Heavy Quark Dynamics in the QGP

Steffen A. Bass
Duke University

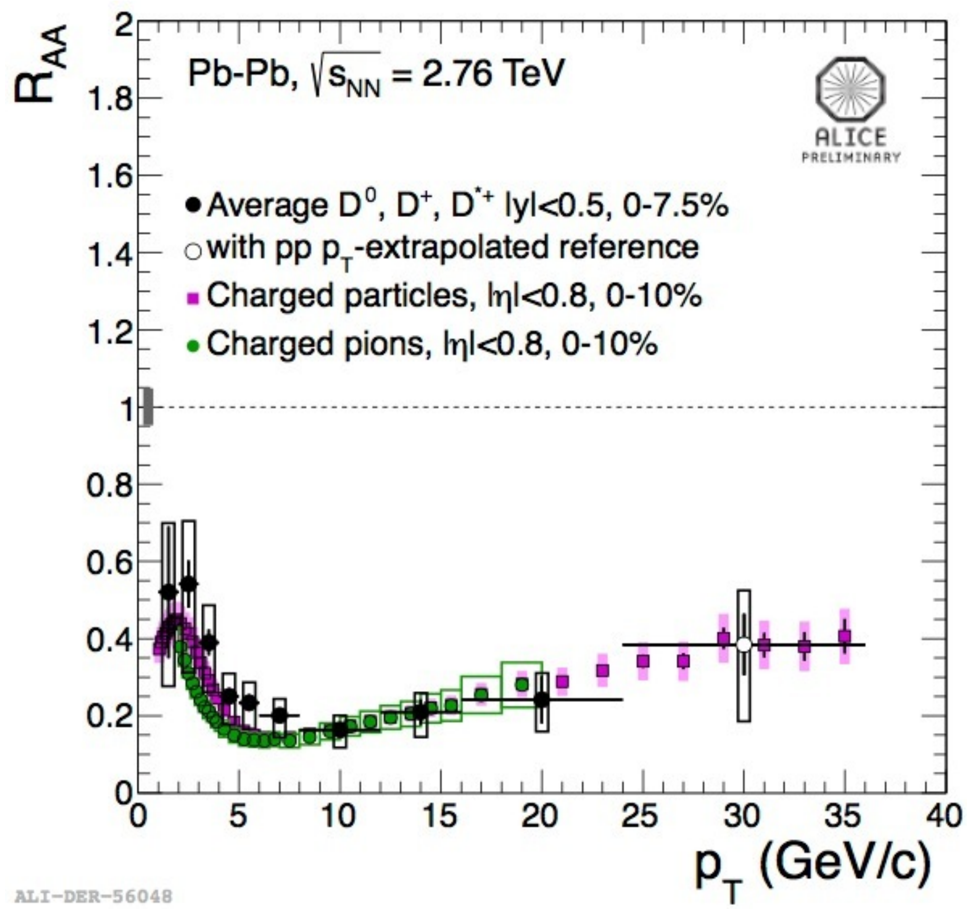
- Introduction
- Langevin Dynamics
- HQ Thermalization
- Langevin with Radiation
- HQ Correlations

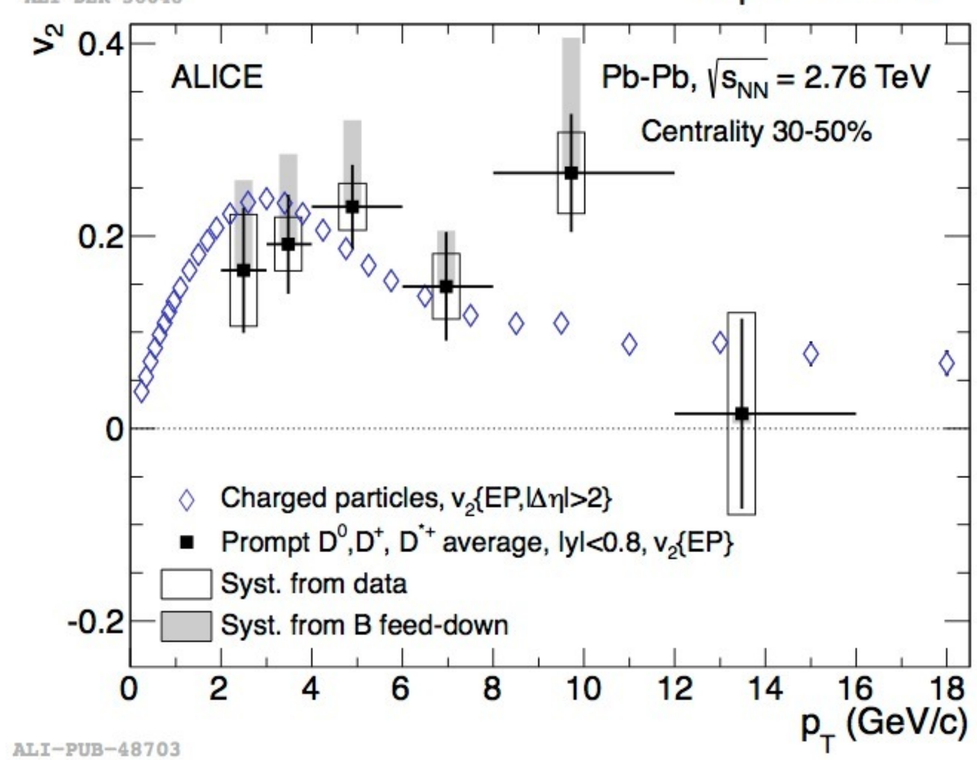
Collaborators:
★Shanshan Cao
★Guang-You Qin
★Scott E. Moreland
★Marlene Nahrgang

S. Cao & S.A. Bass: Phys. Rev. **C84** (2011) 064902
S. Cao, G. Qin & S.A. Bass: J. Phys. **G40** (2013) 085103
S. Cao, G. Qin & S.A. Bass: Phys. Rev. **C88** (2013) 044907

Introduction:
Why Heavy Quarks?

The Heavy Quark Puzzle





- HQ are produced via hard processes early in the collision
- act as probe for medium that they traverse

Folklore:

- large mass should lead to small medium modifications

Observation:

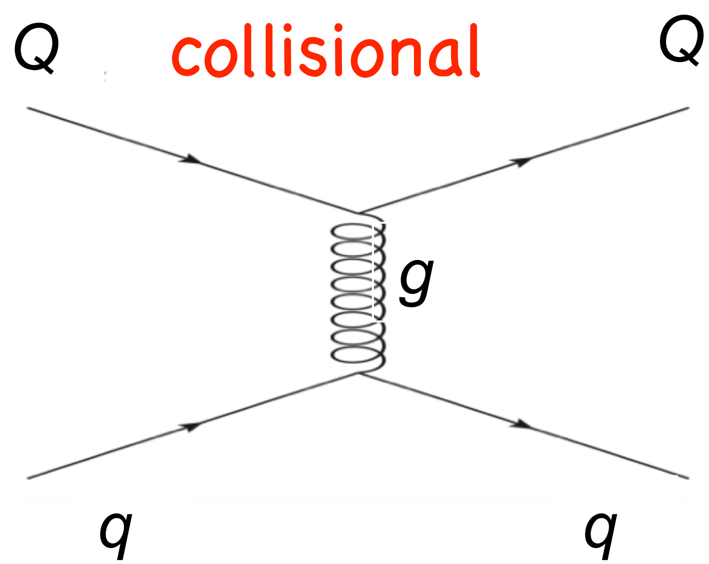
- strong medium modifications, similar to that for light quarks
- evidence for thermalization?

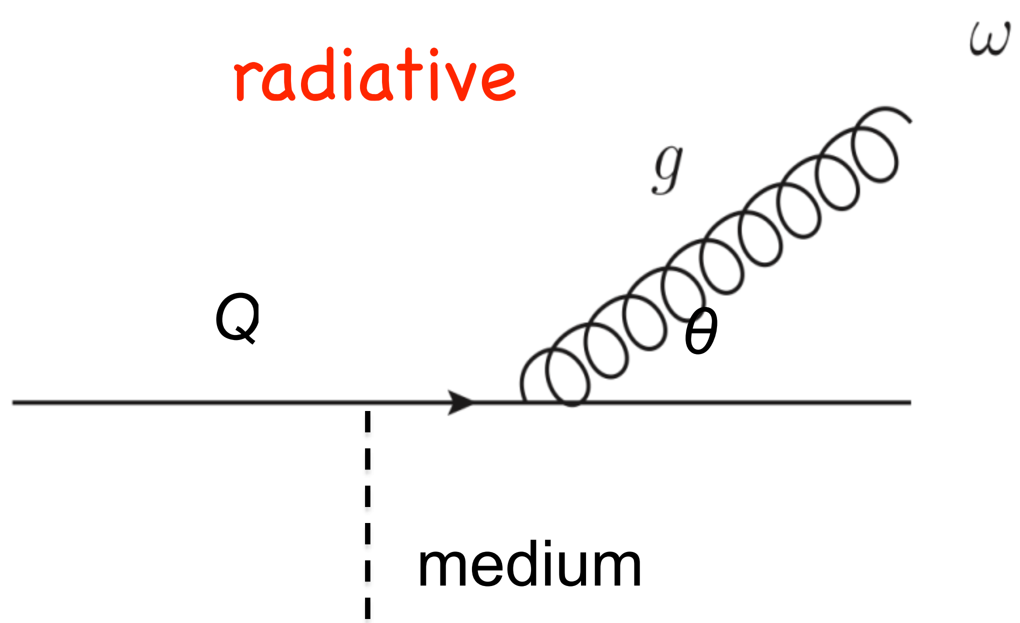
Exploring HQ dynamics in the QGP:

- Langevin approaches very successful
- other approaches: PCM, linearized Boltzmann w/ RFD medium, DGLV...

Collisions vs. Radiation: Dead Cone Effect

partons propagating through a QGP medium loose energy via two mechanisms:





gluon radiation is suppressed at angles smaller than the ratio of the quark mass M to its energy E ("dead cone effect"):

$$dP_{HQ} = dP_0 \left(1 + \frac{\theta_0^2}{\theta^2}\right)^{-2}$$

with $\theta \simeq \frac{k_\perp}{\omega}$ and $\theta_0 = \frac{M}{E}$

Bremsstrahlung dominates in the ultra-relativistic limit:

$$\gamma v \gg 1/g$$

in our RHIC calculations we consistently have: $\gamma v < 4$

Langevin Approach

If HQ dynamics is dominated by elastic scattering, then consider Brownian Motion as a model for HQ evolution in medium: **Langevin Equation**

$$\frac{d\vec{p}}{dt} = -\eta_D(p)\,\vec{p} + \vec{\xi}$$

noise-term is defined via correlation function:
(assume no momentum-dependence for simplicity)

$$\langle \xi^i(t)\,\xi^j(t')\rangle = \kappa\,\delta^{ij}\,\delta(t-t')$$

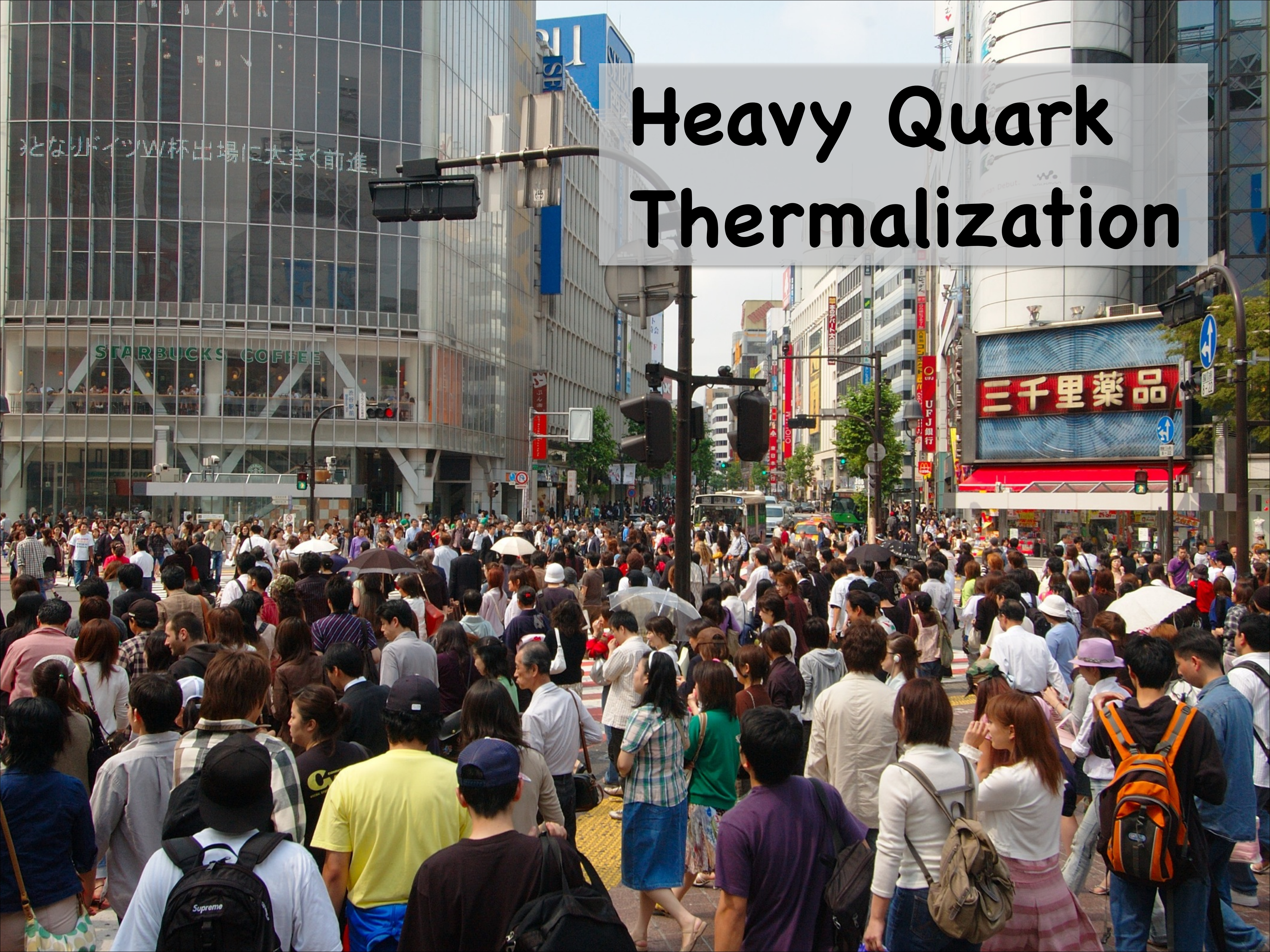
for small individual energy transfers, the Fluctuation-Dissipation Theorem applies, connecting the drag term to the noise term:

$$\eta_D(p) = \frac{\kappa}{2TE}$$

the Heavy Quark diffusion coefficient is related to the drag coefficient via:

$$D = \frac{t}{M\eta_D(0)} = \frac{2T^2}{\kappa}$$

Heavy Quark Thermalization



Thermalization of Heavy Quarks

Previous Studies:

- Moore & Teaney:
  - Langevin approach with 2+1D ideal RFD
  - relaxation time for charm quark thermalization approx. 7 fm/c

- van Hees & Rapp:
  - Fokker-Planck equation with resonant q-Q interaction
  - resonant interaction reduces pQCD based relaxation time of approx. 30 fm/c to a few fm/c

- note that neither study explicitly checked for thermalization (focus was on observables for flow & momentum loss)

This Study:

- explicit focus on thermalization
- follow approach by Teaney & Moore, but with a 3+1D ideal RFD

Thermalization: Methodology

Thermalization of Heavy Quarks can be verified in their local rest-frame either via their energy spectrum or the isotropy and thermal shape of the momentum distributions:

- energy-spectrum: $\frac{d^3N}{dp^3} = Ce^{-E/T}$ and $\frac{d^3N}{dp^3} = \frac{d^2N}{p^2\,dp\,d\Omega} = \frac{d^2N}{pE\,dE\,d\Omega}$

$$\frac{dN}{pE\,dE} = Ce^{-E/T} \quad ★$$

- momentum spectrum:

$$f(p_z) = \frac{V}{Z}\int dp_x\,dp_y\,e^{-\beta\sqrt{p_x^2+p_y^2+p_z^2+m^2}}$$

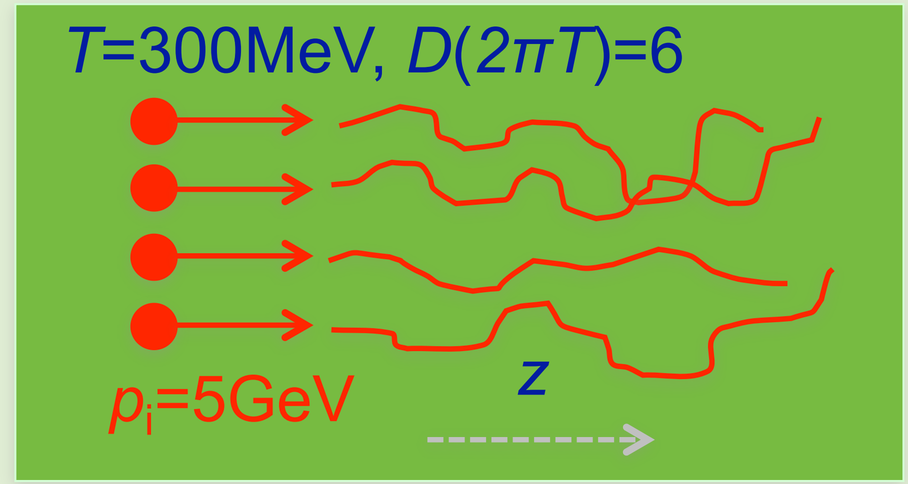
$$= C\cdot T\left(\sqrt{p_z^2+m^2}+T\right)e^{-\sqrt{p_z^2+m^2}/T}$$

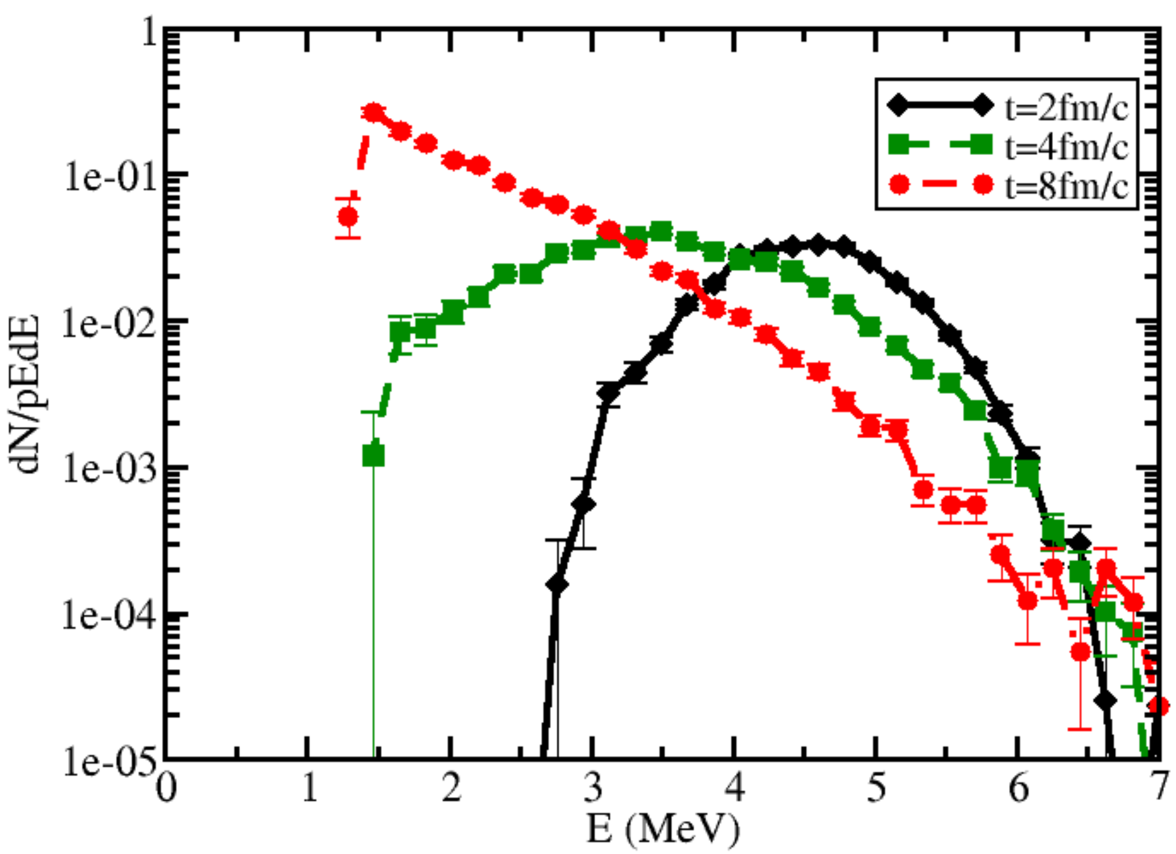
- consider a blue-shift:

$$f(p_z) = C\cdot T\left(\sqrt{(p_z-\tilde{p}_z)^2+m^2}+T\right)e^{-\sqrt{(p_z-\tilde{p}_z)^2+m^2}/T} \quad ★$$

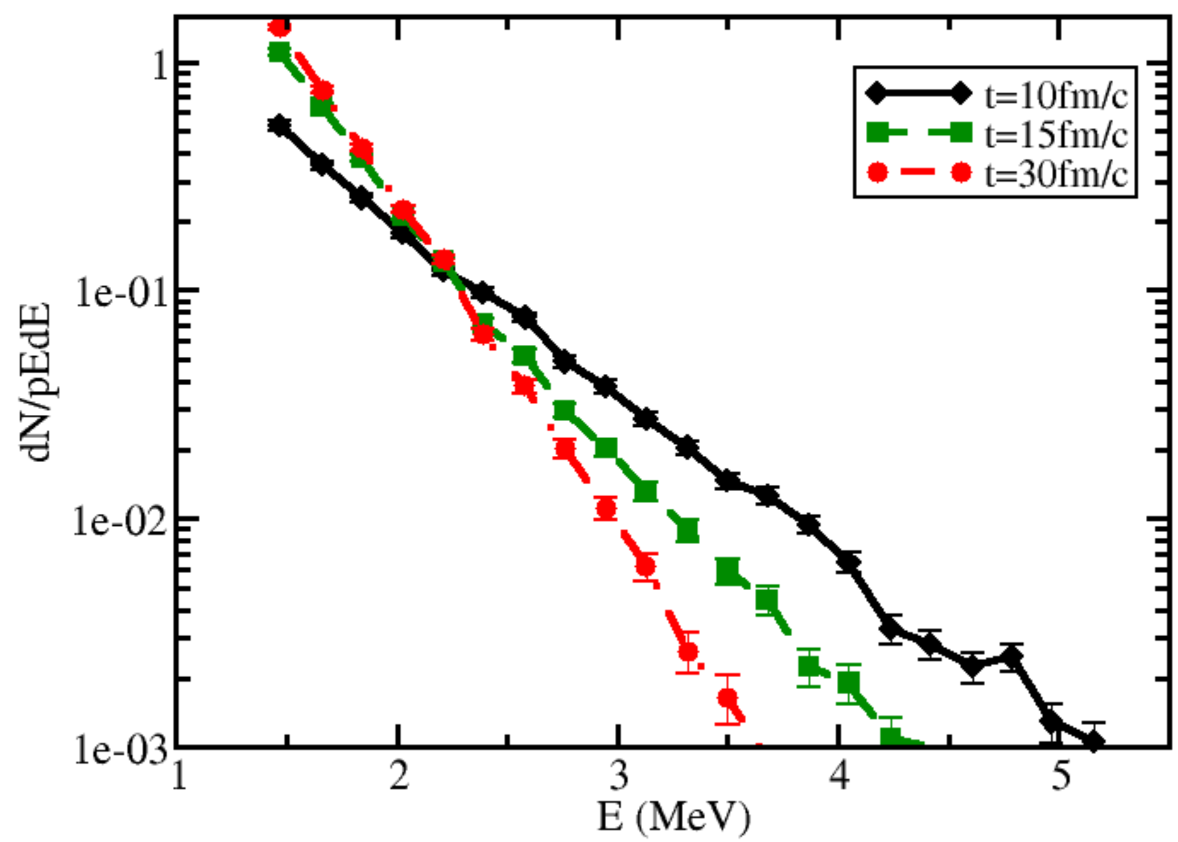
Thermalization: Static Medium

- infinite QGP matter at fixed T
- initialize HQ at fixed momentum
- track evolution towards thermalization
- evaluate energy-spectrum





- no exponential behavior at early times



- onset of exponential behavior at 10 fm/c
- slope converges at approx. 30 fm/c

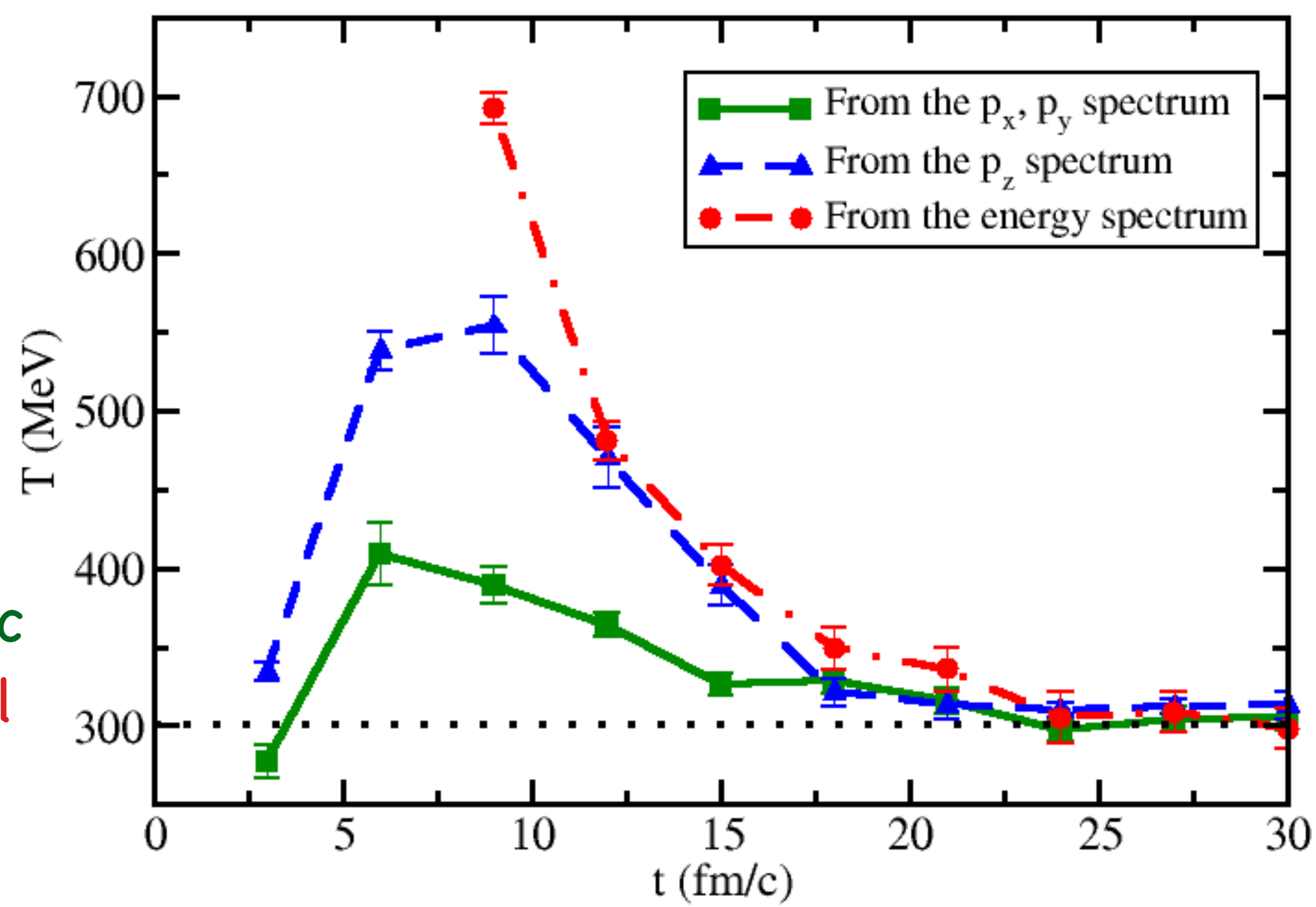
Thermalization: Static Medium II

Equilibration Criterion:

- Temperature parameters extracted from energy spectra and momentum distributions converge and approach the temperature of the medium

comparison of temperature parameters extracted via energy spectra and momentum distributions:

- for this particular set of parameters: $T_{therm} \approx 25$ fm/c
- T_{therm} may depend on initial HQ momentum distribution

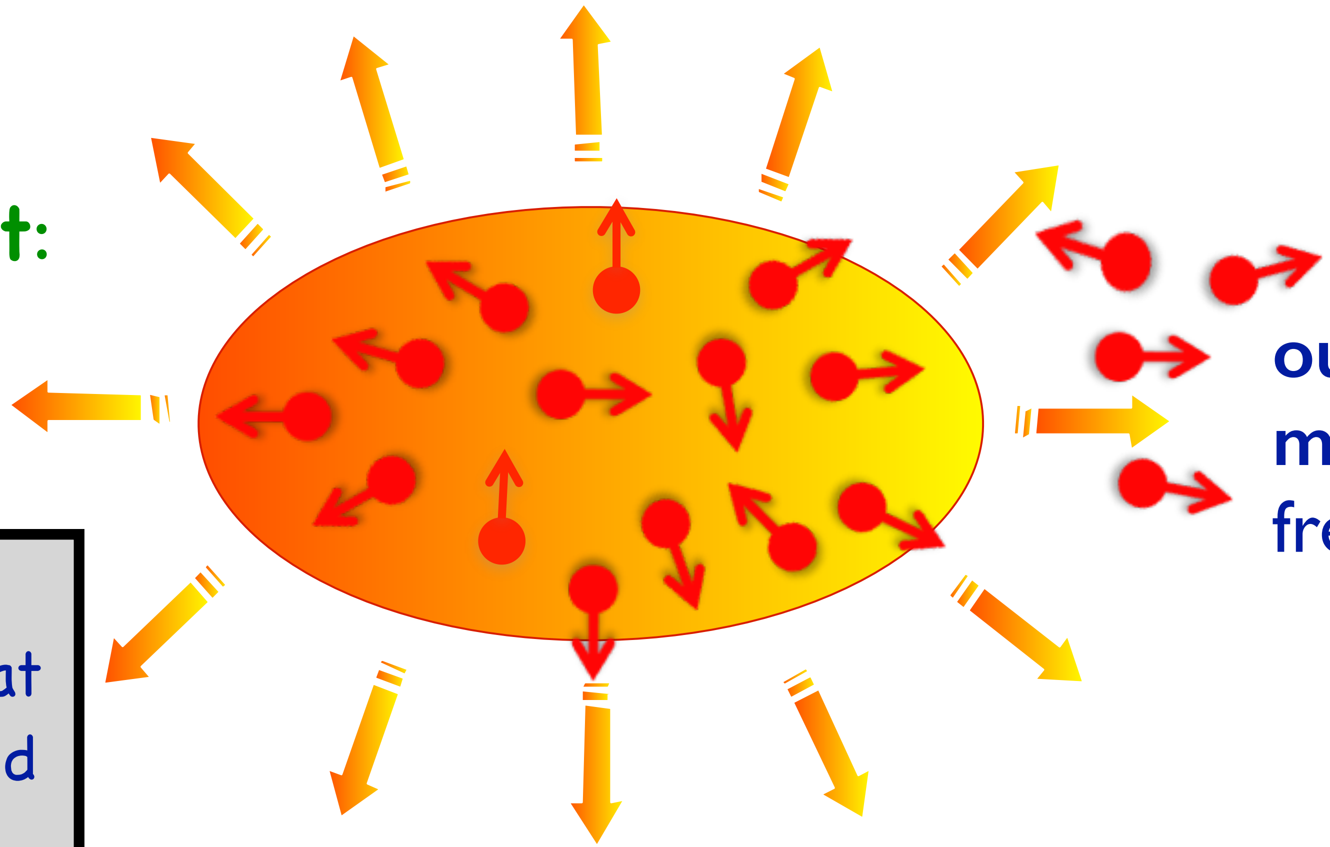


Heavy Quarks in a RFD Medium

Modeling of HQs in a QGP Medium

- QGP medium: Relativistic Fluid Dynamics
  - ideal 3+1D (Nagoya/Duke)
  - viscous 2+1D (VISH - OSU)
- initial distribution of charm quarks:
  - MC Glauber for positions & pQCD for momenta
- Heavy Quark evolution: Langevin algorithm or Langevin+Radiation
- Hadronization: fragmentation, using PYTHIA 6.4

after freeze-out:
free-streaming



outside the medium:
free-streaming

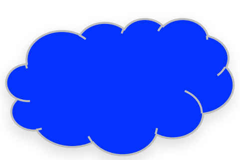
D=7/(2πT),
i.e., qhat ~ 2.6 GeV²/fm at initial temperature (around 400 MeV)

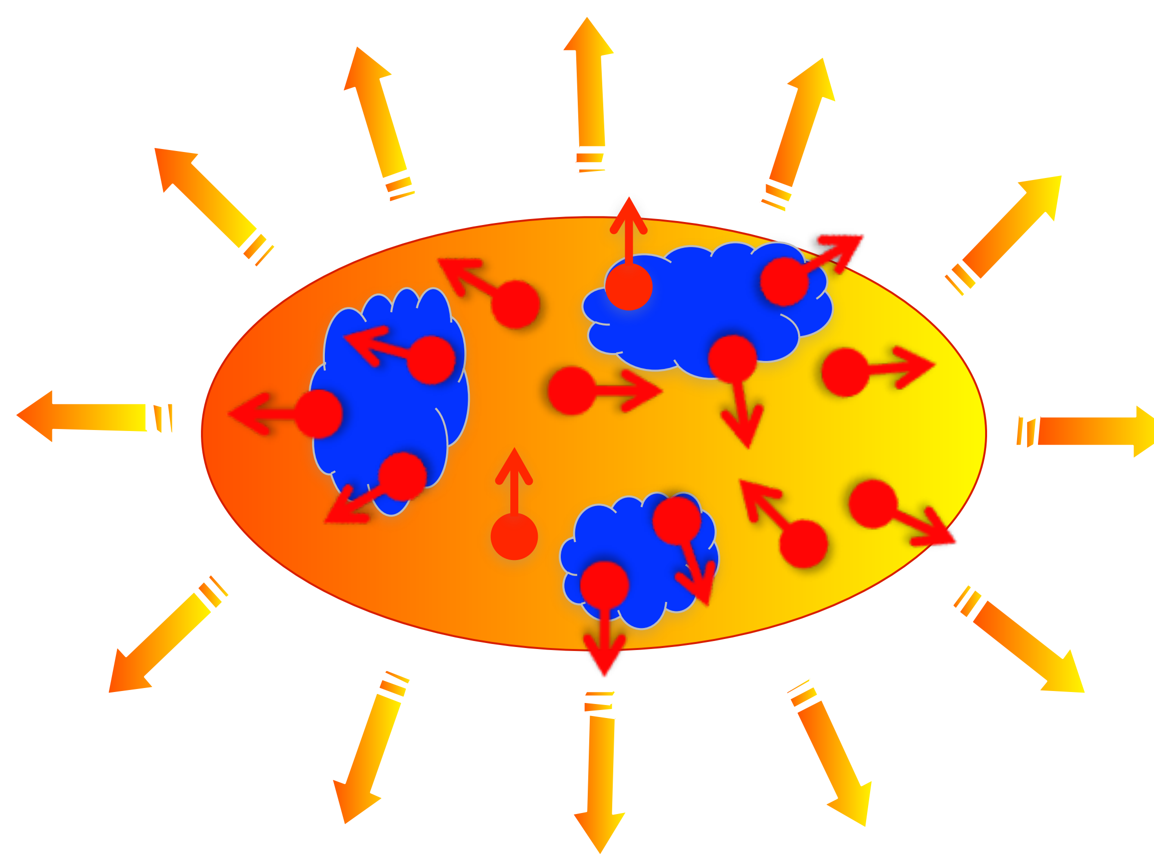
Thermalization in a Dynamic Medium

- at each time step, choose a temperature range of the QGP medium and select charm quarks which happen to be at those locations in the medium (i.e. fluid cells) within that temperature range

- boost the charm quarks into the local rest frame of the medium, extract their temperature and compare it with that of the medium

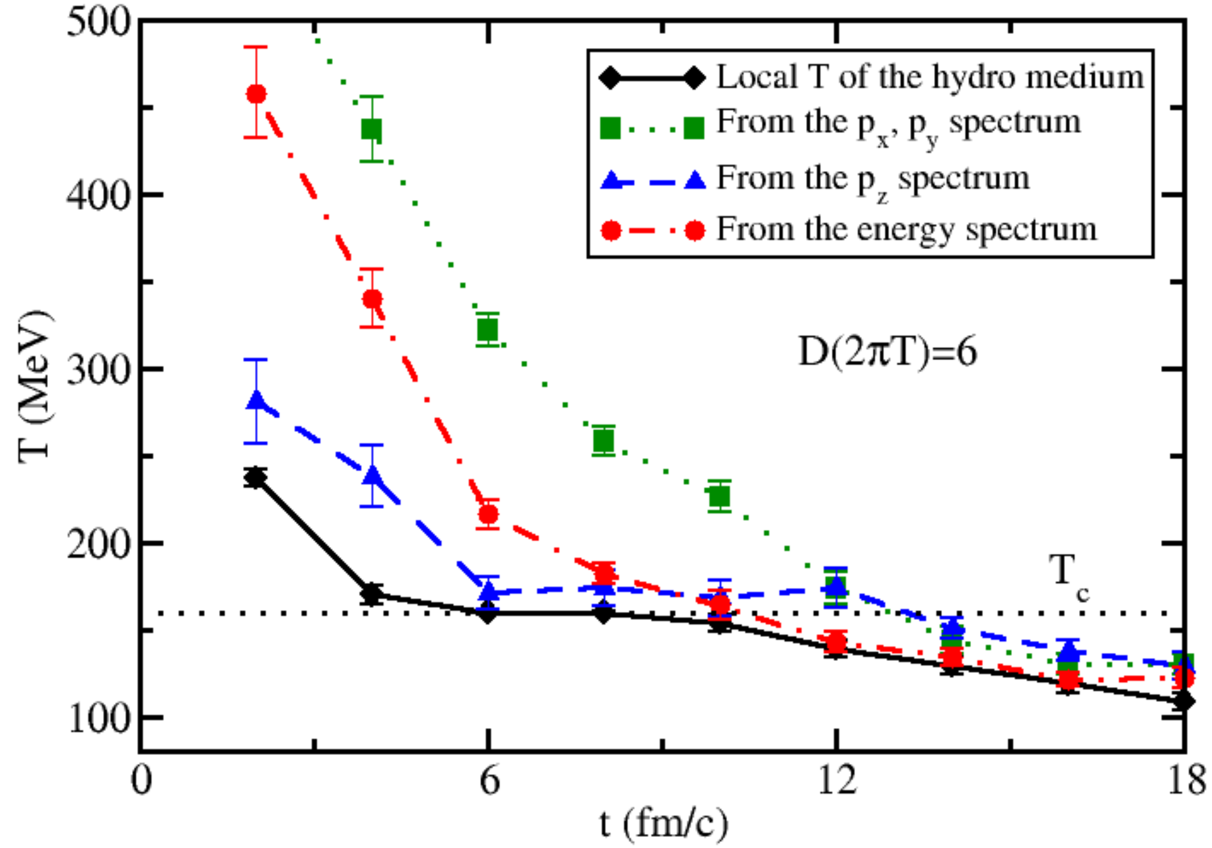
Medium temperature:
temperature range
$T-\Delta T < T < T+\Delta T$



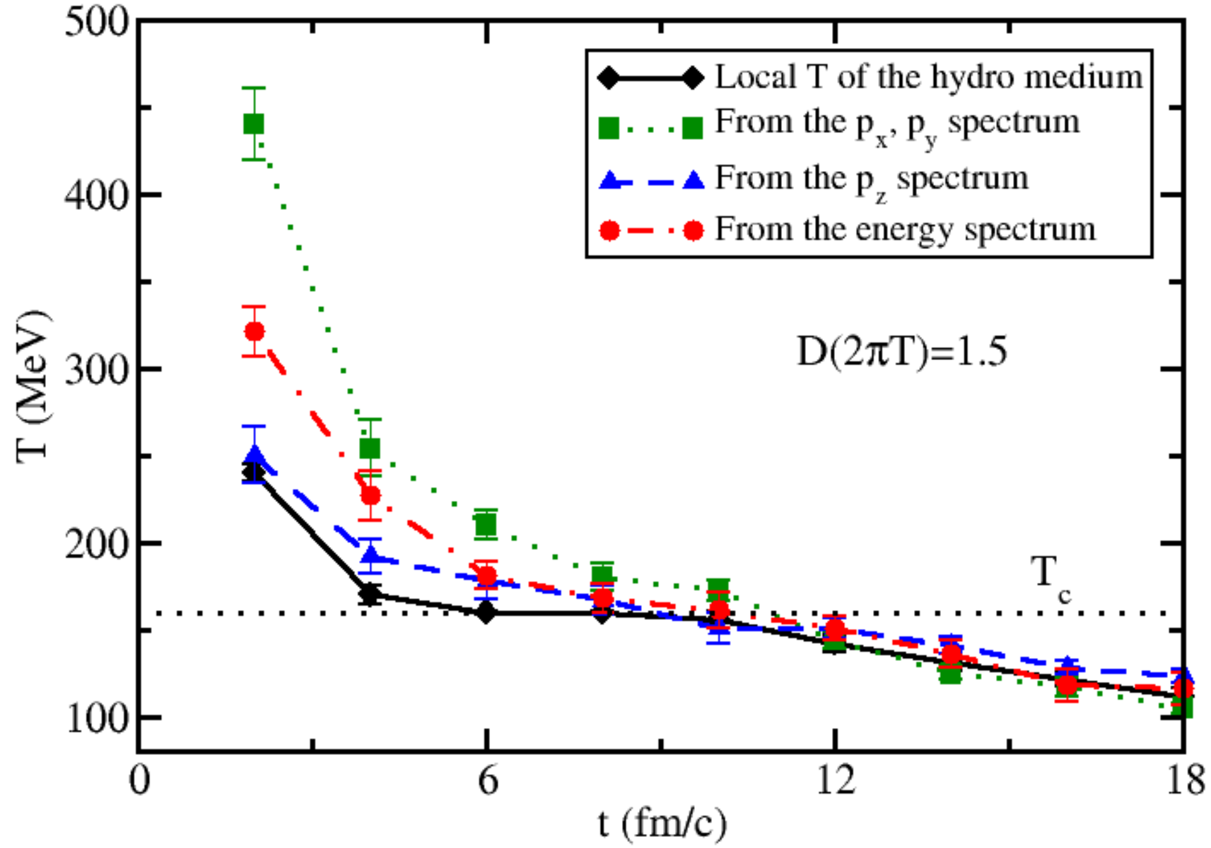


HQ Thermalization in a QGP

run analysis for two different values of the diffusion coefficient:



• no complete thermalization above T_c



• near complete thermalization around T_c

For "reasonable" values of the diffusion coefficient, heavy quarks remain off-equilibrium during the lifetime of the QGP

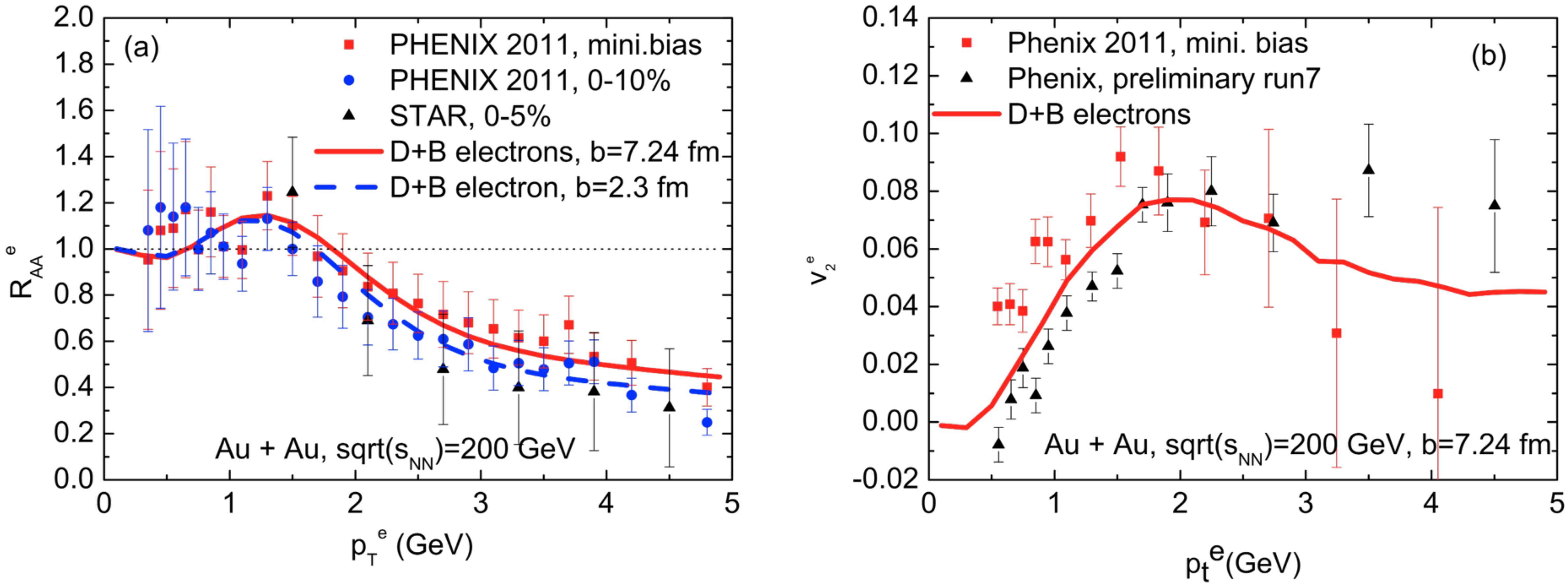
[this particular study was done with an ideal 3+1D RFD and PCM initial conditions for heavy quarks]

Langevin+Radiation

From RHIC to LHC

Current State-of-the-Art:

- Langevin for HQ + coalescence & fragmentation for hadronization + heavy meson diffusion in a hadron gas



He, Fries, Rapp, Phys. Rev **C86**: 014903, arXiv:1208.0256, and private communication with He

From RHIC to LHC:

- Heavy Quarks now (partially) ultra-relativistic:
  - radiative energy-loss
  - fragmentation as dominant hadronization mechanism

Langevin with Radiative Processes

modify Langevin Eqn. with force term due to gluon radiation:

$$\frac{d\vec{p}}{dt} = -\eta_D(p)\,\vec{p} + \vec{\xi} + \vec{f}_g$$

radiation force defined through rate of radiated gluon momenta:

$$\vec{f}_g = \frac{d\vec{p}}{dt}$$

- same noise correlator and fluctuation-dissipation relation still hold:

$$\eta_D(p) = \frac{\kappa}{2TE} \quad \text{and} \quad \langle \xi^i(t)\,\xi^j(t')\rangle = \kappa\,\delta^{ij}\,\delta(t-t')$$

- gluon radiation calculated in Higher Twist formalism:

$$\frac{dN_g}{dx\,dk_\perp^2\,dt} = \frac{2\alpha_s(k_\perp)}{\pi}\,P(x)\,\frac{\hat{q}}{k_\perp^4}\,\sin^2\left(\frac{t-t_i}{2\,\tau_f}\right)\left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2}\right)^4$$

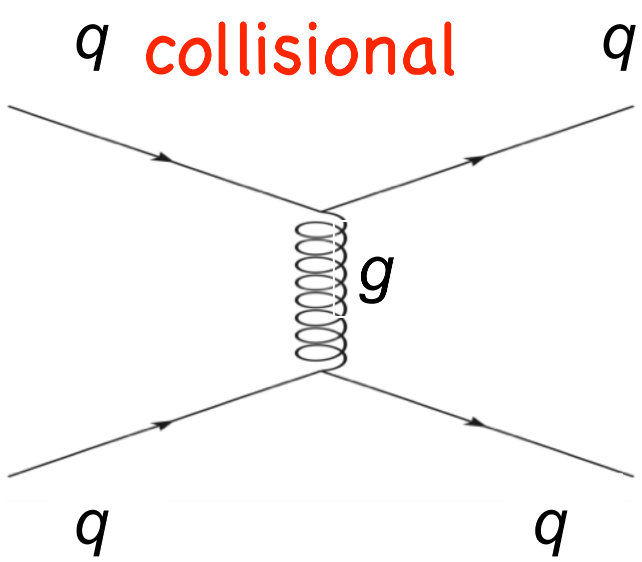
Guo & Wang: PRL 85, 3591
Majumder: PRD 85, 014023
Zhang, Wang & Wang: PRL 93, 072301

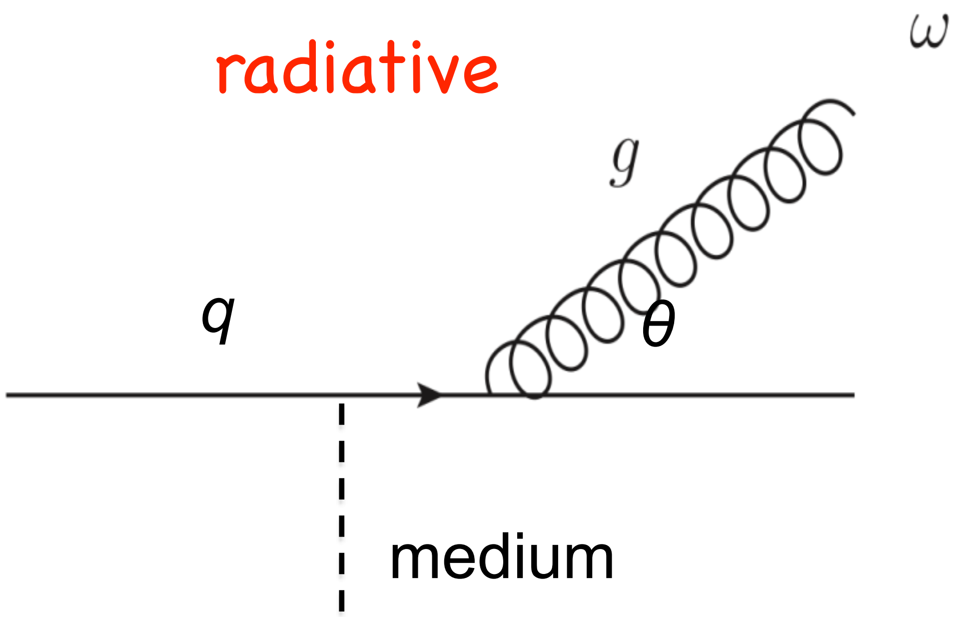
- relevant transport coefficients are now:

$$D = \frac{t}{M\eta_D(0)} = \frac{2T^2}{\kappa} \quad \text{and} \quad \hat{q} = 2\,\kappa\,C_A/C_F$$

Radiative vs. Collisional Energy Loss

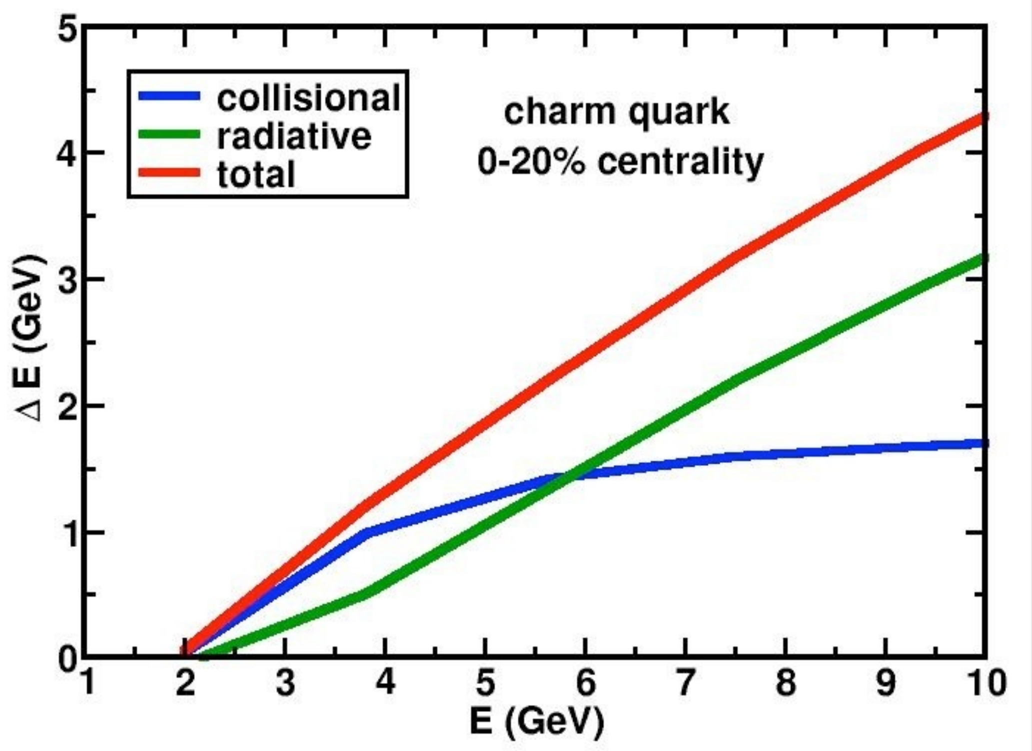
partons propagating through a QGP medium loose energy via two mechanisms:



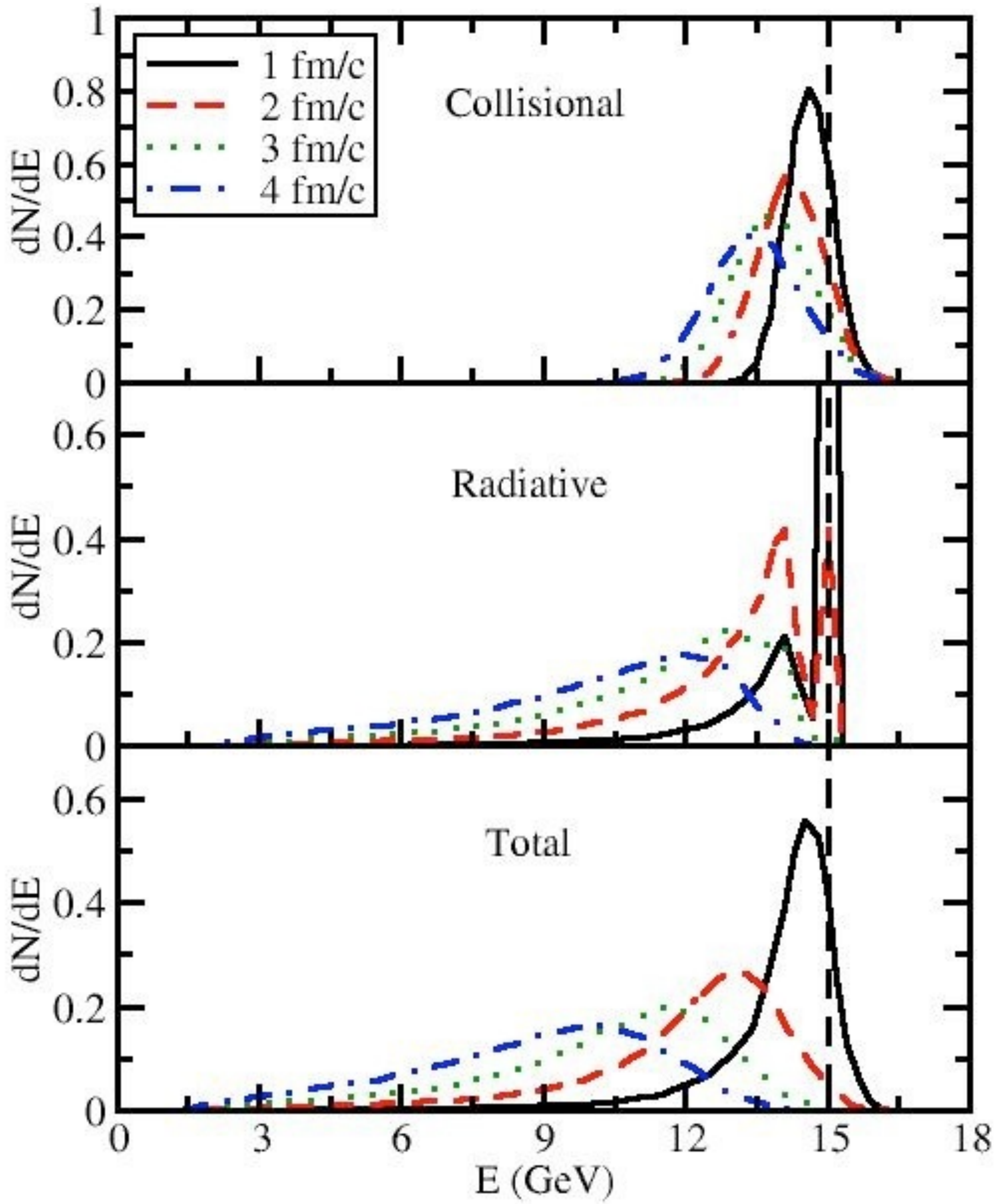


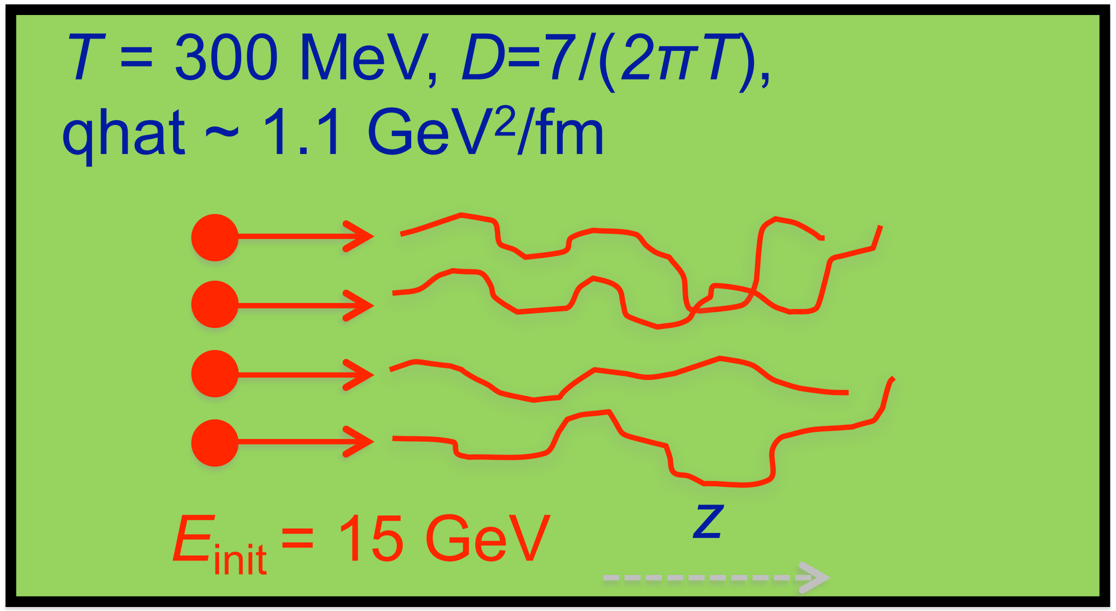
dominant mechanism depends on parton mass and energy:

- collisional energy-loss: heavy quarks at low momenta
- radiative energy loss: light quarks, gluons & heavy quarks at high momenta
- two-particle correlation observables as discriminators?



HQ Evolution in a Static Medium





Evolution of E distribution:

- prior to 2 fm/c, collisional energy loss dominates; after 2 fm/c, radiative dominates;
- collisional energy loss leads to Gaussian distribution, while radiative energy loss generates a long tail in distribution

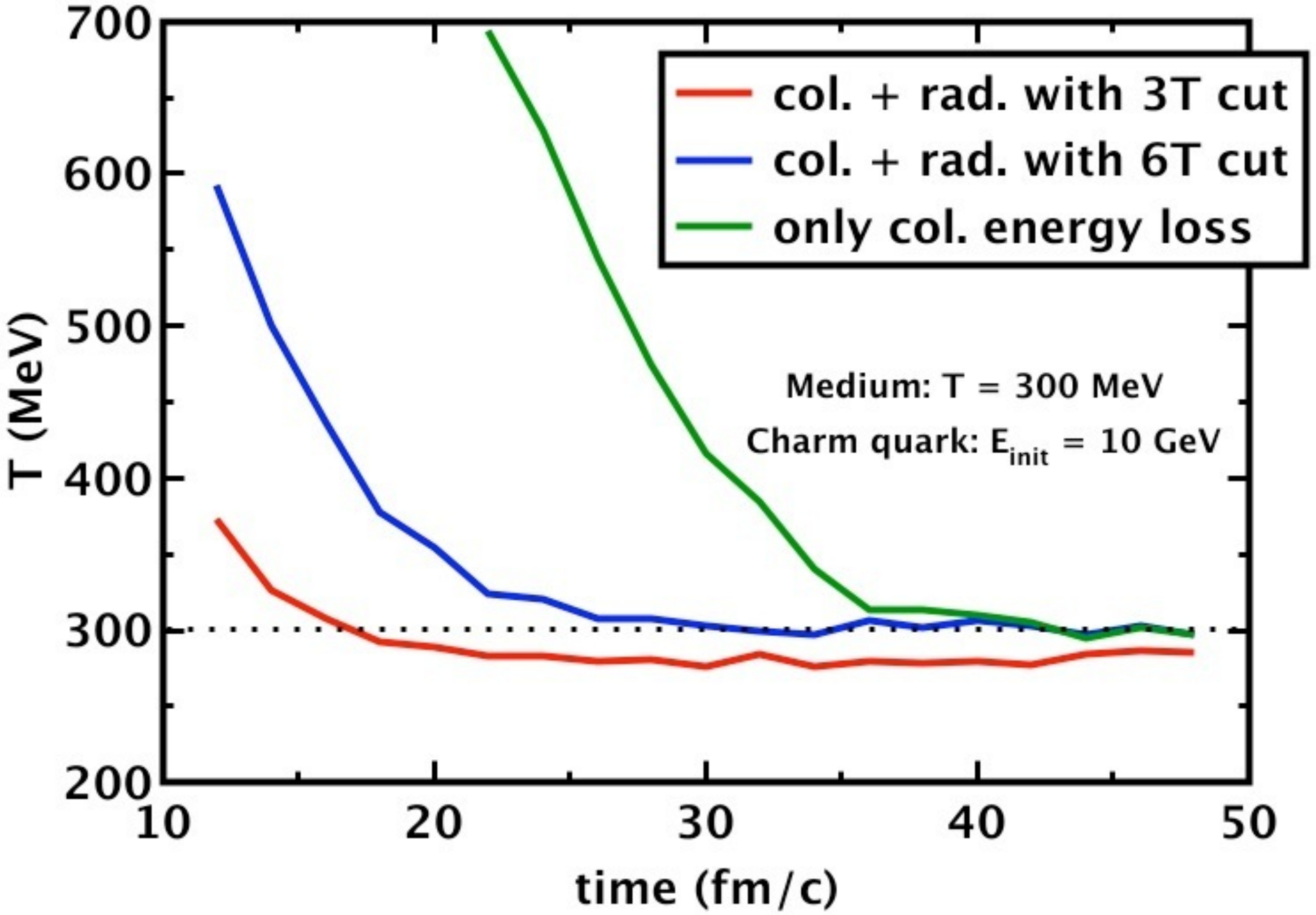
Thermalization in Langevin+Radiation

radiative term in Langevin Equation violates detailed balance:

- radiation should be suppressed for thermal momentum scale
  - introduce low momentum cut-off for gluon radiation: $p_{cut}=\alpha 3T$
- vary parameter α to ensure proper HQ thermalization

redo thermalization analysis in Langevin+Radiation approach:

- system shows proper thermalization dynamics for α≈2
- note that T_{therm} may depend on initial HQ momentum distribution
- for this particular set of parameters thermalization time: T_{therm} is reduced from ≈35 fm/c to ≈25 fm/c



HQ Hadronization:

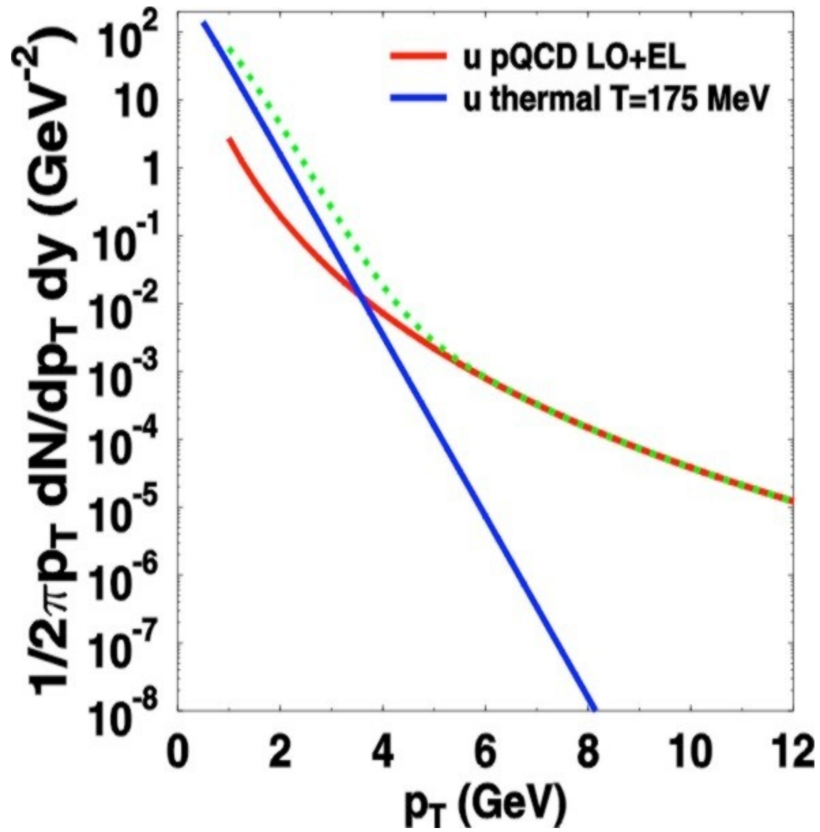
- Recombination + Fragmentation

Recombination+Fragmentation Model

basic assumptions:

- at low p_t, the parton spectrum is thermal and HQs recombine with light quarks into hadrons locally "at an instant":

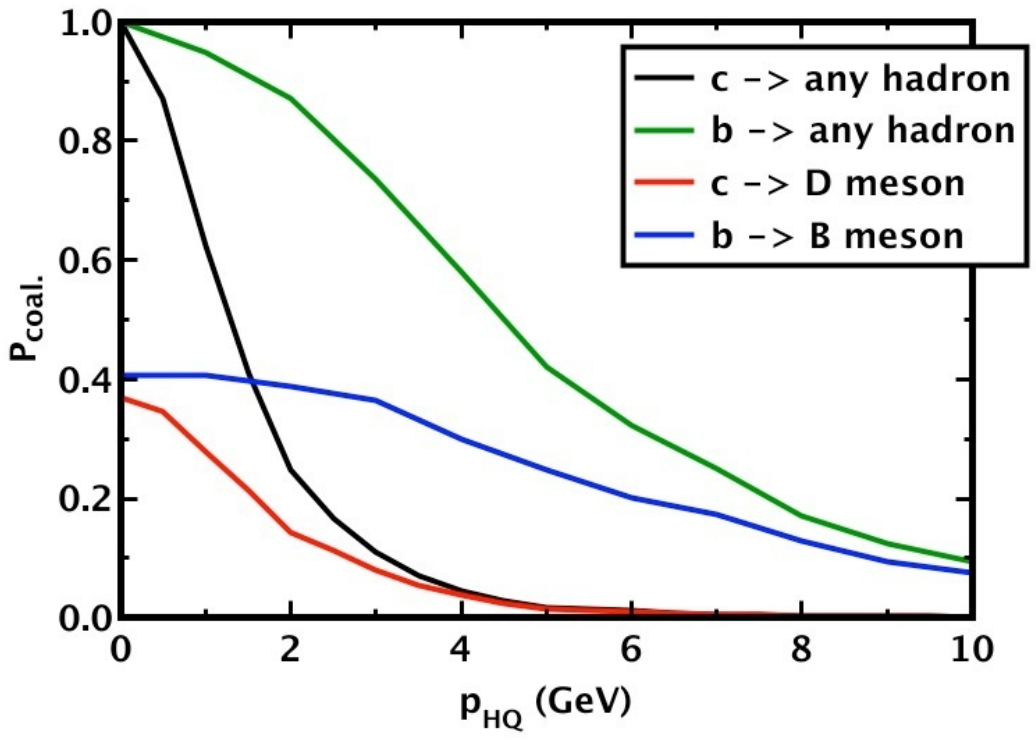
$$\frac{dN_M}{d^3P} = C_M \frac{V}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} w\left(\tfrac{1}{2}P - q\right) w\left(\tfrac{1}{2}P + q\right) \left|\hat{\varphi}_M(q)\right|^2$$



- at high p_t, the parton spectrum is given by a pQCD power law, HQs suffer radiative energy loss and hadrons are formed via fragmentation of HQs:

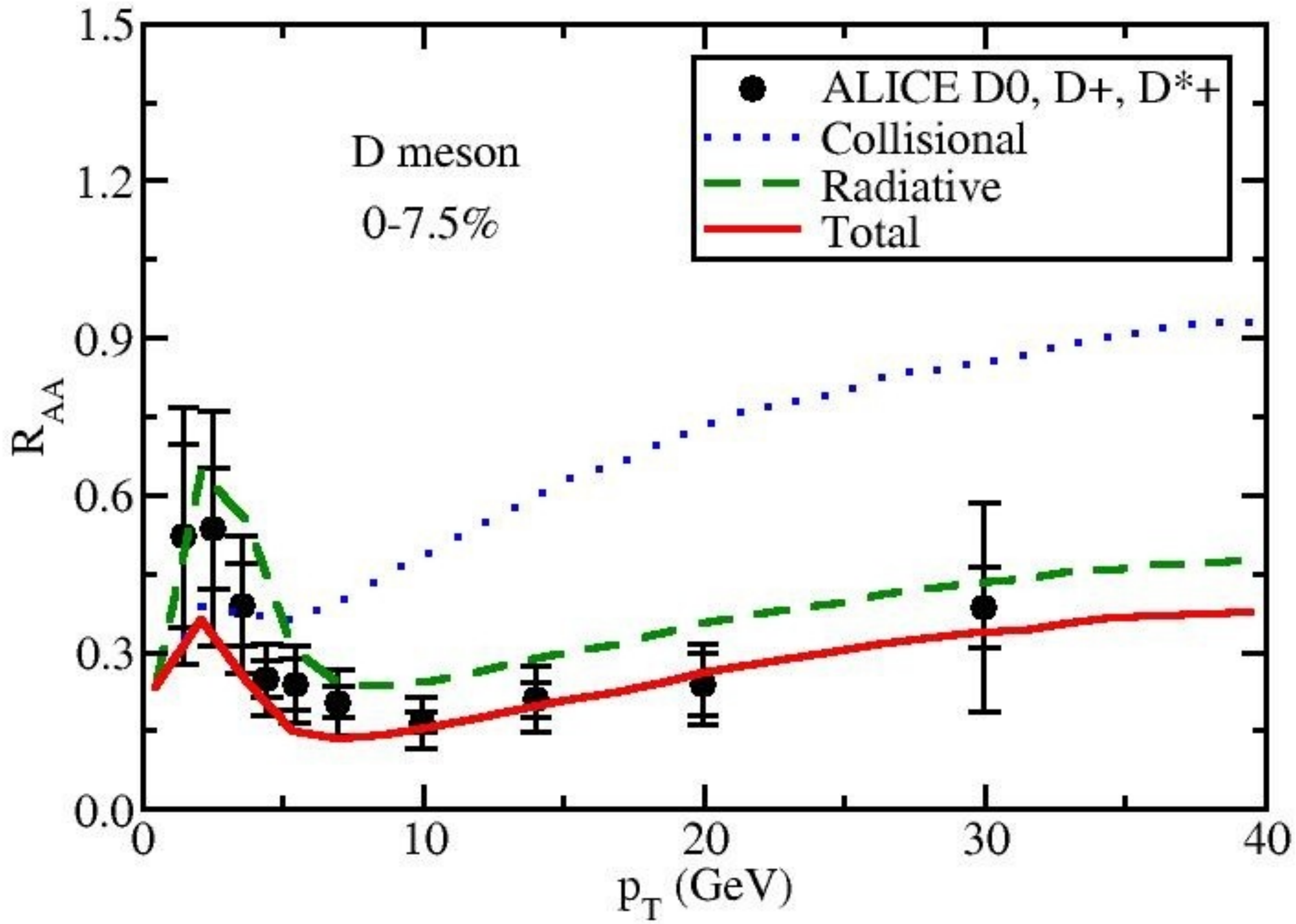
$$E\frac{dN_h}{d^3P} = \int d\Sigma \frac{P\cdot u}{(2\pi)^3} \int_0^1 \frac{dz}{z^2} \sum_\alpha w_\alpha(R, \tfrac{1}{z}P) D_{\alpha\to h}(z)$$

- shape of spectrum determines if reco or fragmentation is more effective:
  - for thermal distribution recombination yield dominates fragmentation yield
  - vice versa for pQCD power law distribution



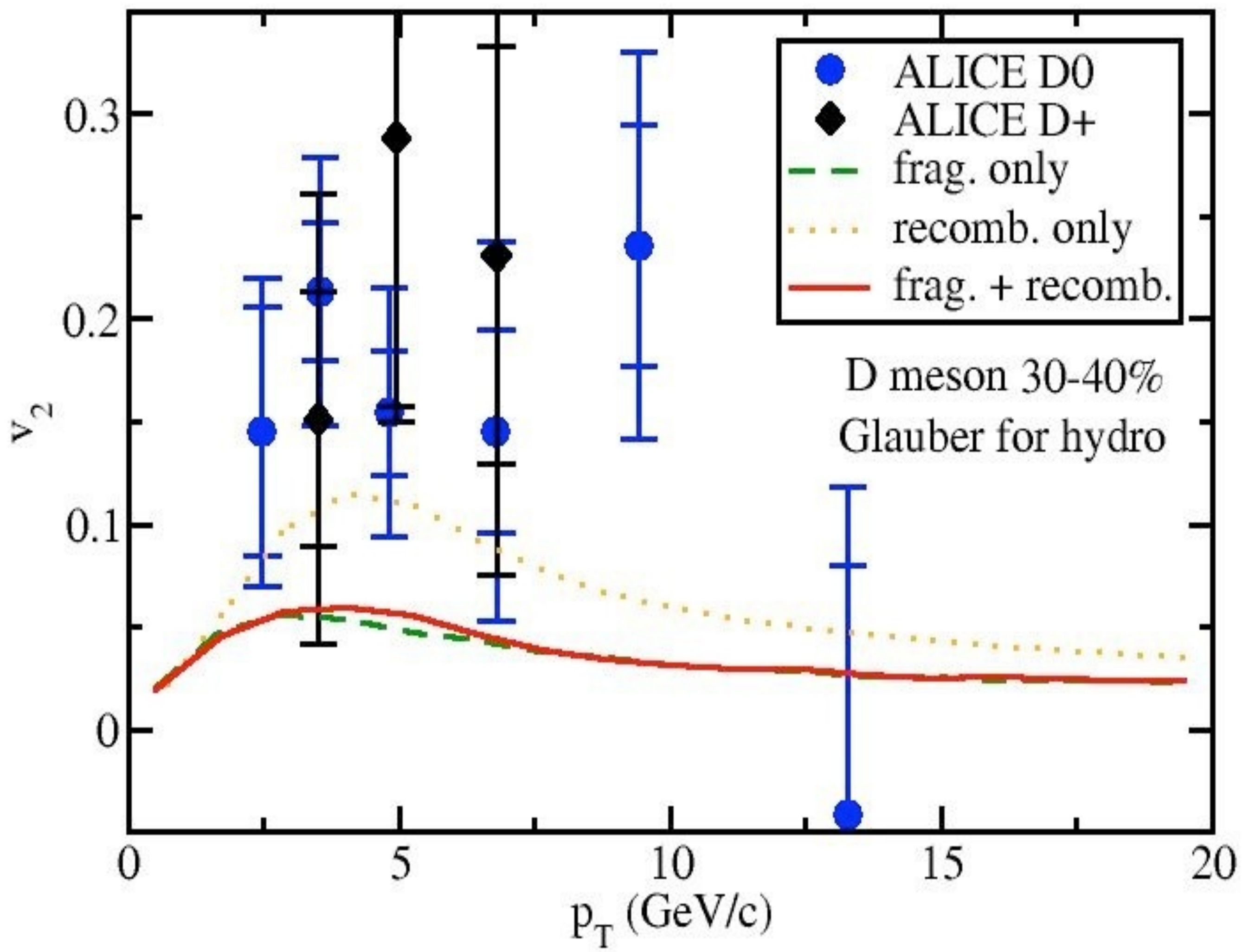
Comparison to Data

Comparison to Data: R_{AA}



- collisional energy loss dominates at low p_T, radiative at high p_T
- recombination important at low momenta
- combination of recombination and fragmentation provides a good description of data

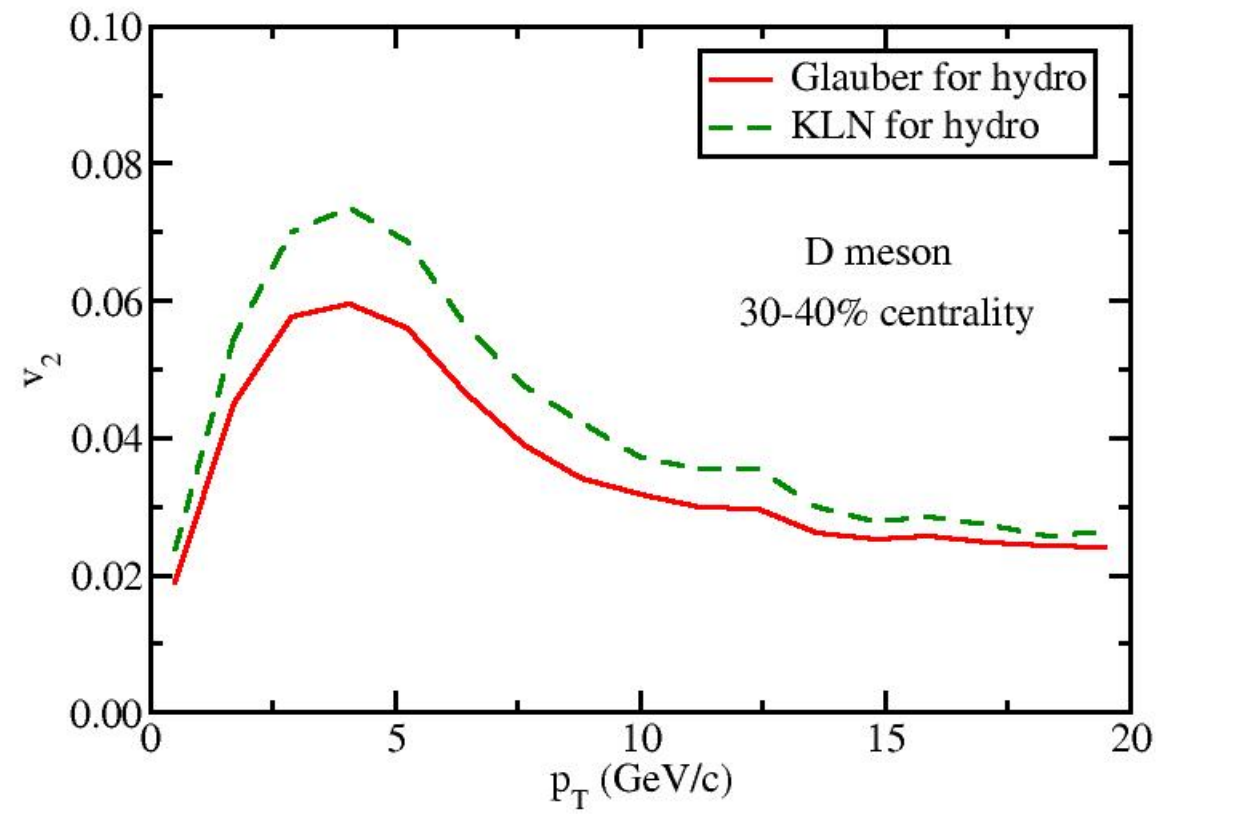
Comparison to Data: Elliptic Flow



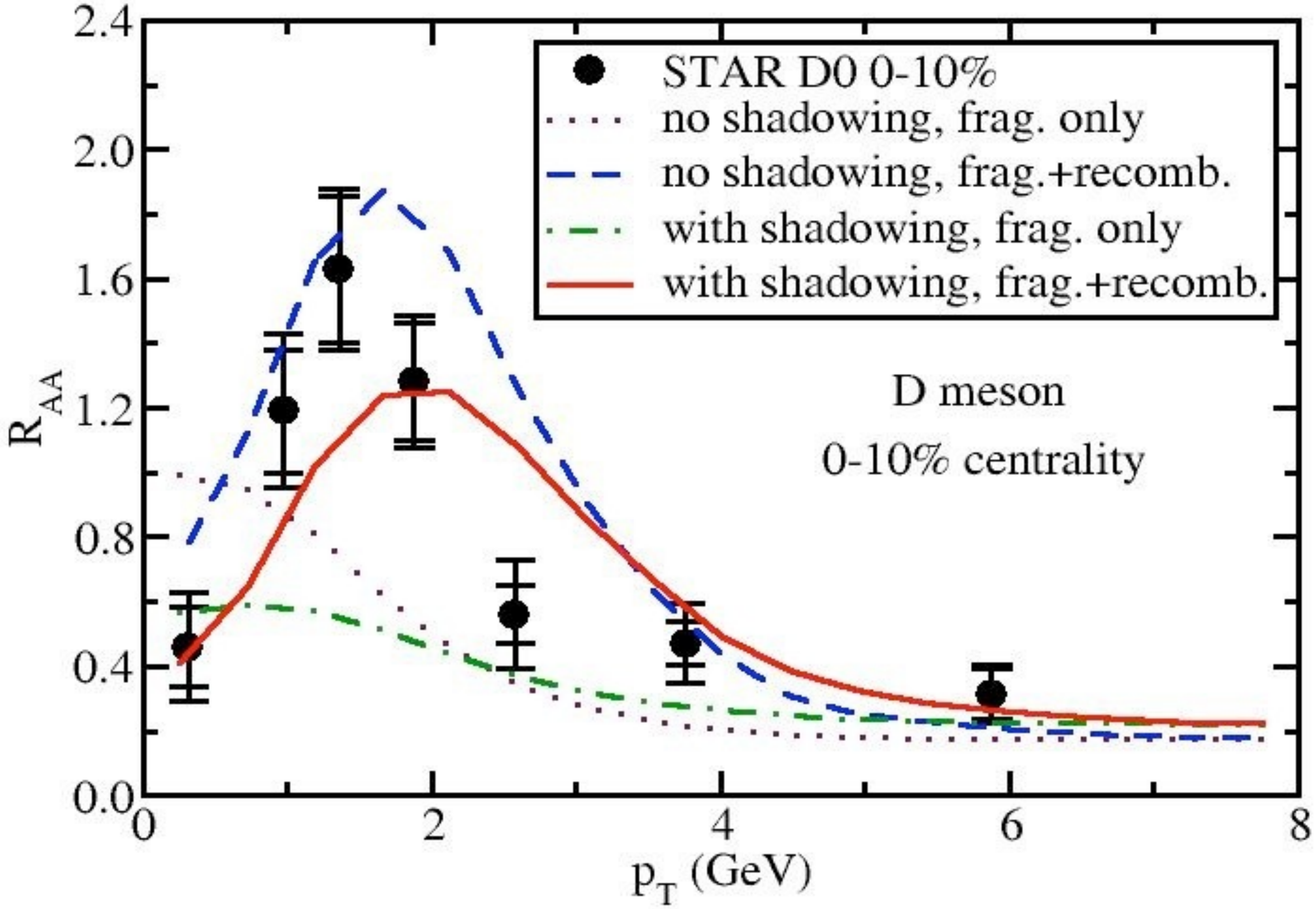
v_2 significantly underpredicted:

- most data in p_T domain already dominated by fragmentation as hadronization mechanism
- even pure recombination underpredicts data - check for initial conditions and EbE effects

- choice of vRFD initial condition more sensitive to v_2 than R_{AA}



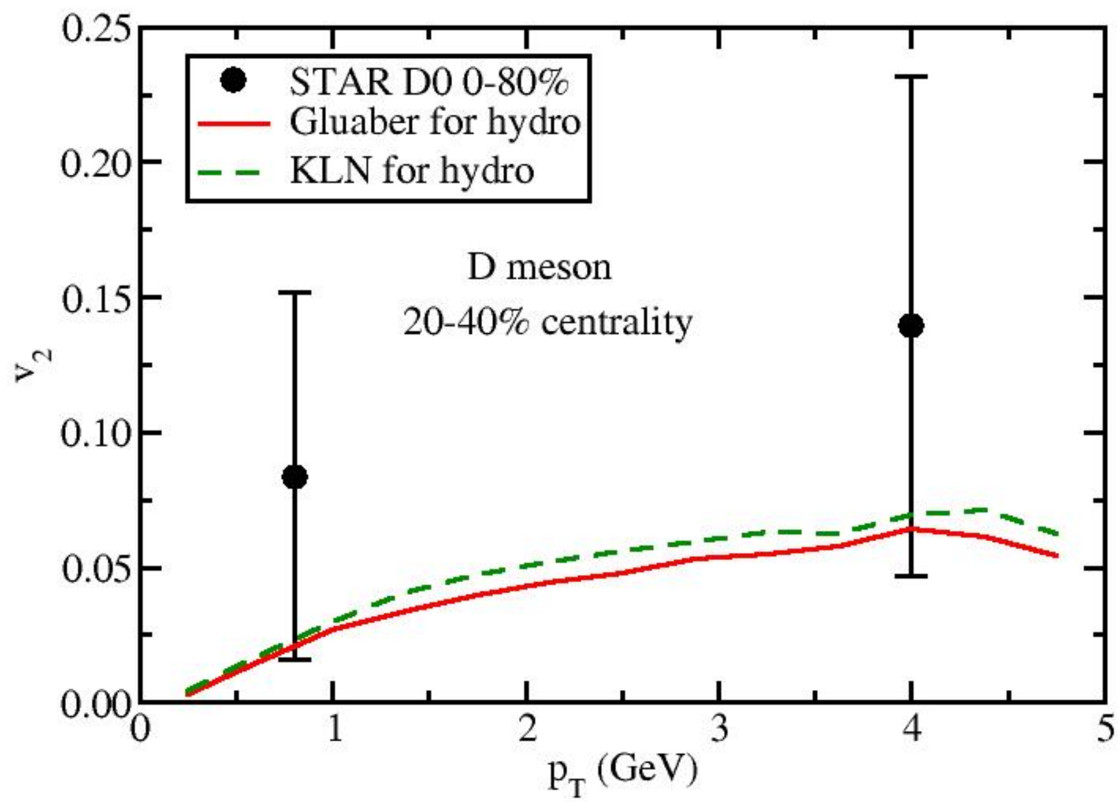
RHIC: D^0 R_{AA} and v_2



- good agreement with R_{AA} data
- shadowing in PDFs provides a degree of uncertainty

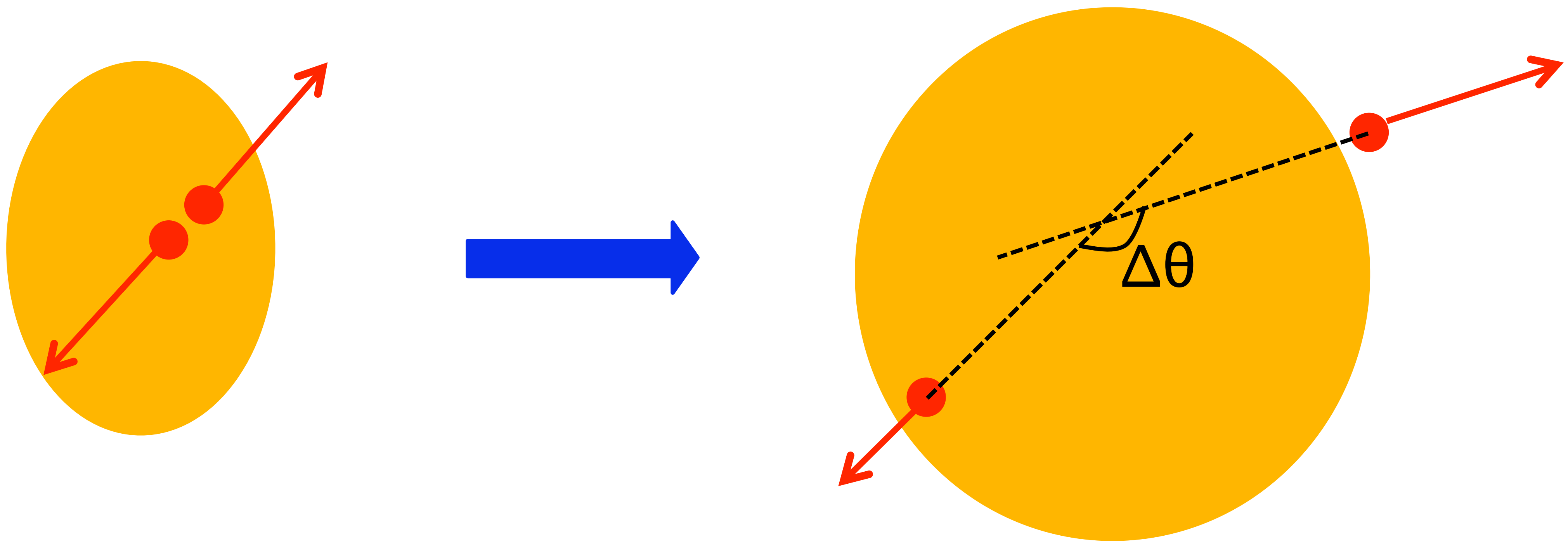
v_2 significantly underpredicted:

- data in p_T domain still dominated by recombination as hadronization mechanism



HQ Correlations

Angular HQ Correlations

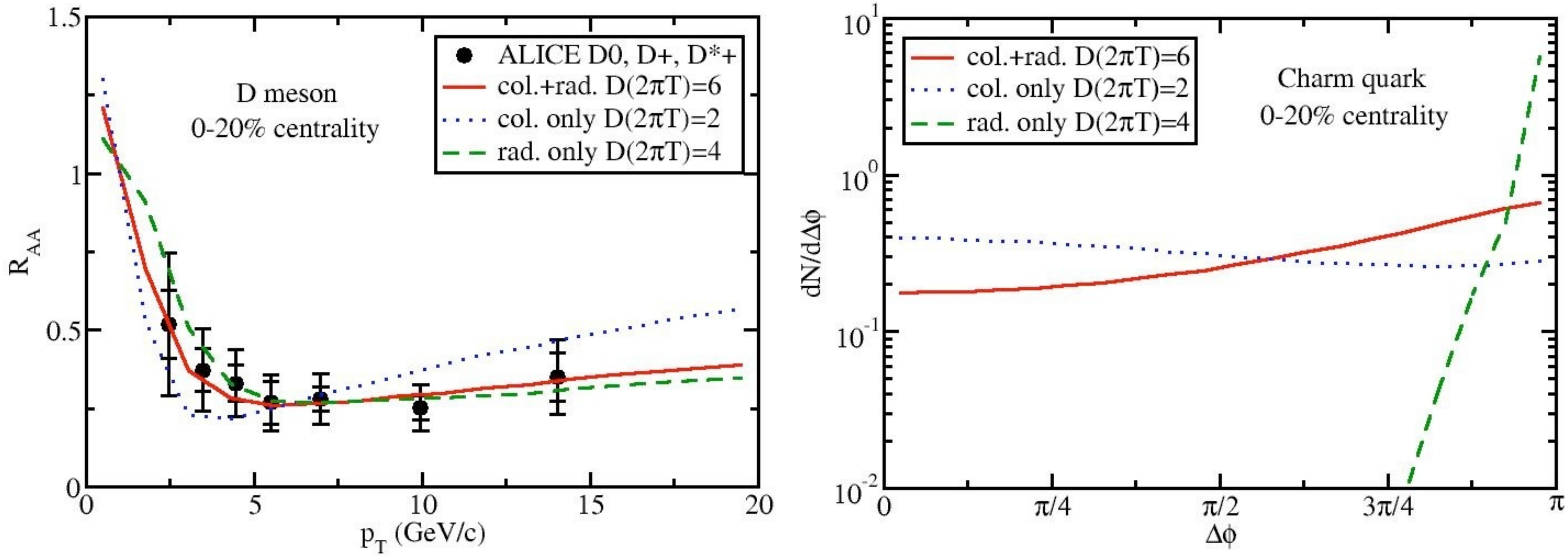


assume back-to-back production of initial Q & Qbar with the same magnitude of momentum

angular correlation of the final state QQbar is sensitive to:

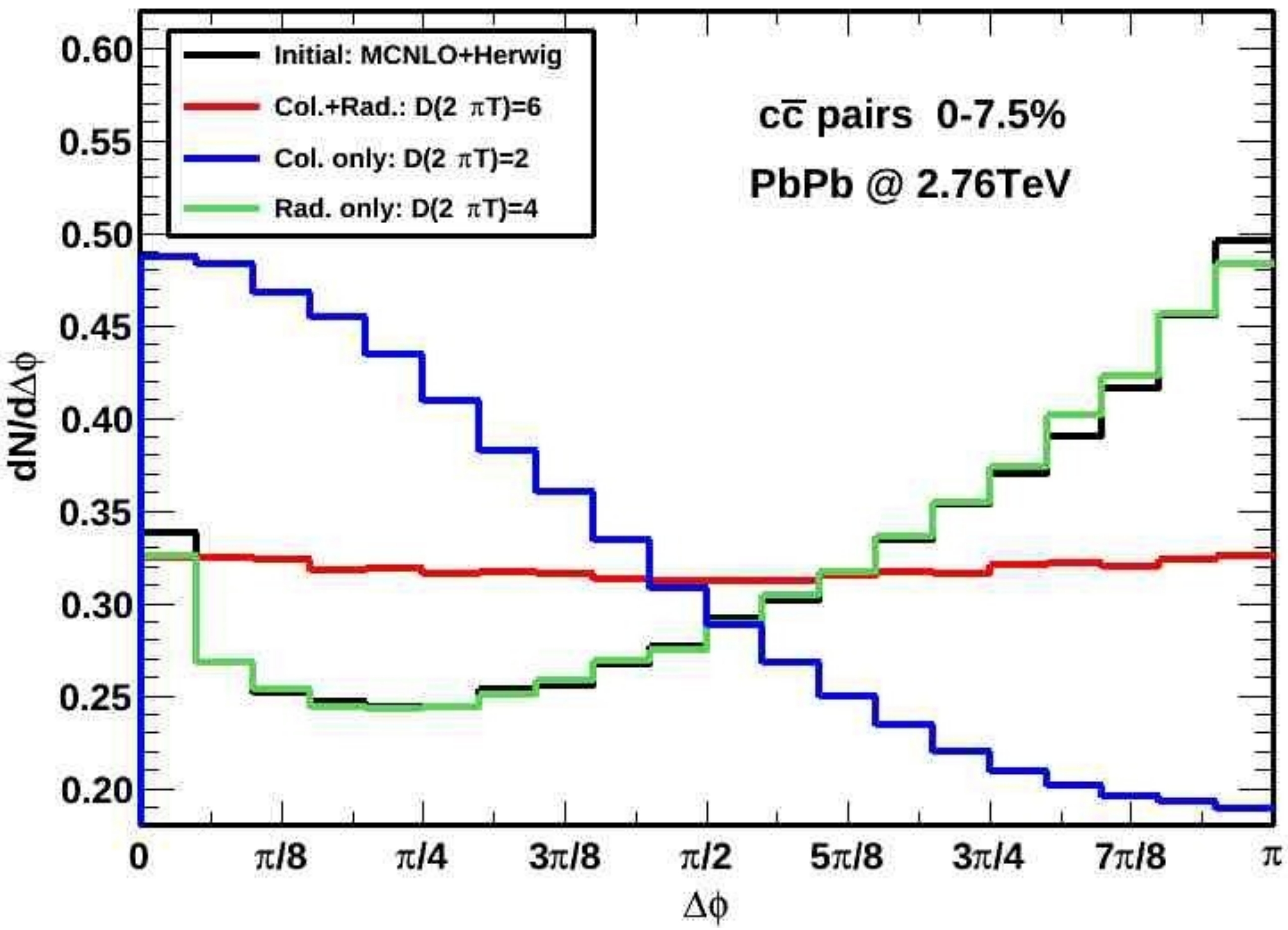
- momentum broadening of heavy quark
- degree of thermalization of heavy quarks
- coupling strength between heavy quarks and the QGP

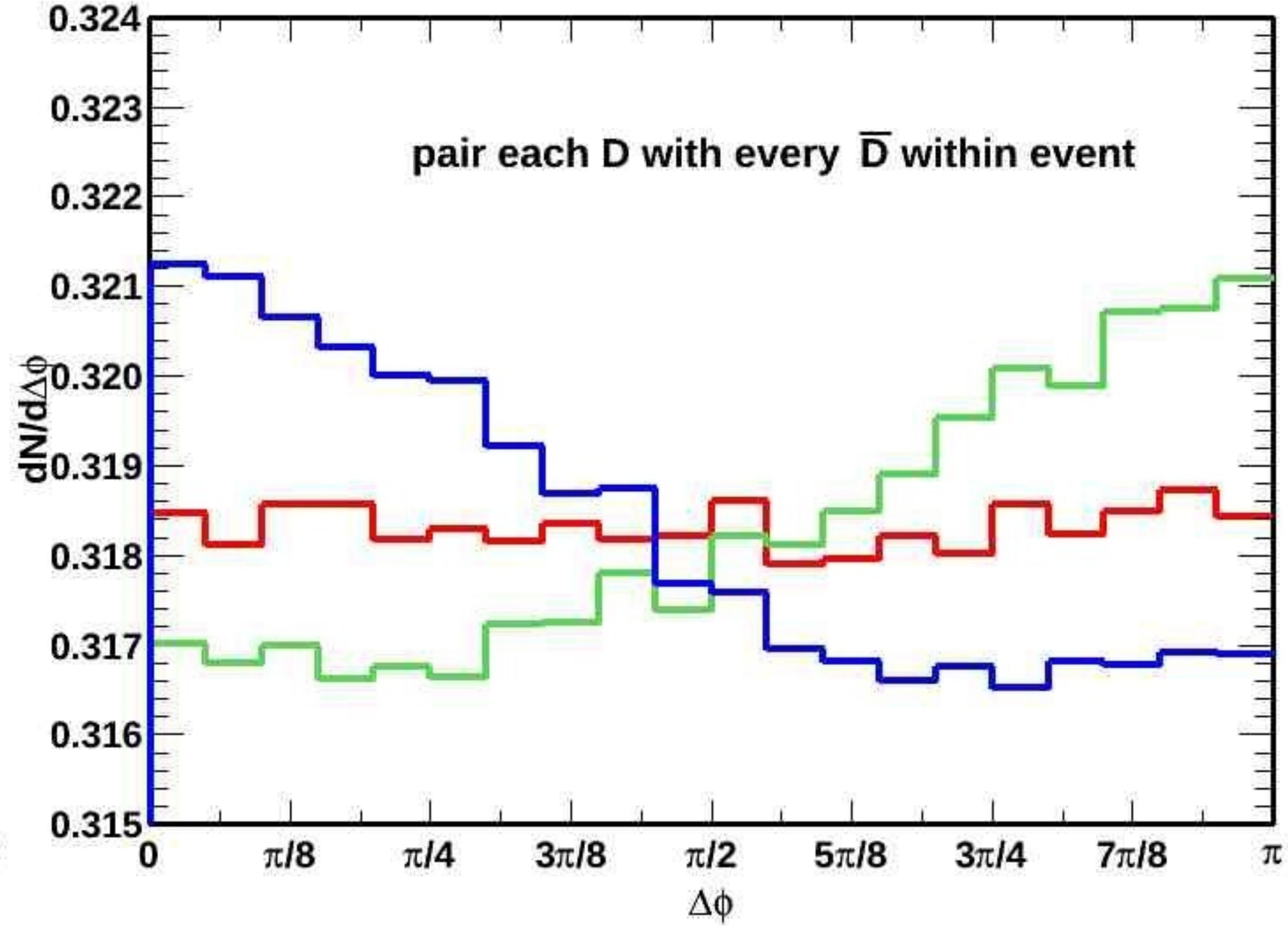
Correlations: Elastic vs. Radiative Processes



- each energy loss mechanism alone can fit R_{AA} with certain accuracy and choice of diffusion coefficient, yet they display very different behavior in the angular correlation function
- experimental observation may discriminate between the energy loss mechanisms of heavy quarks inside the QGP

Correlations II: D Mesons





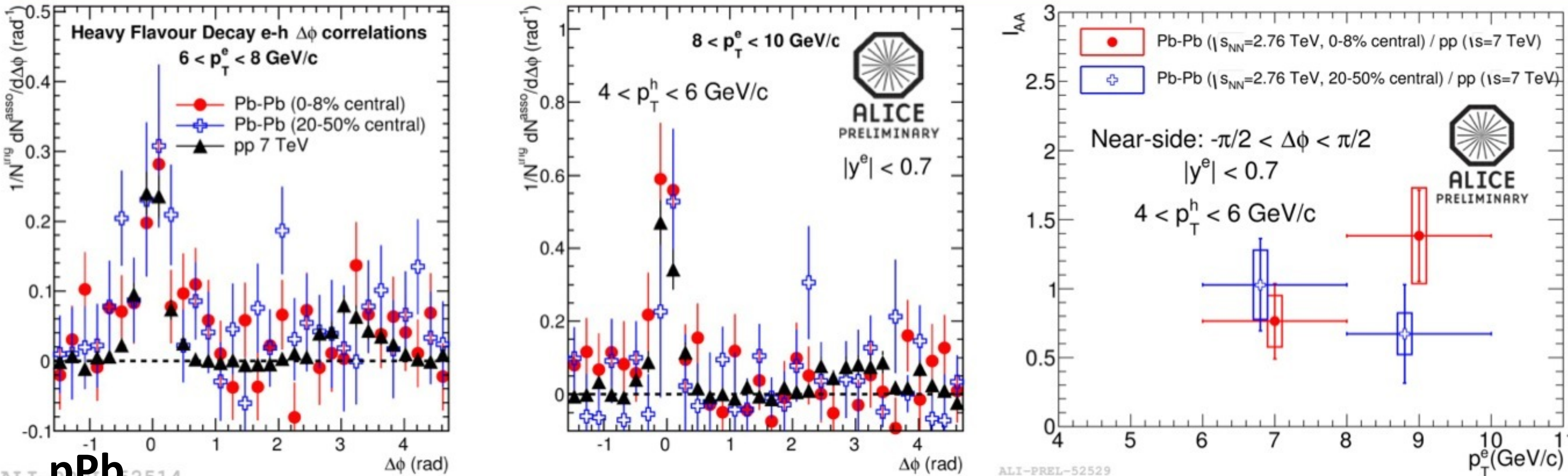
- initial HQ production: MCNLO + Herwig
- calculate angular correlation of final state ccbar pairs

- within each event, correlate each D with all Dbar's
- similar shape as direct ccbar correlation, but on top of a large background

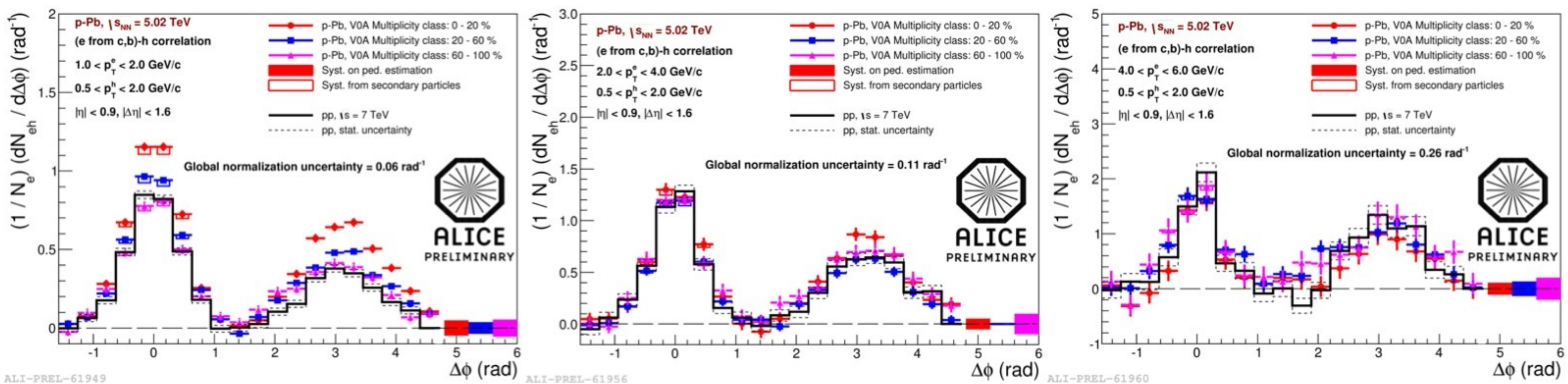
viable signal with good sensitivity to HQ energy loss mechanism if experiments could measure D Dbar angular correlation functions!

Correlations: Experimental Status

PbPb **(e from c,b)-h correlation** (Hard Probes 2013)



pPb



Theoretical challenge: hadronize QGP medium and HQ simultaneously EbE and calculate respective correlation function – will be possible after extending our transport framework to full EbE vRFD+UrQMD (including D meson rescattering in HG)

Next Steps

Transport Models for HQ in Medium

Choice of transport approach allows for study of HQ-medium interactions:
- Langevin+vRFD: sQGP + strong (non-perturbative) HQ-medium interaction
- linearized Boltzmann+vRFD: sQGP + pQCD driven HQ-medium interaction

(viscous) relativistic fluid dynamics:
- transport of macroscopic degrees of freedom
- based on conservation laws:

$$\partial_\mu T^{\mu\nu} = 0$$

$$\begin{aligned} T_{ik} &= \varepsilon u_i u_k + P\left(\delta_{ik} + u_i u_k\right) \\ &- \eta \left(\nabla_i u_k + \nabla_k u_i - \frac{2}{3}\delta_{ik} \nabla \cdot u \right) \\ &+ \varsigma\, \delta_{ik} \nabla \cdot u \end{aligned}$$

(plus an additional 9 eqns. for dissipative flows)

hybrid transport models:
- combine microscopic & macroscopic degrees of freedom
- current state of the art for RHIC modeling

diffusive transport models based on the **Langevin Equation**:
- transport of a system of microscopic particles in a thermal medium
- interactions contain a drag term related to the properties of the medium and a noise term representing random collisions

$$\vec{p}(t+\Delta t) = \vec{p}(t) - \frac{\kappa}{2T}\vec{v}\cdot\Delta t + \vec{\xi}(t)\Delta t$$

microscopic transport models based on the **Boltzmann Equation**:
- transport of a system of microscopic particles
- all interactions are based on binary scattering

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p}}{E}\times\frac{\partial}{\partial \vec{r}}\right] f_1(\vec{p},\vec{r},t) = \sum_{processes} C(\vec{p},\vec{r},t)$$

HQ Transport w/ Linearized Boltzmann

current implementation:

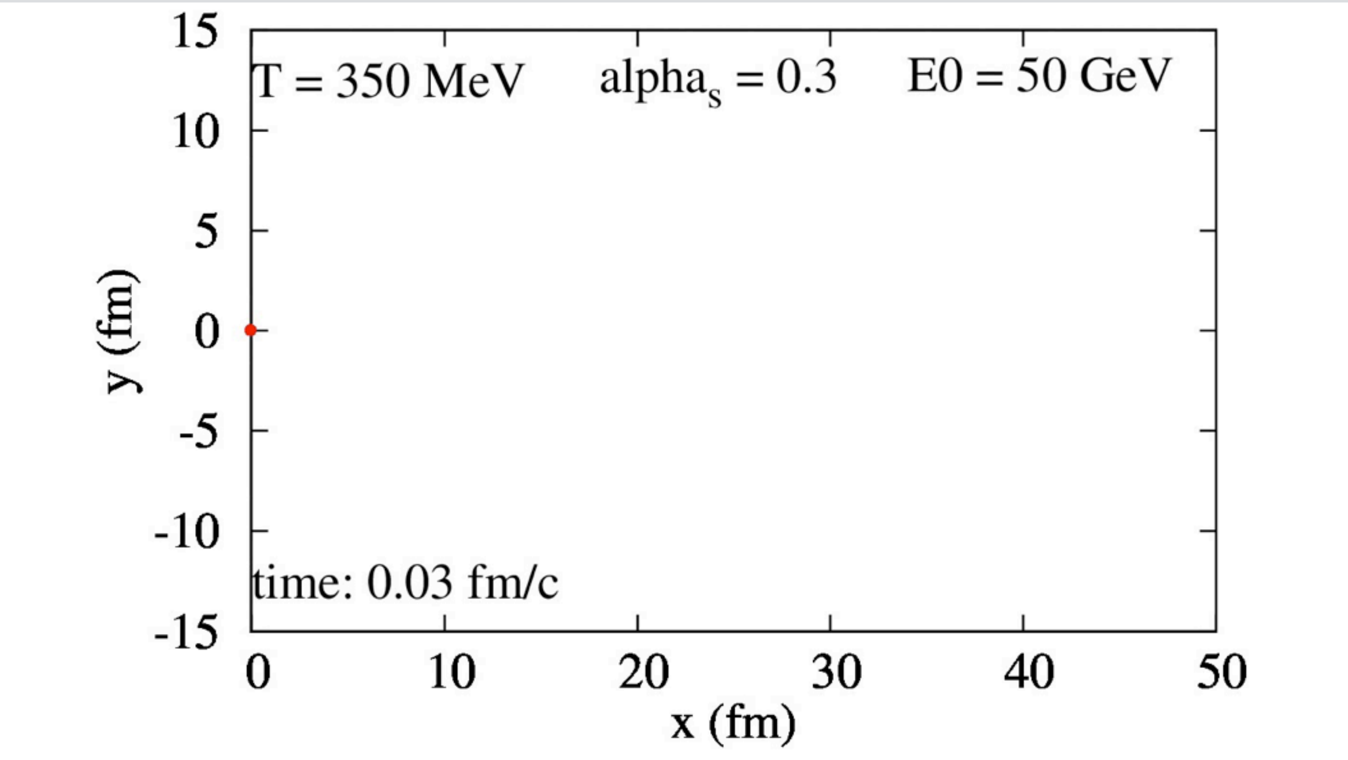
- elastic energy loss: cg➔cg & cq➔cq pQCD matrix elements
- infinite medium at fixed temperature

next steps:

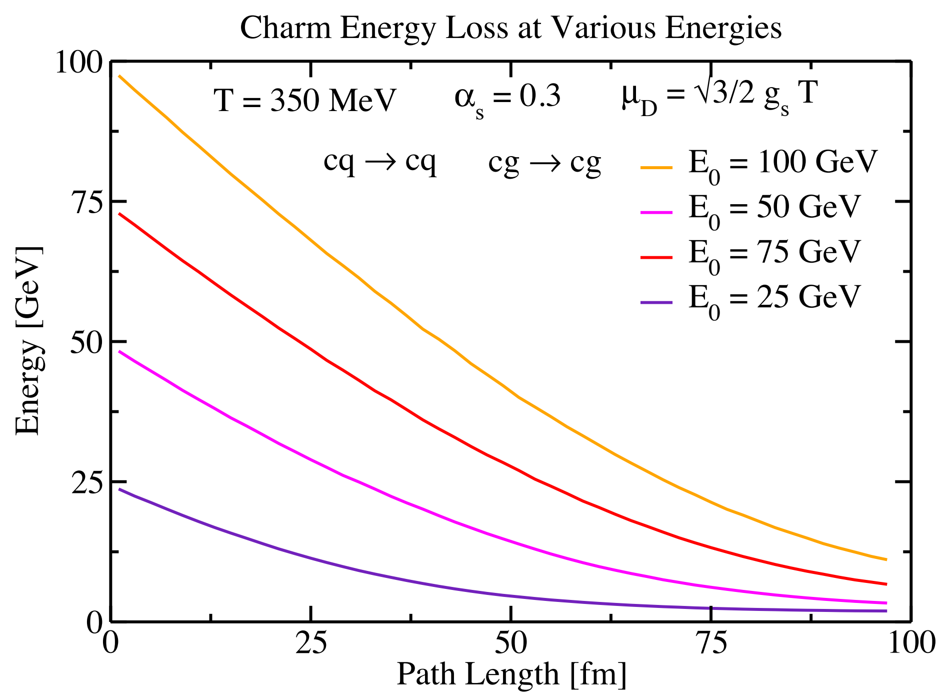
- realistic vRFD medium & initial conditions
- implementation of radiative processes & running coupling
- comparison to data

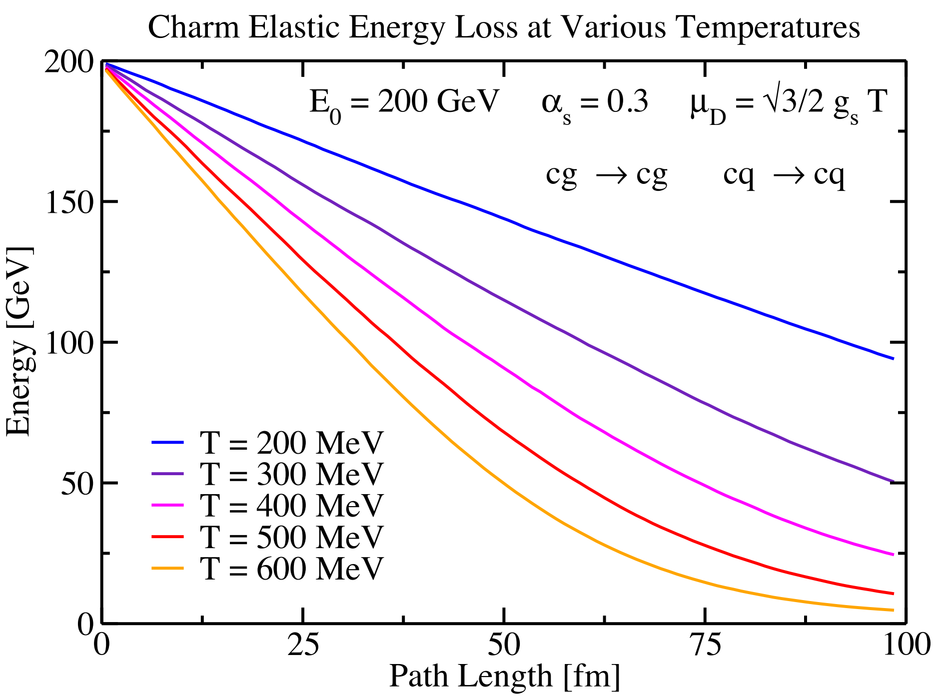
challenges:

- range of applicability



verification & validation: energy loss vs. path length in infinite QGP medium





Summary and Outlook

Heavy Quark Dynamics in a QGP:

- HQ's interact strongly, but do not equilibrate with the QGP
- sensitive to most medium properties - need consistent modeling of medium and HQ evolution
- Langevin with radiation: extend calculations to LHC domain

Work in Progress:

- correlation observables to quantify radiative vs. collisional energy-loss contributions
- Linearized Boltzmann: vary HQ-medium coupling
- D and B meson interactions in a Hadron Gas
- pre-equilibrium dynamics prior to QGP formation (anomalous transport?)


The End

The Parton Cascade Model

The PCM is a microscopic transport model based on the Boltzmann Equation:

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p}}{E} \times \frac{\partial}{\partial \vec{r}}\right] f_1(\vec{p},\vec{r},t) = \sum_{processes} C(\vec{p},\vec{r},t)$$

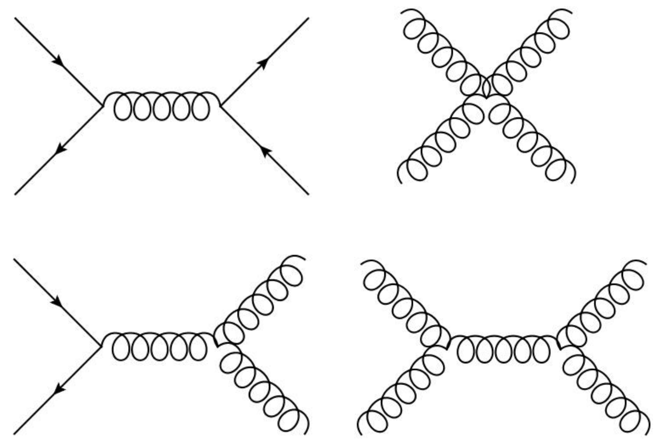
- describes the full time-evolution of a system of quarks and gluons at high density & temperature
- ideally suited for describing the interaction of jet with medium as well as the medium response

- classical trajectories in phase space (with relativistic kinematics)
- interaction criterion based on geometric interpretation of cross section:

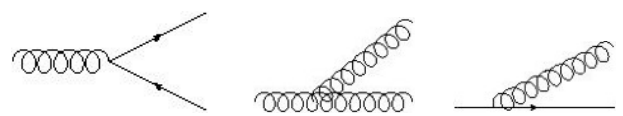
$$d_{\min} \leq \sqrt{\frac{\sigma_{\rm tot}}{\pi}} \qquad \sigma_{\rm tot} = \sum_{p_3,p_4} \int \frac{d\sigma(\sqrt{\hat{s}}; p_1,p_2,p_3,p_4)}{d\hat{t}}\, d\hat{t}$$

- system evolves through a sequence of binary (2↔2) elastic and inelastic scatterings of partons and initial and final state radiations within a leading-logarithmic approximation (2→N)
- guiding scales:
  - initialization scale Q_0
  - IR divergence regularization: p_T cut-off p_0 or Debye-mass μ_D
  - intrinsic k_T
  - virtuality > μ_0

- binary cross sections are calculated in leading order pQCD:



- radiative processes (full DGLAP evolution):



Medium Dependencies

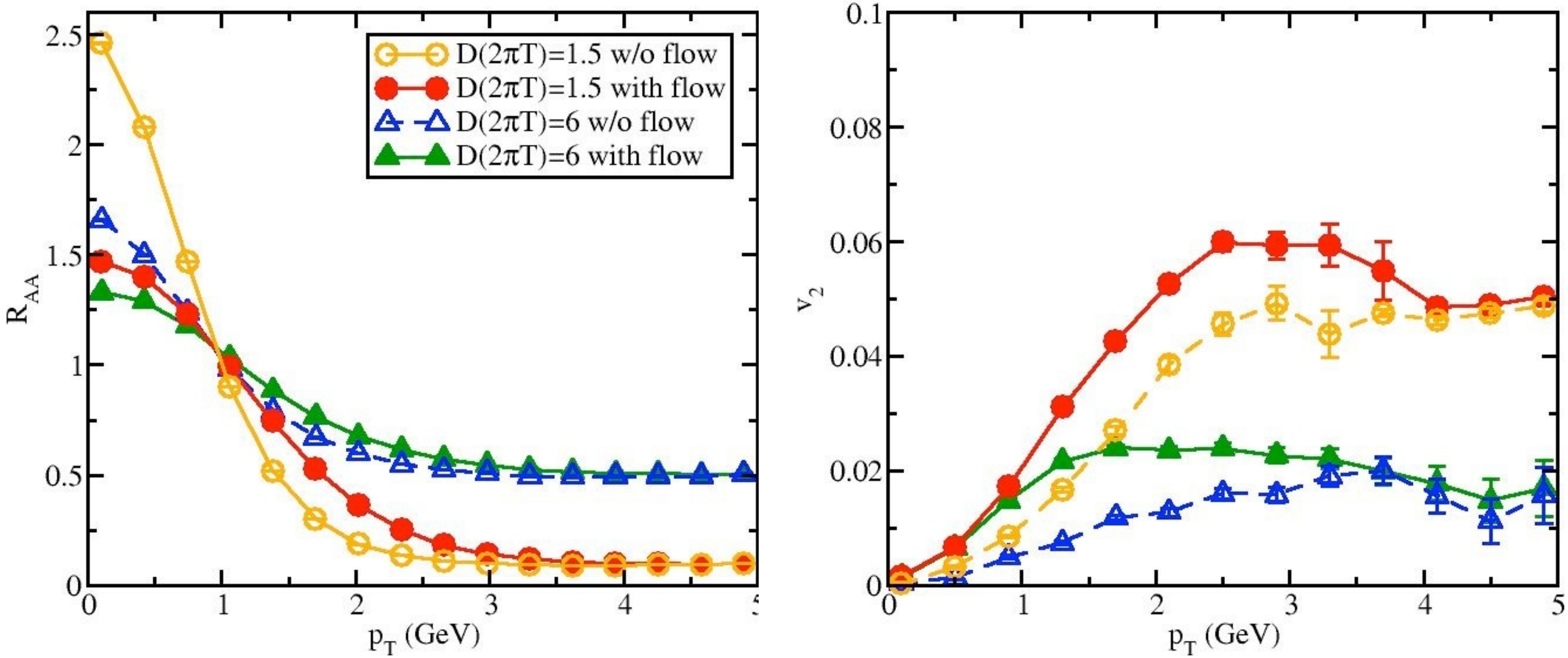
How do HQ observables such as elliptic flow and the nuclear modification factor depend on parameters of the medium and the HQ evolution?

- contributions of medium flow vs. geometry
- RFD initial conditions
- C/B ratio when using non-photonic electrons
- thermalization time of the medium
- ...

Geometry vs. Flow

Both geometric asymmetry and collective flow generate positive v_2:

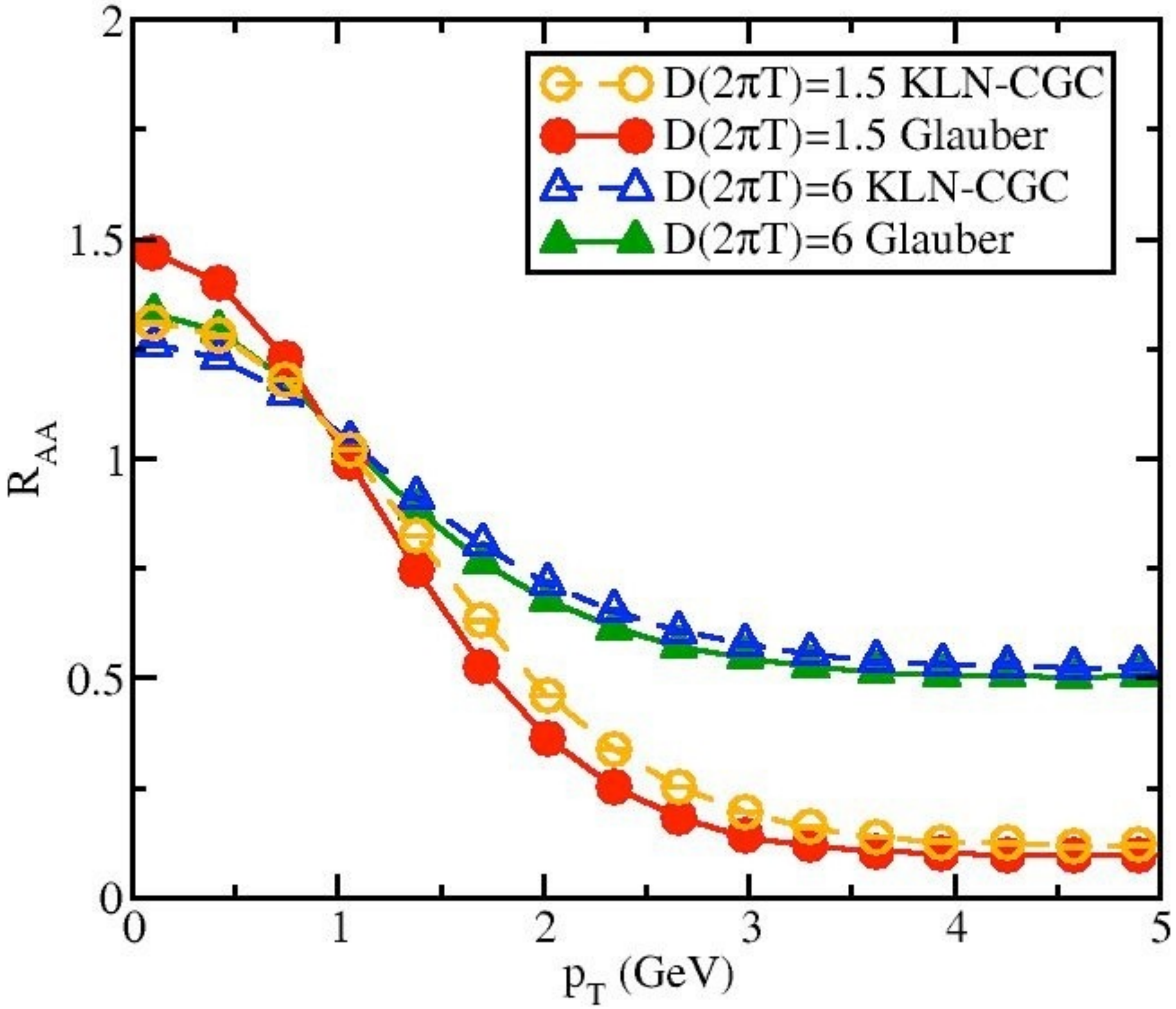
- decouple the influence of QGP collective flow on heavy quark motion by solving Langevin equation in the **global** c.m. frame, instead of the **local** rest-frame
- medium geometry dominates the high p_T region, while the collective flow has a significant impact in the low p_T region

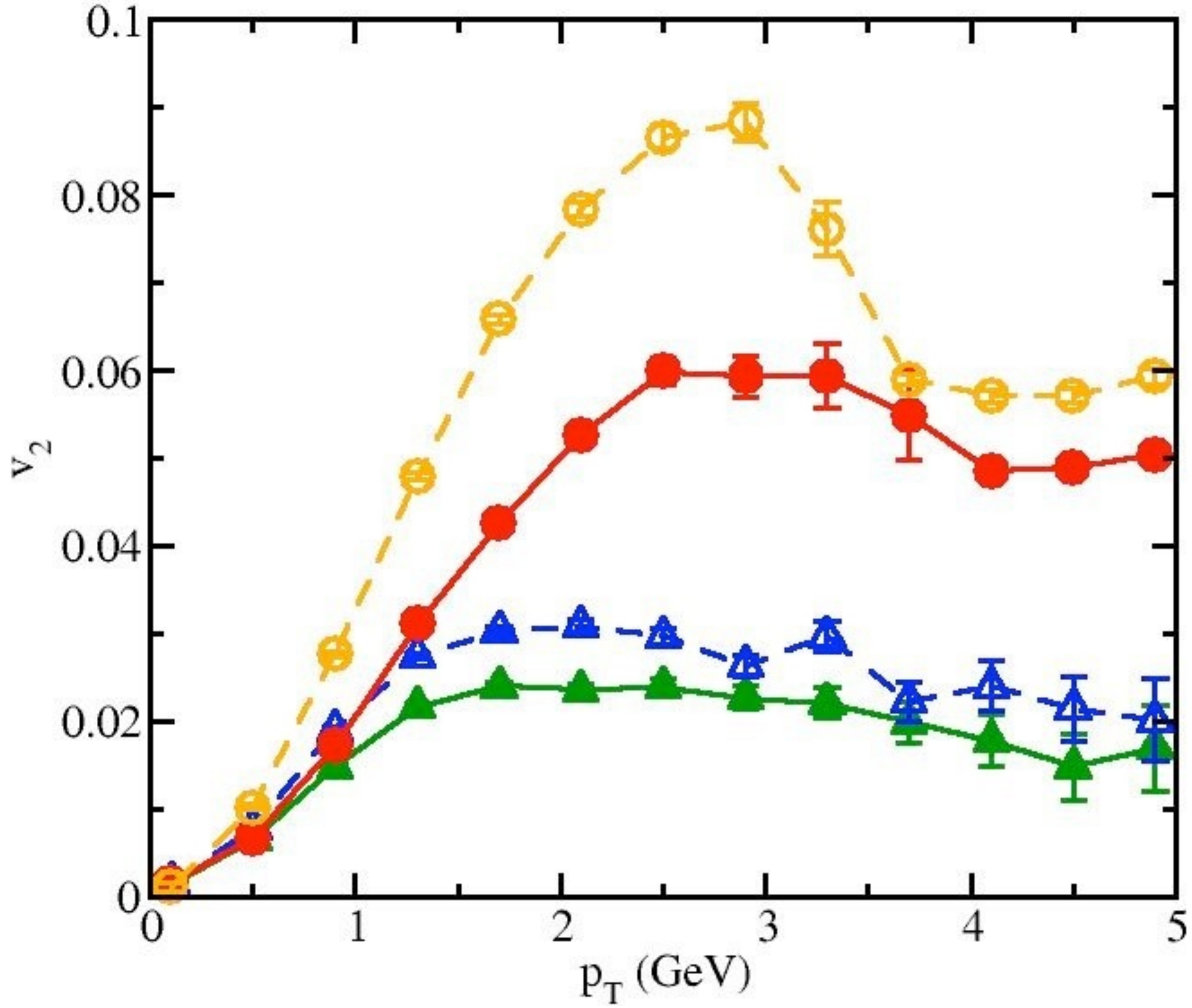


Initial Conditions

KLN-CGC model exhibits a larger eccentricity of the medium:

- no apparent difference in R_{AA}, but significant larger v_2 from KLN-CGC initialization

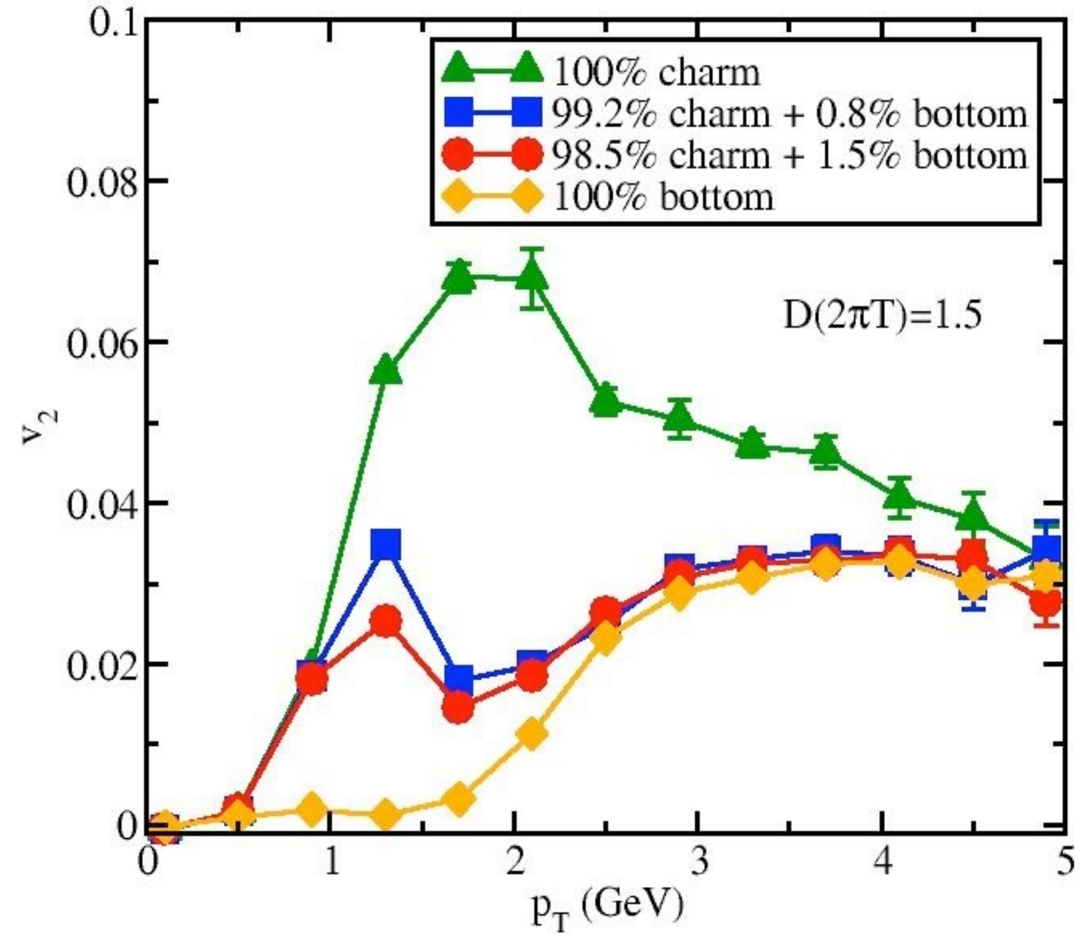


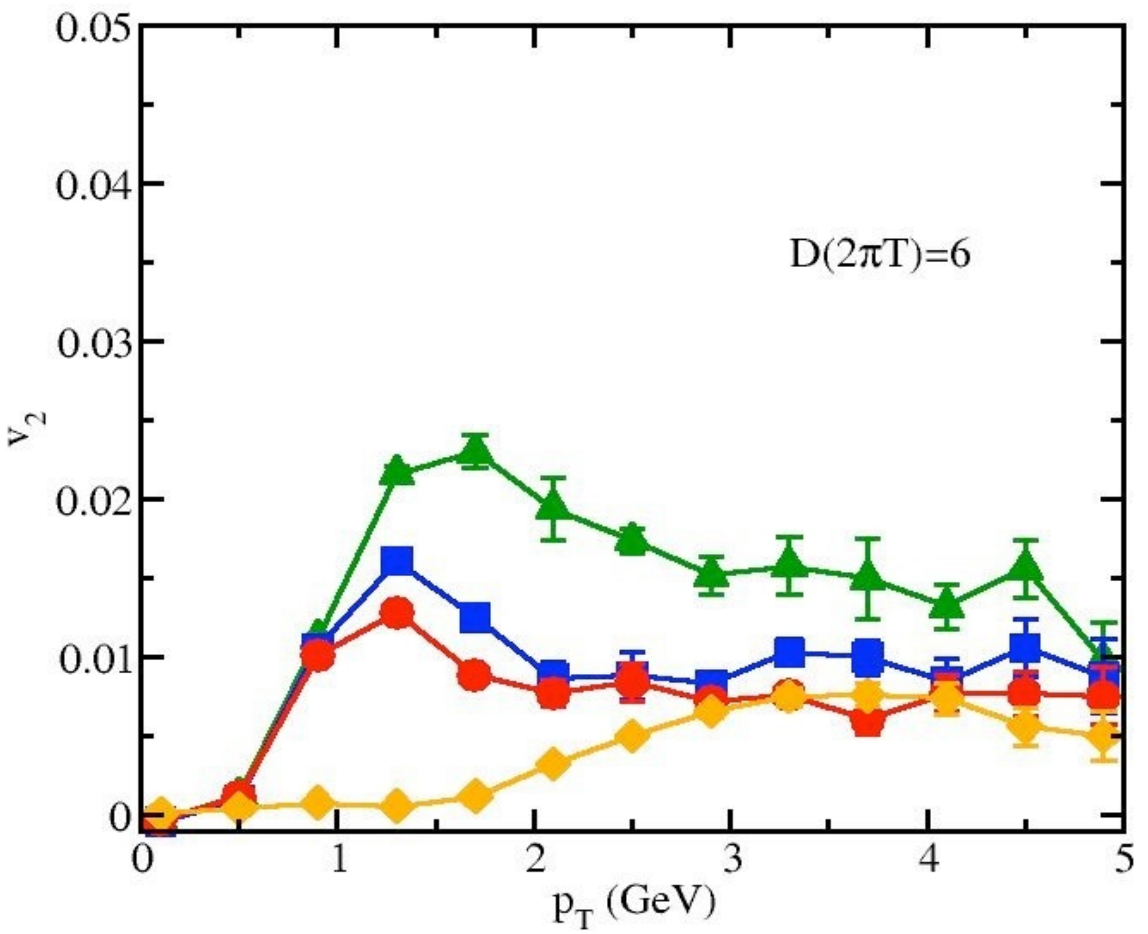


Charm to Bottom Ratio

there still exists an uncertainty in the relative normalization of charm and bottom quark production in pQCD calculations:

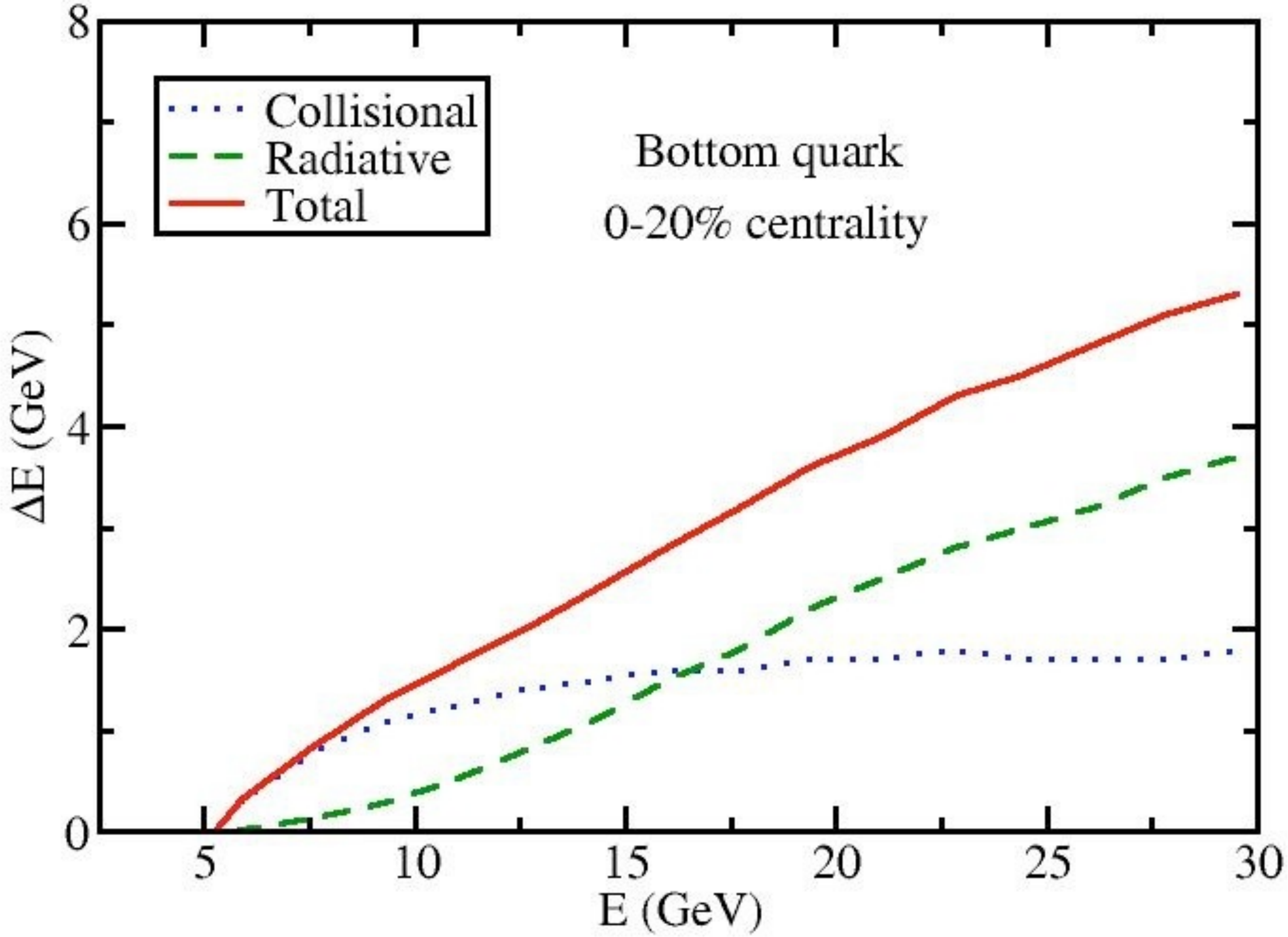
- Choose two mixtures with b/c ratio around 1% in our simulation





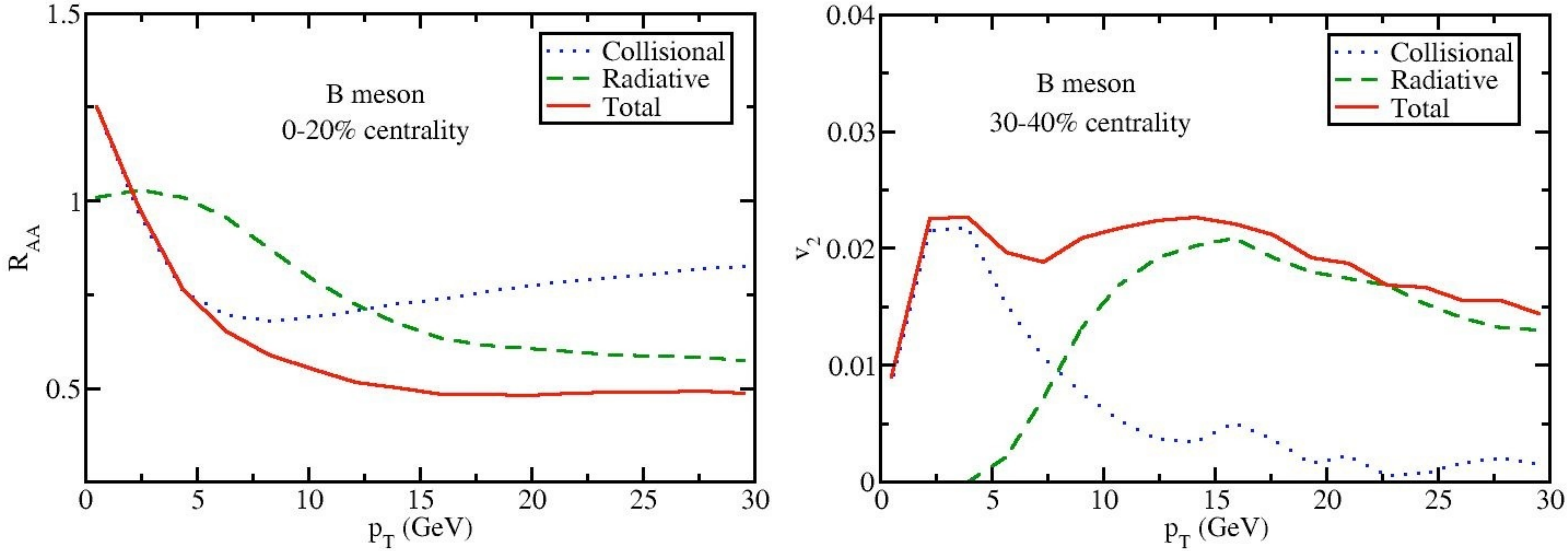
- non-photonic electron spectrum follows c-decay electron behavior at low p_T, but b-decay at high p_T
- v_2 behavior varies with coupling strength and cannot be resolved by current experimental data

Backup: Bottom Quark Energy Loss



- Collisional energy loss dominates low energy region, while radiative dominates high energy region.
- Crossing point: around 17 GeV, much larger than charm quark because of heavier mass.

B-Meson Prediction



- similar behavior as with D mesons: collisional energy loss dominates for the low p_T region, while radiative dominates the high p_T region
- crossing point from collisional to radiative is significantly higher due to the much larger mass of bottom vs. charm quark
- B meson has larger R_{AA} and smaller v_2 than D meson