Convection, Shearbanding and Experiments in Driven Granular Matter

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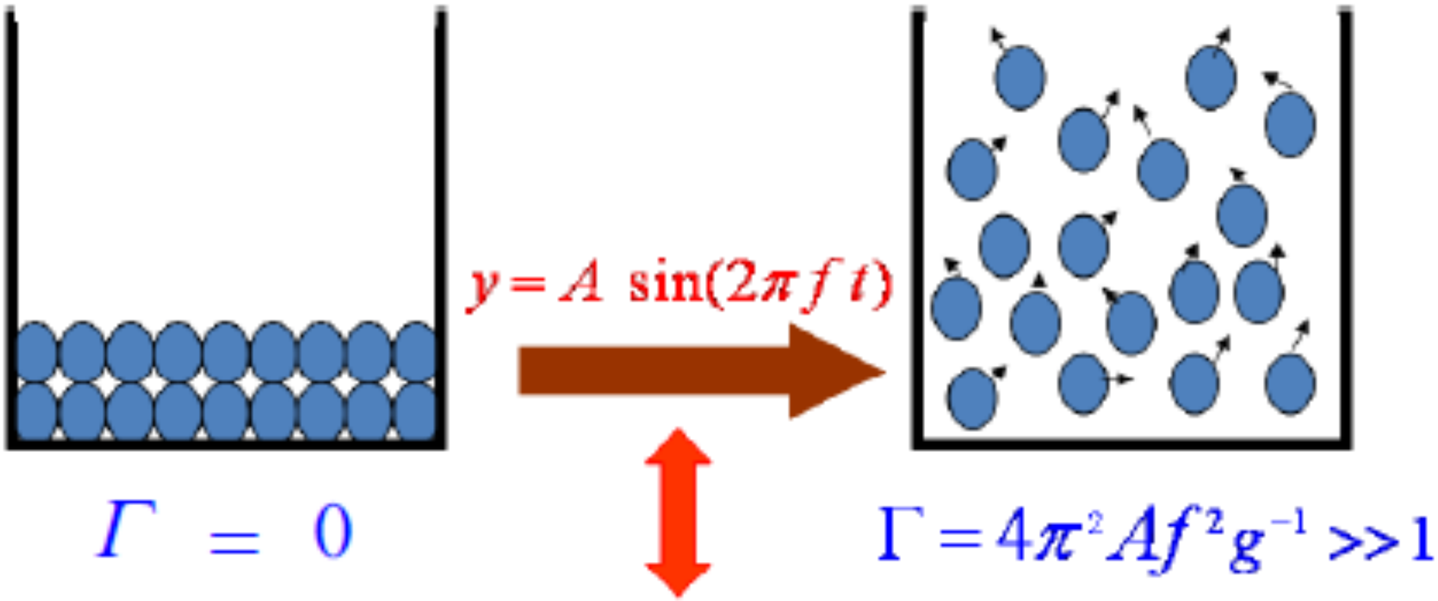
Outline of Talk

- Introduction
- Granular Hydrodynamics
- Nonlinear Stability and Stuart-Landau Eqn
- Results: `Bounded' granular convection
- `Semi-bounded' granular convection
- Gradient and Vorticity Banding
- Experiments on Vibrated Binary Mixtures
- Conclusions

Granular Fluid?



- Athermal system
- Inelastic dissipation ⟹ Microscopic Irreversibility
- Lack of Scale Separation
- Extended Set of Hydrodynamic Fields ?

Oscillons and Faraday Waves (Swinney etal 1996)

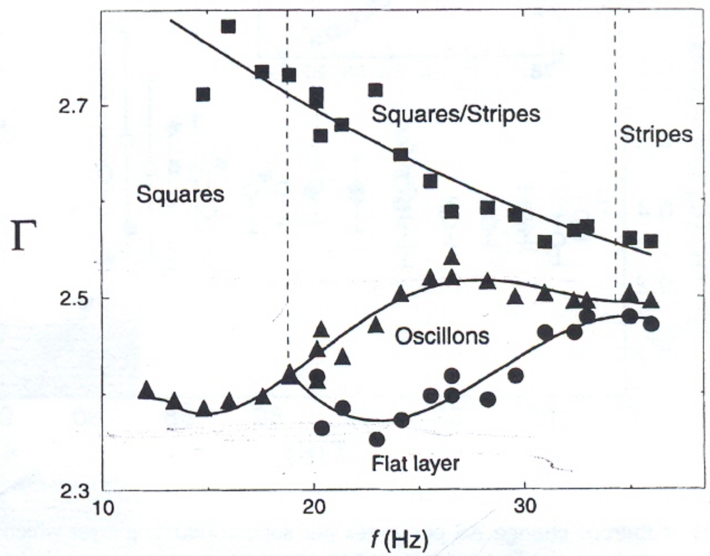
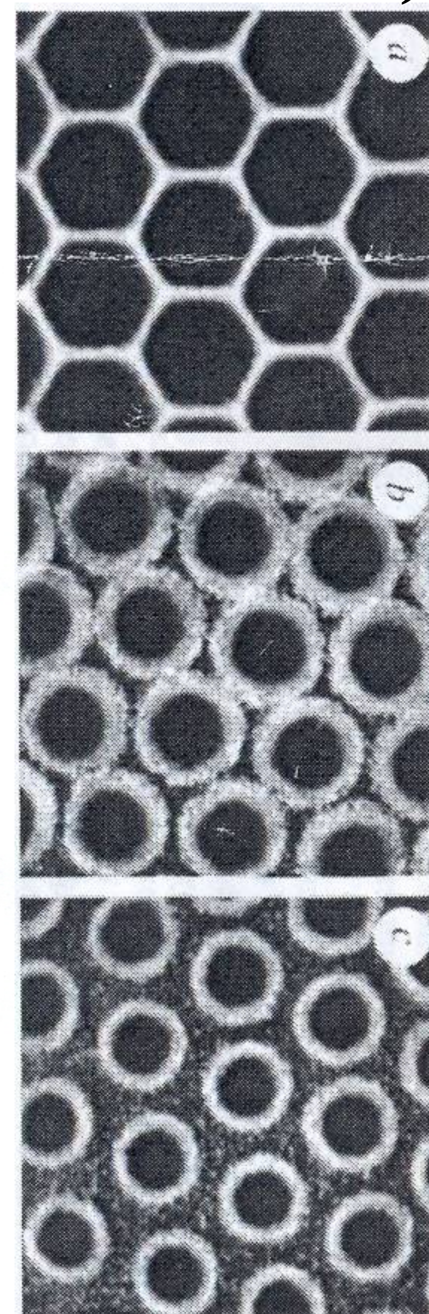
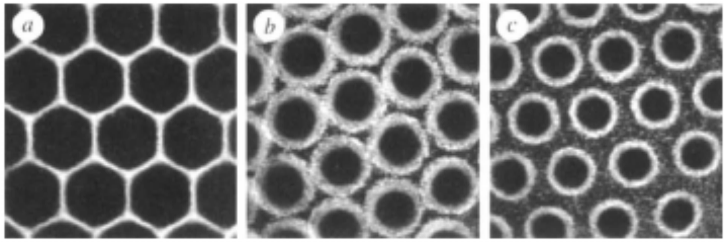


FIG. 2 Diagram showing the stability regions for different states, as a function of *f* and *Γ*, for increasing *Γ* (squares) and decreasing *Γ* (triangles and circles). The transitions from the flat layer to squares and stripes are hysteretic, but the hysteresis is much smaller for stripes. Oscillons are observed for layers greater than 13 particles deep in a range of *f* which increases with increasing depth. For thinner layers, the phase diagram is similar but without the oscillon region.



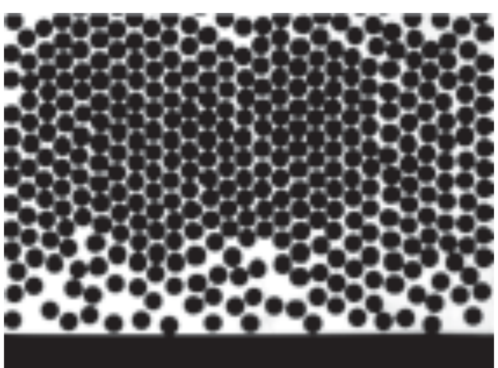
Subharmonic (f/2, f/4, …)



Subharmonic + ???

Oscillons (f/2)
Umbanhower et.al 1996

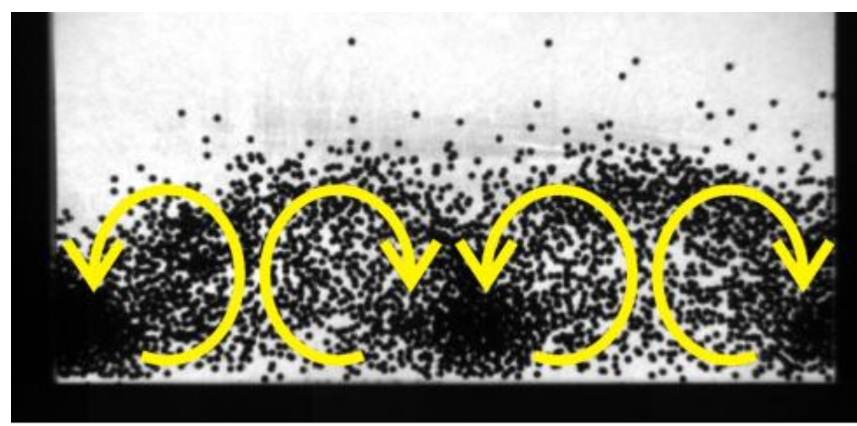
Vibration Driven Granular Matter



Leidenfrost

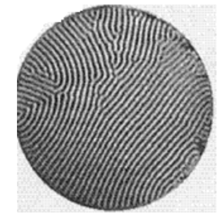
Eshuis et al 2005

Convection
Eshuis et al 2007



Order parameter models for granular Faraday patterns

Patterns can be predicted by the complex Ginzburg-Landau Eqn
(Tsimring and Aranson 1997)



$$\frac{\partial \psi}{\partial t} = \gamma \psi^* - (1 - i\omega)\psi + (1 + ib)\nabla^2 \psi - |\psi|^2 \psi - \rho\psi$$

ψ : complex amplitude of subharmonic pattern (order parameter)

ρ: thickness of the granular layer

$\gamma\psi^*$: parametric driving

γ : normalized amplitude

ω:frequency of driving

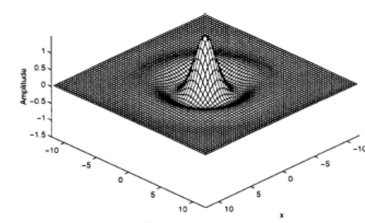
b:ratio of dispersion to diffusion

"Phenomenological model"

Swift-Hohenberg equation describes primary pattern forming bifurcation: square, strips and oscillons *(Crawford and Riecke 1999)*

$$\frac{\partial \psi}{\partial t} = R\psi - \left(\nabla^2 + 1\right)^2 \psi + N(\psi)$$

$$N(\psi) = b_1 \psi^3 - b_2 \psi^5 + \varepsilon \nabla.(\nabla \psi)^3 - \beta_1 \psi (\nabla \psi)^2 - \beta_2 \psi^2 \nabla^2 \psi$$



Landau-type order parameter model for granular patterns?

Granular Hydrodynamic Equations

(Savage, Jenkins, Goldhirsch, …)

Balance Equations

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\mathbf{u}$$

$$\rho\frac{D\mathbf{u}}{Dt} = -\nabla\cdot\boldsymbol{\Sigma}$$

$$\frac{3}{2}\rho\frac{DT}{Dt} = -\nabla\cdot\mathbf{q} - \boldsymbol{\Sigma}:\nabla\mathbf{u} - \mathcal{D}$$

- $\rho = \rho_p\phi$: Bulk density
  - ρ_p : Particle density
  - ϕ : Volume fraction
- $\mathbf{u}$: Bulk velocity
- T : Granular temperature

Navier-Stokes-order Constitutive Model

Stress Tensor

$$\boldsymbol{\Sigma} = [p(\phi,T) - \zeta(\phi,T)\nabla\cdot\mathbf{u}]\,\mathbf{I} - 2\mu(\phi,T)\mathbf{S}$$

$$\mathbf{S} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T) - \frac{1}{3}(\nabla\cdot\mathbf{u})\,\mathbf{I}$$

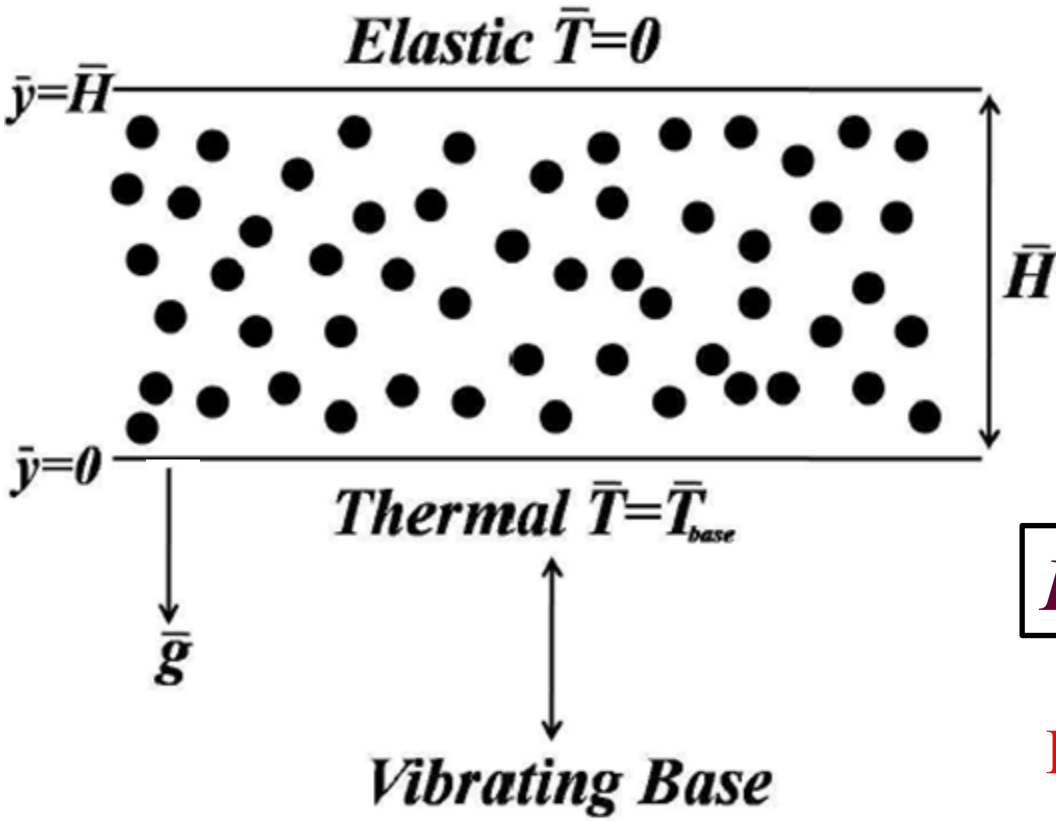
Granular Heat Flux

$$\mathbf{q} = -\kappa(\phi,T)\nabla T$$

Dissipation term or sink of energy

$$\mathcal{D} = \frac{\rho_p}{d}f_5(\phi,e)T^{3/2} \sim (1-e^2)$$

`Bounded' Convection



Reference Scales

•Length:
Gap between two walls

•Number Density:
Average number density

•Temperature:
Base temperature

Dimensionless Parameters

Froude Number $Fr = \dfrac{T_{base}}{mgH}$

Knudsen Number $K = \dfrac{2}{\sqrt{\pi}\langle n\rangle \bar{d}\bar{H}}$

Heat Loss Parameter $R = 4(1-e)K^{-2}$

Khain & Meerson 2003

Steady State + No Flow

Base flow: steady, fully developed flow

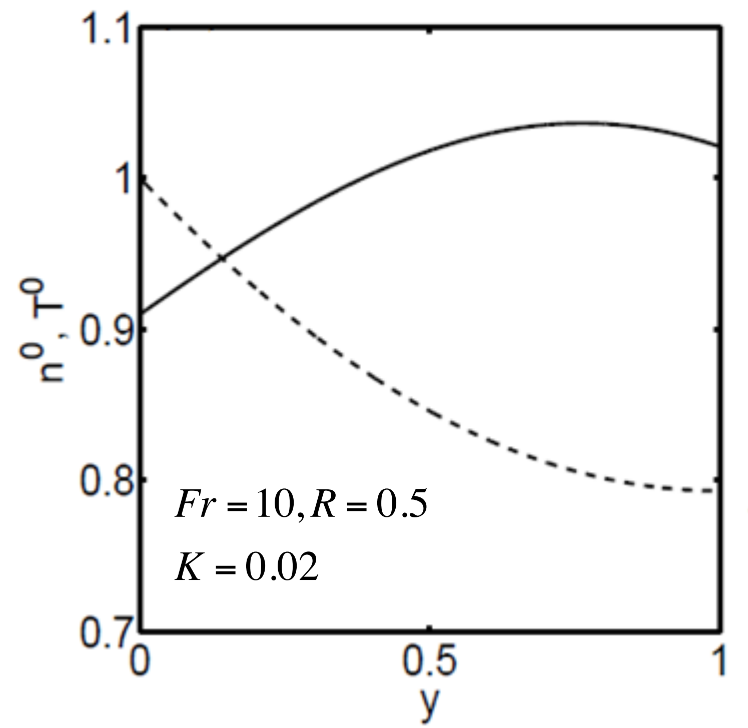
$$\frac{dp^0}{dy} + n^0 / Fr = 0, \quad \frac{d}{dy}\left(\kappa^0 \frac{dT^0}{dy}\right) = \boldsymbol{D}^0$$

Boundary Conditions

Thermal lower wall $\quad T^{(0)}(y=0)=1,$

Adiabatic upper wall $\quad dT^0(y=1)/dy = 0$

Integral relation $\quad \int_0^1 n^0(y)\,dy = 1$



Linear Stability

$$\frac{\partial X'}{\partial t} = LX' + N_2 + N_3 + \ldots \quad \text{where} \quad X = X_{base} + X'$$

Boundary Conditions

$$\frac{\partial u'}{\partial y} = 0, \; v' = 0 \text{ at } y = 0, 1$$

$$\frac{\partial T'}{\partial y} = 0 \text{ at } y = 1 \textit{ and } T' = 0 \textit{ at } y = 0$$

$$\frac{\partial X'}{\partial t} = LX'$$

Linear Problem

⇓

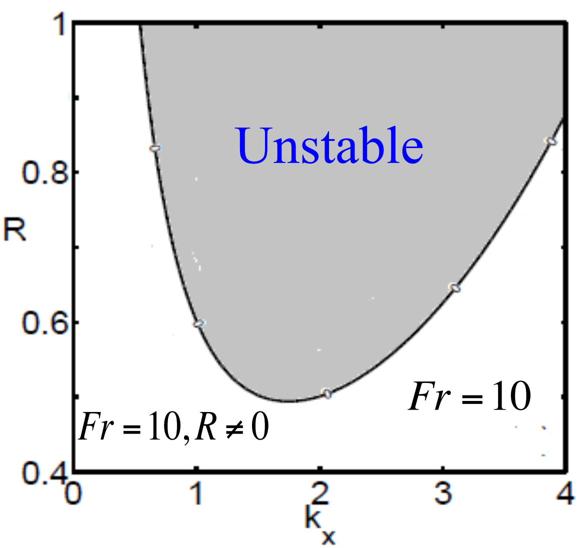
Normal Mode

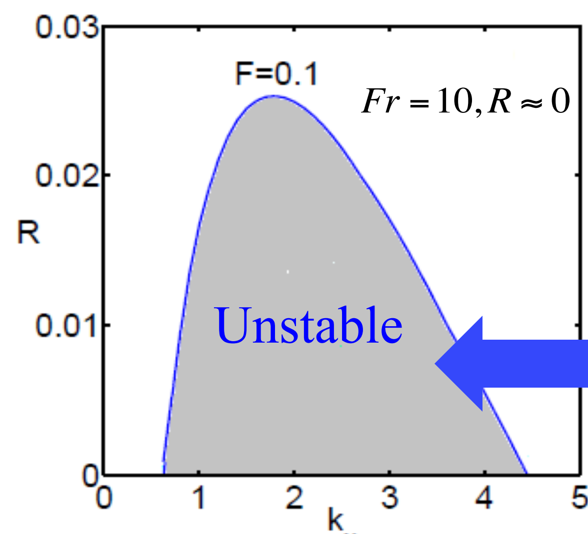
$$L\hat{X} = \omega \hat{X}$$

$$B^{\pm}\hat{X} = 0$$

Eigenvalue Problem

Khain & Meerson 2003





$$R = 4(1-e)K^{-2}$$

$$Fr = \frac{T_{base}}{mgH}$$

New Modes
(Shukla and Alam 2013)

Nonlinear Stability:
Center Manifold Reduction *(Carr 1981; Shukla & Alam, PRL 2009)*

Dynamics close to critical situation is dominated by finitely many "critical" modes.

$$X' = \phi + \psi$$

Disturbance; Critical Mode; Non-Critical Mode

$$\phi = Z X^{[1;1]} + \tilde{Z}\tilde{X}^{[1;1]}$$

Amplitude; Linear Eigenvector

Z : complex amplitude of finite-size perturbation

$$\left(\frac{\partial}{\partial t} - L\right)\phi = N_2 + N_3 \quad\longrightarrow\quad \left(\frac{\partial}{\partial t} - \omega\right) Z X_{11} = N_2 + N_3$$

$$\left(\frac{\partial}{\partial t} - L\right)\psi = N_2 + N_3 \qquad\qquad \left(\frac{\partial}{\partial t} - L\right)\psi = N_2 + N_3$$

Taking the inner product of slow mode equation with adjoint eigenfunction of the linear problem and separating the like-power terms in amplitude, we get an amplitude equation

$$c^{(0)} = a^{(0)} + ib^{(0)} = \omega$$

$$\left(\frac{\partial}{\partial t} - \omega\right) Z X_{11} = N_2 + N_3 \longrightarrow \frac{dZ}{dt} = c^{(0)} Z + c^{(2)} Z|Z|^2 + c^{(4)} Z|Z|^4 + ..$$

First Landau Coefficient

$$c^{(2)} = a^{(2)} + ib^{(2)}$$

Second Landau Coefficient

$$c^{(4)} = a^{(4)} + ib^{(4)}$$

Cont…

Adjoint — Distortion of mean flow — Second harmonic

$$c^{(2)} = \frac{\langle Y,\ N_2(X^{[0;2]}, X^{[1;1]}) + N_2(X^{[2;2]}, \tilde{X}^{[1;1]}) + N_3(X^{[1;1]}, X^{[1;1]}, \tilde{X}^{[1;1]})\rangle}{\langle Y, X^{[1;1]}\rangle}$$

$$\left(\frac{\partial}{\partial t} - L\right)\psi = \text{Nonlinear terms}$$

Slaved Equations

Represent all non-critical modes

$$c^{(4)} = \frac{\langle Y,\ \theta(X^{[1;1]}, X^{[0;2]}, X^{[2;2]}, X^{[1;3]}, X^{[3;3]}, X^{[2;4]}, X^{[0;4]})\rangle}{\langle Y, X^{[1;1]}\rangle}$$

Other perturbation methods can be used:

e.g. Amplitude expansion method (Shukla & Alam, 2011a, JFM)

Multiple scale analysis, (TDGL eqn., Saitoh & Hayakawa 2011)

Caution: Ignoring `Slaved' Equations will lead to qualitatively wrong result!

Equilibrium Amplitude and Bifurcation

Cubic Landau Eqn $Z = Ae^{i\theta}$

$$\frac{dZ}{dt} = c^{(0)}Z + c^{(2)}Z|Z|^2$$

$$\frac{dA}{dt} = a^{(0)}A + a^{(2)}A^3,$$

$$\frac{d\theta}{dt} = b^{(0)} + b^{(2)}A^2$$

Real amplitude eqn.

Phase eqn.

Cubic Solution

$$\frac{dA}{dt} = 0 \Rightarrow A = 0, \quad A = \pm\sqrt{-\frac{a^{(0)}}{a^{(2)}}}$$

Supercritical Bifurcation $a^{(0)} > 0, a^{(2)} < 0$

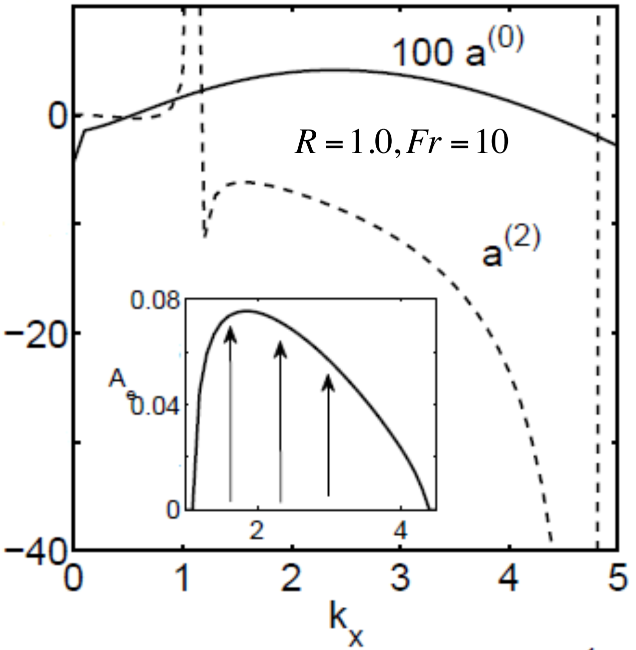
Subcritical Bifurcation $a^{(0)} < 0, a^{(2)} > 0$

$b^{(0)} = 0$ — Pitchfork (stationary) bifurcation

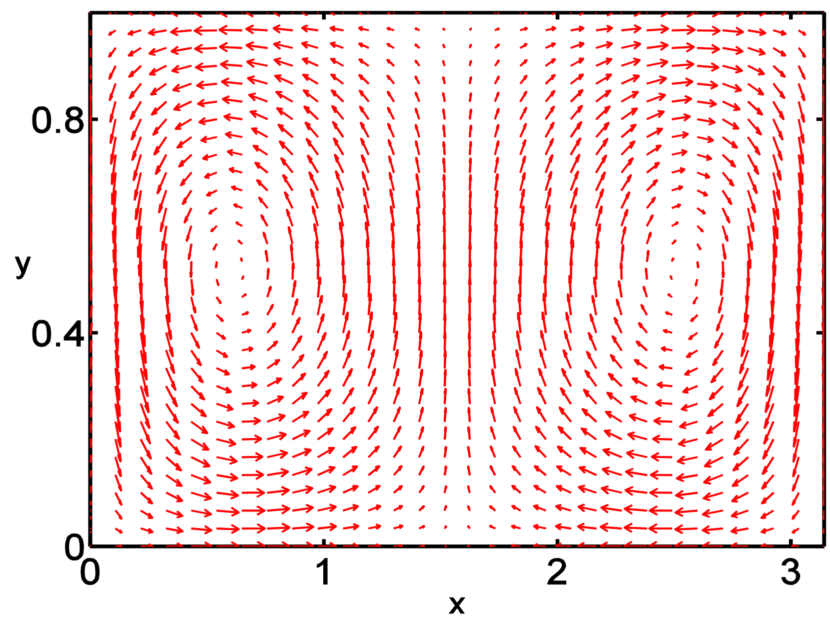
$b^{(0)} \neq 0$ — Hopf (oscillatory) bifurcation

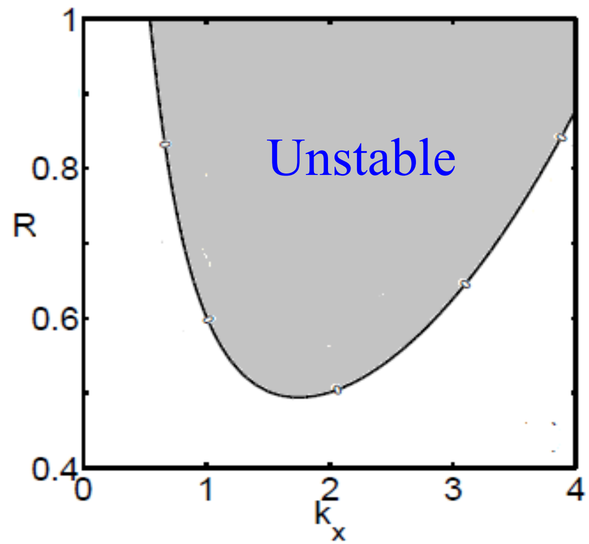
Results: Nonlinear Convection *(Shukla and Alam, 2013)*

Supercritical



$k_x = 2$





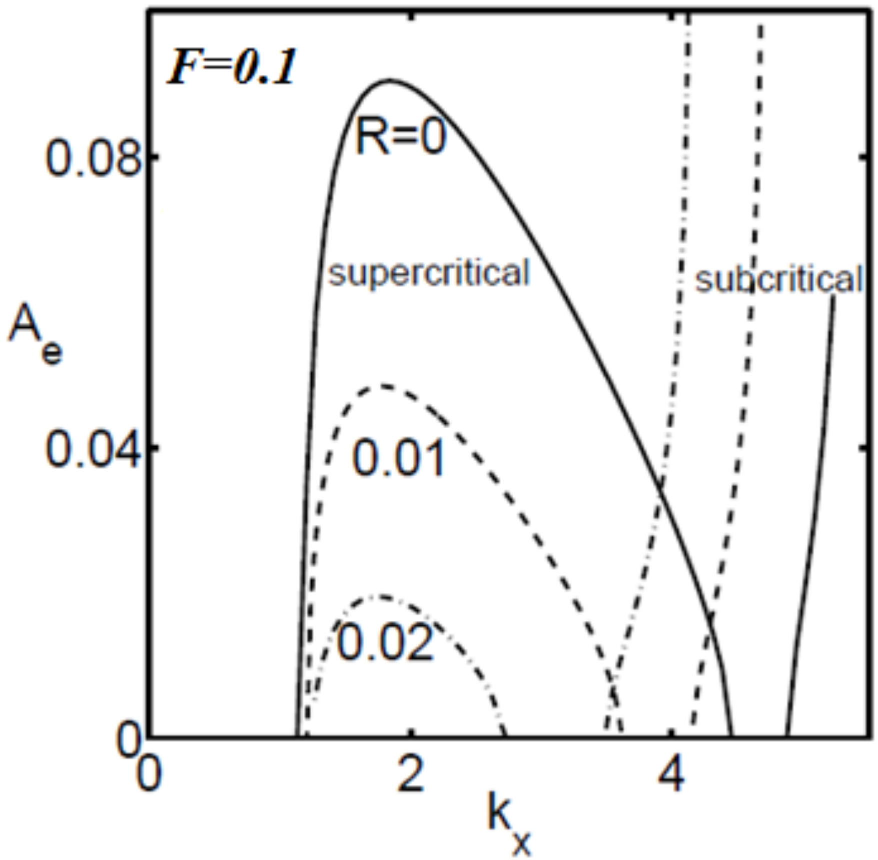
$R = 1$

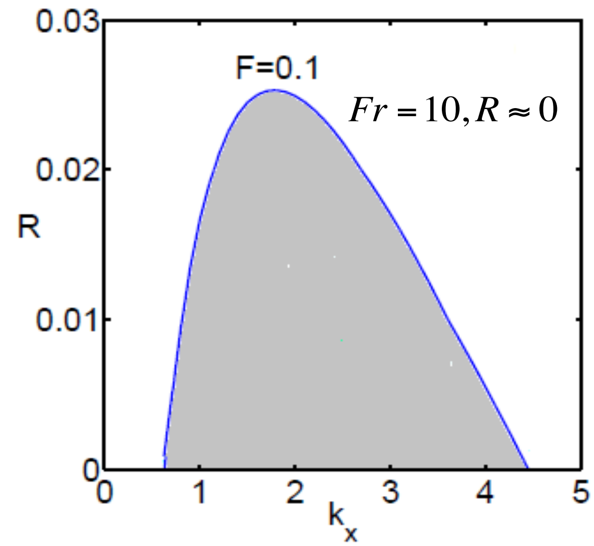
$Fr = 10$

$$\begin{cases} R = 4(1-e)K^{-2} \\ Fr = \frac{T_{base}}{mgH} \end{cases}$$

Elastic and quasi-elastic collisions $(R \approx 0)$

$$\begin{cases} R = 4(1-e)K^{-2} \\ Fr = \frac{T_{base}}{mgH} \end{cases}$$

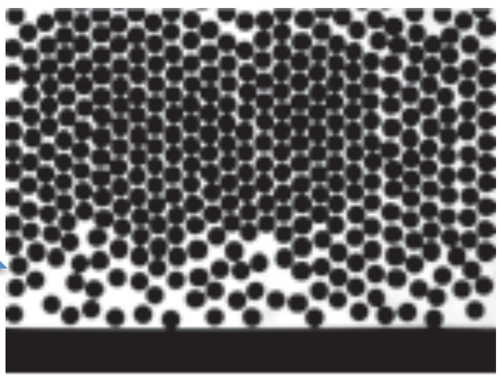




"Subcritical" and "supercritical" bifurcations in "elastic" limit

→ **Classical Rayleigh-Benard Convection**

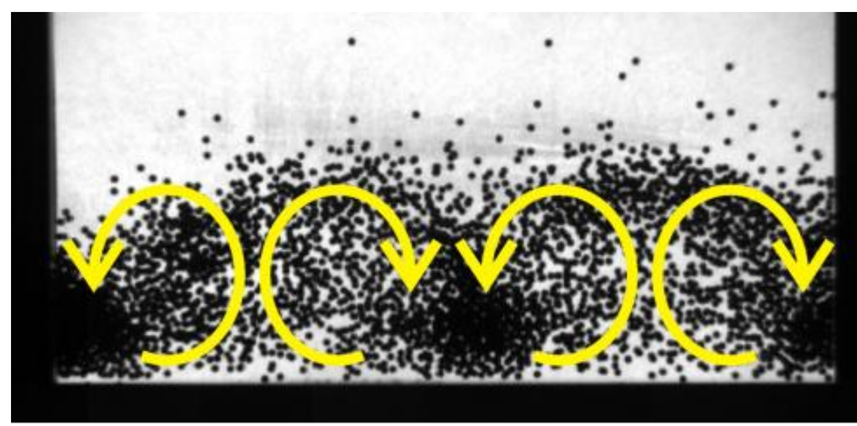
``Leidenfrost State" to ``Convection"



Leidenfrost

(Eshuis et al 2005)

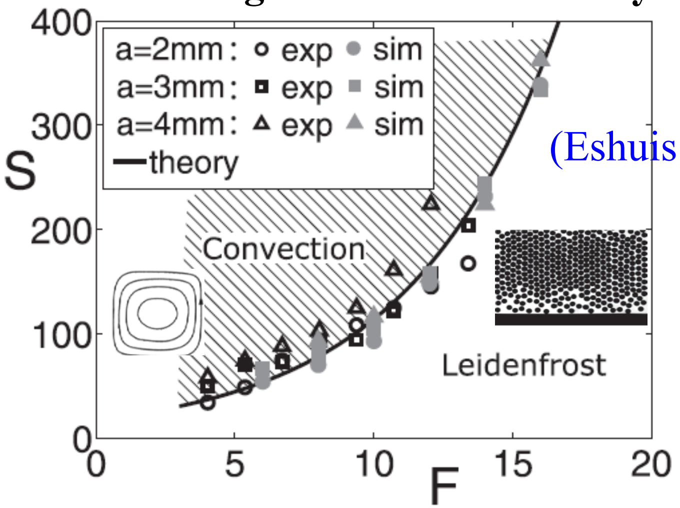
Convection
(Eshuis et al 2007)



Comparison of density patterns from Experiment, Simulation and Nonlinear Theory

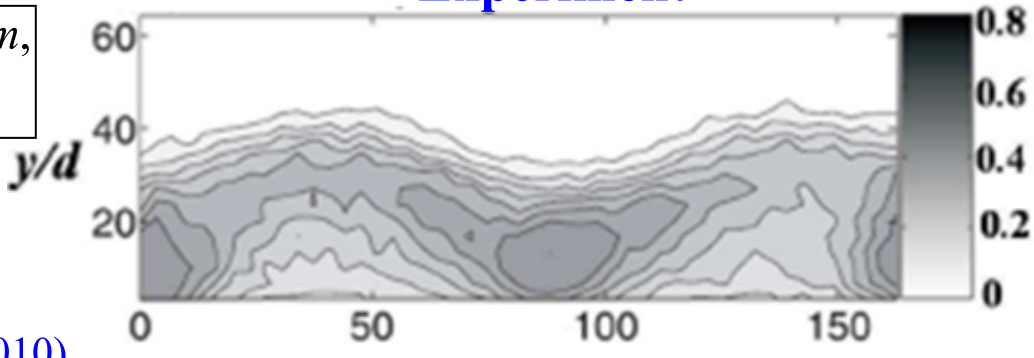
$$F = 6.2, L = 164, a = 4.0mm,$$
$$f = 52Hz \; or \; (S = 174)$$

Phase diagram: Linear Stability

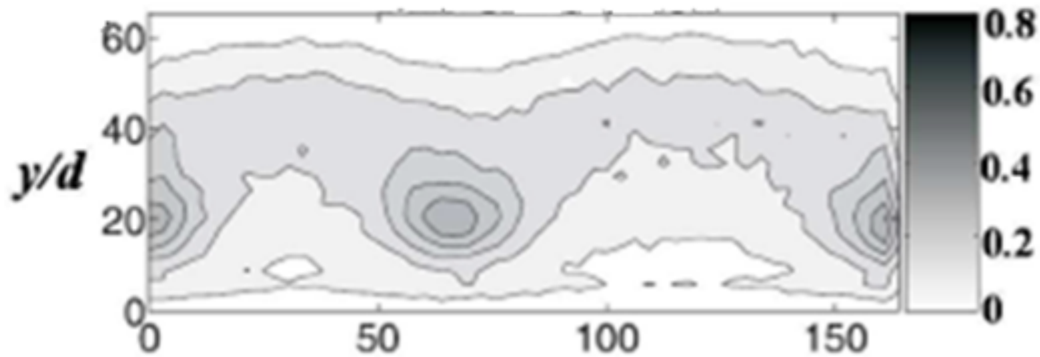


(Eshuis et al 2010)

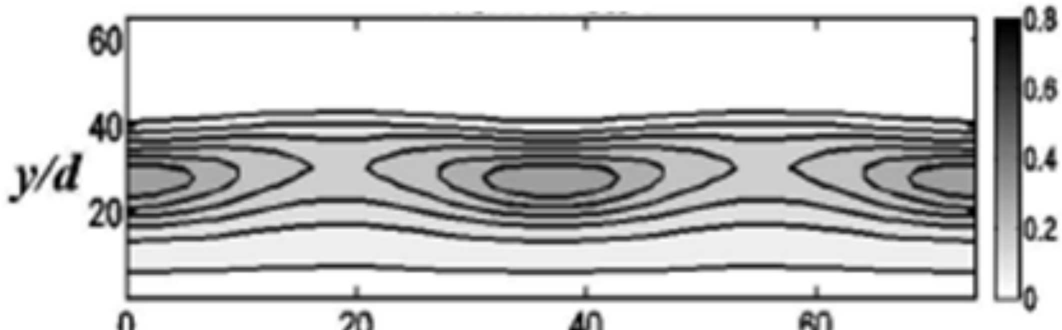
Experiment



Particle Simulation



Nonlinear Solution



$F = 6, S = 170, k_x = 0.17$

Convection rolls are subcritical or supercritical?

Shukla, van der Meer, Lohse and Alam, (2013, Preprint)

Conclusions

- ``Double'' roll (subcritical solution) convection
  (needs verification from simulation)

- New solutions in the quasi-elastic limit
  (related to classical Rayleigh-Benard convection)

- For semibounded convection, theory agrees with experiment and simulation (qualitatively)

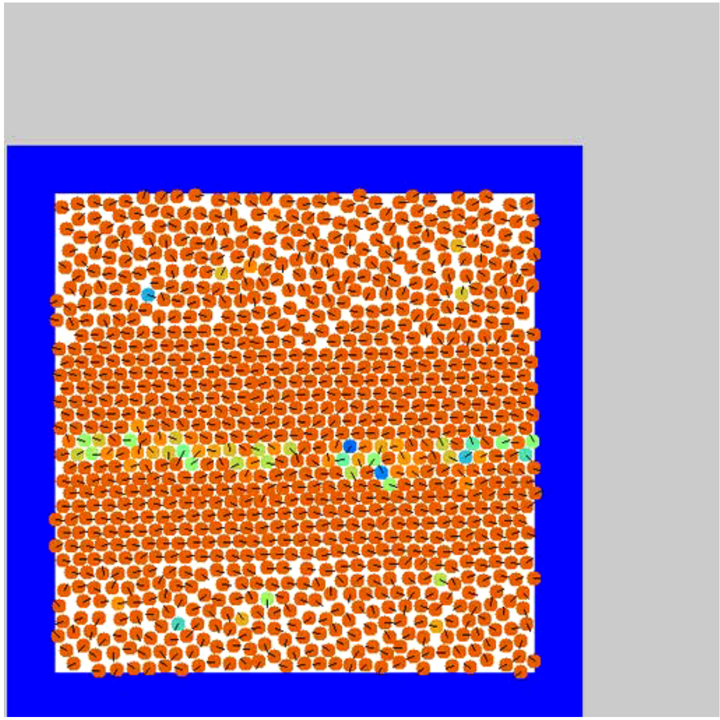
References

Shukla & Alam (2013) Preprint

Shukla , van der Meer, Lohse & Alam (2013), Preprint

Thank you

Gradient and Vorticity-Banding Phenomena in a Sheared Granular Fluid



Meheboob Alam
(with Priyanka Shukla)

Outline of Talk

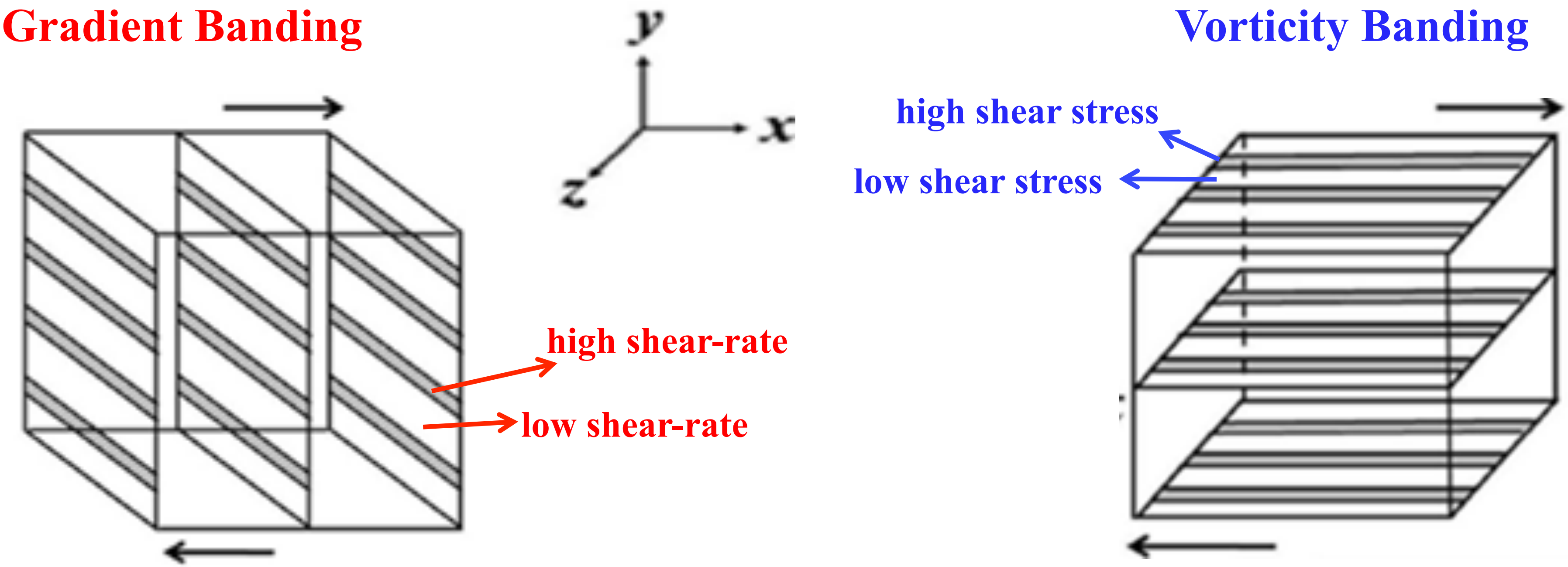
- **Shear-Banding?**
- **Granular Hydrodynamic Equations**
- **Sturat-Landau Equation**
- **Results for Gradient Banding**
- **Results for Vorticity Banding**
- **Summary**

Shear-banding: A misnomer?

Homogeneous/uniform shear flow is **unstable** above some critical applied **shear-rate** or **shear stress** (Hoffman 1972, Olmsted 2008).

Flow becomes inhomogeneous/non-uniform characterized by **coexisting-bands** of different **shear-rate** or **shear stress** (rheological properties).

Gradient Banding **Vorticity Banding**



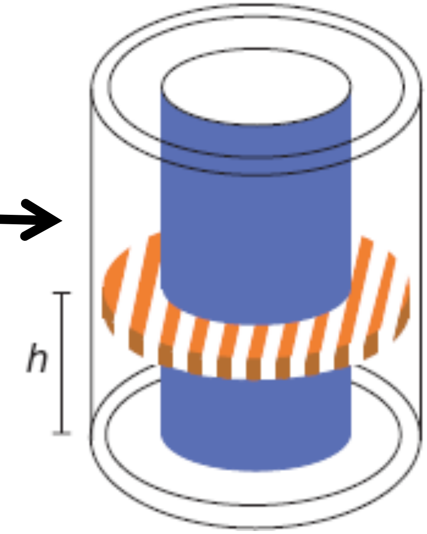
Shear-Cell Experiments

Shear-Banding in `Dense' Granular Flow

(Savage & Sayed 1984; Mueth et.al. 2000)

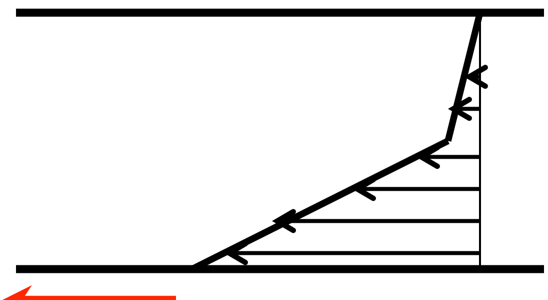
Couette cell

Particle tracking

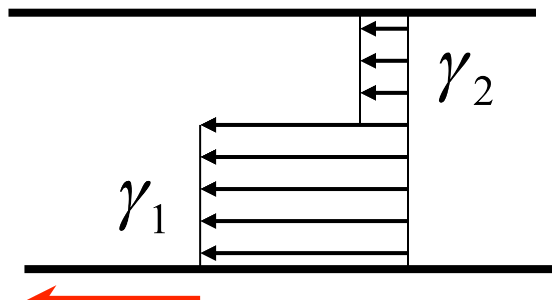


- Granular material does not flow homogeneously like a fluid, but usually forms solid-regions that are separated by ``narrow'' bands where material yields and flow.

- Shear-bands are narrow and localized near moving boundary.

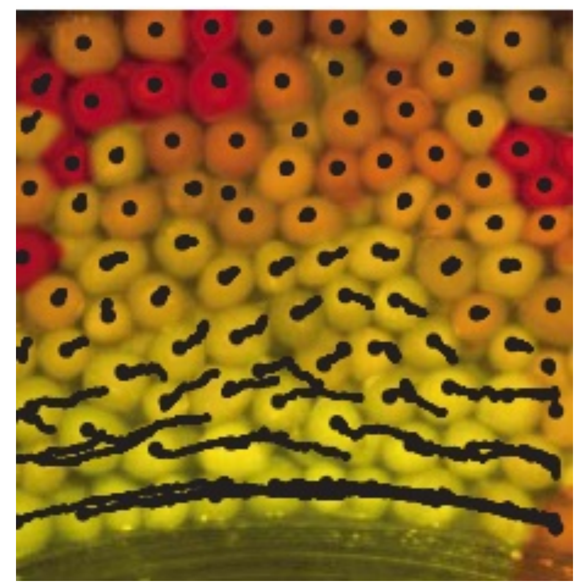


Velocity profile of shear-banded state



Two different shear rates

Fast particles (yellow) near the inner wall appear to move smoothly while the orange and red particles display more irregular and intermittent motion



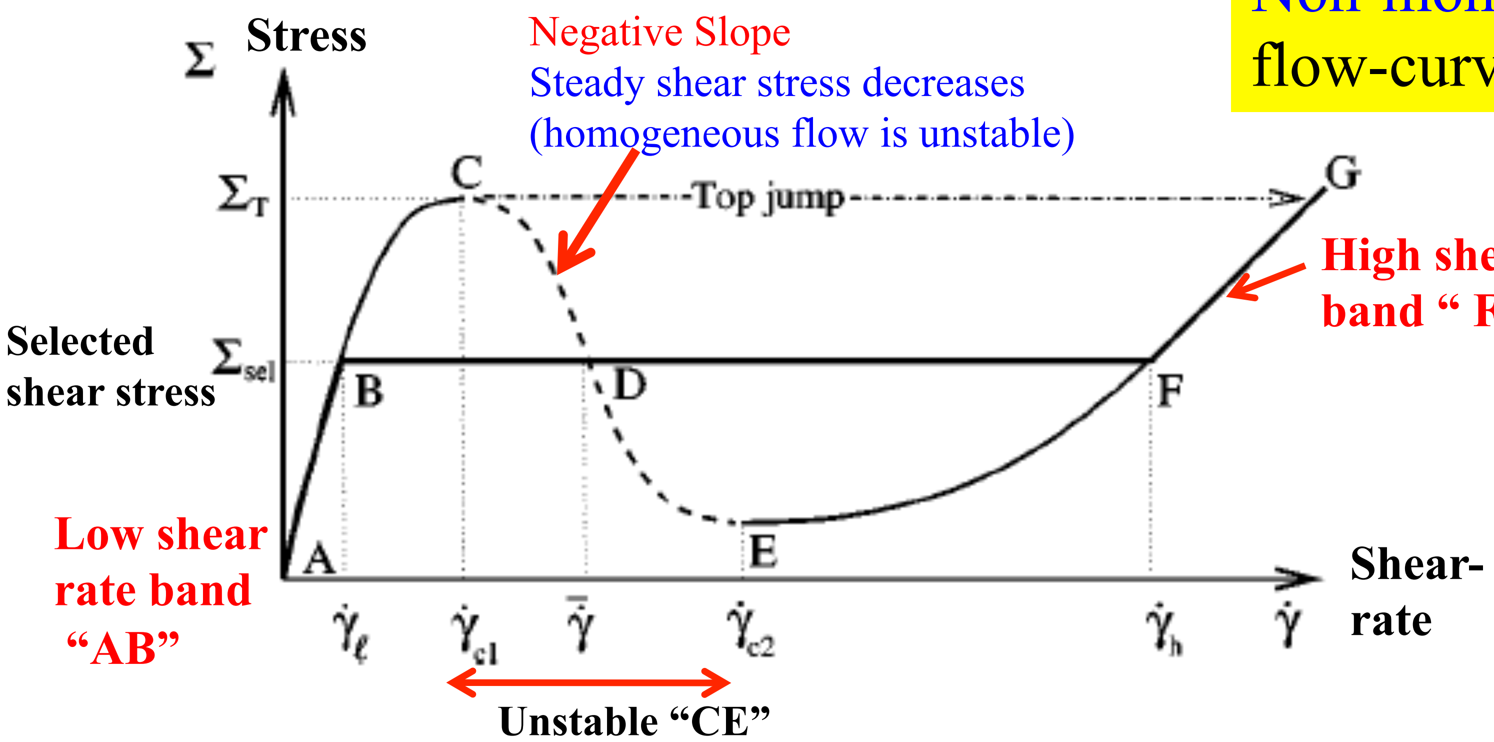
Mueth et al. 2000

Rheological Signature of Banded States

- Multiple(*) Branches of flow-curve

- **Gradient Banding :** Shear-rate > critical shear-rate

Non-monotonic flow-curve



* Banding also occurs for monotonic flow-curves (Olmsted 2008)

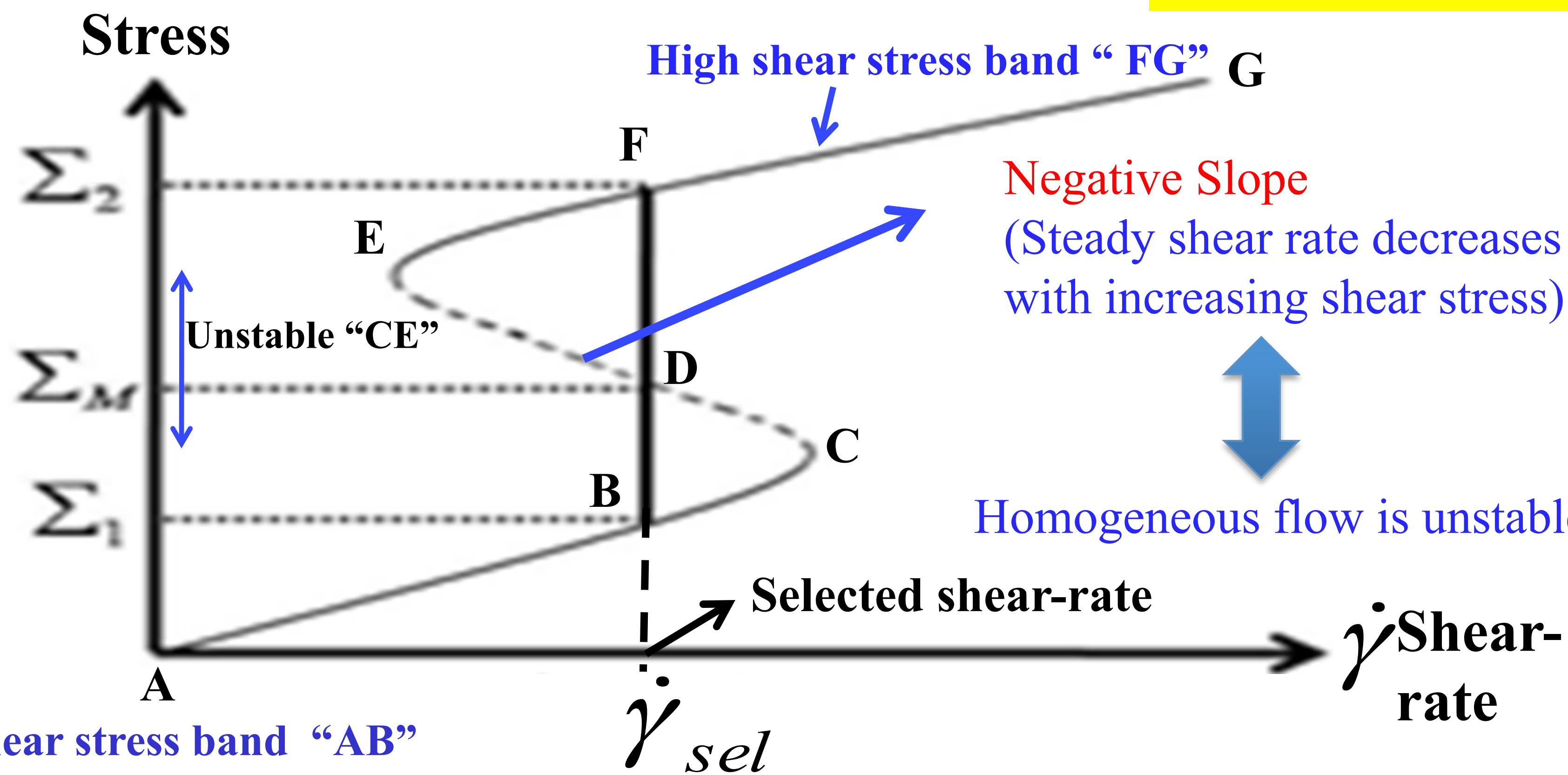
Rheological Signature….

Multiple Branches in flow curve

Vorticity Banding

shear stress > critical shear stress

Non-monotonic flow curve

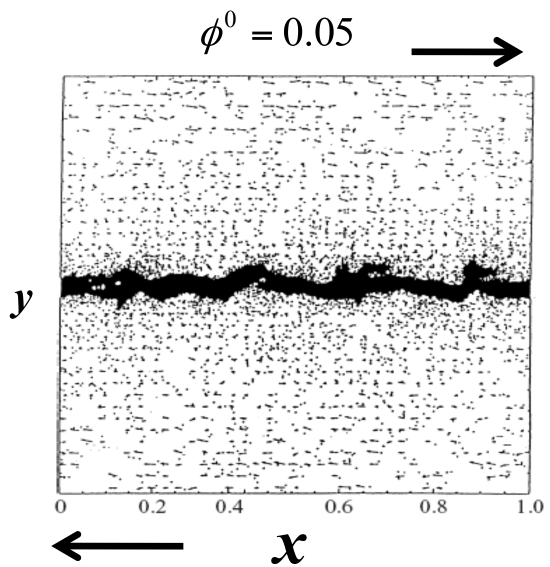


Negative Slope
(Steady shear rate decreases with increasing shear stress)

Homogeneous flow is unstable

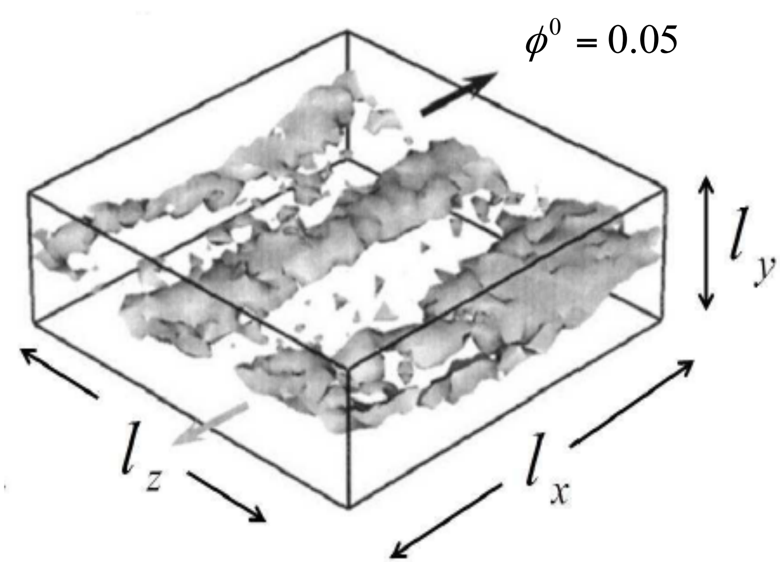
Particle Simulations

Gradient banding
in 2-dimensional granular shear flow at low density



Tan & Goldhirsch 1997

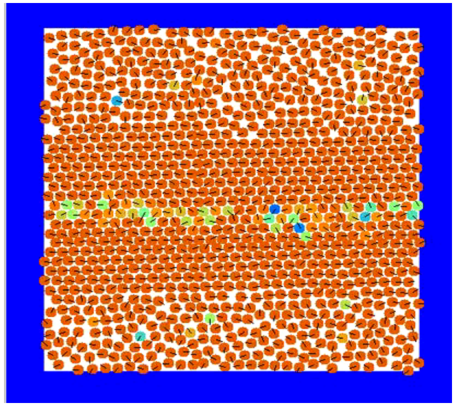
Vorticity banding
in 3-dimensional granular shear flows at low density



Conway & Glasser 2004

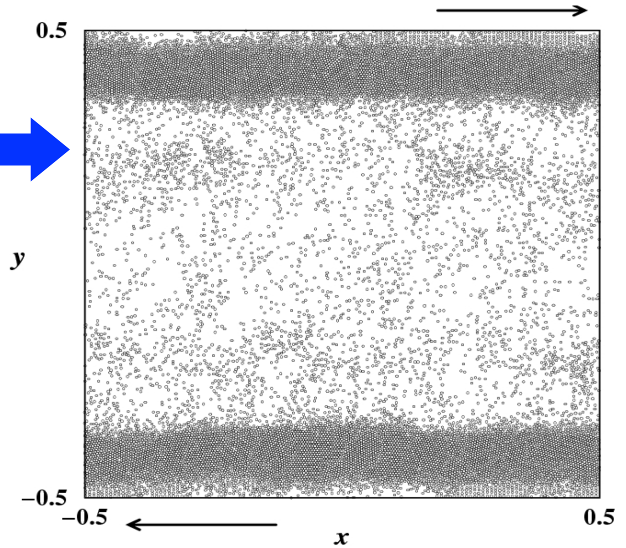
Gradient Banding in Granular Shear Flow

Plane Couette Flow



$\phi = 0.8$

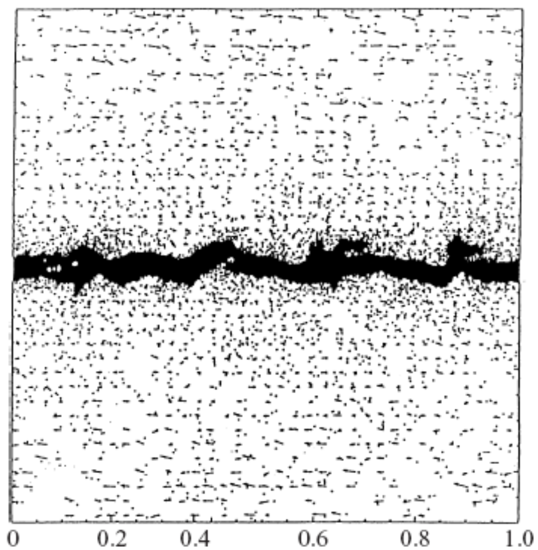
(Alam 2003)

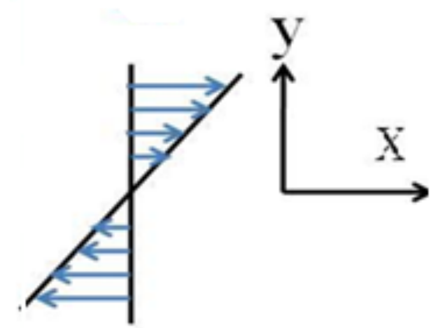


$\phi = 0.3$ (Alam 2003)

$\phi = 0.05 \longrightarrow$

(Tan & Goldhirsch 1997)





Order-Parameter Description Of gradient-banding?

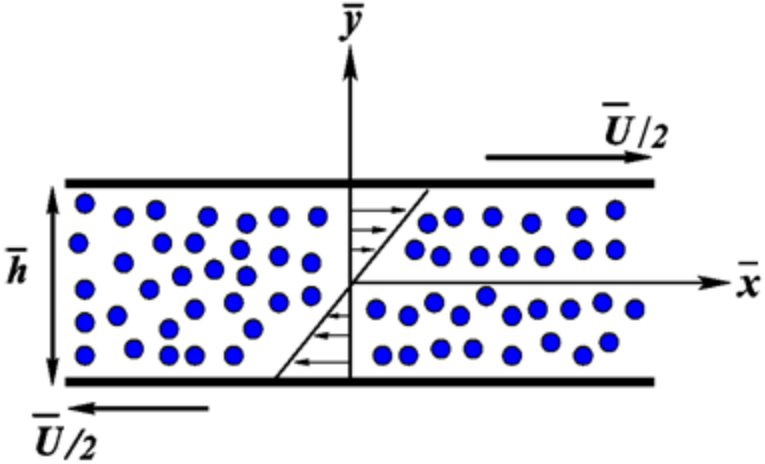
$$\frac{dA}{dt} = f(A,t)$$

$$\frac{\partial A}{\partial t} + a_1 \frac{\partial A}{\partial x} + a_2 \nabla^2 A = g(A,t,x,\cdots)$$

Order-parameter description of gradient-banding?

Shukla & Alam (2009, 2011, 2012)

Uniform Shear Flow (homogeneous state)



- Reference Length: $\overline{h}$
- Reference Velocity: $\overline{U}$
- Reference Time: $\overline{h}/\overline{U}$

➢**uniform flow :** Steady, Fully developed.
➢**boundary condition** No-slip, zero heat flux

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) = 0, \quad \frac{\partial p}{\partial y} = 0$$

$$\frac{1}{H^2}\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 - \mathcal{D} = 0$$

Uniform Shear Solution

- $\phi(y) = \phi^0 = \text{const.}$
- $u(y) = y$
- $T(y) = T^0 = \text{const.}$

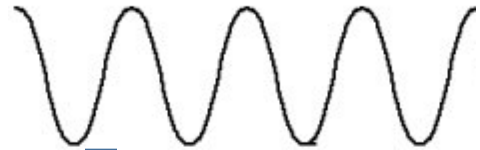
d : diameter

Control Parameters ⇨ • $H = \overline{h}/d, \quad \phi^0, \quad e$

Linear Stability Analysis

- $\phi(y) = \phi^0 = \text{const.}$
- $u(y) = y$
- $T(y) = T^0 = \text{const.}$

+ Finite-size Perturbation (X')



$= X_{total}$

Gradient Banding

$$\frac{\partial}{\partial x}(.) = 0, \frac{\partial}{\partial z}(.) = 0, \frac{\partial}{\partial y}(.) \neq 0$$

Vorticity Banding

$$\frac{\partial}{\partial x}(.) = 0, \frac{\partial}{\partial y}(.) = 0, \frac{\partial}{\partial z}(.) \neq 0$$

If the disturbances are infinitesimal 'nonlinear terms' of the perturbation eqns. can be 'neglected'.

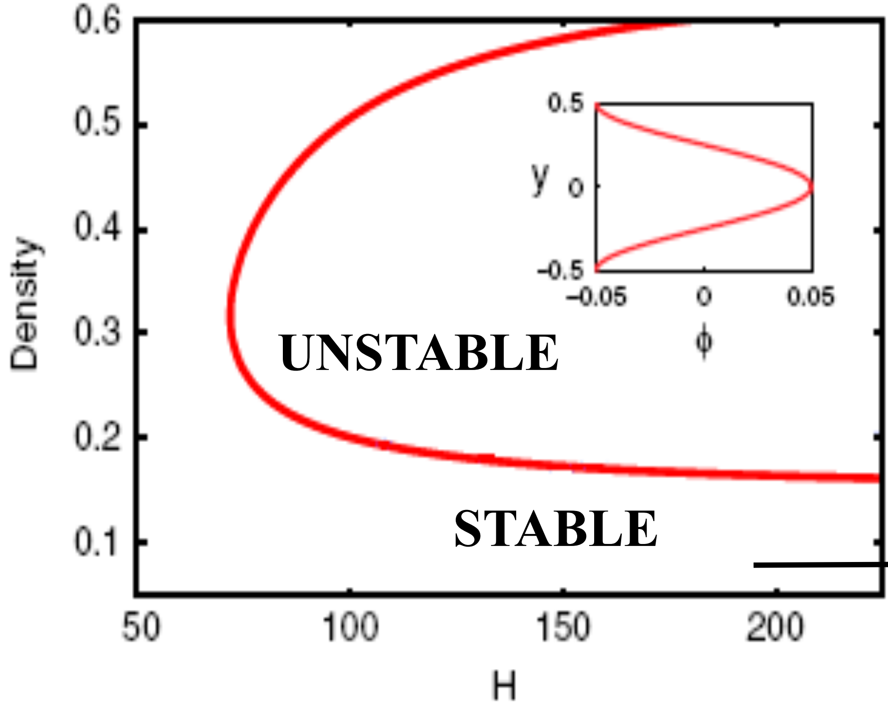
$$\frac{\partial X'}{\partial t} = LX'$$ Linear Problem

Normal Mode

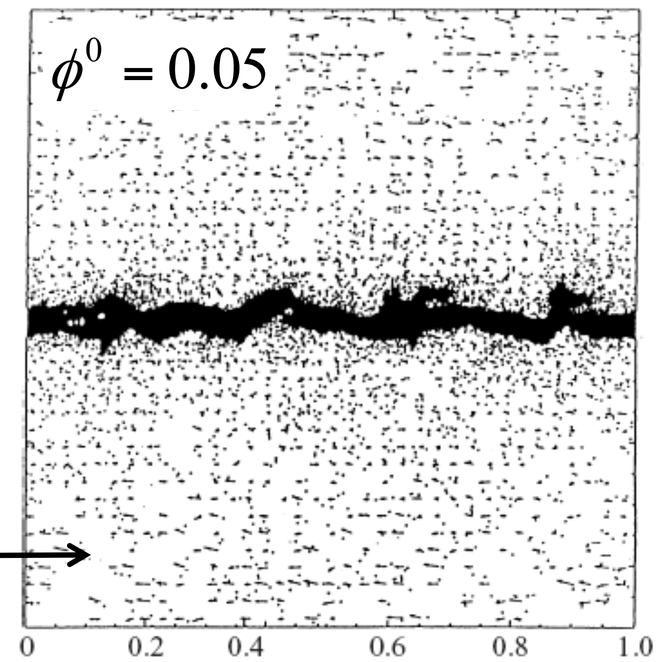
$$L\hat{X} = \omega \hat{X}$$
$$B^{\pm}\hat{X} = 0$$
Eigenvalue Problem

Linear stability theory fails in `dilute' limit!

Linear Theory



Particle Simulation



Tan & Goldhirsch 1997 Phys. Fluids, 9

Flow remains 'uniform' in dilute limit

↓

Density segregated solutions are not possible in dilute limit

Flow is 'non-uniform' in dilute limit

↓

Density Segregated solutions are possible in dilute limit

Nonlinear Analysis:

Center Manifold Reduction *(Carr 1981; Shukla & Alam, PRL 2009)*

Dynamics close to critical situation is dominated by finitely many "critical" modes.

Z(t) : complex amplitude of finite-size perturbation

Disturbance, Critical Mode, Non-Critical Mode

$$X' = \phi + \psi$$

$$\phi = Z X^{[1;1]} + \tilde{Z}\tilde{X}^{[1;1]}$$

Amplitude, Linear Eigenvector

$$\left(\frac{\partial}{\partial t} - L\right)\phi = N_2 + N_3 \longrightarrow \left(\frac{\partial}{\partial t} - \omega\right) Z X_{11} = N_2 + N_3$$

$$\left(\frac{\partial}{\partial t} - L\right)\psi = N_2 + N_3 \qquad \left(\frac{\partial}{\partial t} - L\right)\psi = N_2 + N_3$$

Taking the inner product of slow mode equation with adjoint eigenfunction of the linear problem and separating the like-power terms in amplitude, we get an amplitude equation

$$\left(\frac{\partial}{\partial t} - \omega\right) Z X_{11} = N_2 + N_3 \longrightarrow \frac{dZ}{dt} = c^{(0)} Z + c^{(2)} Z|Z|^2 + c^{(4)} Z|Z|^4 + ..$$

$$c^{(0)} = a^{(0)} + i b^{(0)} = \omega$$

First Landau Coefficient

$$c^{(2)} = a^{(2)} + i b^{(2)}$$

Second Landau Coefficient

$$c^{(4)} = a^{(4)} + i b^{(4)}$$

Analytical Method

Shukla & Alam, J. Fluid Mech. (2011a)

Linear Problem $LX^{[1;1]} = c^{(0)}X^{[1;1]}$

Second Harmonic $L_{22}X^{[2;2]} = G_{22}$

Distortion to mean flow $L_{02}X^{[0;2]} = G_{02}$

Distortion to fundamental

$$L_{13}X^{[1;3]} = c^{(2)}X^{[1;1]} + G_{13}$$

Analytically solvable

Expression for first Landau coefficient

$$c^{(2)} = \frac{\phi^a G_{13}^1 + u^a G_{13}^2 + v^a G_{13}^3 + T^a G_{13}^4}{\phi^a \phi^{[1;1]} + u^a u^{[1;1]} + v^a v^{[1;1]} + T^a T^{[1;1]}}$$

Analytical solution exists at any order!

We have developed a spectral based numerical code to calculate Landau coefficients.

Equilibrium Amplitude and Bifurcation

Cubic Landau Eqn $Z = Ae^{i\theta}$

$$\frac{dZ}{dt} = c^{(0)}Z + c^{(2)}Z\,|\,Z\,|^2$$

$$\frac{dA}{dt} = a^{(0)}A + a^{(2)}A^3,$$ Real amplitude eqn.

$$\frac{d\theta}{dt} = b^{(0)} + b^{(2)}A^2$$ Phase eqn.

Cubic Solution

$$\frac{dA}{dt} = 0 \Rightarrow A = 0, \quad A = \pm\sqrt{-\frac{a^{(0)}}{a^{(2)}}}$$

Supercritical Bifurcation $a^{(0)} > 0, a^{(2)} < 0$

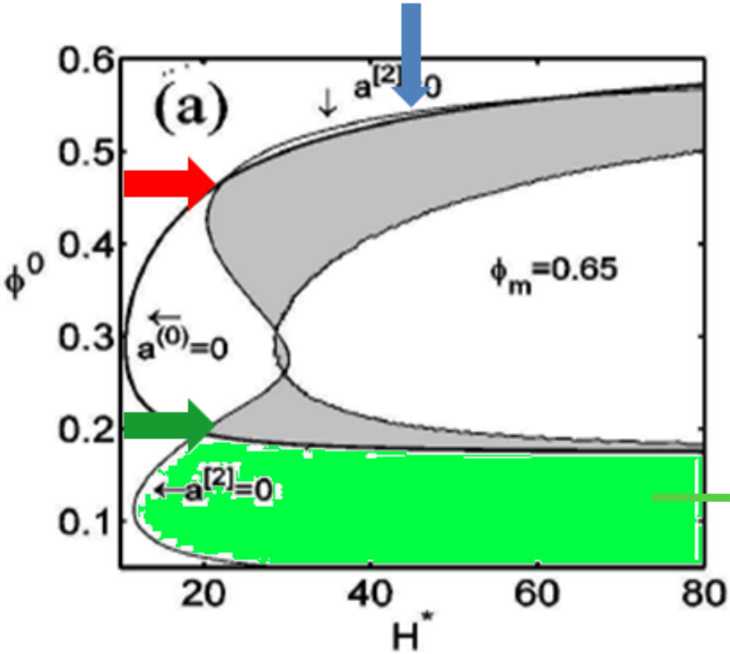
Subcritical Bifurcation $a^{(0)} < 0, a^{(2)} > 0$

$b^{(0)} = 0$: Pitchfork (stationary) bifurcation

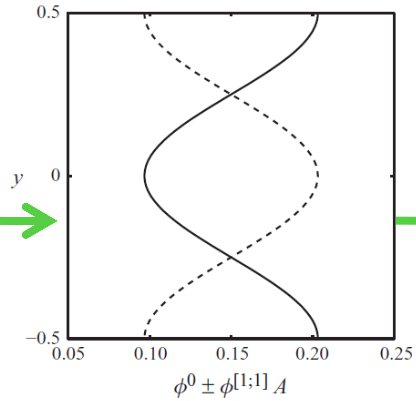
$b^{(0)} \neq 0$: Hopf (oscillatory) bifurcation

Phase Diagram

Shukla & Alam, J. Fluid Mech. (2011a)



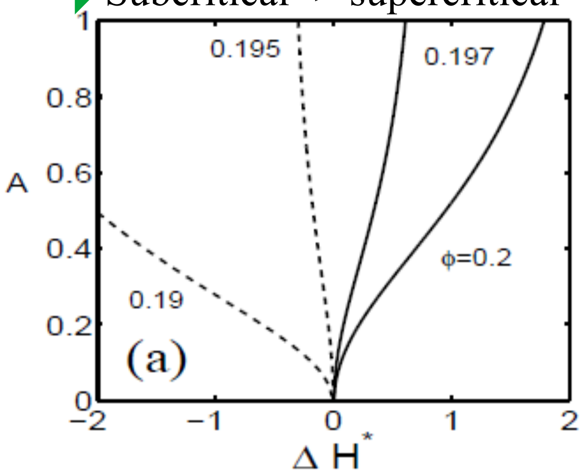
Nonlinear Stability theory and *MD simulations* both supports gradient banding in 2D-GPCF ***(PRL 2009)***



"Density Segregation
and
Shear Localization"
in dilute flows too !
(Tan & Goldhirsch 1997)

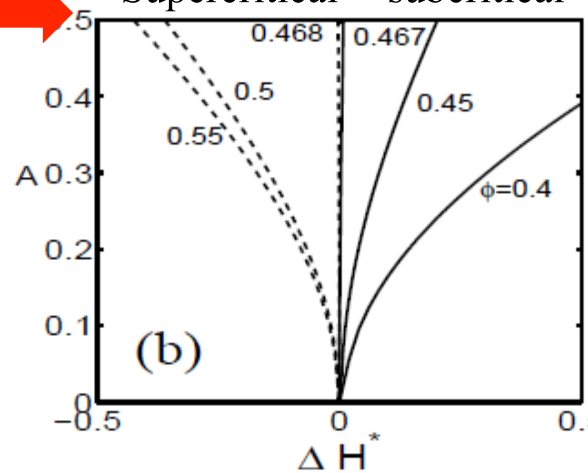
0.196

Subcritical -> supercritical



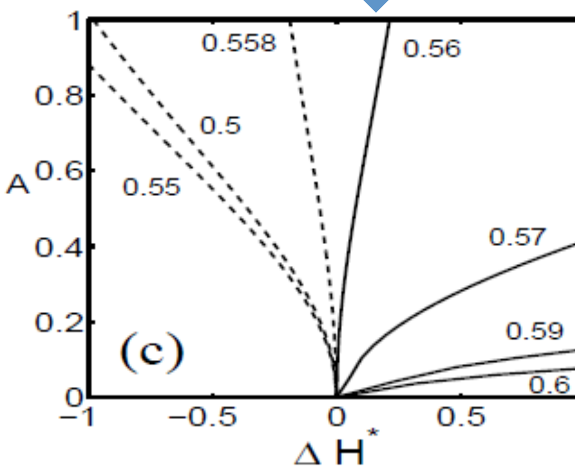
0.467

Supercritical-> subcritical



Subcritical -> supercritical

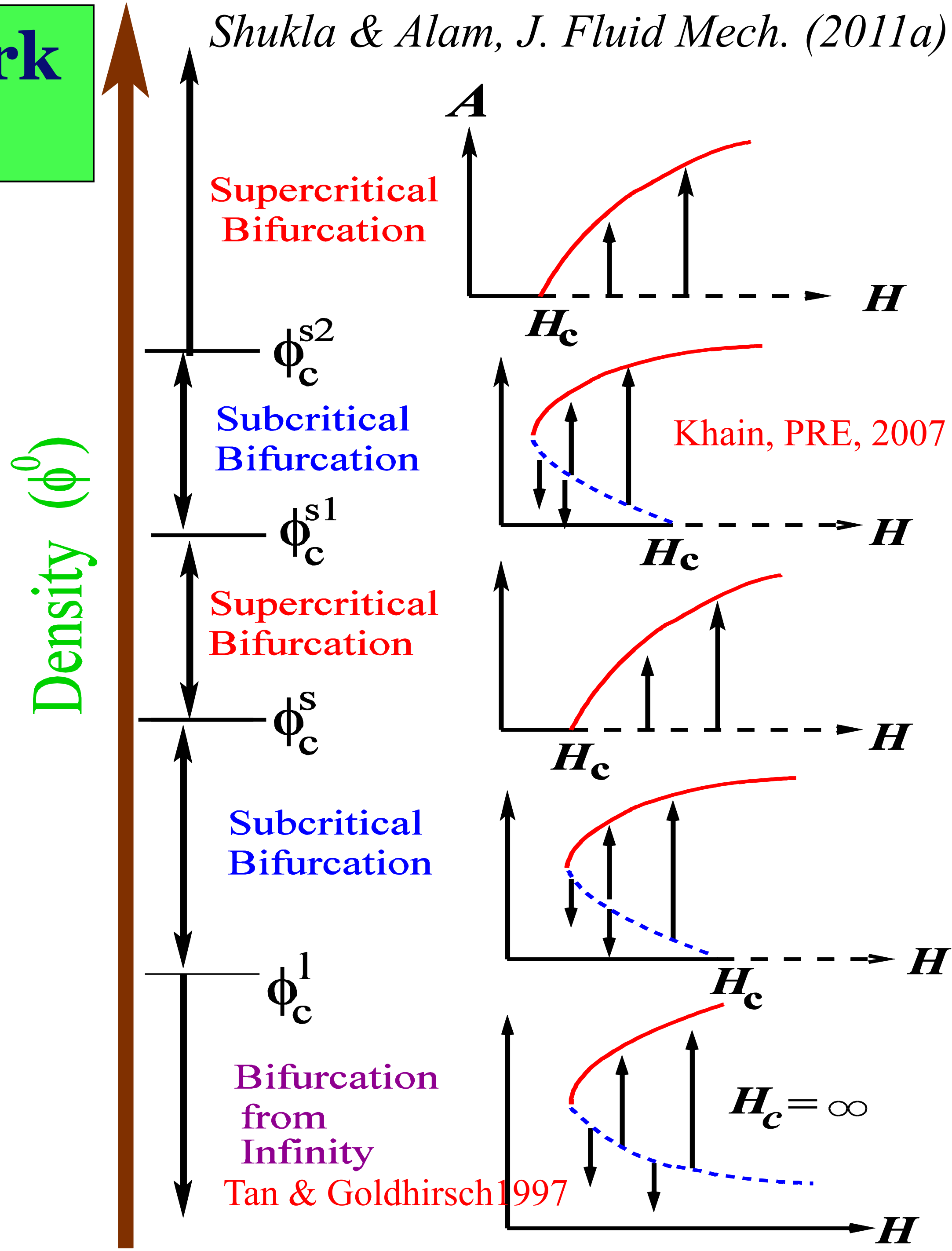
0.559



Paradigm of Pitchfork Bifurcations

- Supercritical:
  $\phi^0 > \phi_c^{s2}$
- Subcritical:
  $\phi^{s1} < \phi^0 < \phi_c^{s2} \approx 0.559$
- Supercritical:
  $\phi^s < \phi^0 < \phi_c^{s1} \approx 0.467$
- Subcritical:
  $\phi^l < \phi^0 < \phi_c^{sl} \approx 0.196$
- Bifurcation from infinity:
  $\phi^0 < \phi_c^l \approx 0.174$

Shukla & Alam, J. Fluid Mech. (2011a)



Conclusions from nonlinear gradient banding

- Problem is analytically solvable

- Landau coefficients suggest that there is a "sub-critical" (bifurcation from infinity) finite amplitude instability for "dilute" flows even though the dilute flow is stable according to linear theory.

- This result agrees with previous MD-simulation of granular plane Couette flow

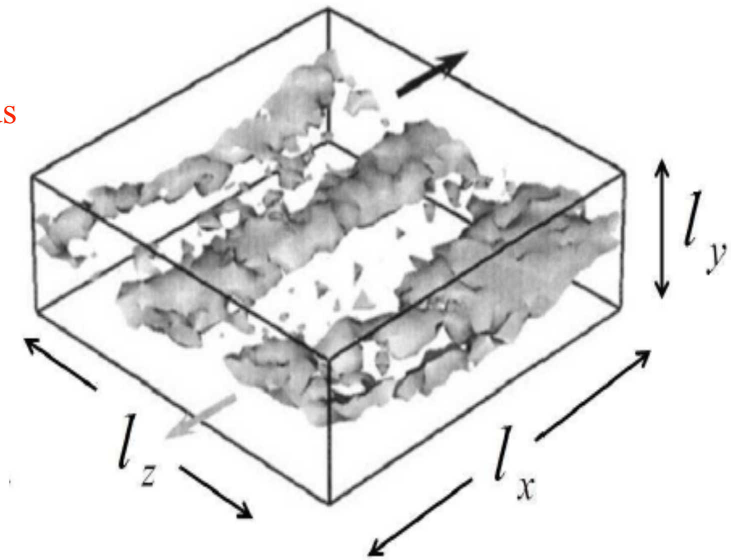
- GCF serves as a **paradigm** of pitchfork bifurcations.

- Gradient banding corresponds to shear localization and density segregation

- Origin of gradient banding is tied to lower dynamic friction (μ/p)

Vorticity Banding

Conway & Glasser, 2004, Phys. Fluids



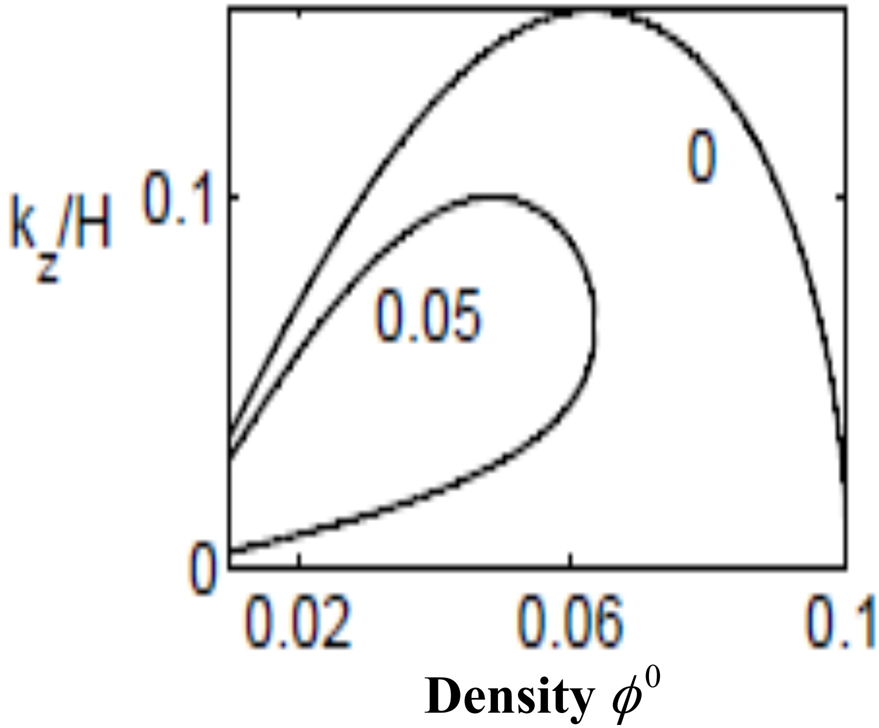
$$\frac{\partial}{\partial x}(.) = 0, \frac{\partial}{\partial y}(.) = 0, \frac{\partial}{\partial z}(.) \neq 0$$

$$\frac{dA}{dt} = c^{(0)}A + c^{(2)}A|A|^2 + c^{(4)}A|A|^4 + \cdots$$

Alam & Shukla, J. Fluid Mech. (2013a) vol 716

Shukla & Alam, J. Fluid Mech. (2013b) vol 718

Vorticity banding instability (linear)



$$\frac{\partial}{\partial x}(.) = 0, \frac{\partial}{\partial y}(.) = 0, \frac{\partial}{\partial z}(.) \neq 0$$

$$\phi^0 = \phi^{vb} \approx 0.1$$

$$k_{zc}^* = \frac{k_{zc}^{vb}}{H} \approx 0.15$$

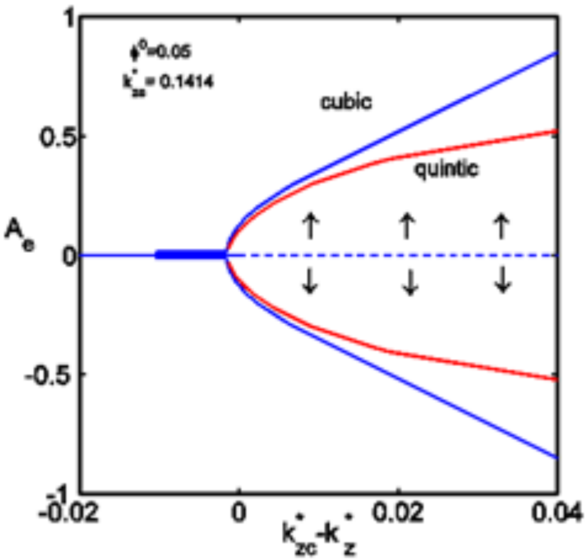
Gradient-banding modes stationary **modes in all the flow density regime.**

Vorticity banding modes stationary **in dilute density limits &** traveling **in moderately dense density limit**

Vorticity…. (nonlinear)

$$A_e = \pm\sqrt{\frac{-a^{(2)} \pm \sqrt{(a^{(2)})^2 - 4a^{(0)}a^{(4)}}}{2a^{(4)}}}$$

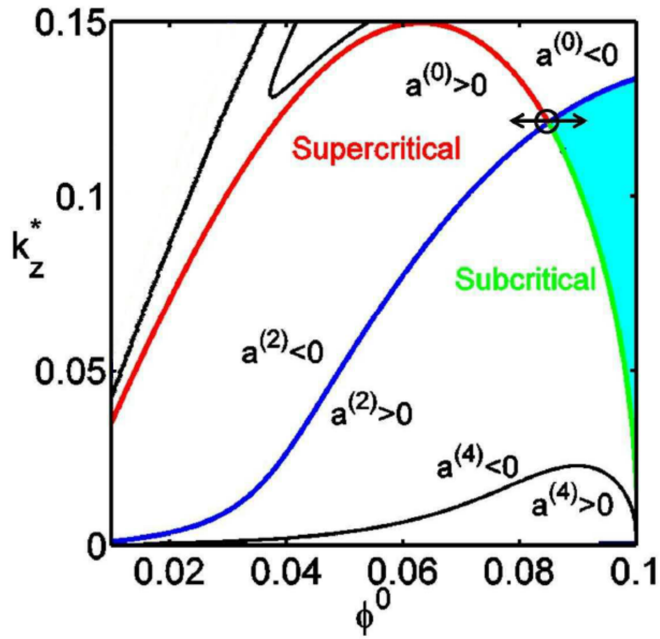
supercritical pitchfork



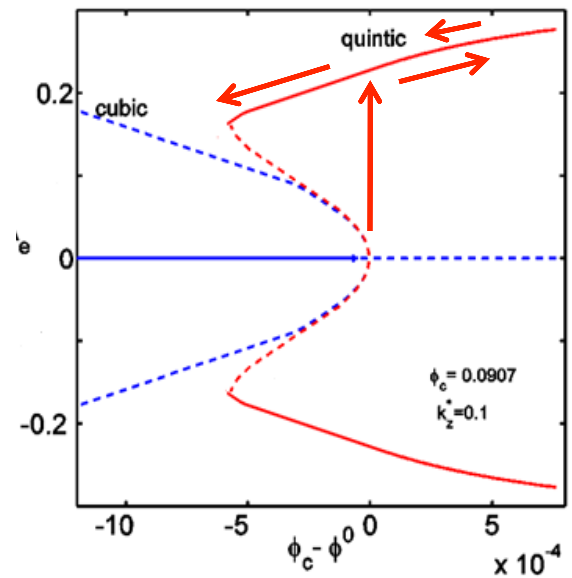
$(\phi^0 < \phi^{bcp})$

Dilute limit

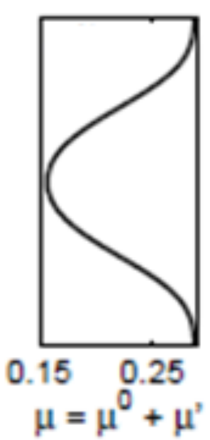
$$\phi^0 \leq \phi^{vb} \approx 0.1$$

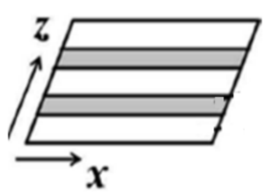


subcritical pitchfork



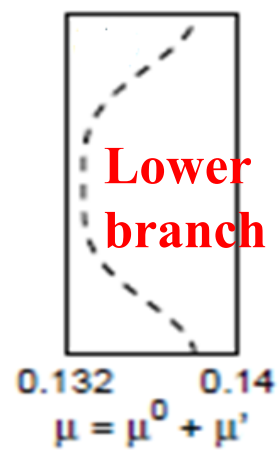
$(\phi^0 > \phi^{bcp})$

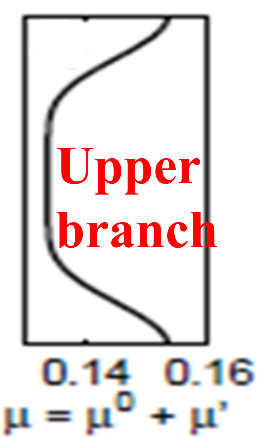




Shear Stress Localization

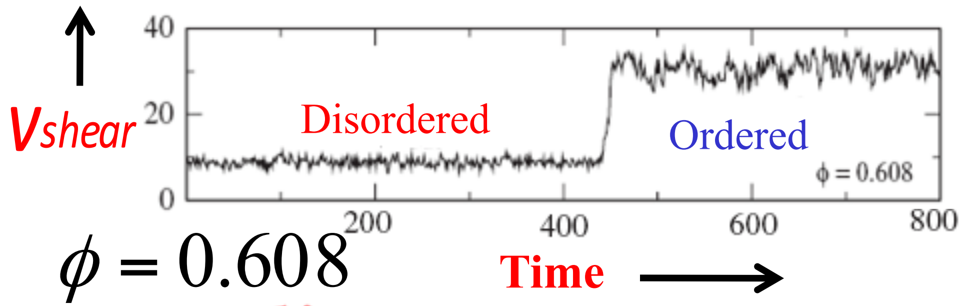
$$\mu = \mu^0 + \mu^0_\phi \phi' + \mu^0_T T'$$

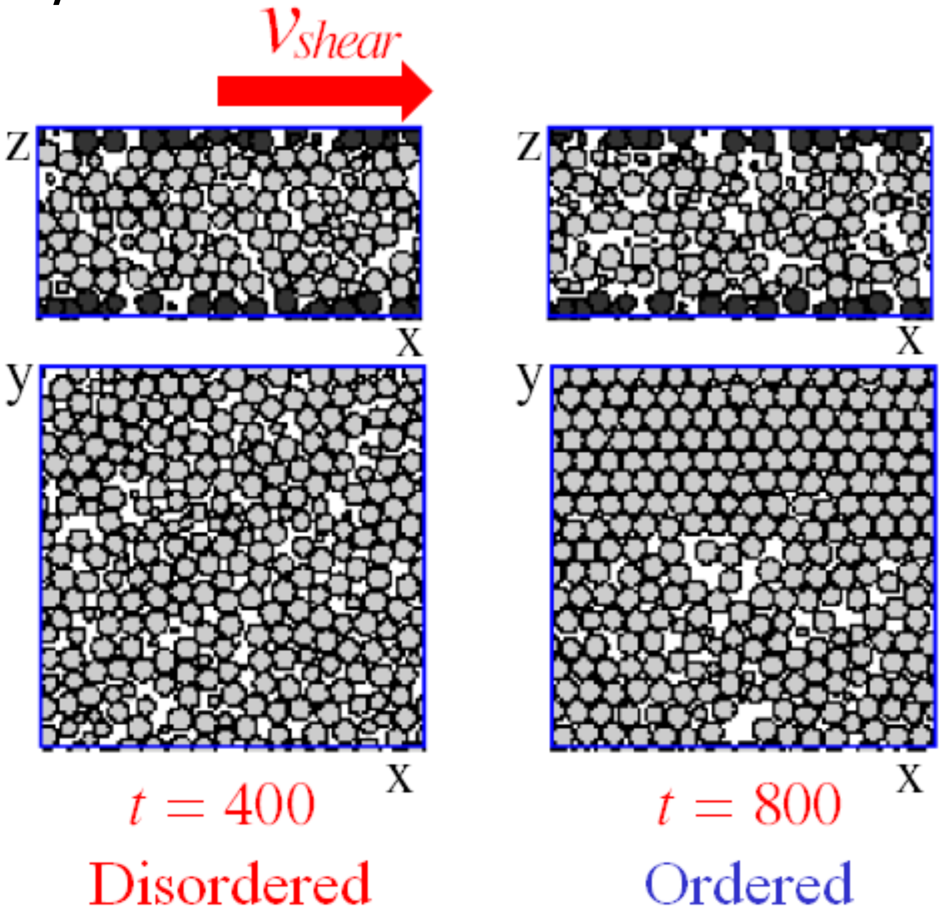


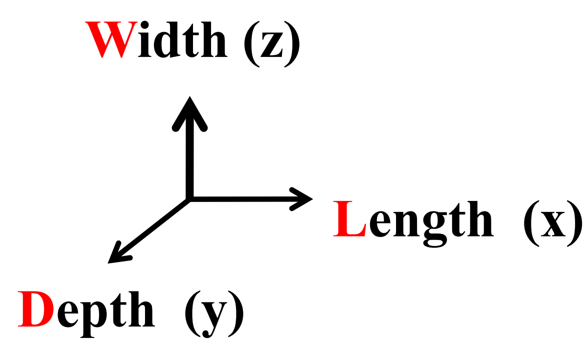


Vorticity Banding in Dense 3D Granular Flow

(Grebenkov, Ciamarra, Nicodemi, Coniglio, PRL 2008, vol 100)







Vorticity Banding in Dense 3D Granular Flow

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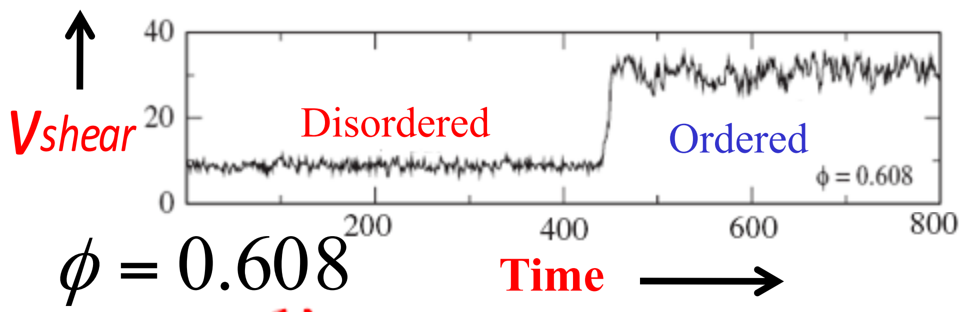
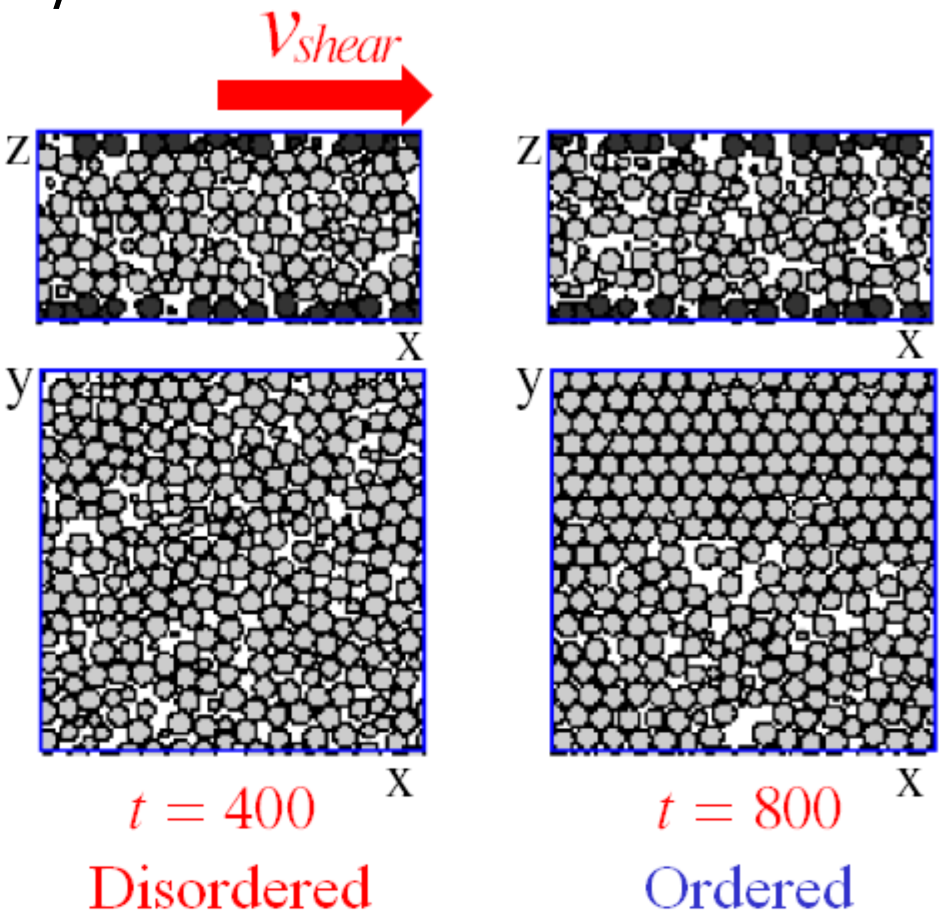
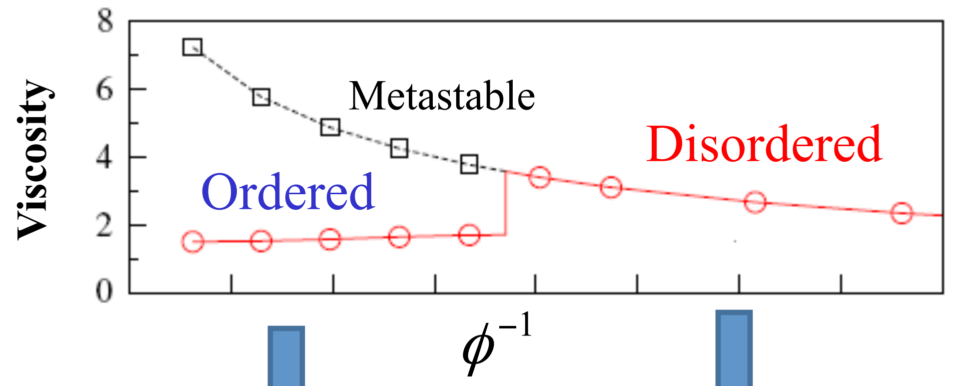






Ordered state

`Lower' viscosity

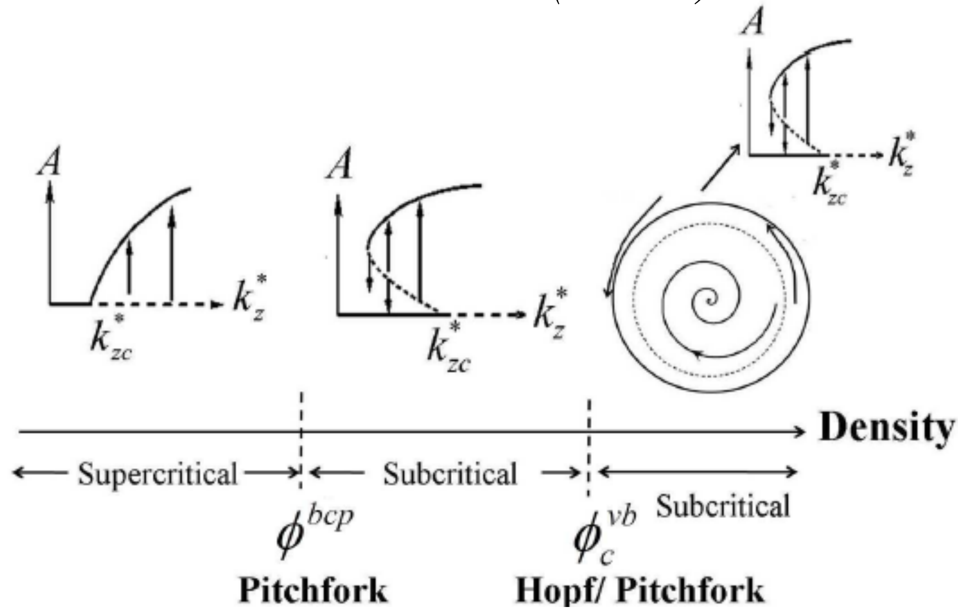
Disordered state

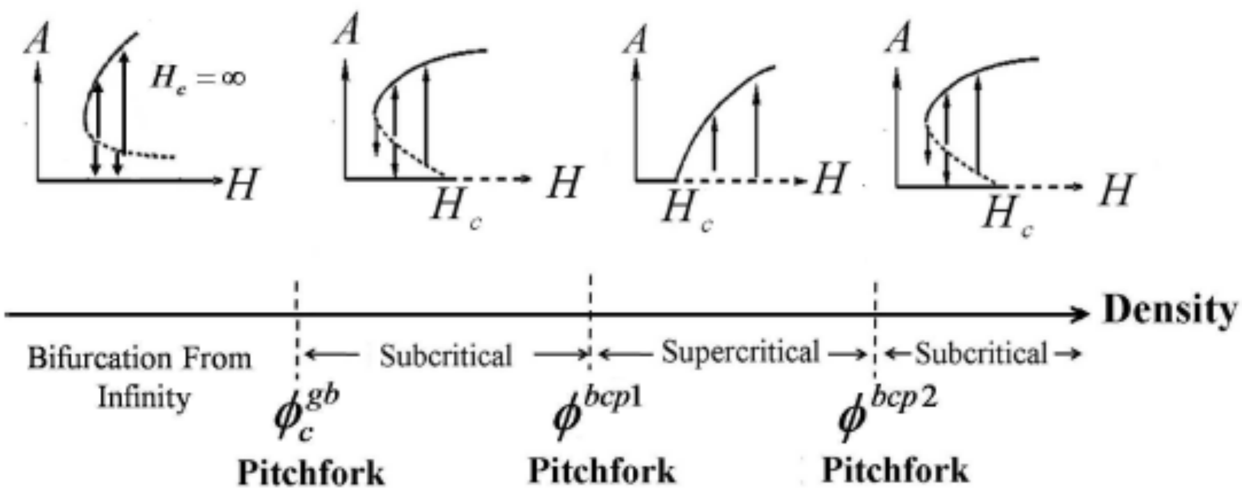
`Higher' viscosity

Bifurcation Scenario in vorticity and gradient banding

Shukla & Alam, J. Fluid Mech. (2013b)

Oscillatory Hopf bifurcation occurs for pure vorticity (spanwise) modes for $\phi^0 \geq \phi^{vb} \approx 0.1$





- Stationary pitchfork bifurcation for all densities
- Two bicritical points exist for gradient banding

Conclusions from nonlinear vorticity banding

- Quintic-order Landau Equation is derived for vorticity banding instability
- Analytical solutions for first and second Landau coefficients have been obtained.
- Bistable nature of nonlinear vorticity banding (stationary & oscillatory) states has been confirmed.
- Localization of shear stress (viscosity) and pressure along the spanwise direction.

Overall Conclusions

➢ **Hydrodynamic justification for gradient and vorticity** banding in a sheared granular fluid using nonlinear stability theory.

➢ **Unified description** of gradient and vorticity banding in terms of shear and viscosity localization, respectively.

References:

*Alam & Shukla (2013a), J. Fluid Mech., vol. **716**, 131-180*

*Shukla & Alam (2013b), J. Fluid Mech., vol. **718**, 349-413*

*Shukla & Alam (2011a), J. Fluid Mech., vol . **666**, 204-253*

*Shukla & Alam (2009) Phys. Rev. Letts , vol. **103** , 068001.*

Shukla & Alam (2013c) Phys Fluid (submitted)

Segregation-driven Patterns, Controlled Convection and Kuramoto's Equation (?)

Meheboob Alam & IstafaulHaque Ansari

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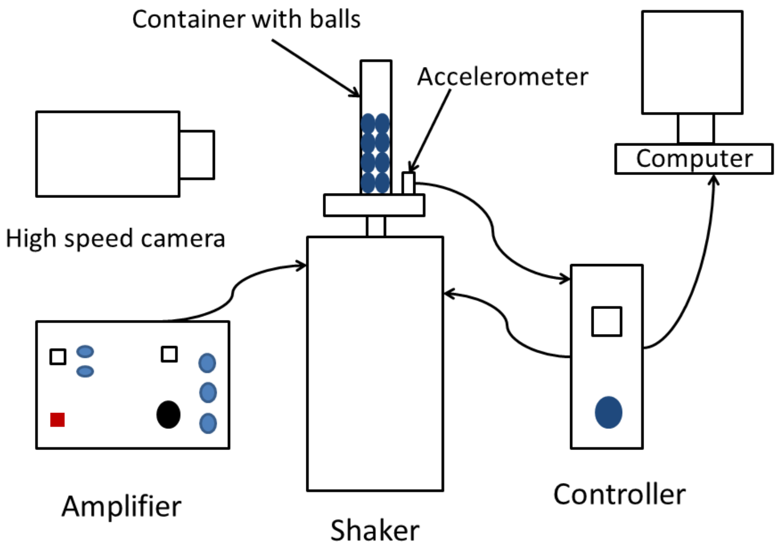
Bangalore, India

Physics of Glassy and Granular Materials (Satellite Meeting of STATPHYS 25)
@YITP, 16-19 July 2013

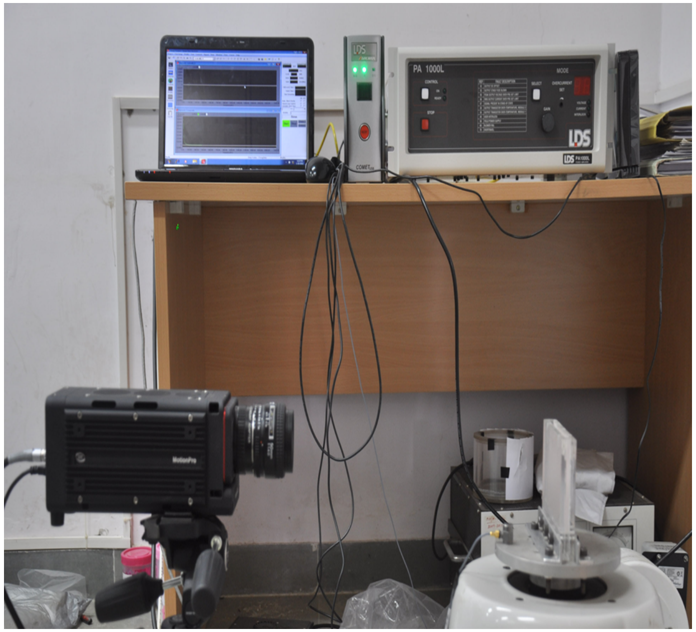
Outline of talk

- Introduction
- Experimental Setup and Procedure
- Particle Image Velocimetry
- Phase Diagram and Patterns
- Conclusions and Outlook

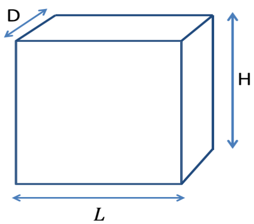
Details of Experimental Setup



Sketch of experimental setup.



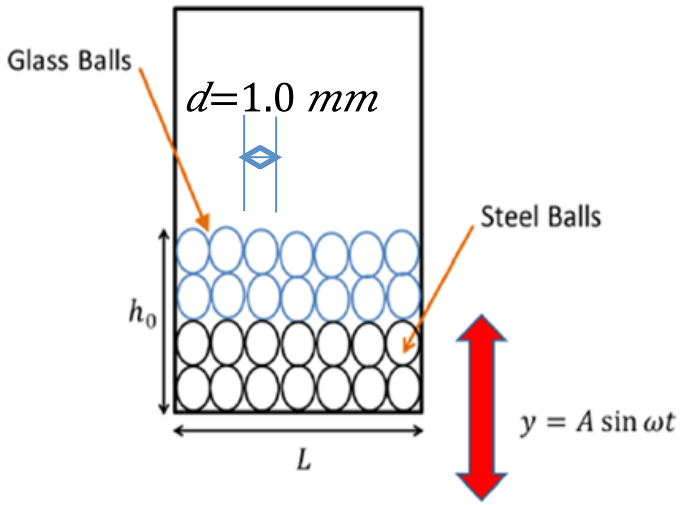
Experimental Setup



$D \sim 5d$

$L \sim 100\ d$ → Quasi-two dimensional box

Experimental Procedure



Dimensionless Control Parameters:

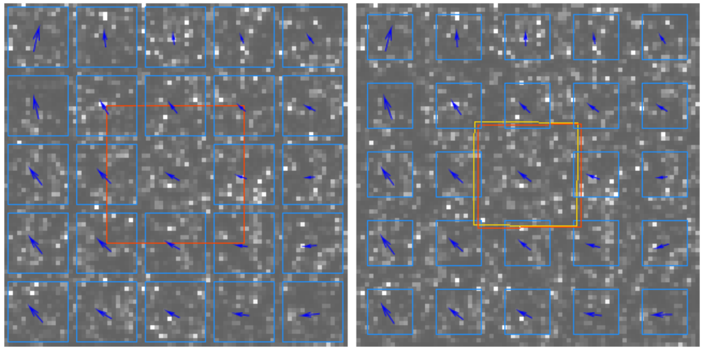
- Shaking accelaration: $\Gamma = A\omega^2/g$
  - A is the shaking amplitude
  - $\omega = 2\pi/\tau$, where τ is the time period.
  - g is the acceleration due to gravity.

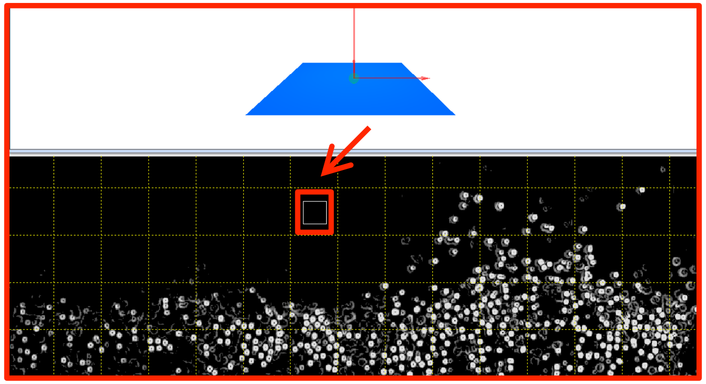
- Number of particle layers at rest

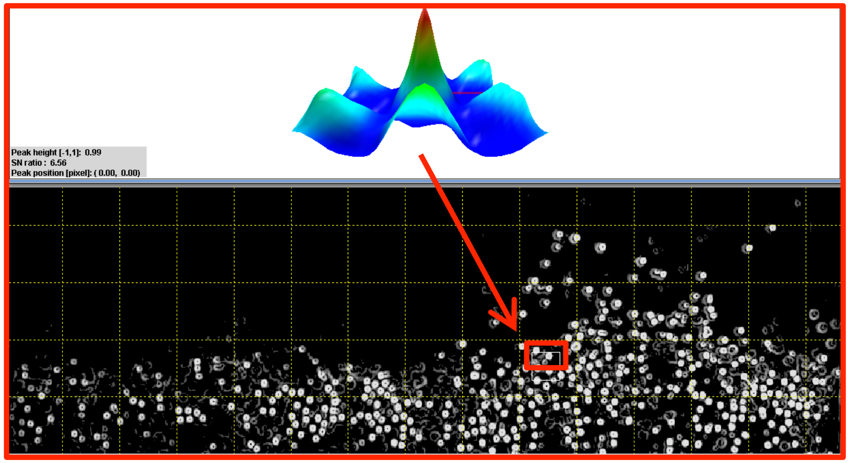
$F = F_g + F_s = h_0/d$

$F \epsilon$ (2.5, 10)

Adaptive PIV (Dantec Dynamics)



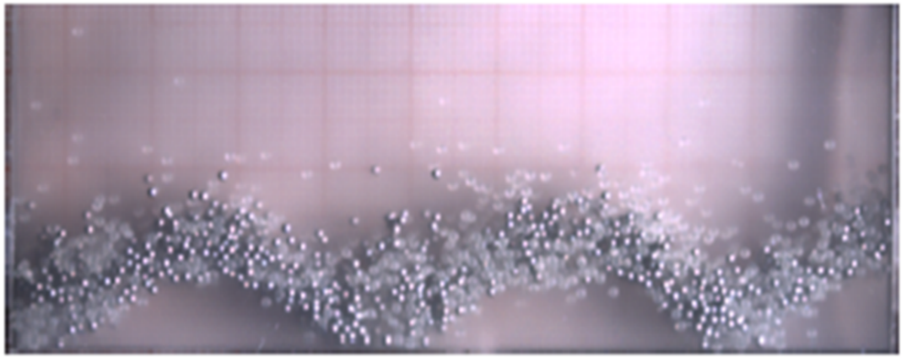




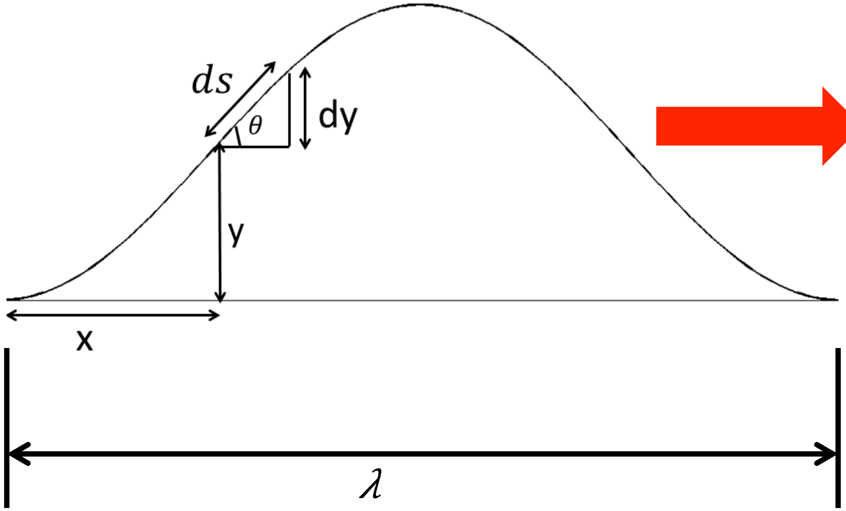
- “Adaptive PIV” iteratively optimizes the size and shape of each interrogation area (IA).

- Interrogation window is chosen iteratively until desired particle density is reached.

Undulation Waves



n=5 mode



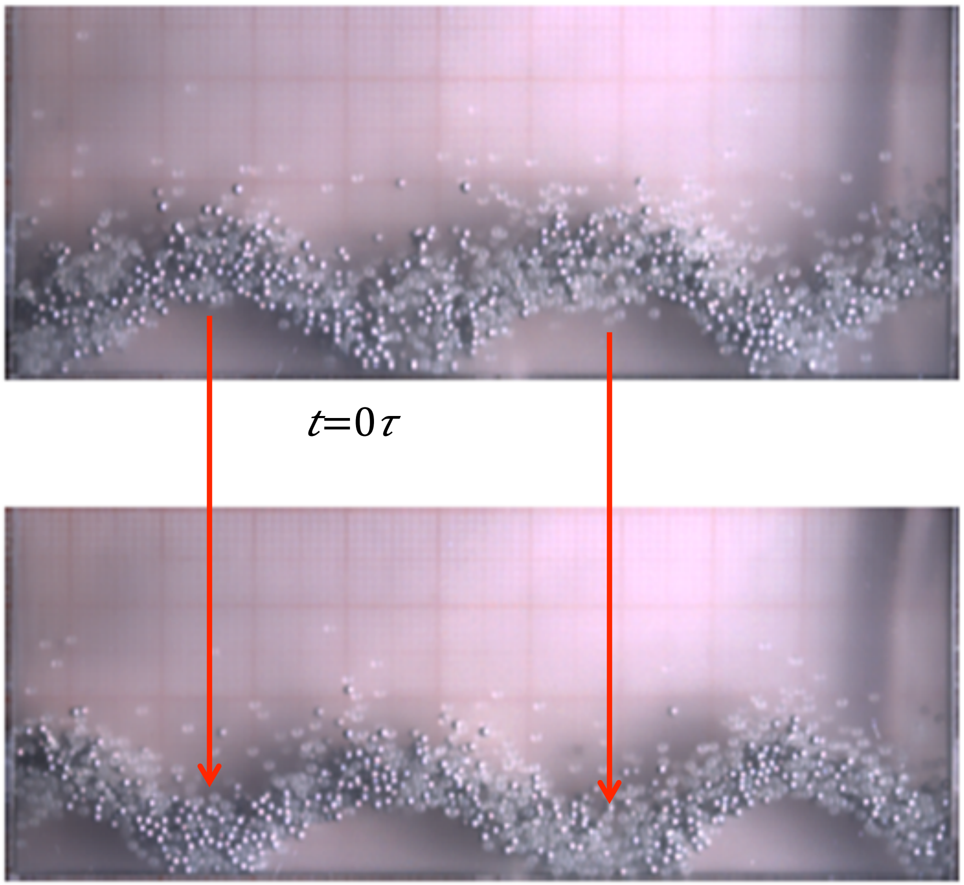
n=2 mode $\Rightarrow$ $L=\lambda$

$L=n\lambda/2$,

n=1,2,3….,

(mode number)

Undulation Waves [$f/2$ Waves]



$t=0\tau$

$t=\tau$

- $F_g=2.5$, $F_s=2.5$
- $A/d=3$
- $\Gamma=8.16$ ($f=26$ Hz)
- $n=5$ mode

Maxima (peak) exchanges with Minima (valley) after each cycle

f/2-waves

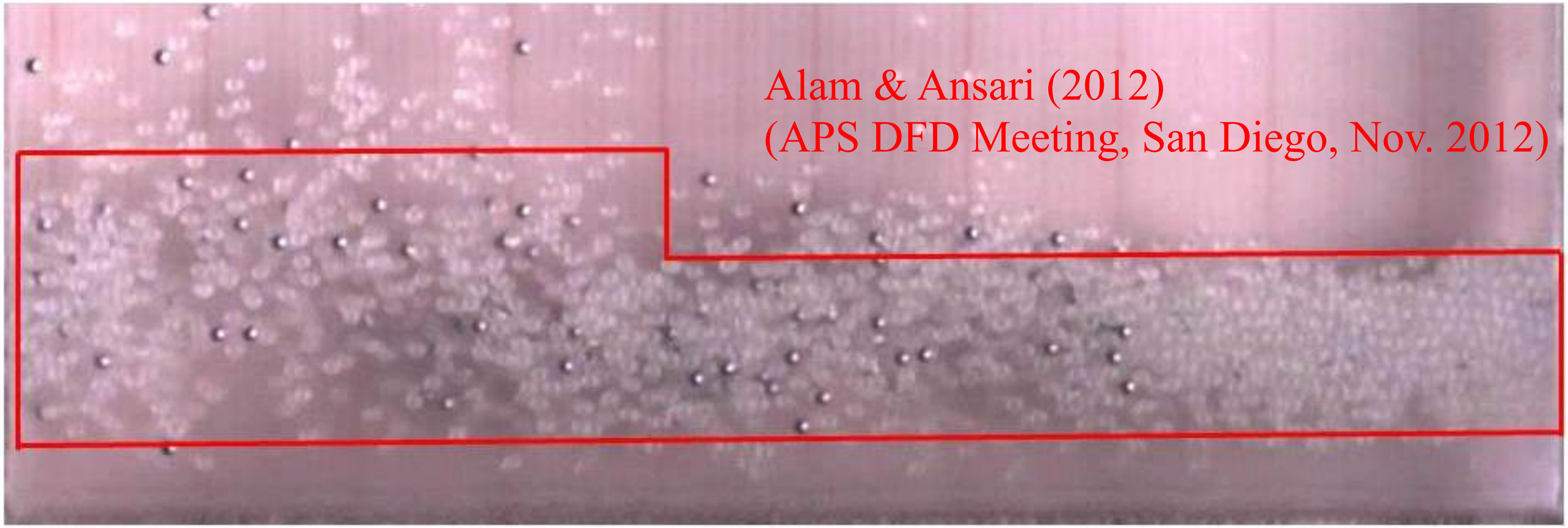
Well-mixed ?
No Segregation ?

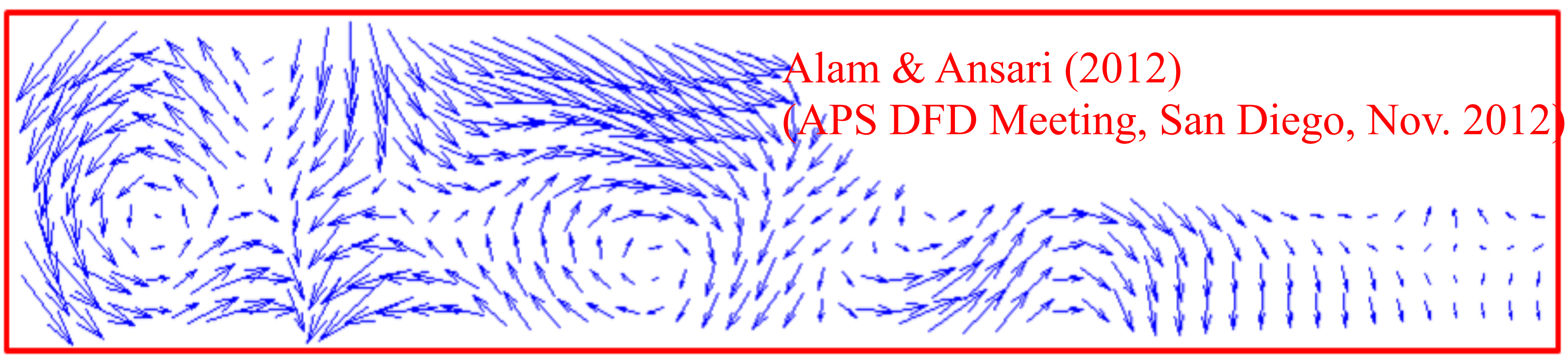
Undulation Wave (UW) + Gas

Alam & Ansari (2012)
(APS DFD Meeting, San Diego, Nov. 2012)

Synchronous (period-2) + Disordered (gas) states

Convection + Leidenfrost





Particle Simulations?

- Only `qualitative' agreement with MD simulation

- All phase-coexisting patterns are found in simulations,
  but (i) the life-time of `UW+Gas' from simulations is found to be orders-of-magnitude lower than that from experiments
  (ii) vertical segregation is not well reproduced by present simulation,
  (iii) ….

§ Simulation help from Mr. Rivas N.

¶ Simulations (impact model, etc) are currently being improved

Ansari, Rivas & Alam (June 2013, submitted)

Conclusions

- We discovered novel phase-coexisting patterns (Alam & Ansari 2012) in vibrated binary granular mixtures:
  - Bouncing bed + Granular gas
  - Undulations + Granular gas
  - Leidenfrost + Granular gas
  - Leidenfrost + Convection
  - Leidenfrost + Horizontal segregation
  - Bouncing Bed + Vertical Segregation

Small (F

all F

Phase Segregation?

Large Γ>10

Species Segregation?

- In all ***phase-coexisting states***, steel balls are in gaseous phase and the glass balls are in a Leidenfrost /Bouncing Bed/Undulation state.

- Segregation is due to non-equipartition of granular energy between heavier and lighter particles.

- Convection can be controlled in a binary mixture

- Alam & Ansari (2012, APS's DFD Meeting, San Diego)

- Ansari, Rivas & Alam (2013, submitted)

Thank You