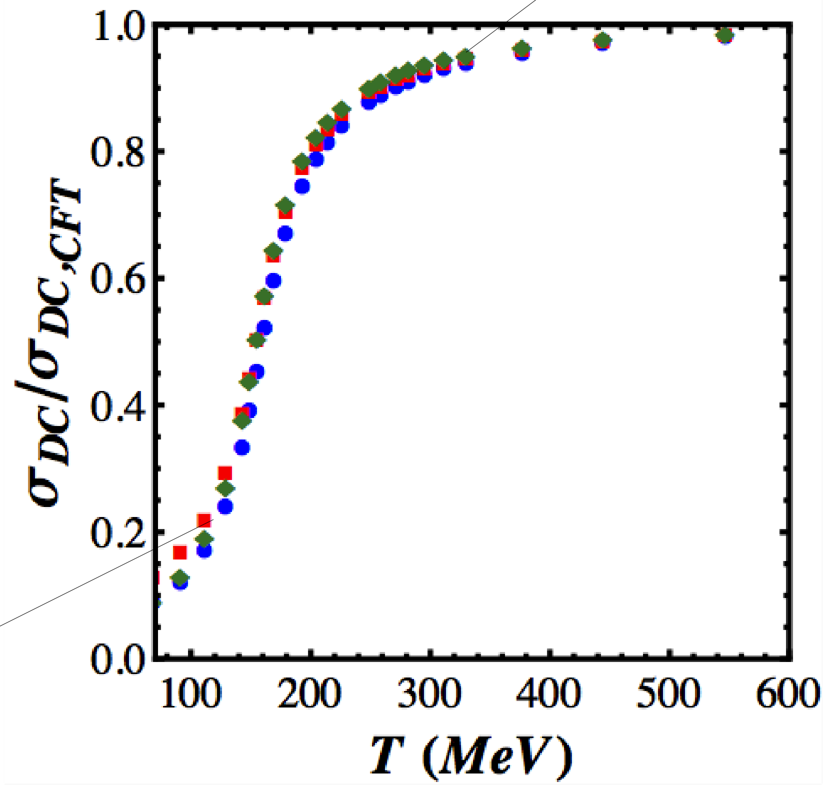
AC conductivity: $\sigma^{ij}(\omega) = -\frac{G_R^{ij}(\omega, \mathbf{k} = 0)}{i\omega}$;

$$\sigma_{DC} = \lim_{\omega \to 0} \sigma(\omega)$$

Isotropic medium

Maximal conductivity reached above 300 MeV

DC conductivity:



Lousy conductor

Note here that, differently than the CFT limit of weak coupling QCD, the holographically computed conductivity/T of a CFT is finite.

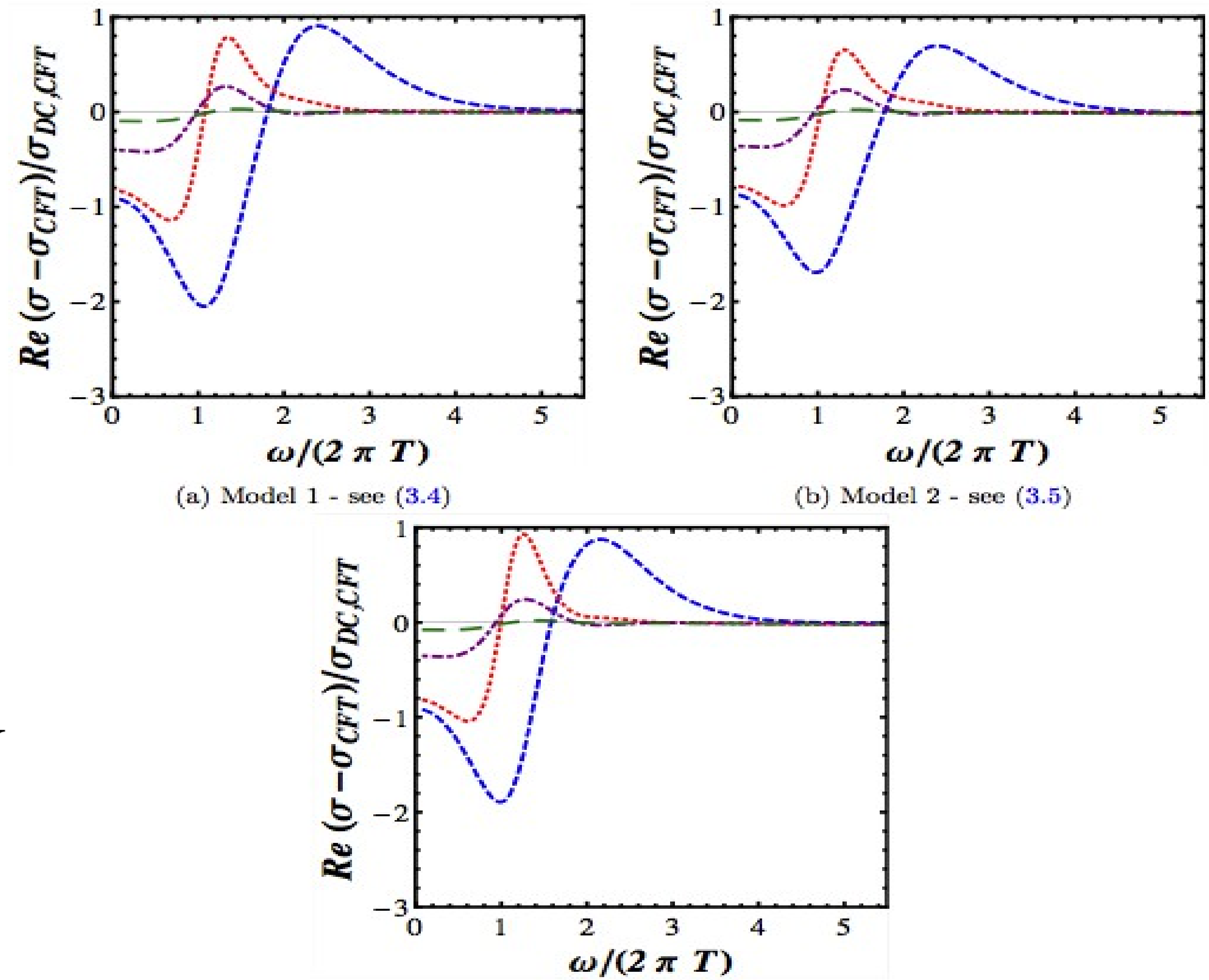
T/Tc = 0.45

T/Tc=0.74

T/Tc=1.13

T/Tc=1.81

$$T_c \equiv 150\,\mathrm{MeV}$$



(a) Model 1 - see (3.4)

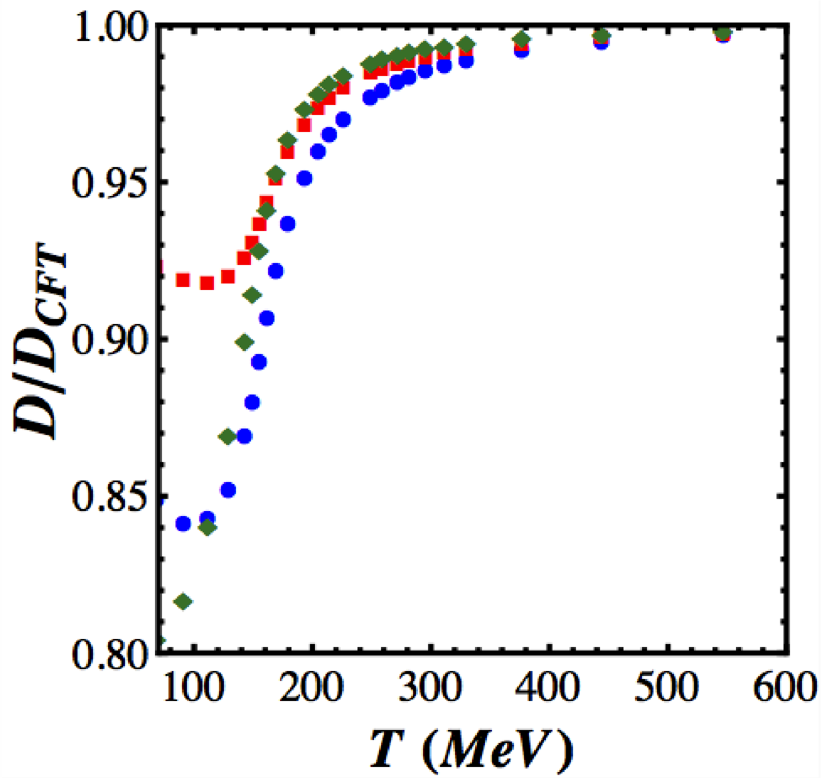
(b) Model 2 - see (3.5)

Interesting frequency dependence near Tc.

Electric charge diffusion:

Einstein relation is valid:

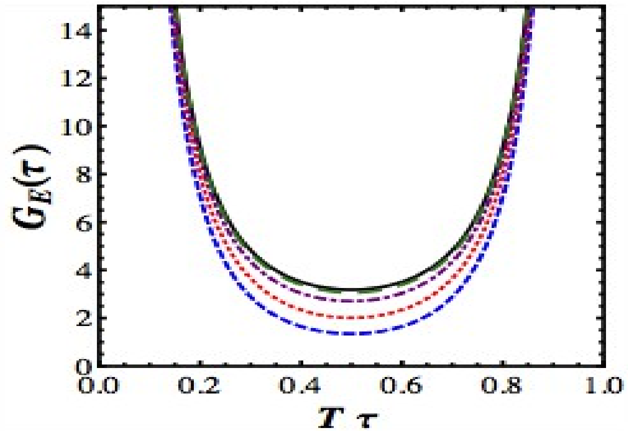
$$D = \frac{\sigma_{DC}}{\chi_2^Q}$$



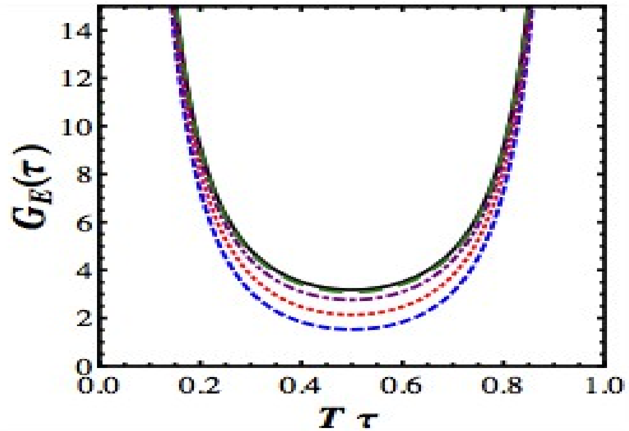
In this case there is a dependence at low T on the choice of $f(\phi)$

Euclidean correlator:

$$G_E(T\tau) = \int_0^\infty d\omega\, \rho(\omega) \frac{\cosh\left[\omega\left(T\tau - \frac{1}{2}\right)/T\right]}{\sinh(\omega/2T)}$$



(a) Model 1 - see (3.4)



(b) Model 2 - see (3.5)

T/Tc = 0.45

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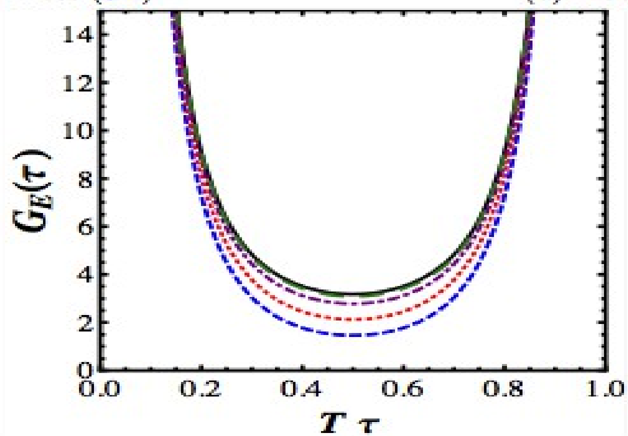

(c) Model 3 - see (3.6)

Structures vanished.

Downward shift gives the conductivity.

$$T_c \equiv 150\,\mathrm{MeV}$$

Conclusions and Outlook

- If the QGP is truly strongly coupled, the coarse-grained equations of motion may actually be very different than the currently relaxation time equations used in simulations. **No quasiparticles → not obvious if Israel-Stewart-like hydro is applicable**.

- It should be possible to figure out the appropriate equations of motion of a strongly coupled fluid from holography. Current holographic methods, which lead to the gradient expansion, must be generalized.

- Non-conformal models of the QGP (in the “Goldilocks” zone) may be a good tool to understand the near Tc transport properties of the QGP involving conserved currents. Calculations of photon production near Tc are being currently pursued.

EXTRA SLIDES

At the fixed point then the geometry in the bulk theory should be very special to encode all the symmetries present in the QFT conformal group.

Indeed, the bulk geometry that describes a 4d CFT state must have:

- At least 5 dimensions (where “r” is the extra holographic coordinate).
- 4d translations, rotations, and boosts as isometries for each “r” slice.
- Rotations and boosts that mix up “r” with the other coordinates.
- Scaling transformations

$$(r, \vec{x}, t) \to (\alpha r, \alpha \vec{x}, \alpha t)$$

The only bulk geometry that fulfills all of these requirements is AdS5.

Maldacena, 1998

AdS5: $$ds^2 = \frac{L^2}{r^2}\left[-dt^2 + d\vec{x}^2 + dr^2\right]$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu}$$

Einstein's equations

$$\Lambda = -\frac{d(d-2)}{2L^2}$$

Negative cosmological constant

$$L$$

AdS radius

$$R \sim \frac{1}{L^2}$$

Ricci scalar

Is there a gauge theory in d = 3+1 which is conformally invariant (after quantization)?

YES!!!! $\boxed{\mathcal{N} = 4 \quad \text{SU(Nc) Supersymmetric Yang-Mills}}$

- 16 supercharges + extra 16 due to conformal invariance.
- SU(4) R-symmetry (rotates the scalars and the fermions).
- Global SO(6) symmetry.
- Field content: massless

A_μ^a	ψ	ϕ^I	$I = 1, \ldots, 6$
1 Gauge boson	4 fermions	6 Scalars	

All in the adjoint representation of SU(Nc)

$$\mathcal{L} = \frac{1}{g^2} Tr \left[F^2 + (D\phi)^2 + \bar{\psi} \not{D} \psi + \sum_{I,J} \left(\phi^I \phi^J\right)^2 + \bar{\psi} \Gamma^I \phi^I \psi \right]$$

The cool thing about this theory is that it comes up quite naturally in string theory …

Black holes in AdS

- If pure empty AdS gives the ground state for a CFT in the vacuum, finite temperature CFTs should correspond to asymptotically AdS spacetimes with black holes (black branes – translational invariant in “x” and “t” but not in “r”).

The simplest asymptotically AdS-Schwarzchild d+1-dimensional, black brane is given by

$$ds^2 = \frac{L^2}{r^2}\left[-f(r)\,dt^2 + d\vec{x}^2 + \frac{1}{f(r)}dr^2\right]$$

Also solution of

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\Lambda g_{\mu\nu}$$

Blackening factor: $f(r) = 1 - \frac{r^d}{r_H^d}$.

Horizon $r_H \to f(r_H) = 0$

boundary is at $r \to 0$ and $\lim_{r\to 0} f(r) = 1$