Theory and phenomenology of quarkyonic matter

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Synopsis

What is quarkyonic matter (my definition)

Large N_c : A short introduction

An estimate from percolation theory

Towards a pheonomenolgy of quarkyonic matter in supernova and at FAIR

What is "Quarkyonic matter"

Name introduced in L.MacLerran,R.Pisarski, NPA796 (2007) 83-100
300 Citations, 5 conferences, 1 wikipedia entry. So its a big deal! but definitions found in the literature so far include,these and more...

- Coexistance between Confinement+pQCD (Mclerran,Pisarski,2007)
- Confinement+Chiral restoration (Fukushima,McLerran, 2008)
- Deconfinement+Chirally broken (Satz, Csernai,...)
- Chiral spiral inhomogeneities (Kojo,Pisarski,Tsvelik, 2009)
- Generic chirally inhomogeneus regions (Buballa et al)
- Condensation of "baryons" in 2-color QCD (Hands,Skullerud,Giudice)

All relevant for “high density low temperature” matter, produced in neutron stars or “low energy” uRHICs

- RHIC low energy scan
- SPS experiment NA61
- FAIR
- NICA

The issue: QCD at $\mu_Q \geq \Lambda_{QCD}, T < T_c$ is really not understood

Hadronic or EFTs (σ,NJL,PNJL etc): based under the assumption that $p_i - p_j \ll \Lambda_{funamental}$
Only scale in QCD is $\Lambda_{fundamental} = \Lambda_{QCD}$, and $p_i - p_j \sim \mu_Q \sim \Lambda_{QCD}$

So EFT at $\mu_Q \simeq \Lambda_{QCD}$ means Taylor-expanding around 1!
For any operator $\hat{O}(x)$ (e.g. $q, P, ...$) Not quaranteed $\hat{O}^n \ll \hat{O}^{n-1}$ for any N

Lattice QCD has the sign problem, any expansion is good for $\mu_q \ll T$

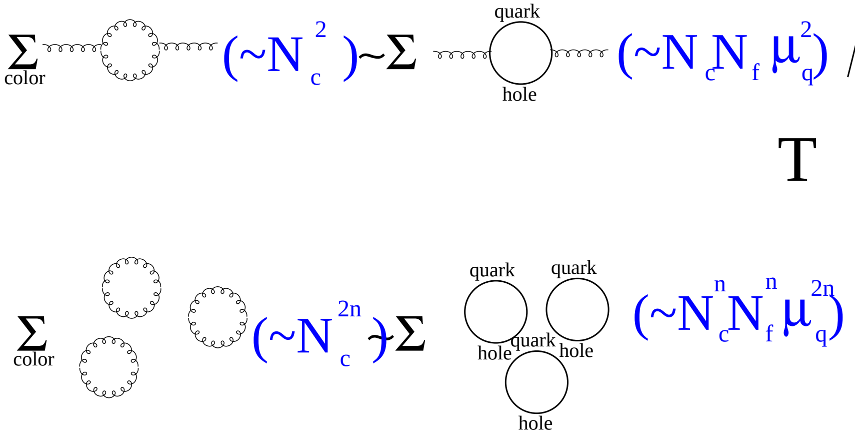
AdS/CFT apart from the many unrealistic assumptions, classical Gauge dual depends on $N_c \to \infty$, on which more later

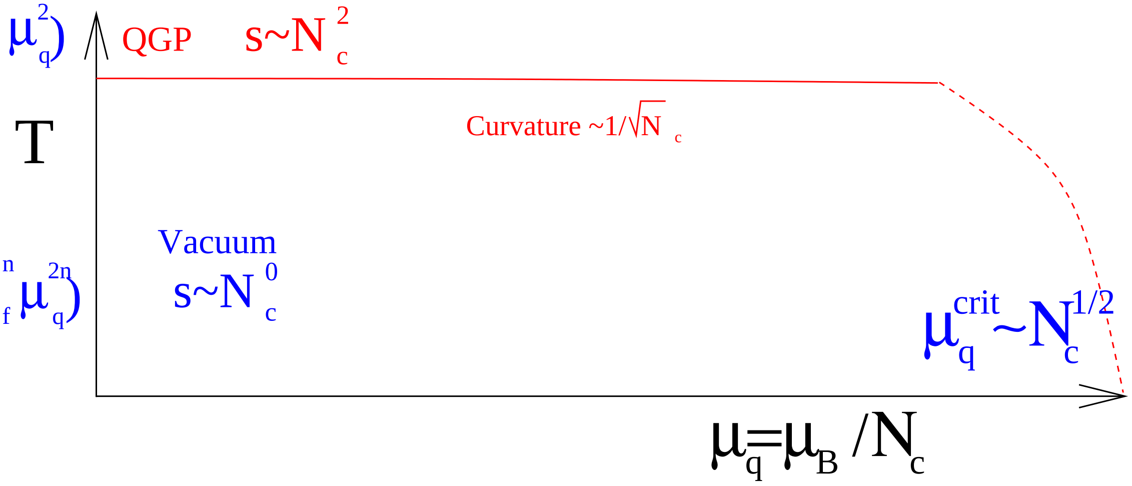
The only hyerarchy that seems to be roughly correct is the large N_c limit
't Hooft, over 20 years ago, showed that provided a continuus limit exists where $N_c \rightarrow \infty, g_{YM} \rightarrow 0, g_{YM}^2 N_c \rightarrow \lambda$
Theory still strongly coupled and confining below Λ_{QCD} but in this limit drastic semplifications are possible, as some observables $\sim N_c^2$, some $\sim N_c^0$ etc. Plugging in $N_c = 3 \rightarrow \mathcal{O}\,(10)$ hierarchy

- Quasi-particle picture of “light” mesons
- Quasi-classical structure of “heavy” baryons (Skyrme model)
- OZI rule, planar diagrams (strings,AdS/CFT) etc

all compatible with this hyerarchy

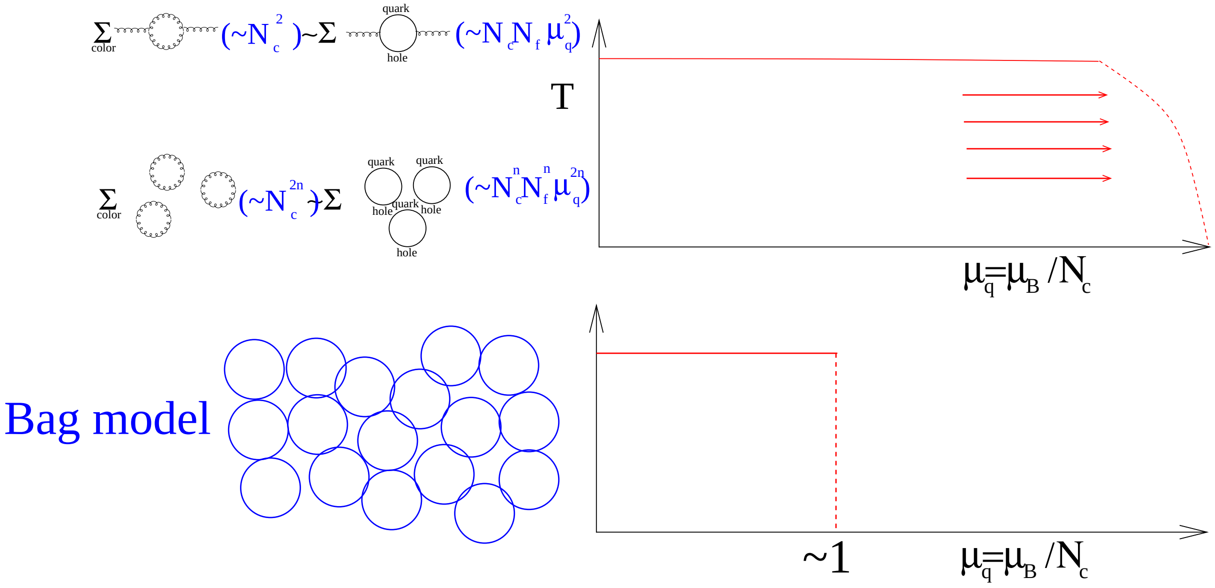
The phase diagram: if deconfinement ⇔ quark-hole loops “beat” gluon antiscreening...





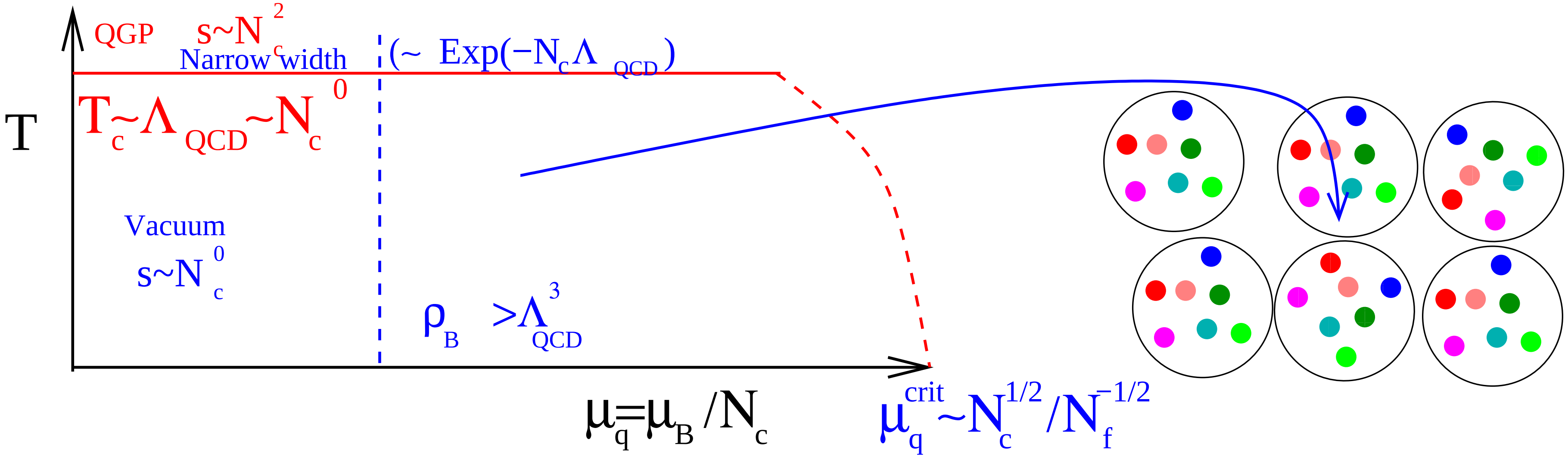
Deconfinement line flattens,for deconfinement $\mu_Q \sim N_c^{1/2} N_f^{-1/2} \Lambda_{QCD}$

NB: higher n order hyerarchy $\sim (N_c/N_f)^{n(n-1)}$, does not help!



Note: Above is a big if

Above reasoning contradicts, for example, bag model intuition, where $\mu_Q^{crit} \sim T_c \sim \Lambda_{QCD} \sim N_c^0$. The "trick is" it assumes non-perturbative contributions to β-function/confinement order parameters don't have a different N_c dependence, which could dominate at $N_c = 3$. Lets continue to assume this , but its unproven! either alternative is instersting



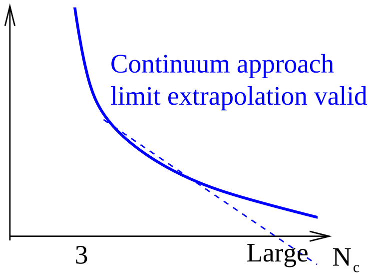
Inter-quark distance in this phase $\sim N_c^{-1/3} \to 0$, asymptotic freedom in configuration space! . Confined but quasi-free quarks below fermi surface and $P \sim N_c$ (quark-hole?)

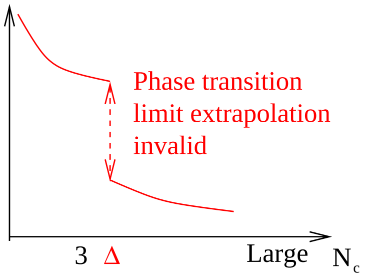
NB: If color can propagate at inter-baryonic distances, “quarkyonic matter”$\equiv$ QGP, “bag model intuition” correct). otherwise , A new phase to look for at low energy, high density (Neutron stars, FAIR, NICA, etc.), In alternative to critical point,but...

Can we exclude phase transitions in N_f/N_c?

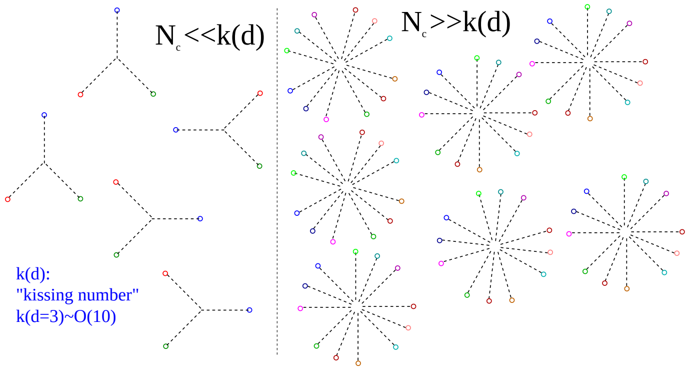
Quantity	$N_c \to \infty$	QCD
$E^{binding}_{Nucleus}$	$N_c \Lambda_{QCD}$	$\ll \Lambda_{QCD}, m_\pi$
$\Delta E_{spin-flip}$	$\sim \Lambda_{QCD}/N_c$	$\sim \Lambda_{QCD}$
Ground state	Crystal	Liquid

When you are expanding around the right vacuum, a $\sim 30\%$ correction is OK. When you are expanding around the wrong vacuum, any correction is catastrophic, and not always easy to see (Confinement!).



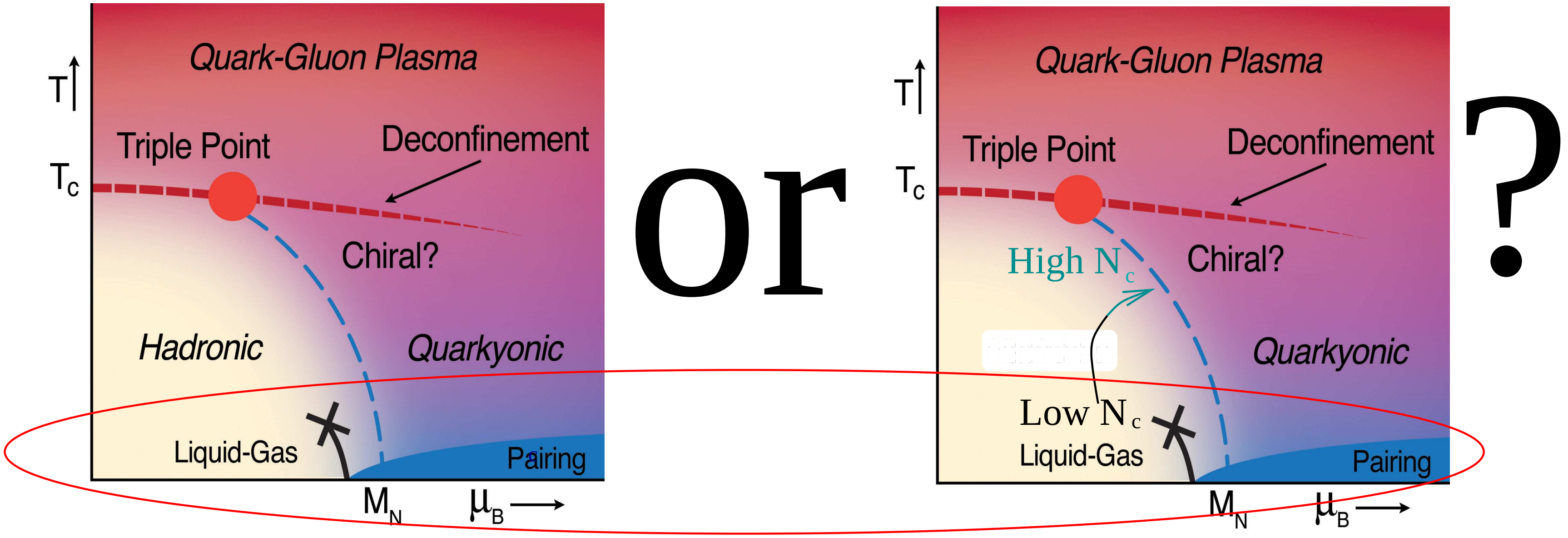


In fact, phase transitions in N_c are certain to happen!



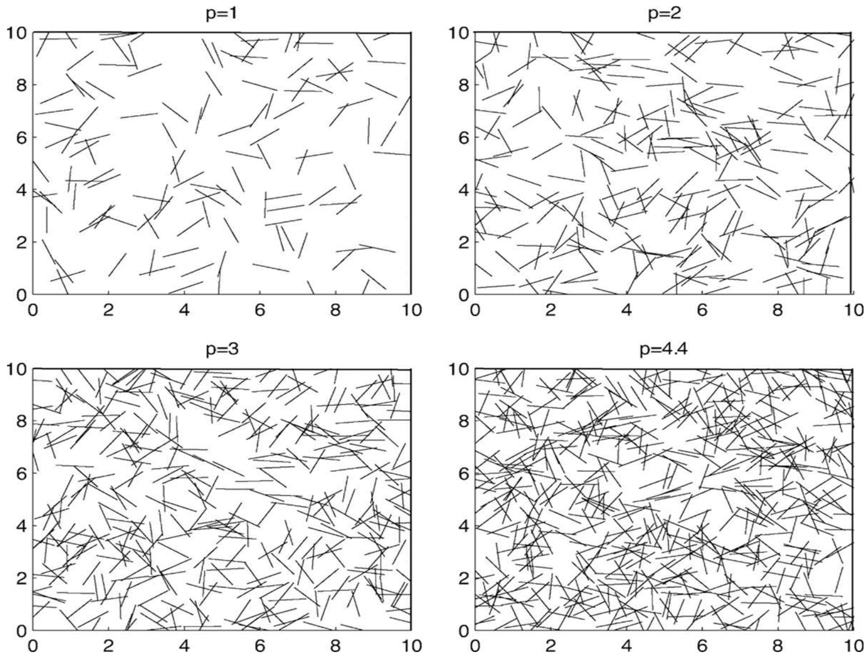
Confinement Symmetry principles (Z_N) dictate that deconfinement is a phase transition, at $N_f \ll N_c$. Critical point in N_f/N_c !

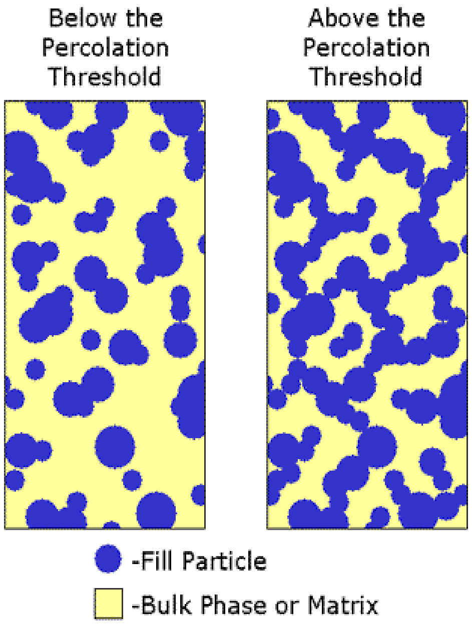
Baryon crystals At $N_c \to \infty, \mu_B/N_c \sim \Lambda_{QCD}$, the ground state of nuclear matter is widely understood to be a Skyrme crystal I.Klebanov, Nucl.Phys.B262:133,1985 From that paper... *Of course , this treatment ignores the kinetic energy of skyrmions. It can be roughly estimated to be* $1/Mca^2 \sim 100\ MeV$*. Energy of this order is enough to unbind the crystal at* $N_c = 3$. Pauli exclusion principle $\Rightarrow$ $N_c, N_{neighbors}$ interplay



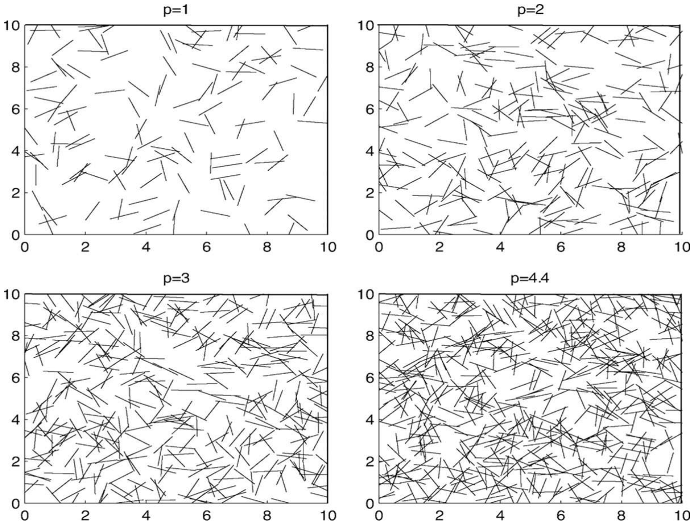
GT,I.Mishustin, PRC82 055202 “quarkyonic matter” might be nuclear matter at $N_c \gg N_{neighbours}$. Or not as depedence on flavor, density not so clear. But $N_{neighbors}$ scaling motivates percolation.

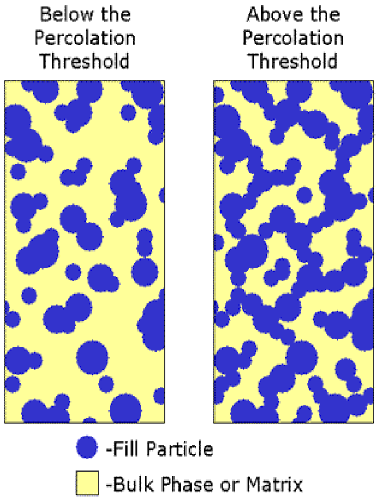
Percolation: the archetypal 2nd order transition





Basic idea: You have a (regular or irregular) lattice of sites, which can be ”on” and ”off” (links ”switched on”, particles ”in sites”, etc), with probability p . Count adjacent sites $\langle N_{sites} \rangle$. When $p \simeq p_c$, $\langle N_{sites} \rangle \to \infty$





- second order transition ($\langle N_{sites} \rangle \equiv$ correlation), with critical behavior.

- $p_c(1D) = 1, p_c(2D) \sim \mathcal{O}(0.5), p_c(3D) \sim \mathcal{O}(0.2)$ (depends on $N_{neighbors}$). So "small" $\sim N_c^{-1}$ correction could trigger it.

Some people have tried to describe deconfinement by percolation of strings/bags, but order of phase transition missed.

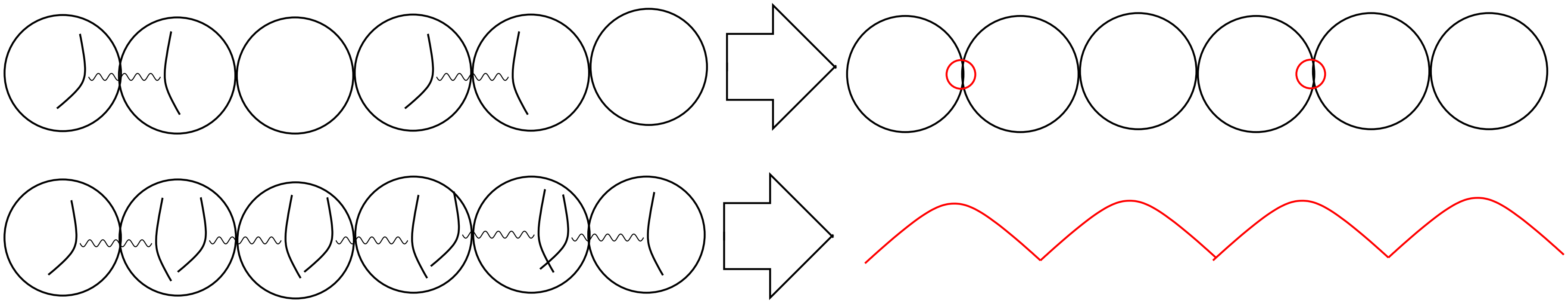
an EFT of $\mu_Q \sim \Lambda_{QCD}, N_c \gg 1$ matter

Baryons are heavy and immobile "background"

Quarks are delocalized, since $\rho_{baryon}^{-1/3} \leq R_{baryon}$ Such delocalization compatible with confinement

An immediate physical analogy: conductor in QED, with baryons playing the role of atoms.

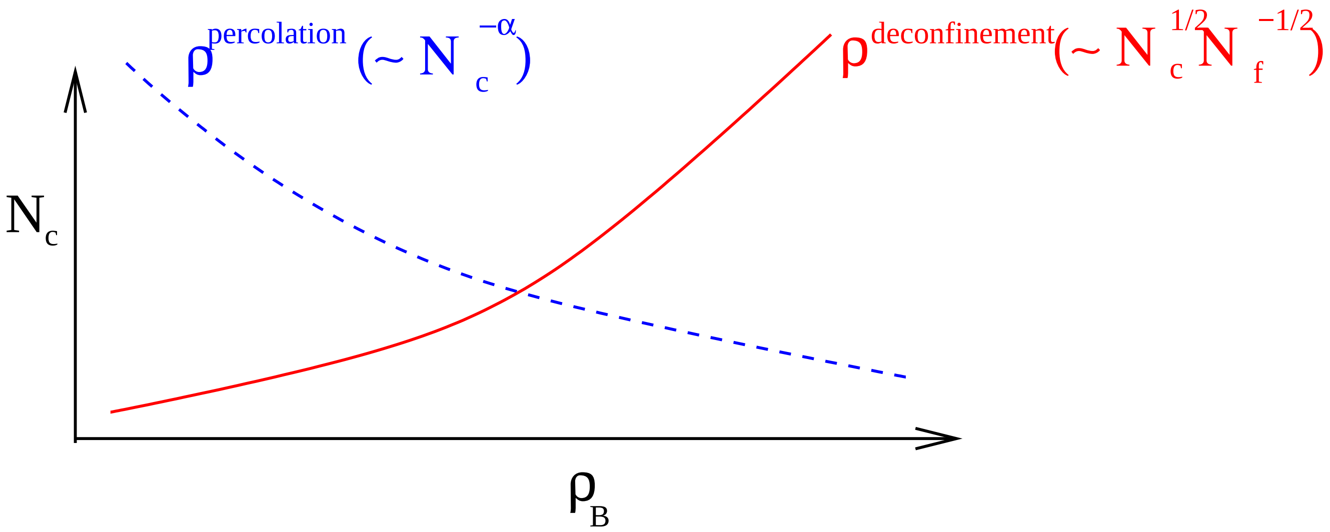
Such a “conducting phase”, not predicted by any EFT, could be the “surprise” we were looking for



But remember, conductor insulator phase transition is governed by number of electrons in the “conducting band”.

However , since Quark/baryon $\sim N_c$, conductor/insulator transition in full $T-\mu_Q-N_c$ space!

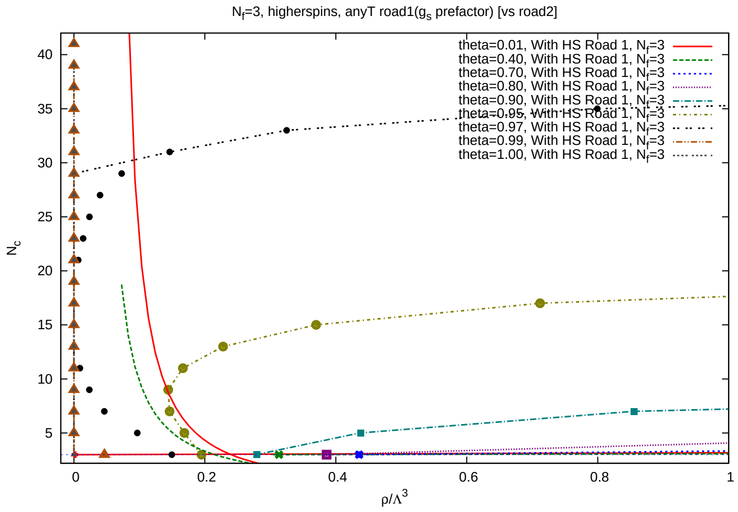
Percolation and deconfinement

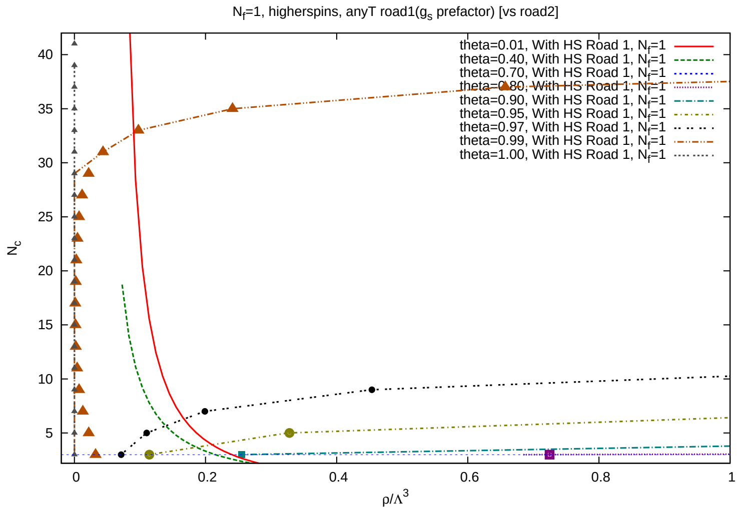


$N_c \leq N_c^{crit}$ Deconfinement happens below percolation, ie percolation transition does not exist separately from deconfinement

$N_c \geq N_c^{crit}$ Percolation, deconfinement separate (Quarkyonic phase?)

Numerical simulation with percolation model


N_f=3, higherspins, anyT road1(g_s prefactor) [vs road2]

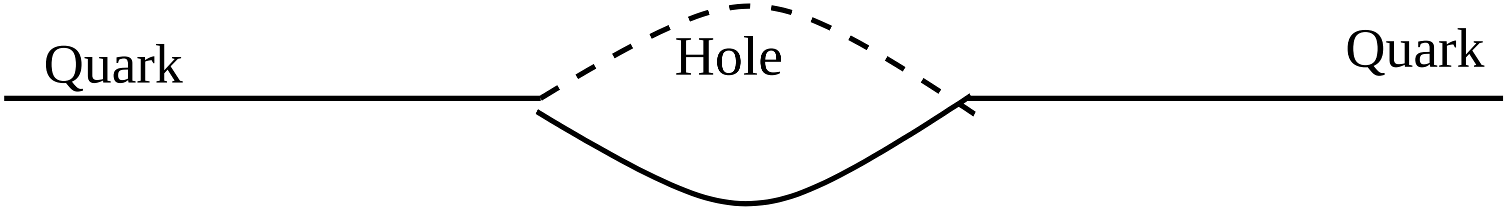

N_f=1, higherspins, anyT road1(g_s prefactor) [vs road2]

A sliver of $n - \rho - N_c = 3$ space which is percolating but confined seems to be there, but... depends a lot on $N_f = 2$ or 3.

Observing such a percolating phase: What does it look like?

How do confinement and free quarks coexist? McLerran,Pisarski,Kojo : quark Fermi surface and baryonic excitations. But..

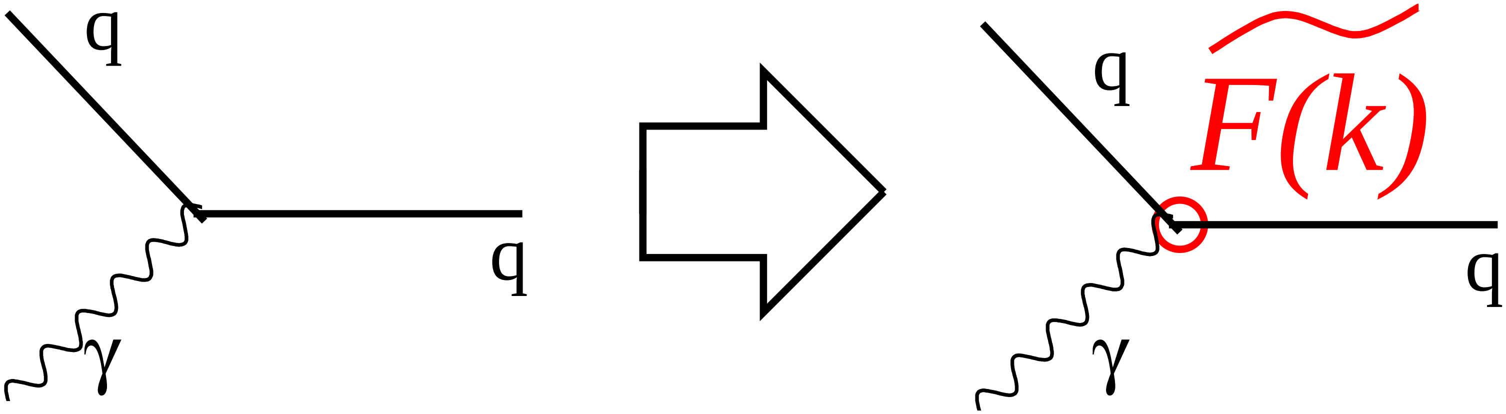
$$\frac{dS}{dV}=\frac{dP}{dT}=\frac{P+\rho-\mu n}{T}$$



And any diagrams of this type will give $T\mu_B$ contributions to pressure, and hence dS/dV . So need theory with confinement but free quarks! Physical example: Electrons in a metal

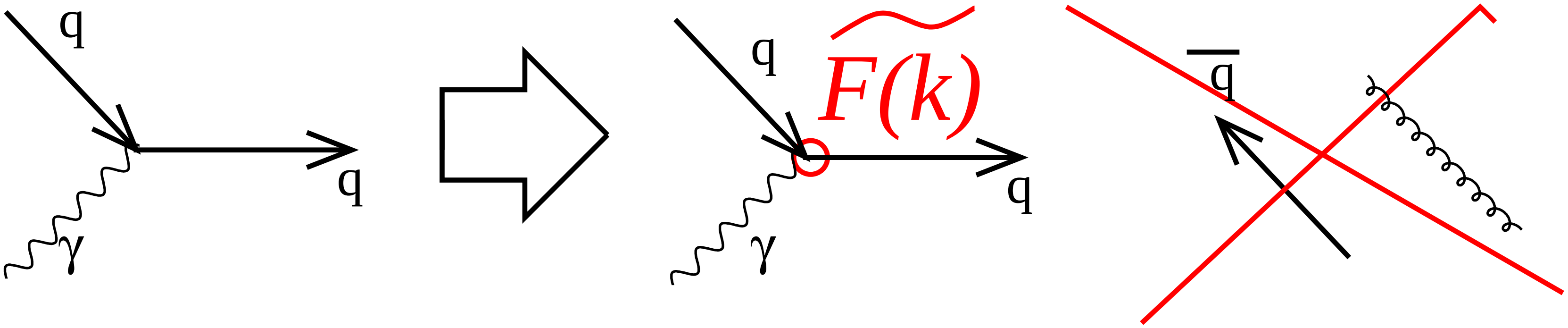
pQCD but not quite: the role of baryons

Unlike pQCD, quarkyonic matter's "vacuum" is a classical dense baryon state. Treating baryons as mean fields will give a momentum-dependent form factor



$F(k)$ gives the F.T. of the baryonic gluon content. For the equation of state, it should just be a $\mathcal{O}(1)$ normalization factor, but for scattering processes it is a qualitative difference from naive QCD. Spin-color-flavor separation can ensure color neutrality with quark-like degrees of freedom. Baryons motion doesent influence quarks up to N_c^{-1} corrections

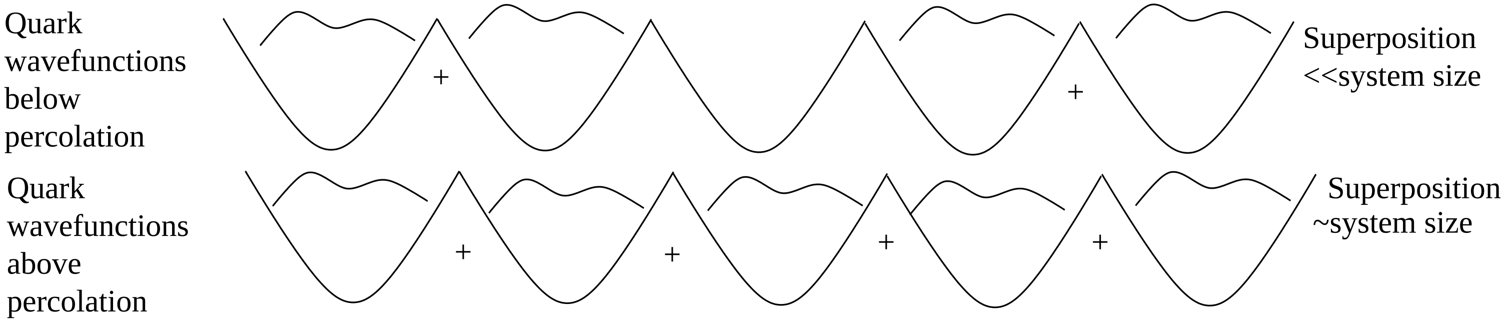
NB: Quarks delocalized by tunneling, not confinement



Gluons, antiquarks still confined, only processes with outgoing quarks allowed!

In particular, $gg \to s\bar{s}$ not expected. If these are responsible for strangeness enhancement, rate is comparable to hadronic rate

From EoS to dynamics: An EFT of percolating matter

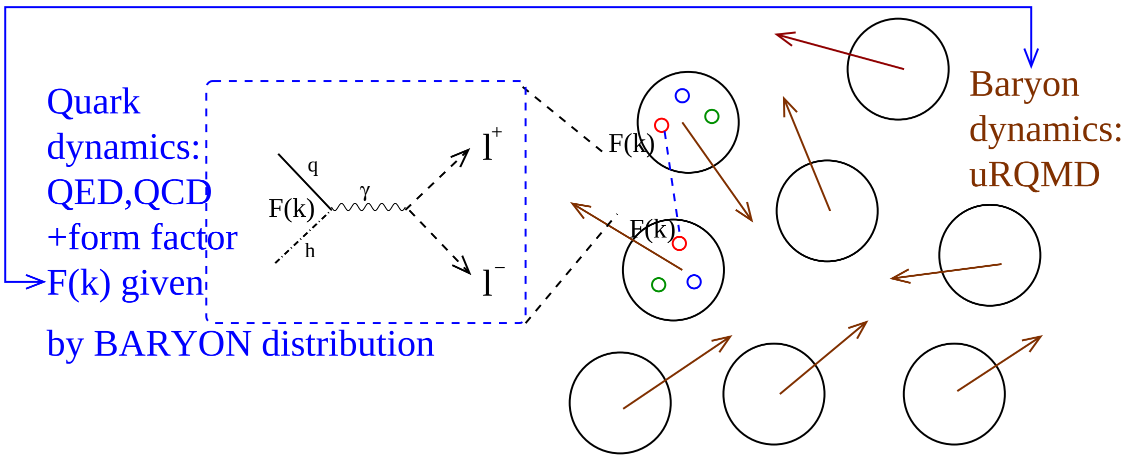


In percolation regime, asymptotically free quark wavefunctions of different baryons can superimpose across large distances.

Thus, even if $E_{state} \sim 1/L_{baryon} \sim N_c^0 \ll N_c^{1/2}\Lambda_{QCD}\Big|_{deconfinement}$ degrees of freedom quark-like, so $P \sim N_c, s \sim N_c$ (In the same way electrons in a metal have a much lower energy than ionization).

Periodic wavefunctions $\Rightarrow$ leading component always $p \geq \Lambda_{QCD}^{-1}$

Modeling quarkyonic matter for RHIC/NICA/FAIR



Quark
dynamics:
QED,QCD
+form factor
F(k) given
by BARYON distribution

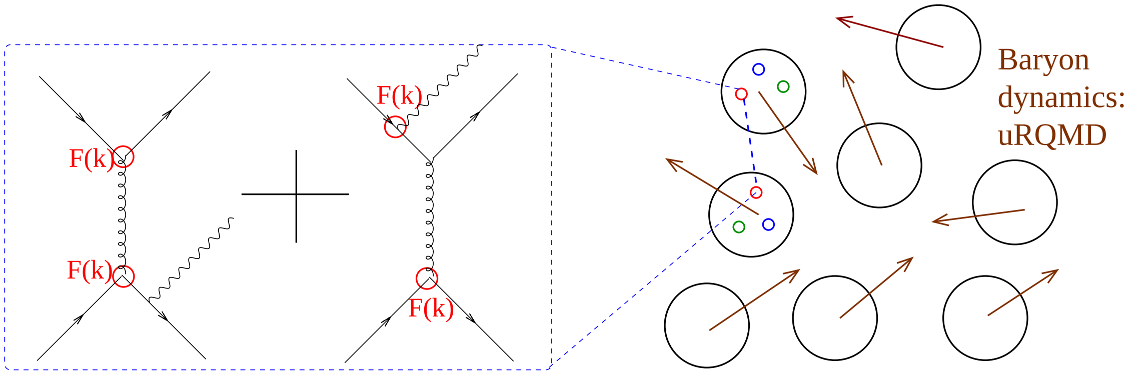
Baryon
dynamics:
uRQMD

$R_{qq\to X} = \Psi(k)\Psi(k')M^2_{qq\to X}$ Where $M_{qq\to X}$ is the pQCD matrix element

$$\Psi(k) \sim \exp\sum_i \left[ikx_{0i}\right] F(k) \sim \exp\left[ikx_{0i} - \frac{k^2}{\Lambda_{QCD}}\right]$$

$F(k)$ is the quark function inside a "classical" proton potential well ($\sim$ Gaussian) and x_{oi} are the baryon locations. The latter is given by uRQMD.

Photon production in this approach



As antiquarks, gluons suppressed leading channel is quark Brehmsstrahlung.

$$\mathcal{M}^2 = L^2(k_1, k_2 \to k_3, k_4, p) + L^2(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_4)$$

$$L^2 = -\frac{1}{4} e^2 \lambda^2 N_c^{-2} (k_2 - k_4)^{-4} Tr\left[\not{k}_4 \gamma^\sigma \not{k}_2 \gamma_\rho\right] Tr\left[\not{k}_3 Z^\mu_\sigma \not{k}_1 Z^\rho_\mu\right]$$

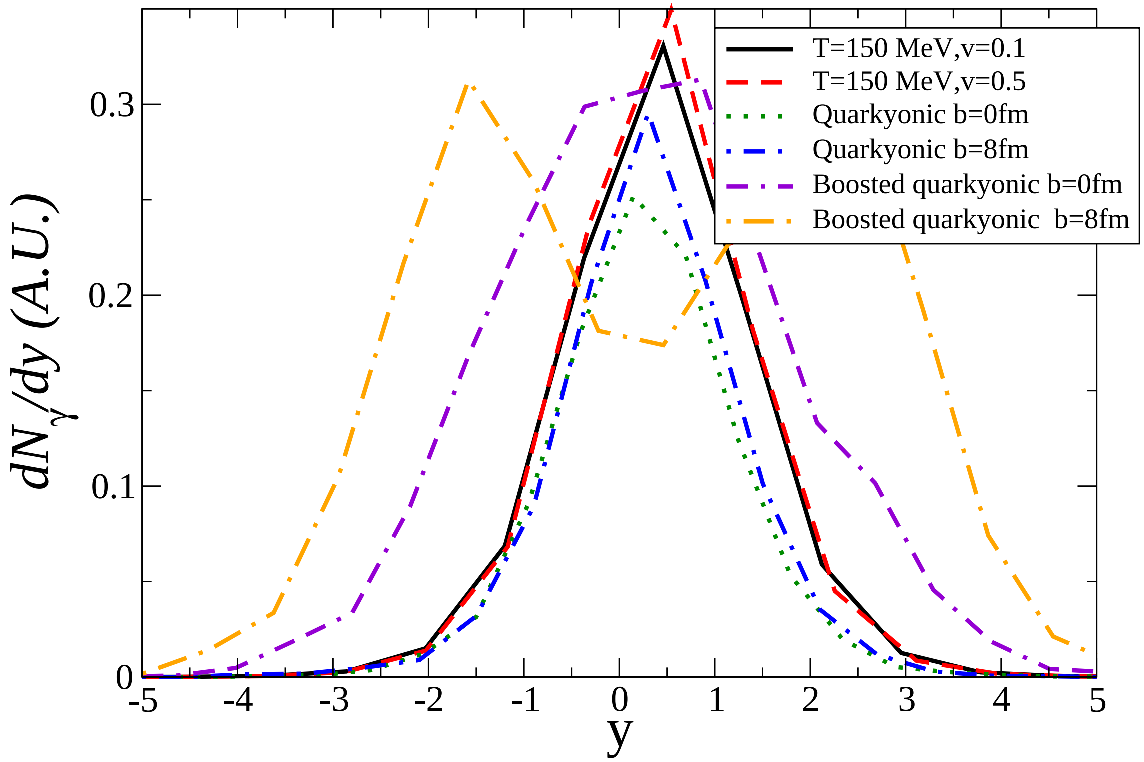
$$Z^\beta_\alpha = \gamma_\alpha (k_1 - p)^{-1} \gamma^\beta + \gamma^\beta (k_3 + p)^{-1} \gamma_\alpha$$

$$\frac{dN_\gamma}{dyp_Tdp_Td\phi} = \int \frac{d^4k_1}{k_1^0}\frac{d^4k_2}{k_2^0}\frac{d^4k_3}{k_3^0}\frac{d^4k_4}{k_4^0}\left(\mathcal{M}\left(k_1,k_2 \to k_3,k_4,p\right)\Psi(k_1)\Psi(k_2)\right)^2$$

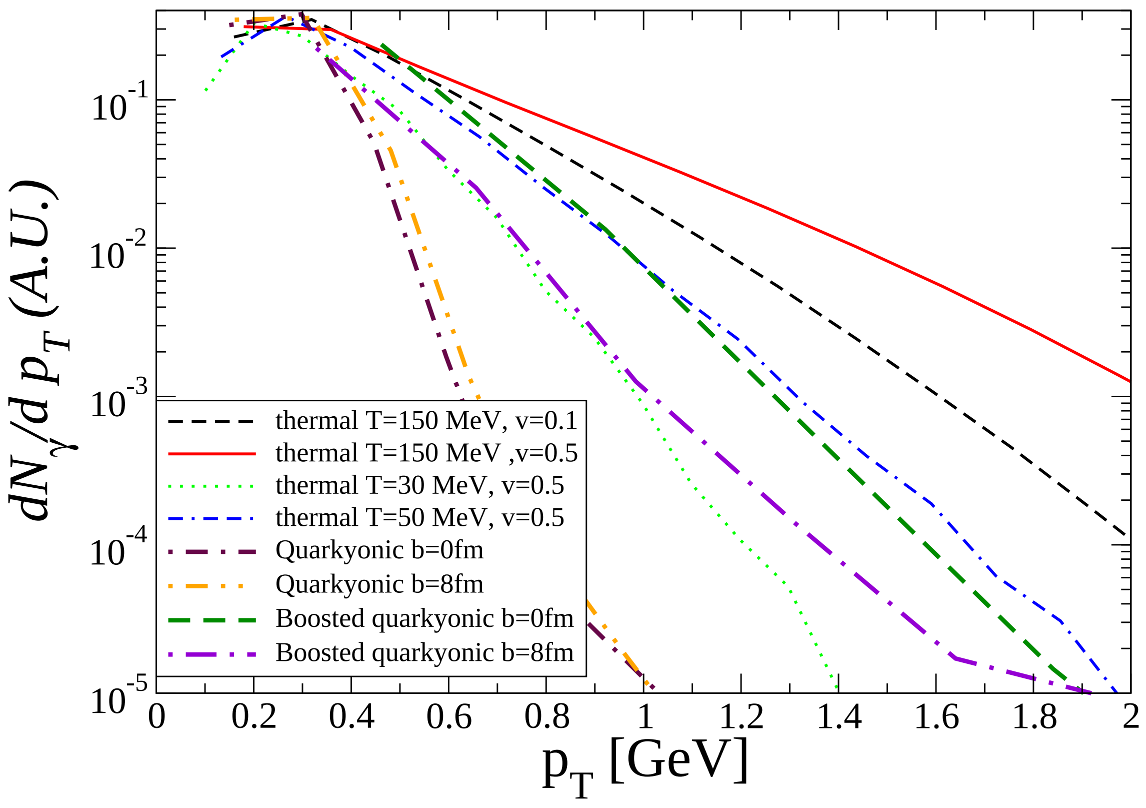
- Quarkyonic quark wavefunctions
  $\Psi(k) \sim \exp\sum_i \left[ikx_{0i}\right] F(k) \sim \exp\left[ikx_{0i} - \frac{k^2}{\Lambda_{QCD}}\right], uRQMD \Rightarrow x_{0i}$

- "Boosted quarkyonic": Same wavefunction as above boosted to flow of a "random" baryon: An upper limit to N_c^{-1} backreaction (effect of baryon flow on quark wavefunction)

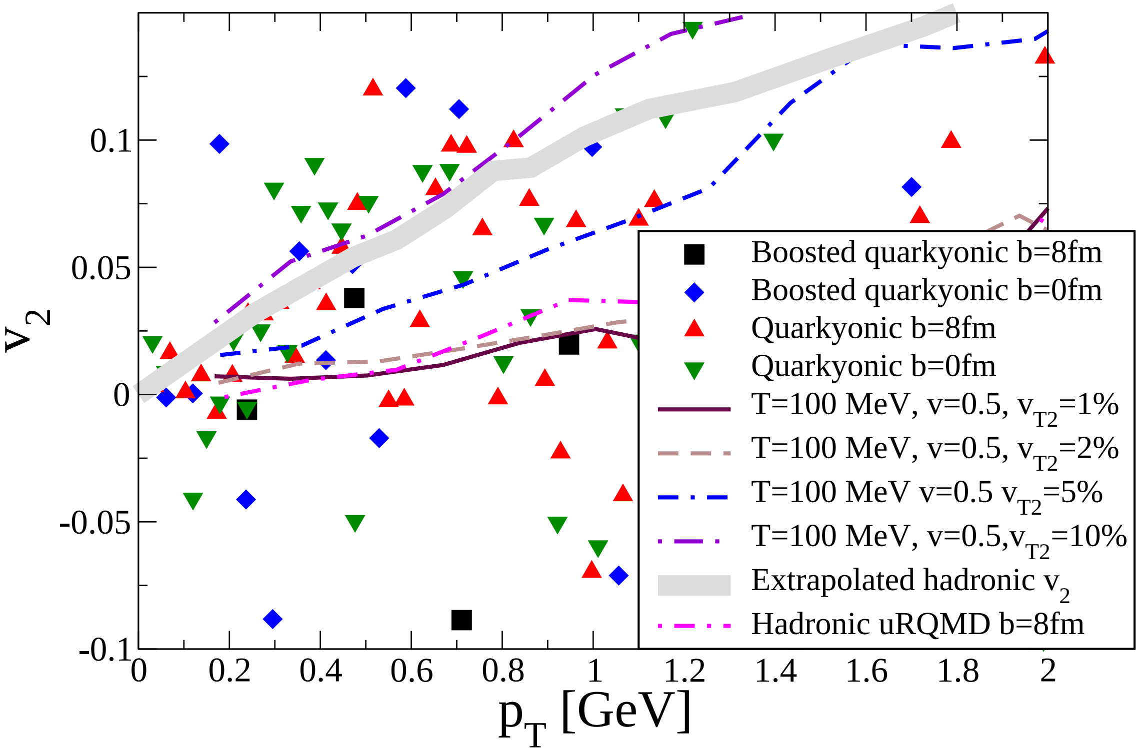
- "QGP" quark wavefunctions $\Psi(k)\Psi(k') = \delta(k'-k)\exp\left[-k_\mu u^\mu/T\right]$



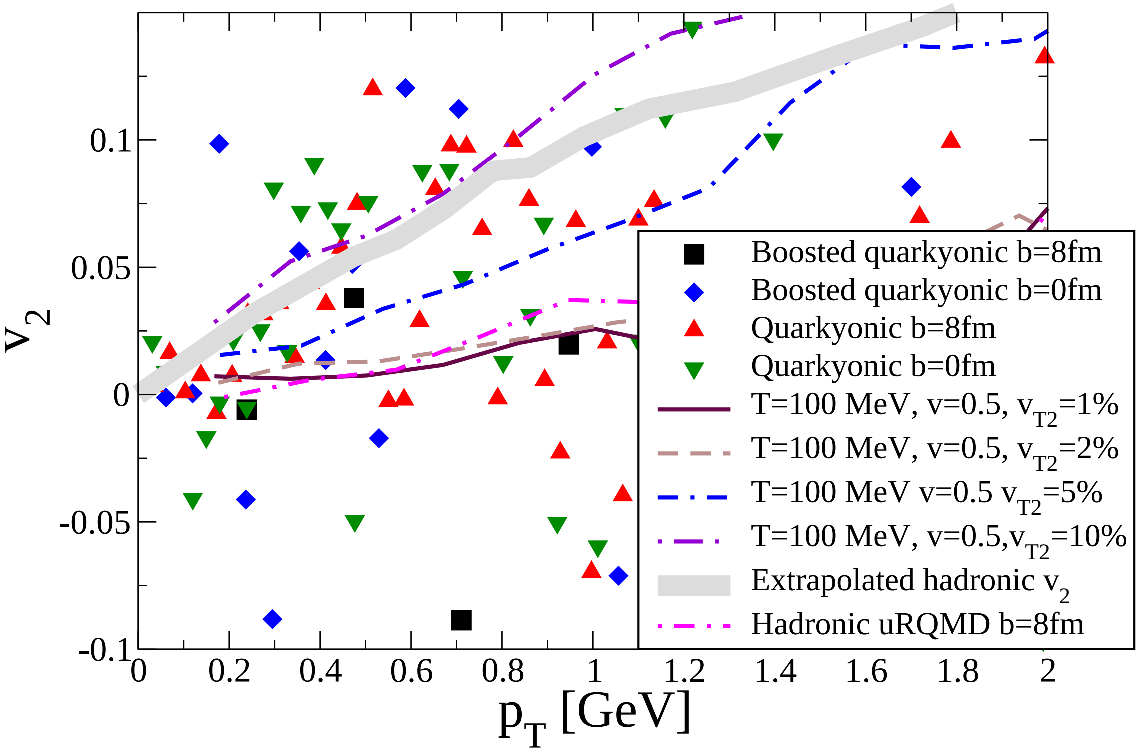
Very little difference. NB “static baryon” approximation breaks down away from mid-rapidity



Quarkyonic wavefunction similar to <u>cold</u> quark gluon plasma, unrealistic temperatures. NB: “boosted quarkyonic” increases flow, but still cold!



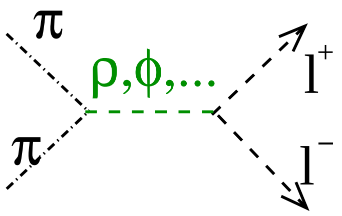
Random distribution of quark wavefunctions quenches total v_2 but produces big fluctuation in event and p_T : oscillation frequency $\sim p_T \rho_B^{-1/3}$



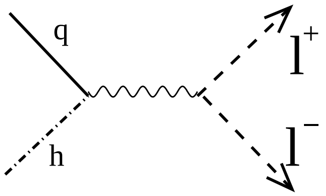
“pure” quarkyonic effect, it is due to sensitivity of quark wavefunctions to baryon location. signature?

dileptons potentially more direct probe but more complicated

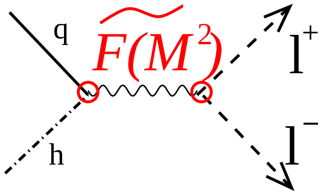
Both quarks and holes needed Sensitivity to equilibration



Hadron gas



QGP



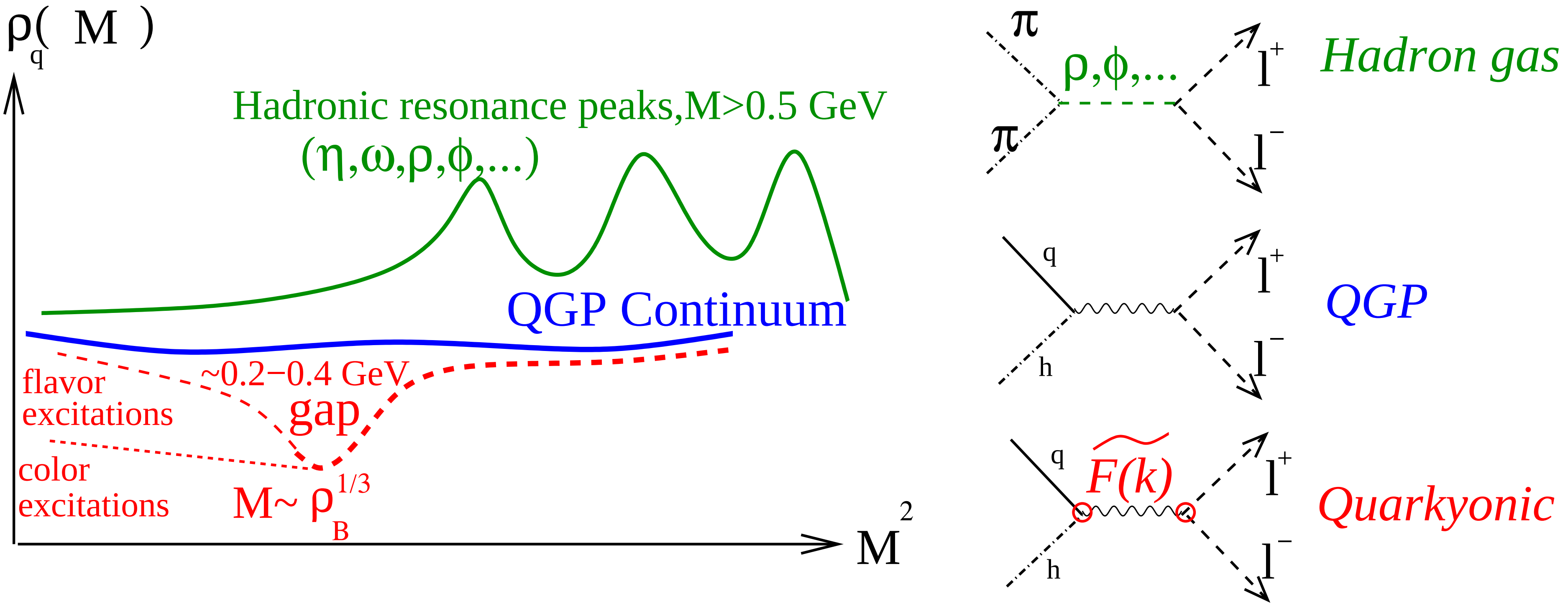
Quarkyonic

$$\frac{dN_{l+l-}}{d^3P_{l+l-}d^3M_{l+l-}} = \int \frac{d^4k_1}{k_1^0}\frac{d^4k_2}{k_2^0}\frac{d^4k_3}{k_3^0}\frac{d^4k_4}{k_4^0}\left(\mathcal{M}\left(k_1,k_2 \to k_3,k_4,p\right)\Psi(k_1)\Psi(k_2)\right)^2 \times$$

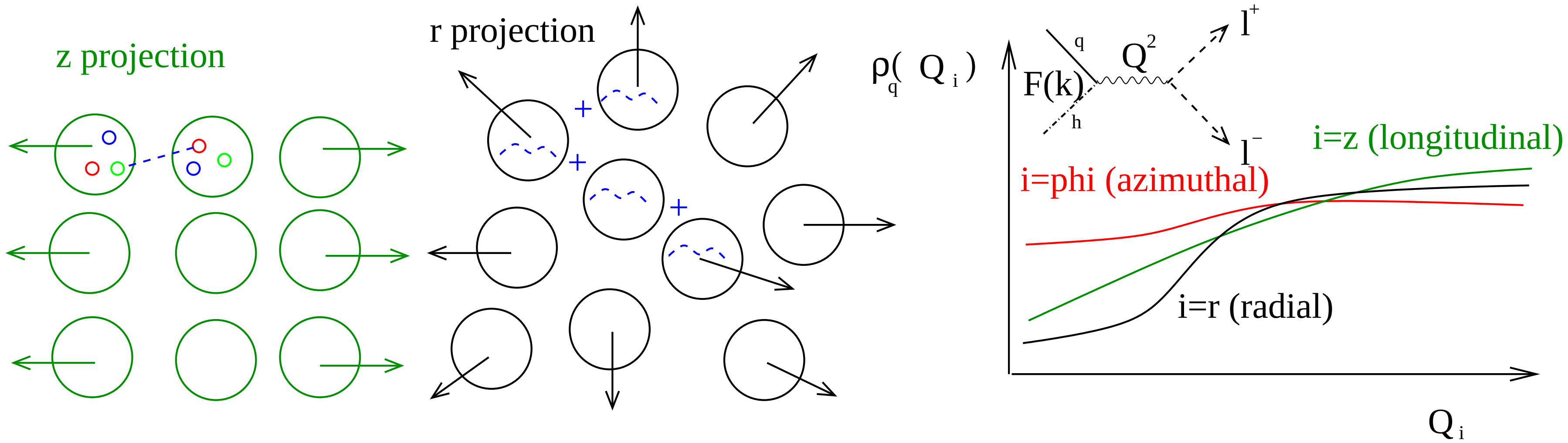
$$\times\delta\left((k_1-k_2)_\mu(k_1-k_2)^\mu - M_{l+l-}^2\right)\delta\left((k_1+k_2)_\mu(k_1+k_2)^\mu - P^2\right)$$

But , extending the idea used for photons

$\tilde{F}(M^2)$ connects baryon distribution to M^2 dilepton spectrum



If baryons were regular (pasta phase?) one could observe bloch waves!
(“upside down resonance”?)



Event by event fireball structure not regular, but Collective structures exist in events flow profile (radial, longitudinal flow) and baryons have repulsive potential, soo structures in 3D dilepton spectral function $Q_{z,r,\phi}$ bound to exist!

Conclusions

- Exotica at $\mu_Q \simeq \Lambda_{QCD}$ possible, "simple models" can serve as guide

The best physicist in the USSR is Yakov Frenkel, who uses
in his papers only quadratic equations.
I am slightly worse, I sometimes use differential equations.

L.D.Landau, quoted in
BULLETIN OF THE American Mathematical Society
Volume 43, Number 4, October 2006, Pages 563−565

- Large N_c expansion tells us quark DoFs could appear even at confinement!

- On the other hand, not at all clear $N_c \simeq \infty$

- Phenomenology of quarkyonic matter needed: EM probes promising

Spare slides

Confinement and quasi-free quarks: spin-color-flavor separation?

Confinement remains, so regions above $\sim 1 fm$ can-not be color charged. (Same problem at $T \geq T_c$, but correlations required to maintain confinement can be $\left(N_c^0 \Lambda_{QCD}^{-1} \right) \ll s\left(T \geq T_c\right) \sim N_c^2 T^3$

Spin-color-flavor separation can achieve this and maintain N_c, N_f scaling! Pisarski,McLerran,Kojo, NPA843 (2010) 37-58 and subsequent works: implement this by 1D WZW model.

$$S = S_{2N_f}^{WZW}[h_{\mathrm{color}}] + S_{N_c}^{WZW}[h_{\mathrm{flavor}}]$$

which generalized spin-charge separation to $SU(N_f), SU(N_c)$.
Modifications to S_{2N_f} could localize color, maintain $\sim N_c$ degeneracy.
“Naively” WZW incompatible with percolation (1D) , but could work as EFT in percolation regime. Work in progress .