Exploring the phase structure and dynamics of QCD

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Universität Heidelberg & ExtreMe Matter Institute

Kyoto, December 5th 2013

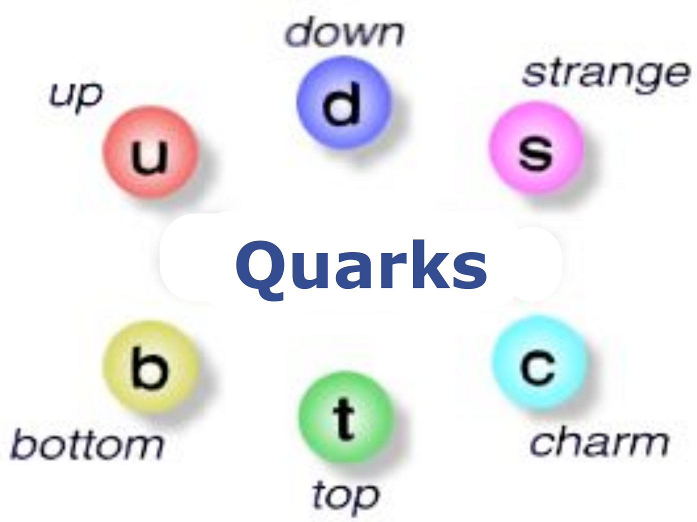
Outline

- **Phase structure & thermodynamics**

- **Transport coefficients**

- **Outlook**

Functional Methods for QCD



Gluons

FunMethods: FRG-DSE-2PI-...

FRG QCD survey

JMP, Aussois '12

Phase diagram survey

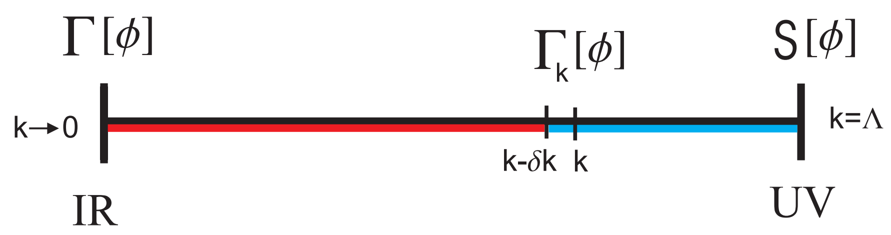
JMP, Schladming '13

Functional Methods for QCD

Functional RG

JMP, AIP Conf.Proc. 1343 (2011)

free energy at momentum scale k



RG-scale k: $t = \ln k$

DSE: see talk of C. Fischer

Functional Methods for QCD

Functional RG

JMP, AIP Conf.Proc. 1343 (2011)

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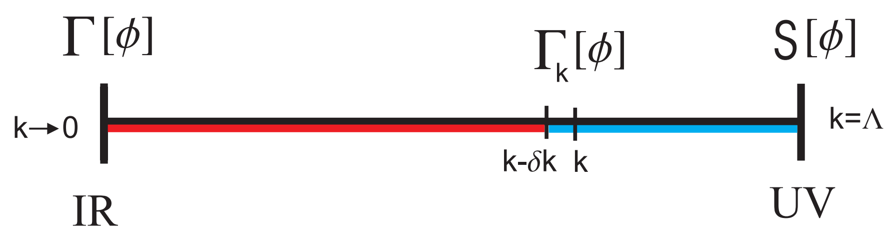
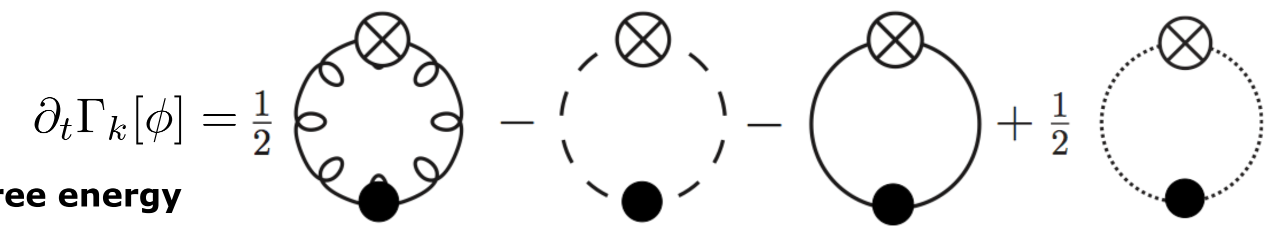


RG-scale k: $t = \ln k$

QCD

glue quantum fluctuations

hadronic quantum fluctuations



free energy

quark quantum fluctuations

Functional Methods for QCD

Functional RG

JMP, AIP Conf.Proc. 1343 (2011)

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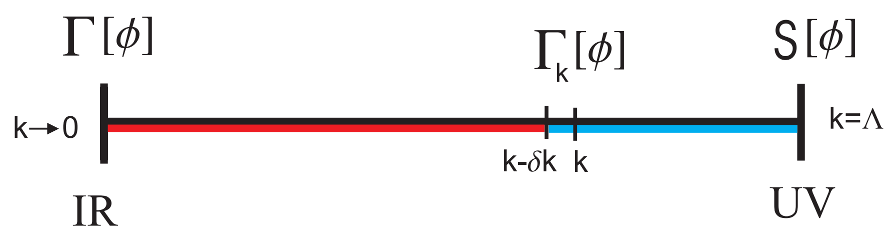

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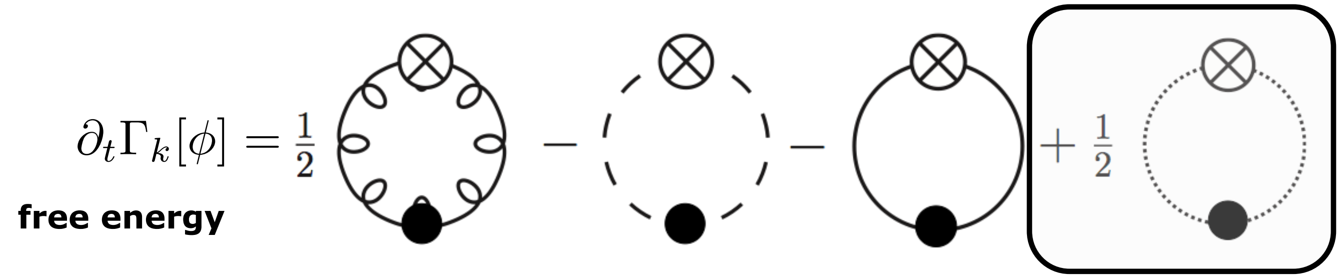
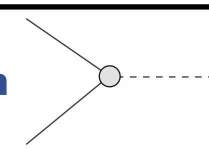


quark quantum fluctuations

Dynamical hadronisation



dynamical

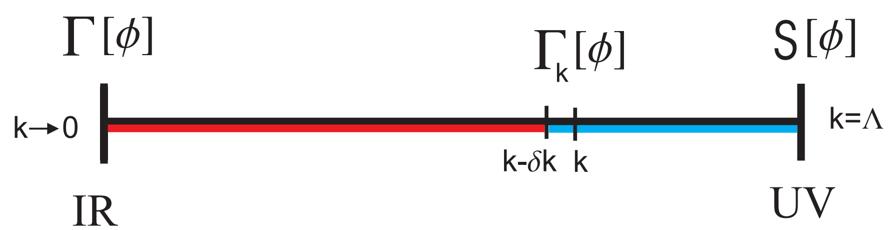
Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

Functional Methods for QCD

Functional RG

JMP, AIP Conf.Proc. 1343 (2011)

free energy at momentum scale k



RG-scale k: $t = \ln k$

QCD

glue quantum fluctuations

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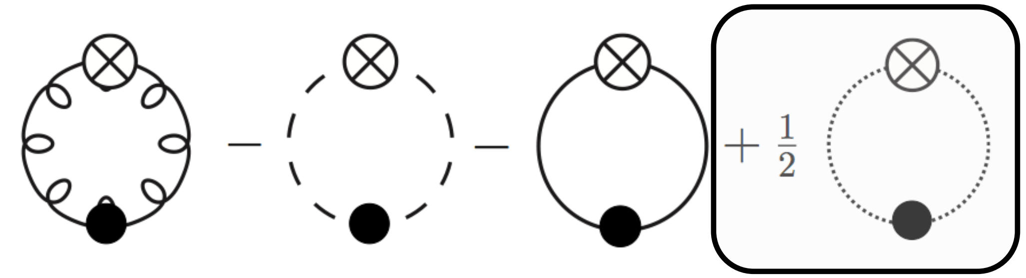
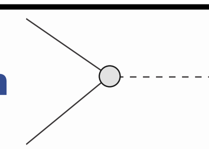
$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$



free energy

quark quantum fluctuations

Dynamical hadronisation



dynamical

Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

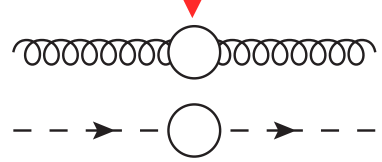
Yang-Mills

$$\partial_t \Gamma_k[A,\bar{c},c] = \frac{1}{2}\mathrm{Tr}\left\{\frac{1}{\Gamma^{(2)}[A,\bar{c},c]+R_k}\,\partial_t R_k\right\} - \partial_t C_k$$

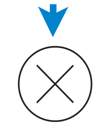
$\partial_t = k\,\partial_k$

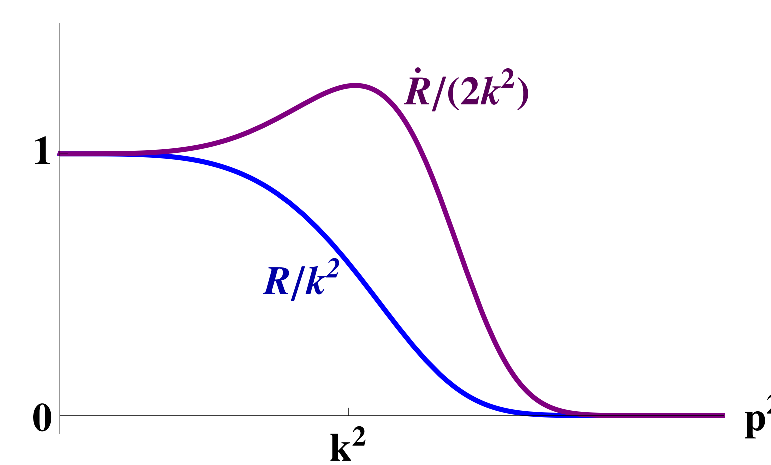
by L. Fister

full



regulator

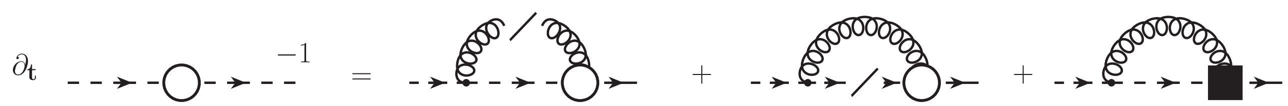




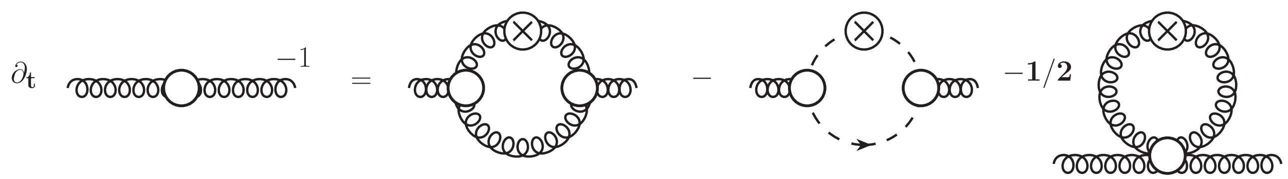
Functional Methods for QCD

Fister, JMP ’11, 13

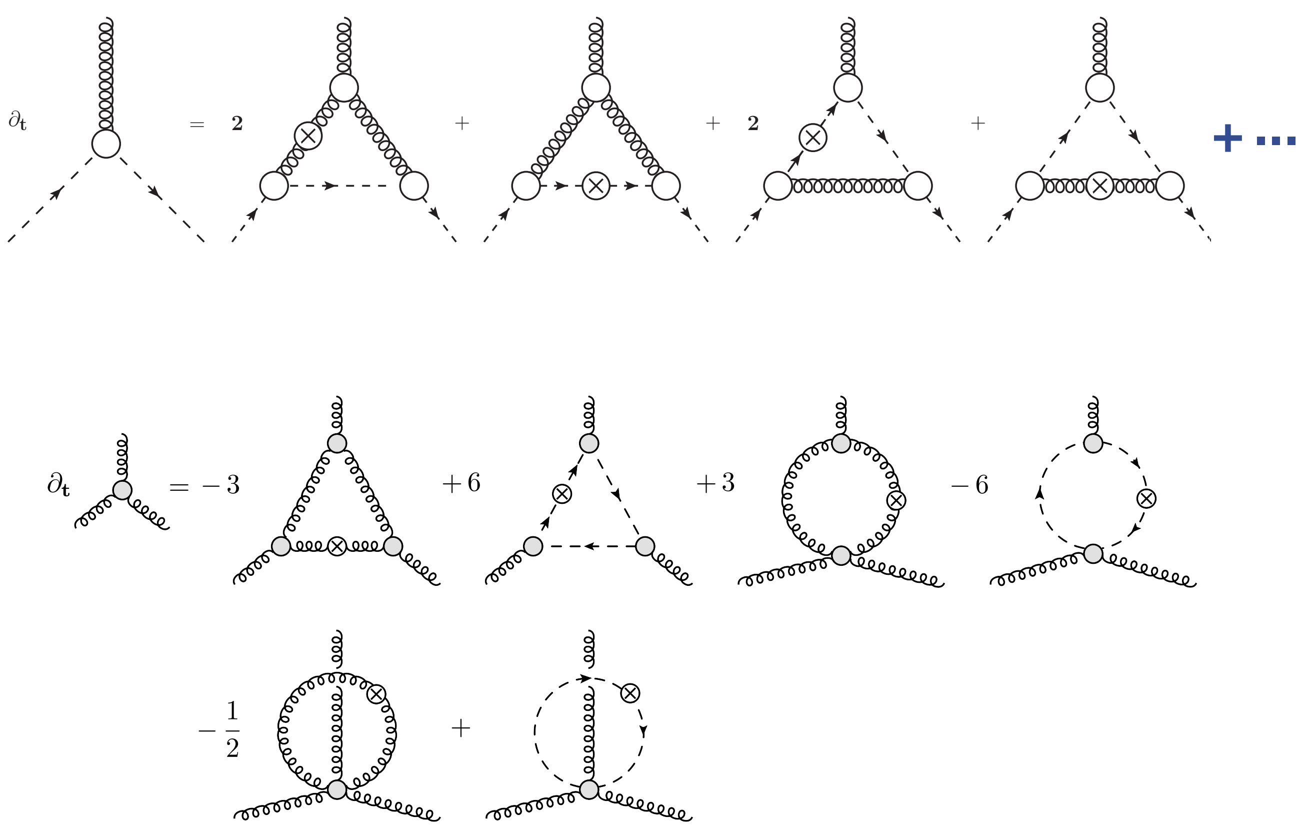
Yang-Mills



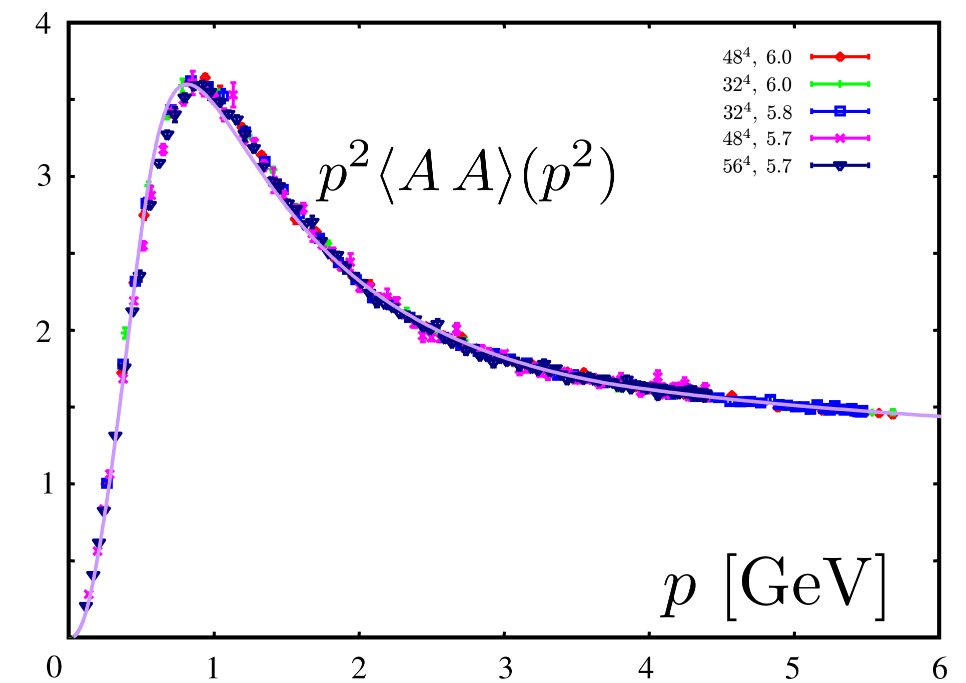
DSE-flow



2PI-resummation



Yang-Mills propagators



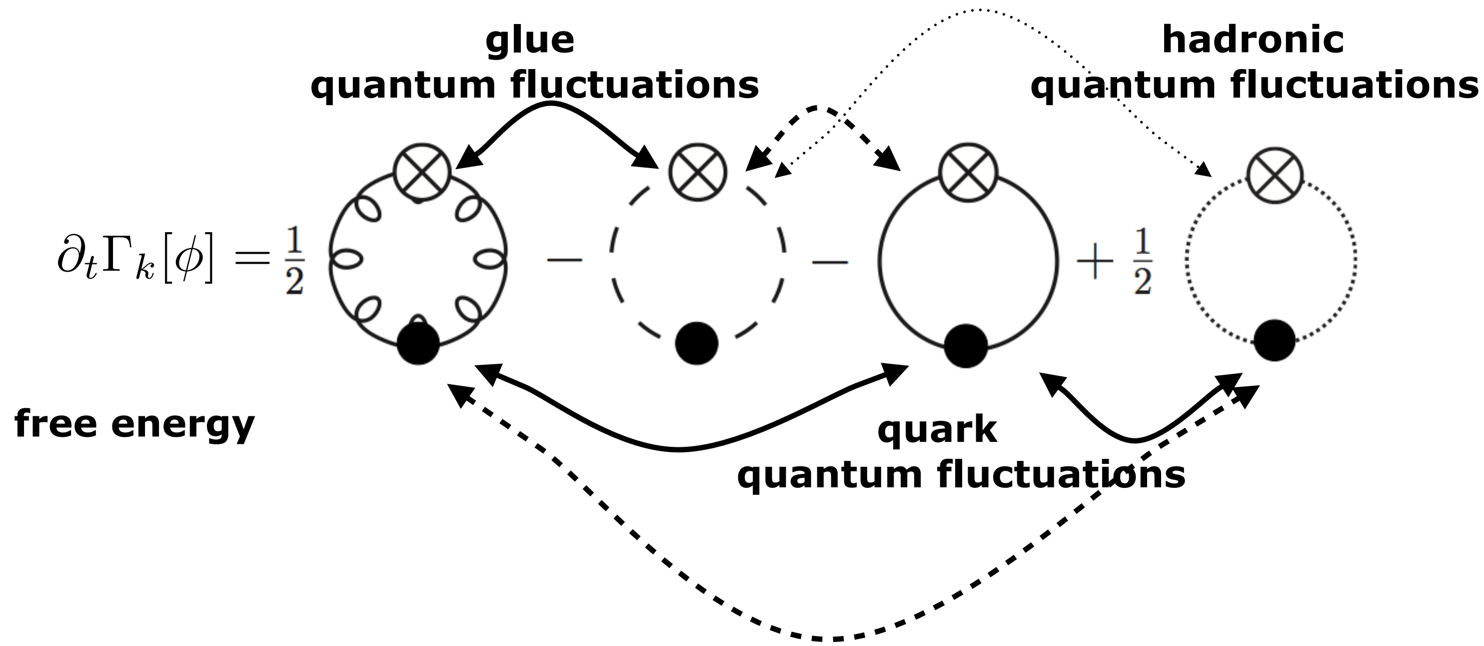
FRG: Fischer, Maas, JMP ’08

lattice: Sternbeck et al. ’06

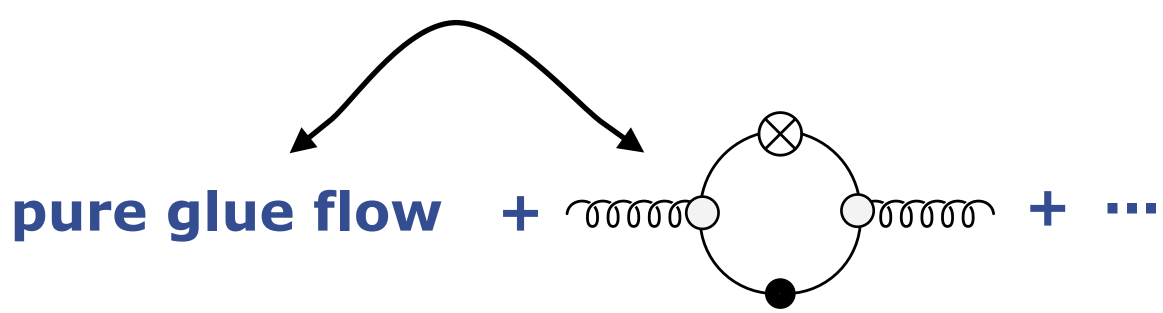
Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

QCD

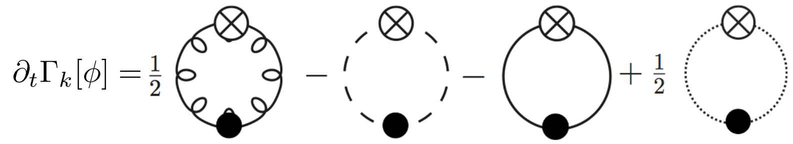


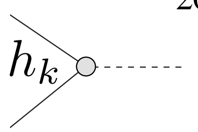
flow of gluon propagator

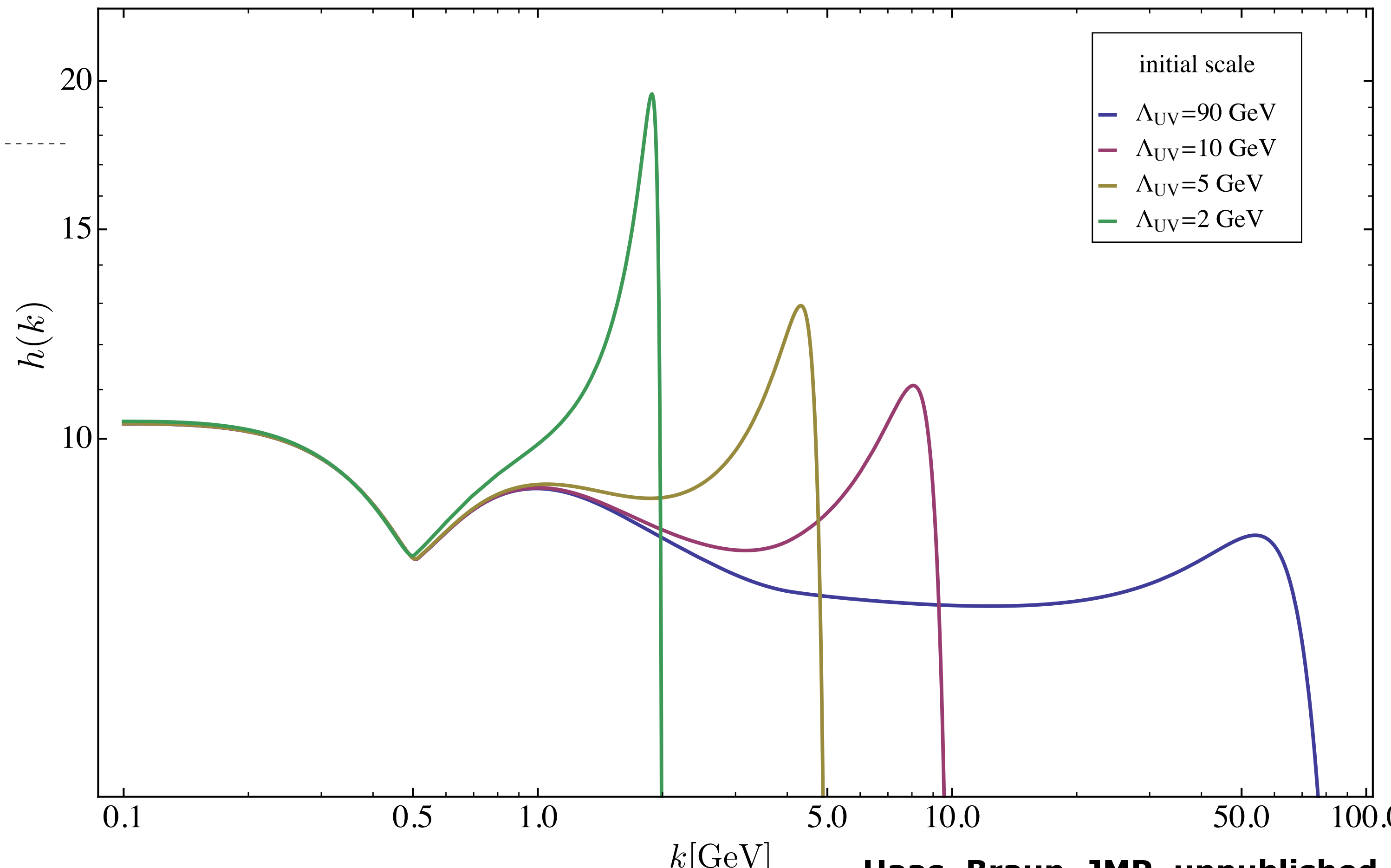


Naturally encorporates PQM/PNJL models as specific low order trunations

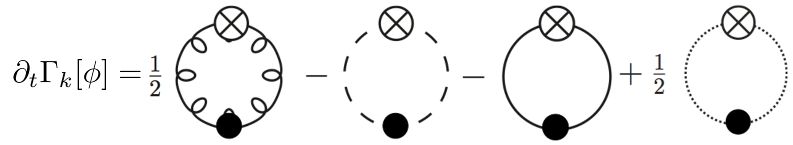
QCD



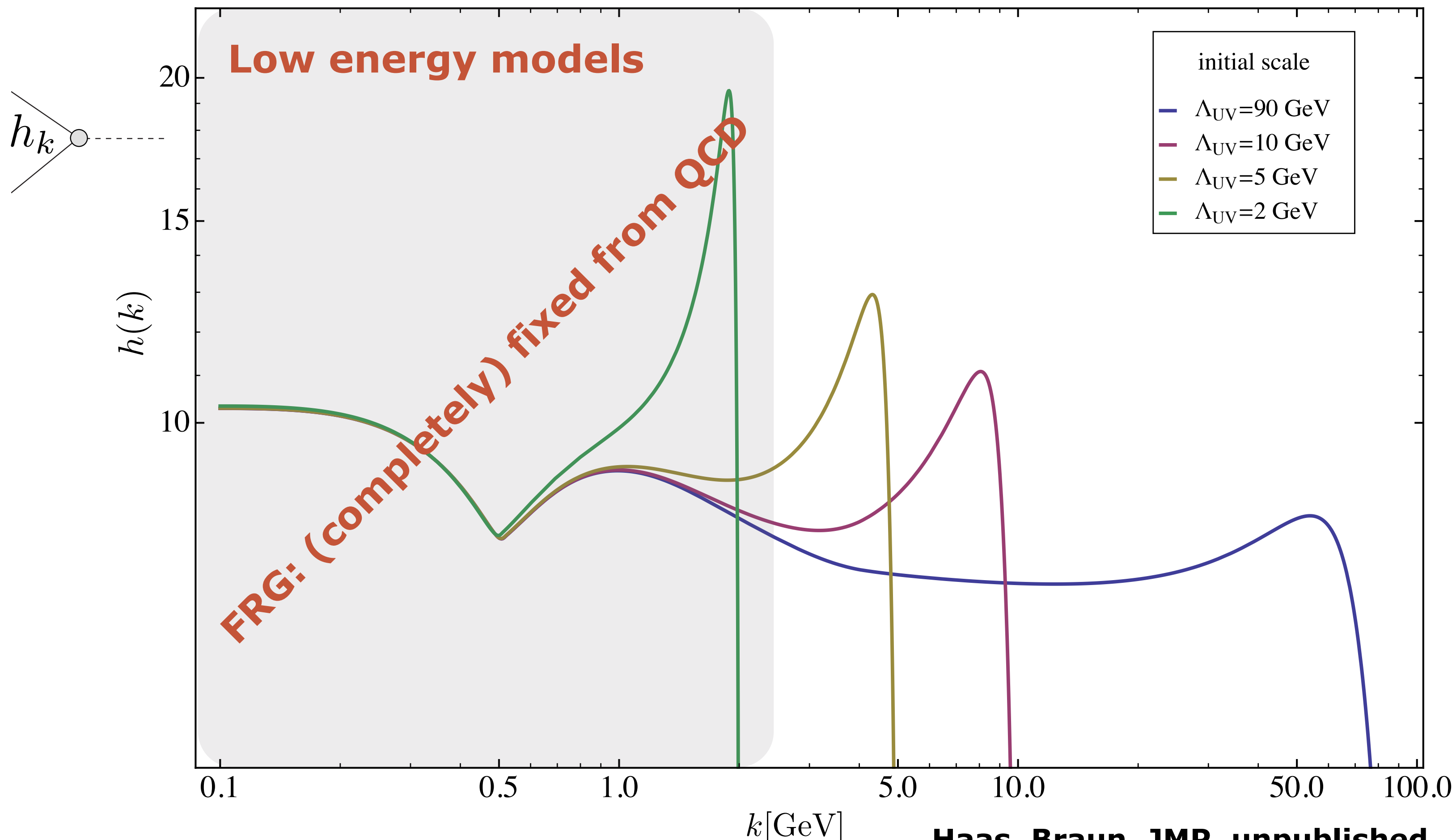




Haas, Braun, JMP, unpublished



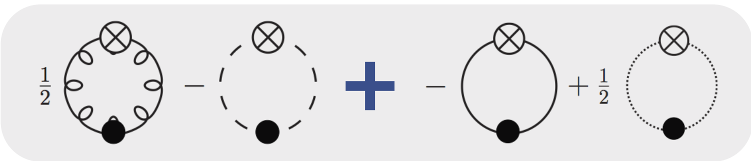
QCD



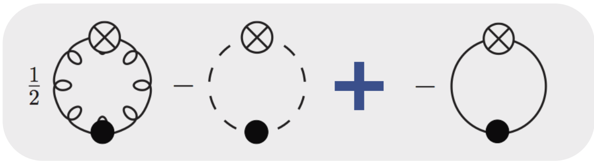
Haas, Braun, JMP, unpublished

Model results on the phase structure of QCD

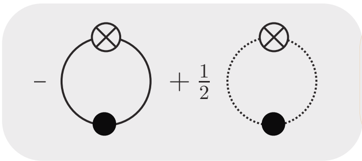
PQM-model



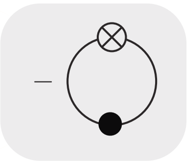
PNJL-model



QM-model



NJL-model



FRG QCD survey

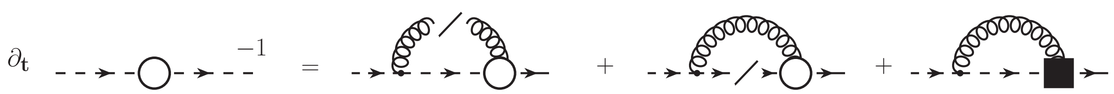
JMP, Aussois '12

Phase diagram survey

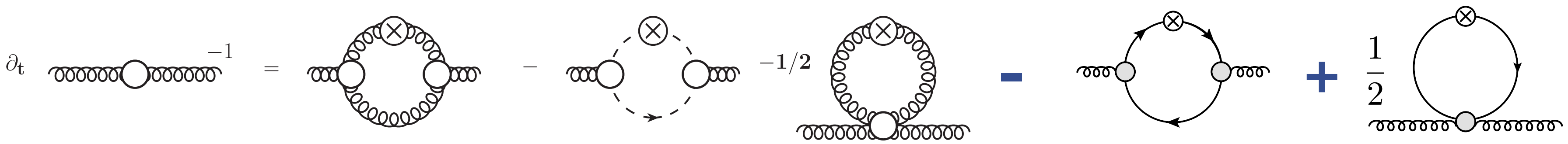
JMP, Schladming '13

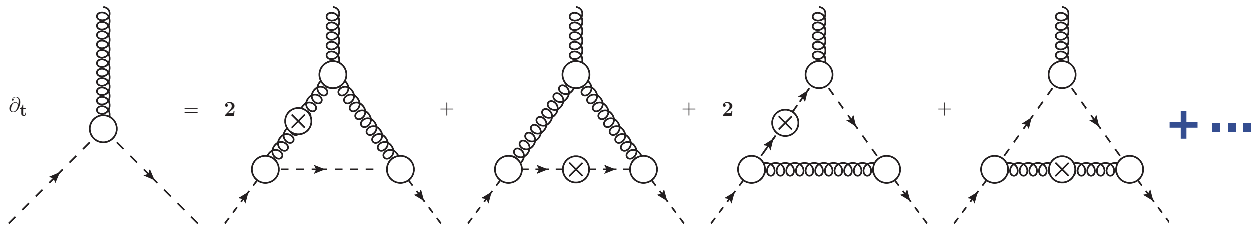
Functional Methods for QCD

present best approximation

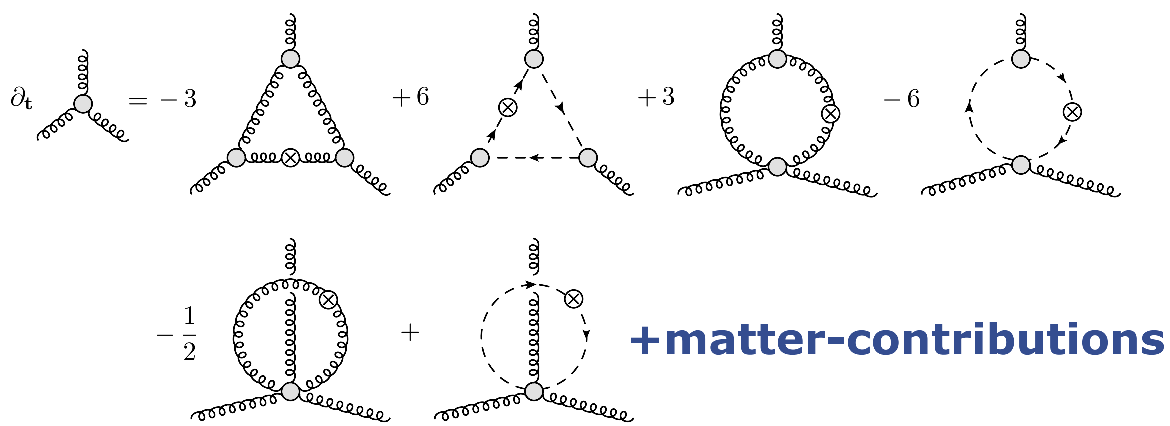


full momentum dependence

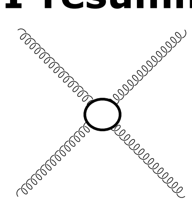




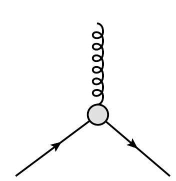
evaluated at symmetric point



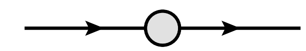
2PI-resummed



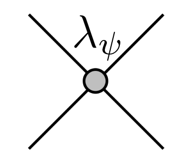
RG-dressed



full momentum dependence

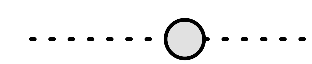


s-channel-hadronised

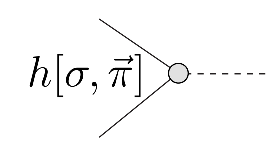


all tensor structures

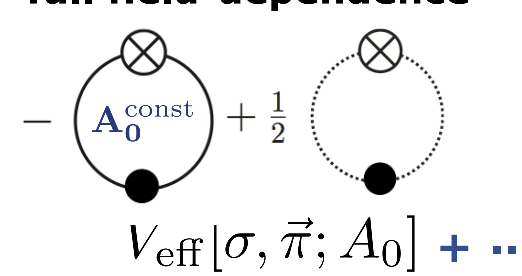
full momentum dependence



full mesonic field-dependence



full field-dependence



Phase structure and thermodynamics

Confinement

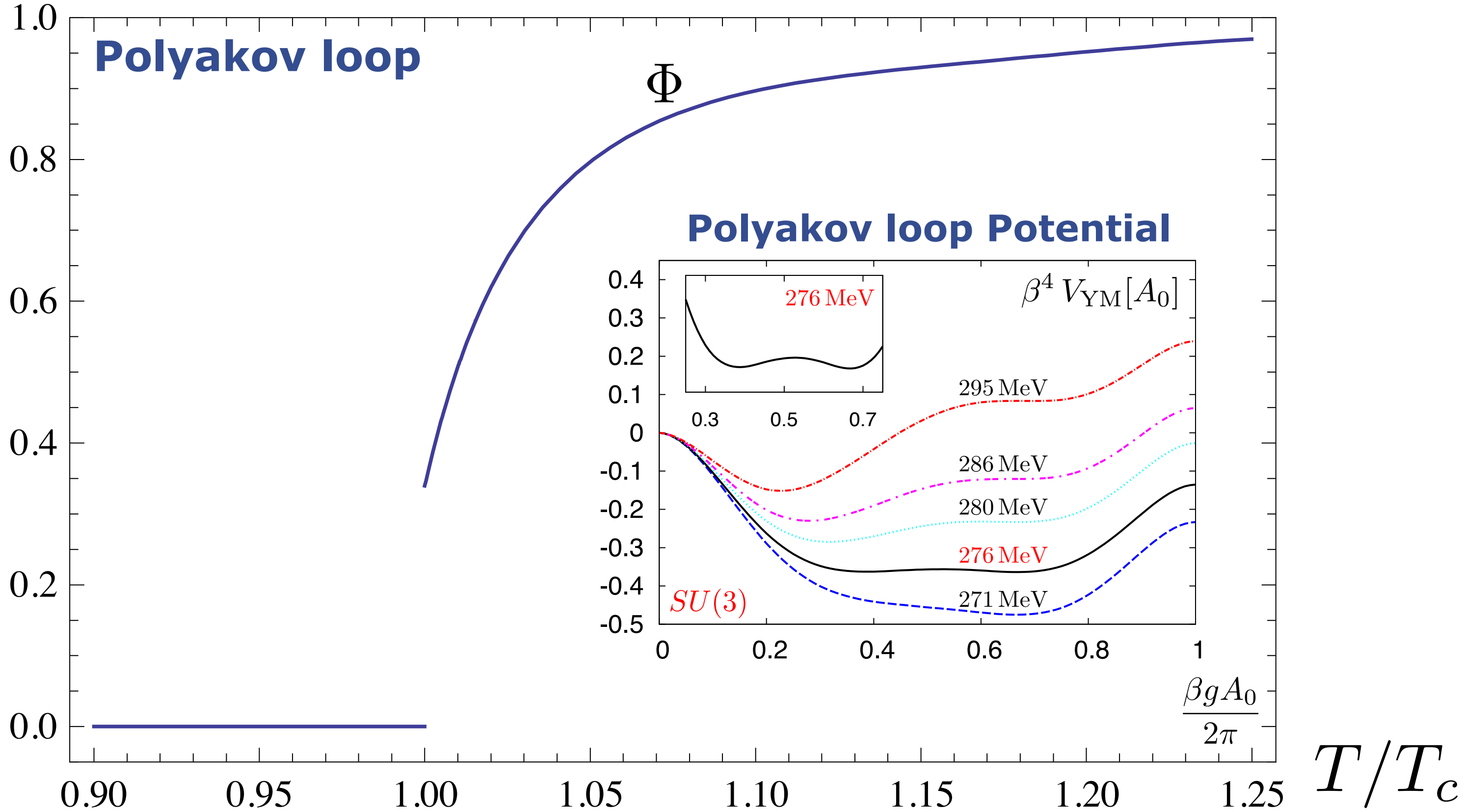
FRG: **Braun, Gies, JMP '07**

FRG, DSE, 2PI: **Fister, JMP '13**

$$T_c = 276 \pm 10\,\mathrm{MeV}$$

$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

$$\text{lattice}: T_c/\sqrt{\sigma} = 0.646$$



$$\Phi[A_0] = \frac{1}{3}(1 + 2\cos\frac{1}{2}\beta g A_0)$$

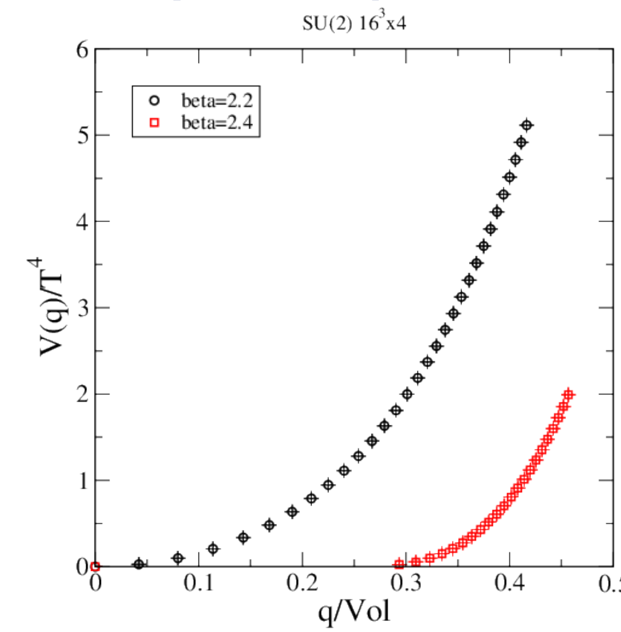
1st Lattice results

Diakonov, Gattringer, Schadler '12
Greensite '12
Greensite, Langfeld '13

Strong coupling expansion

Langelage, Lottini, Philipsen '10

Polyakov loop Potential



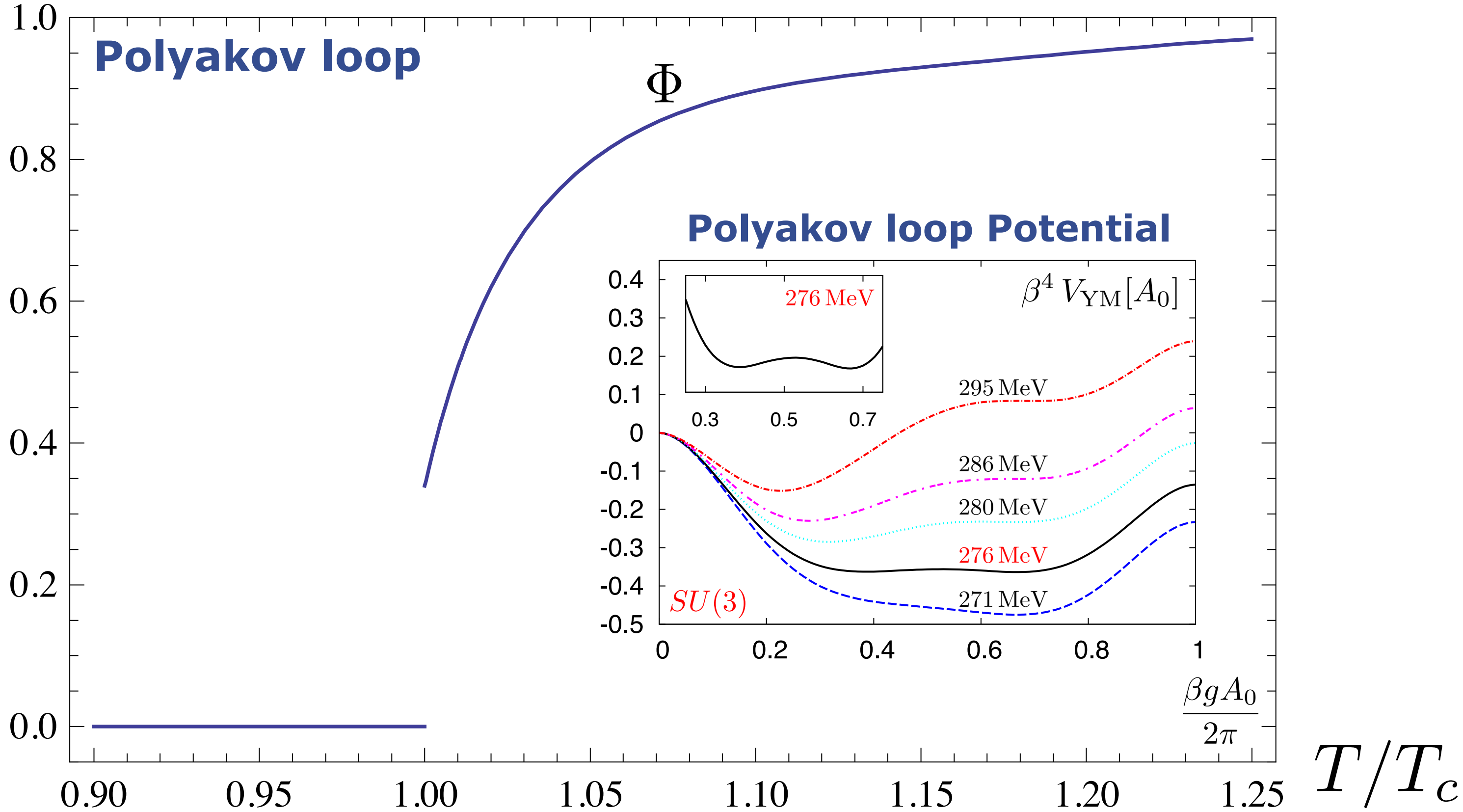
Langfeld, JMP '13

$$q \simeq \Phi$$

Confinement

FRG: Braun, Gies, JMP '07

FRG, DSE, 2PI: Fister, JMP '13



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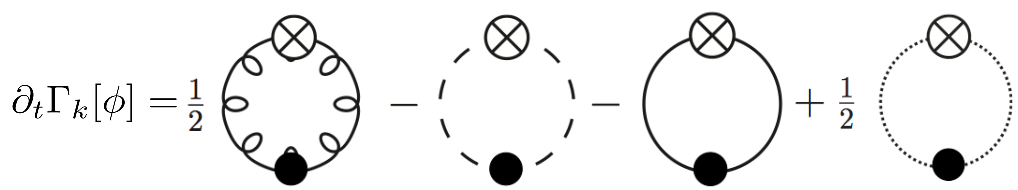
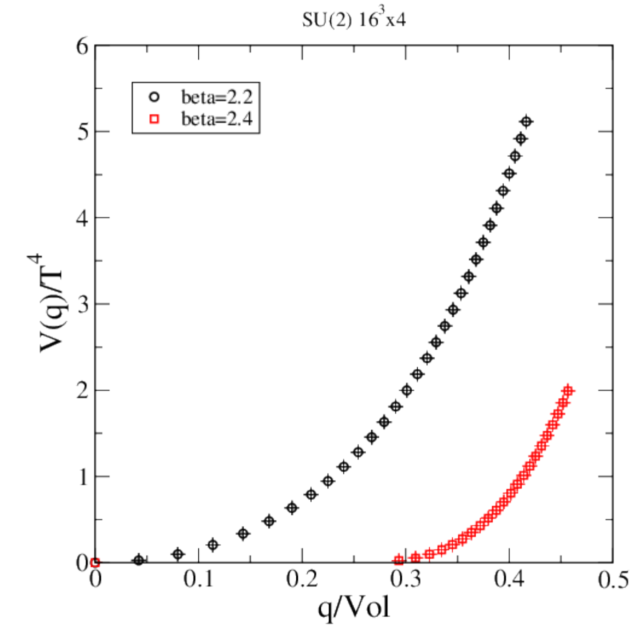
Functional RG



$$\text{RG-scale k:}\ t = \ln k$$

Polyakov loop Potential



Langfeld, JMP '13

$$q \simeq \Phi$$

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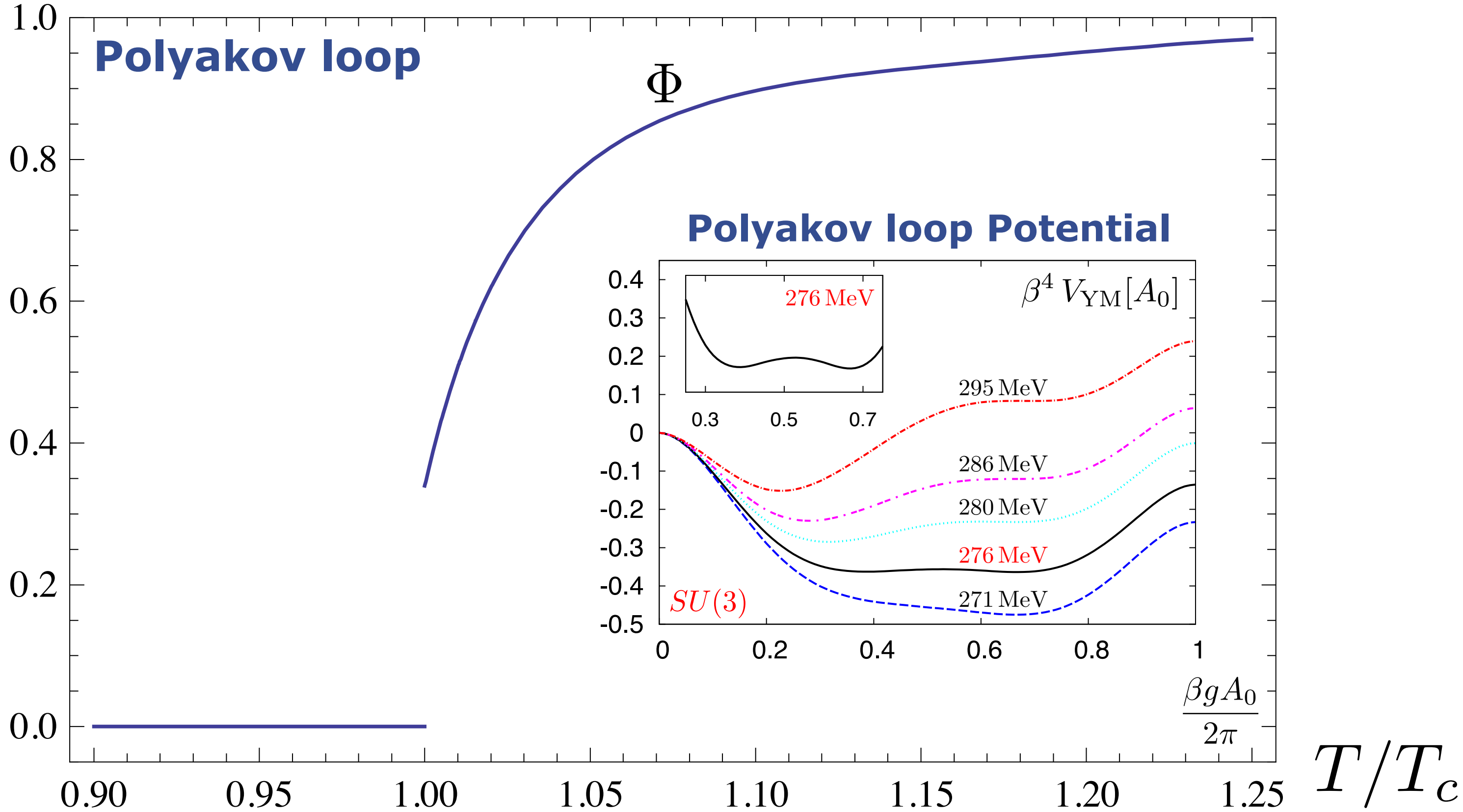
FRG, DSE, 2PI: Fister, JMP '13



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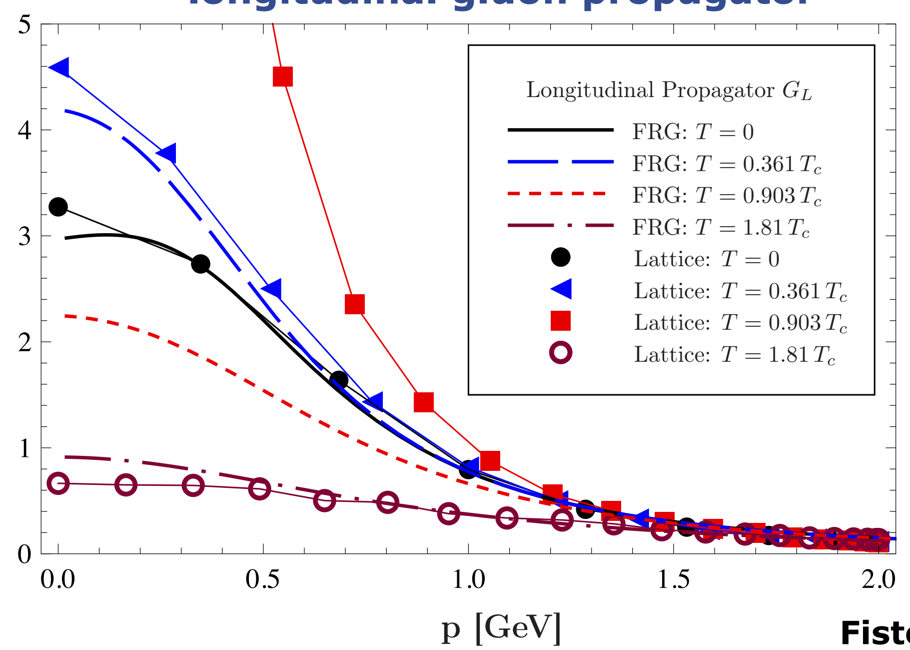
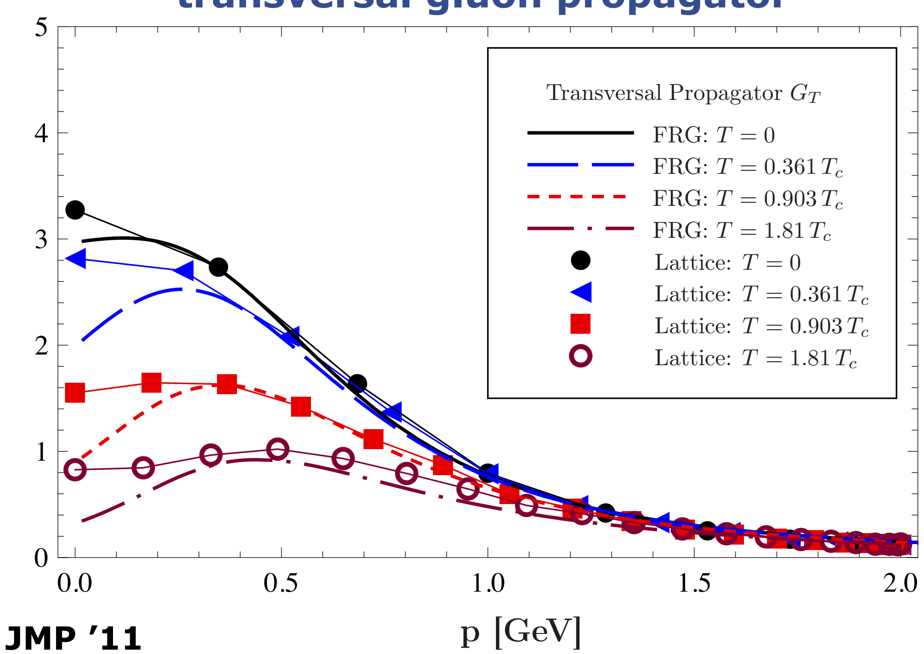
- from the full propagators
- gauge independence
- confinement criteria

longitudinal gluon propagator



transversal gluon propagator

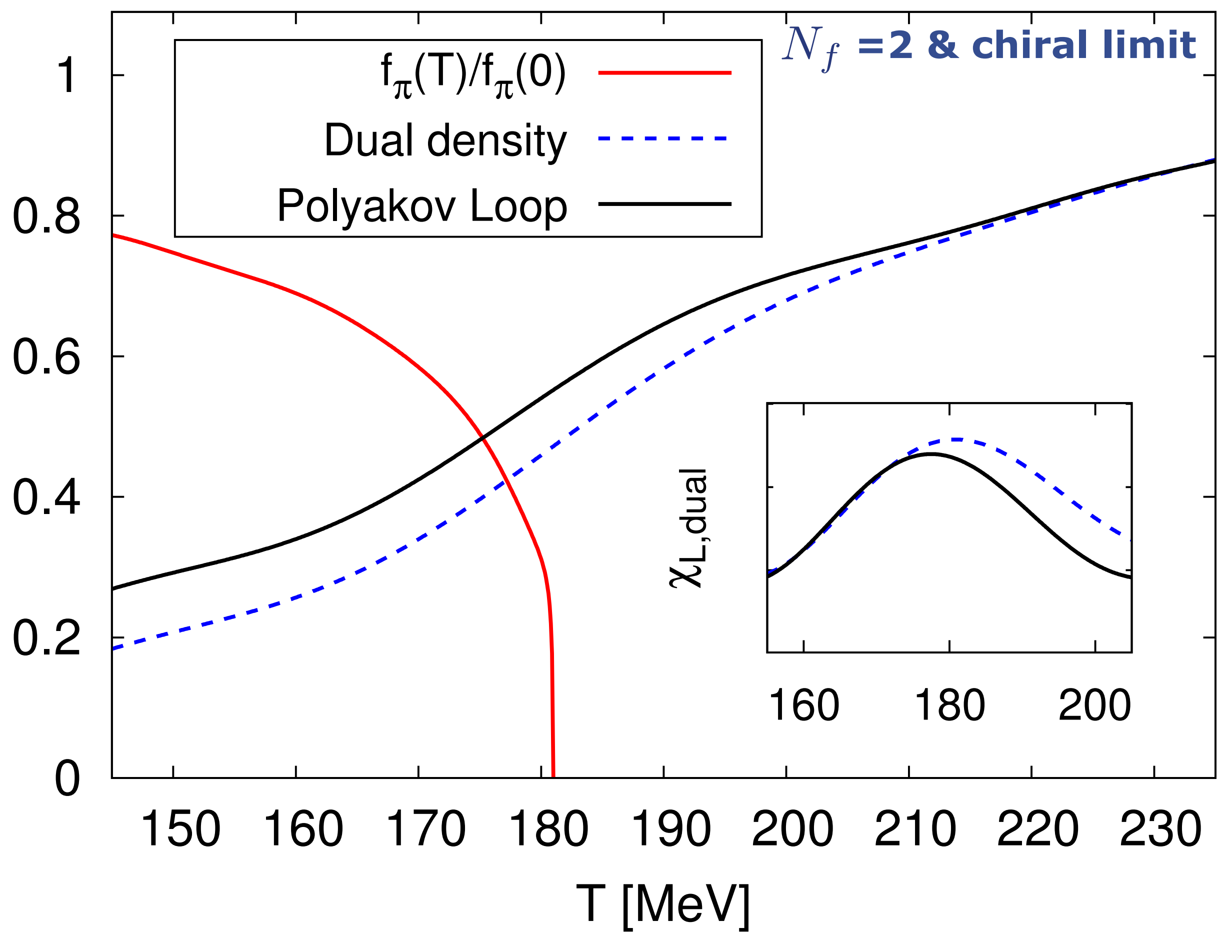


Fister, JMP '11

Full dynamical QCD

Phase structure

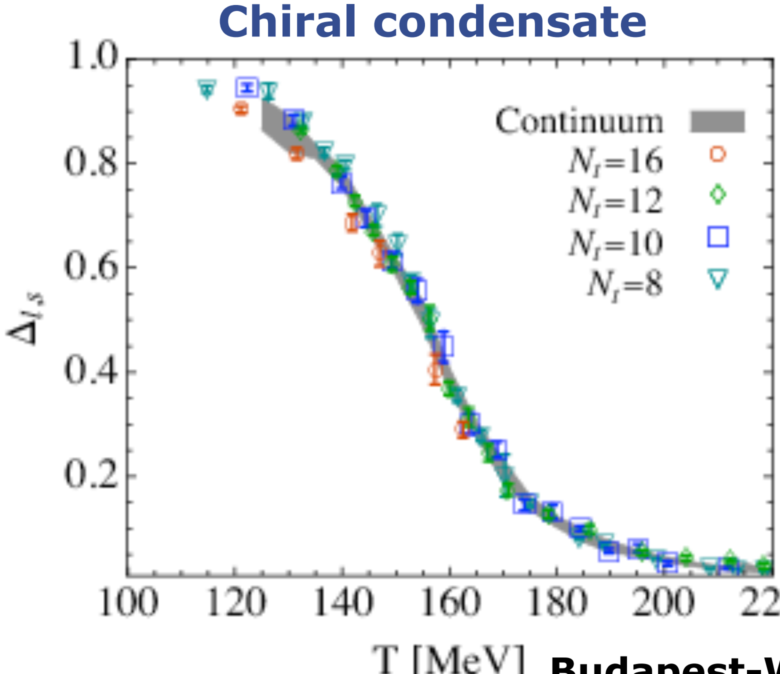
Braun, Haas, Marhauser, JMP '09



$$T_\chi \simeq T_{\rm conf} \simeq 180{\rm MeV}$$

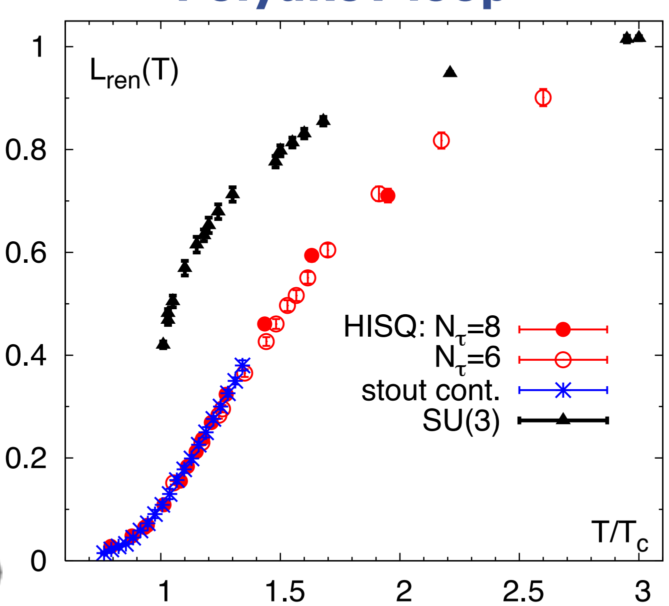
Width $\Delta T_{\rm conf} \simeq \pm 20{\rm MeV}$

Chiral condensate



Budapest-Wuppertal '10

Polyakov loop



hotQCD '10

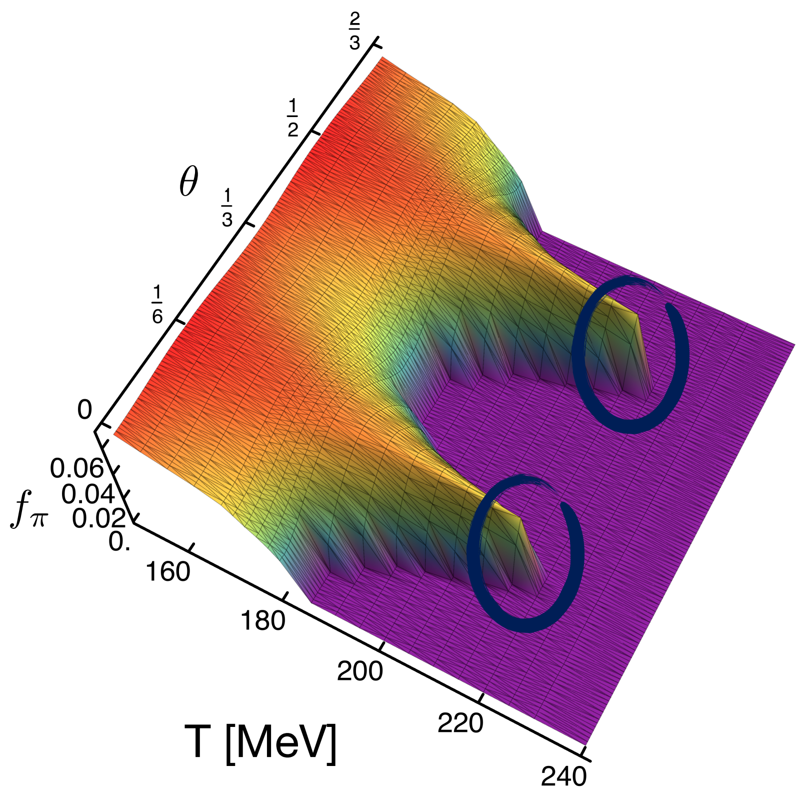
N_f=2+1

DSE: Fischer, Lücker, Mueller '11 (2 flavour)
Fischer, Lücker '12 (2+1 flavour)

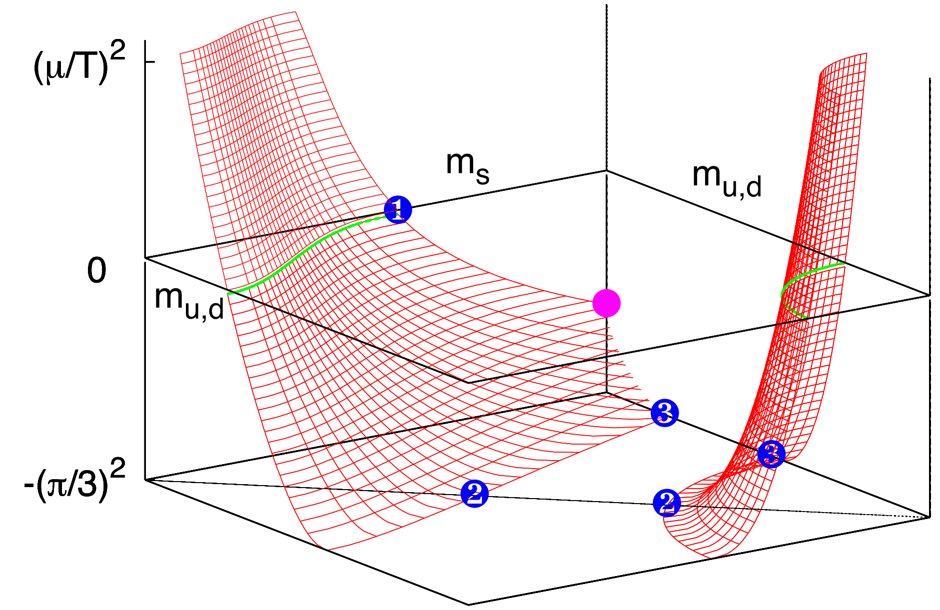
Imaginary chemical potential

Nature of the RW endpoint

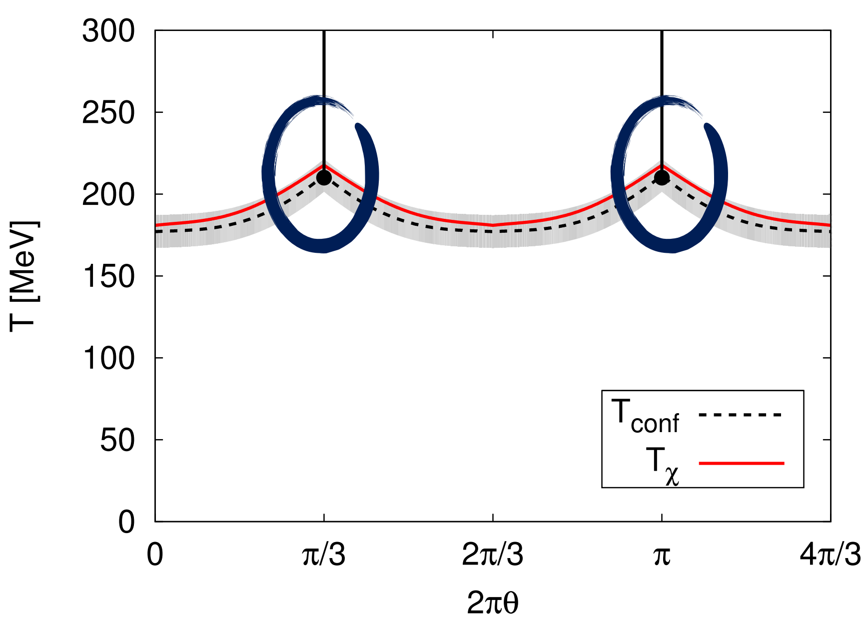
$$\mu = 2\pi T \theta\, i$$



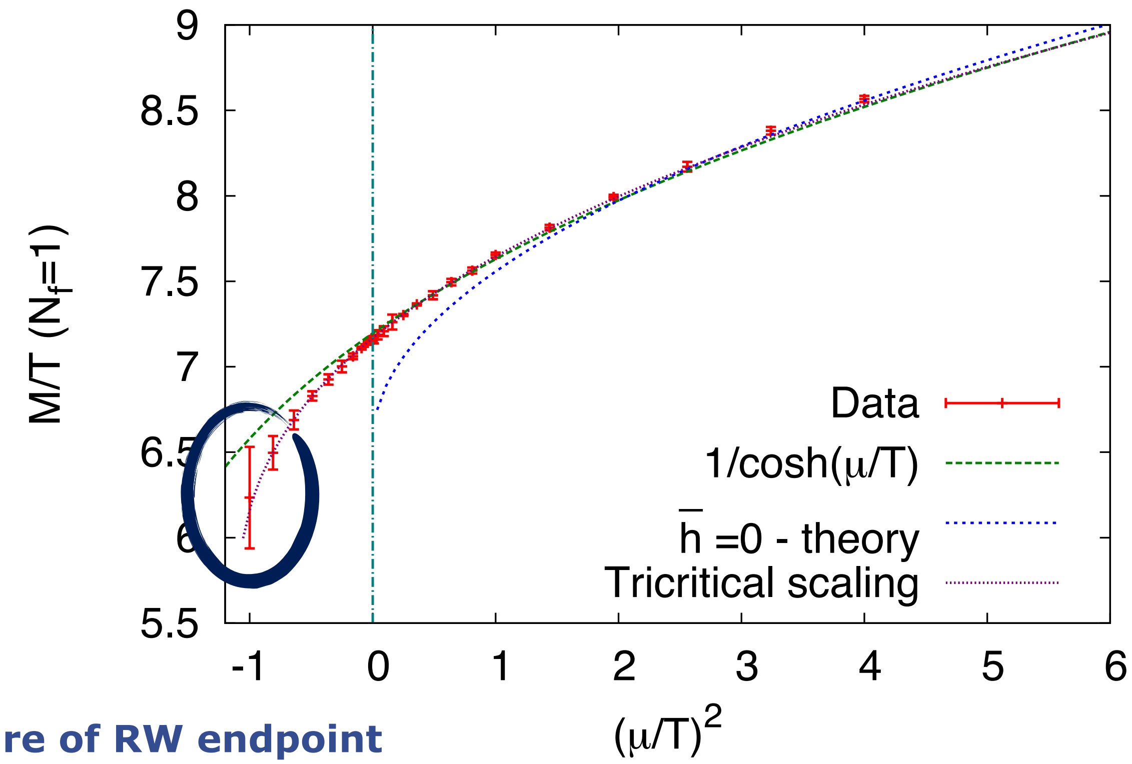
Braun, Haas, Marhauser, JMP '09



O. Philipsen '11



RW endpoint



Nature of RW endpoint

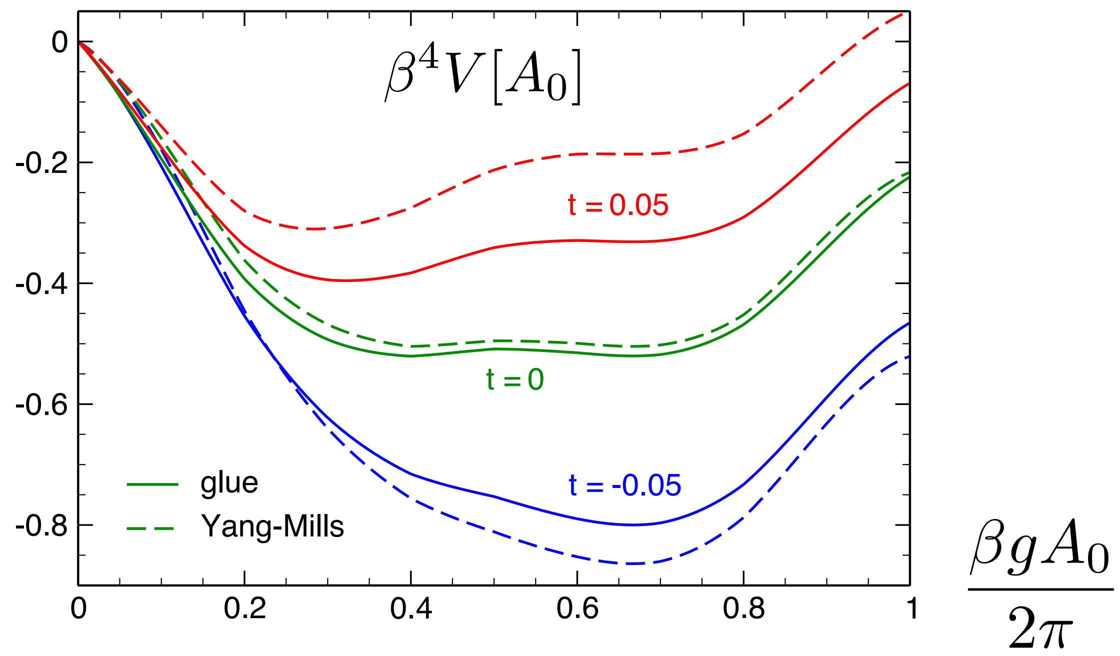
lattice: D'Elia, Sanfilippo '09
de Forcrand, Philipsen '10
...

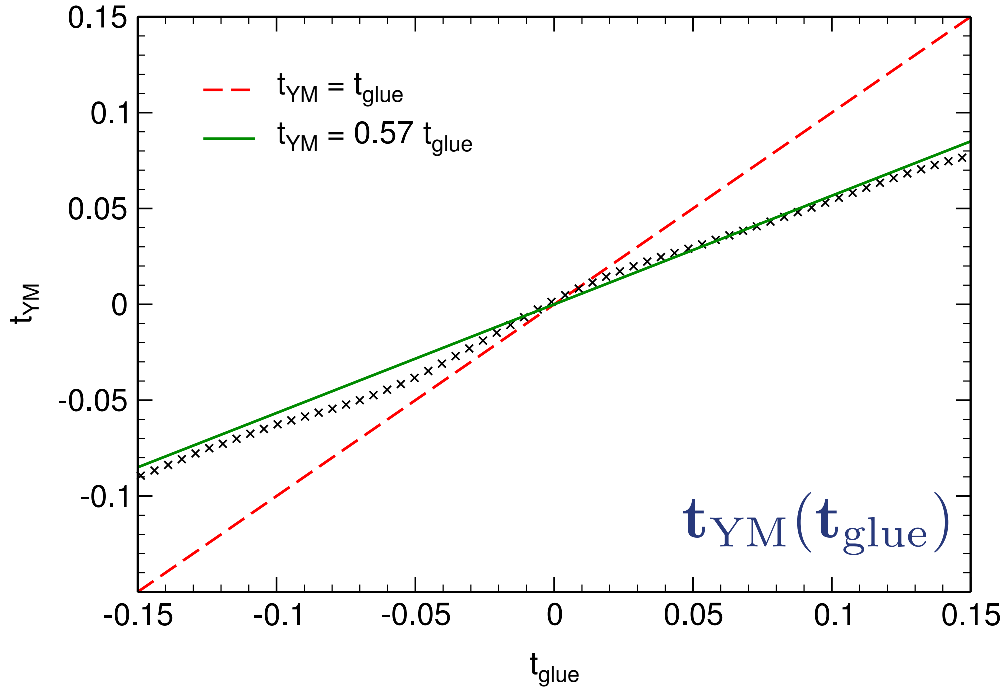
PNJL: Sakai et al '10
Morita et al '11

Full dynamical QCD

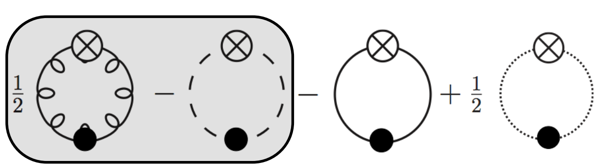
Improving models towards full QCD

Polyakov loop potential in full QCD

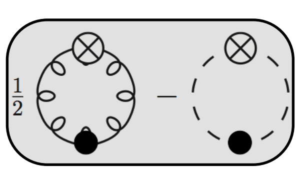




Glue Potential



Yang-Mills Potential

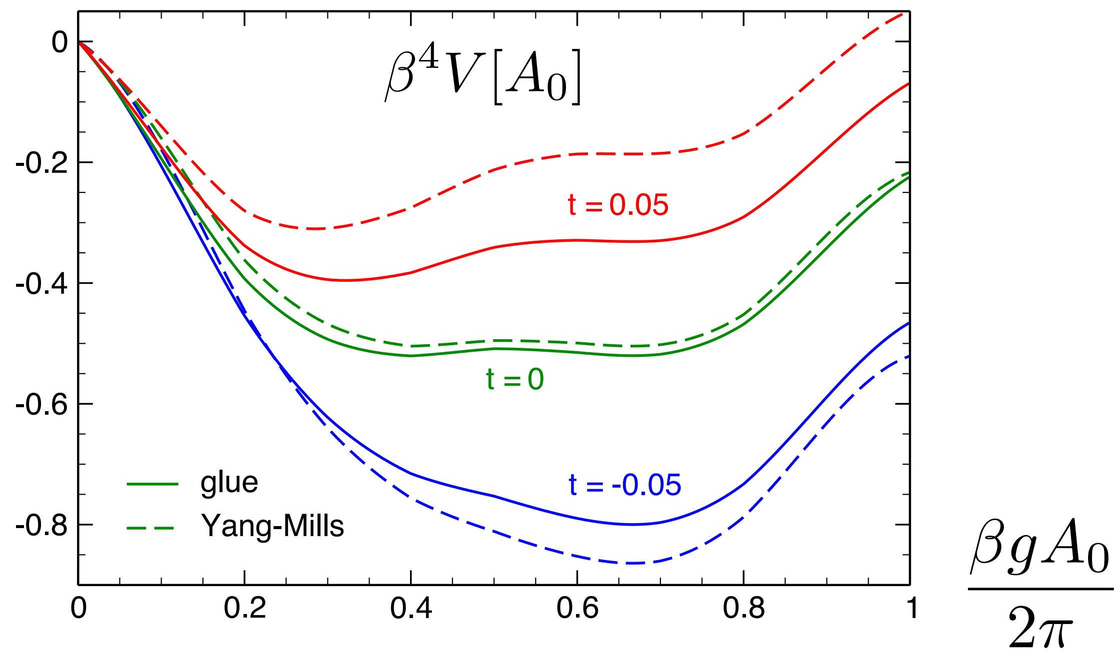


JMP '10
Haas, Stiele, Braun, JMP, Schaffner-Bielich '13
Herbst, Mitter, JMP, Schaefer, Stiele '13

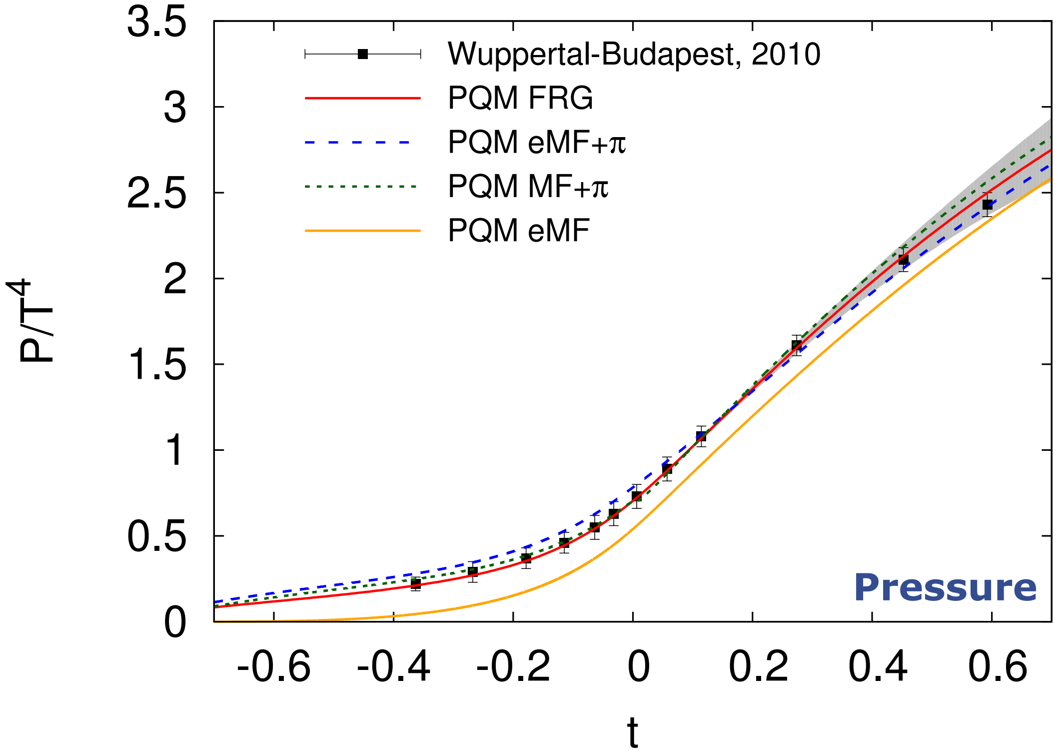
Full dynamical QCD

Improving models towards full QCD

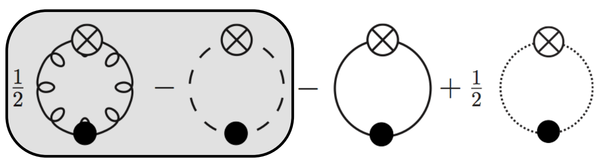
Polyakov loop potential in full QCD



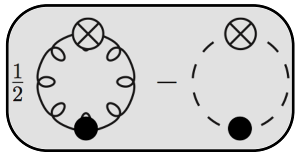
2+1 flavor Polyakov-loop - enhanced QM-model

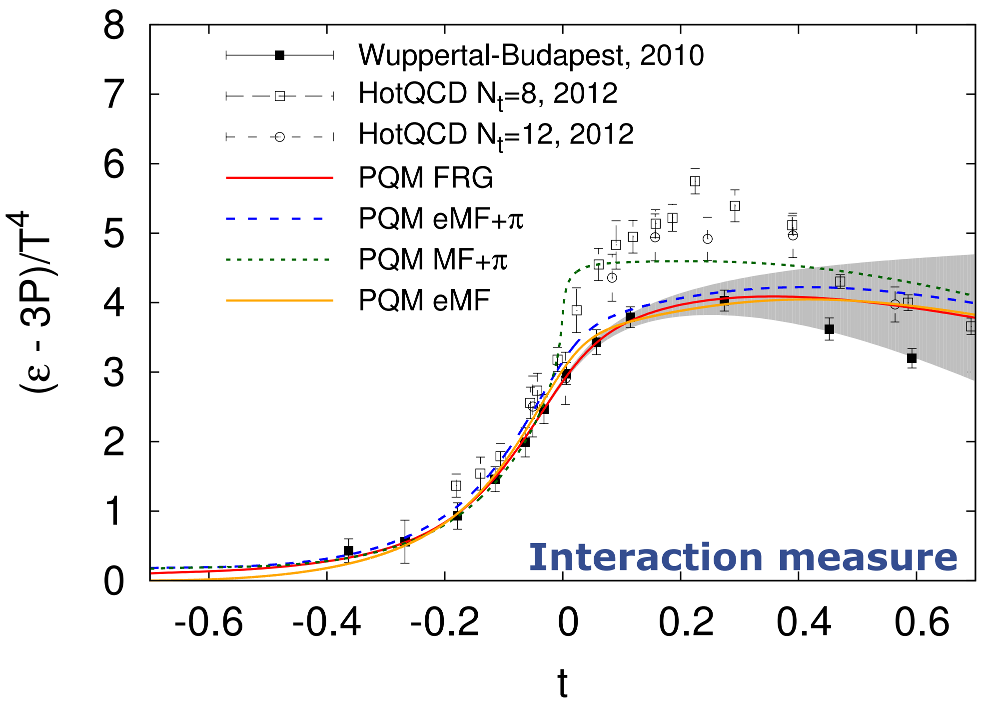


Glue Potential



Yang-Mills Potential





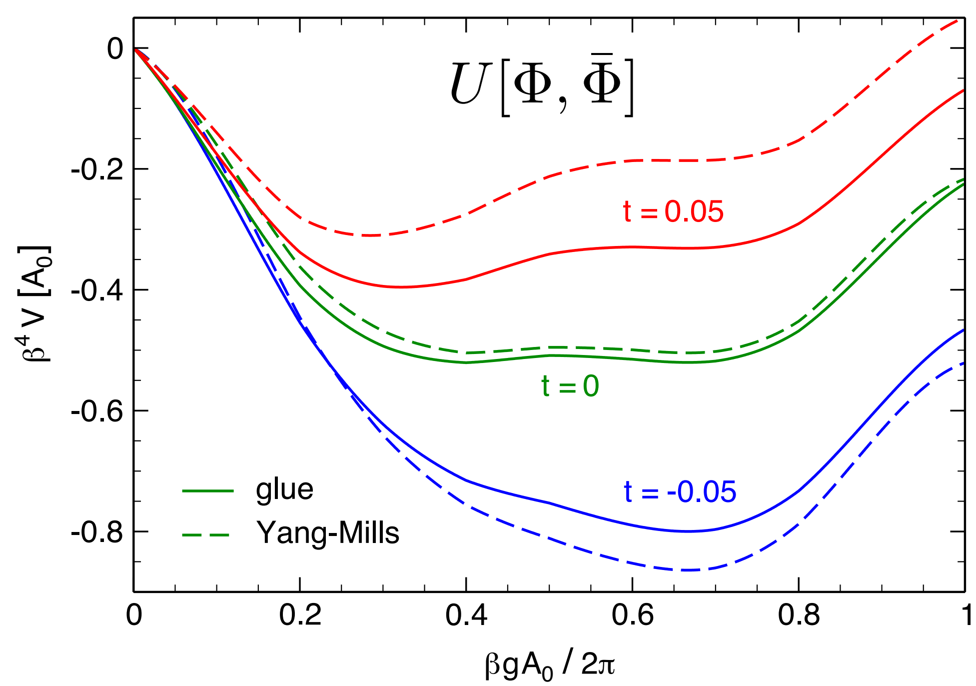
JMP '10
Haas, Stiele, Braun, JMP, Schaffner-Bielich '13
Herbst, Mitter, JMP, Schaefer, Stiele '13

Shaded area: systematic error estimate due to low initial scale 1 GeV

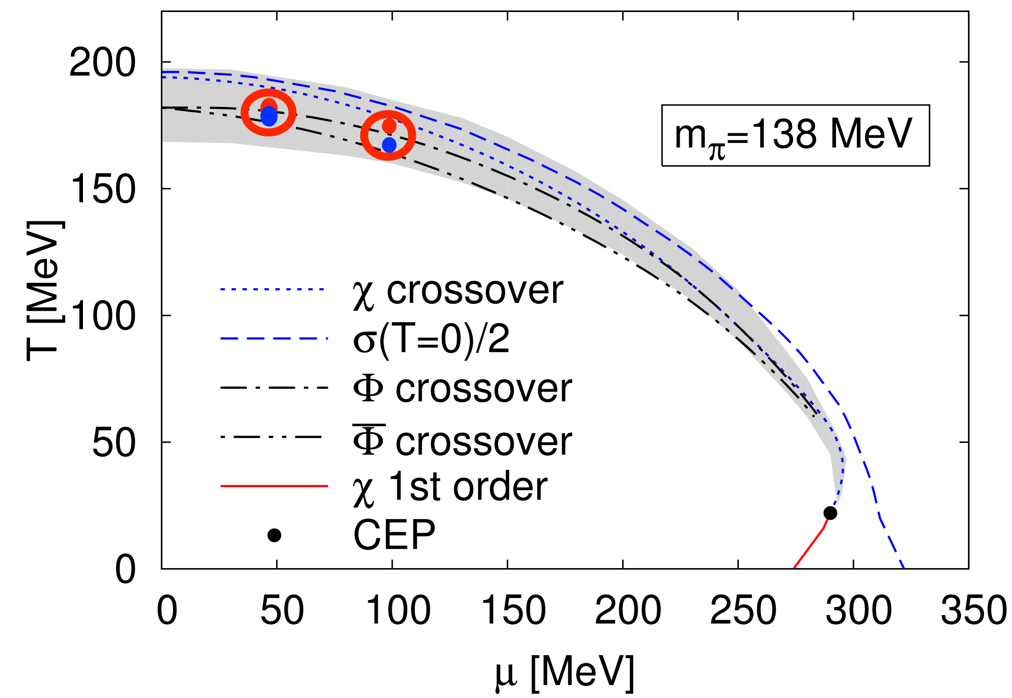
Full dynamical QCD

Phase structure

Polyakov loop potential in full QCD



Phase diagram of quantised PQM-model

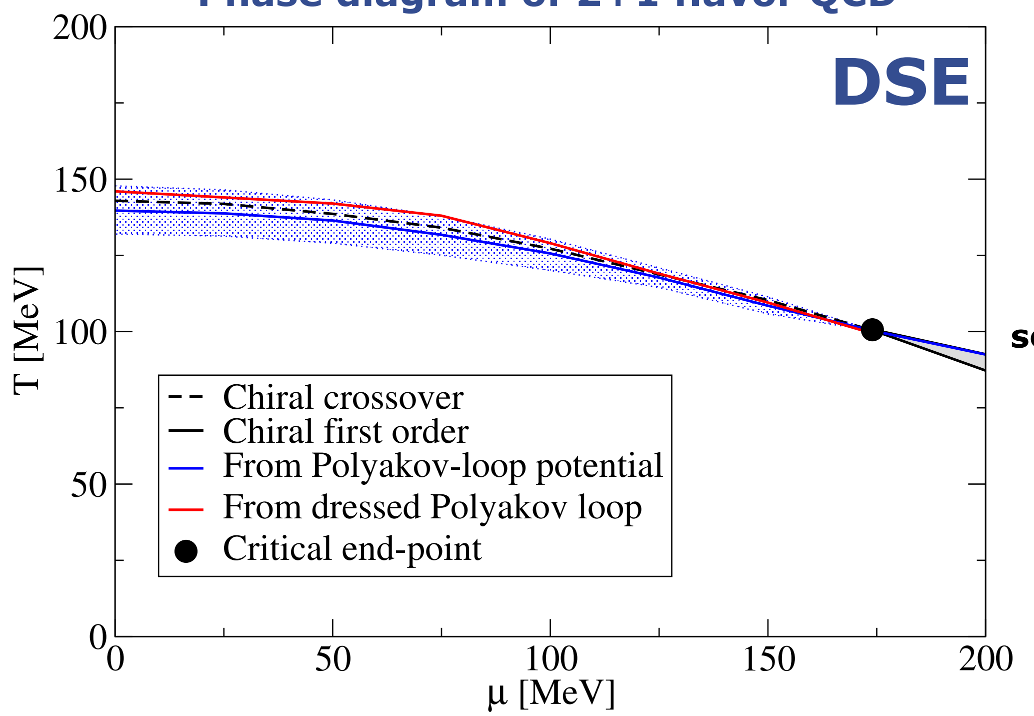


Herbst, JMP, Schaefer '13

FRG QCD results at finite density

Haas, Braun, JMP, unpublished

Phase diagram of 2+1 flavor QCD



see talk of C. Fischer

Fischer, Lücker '12
Fischer, Fister, Lücker, JMP '13

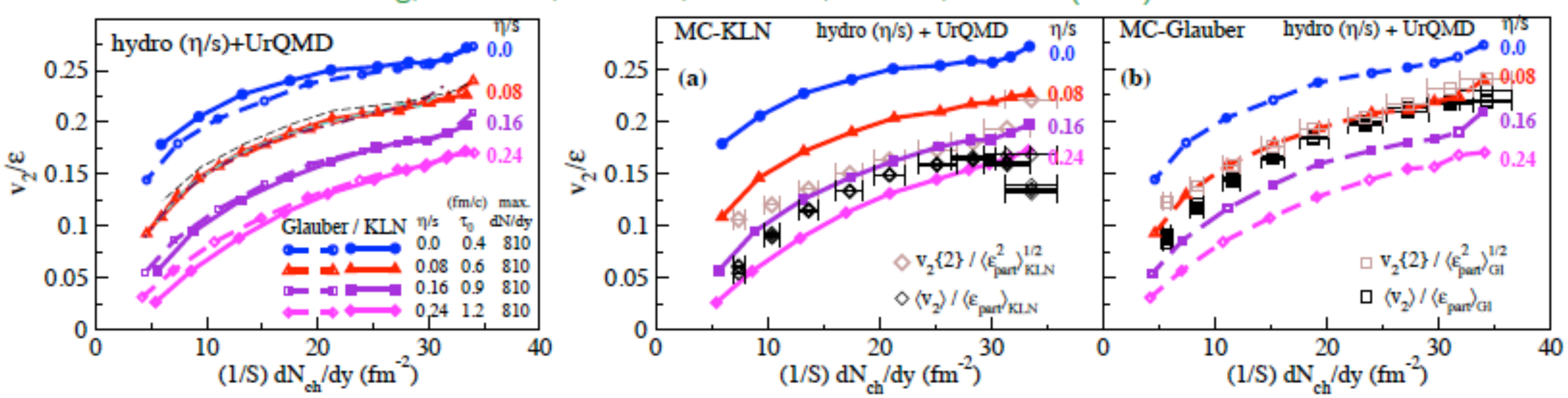
Critical point unlikely for

$$\frac{\mu_B}{T} < 2$$

Spectral functions & transport coefficients

Extraction of $(\eta/s)_{QGP}$ from AuAu@RHIC

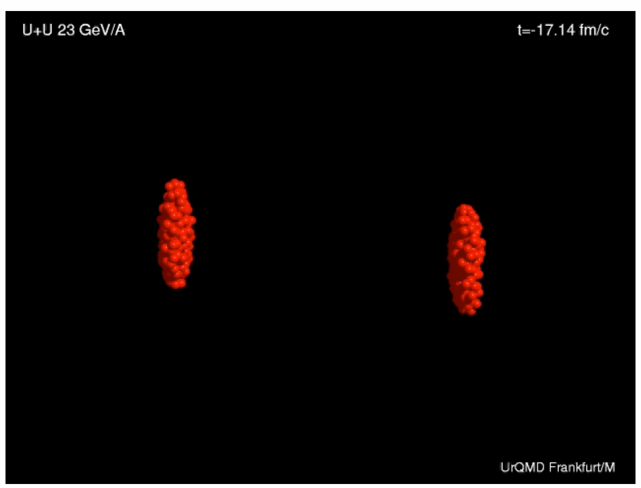
H. Song, S.A. Bass, U. Heinz, T. Hirano, C. Shen, PRL106 (2011) 192301



$1 < 4\pi(\eta/s)_{QGP} < 2.5$

UrQMD Frankfurt/M

Spectral functions & transport coefficients



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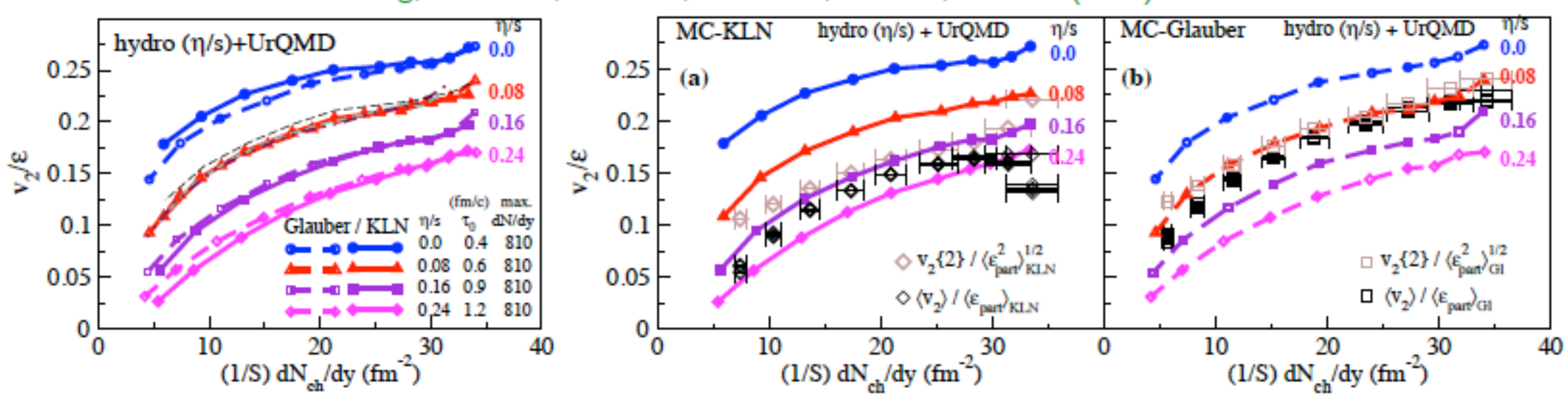

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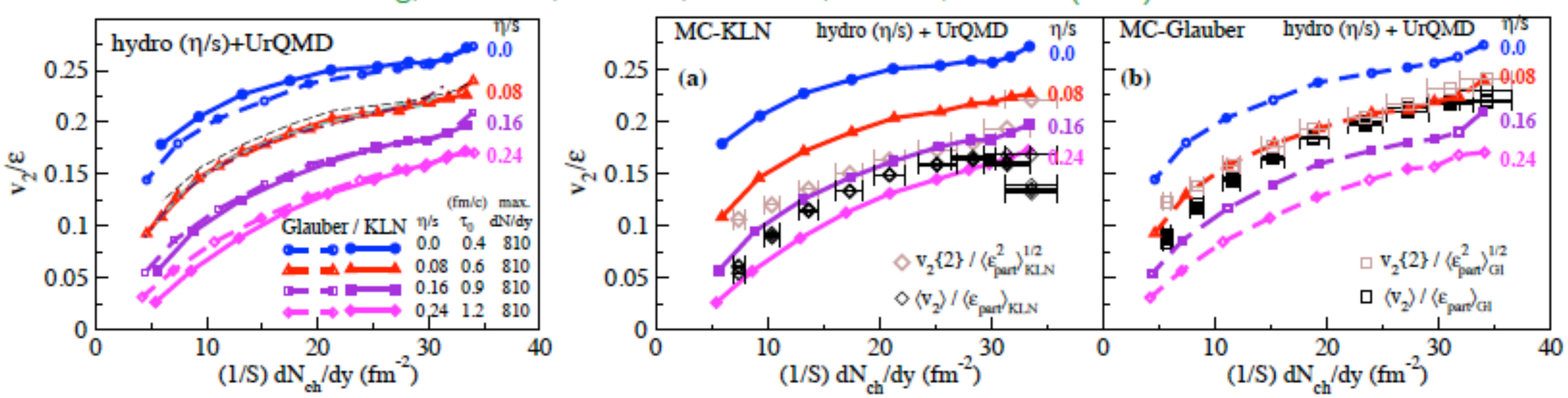


UrQMD Frankfurt/M

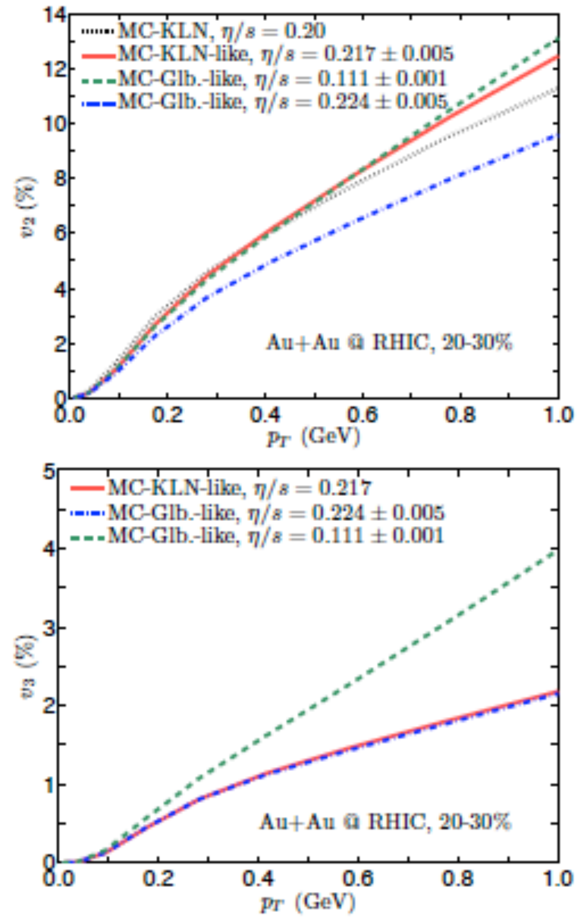
$$1 < 4\pi(\eta/s)_{QGP} < 2.5$$

Heavy ion collisions

Shooting the elephant

Proof of principle calculation:

Zhi Qiu and U. Heinz, to be published



- Take ensemble of sum of deformed Gaussian profiles, $s(\boldsymbol{r}_\perp) = s_2(\boldsymbol{r}_\perp; \bar{\varepsilon}_2, \psi_2) + s_3(\boldsymbol{r}_\perp; \bar{\varepsilon}_3, \psi_3)$, with
  1. equal Gaussian radii $R_2^2 = R_3^2 = 8\,\mathrm{fm}^2$ to reproduce $\langle r_\perp^2 \rangle$ of MC-KLN source for 20-30% AuAu
  2. $\bar{\varepsilon}_2$ and $\bar{\varepsilon}_3$ adjusted such that
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{KLN}}^{20-30\%}$ ("MC-KLN-like")
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{Gl}}^{20-30\%}$ ("MC-Glauber-like")
  3. $\psi_2 = 0$, ψ_3 (direction of triangularity) distributed randomly
- Use $v_2^\pi(p_T)$ from VISH2+1 for $\eta/s = 0.20$ with MC-KLN initial conditions for 20-30% AuAu as "mock data"
- Fit mock $v_2^\pi(p_T)$ data with VISH2+1 for "MC-Glauber-like" or "MC-KLN-like" Gaussian initial conditions with both elliptic and triangular deformations by adjusting η/s
  $\Longrightarrow$ $(\eta/s)_{\mathrm{KLN}} = 0.217 \pm 0.005$ for "MC-KLN-like",
  $(\eta/s)_{\mathrm{Gl}} = 0.111 \pm 0.001$ for "MC-Glauber-like"
- Compute $v_3^\pi(p_T)$ for "MC-KLN-like" fit with $(\eta/s)_{\mathrm{Gl}}$=0.217 and reproduce it with "MC-Glauber-like" initial condition by readjusting η/s
  $\Longrightarrow$ $(\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224 \pm 0.005$ for "MC-Glauber-like"
- Compute $v_2^\pi(p_T)$ for "MC-Glauber-like" initial profiles with readjusted $(\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224$ and compare with "MC-Glauber-like" fit to original mock data $\Longrightarrow$ clearly visible (and measurable) difference!

This exercise proves: (i) Fitting $v_3(p_T)$ data with MC-Glauber and MC-KLN initial conditions yields the same η/s (within narrow error band); (ii) The corresponding $v_2(p_T)$ fits are quite different, and only one (more precisely: at most one!) of the models will fit the corresponding $v_2(p_T)$ data.

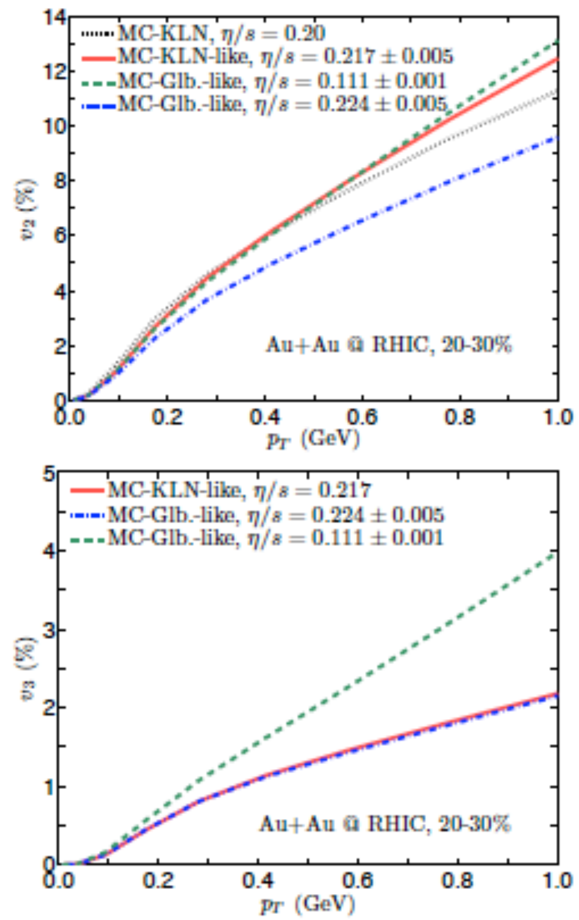
U. Heinz, talk at RETUNE '12

Heavy ion collisions

Computing the elephant

Proof of principle calculation:

Zhi Qiu and U. Heinz, to be published



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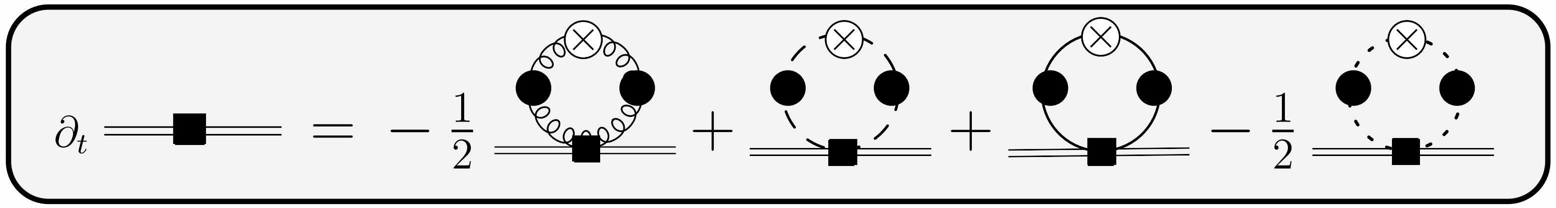
Transport in QCD

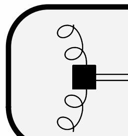
correlations of energy-momentum tensor

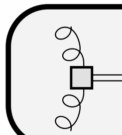
M. Haas, Fister, JMP ´13

Flow

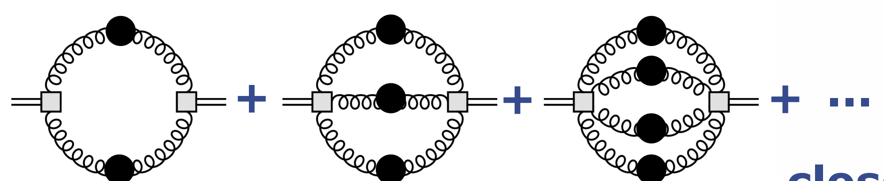


$\rho_{\pi\pi}$





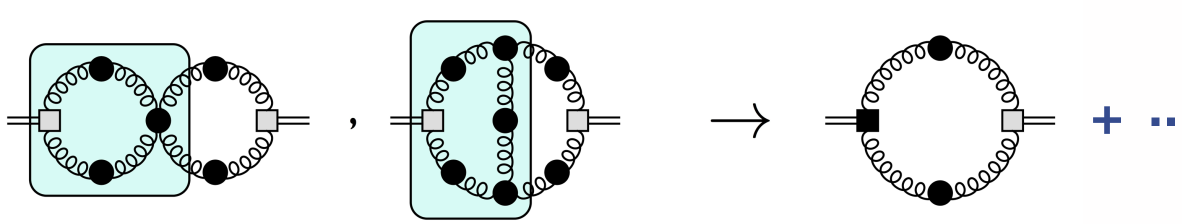
Diagrammatic representation

$\rho_{\pi\pi} =$ 

closed form

full computation Christiansen, Haas, JMP, Strodthoff, in prep.

Vertex corrections



Transport in QCD

correlations of energy-momentum tensor

M. Haas, Fister, JMP ´13

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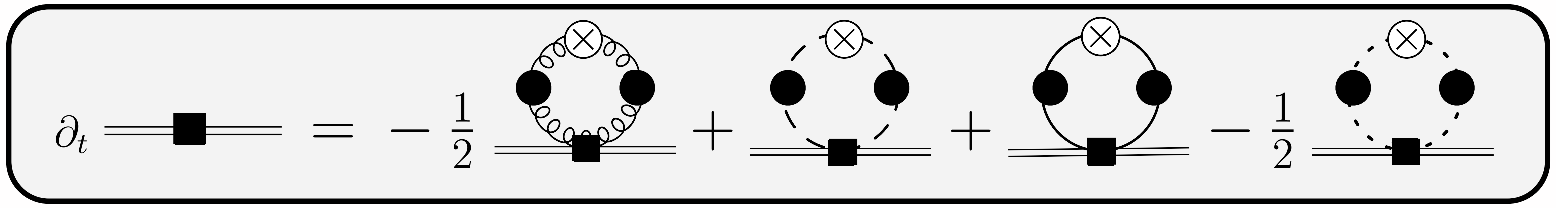
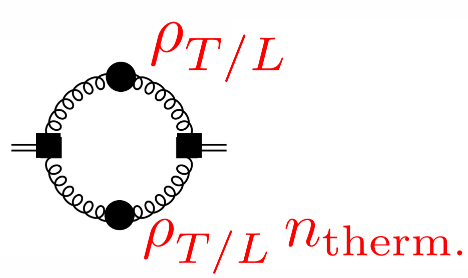
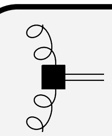


$\rho_{\pi\pi}$

Current approximation

$\rho_{\pi\pi} =$





full vertex

with optimised RG-scheme from Fister, JMP ’13

$\rho_{T/L}$ with MEM

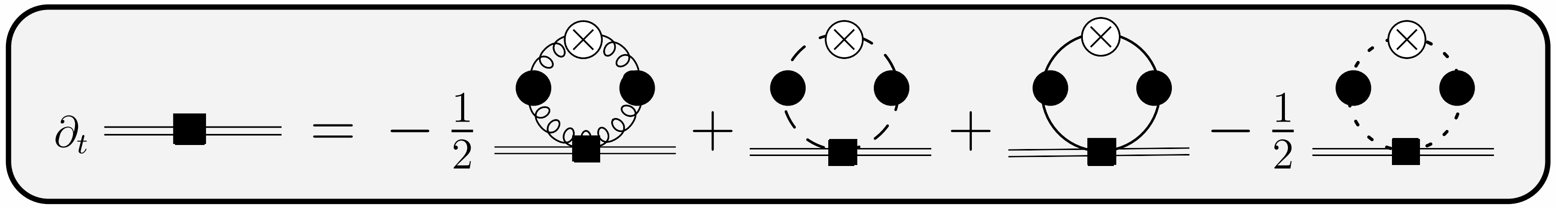
$$\rho_{\pi\pi}(p) = \frac{2d_A}{3} \int \frac{d^4k}{(2\pi)^4} \left[n(k^0) - n(k^0 + p_0)\right] \left(V_{TT}\rho_T(k)\rho_T(k+p) + V_{TL}\rho_T(k)\rho_L(k+p) + V_{LL}\rho_L(k)\rho_L(k+p)\right)$$

Transport in QCD

correlations of energy-momentum tensor

M. Haas, Fister, JMP ´13

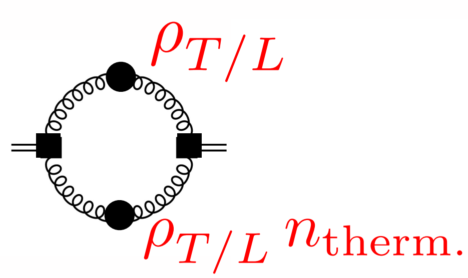
Flow

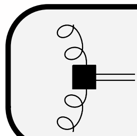


$\rho_{\pi\pi}$

Current approximation

$\rho_{\pi\pi} =$





full vertex

with optimised RG-scheme from Fister, JMP ’13

$\rho_{T/L}$ with MEM

‘Those are my methods (principles),
and if you don’t like them…well, I have others’

direct computation

Groucho Marx

$$\rho_{\pi\pi}(p) = \frac{2d_A}{3}\int \frac{d^4k}{(2\pi)^4}\left[n(k^0) - n(k^0+p_0)\right]\left(V_{TT}\rho_T(k)\rho_T(k+p) + V_{TL}\rho_T(k)\rho_L(k+p) + V_{LL}\rho_L(k)\rho_L(k+p)\right)$$

Transport in QCD

correlations of energy-momentum tensor

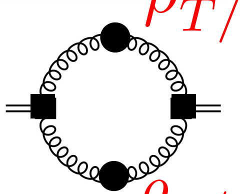
M. Haas, Fister, JMP ´13

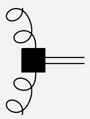
Shear viscosity

$$\eta = \frac{1}{20} \frac{d}{d\omega}\bigg|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

Kubo relation

Current approximation

$\rho_{\pi\pi} =$ 

 full vertex

$\rho_{T/L}$ **with MEM**

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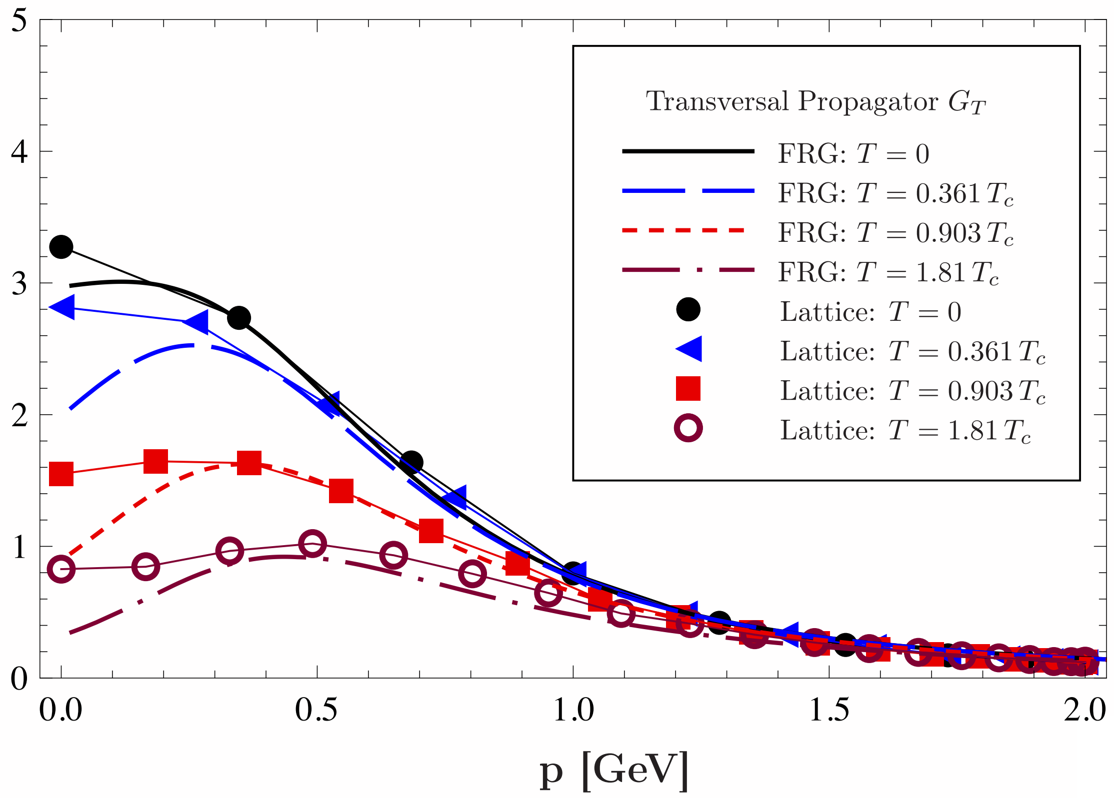
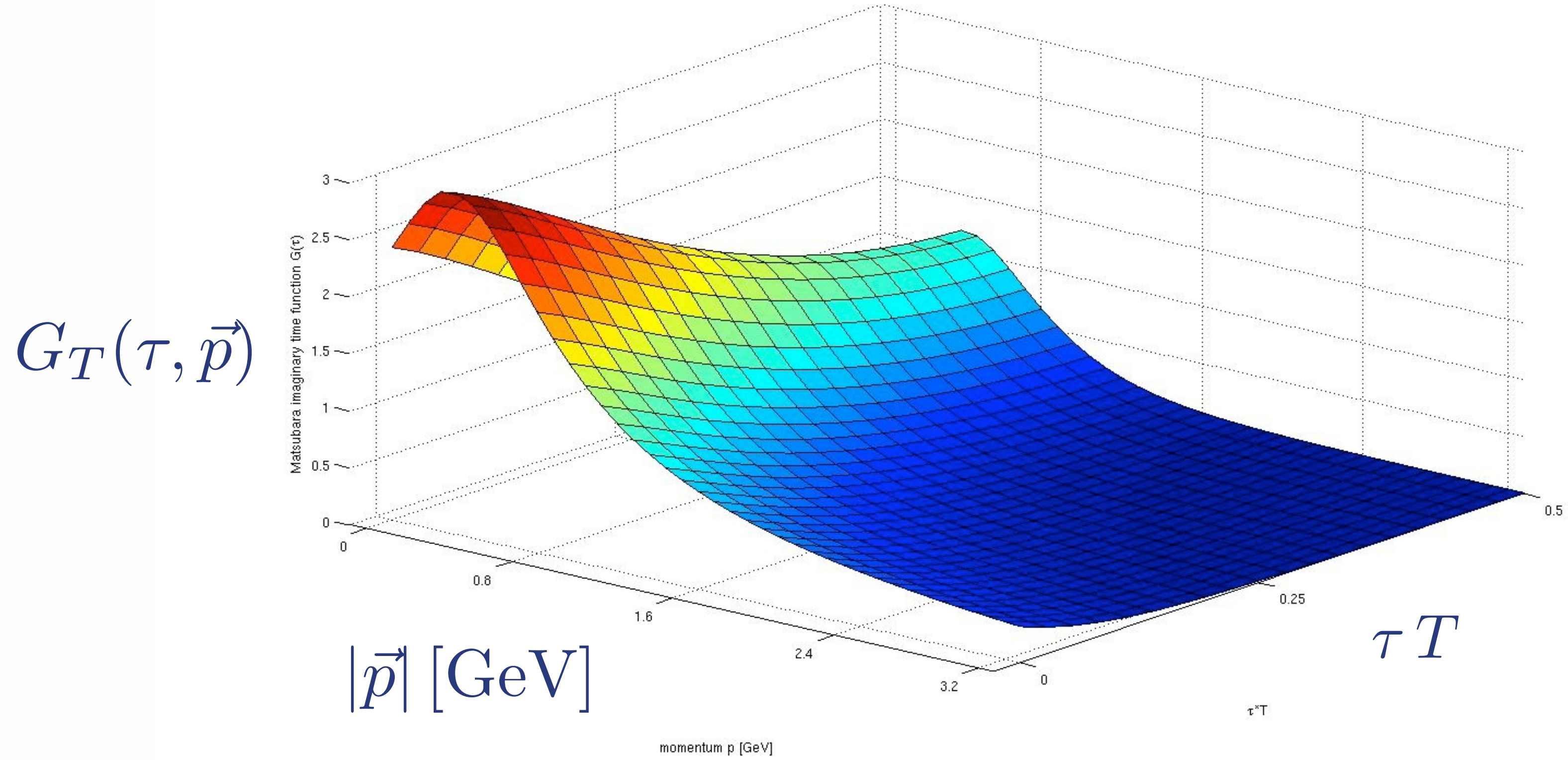
Viscosity in pure glue

imaginary time correlations

M. Haas, Fister, JMP ´13



transversal gluon propagator

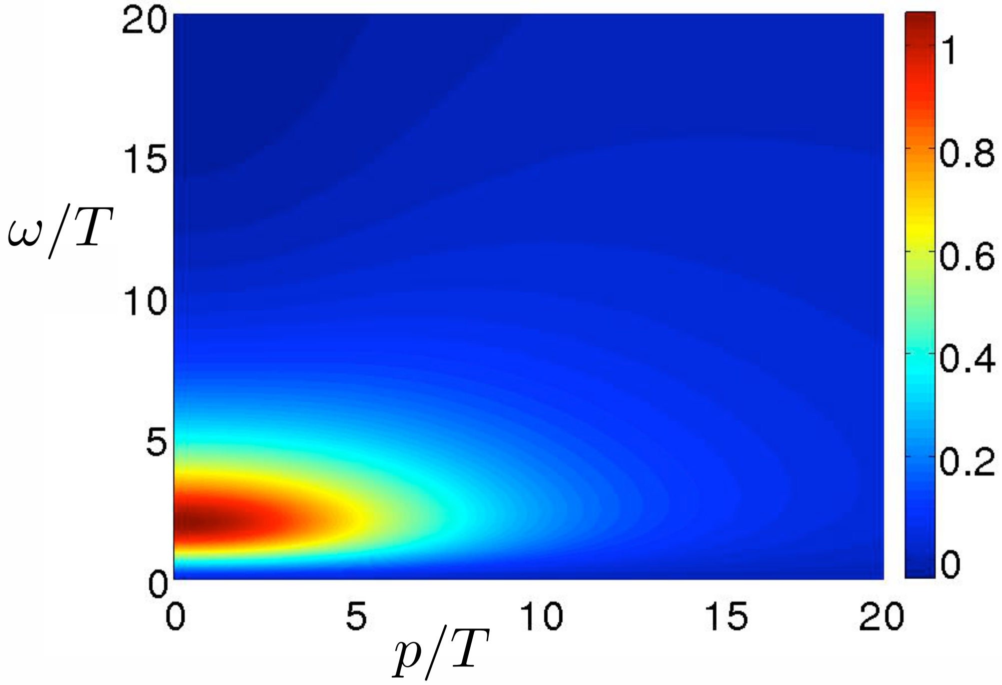


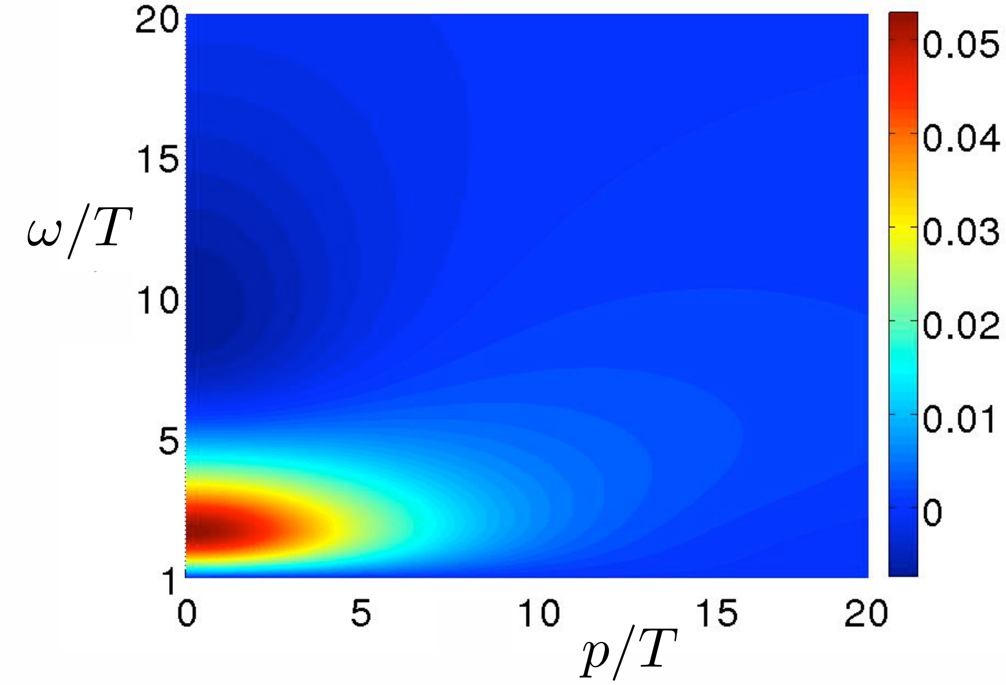
Viscosity in pure glue

spectral functions

M. Haas, Fister, JMP ´13

transversal

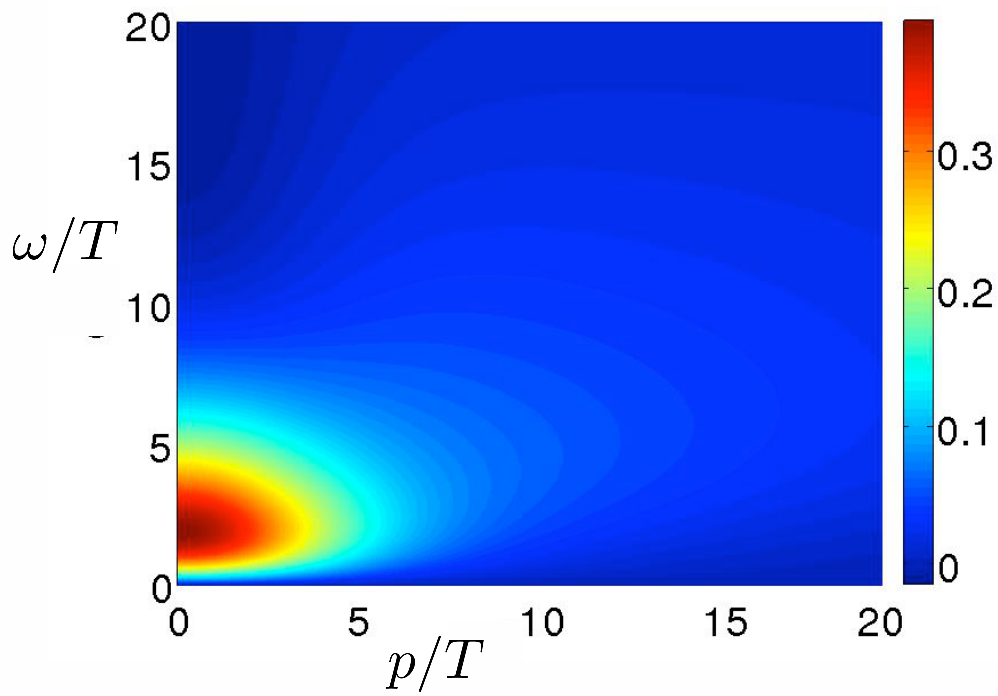


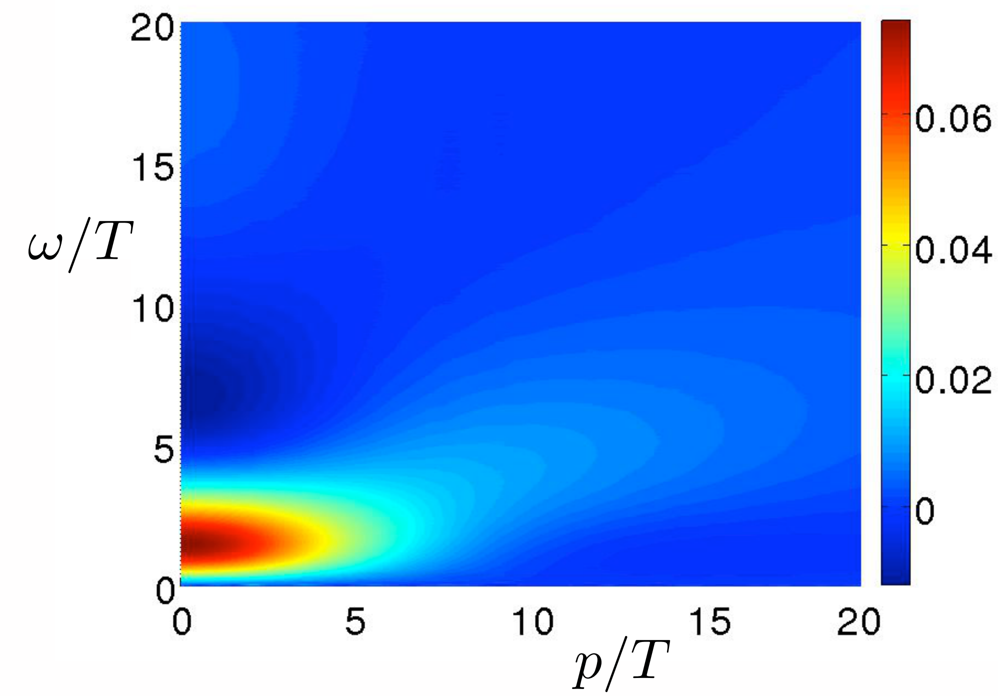


$\mathbf{T = 0.36T_c}$

longitudinal

$\mathbf{T = 1.8T_c}$

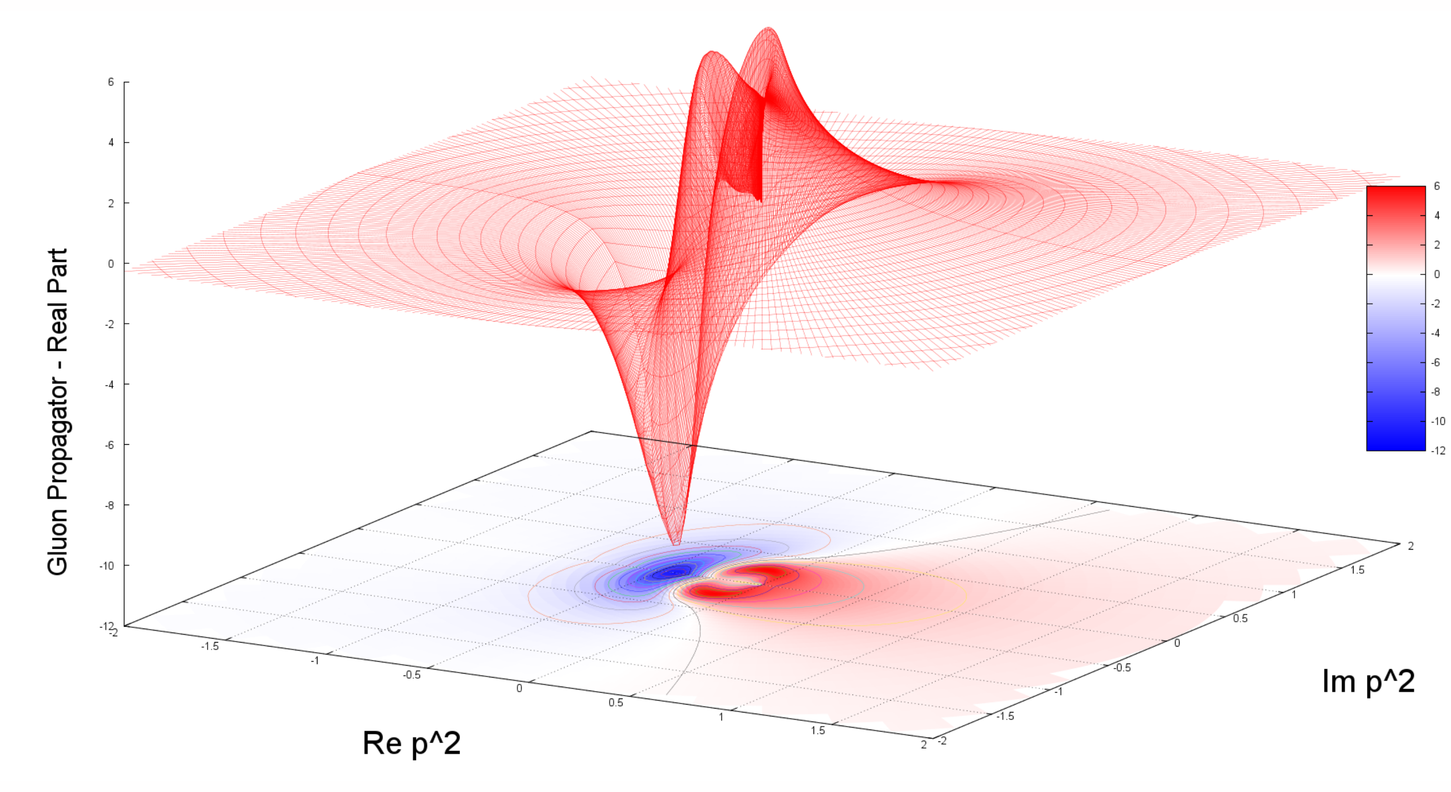


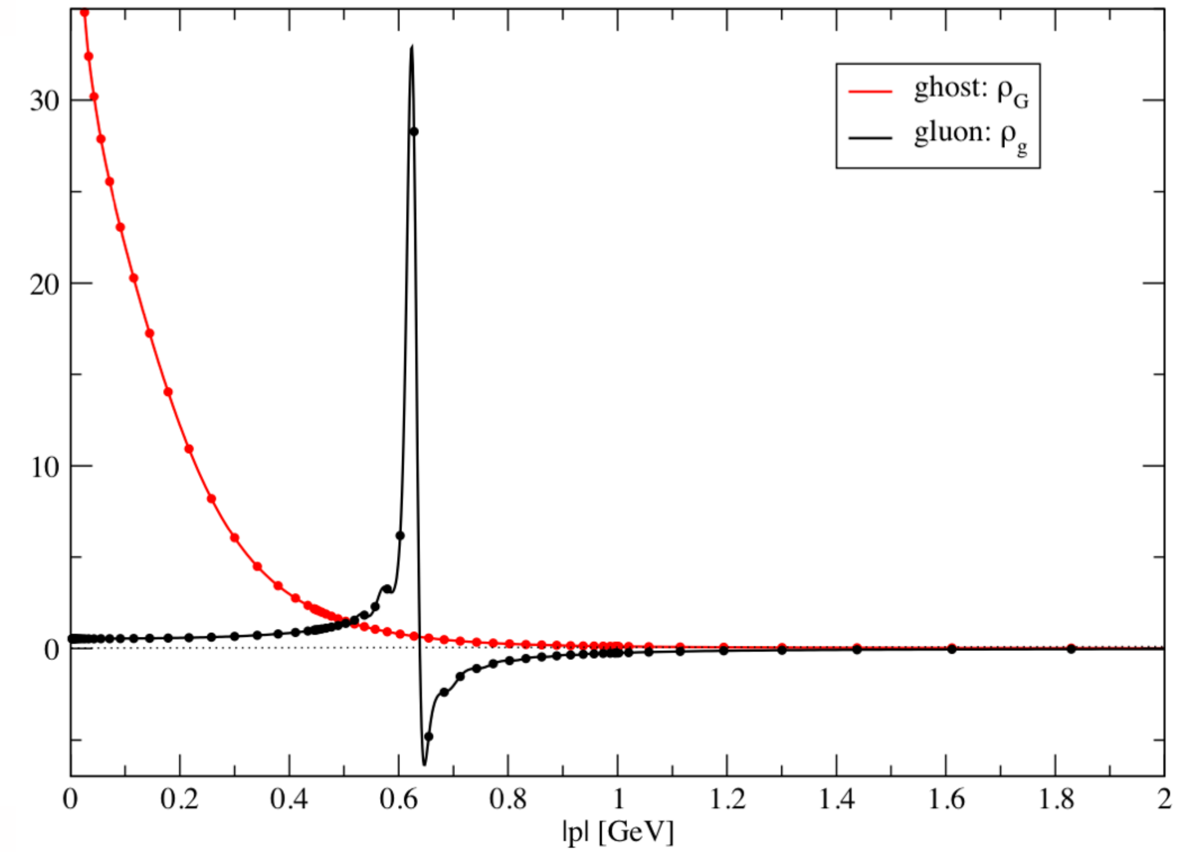


Viscosity in pure glue

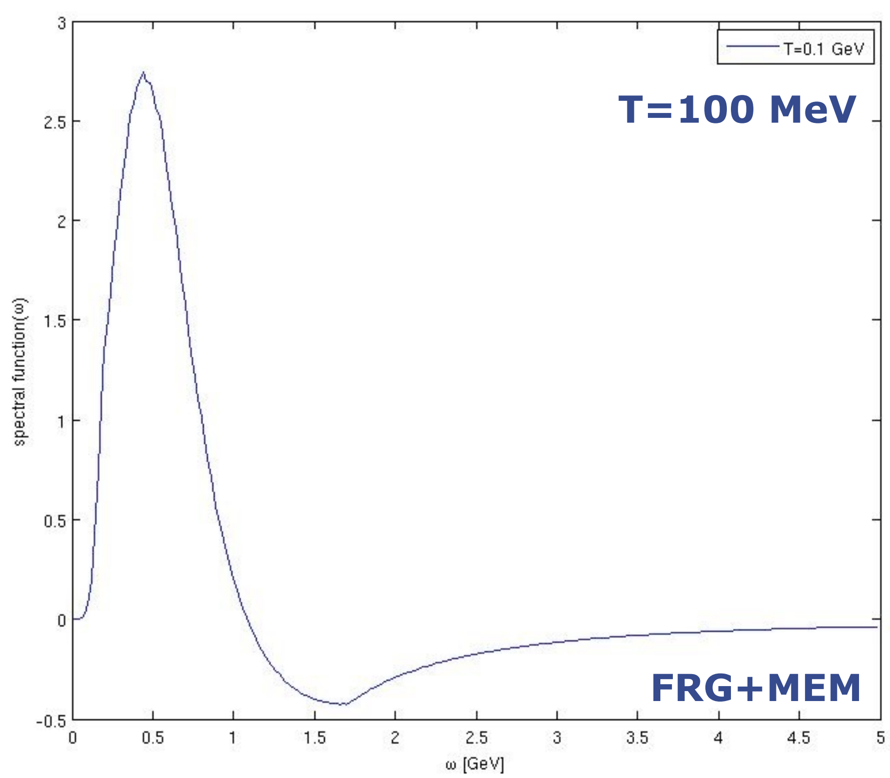
spectral functions

Complex DSEs





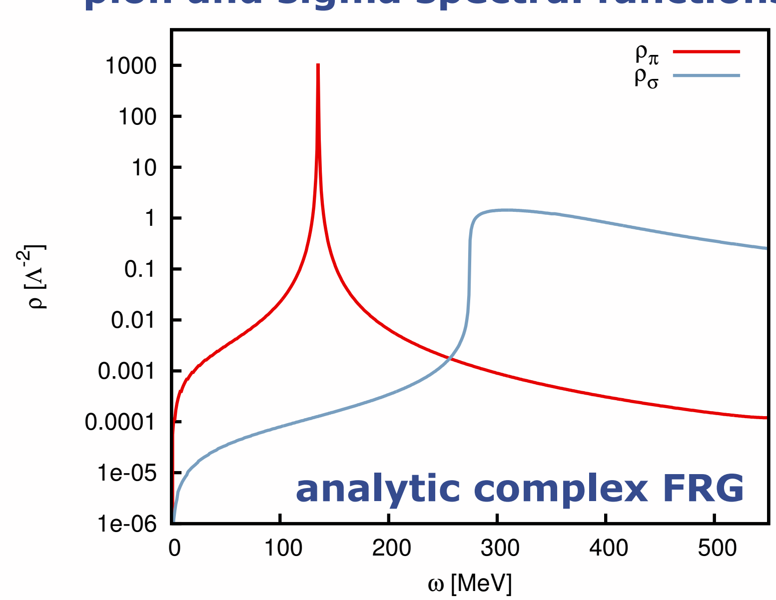
Strauss, Fischer, Kellermann ’12



transversal spectral function

M. Haas, Fister, JMP ´13

pion and sigma spectral functions

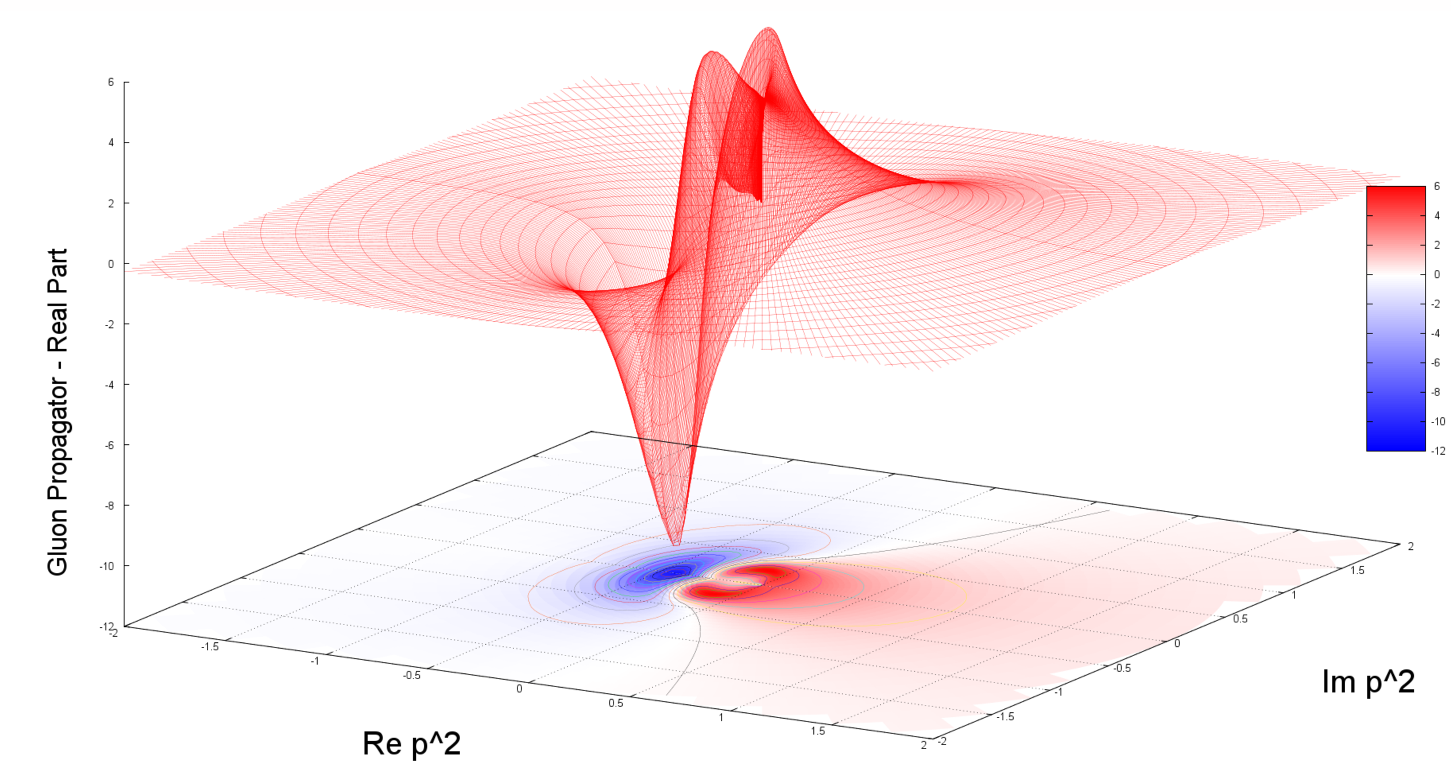


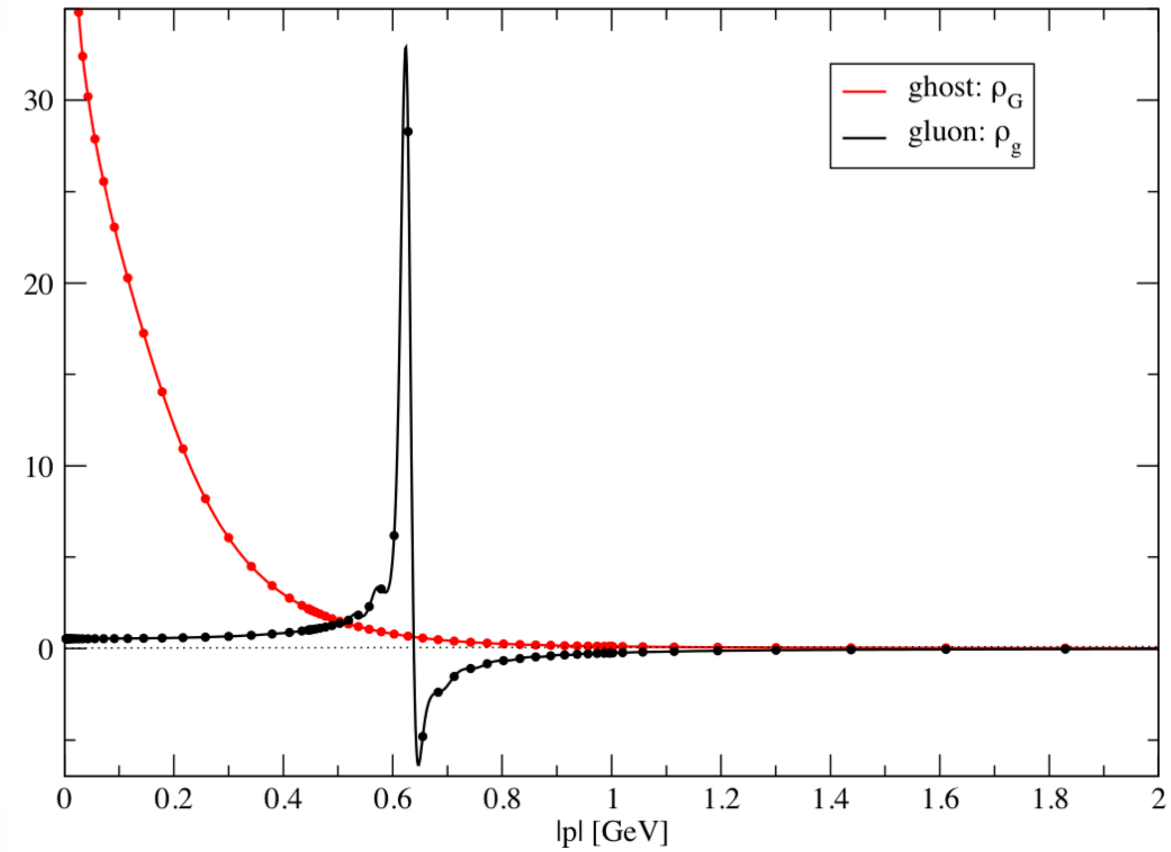
Kamikado, Strodthoff, von Smekal, Wambach ’13

Viscosity in pure glue

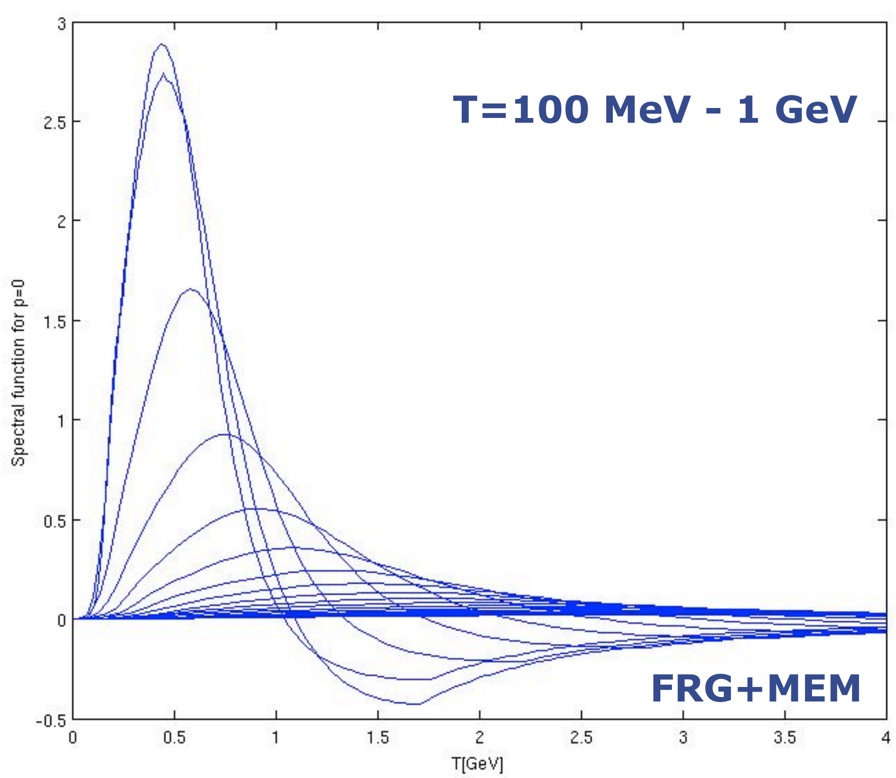
spectral functions

Complex DSEs





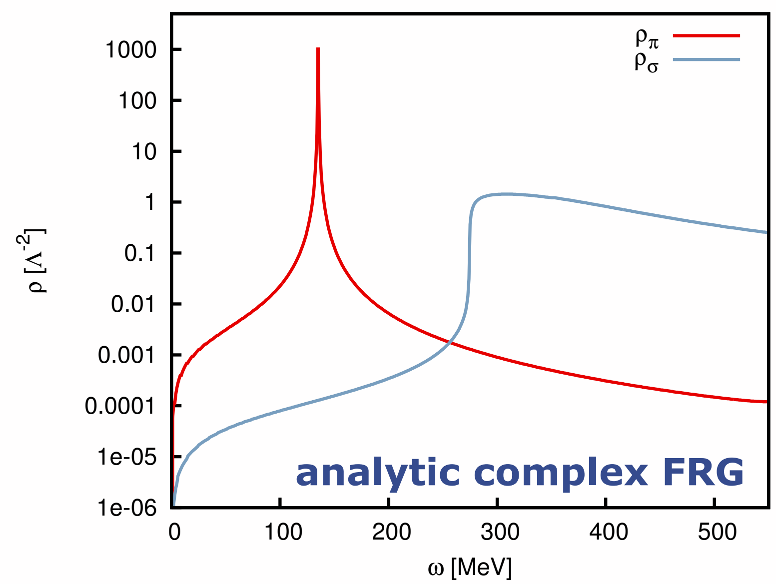
Strauss, Fischer, Kellermann ’12



transversal spectral function

M. Haas, Fister, JMP ´13

pion and sigma spectral functions

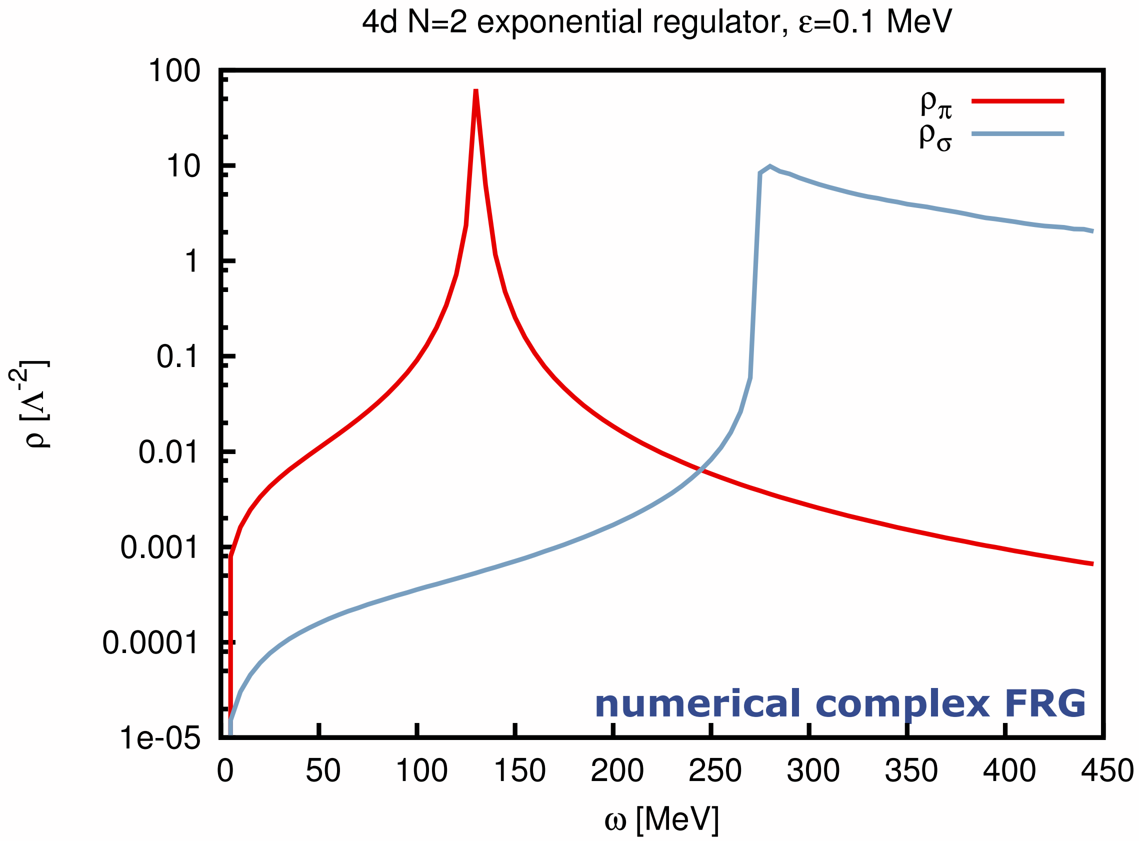


Kamikado, Strodthoff, von Smekal, Wambach ’13

Viscosity in pure glue

spectral functions

pion and sigma spectral functions



JMP, Strodthoff, in preparation

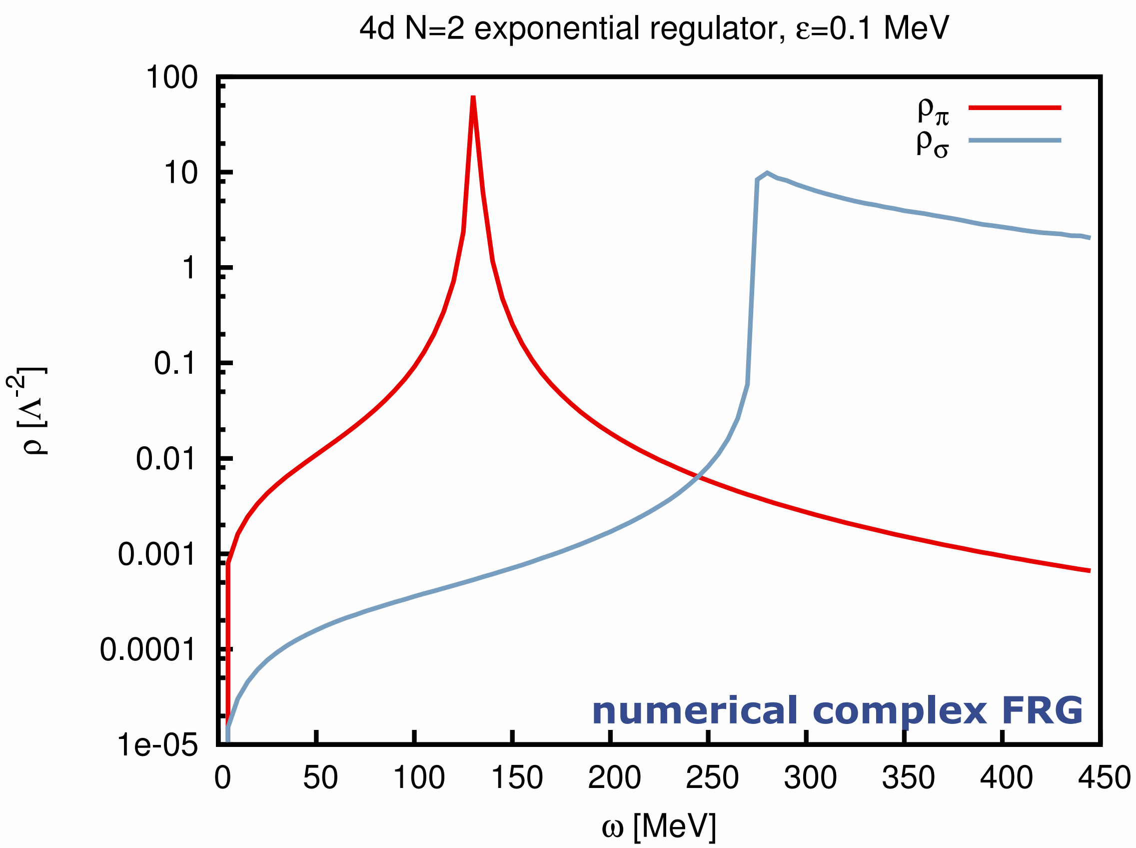
'Those are my methods (principles), and if you don't like them...well, I have others'

direct computation **Groucho Marx**

Viscosity in pure glue

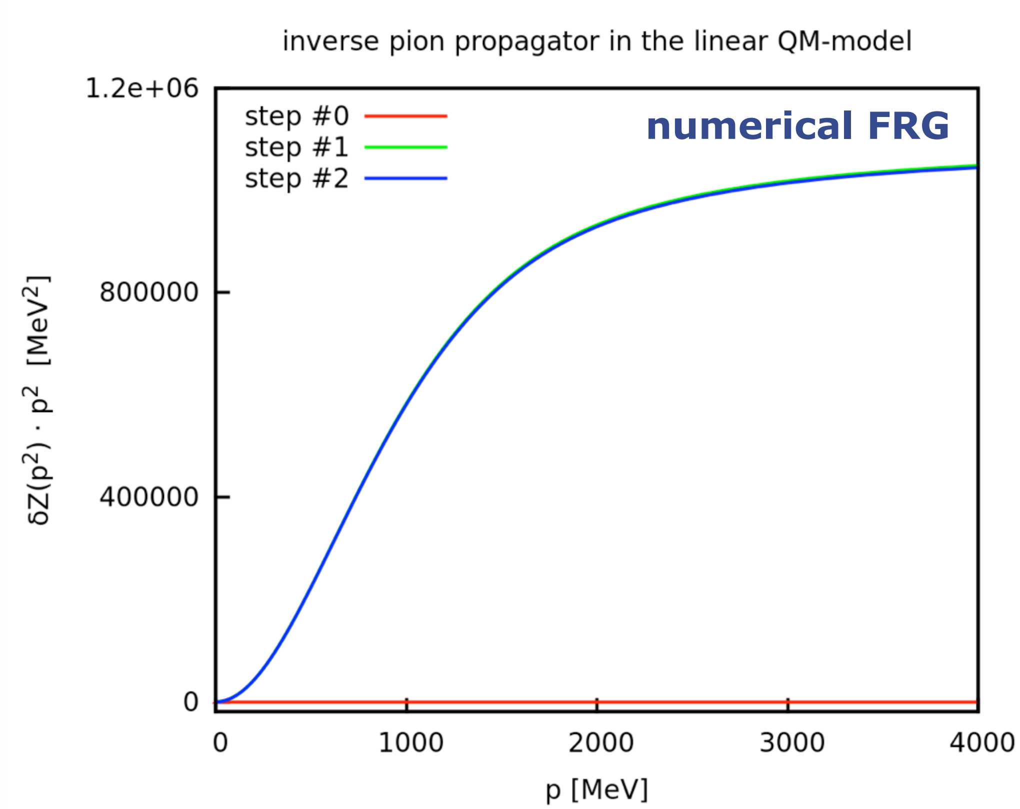
spectral functions

pion and sigma spectral functions



JMP, Strodthoff, in preparation

QM-model

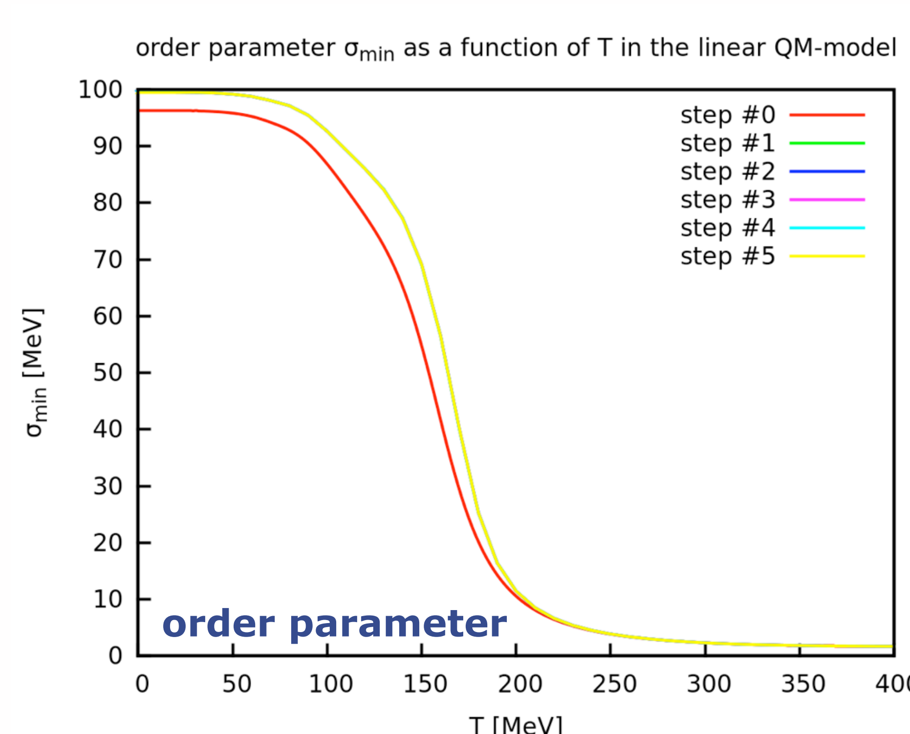


O(N)-model

iteration step	σ_0 [MeV]	δ_ρ [%]	m_{pole} [MeV]	m_{screen} [MeV]	δ_m [%]
0	93.55	0.0043	130.3113	136.7593	4.9
1	100.05	0.0028	126.6390	126.4590	0.14
5	99.38	0.0043	127.0347	127.0110	0.019

iteration step	σ_0 [MeV]	δ_ρ [%]	m_{pole} [MeV]	m_{screen} [MeV]	δ_m [%]
0	96.25	0.0052	91.4911	134.8281	47
1	99.56	0.0044	90.8841	91.1611	0.30
5	99.56	0.0073	90.9244	91.1551	0.25

QM-model



Helmboldt, JMP, Strodthoff, in preparation

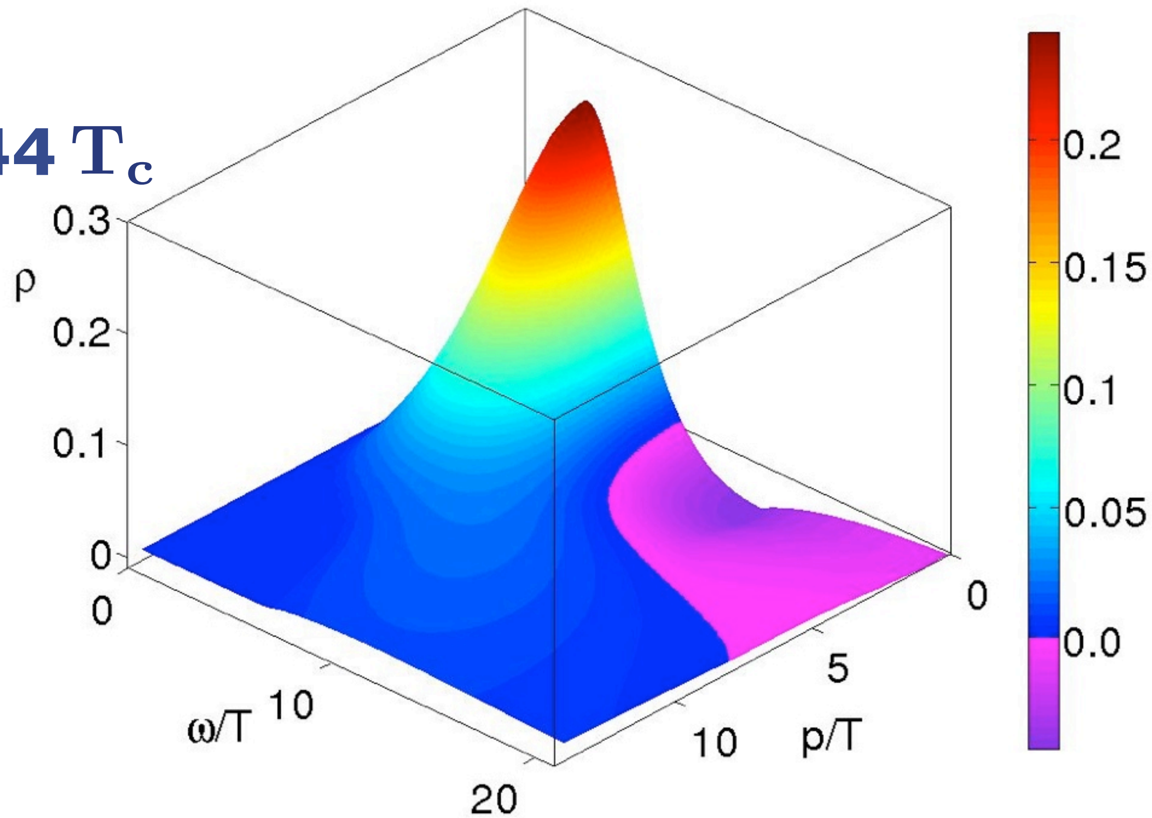
Viscosity in pure glue

spectral functions

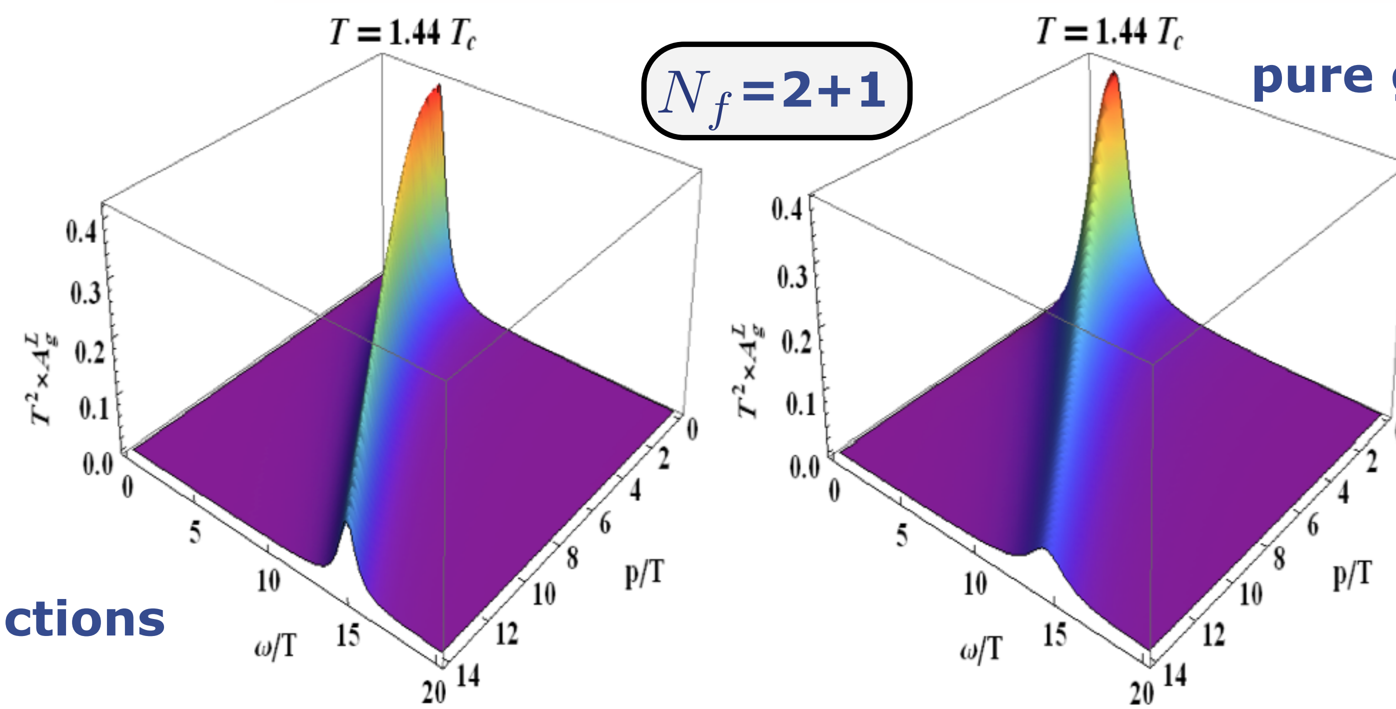
M. Haas, Fister, JMP ´13

transversal

T=1.44 $\mathrm{T_c}$



T=1.44 $\mathrm{T_c}$



N_f=2+1

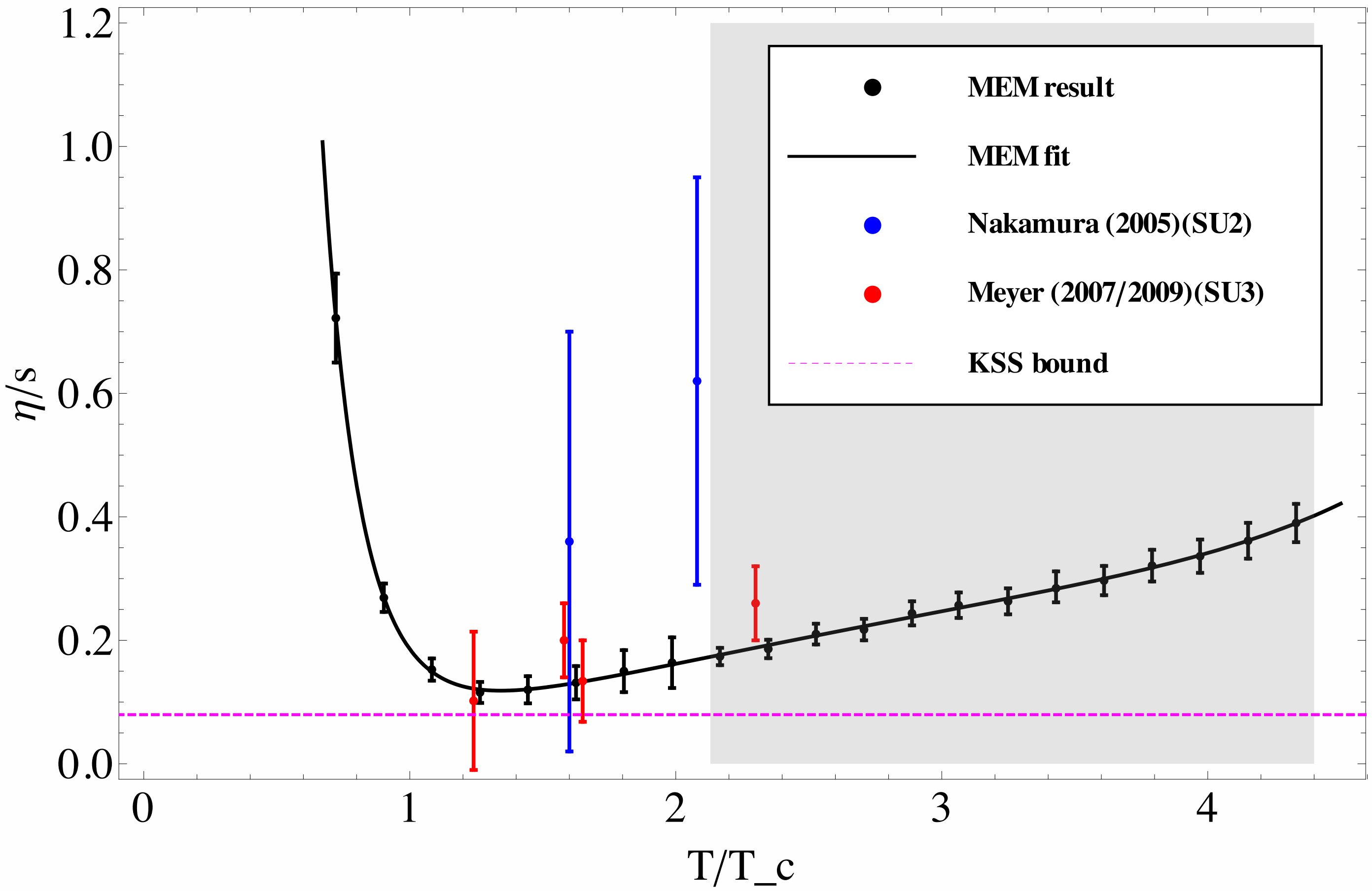
pure glue

PHSD spectral functions

Viscosity in pure glue

shear viscosity

M. Haas, Fister, JMP ´13



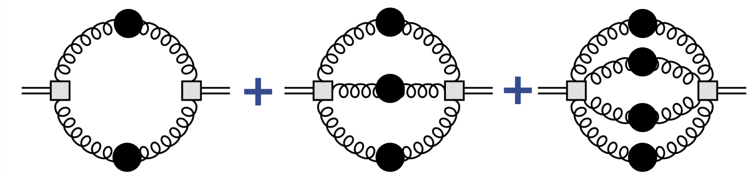
$T \lesssim 2T_c$: **MEM+optimised RG-scheme systematic error estimates**

Shaded area: MEM error estimates

Kubo relation

$$\eta = \frac{1}{20}\, \frac{d}{d\omega}\bigg|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

Diagrammatic representation

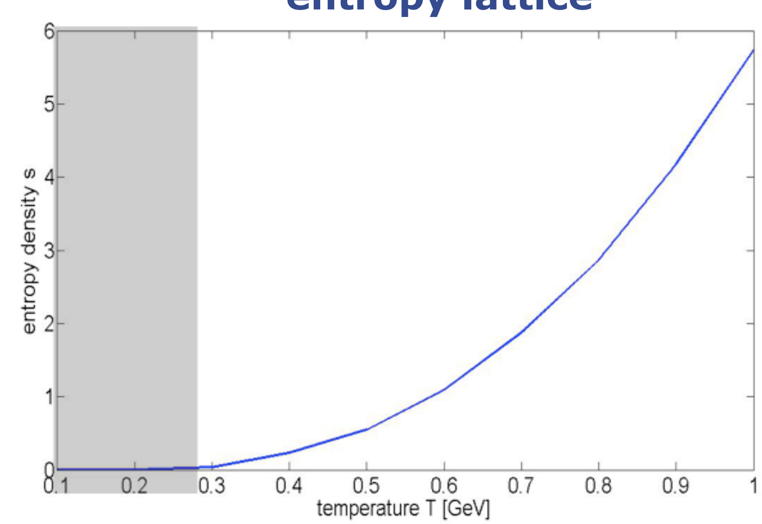
$\rho_{\pi\pi} =$ 

+ ··· closed form

minimum at T = 1.25T_c: $\frac{\eta}{s} = 1.45\frac{1}{4\pi}$

scale matching with QCD: $\frac{\eta}{s} = 2.27\frac{1}{4\pi}$

entropy lattice



H. Meyer '09

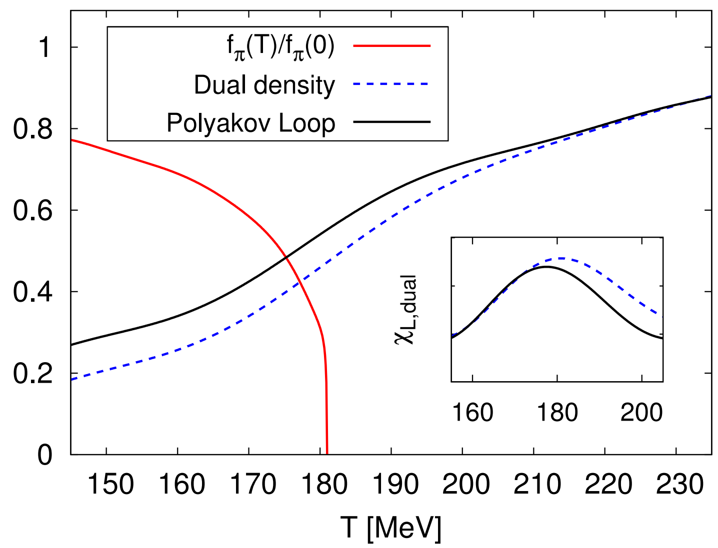
Boyd, Engels, Karsch '95

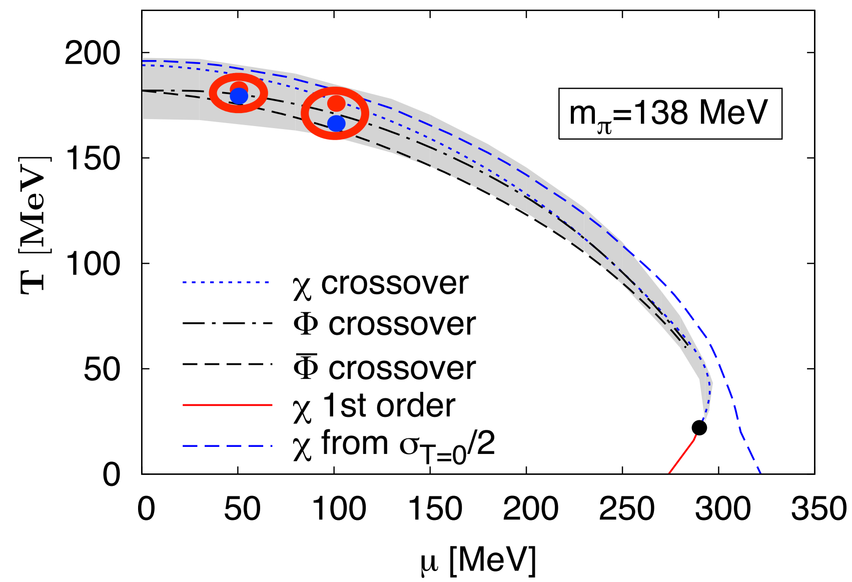
Summary & Outlook

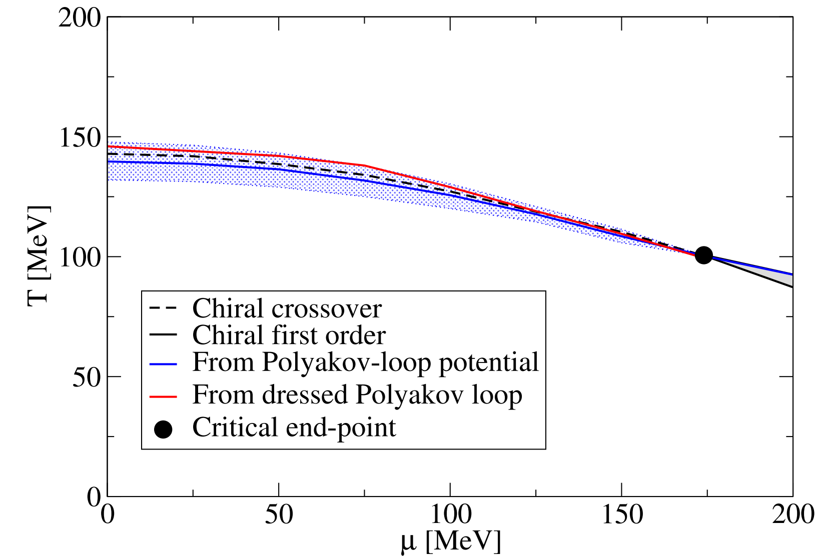
Summary & outlook

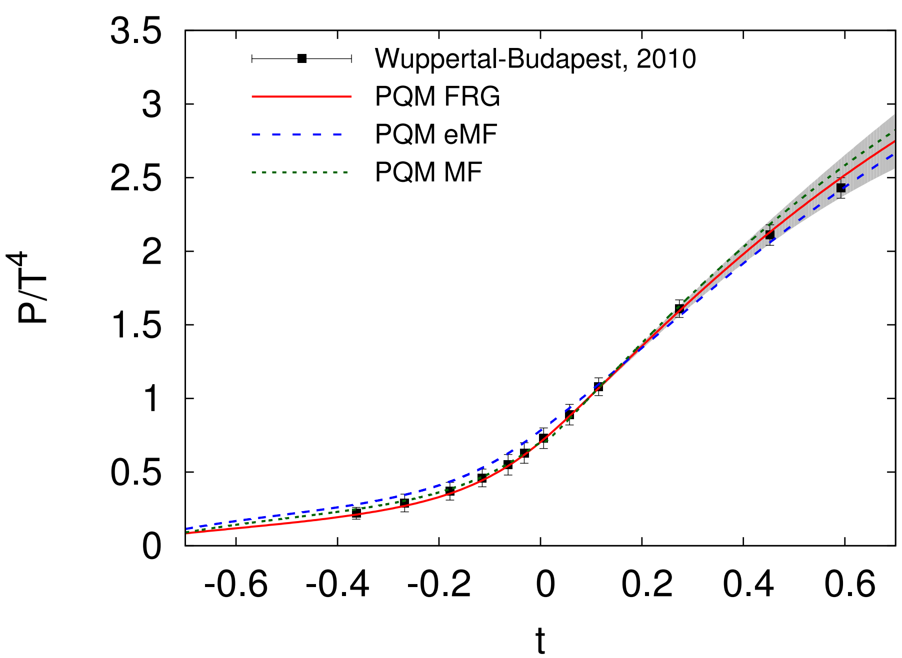
▪ Phase diagram of QCD

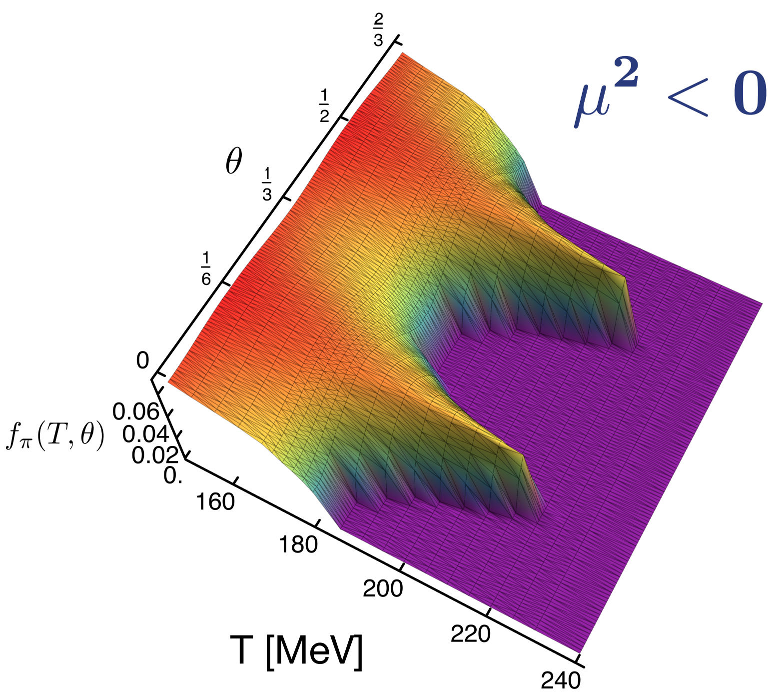
▪ Phase structure and thermodynamics at finite T & μ







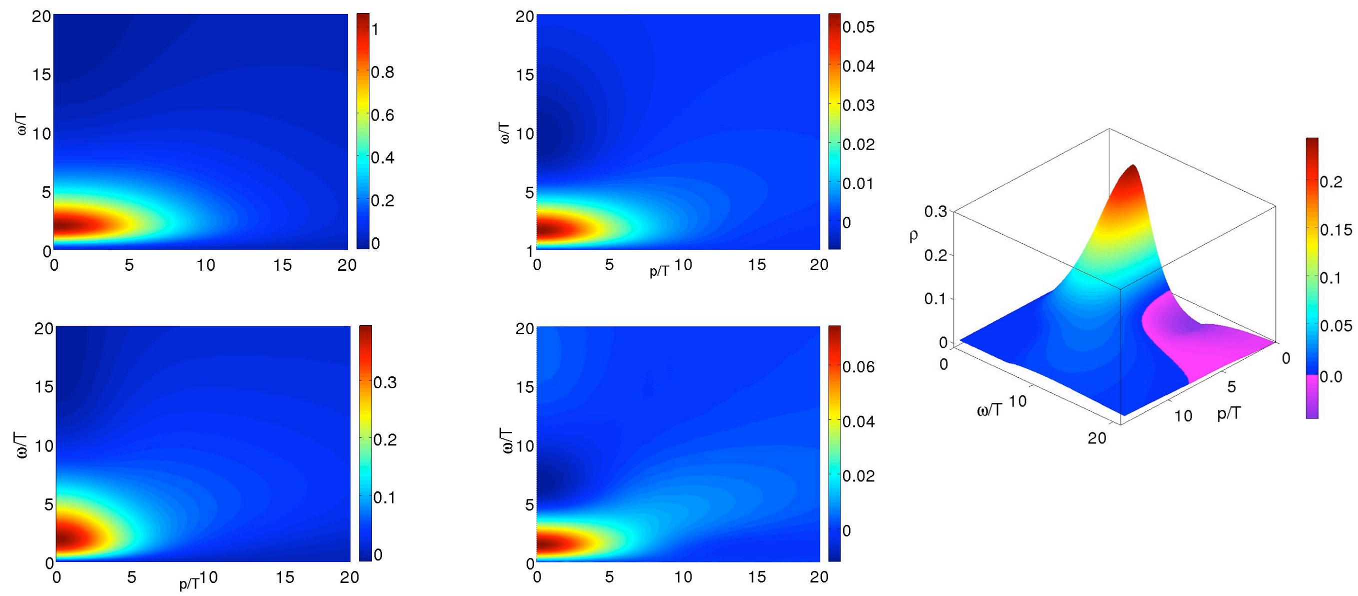




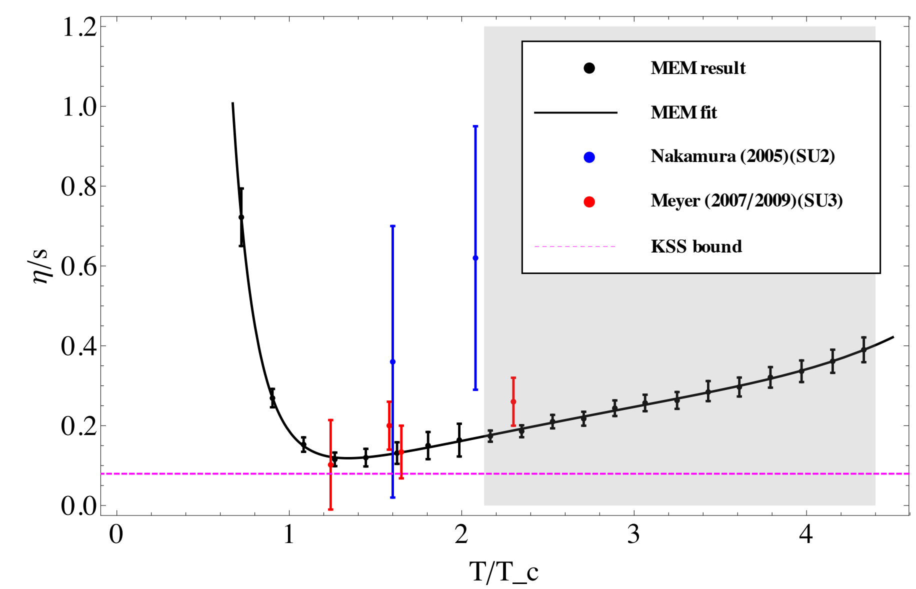
Summary & outlook

- **Phase diagram of QCD**
  - **Phase structure and thermodynamics at finite T & μ**
  - **2+1 flavours, baryons, phenomenology, non-eq. dynamics**

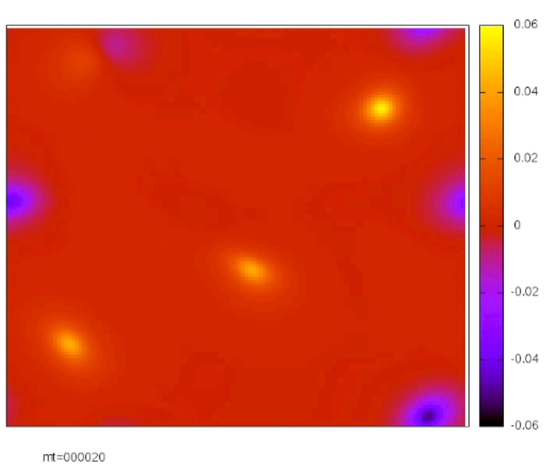
spectral functions



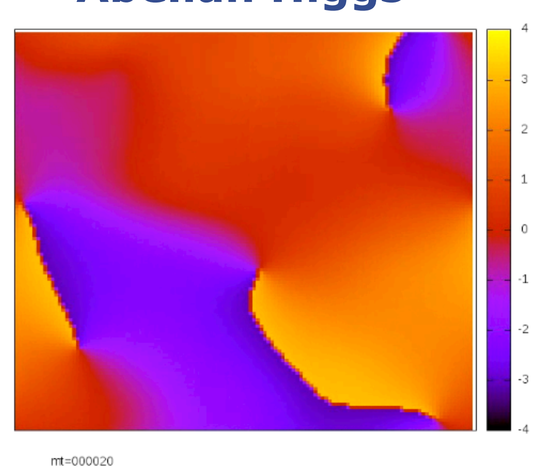
viscosity over entropy ratio



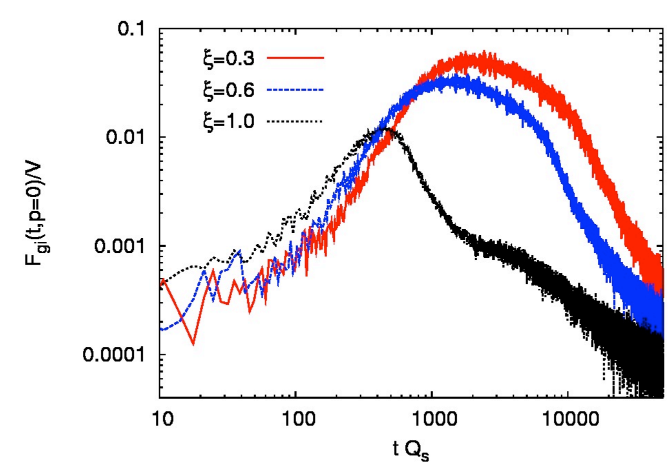
Abelian Higgs



magnetic field



phase of Higgs



talk to Larry

Summary & outlook

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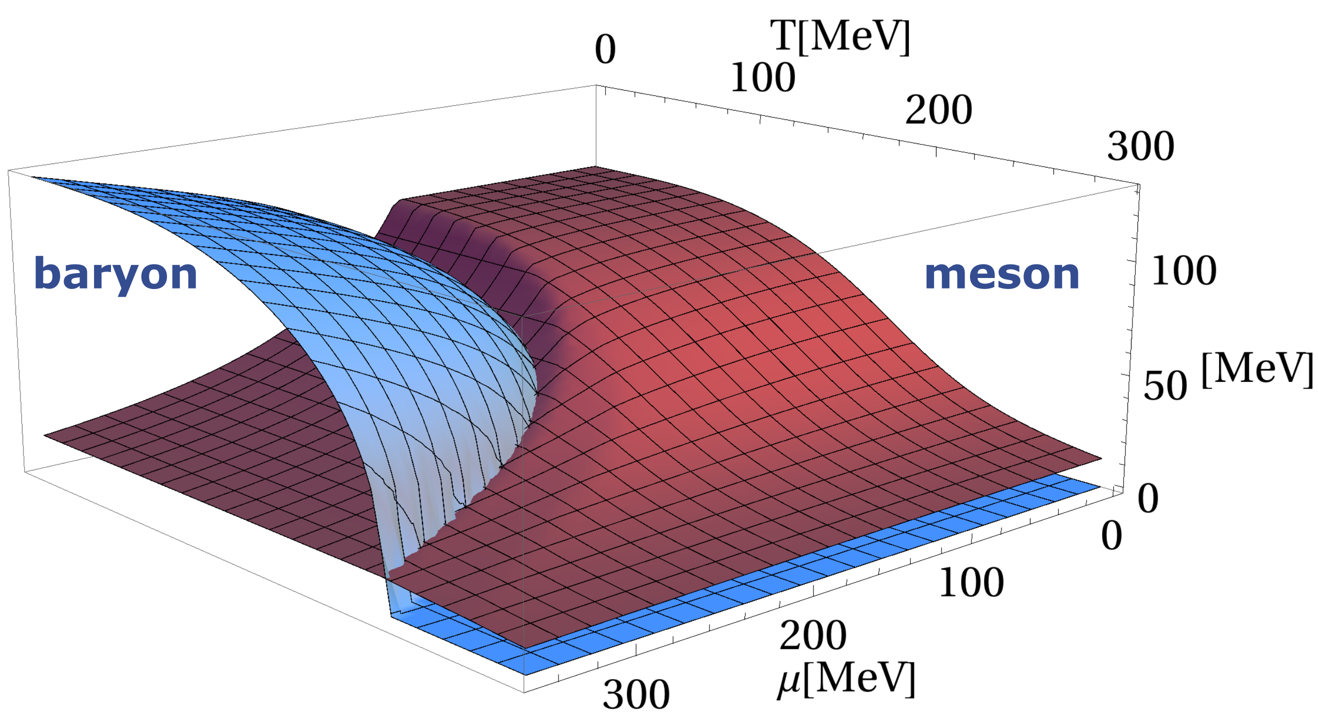
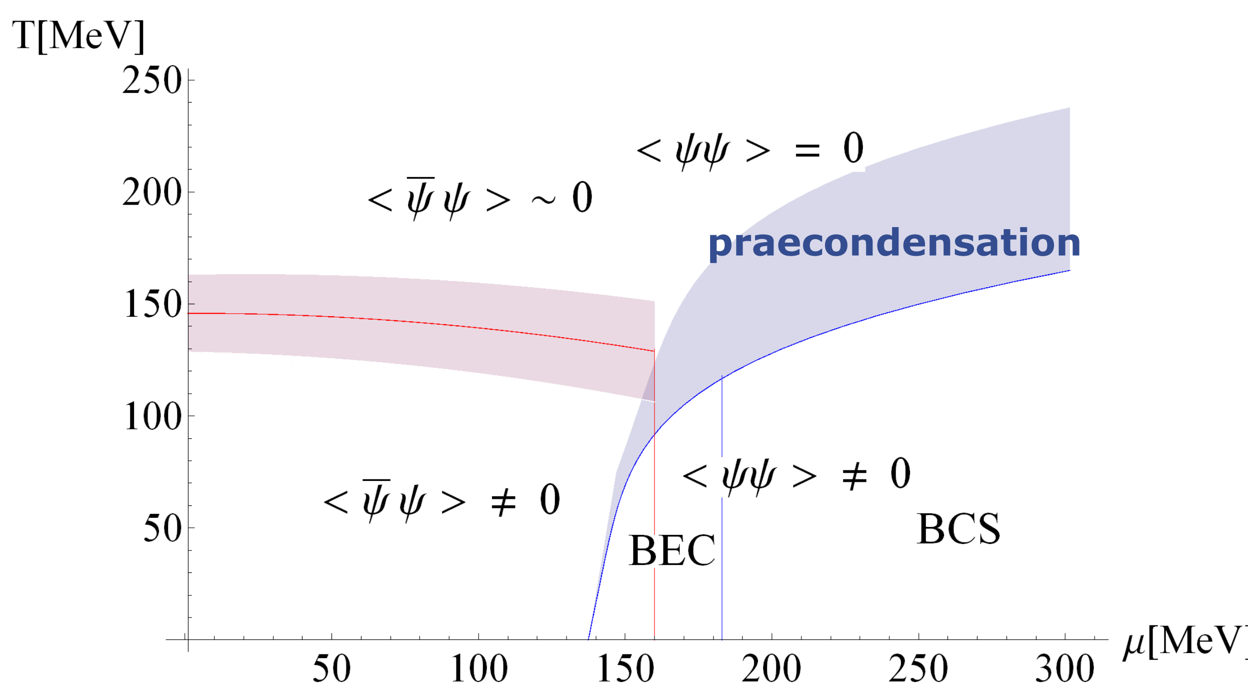
QCD meets cold quantum gases: two-colour QCD





mesons & baryons

Summary & outlook

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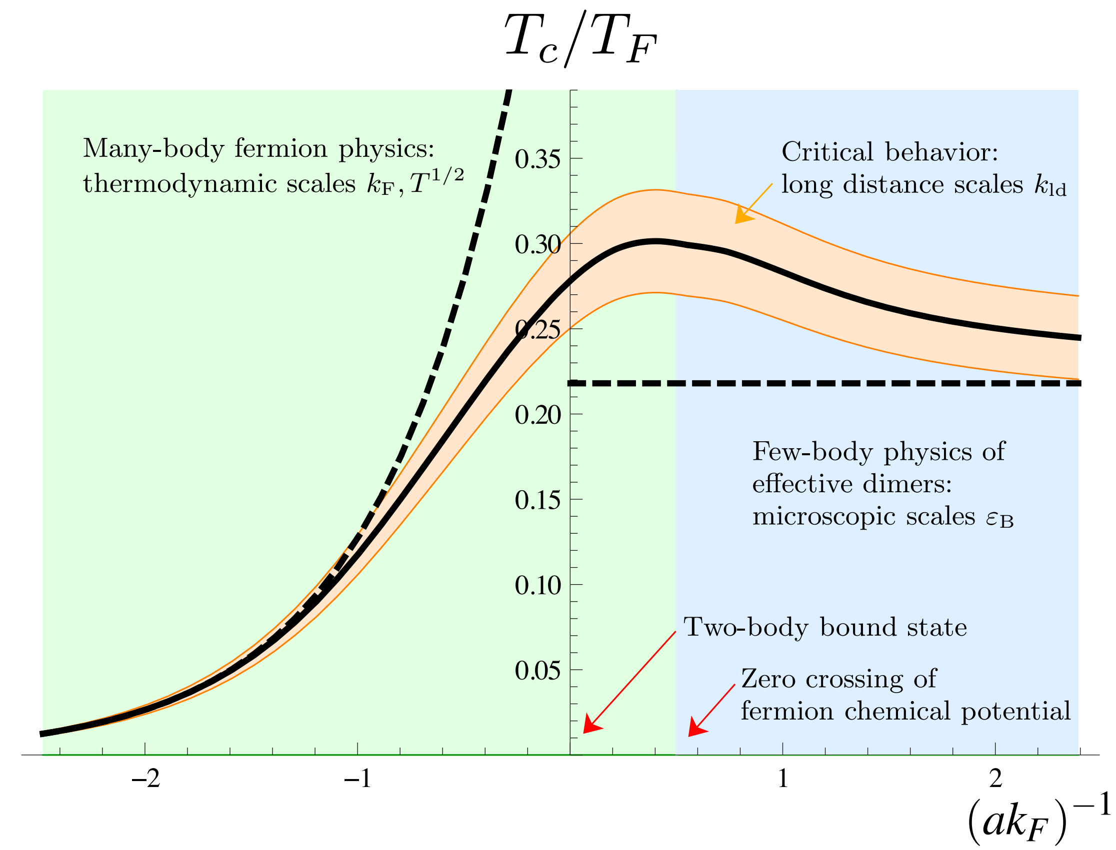
- **Phase diagram of cold quantum gases**
  - **quantitative precision, dynamics**
    - **close collaborations with experimental groups**
    - **close links to QCD**



Summary & outlook

- **Phase diagram of QCD**
  - **Phase structure and thermodynamics at finite T & μ**
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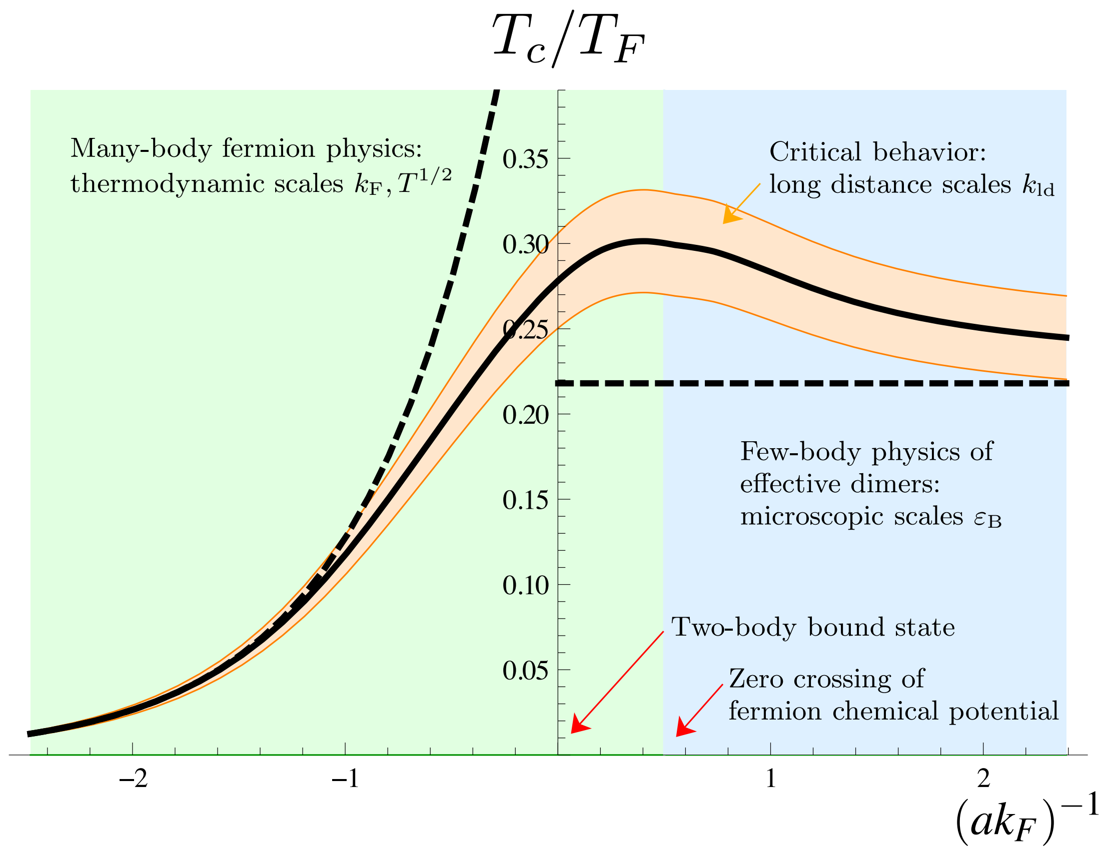
    - **close collaborations with experimental groups**

      'You name it, we do it'
      John Thomas
      QGP meets cold atoms-Episode III

    - **close links to QCD**



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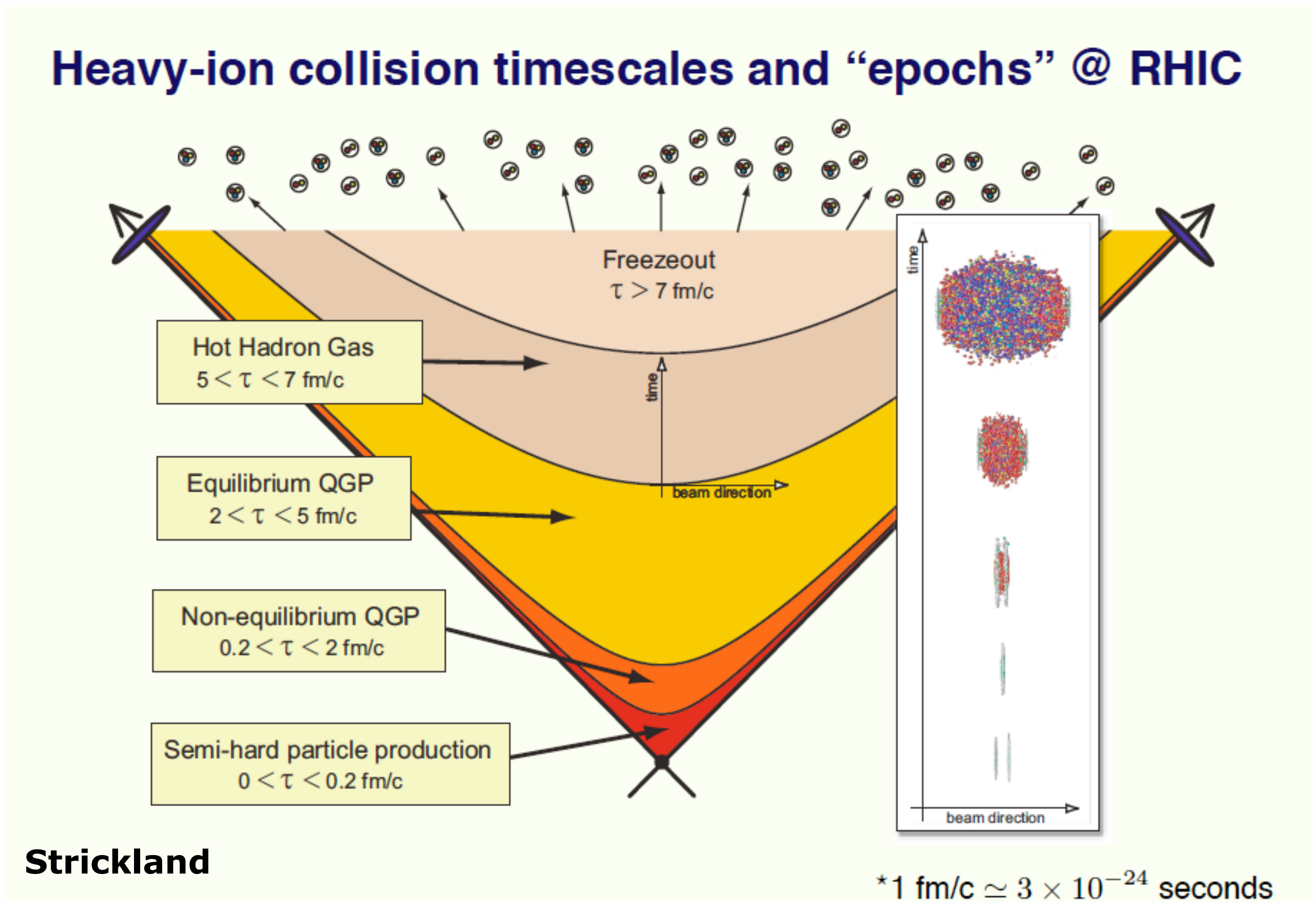
- **Hadronic properties**
  - **hadron spectrum & in medium modifications**
  - **low energy constants**

Additional material

Gauge dynamics far from equilibrium



Gauge dynamics far from equilibrium

Abelian Higgs model in 2+1 dim

Gasenzer, McLerran, JMP, Sexty '13

Classical action:

$$S[A_\mu, \phi] = -\int_x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* D^\mu \phi + V(\phi)\right]$$

ϕ **Higgs**

phase $\dfrac{\phi}{|\phi|} = e^{i\,\varphi}$

Gauge dynamics far from equilibrium

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ϕ Higgs

phase $\dfrac{\phi}{|\phi|} = e^{i\,\varphi}$

Classical action of Yang-Mills theory in diagonalisation gauges:

$$S_{\mathrm{YM}} \simeq \frac{1}{2}\int_x \mathrm{tr} F^2_{\bar\mu\bar\nu} + \frac{1}{2}\int_x \mathrm{tr}\,(D_{\bar\mu}A_2)^2$$

$$A_2 = A_2^c(x_0,x_1)$$

Wilson loop

$$\mathcal{W}_2 = \mathcal{P}\exp\left\{i\int_0^{L_2} dx_2\, A_2(x)\right\} = \exp\{i\,\phi\}$$

Vortex winding **phase**

$$n(\mathcal{S}) = \frac{1}{16\pi\, i}\oint_{\mathcal{S}} d^2x\, \epsilon_{ij}\mathrm{tr}\,\hat\phi\,\partial_i\hat\phi\,\partial_j\hat\phi \qquad \hat\phi = \frac{\phi}{||\phi||}$$

Quiz

Complex scalar vs Abelian Higgs

Gasenzer, McLerran, JMP, Sexty '13

phase φ of scalar field

'tachyonic' initial conditions

classical statistical lattice simulations

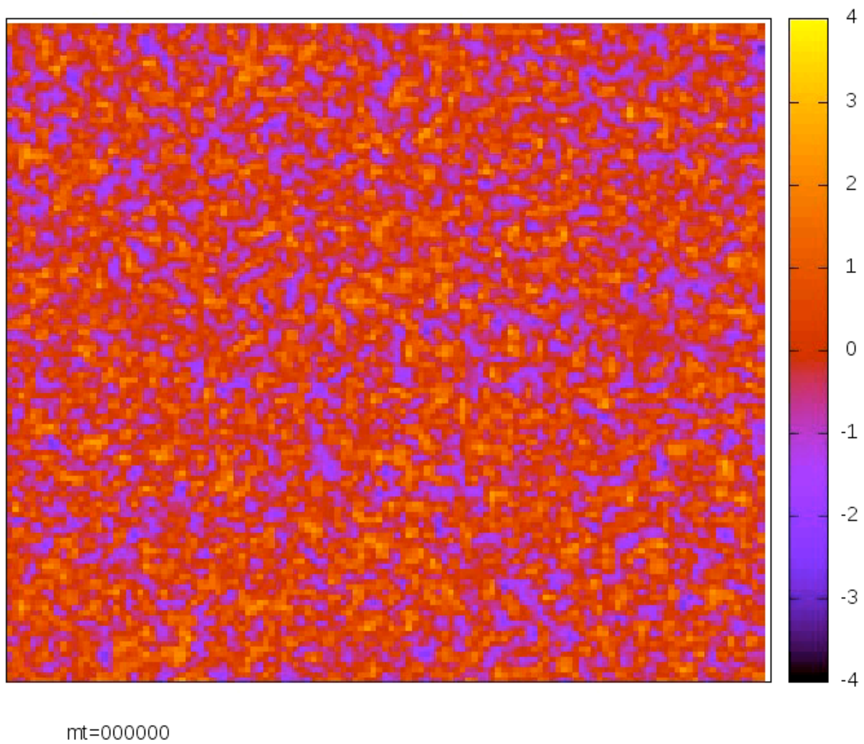
Which is which?

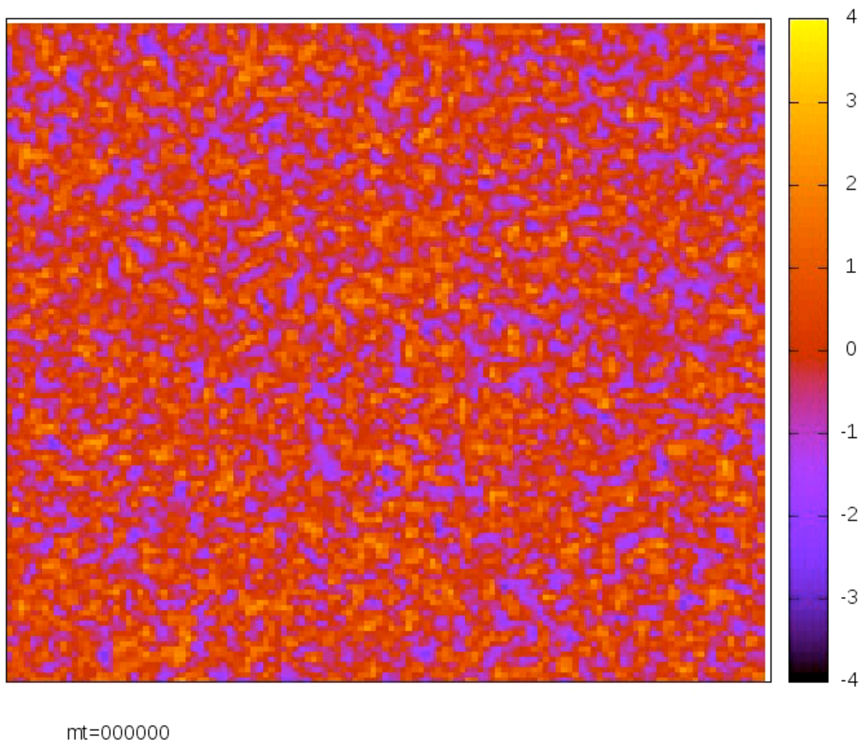
Quiz

Complex scalar vs Abelian Higgs

Gasenzer, McLerran, JMP, Sexty '13

phase φ of scalar field





'tachyonic' initial conditions

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Which is which?

Gauge dynamics far from equilibrium

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magnetic field

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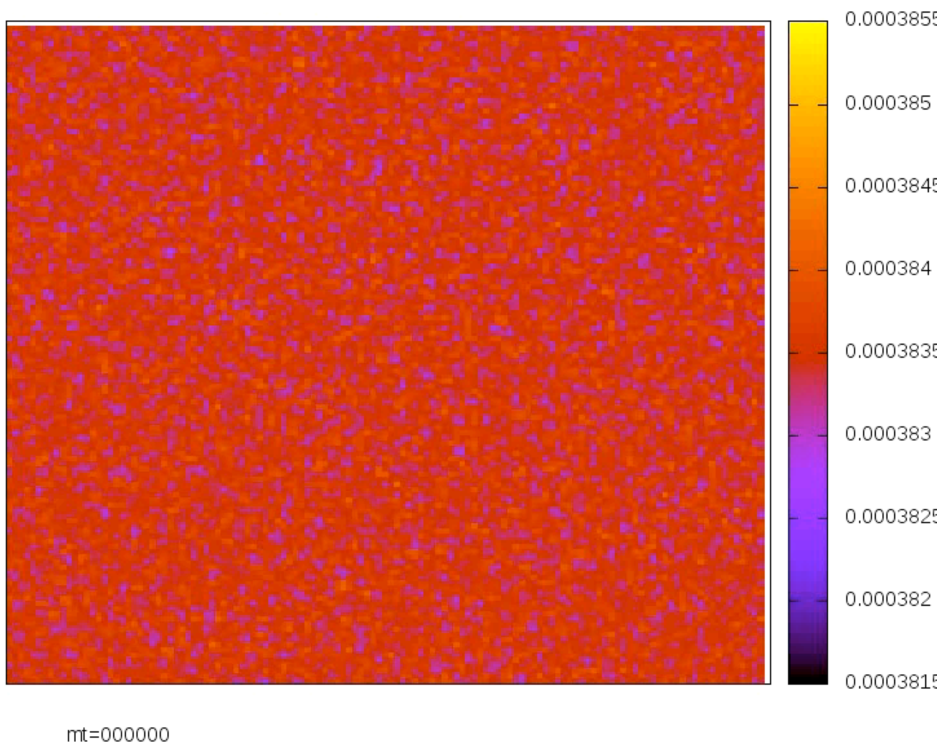
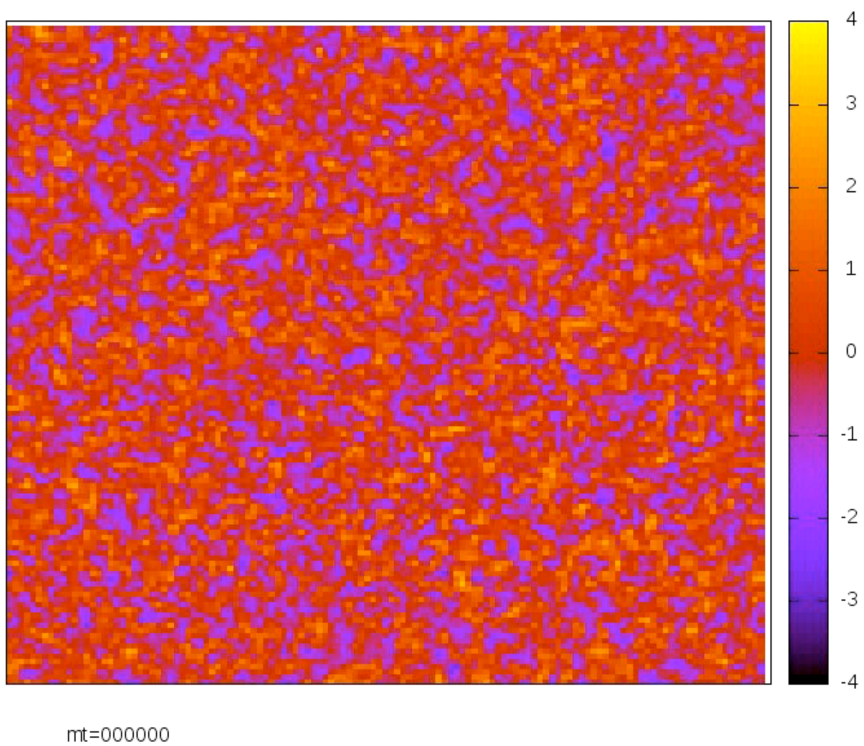


phase of Higgs



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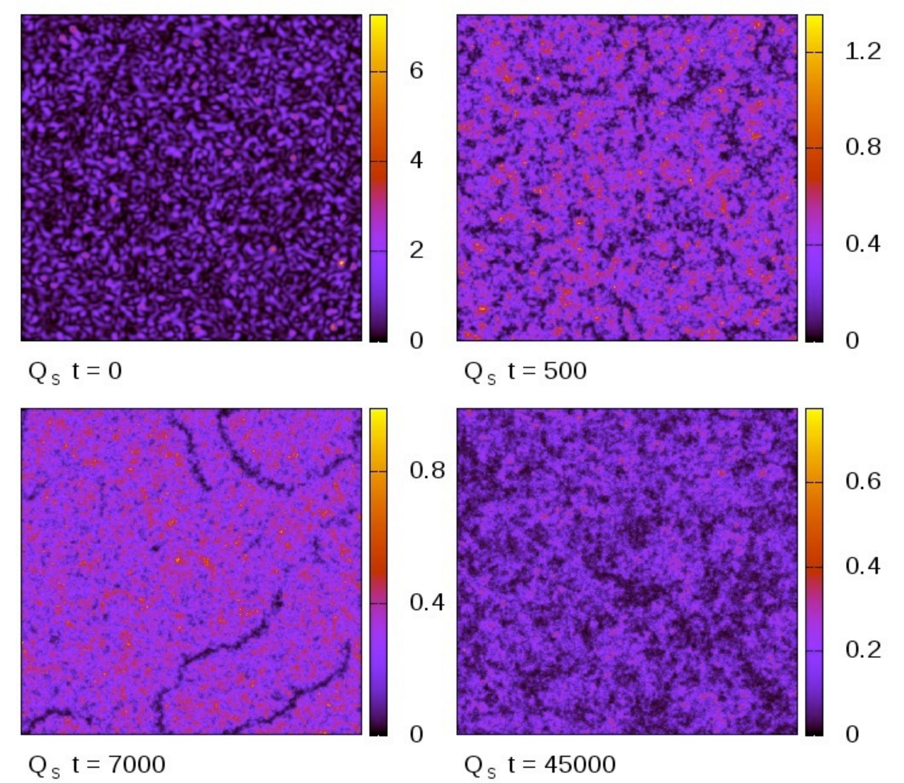
'overpopulation' initial conditions

modulus of Higgs

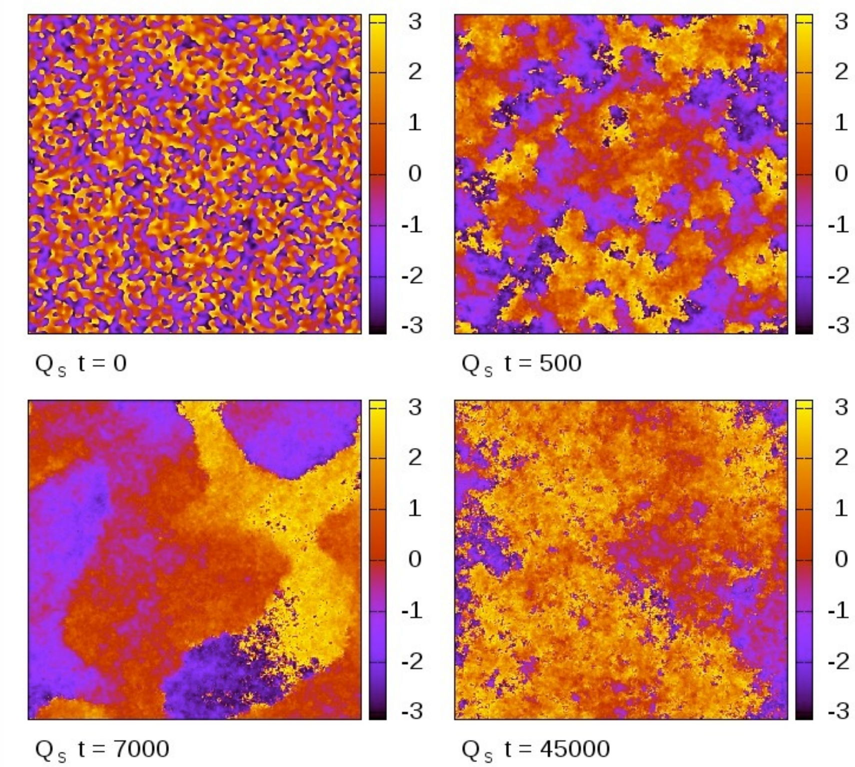


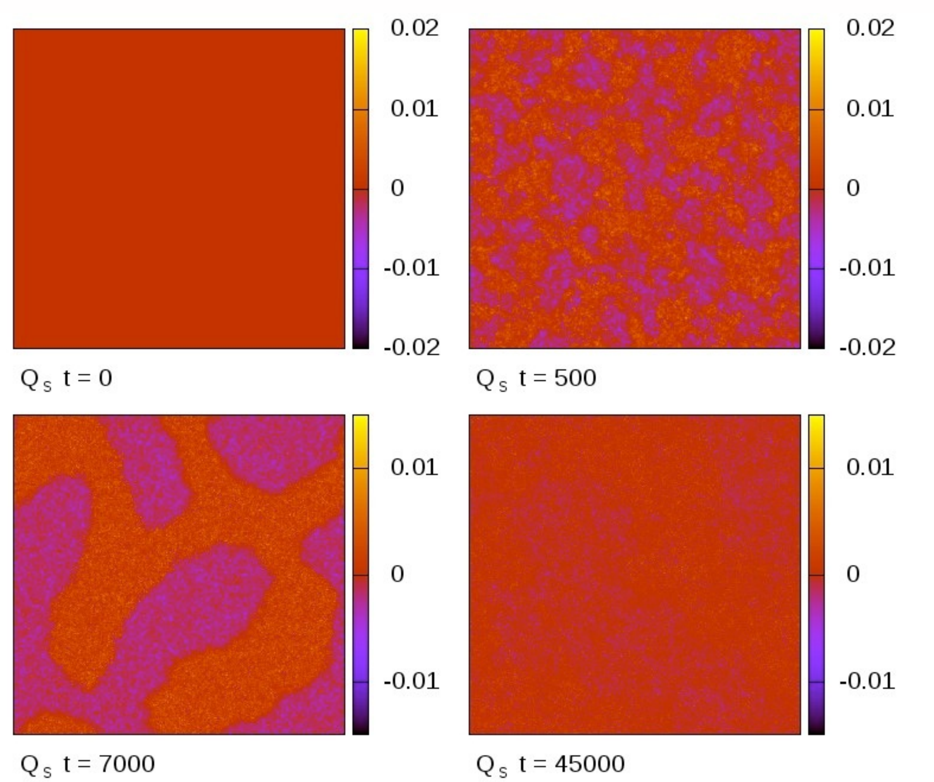
$$\xi = 0.025$$

coupling

$$\xi = \frac{6e^2}{\lambda}$$

relative phase





charge

$$\varphi^U(\vec{x}, t) = \arg(G^U(\vec{0}, \vec{x}, t))$$

relative phase

$$G^U(\vec{x}, \vec{y}, t) = \langle \phi(\vec{x}, t) U(\vec{x}, \vec{y}, t) \phi(\vec{y}, t)^* \rangle_{\mathrm{cl}}$$

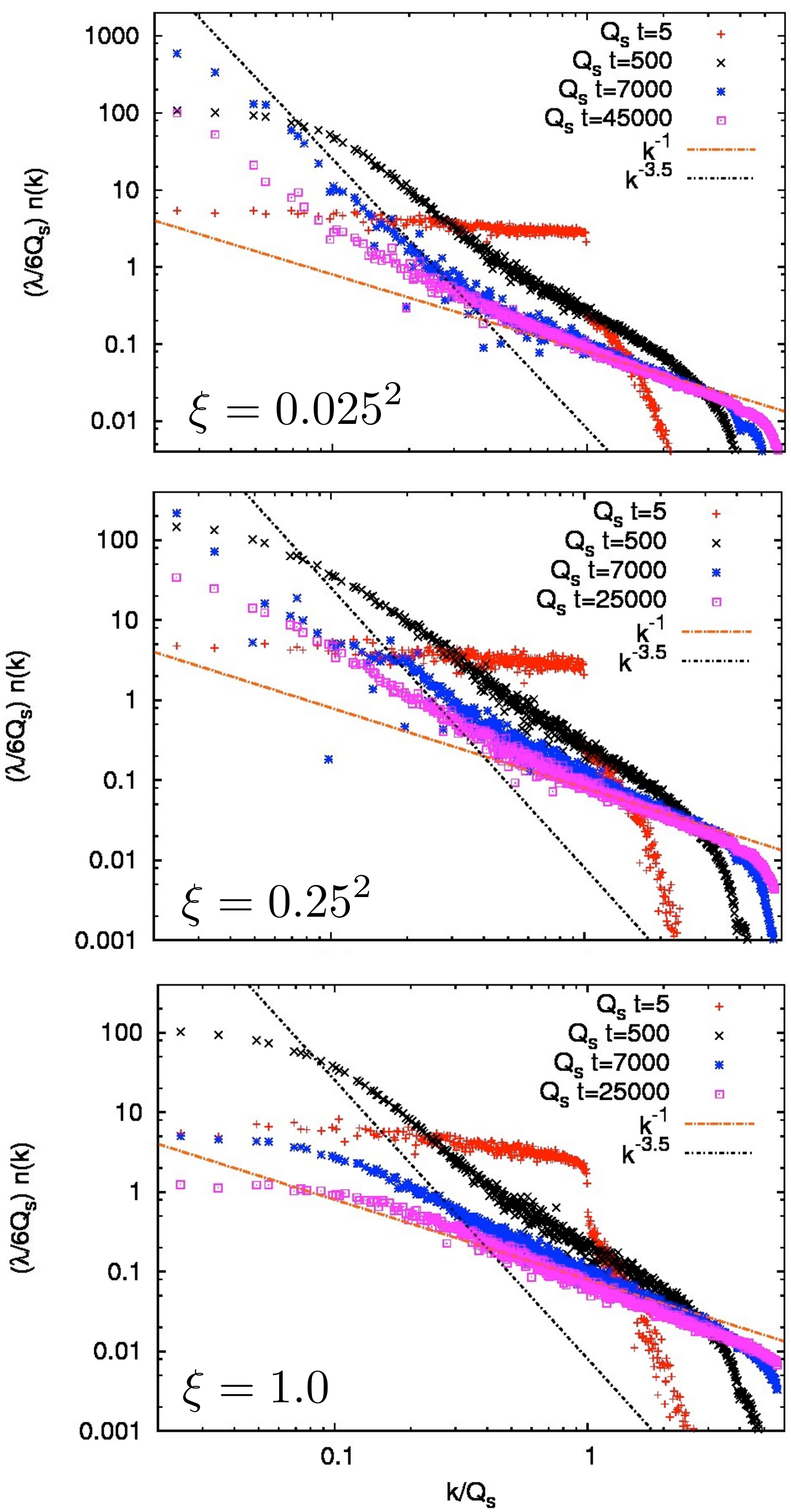
parallel transport U

Gasenzer, McLerran, JMP, Sexty '13

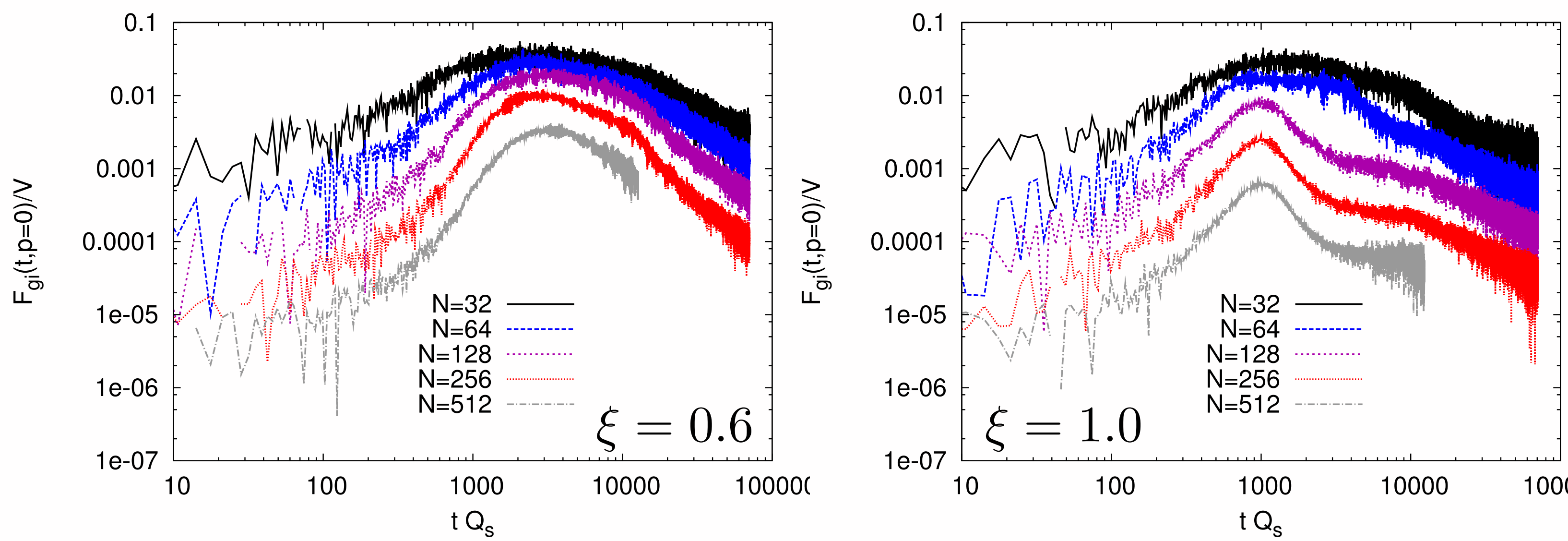
Gauge dynamics far from equilibrium

Abelian Higgs model in 2+1 dim

'overpopulation' initial conditions **Gasenzer, McLerran, JMP, Sexty '13**



$$\frac{F_{\rm gi}(p=0)}{V} = \frac{1}{V^2}\int dxdy\phi^*(x)U(x,y)\phi(y)$$



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$$\xi = \frac{6e^2}{\lambda}$$