

# **The non-equilibrium dynamics of strongly correlated glue**

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**Brookhaven National Laboratory**

**YITP symposium, Kyoto, December 2-6, 2013**



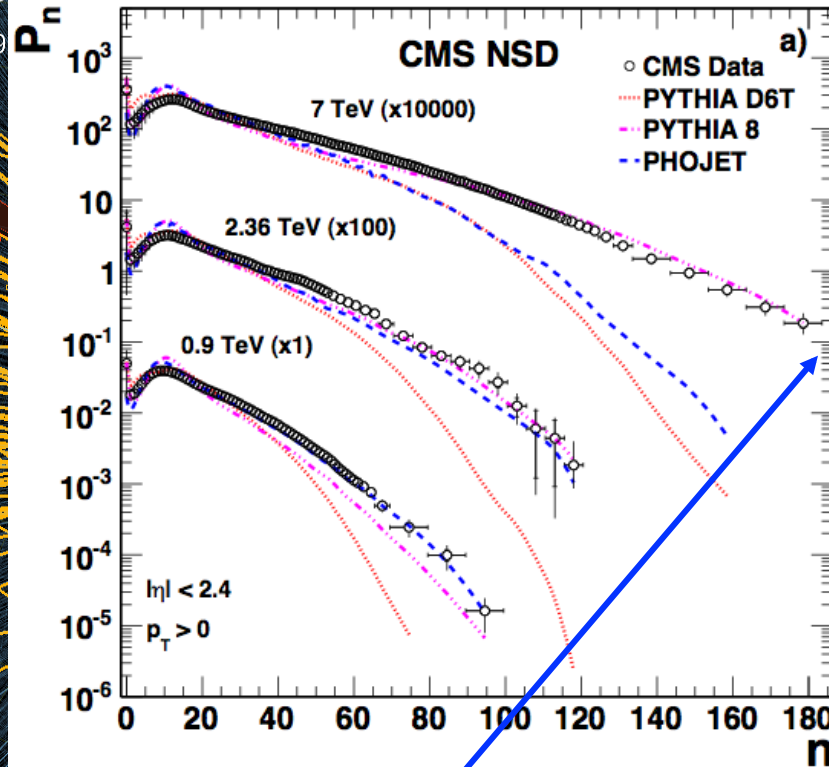
# High Multiplicity pp collisions



CMS Experiment High Multiplicity events are rare in nature

Data recorded: 2010-Jul-0

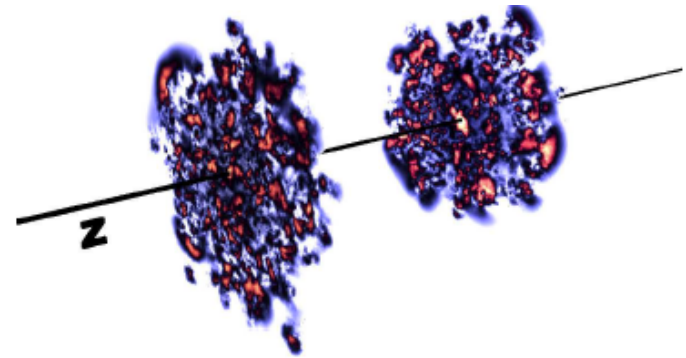
Run / Event: 139779 / 499



Very high particle density regime  
*Is there anything peculiar happening there?*

# Multiparticle production

How are  $\sim 150$  hadrons per 5 units of rapidity produced in a single p+p event ?

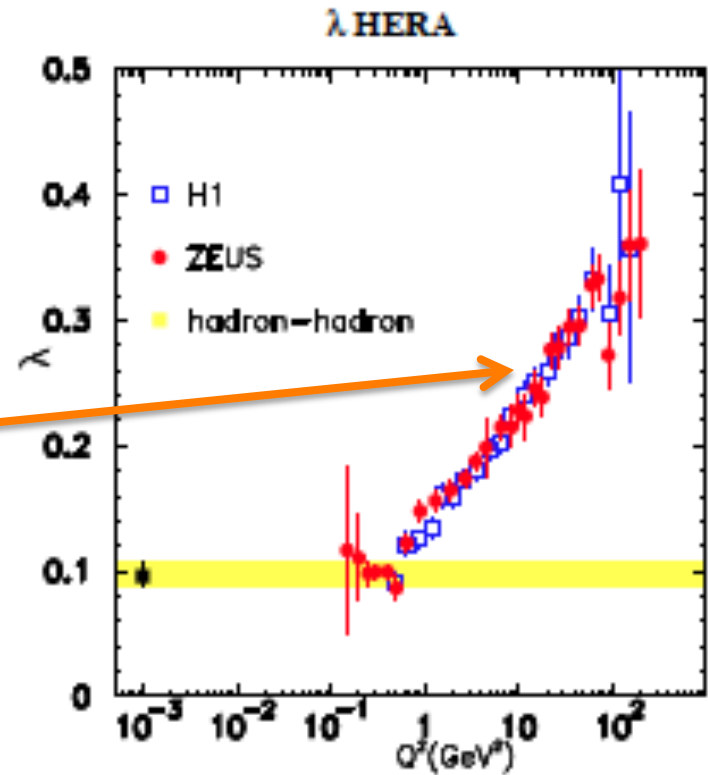


What's the guidance from HERA?

$$\begin{aligned}
 N_{\text{glue}}^{\text{proton}}(Q^2) &= \int_{x_{\text{min}}}^{x_{\text{max}}} dx \frac{dN}{dx}(x, Q^2) \\
 &\sim \int_{x_{\text{min}}}^{x_{\text{max}}} dx \frac{1}{x^{1+\lambda(Q^2)}} \\
 &= \frac{1}{\lambda(Q^2)} \left( \frac{1}{x_{\text{min}}^{\lambda(Q^2)}} - \frac{1}{x_{\text{max}}^{\lambda(Q^2)}} \right)
 \end{aligned}$$

$$x_{\text{min}}(Q^2 = 2, y = 0) = 2 \cdot 10^{-4}$$

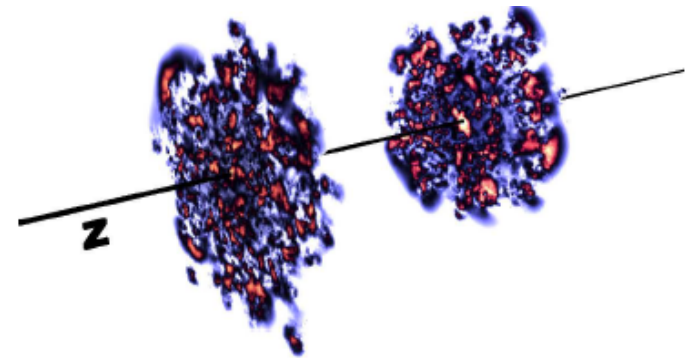
$$x_{\text{max}}(Q^2 = 2, y = 2.4) = 2.2 \cdot 10^{-3}$$





# Multiparticle production

How are  $\sim 150$  hadrons per 5 units of rapidity produced in a single p+p event ?

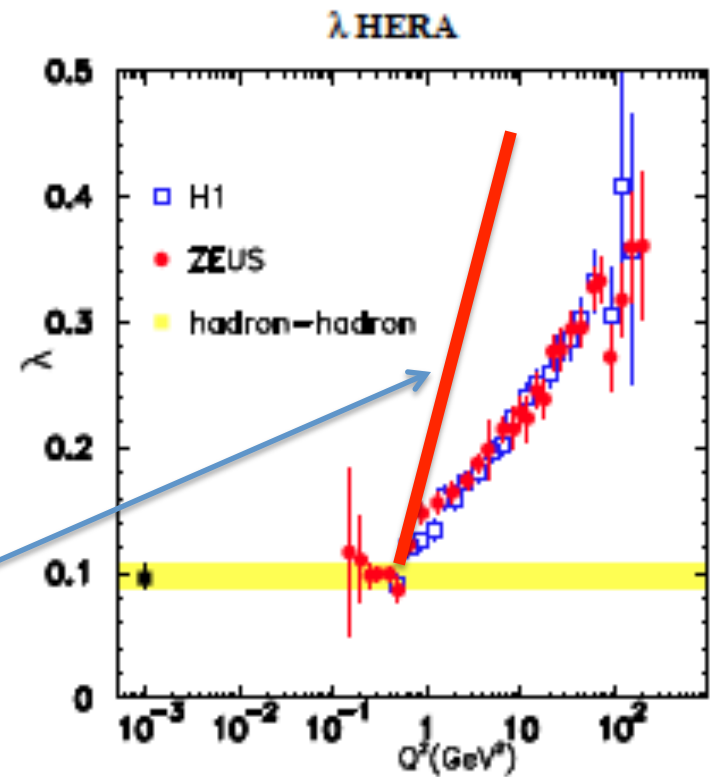


For  $\lambda=0.14$ ,  **$\sim 13$  gluons produced** in 5 units  
 $\sim$  min.bias hadron multiplicity

$\lambda=0.3$ :  **$\sim 45$  gluons** in 5 units,

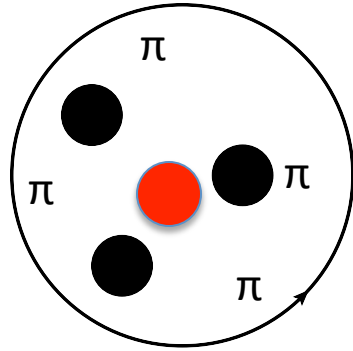
$\lambda=0.4$ :  **$\sim 90$  gluons** in 5 units, **in ball park...**

Very rapid effective growth of  
gluon dists. in such events...

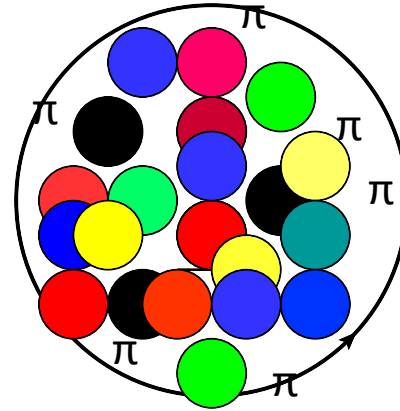


# The proton in a high multiplicity event

$Y=y_{\text{beam}}$

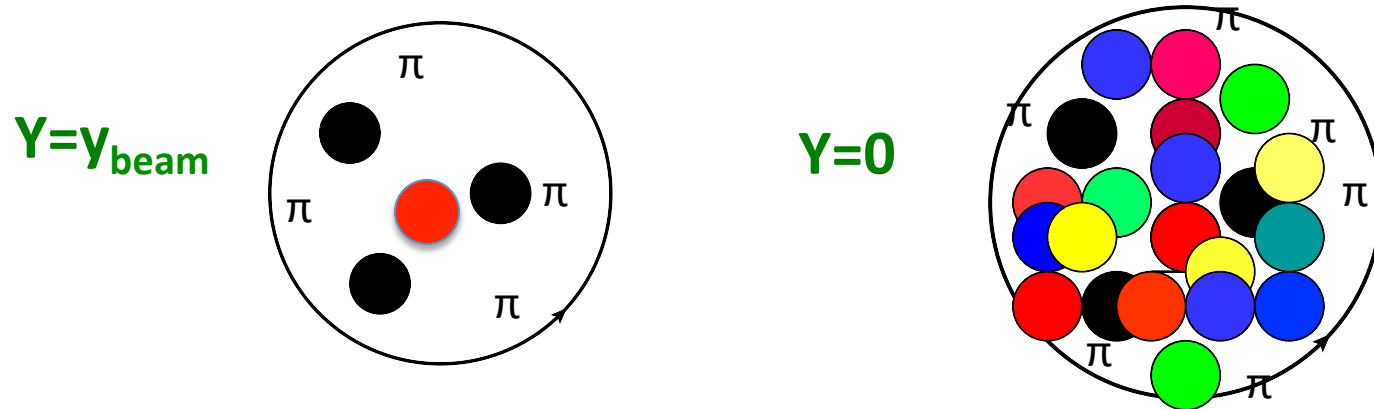


$Y=0$



For  $Q^2=2\text{ GeV}^2$ , what's the proton's gluon radius?

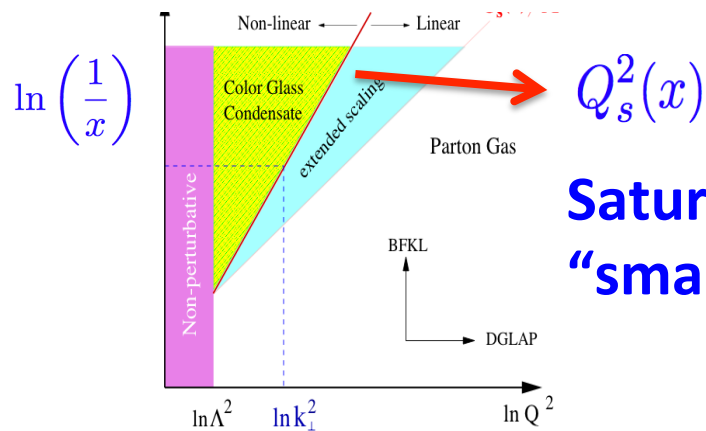
# The proton in a high multiplicity event



For  $Q^2=2 \text{ GeV}^2$ , what's the proton's gluon radius?

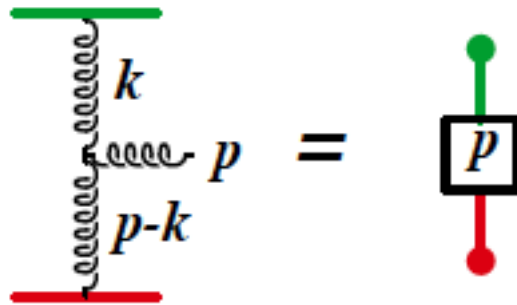
$$\frac{4\pi}{Q^2} * N_g(Q^2) = \pi R_{\text{glue}}^2$$

Gribov diffusion  $\Rightarrow R^2 \sim \ln(s)$   
 $R_g$  grows much faster depending on  $N_g$   
 rate--will violate unitarity

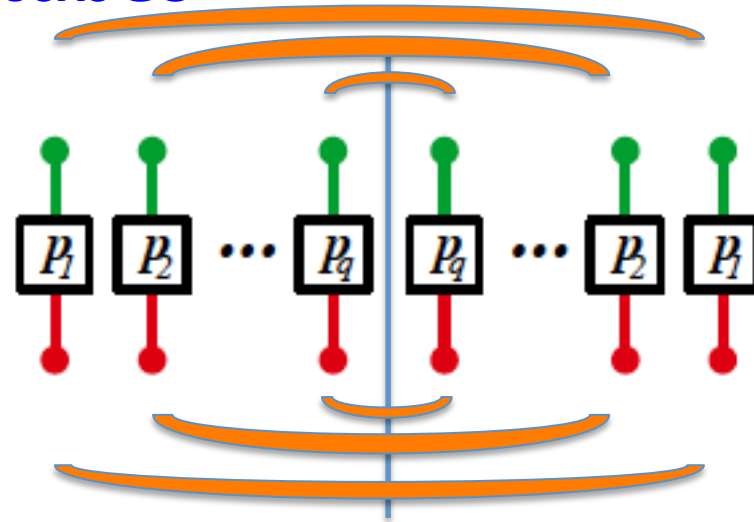


Saturation regulates this by adding increasingly  
 "smaller" gluons of size  $1/Q_s(x)$  as  $x \rightarrow 0$

# Lasing gluons: Stimulated emission from Glasma flux tubes

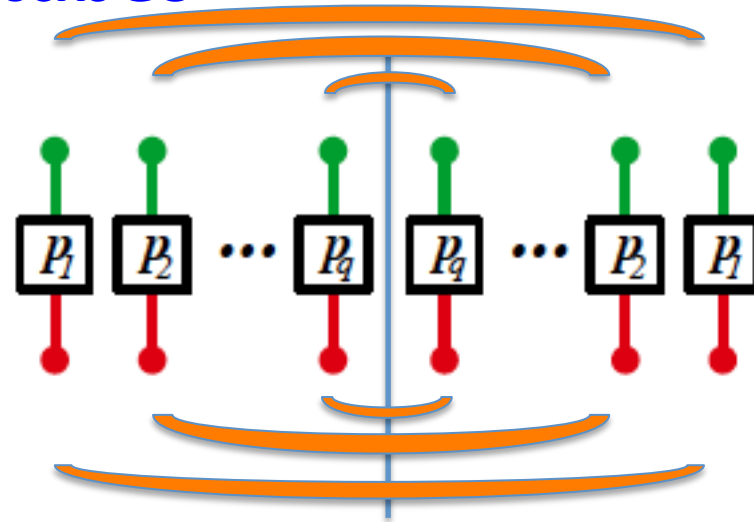
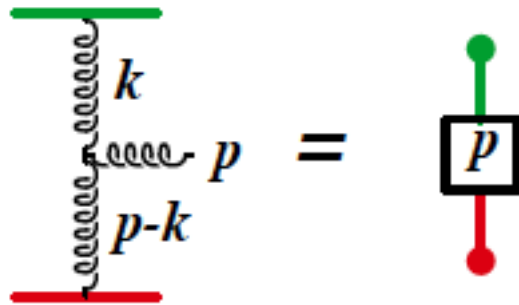


Dumitru, Gelis, McLerran, RV (2008)  
Dusling, Fernandez-Fraile, RV (2009)  
Gelis, Lappi, McLerran (2009)



**Color combinatorics of cut graphs: a negative binomial distribution**

# Lasing gluons: Stimulated emission from Glasma flux tubes



Dumitru, Gelis, McLerran, RV (2008)  
 Dusling, Fernandez-Fraile, RV (2009)  
 Gelis, Lappi, McLerran (2009)

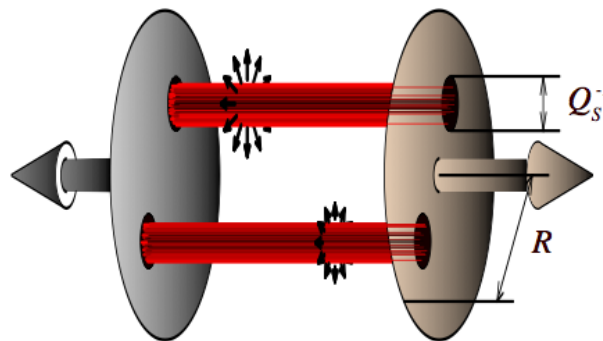
## Color combinatorics of cut graphs: a negative binomial distribution

$$P_n^{\text{NB}} = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

$k=1$ : Bose-Einstein dist.

$k=\infty$ : Poisson distribution

$$k = \kappa \frac{(N_c^2 - 1) Q_s^2 S_\perp}{2\pi}$$

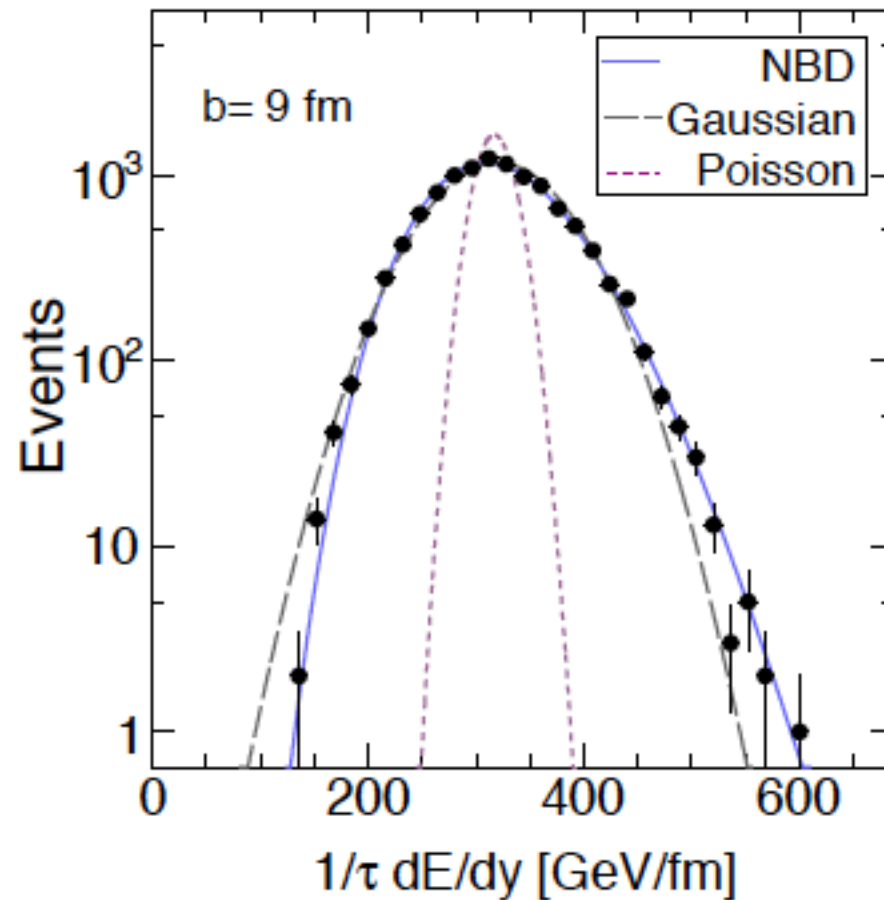


For  $Q_s^2 \approx 1/S_\perp$   
 close to a Bose-dist!

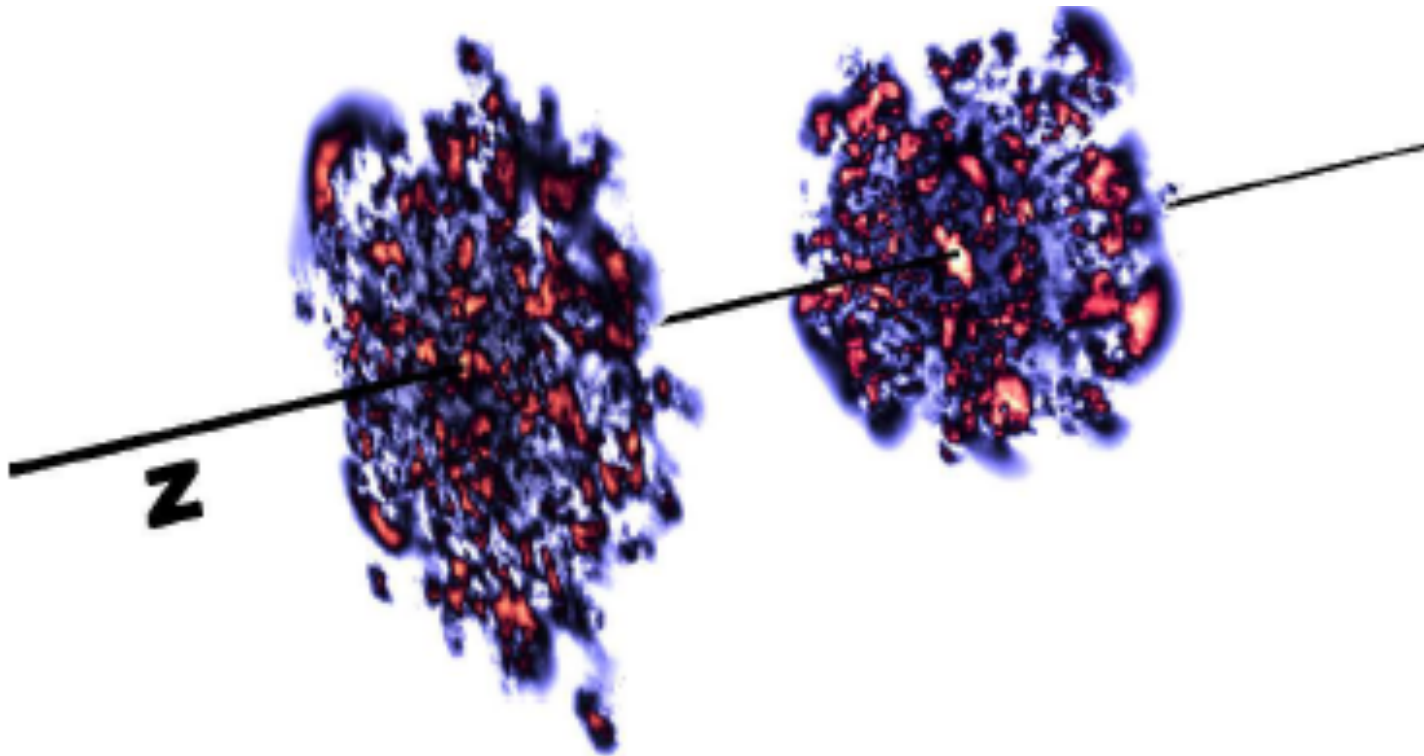


# Negative Binomial Distributions from non-perturbative Yang-Mills dynamics

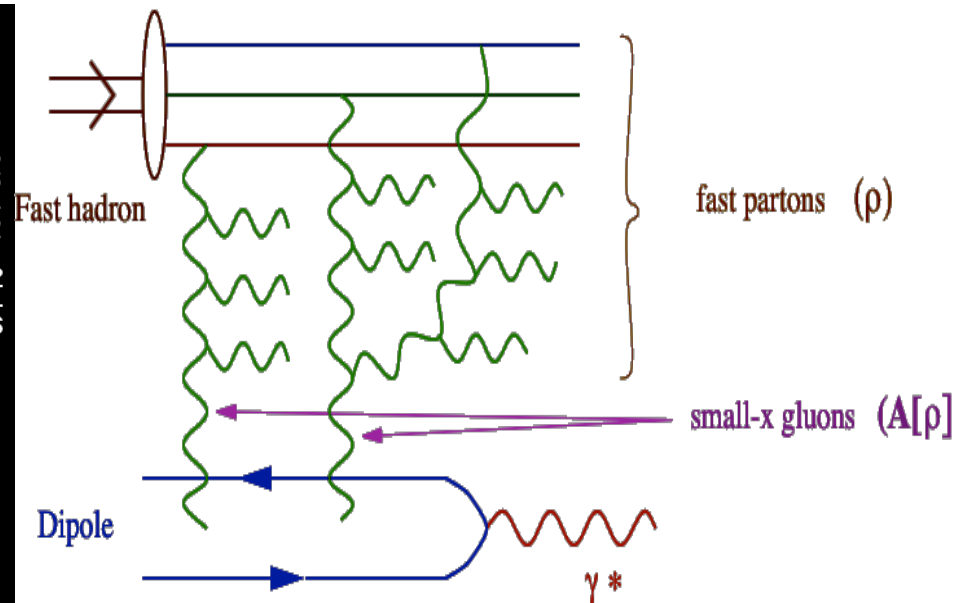
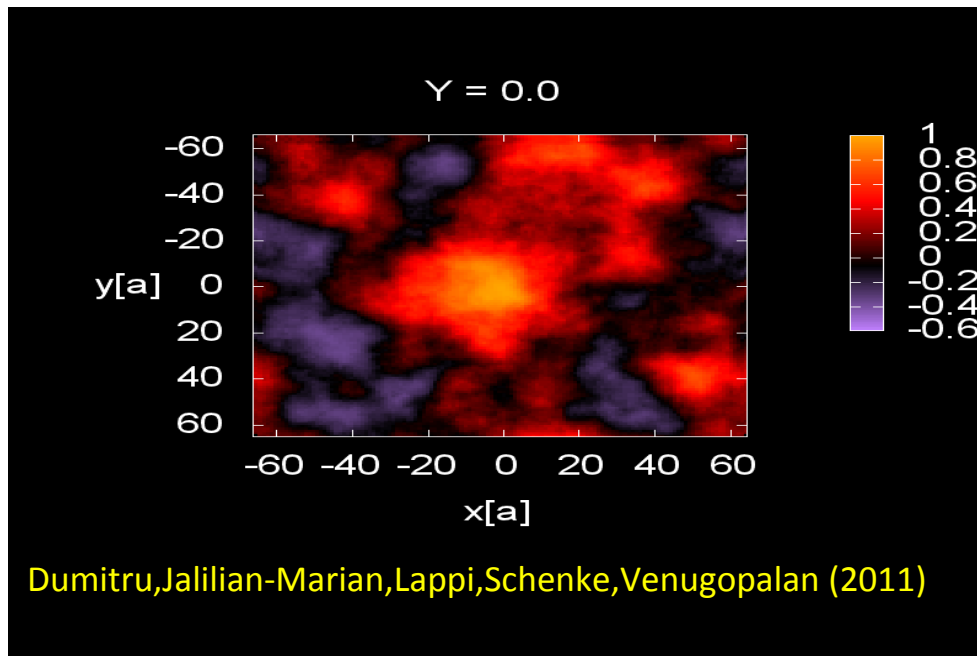
Schenke, Tribedy, RV:1202.6646



# Colliding lumpy glue



# Extracting lumpy glue in the proton

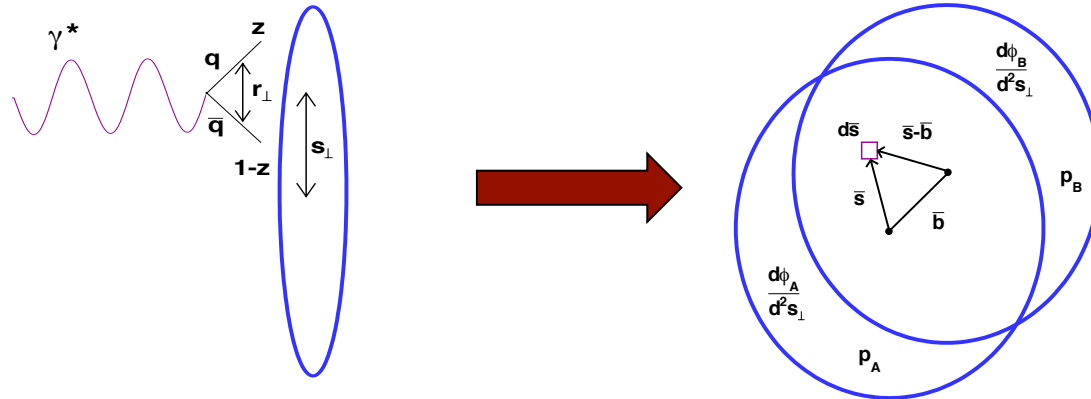


**B-JIMWLK: the BBGKY hierarchy of gluodynamics**

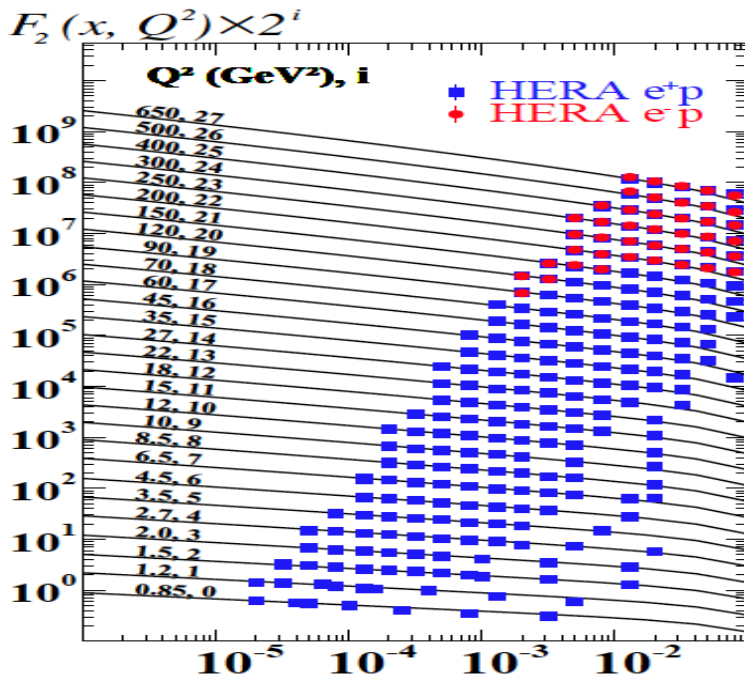
For recent status, see Tuomas Lappi's talk

# Extracting lumpy glue in the proton-IPSat model

Bartels, Golec-Biernat, Kowalski  
 Kowalski, Teaney  
 Kowalski, Motyka, Watt

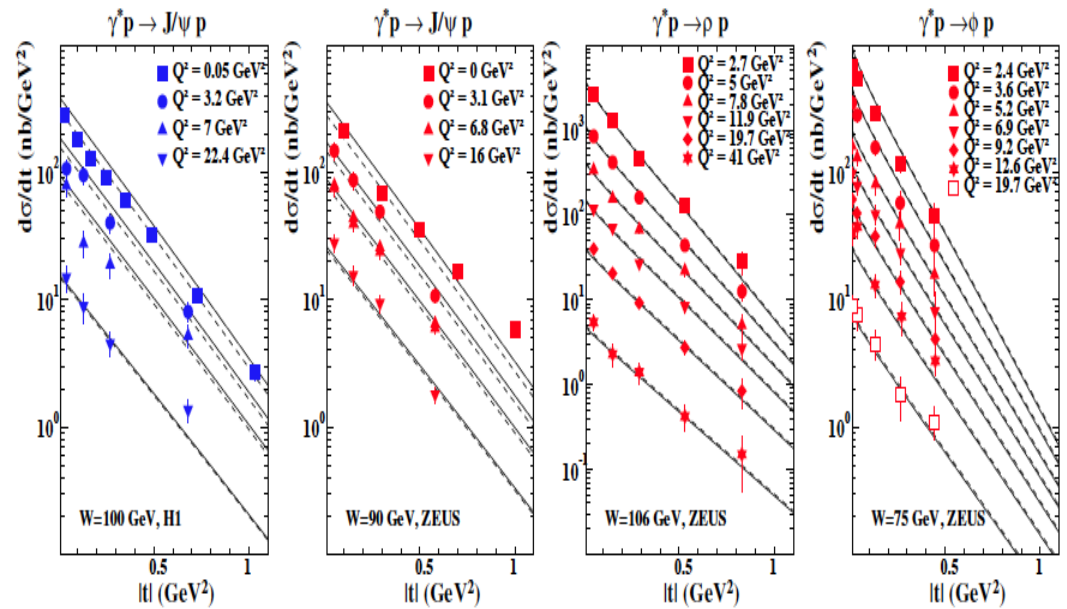


Very good agreement of IPSat model with combined HERA data



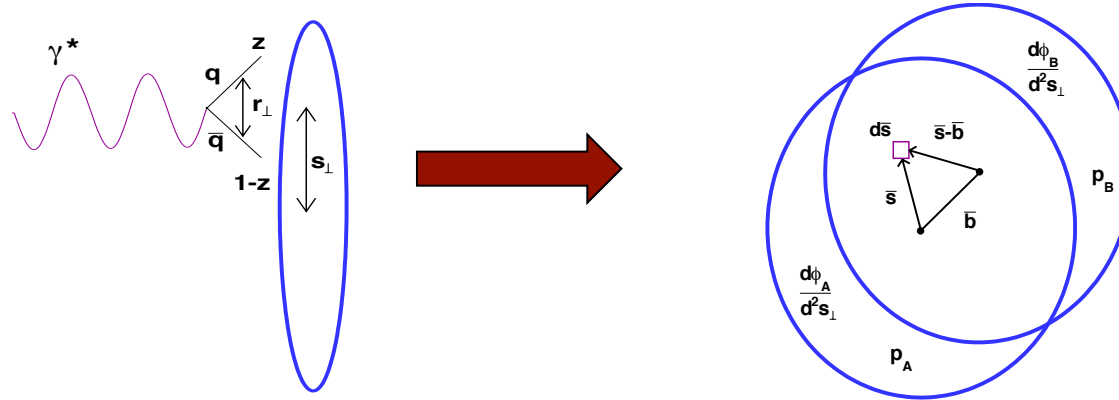
Inclusive DIS off proton

Rezaiean, Siddikov, Van der Klundert, RV:1212.2974



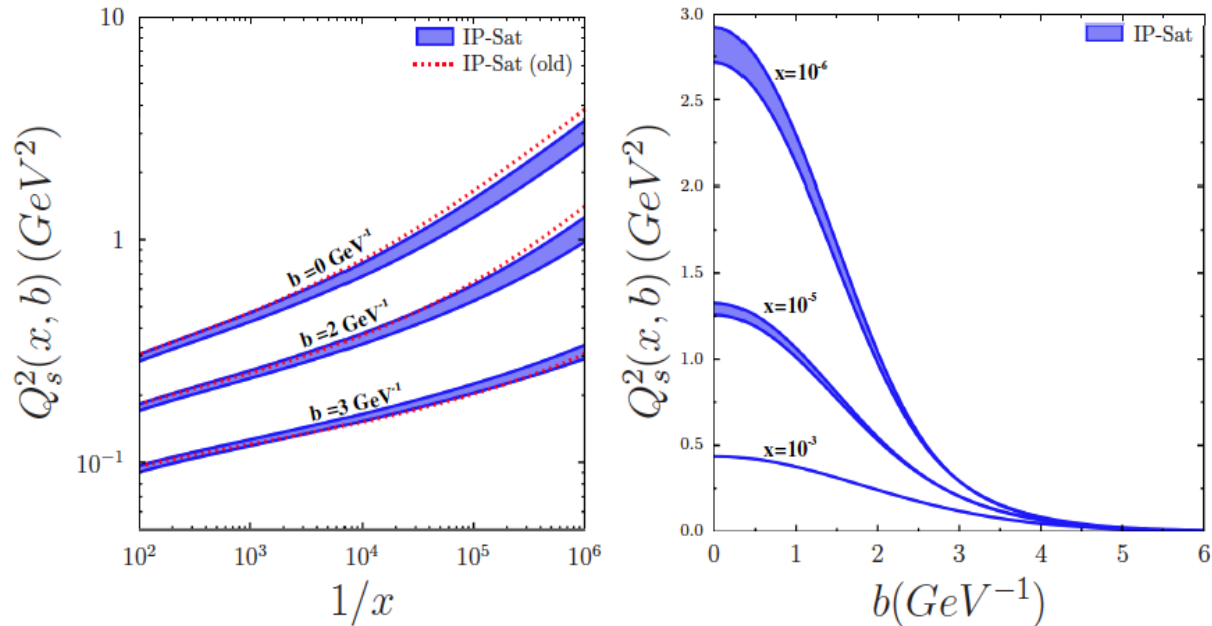
Exclusive DIS off proton

# Colliding lumpy glue-IPSat model



## Lumpy $Q_s(b)$ profile from fits to combined HERA DIS data

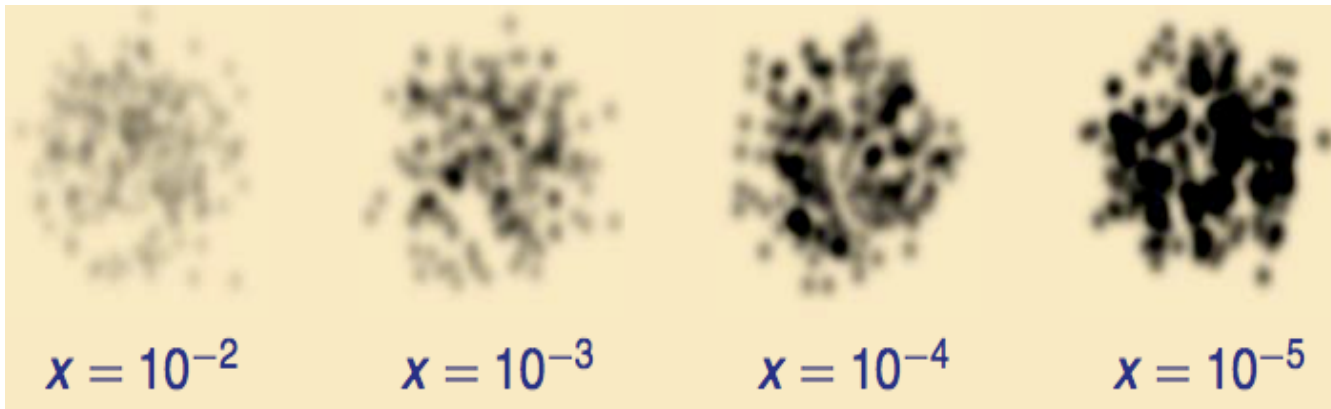
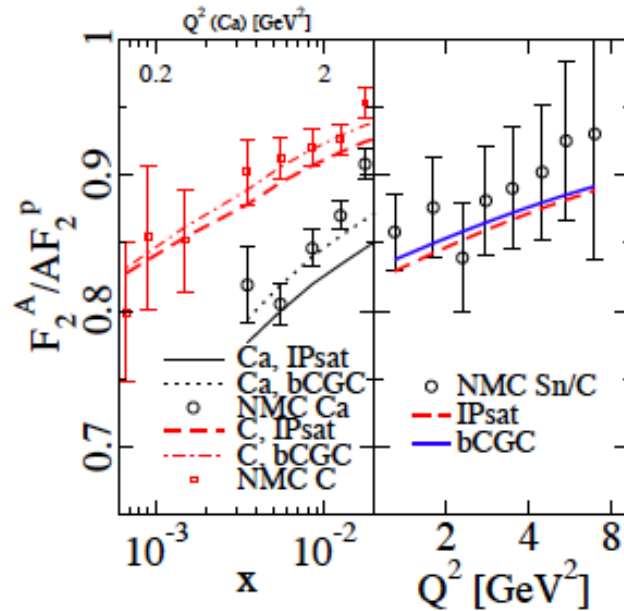
Rezaiean, Siddikov, Van der Klundert, RV:1212.2974





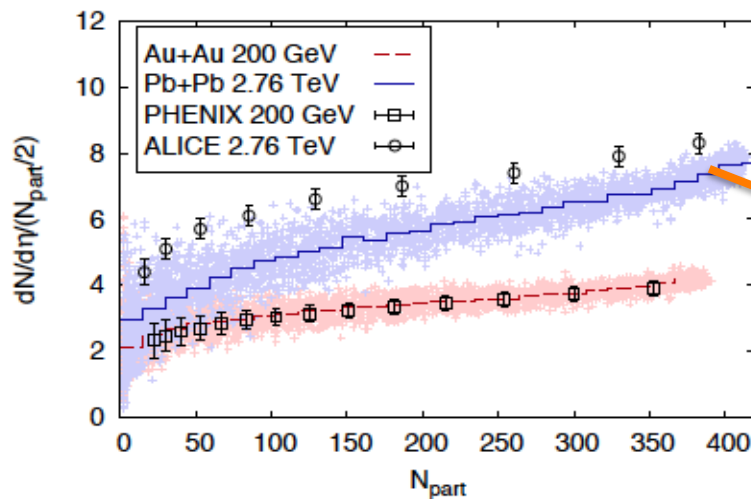
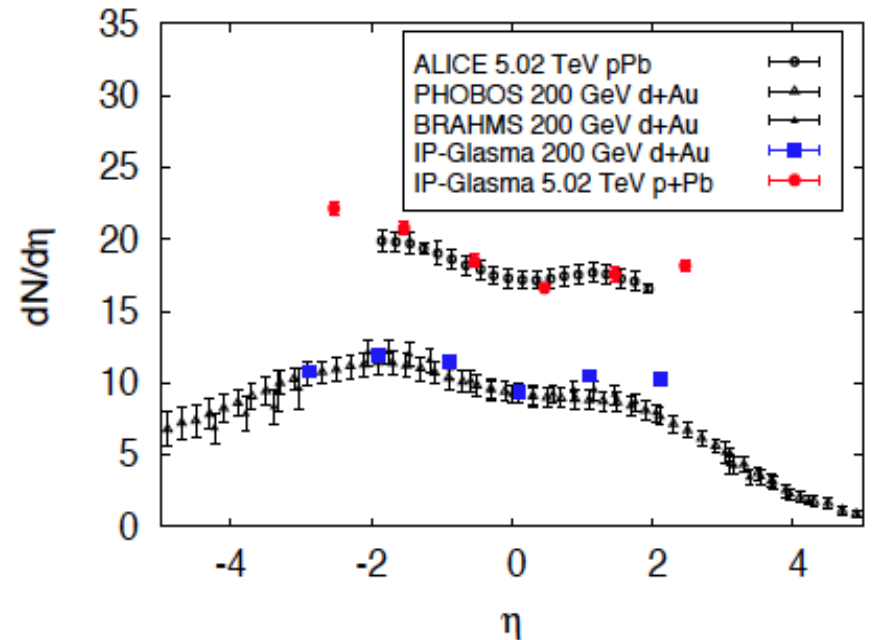
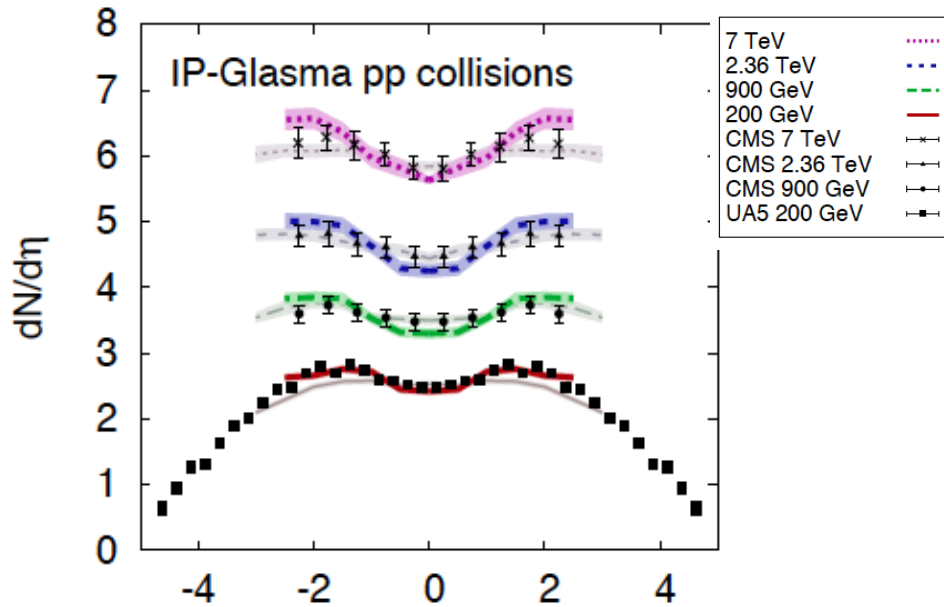
# Lumpy nuclei: constrained by (limited) DIS data

Kowalski, Lappi, RV, PRL (2008)



# Multiplicities from Yang-Mills dynamics

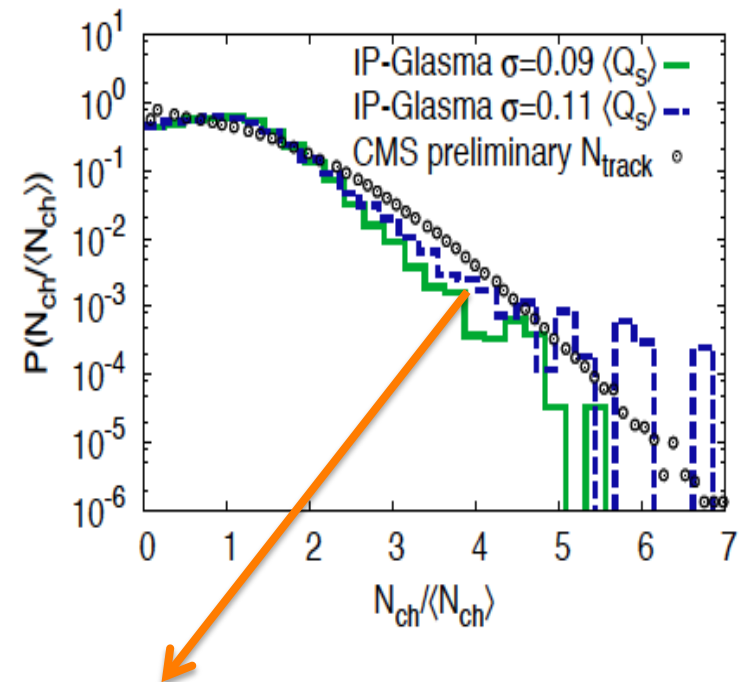
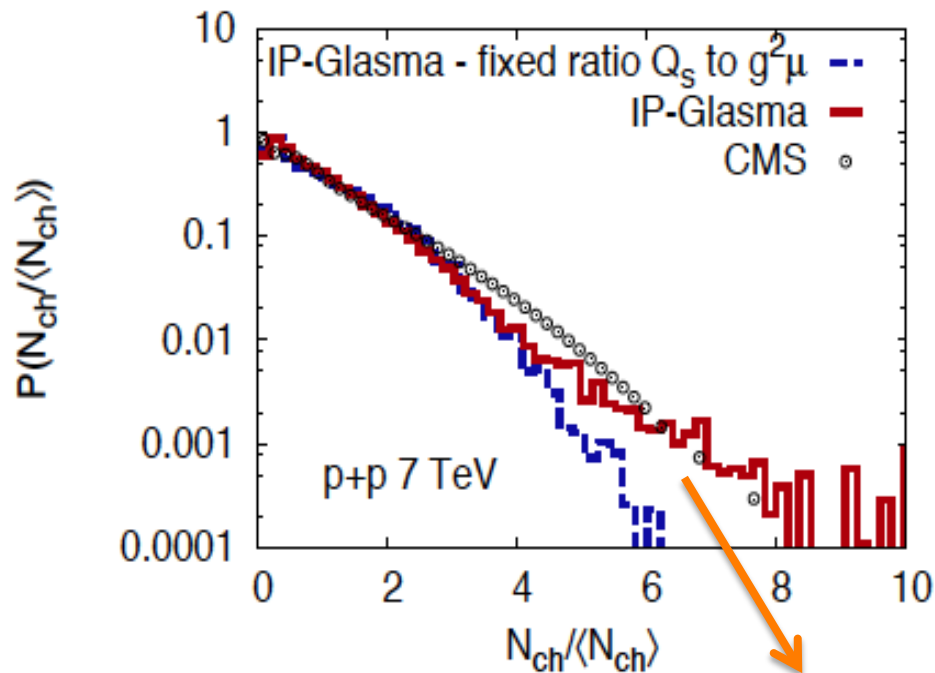
Schenke, Tribedy, RV:1311.3636



15% (30%) shortfall for large  $N_{part}$  (low  $N_{part}$ ) at LHC possibly due to the  $\sim 70\%$  greater  $\eta/s$

# Multiplicity distributions from Yang-Mills dynamics

Schenke, Tribedy, RV:1311.3636

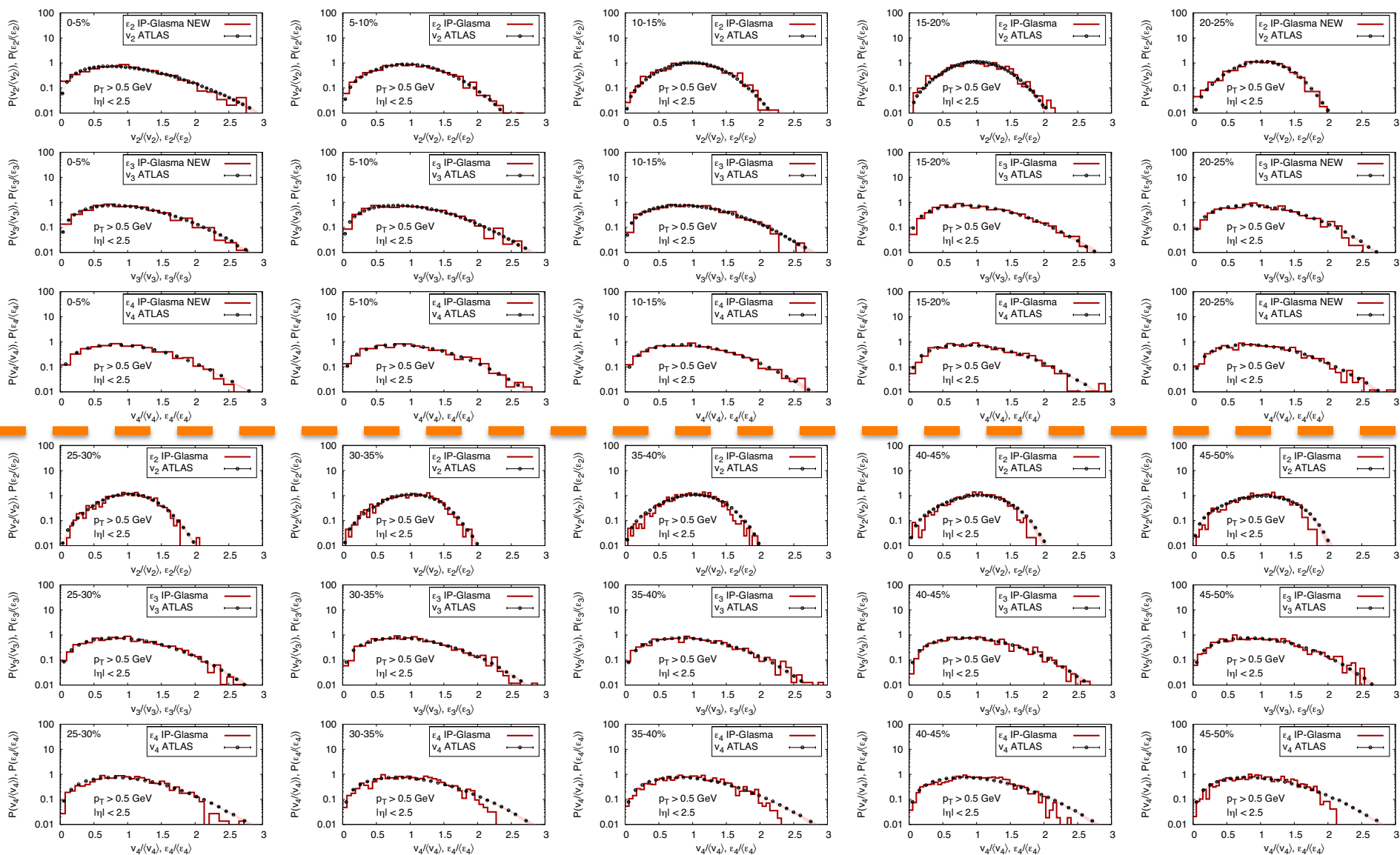


**Additional gluon # fluctuations (beyond color charge fluctuations) appear to be necessary to describe multiplicity distributions in p+p and p+A**  
**-much smaller role in A+A**

See also, Dumitru and Petreska, 1209.4105

# A+A eccentricity fluctuations from IP-Glasma

Beyond results presented in Gale et al 1209.6330



# Mission accomplished ?

**Far from it...many (possibly  $O(1)$  ) systematic uncertainties**

**On the hydro side: bulk viscosity, hydrodynamic fluctuations**

**Non-equilibrium dynamics: big source of uncertainty**

**– especially important in peripheral events/small sized systems...**



# LHC p+A vs A+A collisions

$N_{\text{trk}}^{\text{offline}}$ bin	PbPb data			pPb data		
	$\langle \text{Centrality} \rangle$ $\pm \text{RMS} (\%)$	$\langle N_{\text{trk}}^{\text{offline}} \rangle$	$\langle N_{\text{trk}}^{\text{corrected}} \rangle$	Fraction	$\langle N_{\text{trk}}^{\text{offline}} \rangle$	$\langle N_{\text{trk}}^{\text{corrected}} \rangle$
[0, $\infty$ )				1.00	40	50 $\pm$ 2
[0, 20)	92 $\pm$ 4	10	13 $\pm$ 1	0.31	10	12 $\pm$ 1
[20, 30)	86 $\pm$ 4	24	30 $\pm$ 1	0.14	25	30 $\pm$ 1
[30, 40)	83 $\pm$ 4	34	43 $\pm$ 2	0.12	35	42 $\pm$ 2
[40, 50)	80 $\pm$ 4	44	55 $\pm$ 2	0.10	45	54 $\pm$ 2
[50, 60)	78 $\pm$ 3	54	68 $\pm$ 3	0.09	54	66 $\pm$ 3
[60, 80)	75 $\pm$ 3	69	87 $\pm$ 4	0.12	69	84 $\pm$ 4
[80, 100)	72 $\pm$ 3	89	112 $\pm$ 5	0.07	89	108 $\pm$ 5
[100, 120)	70 $\pm$ 3	109	137 $\pm$ 6	0.03	109	132 $\pm$ 6
[120, 150)	67 $\pm$ 3	134	168 $\pm$ 7	0.02	132	159 $\pm$ 7
[150, 185)	64 $\pm$ 3	167	210 $\pm$ 9	$4 \times 10^{-5}$	162	195 $\pm$ 9
[185, 220)	62 $\pm$ 2	202	253 $\pm$ 11	$5 \times 10^{-4}$	196	236 $\pm$ 10
[220, 260)	59 $\pm$ 2	239	299 $\pm$ 13	$6 \times 10^{-5}$	232	280 $\pm$ 12
[260, 300)	57 $\pm$ 2	279	350 $\pm$ 15	$3 \times 10^{-6}$	271	328 $\pm$ 14
[300, 350)	55 $\pm$ 2	324	405 $\pm$ 18	$1 \times 10^{-7}$	311	374 $\pm$ 16

Peripheral A+A

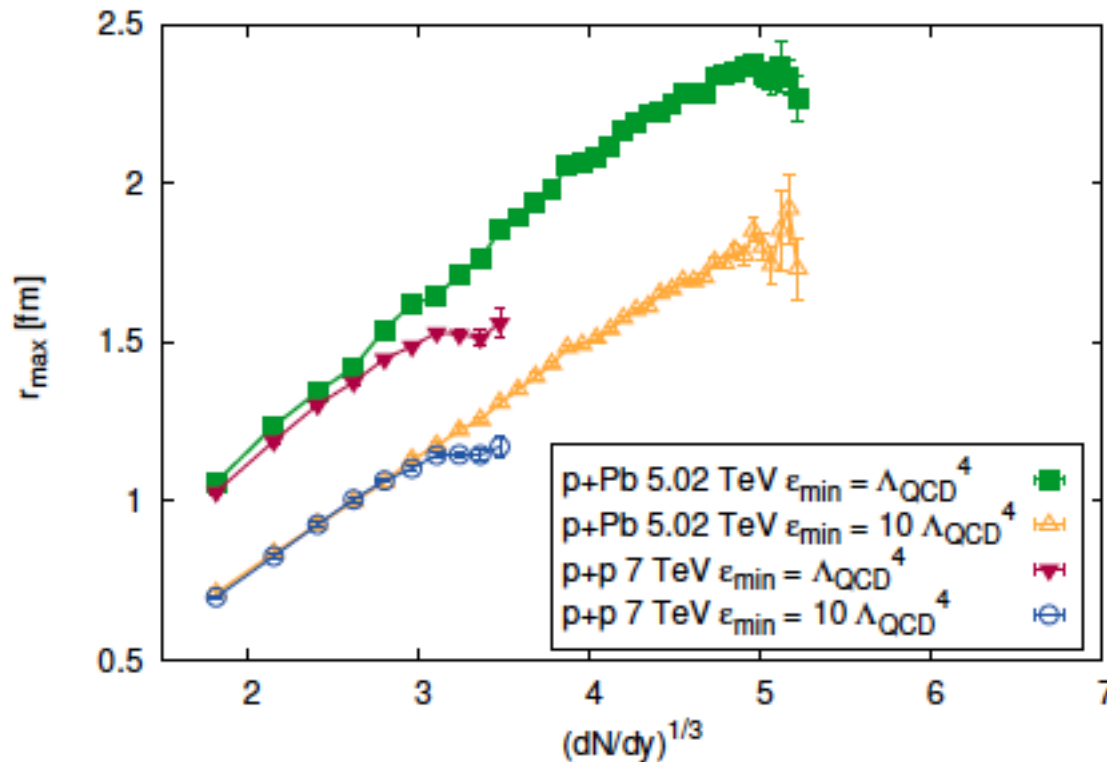
Central p+A

# Flow in p+A? IP-Glasma+MUSIC model

CGC initial conditions: IP-Glasma model with Yang-Mills dynamics

Event-by-event viscous hydro with MUSIC

Bzdak,Schenke,Tribedy,RV:1304.3403

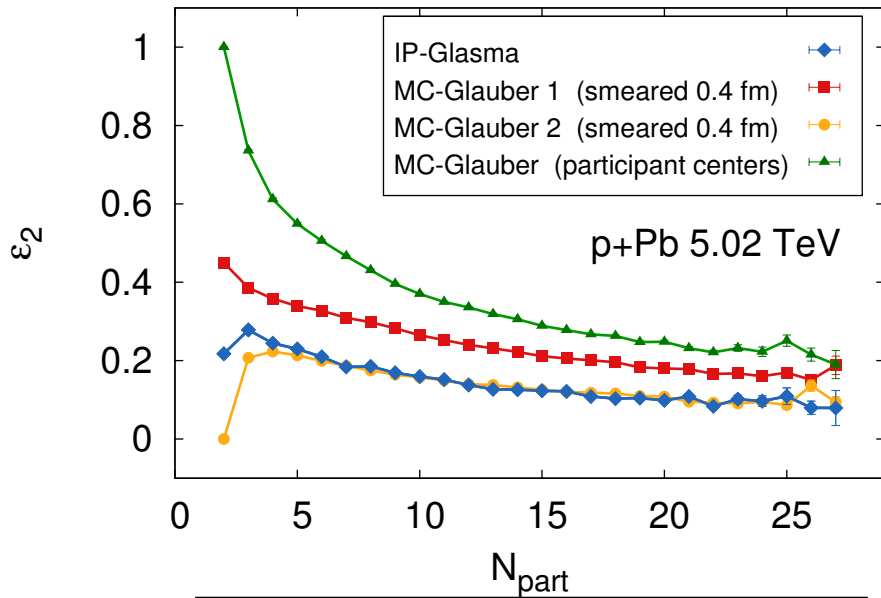


Initial radii are nearly identical in p+p and p+A until large multiplicity

See McLerran,Praszalowicz,Schenke,1306.2350

& Rezaiean,Schmidt 1307.0825 for discussion of multiplicities

# Flow in p+A? IP-Glasma vs “Glauber”



IP-Glasma, Bzdak,Schenke,Triedy,RV:1304.3403

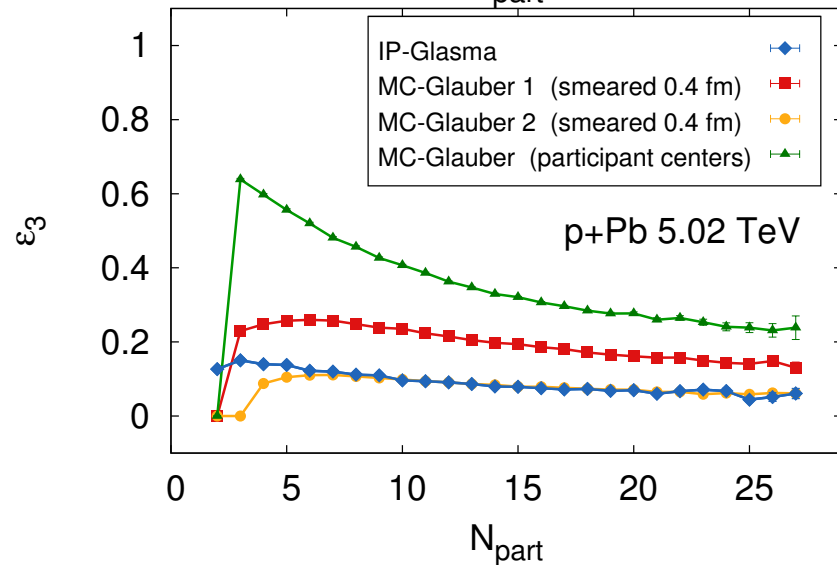
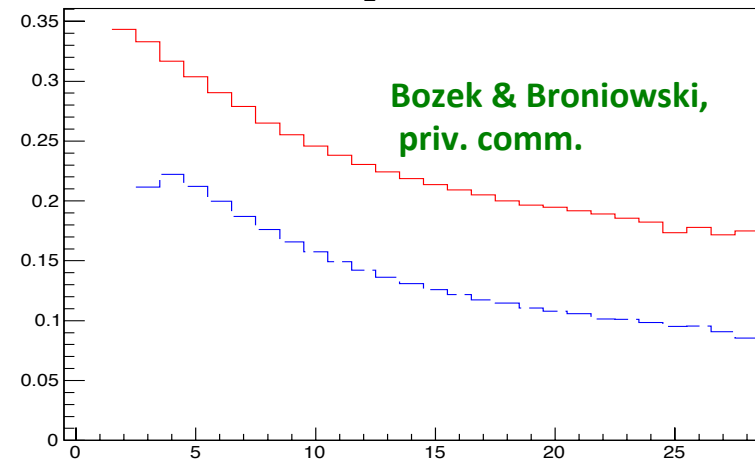
MC-Glauber: Bozek,Broniowski:1304.3044



MC-Glauber 1

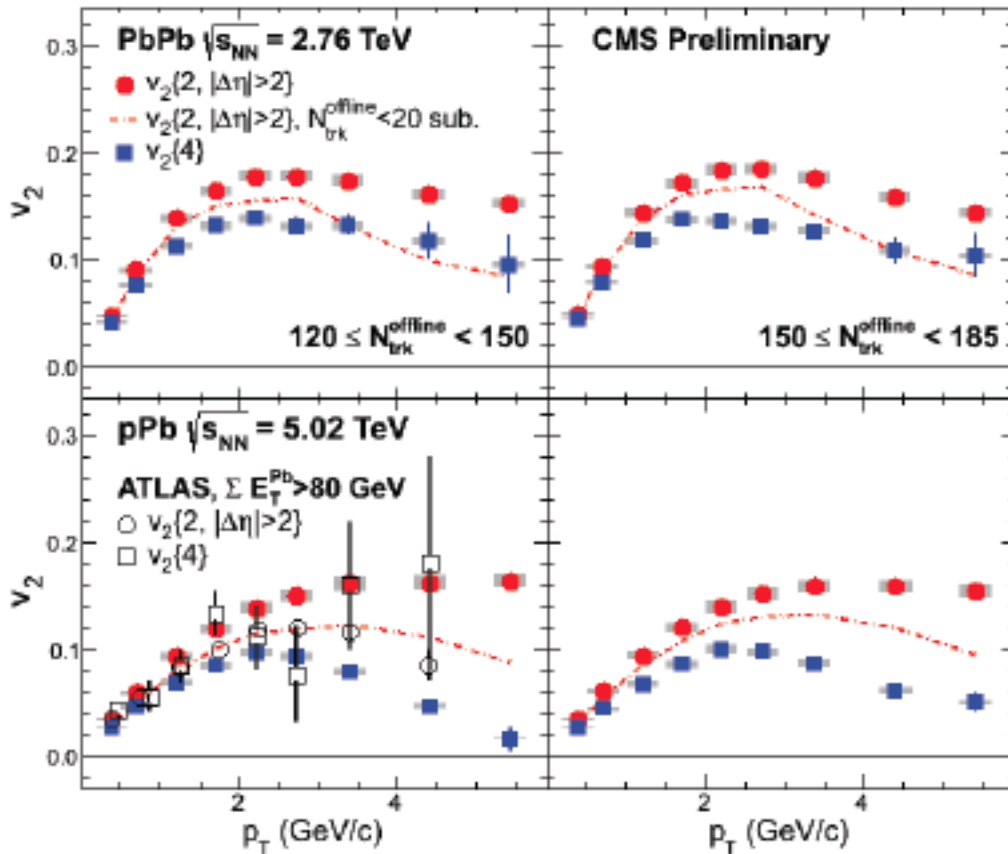
MC-Glauber 2

$\epsilon_2$  vs.  $N_w$

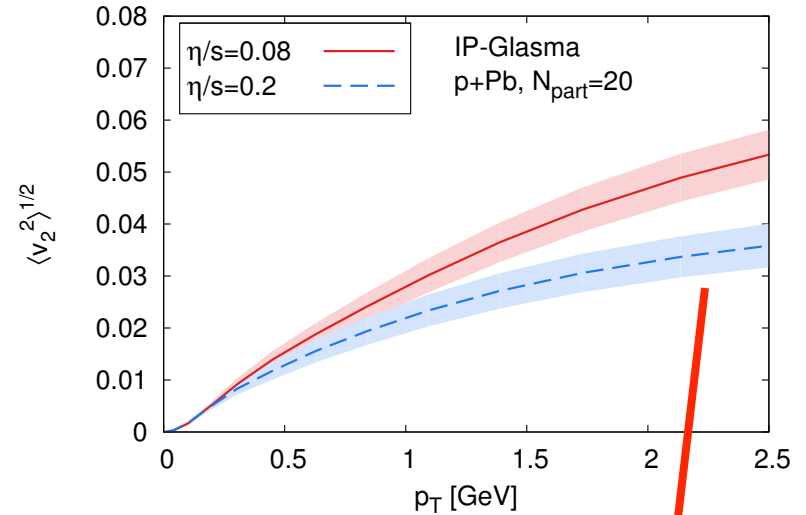


Hydro results **VERY** sensitive to  
initial conditions

# Flow in p+A: IP-Glasma+MUSIC



IP-Glasma, Bzdak, Schenke, Tribedy, RV:1304.3403

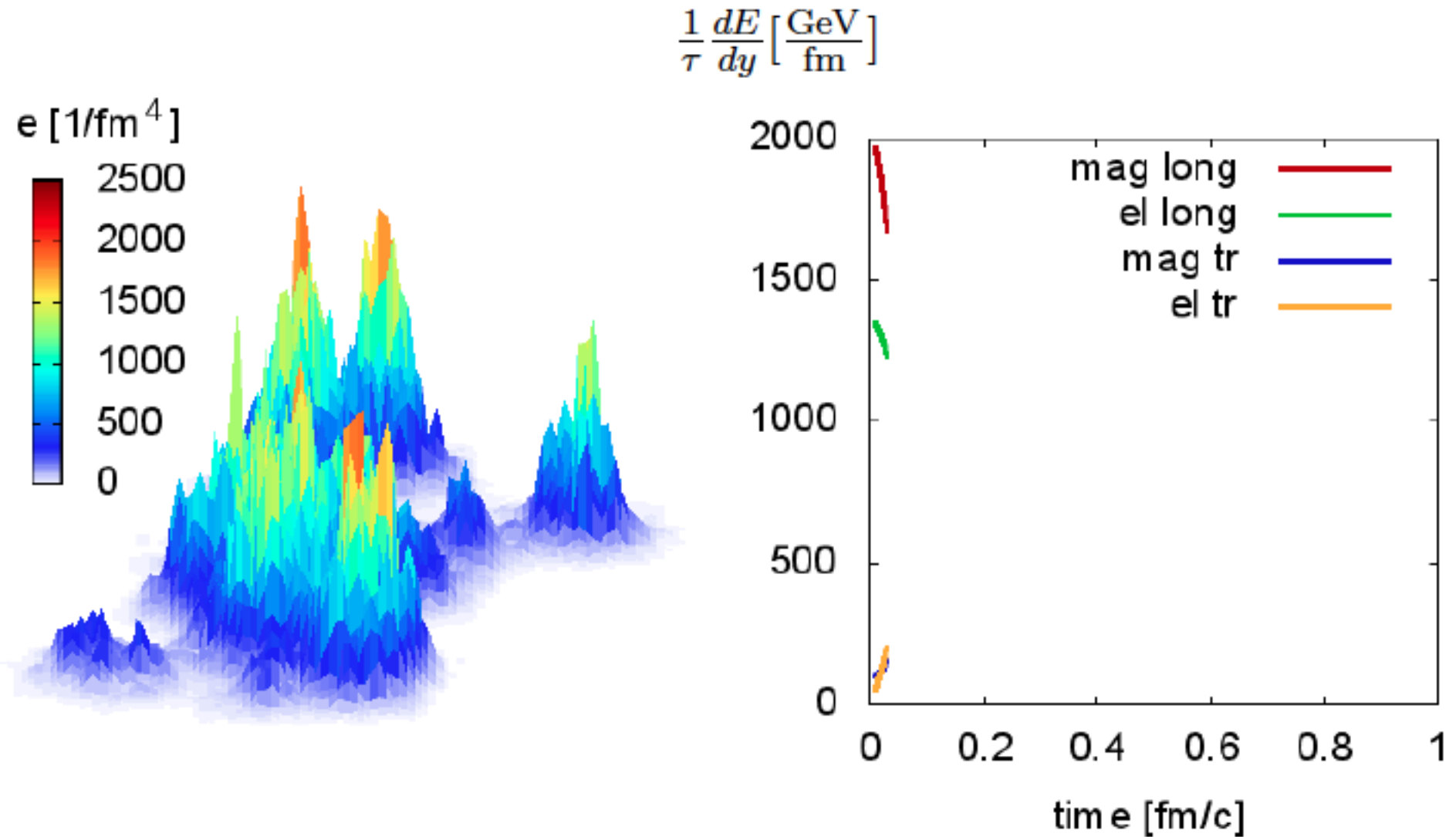


IP-Glasma result nearly  
 3 times smaller than  
 data for  $\eta/s=0.2$

◆ Extend this framework to JIMWLK+Yang-Mills to include  
**BOTH initial state AND final state event-by-event hydro**  
 with apples to apples centrality selection in p+p, p/d+A and A+A

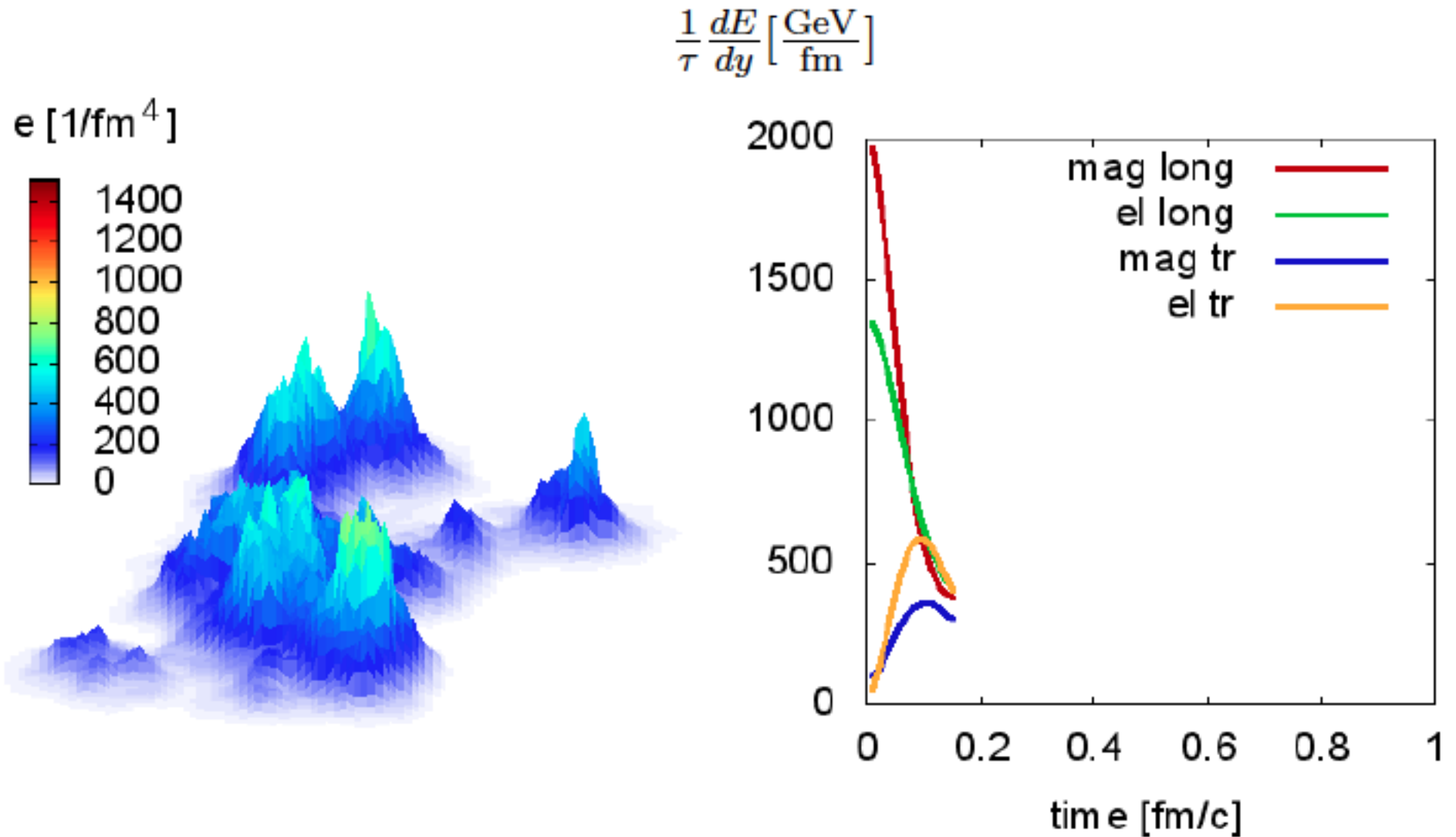
The ridge: my take, see 1312.0113

# Classical Yang-Mills in IP-Glasma





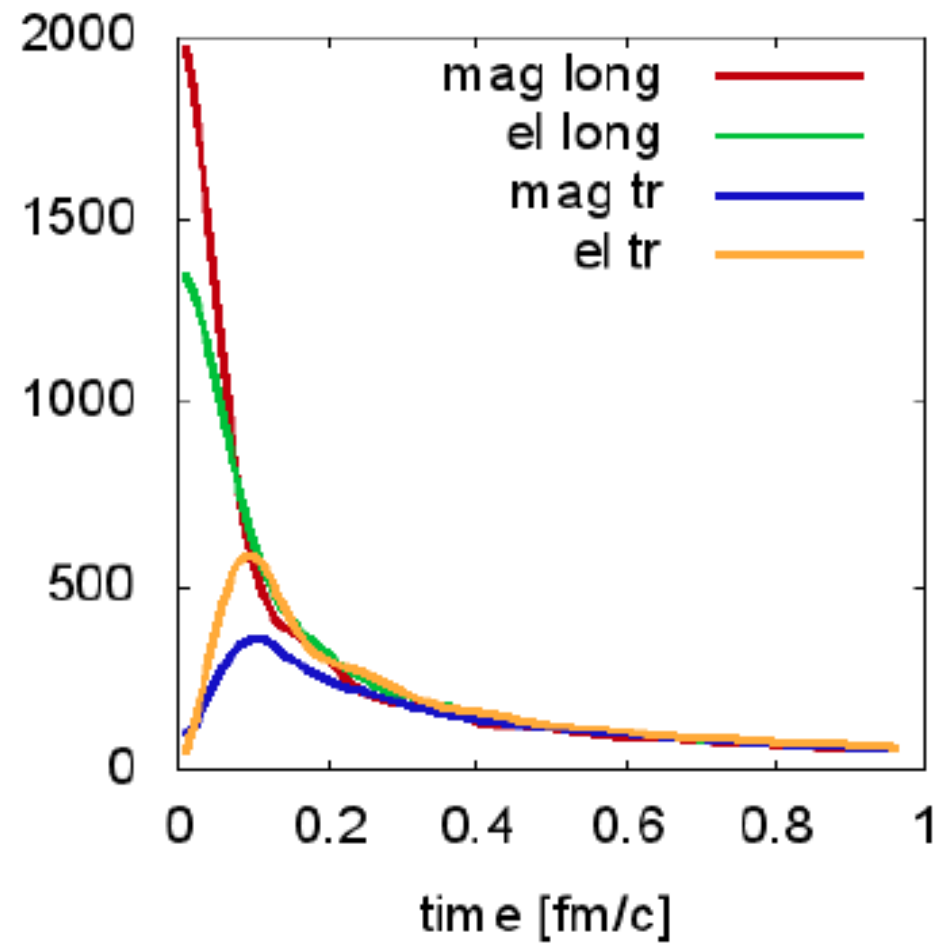
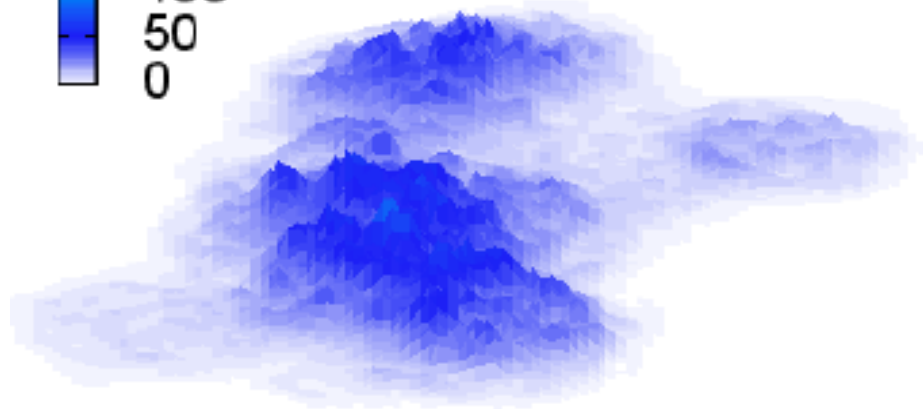
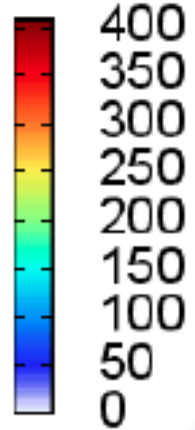
# Classical Yang-Mills in IP-Glasma



# Classical Yang-Mills in IP-Glasma

$$\frac{1}{\tau} \frac{dE}{dy} \left[ \frac{\text{GeV}}{\text{fm}} \right]$$

$e [1/\text{fm}^4]$



# IP-Glasma: Boost invariant Classical Yang-Mills

Compute all components of  $T_{\mu\nu}$

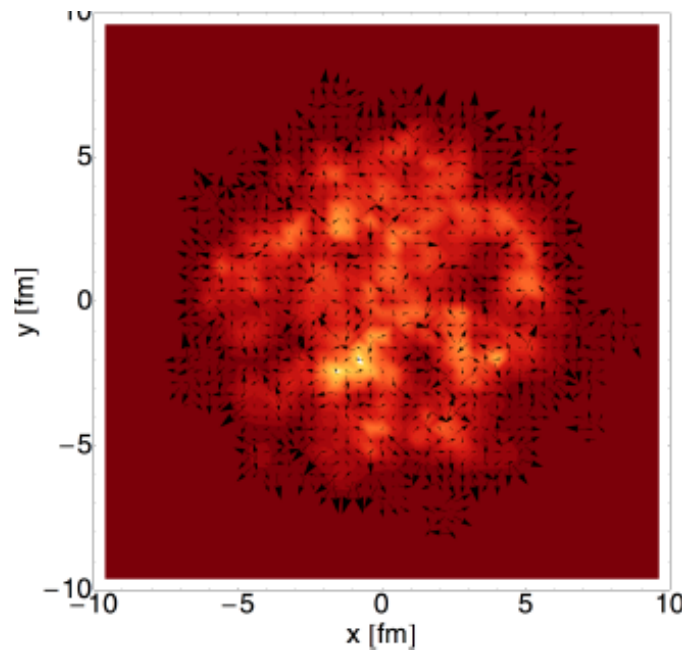
$$T_{\mu\nu}(\tau = 0) = \frac{1}{2}(B_z^2 + E_z^2) \times \text{diag}(1, 1, 1, -1)$$

Initial longitudinal pressure is negative:

Goes to  $P_L = 0$  from below with time evolution

# IP-Glasma: Boost invariant Classical Yang-Mills

Compute all components of  $T_{\mu\nu}$

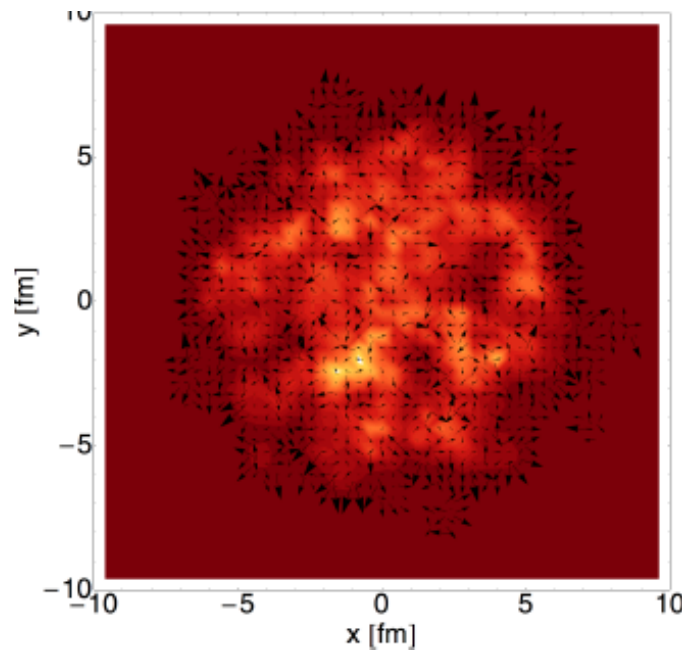


Energy density  
and  $(u_x, u_y)$   
at  $\tau = 0.4 \text{ fm}/c$

Energy density and  $(u_x, u_y)$  from  $u_\mu T^{\mu\nu} = \varepsilon u^\nu$

# IP-Glasma: Boost invariant Classical Yang-Mills

Compute all components of  $T_{\mu\nu}$



Energy density  
and  $(u_x, u_y)$   
at  $\tau = 0.4 \text{ fm}/c$

Matching to viscous hydro is “brutal” : assume very rapid isotropization at initial hydro time



# Sturm und drang



Initial state:  
Far from equilibrium



*Non-equilibrium  
dynamics*



Final state:  
Thermal equilibrium



*How is thermal equilibrium achieved?*

# 3+1-D classical expanding Yang-Mills simulations

◆ Epelbaum, Gelis; Fukushima

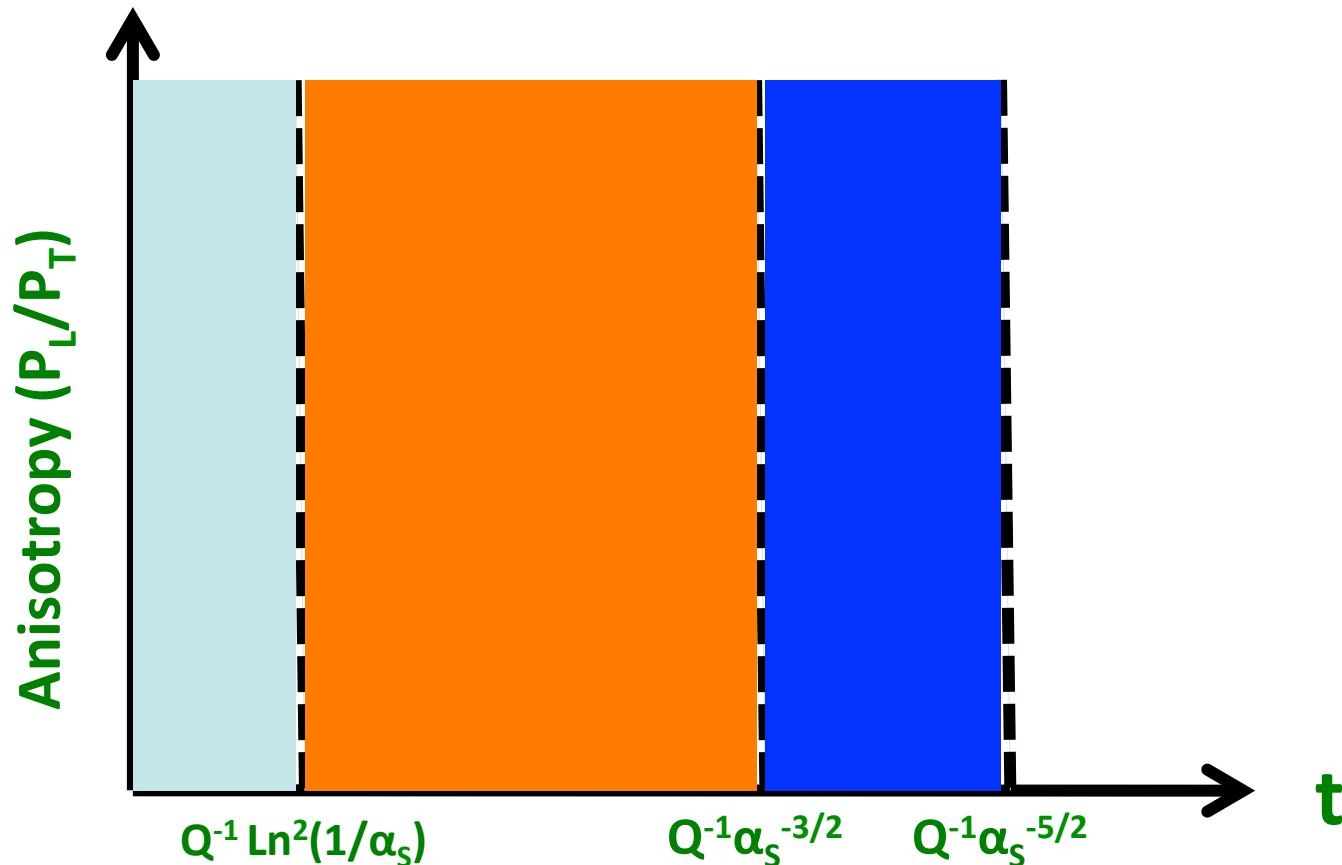
◆ Berges, Boguslavski, Schlichting, Venugopalan

◆ Attems, Rebhan, Strickland

❖ Very significant progress in large scale numerical YM simulations of strongly correlated dynamics that could generate significant flow —and in kinetic descriptions of such overoccupied systems

(Blaizot,Liao,McLerran)

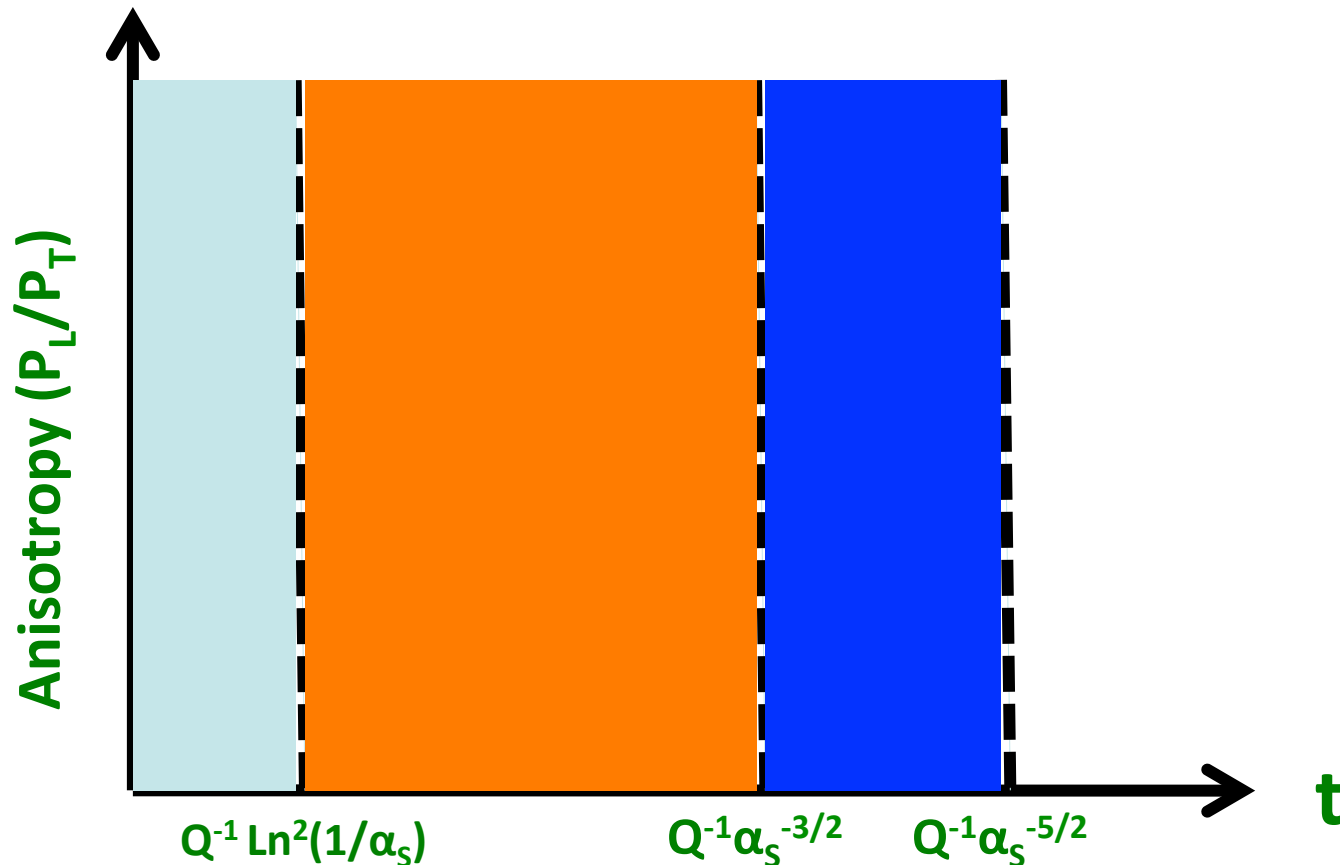
# 3+1-D classical Yang-Mills simulations



**Instability dominated region**

-quantum fluctuations become as large as the classical background – can lead to phase decoherence

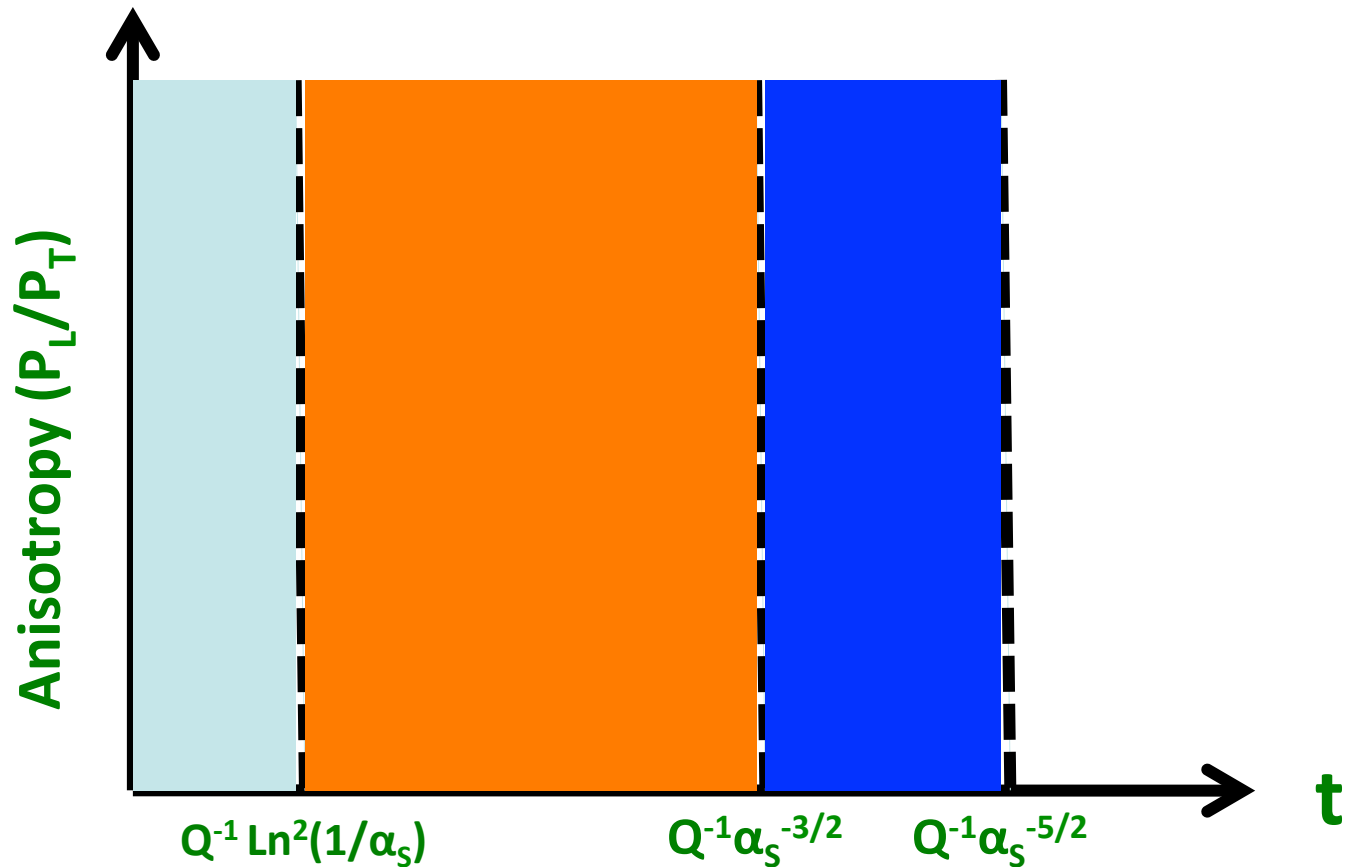
# 3+1-D classical Yang-Mills simulations



Instability dominated region

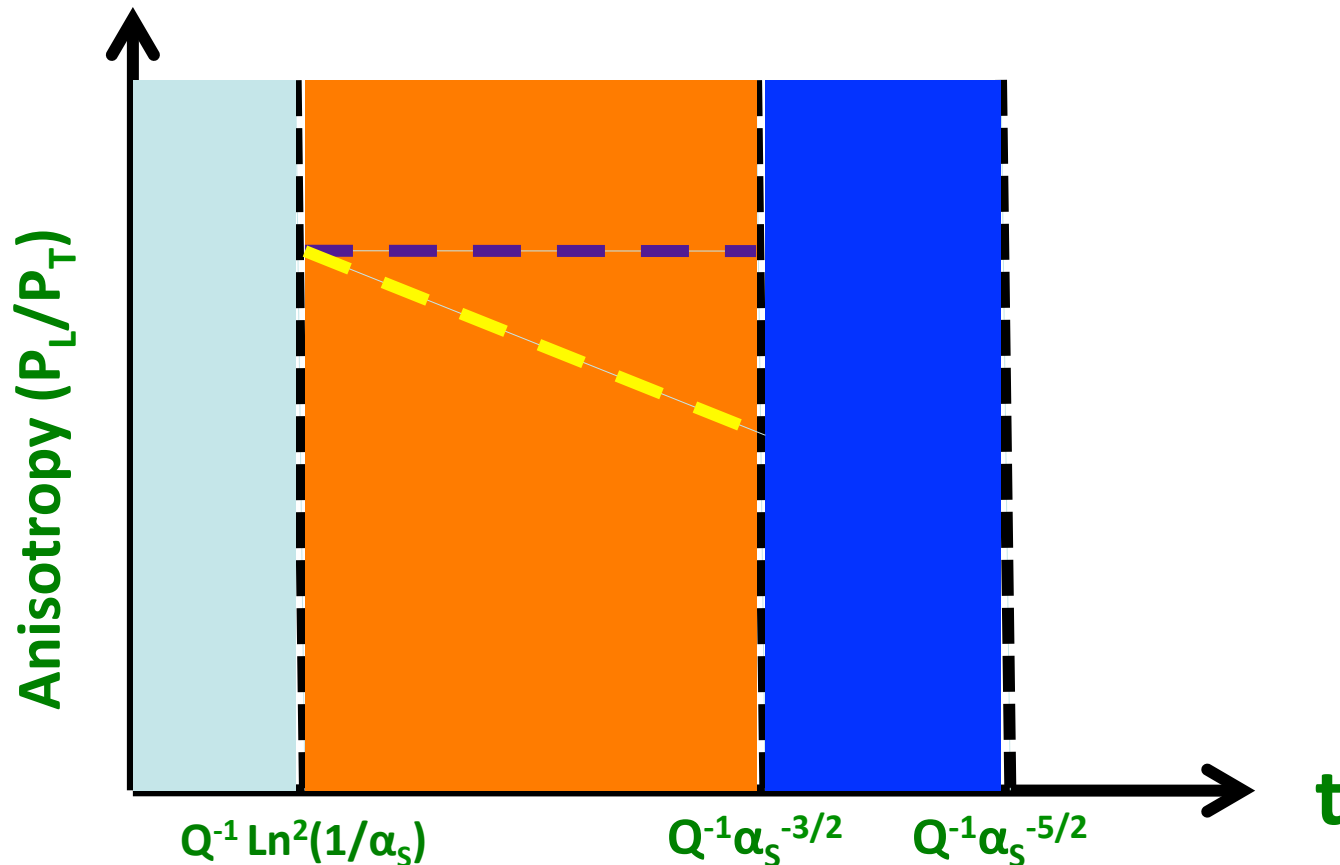
-quantum fluctuations become as large as the classical background – can lead to phase decoherence

# 3+1-D classical Yang-Mills simulations



Strongly correlated dynamics up to here  
-- strongest possible in QCD – for arbitrarily small  $\alpha_s$

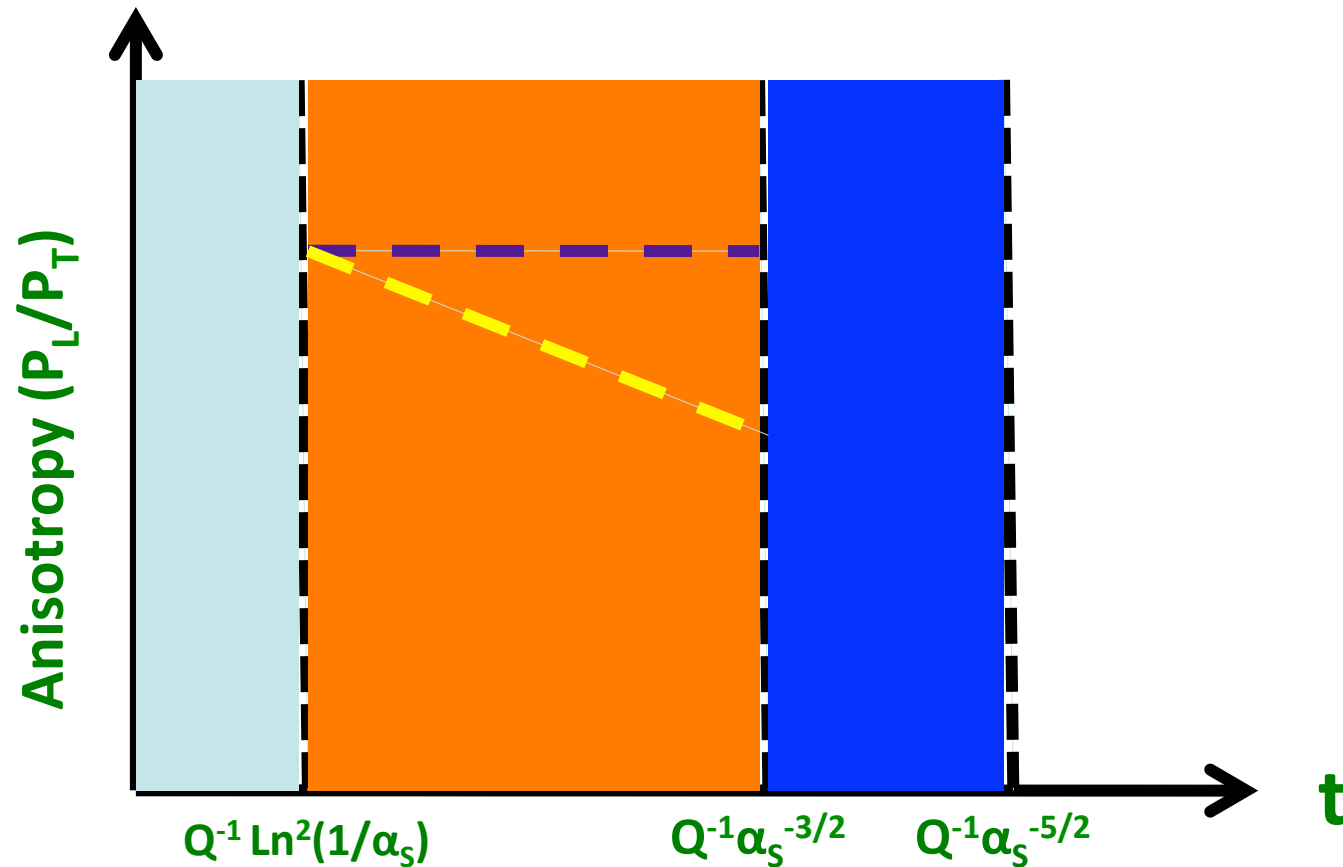
## 3+1-D classical Yang-Mills simulations



Open question: a) what happens when one cranks up the coupling ?  
--all of these regions shrink  
-- is there a reliable weak coupling regime where transient dynamics dominates through to isotropization?

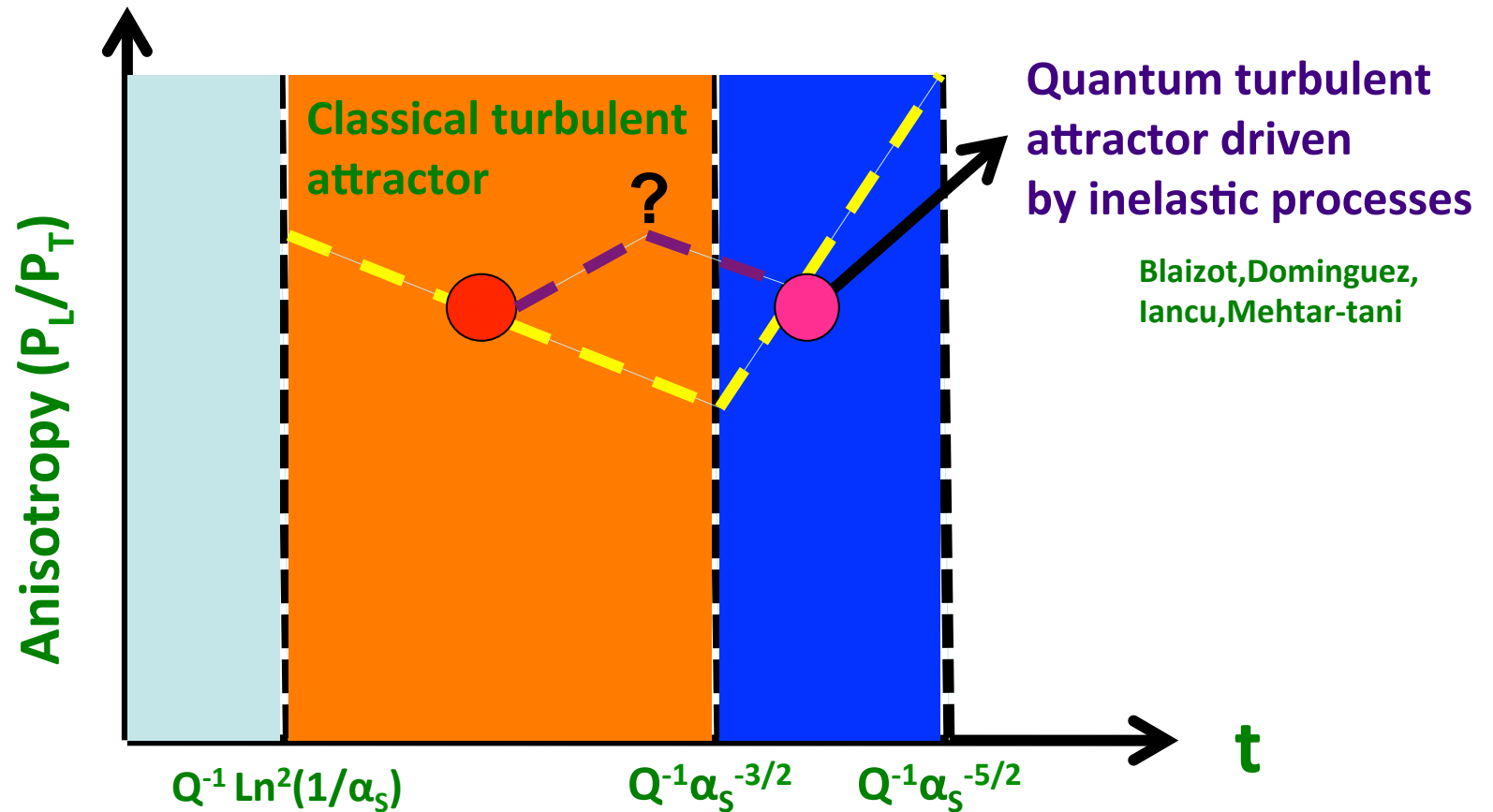


## 3+1-D classical Yang-Mills simulations



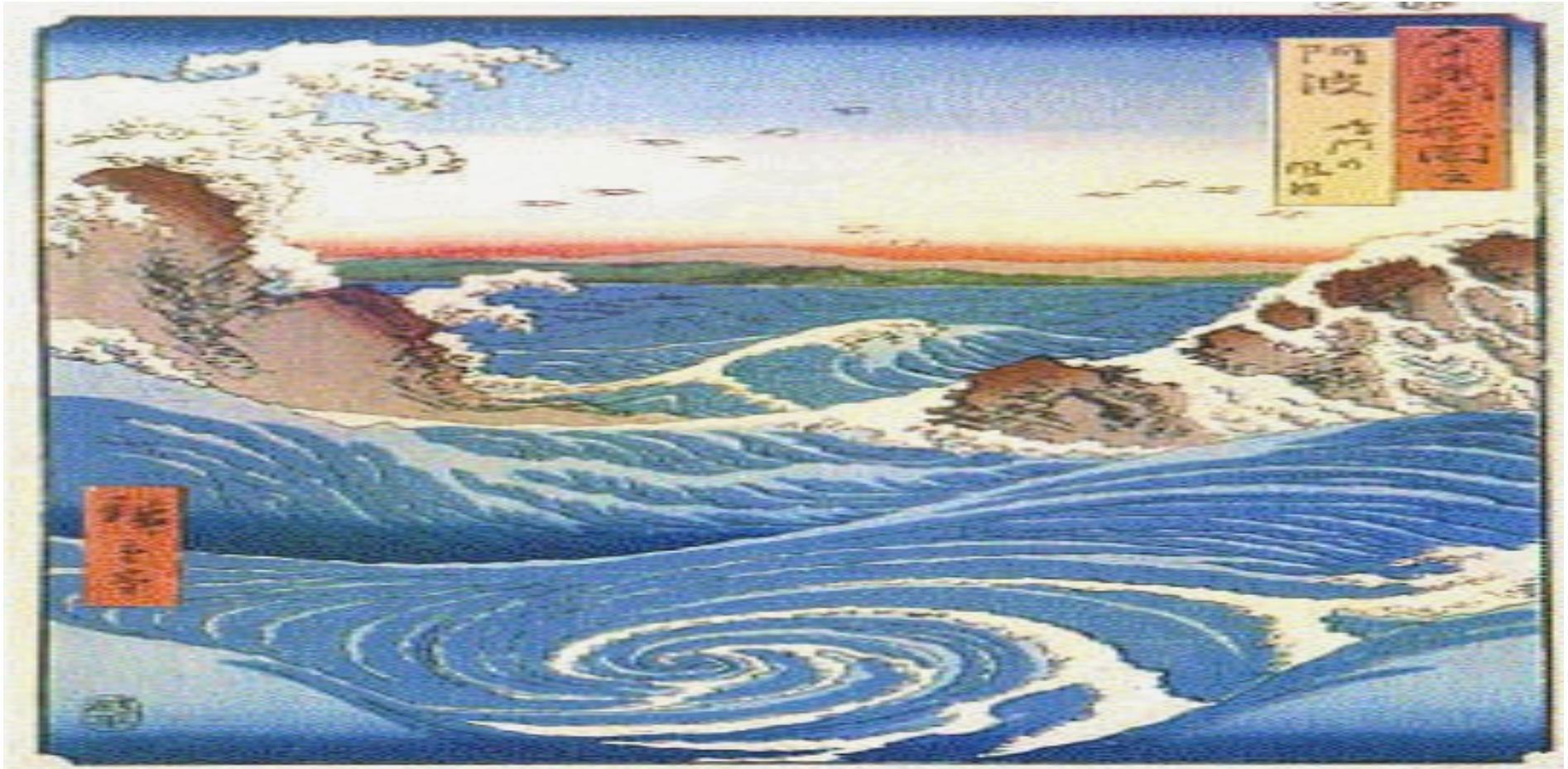
This can be settled conclusively with existing “technology” + really smart young folks...

# Quo vadis, thermalization ?



Open question: can we compute the prefactors reliably ?

# Universal non-thermal attractors in QCD



**“Big whorls have little whorls, which feed on their velocity,  
And little whorls have lesser whorls, and so on to viscosity.”**





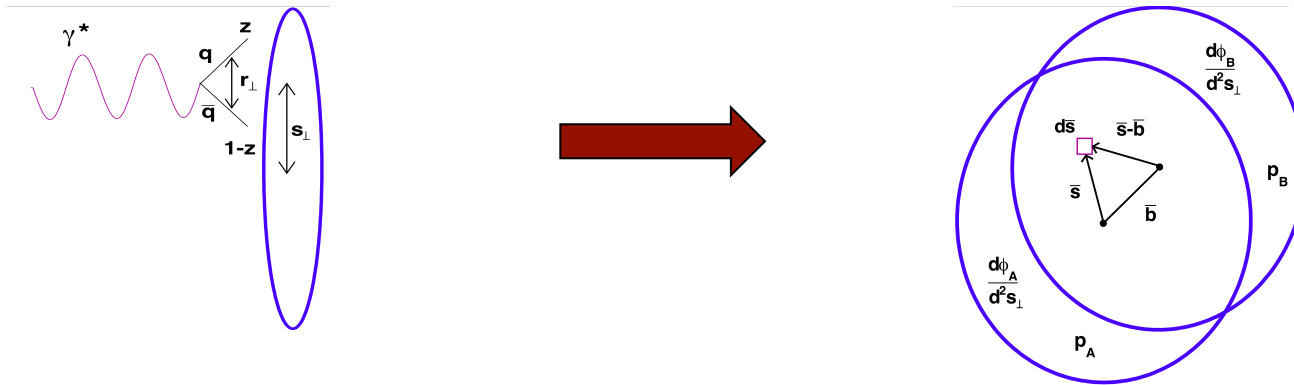
**Many thanks to the organizers for this very enjoyable meeting!**

**Backup slides**

# From nuts to soup: I. constraining initial conditions

First understand e+p and p+p:

Global analysis of HERA data thus far performed only in the IP-Sat, b-CGC and rcBK saturation models - more detailed JIMWLK analysis is desirable and likely



Unintegrated proton gluon dist. from dipole cross-section:

$$\frac{d\phi(x, k_\perp | s_\perp)}{d^2 s_\perp} = \frac{k_\perp^2 N_c}{4 \alpha_s} \int_0^\infty d^2 r_\perp e^{ik_\perp \cdot r_\perp} \left[ 1 - \frac{1}{2} \frac{d\sigma_{\text{dip.}}^p}{d^2 s_\perp}(r_\perp, x, s_\perp) \right]^2$$

$k_\perp$  factorization: compute inclusive dist. of produced gluons at given impact par. :

$$\frac{dN_g(b_\perp)}{dy d^2 p_\perp} = \frac{16 \alpha_s}{\pi C_F} \frac{1}{p_\perp^2} \int \frac{d^2 k_\perp}{(2\pi)^5} \int d^2 s_\perp \frac{d\phi_A(x, k_\perp | s_\perp)}{d^2 s_\perp} \frac{d\phi_B(x, p_\perp - k_\perp | s_\perp - b_\perp)}{d^2 s_\perp}$$

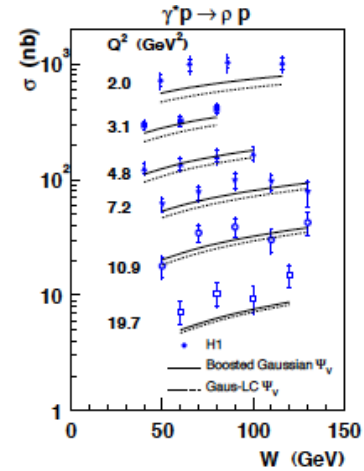
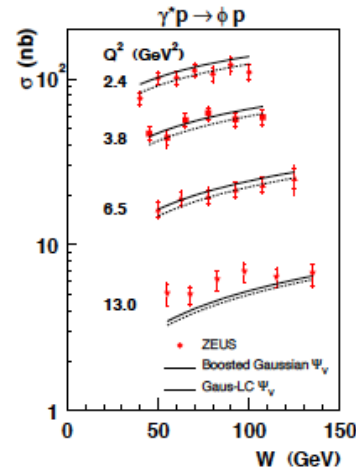
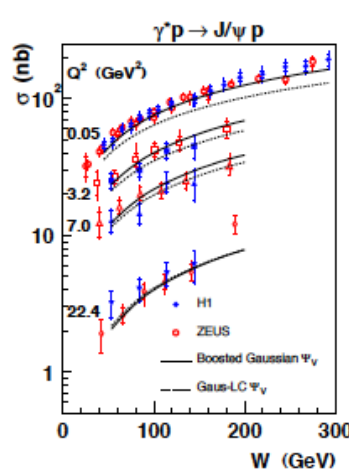
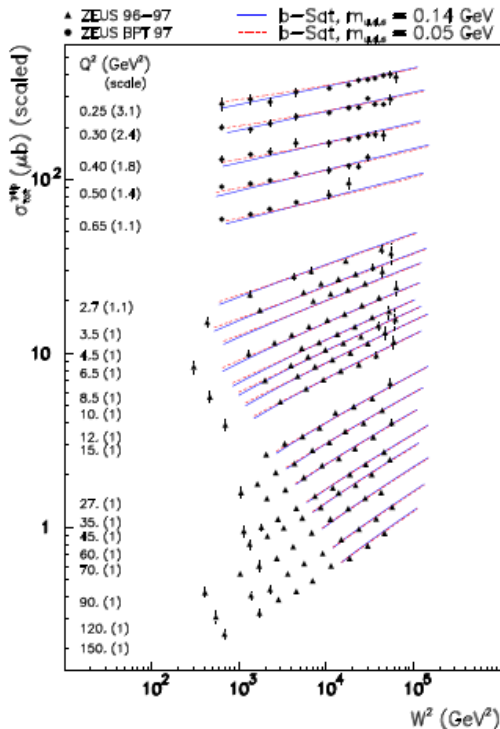
# The IP-Sat model

Bartels, Golec-Biernat, Kowalski  
Kowalski, Teaney  
Kowalski, Motyka, Watt

$$\frac{d\sigma_{\text{dip}}^p}{d^2b_{\perp}}(\mathbf{r}_{\perp}, x, \mathbf{b}_{\perp}) = 2\mathcal{N}(\mathbf{r}_{\perp}, x, \mathbf{b}_{\perp}) = 2 \left[ 1 - \exp \left( -\frac{\pi^2}{2N_c} \mathbf{r}_{\perp}^2 \alpha_s(\tilde{\mu}^2) x g(x, \tilde{\mu}^2) T_p(\mathbf{b}_{\perp}) \right) \right]$$

MV model extended to small x + impact parameter dependence  $\tilde{\mu}^2 = \mu_0^2 + \frac{4}{r_{\perp}}^2$

$$T_p(b_{\perp}) = e^{-\frac{b_{\perp}^2}{2B_G}} \rightarrow \text{Average gluon radius of the proton extracted from HERA diffractive data}$$



$\chi^2 \sim 1$  fits to HERA inclusive, diffractive and exclusive small x data with few parameters



# From nuts to soup: II. the IP-Glasma model

Schenke, Tribedy, RV:1202.6646

## A. Construct color charge distributions, event-by-event:

- Positions of nucleons sampled from the Woods-Saxon distribution of each nucleus A and B
- IP-Sat provides  $Q_s^2(x, b_T)$  for each nucleon – proportional to color charge squared per unit area  $g^2\mu_p^2$  (details, see T. Lappi, arXiv:0711.3039)
- Add all  $g^2\mu_p^2(x_T)$  to obtain  $g^2\mu_A^2(x_T)$  and  $g^2\mu_B^2(x_T)$
- Sample  $\rho_{A,B}^a$  from local Gaussian distribution for each nucleus:

$$\langle \rho_k^a(x_\perp) \rho_l^b(y_\perp) \rangle = \delta_{kl} \delta^{ab} \delta^{(2)}(x_\perp - y_\perp) g^2 \mu_{A,B}^2(x_\perp)$$

*This gives the random static source distribution for event-by-event multi-particle production*