Spatial Wilson loops and domains at the initial time in heavy-ion collisions

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One of the most fundamental questions in QCD:

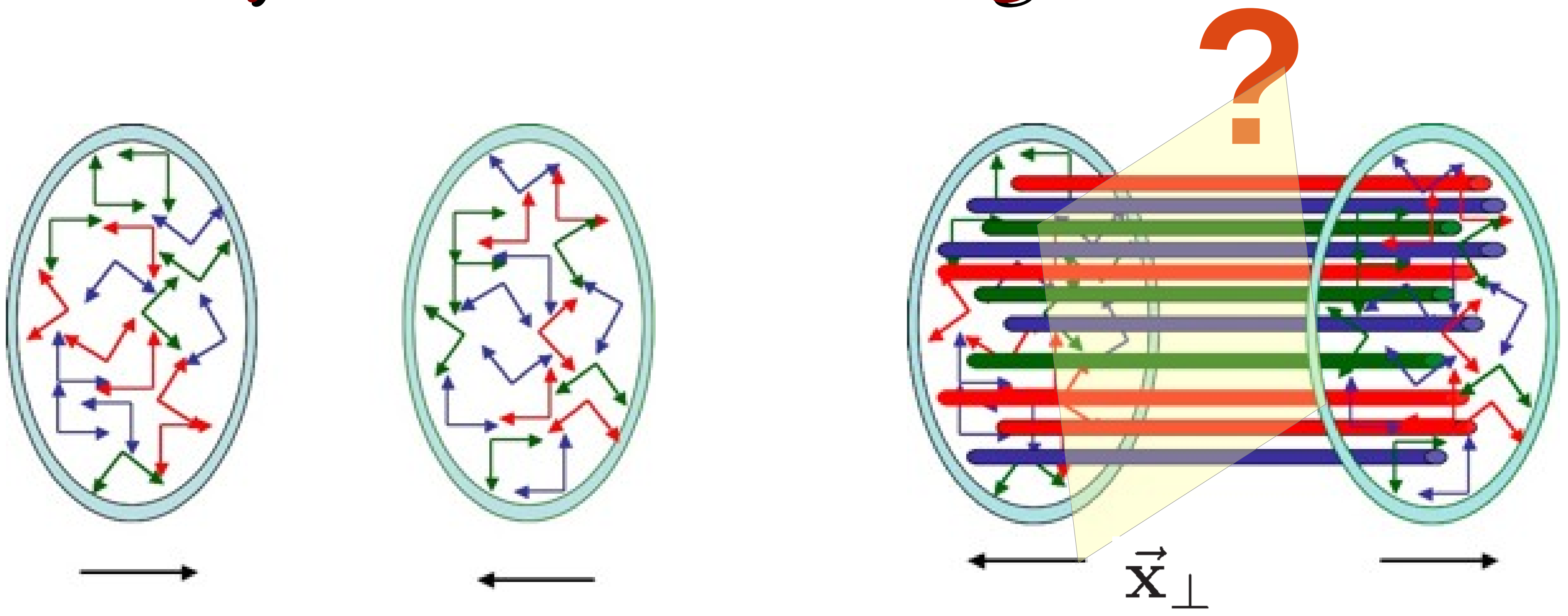
- does eff. classical color field exist ?
  (coherent field, high occupation number)

- scale separation:
  hard field modes ⟶ eff. charges ρ
  soft modes ⟶ cl. field A^μ
  (⟶ Q_s)

McLerran & Venugopalan 1993

this talk: if this is so, initial state of HIC is most interesting:
magnetic vortices, condensate of A^i,
magnetic screening (massive gauge fields)

Heavy-ion collision: long. initial fields



before collision

right after impact

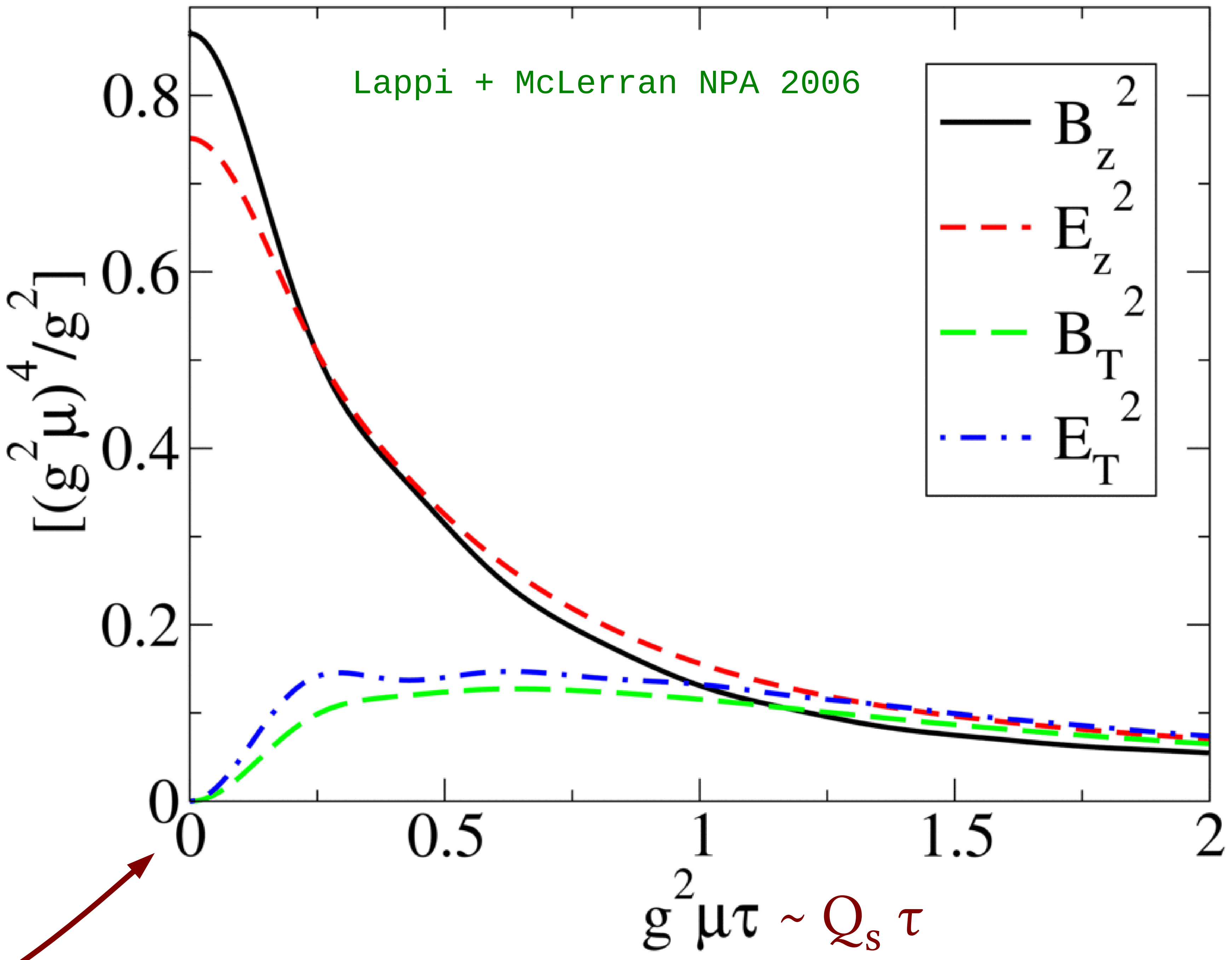
$$\partial^i \alpha^i = g\,\rho$$

$$E^z = ig\left[\alpha_1^i, \alpha_2^i\right] \quad , \quad B^z = ig\epsilon^{ij}\left[\alpha_1^i, \alpha_2^j\right]$$

$$\nabla\cdot\mathbf{B} = ig\left[A^i, B^i\right]$$

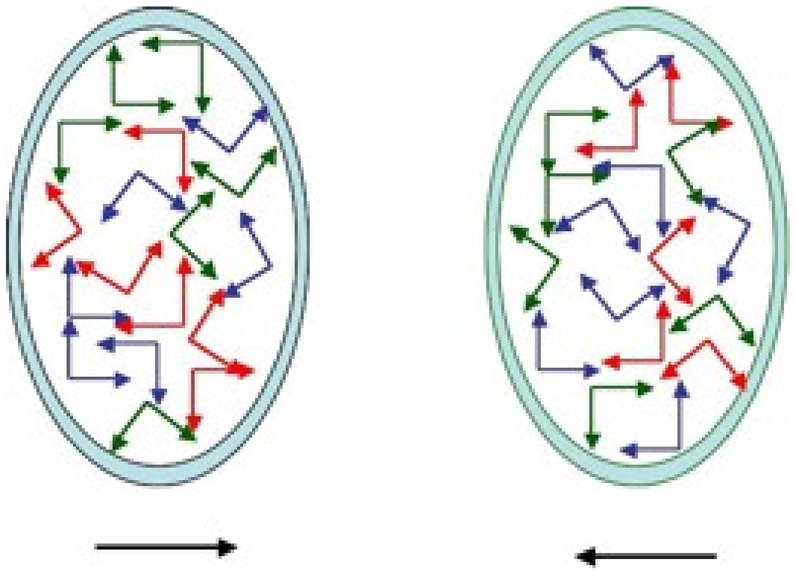
Kharzeev, Krasnitz, Venugopalan: PLB 2002
R. Fries, J. Kapusta, Y. Li: nucl-th/0604054
Lappi + McLerran NPA 2006

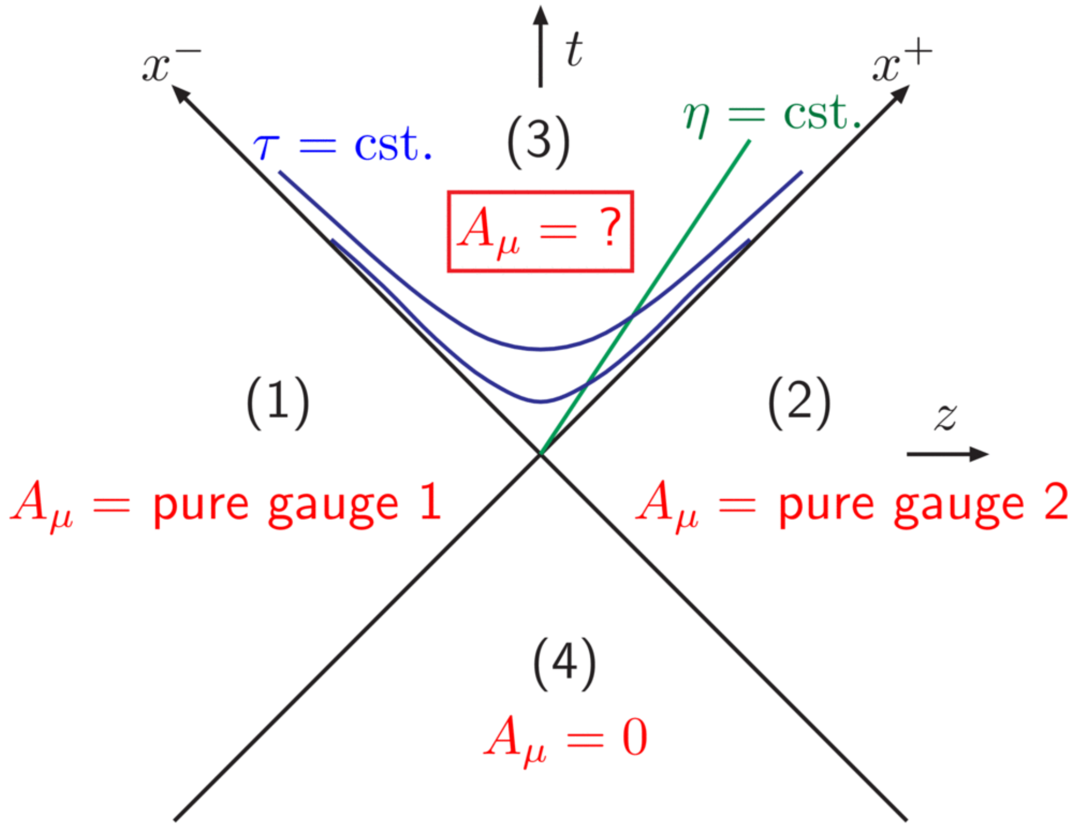
Non-perturbative solution (using a lattice)



This talk: t=0

Heavy-ion collisions





before collision

$$\begin{aligned} \alpha^i_m &= \frac{i}{g} U_m \, \partial^i U^\dagger_m \qquad SU(N) \text{ 2D pure gauge} \\ \partial^i \alpha^i_m &= g\rho_m \qquad (m = 1,2 \ \ \text{Proj/Targ}) \\ A^i &= \alpha^i_1 + \alpha^i_2 \qquad (\tau = +0) \end{aligned}$$

invert numerically (lattice)
or via perturbative expansion

gauge : $x^+A^- + x^-A^+ = 0$

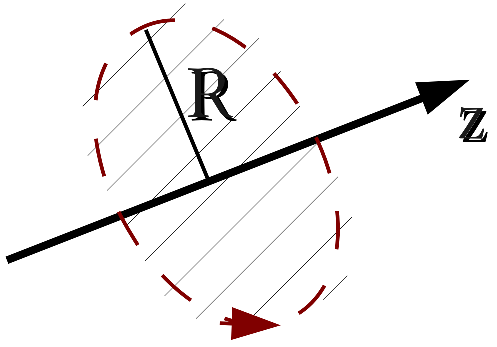
avg configurations: $S_{\mathrm{MV}} = \int d^2x_\perp \, \frac{1}{2\mu^2} \rho^a \rho^a$

- ρ random from point to point
- NOT so for A^i !

Analyze classical field configurations at midrapidity: η=0, 2D, SU(2)

what is structure of B_z field ?

magnetic flux loop in x-y plane:

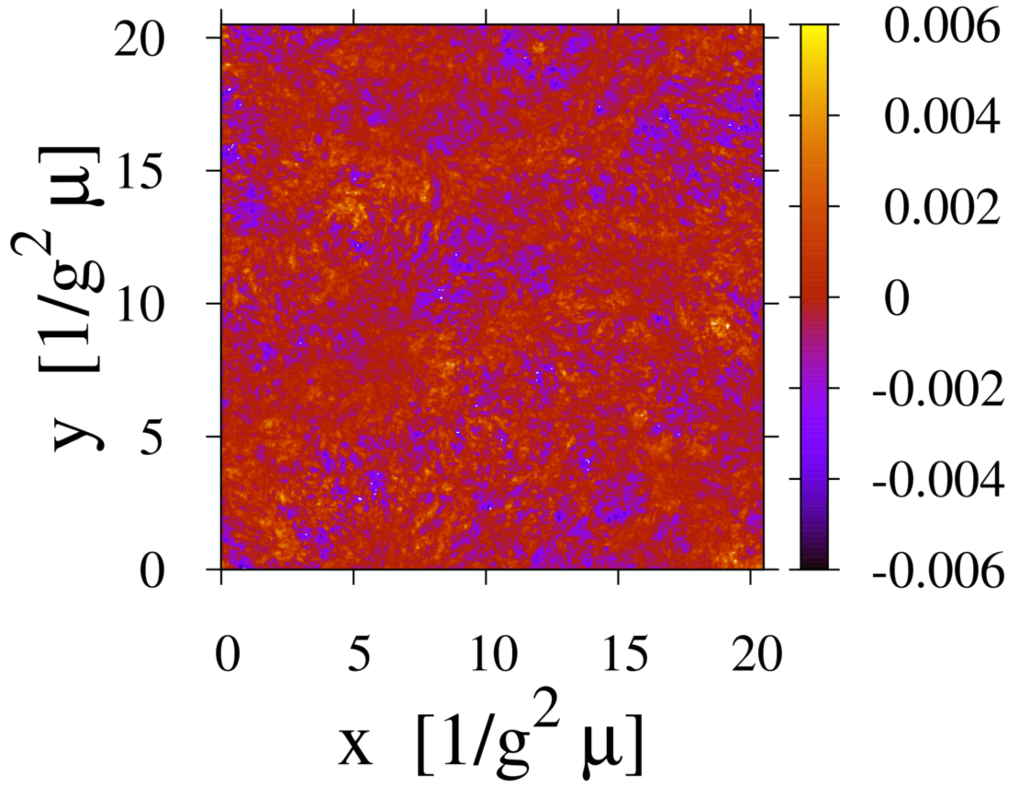


$$M(R) = \mathcal{P} \exp\left(ig \oint dx^i A^i \right)$$

$$W_M(R) = \frac{1}{N_c} \langle \mathrm{tr}\ M(R) \rangle$$

$$W_M^{Z(2)}(R) = \langle \mathrm{sgn}\ \mathrm{tr}\ M(R) \rangle$$

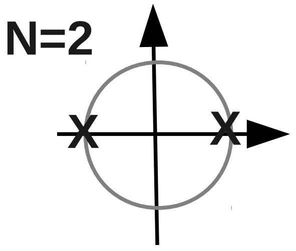
actual field configuration B_z^3

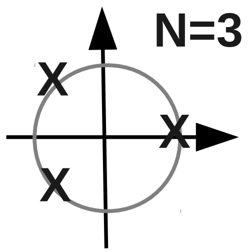


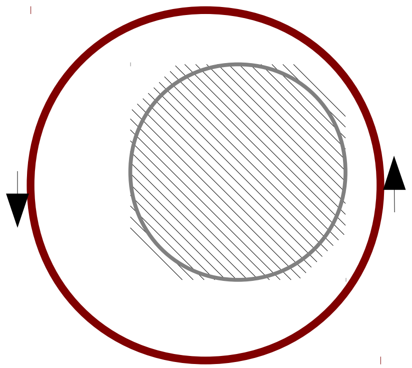
Magnetic flux loop: non-Abelian adjoint fields

transform under SU(N) / Z(N)

$$Z(N) = \left\{ e^{2\pi i\, n/N}\, \mathbb{1},\ n = 0 \cdots N-1 \right\}$$





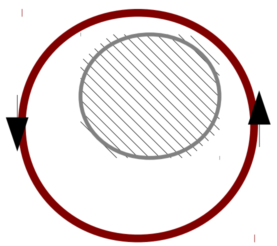


$$\mathcal{P} \exp\ ig \oint \vec{A} \cdot d\vec{\ell} \ = \ e^{2\pi i\, n/N}\, \mathbb{1}_{N\times N}$$

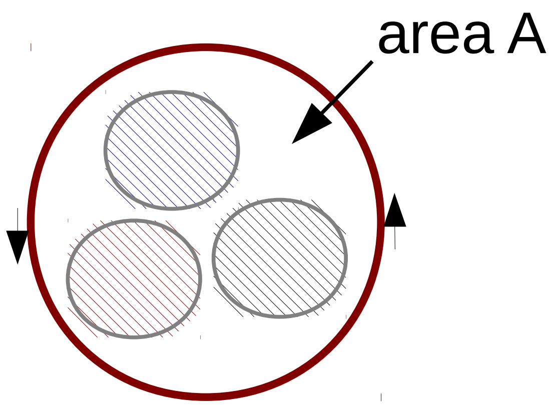
i.e., gauge transformation <u>can</u> be multi-valued by an element of Z(N)

n = Z(N) charge in shaded region

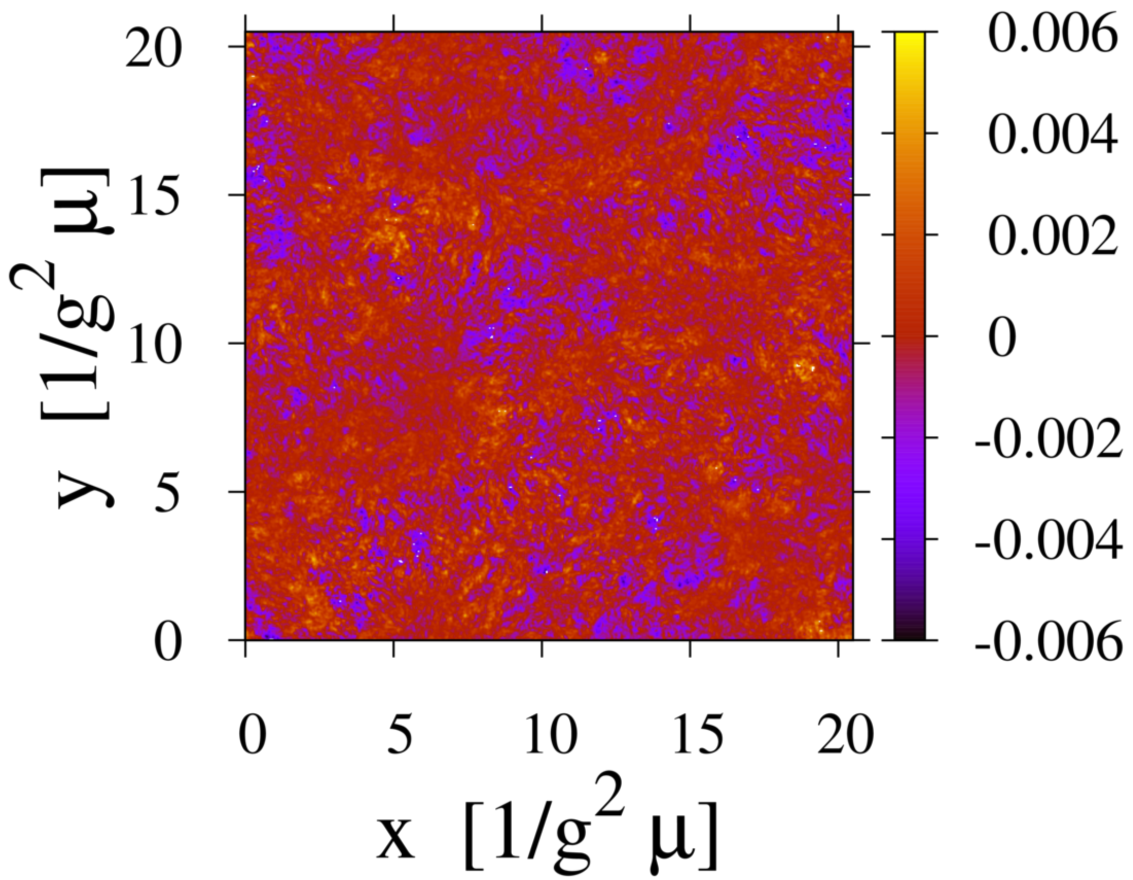
Magnetic Z(N) "vortices":



$e^{2\pi i\, n/N}$



$e^{2\pi i\,(n_1+n_2+n_3)/N}$



do we find :

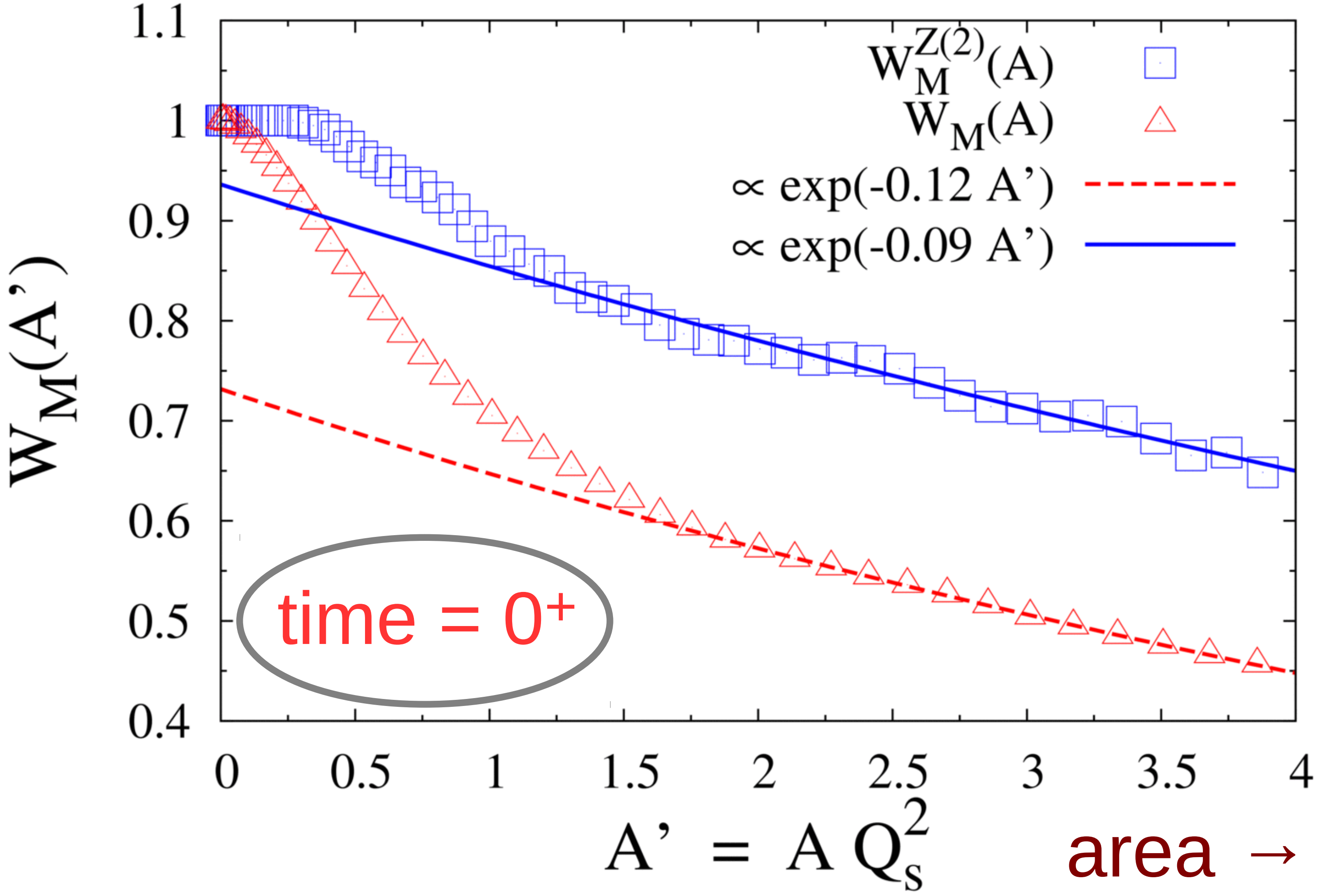
- area law ?

$$W_M(R) \sim e^{-\sigma A}$$

- loop $\in$ Z(N) ?

$$\langle \mathrm{sgn}\,\mathrm{tr}\ M\rangle \sim \frac{1}{N}\langle \mathrm{tr}\ M\rangle$$

SU(2) solution :



area →

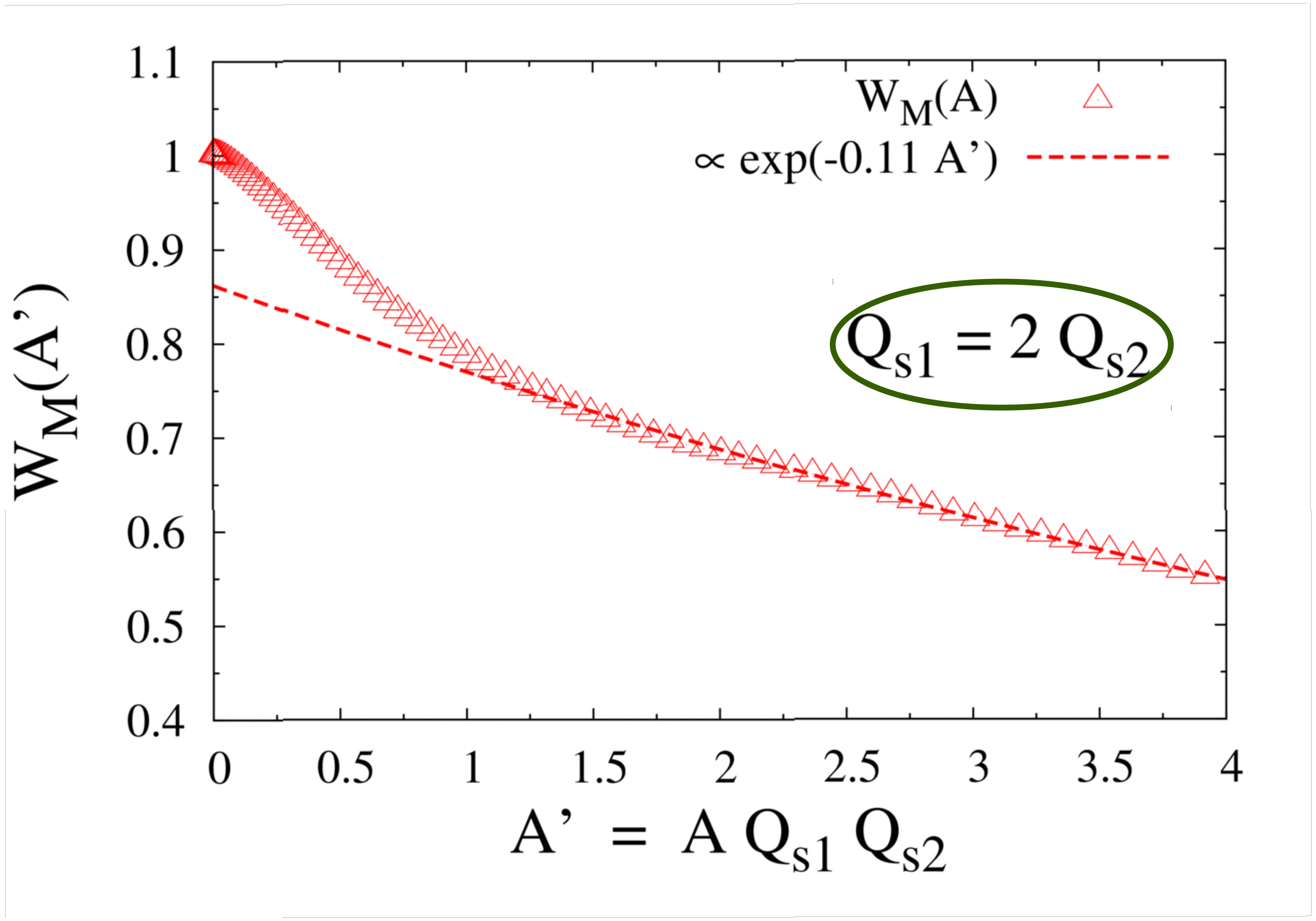
A.D., Y. Nara, E. Petreska, arXiv:1302.2064

- evidence for domain structure / condensate for A^i
- area law for loops with area A ≥ 1.5 – 2
- $\sigma_M \sim 0.12\ Q_s^2$; thermal SU(N): $\sigma_M \sim g_{3D}^2 \sim (g^2 T)^2$
- small loops ∉ Z(2) but roughly ok for large ones!
- structure of B_z ~ uncorrelated vortices
- $R_{vtx} \sim 1/Q_s$ from onset of area law

random uncorrelated vortex fluctuations:

density ≈ 1 / 20 Q_s^2

lattice solution for ***asymmetric*** collision



- ~ same string tension $\sigma_M = 0.11\ Q_{s1}\ Q_{s2}$

Magnetic screening !

A.D., H. Fujii, Y. Nara, arXiv:1305.2780

$$C^{(2)}(r) = \langle \mathrm{tr}\, G(\mathbf{0})\, G(\mathbf{x}) \rangle$$

$$G(\mathbf{x}) = g\, U(\mathbf{0} \to \mathbf{x})\, F_{xy}(\mathbf{x})\, U(\mathbf{x} \to \mathbf{0})$$

expectation: (d=2) $\int d^d p \, \frac{e^{ipx}}{p^2 + m^2} \sim \frac{1}{r^{(d-1)/2}} \exp(-m\, r)$

(A. Abrikosov, '50s)

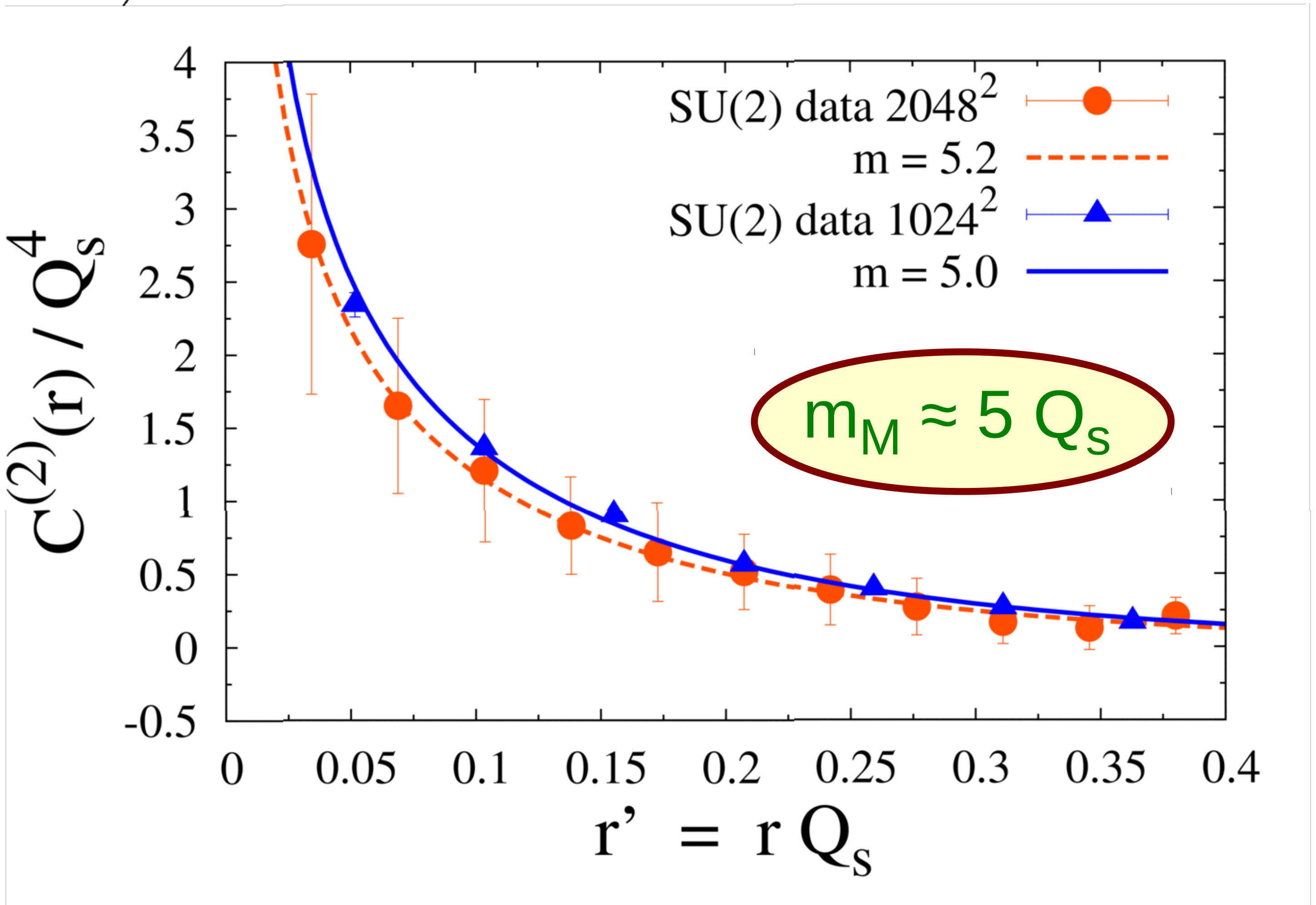
Chern-Simons term
can provide mass to gauge boson,

$$-\frac{1}{2} m\, \epsilon_{\mu\nu\sigma} \mathrm{tr} \left[A^\mu \partial^\nu A^\sigma + \frac{2}{3} A^\mu A^\nu A^\sigma \right]$$

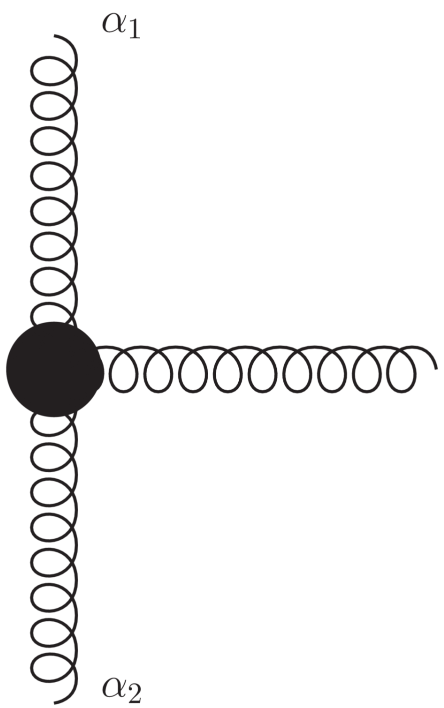
effective dynamics: YM + CS

Note:

- gauge links: interactions of external legs with produced gluons $A^i = \alpha_1^i + \alpha_2^i$



Perturbative analytical calculation :



it usually goes like this :

$$\nabla^2 \, \Phi = g \, \rho$$
$$\alpha^i = -\partial^i \Phi$$

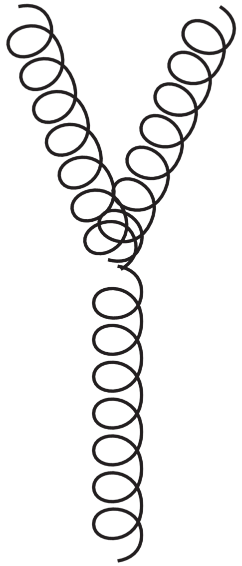
<u>but</u> :

$$\oint dx^i \, \partial^i \Phi = 0$$

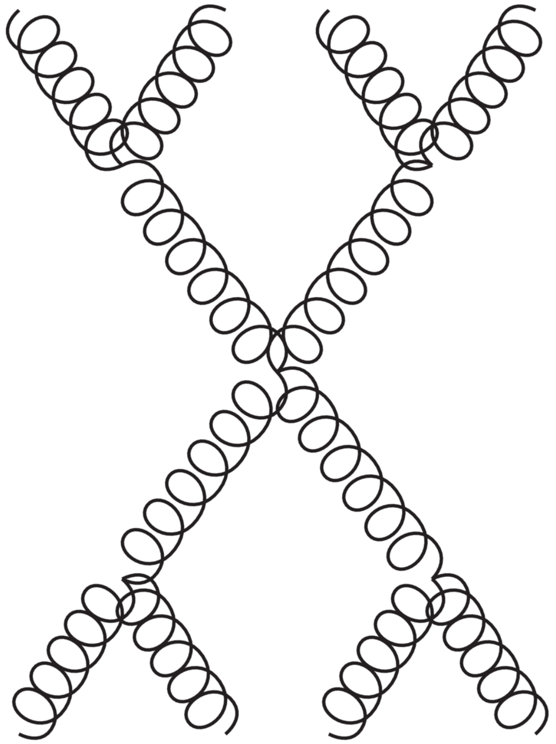
so we need :

$$\alpha^i = -\partial^i \Phi + \frac{ig}{2} \left(\delta^{ij} - \partial^i \frac{1}{\nabla^2} \partial^j \right) \left[\Phi, \partial^j \Phi \right]$$

$$\alpha^{i,a} \sim \; g f^{abc} \, \Phi^b \partial^i \Phi^c$$



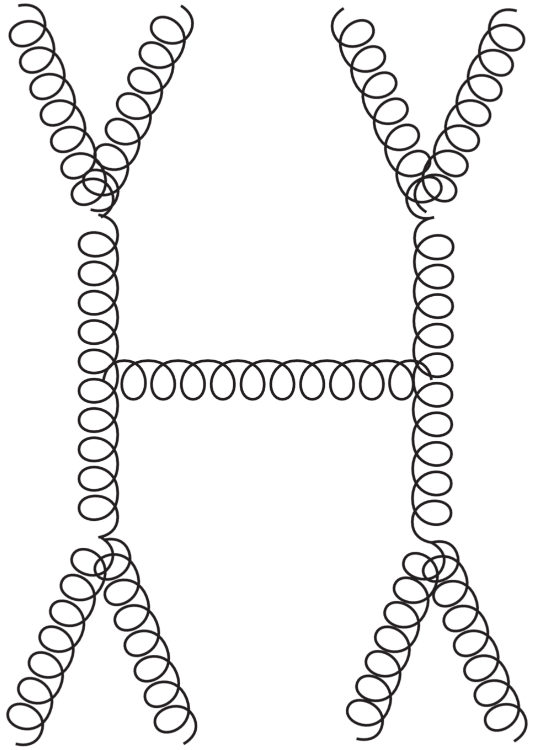
(at this order, field still classical: Yu. Kovchegov) classical diagram:



$$W_M(A) = 1 - \# A^2 Q_s^8 \frac{1}{\Lambda^4}$$

(E. Petreska, arXiv:1311.2066)

quantum diagram:



Field of <u>single</u> domain / vortex might look like this :

(Floerchinger-Wetterich condensate)

$$A_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix} \quad , \quad A_2 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad , \qquad \vec{A} = \sqrt{2}\,\frac{\lambda}{g}\,\left(-A_1 \sin\varphi + A_2 \cos\varphi\right)$$

$$\Rightarrow \; \frac{1}{3}\,\mathrm{tr}\,\exp\, ig \oint dx^i A^i = \frac{1}{3}\left(1 + 2\cos 2\pi R\lambda\right) \quad ,$$

Note: $\lambda \sim Q_s$ and $R \sim 1/Q_s$ implies Z(N) charge ~ O(1) !

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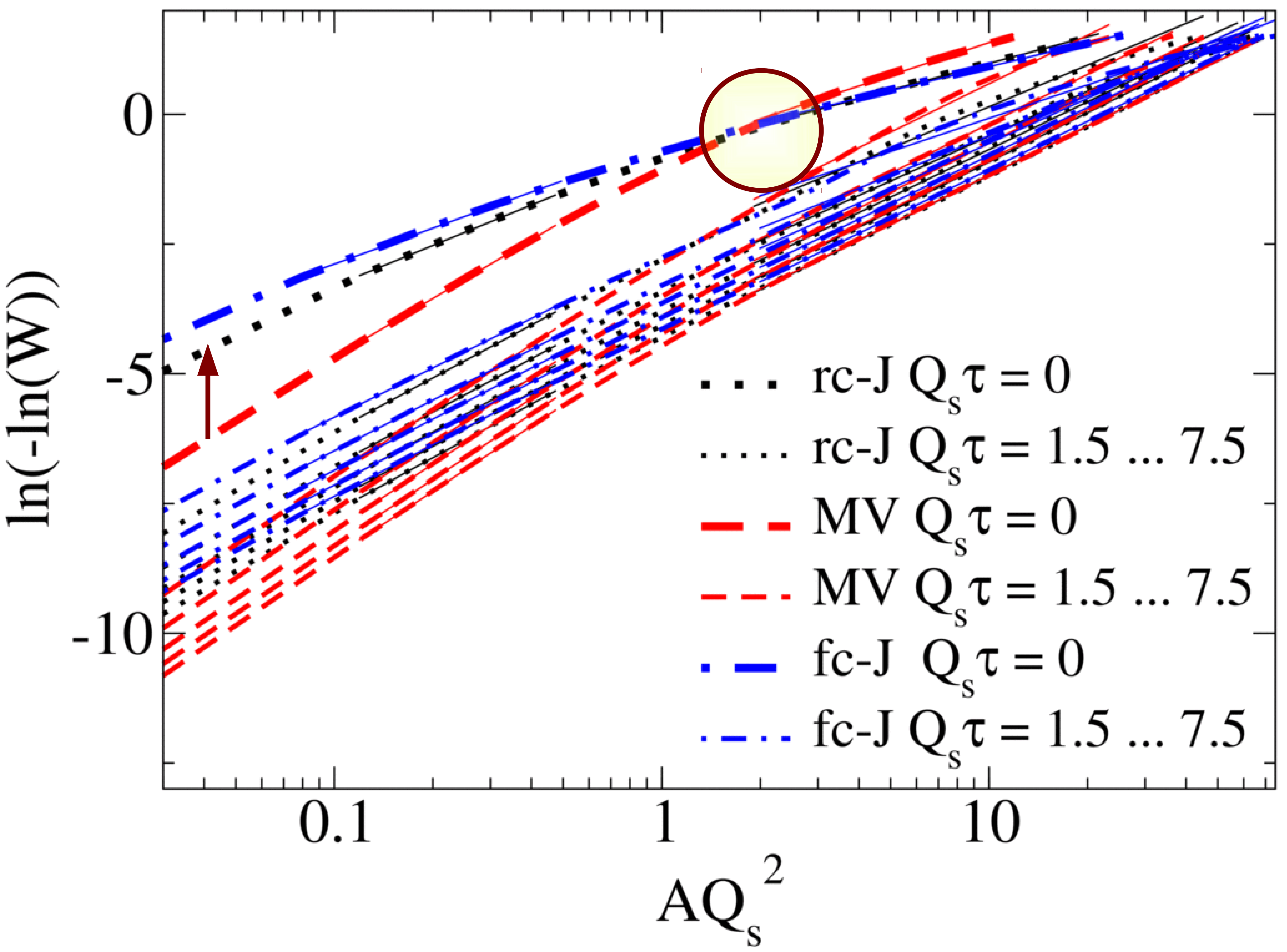
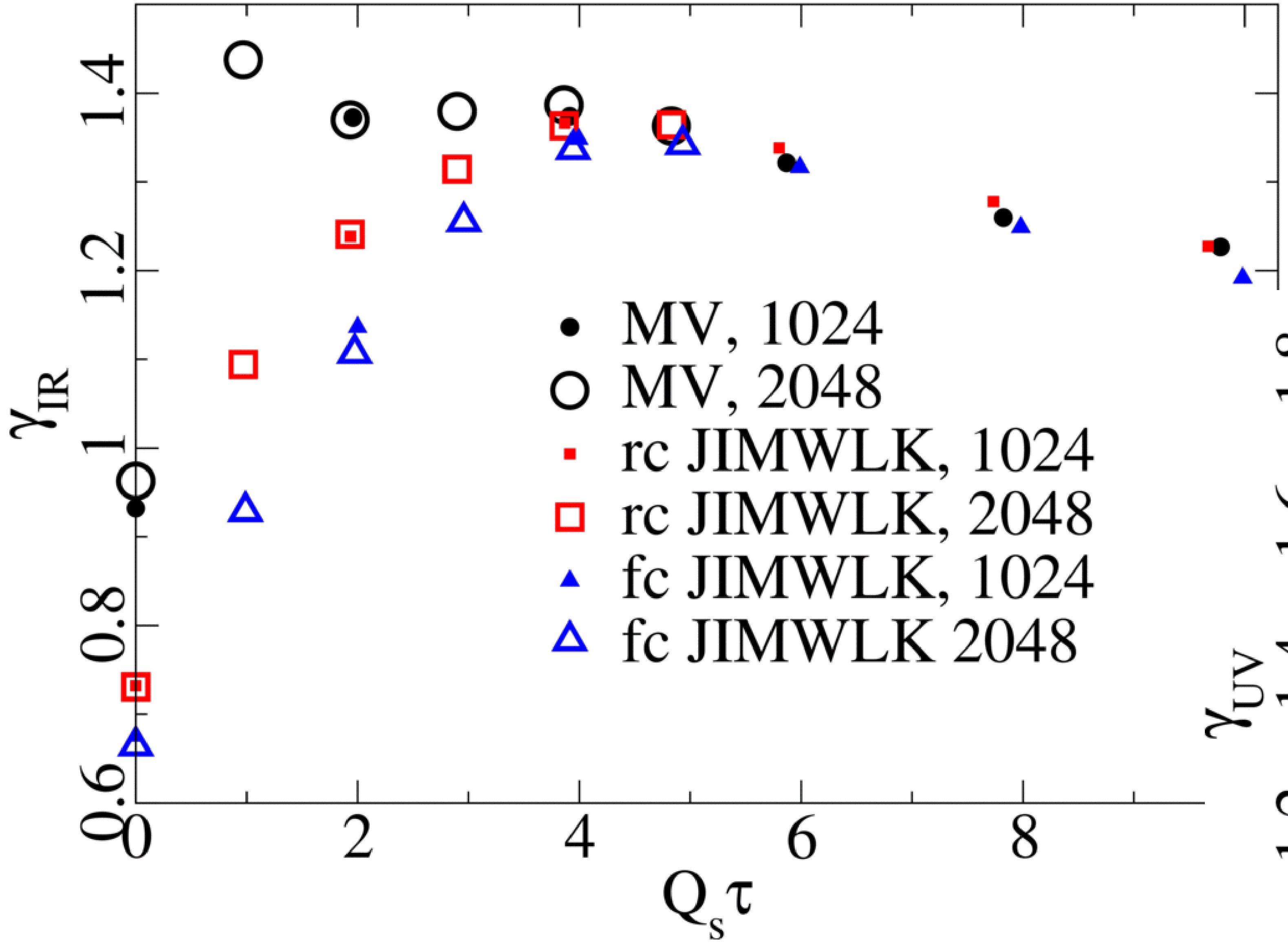
JIMWLK small-x evolution & Wilson loop

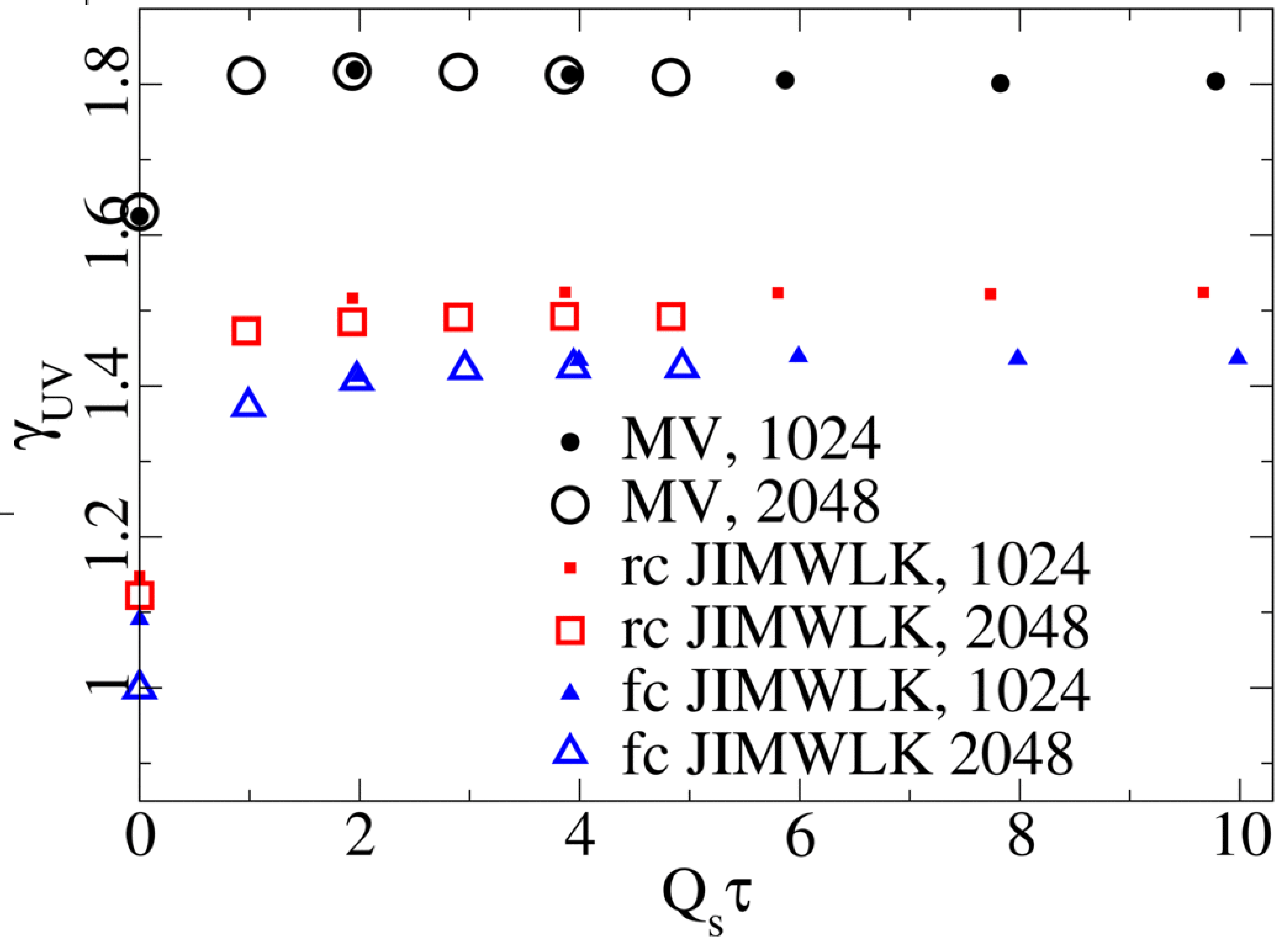
(T. Lappi et al, in preparation)



- evolution increases magn. flux through small loops
- decreases flux through large loops !
- loop with area $A \sim \pi/2Q_s^2$ is RG invariant (spatial string tension σ_M/Q_s^2)

IR / UV fits ~ exp[- (σ A)$^{\gamma}$]

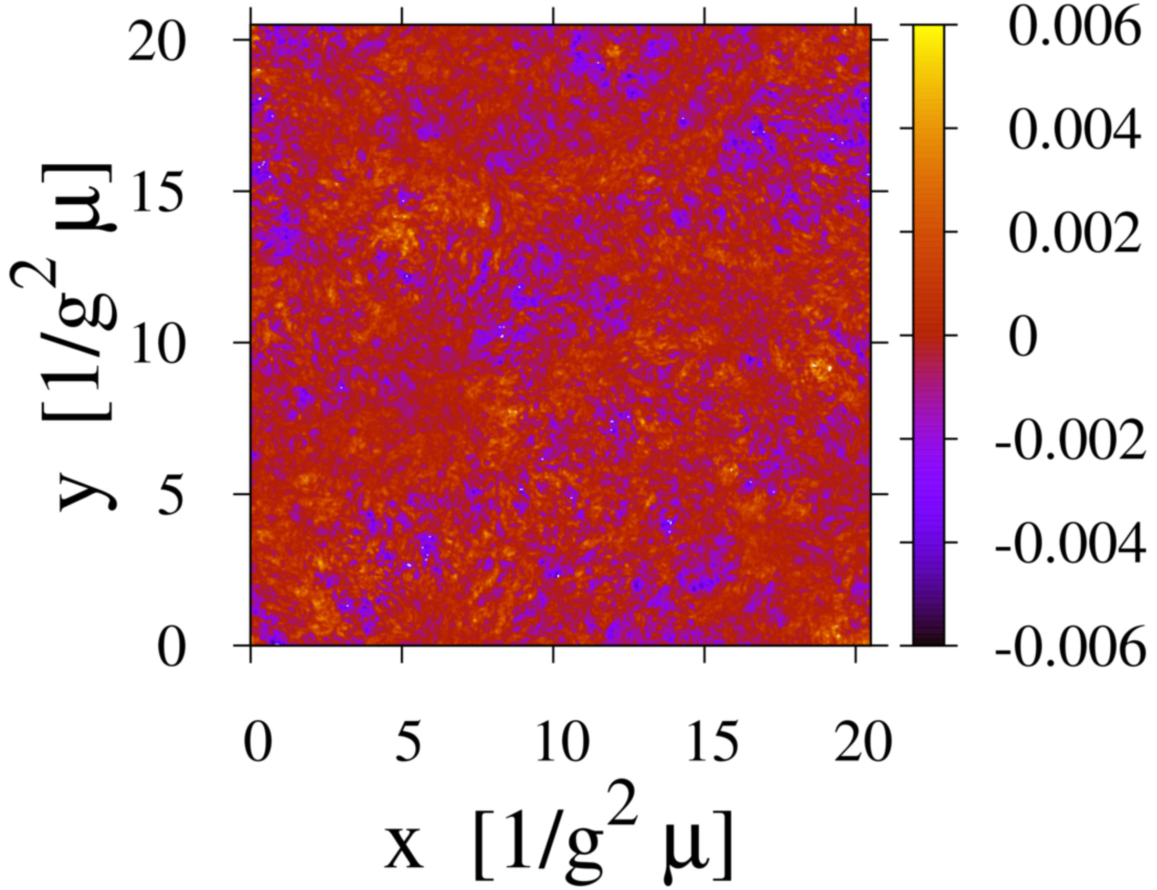




Summary

- "Clumping" of magnetic field / domains
- area law: $W_M(A) \sim \exp(-\sigma_M A)$ for loop radius $R \sim 1/Q_s$
- $\sigma_M \cong 0.1\, Q_{s1}\, Q_{s2}$
- Z(2) projected loop gives similar σ_M
- magnetic screening at scale $m_M \cong 5\, Q_s$

B_z

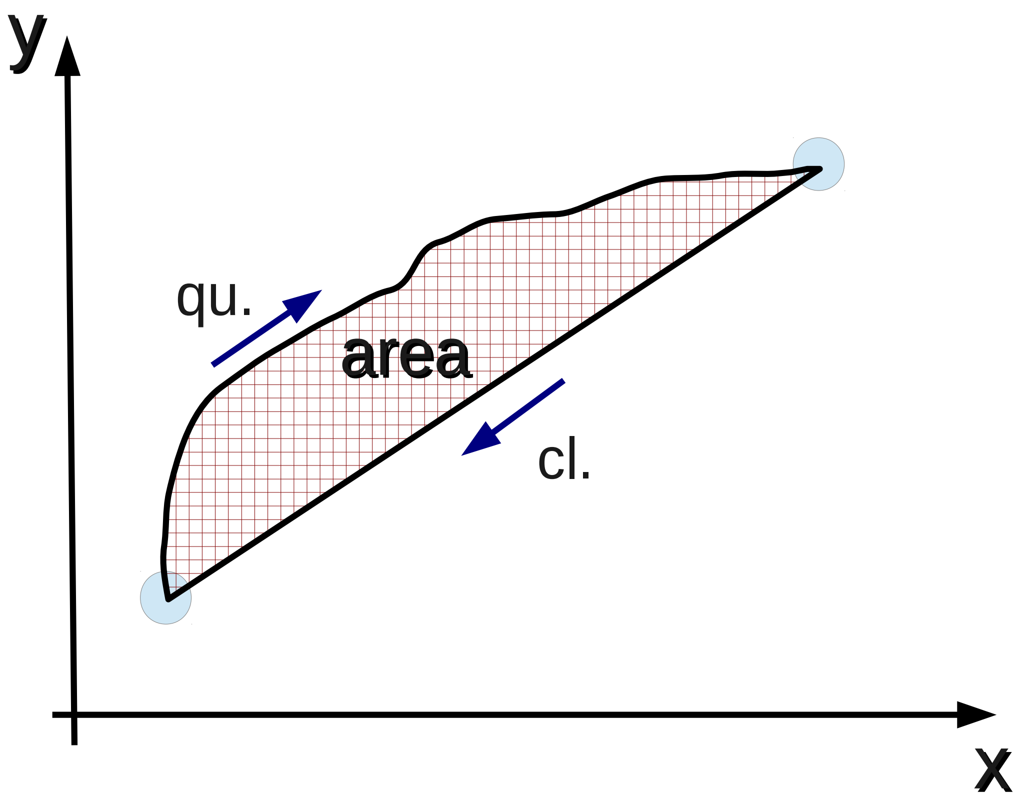


In preparation (T. Lappi et al):

- JIMWLK evolution effects on $W_M(R)$
- time > 0

Backup Slides

Propagation of hard particles in background of magnetic Z(N) vortices



classical trajectory ?

- only if paths within one de Broglie length ($1/p_T$) have *same Aharonov-Bohm phase*
- destructive interference leads to Anderson localization

$$\int_0^\infty ds \int \mathcal{D}x^\mu \left\langle \exp\, i \int_0^s d\tau \left(m\dot{x}^2 + gA_\mu \dot{x}^\mu\right) \right\rangle \sim$$

$$\int_0^\infty ds \int \mathcal{D}x^\mu \exp\left(i \int_0^s d\tau\, m\dot{x}^2 \right) \exp(-\sigma_M A) = \frac{i}{p^2 + i\sigma_M \frac{m}{p_T}}$$

Do we need to care about all of this ?

Correlations ?

$$(v_2[2])^2 = \left\langle e^{2i(\theta_1-\theta_2)} \right\rangle \geq 0 \quad , \qquad (v_2[4])^4 = 2\left\langle e^{2i(\theta_1-\theta_3)} \right\rangle \left\langle e^{2i(\theta_2-\theta_4)} \right\rangle - \left\langle e^{2i(\theta_1+\theta_2-\theta_3-\theta_4)} \right\rangle$$

Borghini, Dinh, Ollitrault, 2001

analogy: $2\left\langle \phi\phi^\dagger \right\rangle \left\langle \phi\phi^\dagger \right\rangle - \left\langle \phi\phi^\dagger\phi\phi^\dagger \right\rangle$

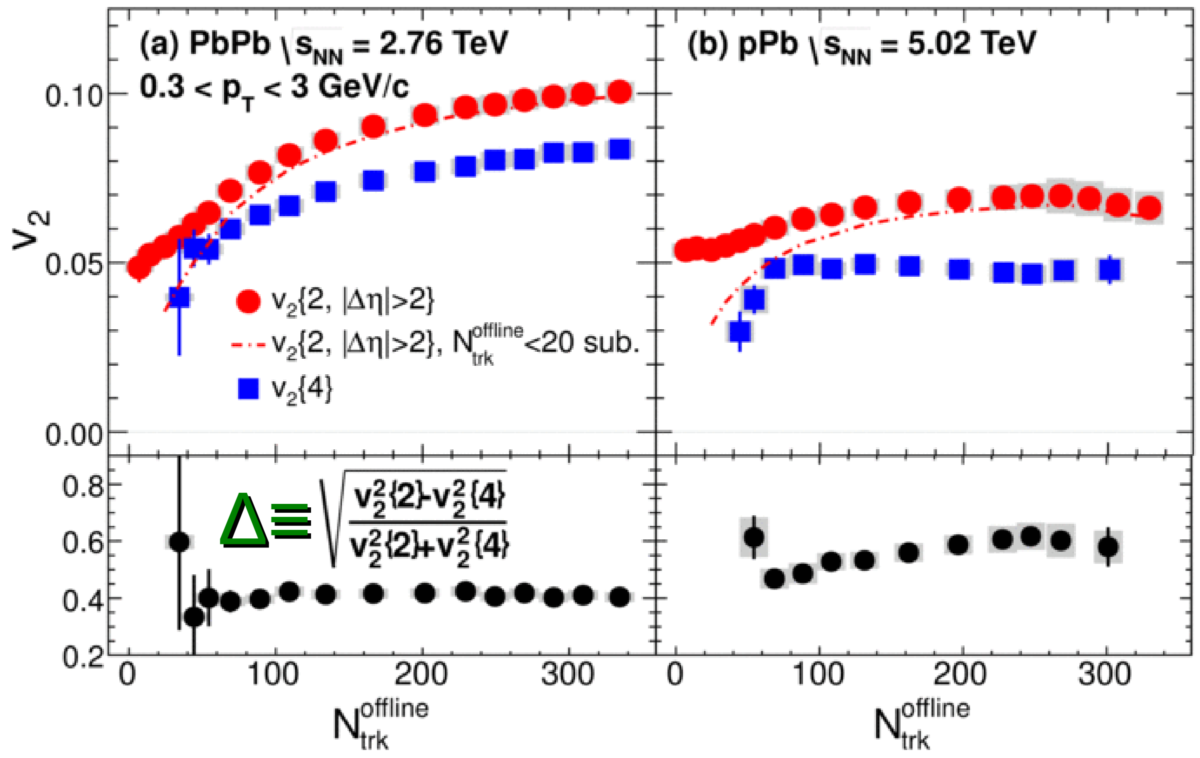
$= \left\langle \phi\phi^\dagger \right\rangle^2$ in Gaussian theory

$> \left\langle \phi\phi^\dagger \right\rangle^2$ in $\lambda\phi^4$ for $\lambda>0$ (suppressed fluct.)

$< \left\langle \phi\phi^\dagger \right\rangle^2$ for $\lambda<0$ (stronger fluct.)

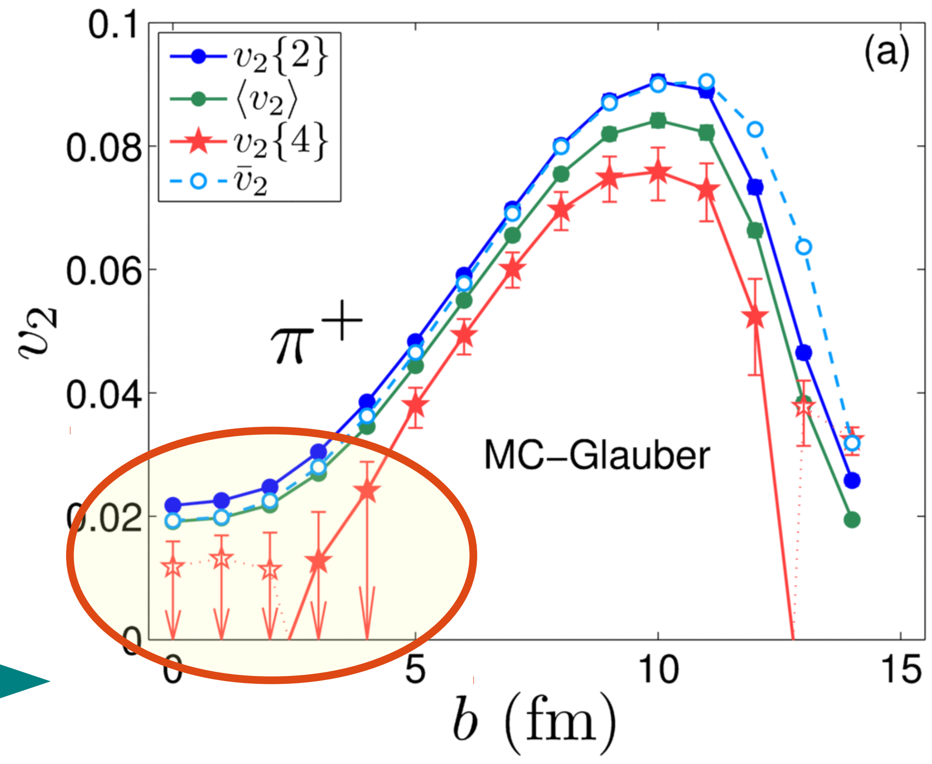
A.D., T. Lappi, L. McLerran,
arXiv:1310.7136

periph. Pb+Pb | very high mult p+Pb



CMS, arXiv:1305.0609

Qiu, Heinz,
arXiv:1104.0650



- peripheral Pb+Pb: mean field dominates ($v_2[4]$~$v_2[2]$) due to geometry of large impact parameter collision
- central Pb+Pb: fluctuation dominated ($v_2[4]$~0 or even <0)
- high-mult p+Pb: does not appear to be fluctuation but mean-field dominated !

single configuration of angular correlations from initial condition ?