Vibrated granular experiments: Probing the vicinity of Jamming

Olivier Dauchot,
Corentin Coulais, Raphael Candelier,
Frederic Lechenault, Antoine Seguin

Coll.: Giulio Biroli, Jean Philippe Bouchaud
Ludovic Berthier, Hajime Yoshino

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Overview

- Vibrated granular experiments: probing the jamming critical regime
  - Two distinct signatures of criticality at finite vibration
  - Approaching the zero vibration limit

- Yielding close to jamming in hard discs
  - Yield stress of “vibration” origin below jamming

- Probing elasticity
  - Inflating an intruder: experimental set up and the linear elastic framework
  - Integrated vs Local measurements
  - Discussion: the interplay between shear and dilatancy
Vibrating soft photo-elastic discs

- 8000 soft discs
- Bi-disperse: $d_s = 4 \& 5$ mm
- Horizontal vibration
  (a=1 cm, $\omega = 5$ to 10 Hz)
- Acquisition:
  - Stroboscopic
  - Fast inside the cycles
Heterogeneous Dynamics of the contacts

\[ \chi^{z,r}_4(\tau) \equiv NVaR \left( \left\langle Q^{z,r}_i \right\rangle_t \right) \]

\[ Q^z_i(t, \tau) = \begin{cases} 
1 & \text{if } |z_i(t+\tau) - z_i(t)| \leq 1 \\
0 & \text{if } |z_i(t+\tau) - z_i(t)| > 1 
\end{cases} \]

\[ Q'_i(t, \tau) \equiv \exp \left( -\frac{\Delta r^2_i}{2\left\langle \Delta r^2_i \right\rangle} \right) \]
Two distinct signatures crossovers

![Graph showing two distinct signatures crossovers](image-url)
Decreasing the vibration

The fluctuations increase

The crossovers merge

Decreasing the vibration !

\[ \gamma \]

\[ \epsilon \times 10^2 \]

\[ \max \chi_4^{z*} \]

|e*| \times 10^2
Hence two crossover lines: Widom lines

How far from the critical point?
Comparison with thermal soft spheres...

Simulation of thermal soft-spheres

Ikeda et al, 2012

Simulation of thermal soft-spheres

Ikeda et al, 2012

Temperature $T$ vs. density $\phi$

$\chi_4$ vs. density $\phi$

$T$ vs. $\phi$

MSD plateau

$\phi$ vs. $T$

$T$ vs. $\phi$

MSD plateau

$\phi$ vs. $T$
In this section, we investigate the heterogeneities of the dynamics, and in particular to probe collective effects, one defines a dynamical structure factor for the contact distribution of the grains.

The contact density fluctuations, which are the random spatial fluctuations of the number of contacts, are a direct consequence of jamming. The contact density fluctuations have been considered in the experiments with soft grains. The authors related these fluctuations to the spatial fluctuations of the contacts number.

In the case of brass discs, it is not possible to measure the structural signature of the jamming transition. In the brass grains experiment and more recently in the experiment with the brass discs, it is possible to probe the jamming transition. The cross-over occurs for lower packing fractions and on a long time (stroboscopic acquisition) experiments. The packing fraction where the transitions take place increases strongly with the packing fraction when crossing the jamming transition. It is much less frequent used when probing the jamming transition, but a puzzling experiment with the brass discs had actually probed the jamming transition in the absence of vibration is expected for lower packing fractions.

One also notices that the range of packing fractions that the energy transfer and dissipation are different from those of soft grains. The friction coefficients made of brass (Young modulus, 70 GPa) is different from those of soft grains. The friction coefficients and the friction force are larger, and that the one associated with the friction force increases with the packing fraction.

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Discussion: Can we go further?

Here dynamics, and in particular to probe collective effects, one defines a dynamical structure factor for the contact distribution of the grains.

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Conclusion of the first part

- Shaken Granular Experiments are in the street lamp halo of the J point:
  - They can constrain existing theories
  - Theories have something to say about the real world…
  - One cannot exclude effects of friction at the quantitative level

=> One step further (in the dark…)
  - Yielding close to jamming
  - A first attempt to probe elasticity close to jamming
Yielding close to jamming: the motion of an intruder ...
Evidence of a fluidization transition

Transition: \( \% \delta x < 0 \rightarrow 0 \)
Indeed two very different rheological behaviors

 Fluidized regime: $F \propto <V>$

 Intermittent regime: $F \propto \ln <V>$
Critical force: “thermal” yield stress

\[ \sigma^3 / \epsilon \]

PNIPAM

Emulsion

Foam

Glass

\[ F_c \sim 1/(\Phi_J - \Phi) \]

\[ \eta \]

\[ \tau \]

\[ T \]

\[ \sigma \]

\[ \epsilon \]

\[ \mu \]

\[ \phi \]

\[ \phi_1 \]

\[ \phi_2 \]

\[ \phi_3 \]

\[ F(N) \]

\[ \text{Fluidisation line} \]

Emulsions with larger droplet sizes [49], but for brevity experimental data from other sources, in particular ultra- and PNIPAM microgels [21, 53]. We have also gathered experimental flow curves obtained for a variety of dense suspensions obtained for harmonic spheres, and by the analysis of the complex features also observed in the simulations. We mentioned that similar indications are also found for PMMA colloids, in particular at large Péclet numbers, after Petekidis et al. [41]. Foam with \( \Phi \) \[\mu\] = 0.64, after Otsubo et al. [44]. Emulsion with \( \Phi \) \[\mu\] = 2, after Koumakis et al. [47] and PNIPAM microgels [21, 53]. Star polymers with \( \Phi \) \[\mu\] = 8, after Nordstrom et al. [55]. Emulsion with \( \Phi \) \[\mu\] = 0.6, after Mason et al. [49]. PNIPAM microgels undergo a colloidal glass transition. This is the case for foams in Fig. 9 for which the glass ‘wing’ has negligible effects. Note that PMMA colloids with \( \Phi \) \[\mu\] = 0.56, see Eqs. (3, 5). These flow curves are collected experimentally, and the measurements would stop as the yield stress surface as a function of the thermodynamic parameters, see Eq. (16). The three lines represent the location of the jamming transition. This is the case for foams in Fig. 9 for which the critical force is “thermal” yield stress.
Probing elasticity : set up

- Prepare the system at large packing fraction under vibration
- Inflate an intruder in the center (the vibration is stopped)
- Decrease the packing fraction while vibrating
- Iterate

\[ R_0 \rightarrow R_0 + a \]
\[ \gamma = \frac{a}{R_0} \]
Probing elasticity: the linear elastic framework

\[ \begin{align*}
\text{div}(\sigma) &= 0 \\
\sigma &= \frac{1}{2} Tr(\sigma) I + \varepsilon \\
\sigma &= K Tr(\varepsilon) I + 2G \varepsilon \\
\varepsilon &= \frac{1}{2} \left[ \nabla U + (\nabla U)^T \right] = \frac{1}{2} Tr(\varepsilon) I + \varepsilon
\end{align*} \]

\[ \begin{align*}
\delta &= Tr(\varepsilon) = -2 \frac{a}{R_0} A \\
\gamma &= J_2(\gamma) = \sqrt{\frac{1}{2} \gamma \circ \gamma} = \frac{a}{R_0} B \left( \frac{R_0}{r} \right)^2 \\
P &= Tr(\varepsilon) = K Tr(\varepsilon) \\
\tau &= J_2(\tau) = \sqrt{\frac{1}{2} \tau \circ \tau} = 2G J_2(\gamma)
\end{align*} \]

**Nota Bene**

- In the limit of large \( R_1 \), A->0, B->1: this is a shear test!
- \( G \) and \( K \) are simply obtained by the ratio of the stress and strain tensor invariants

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EC2M Effets Collectifs & Matière Molle

Gulliver
For each packing fraction and each $a/R_0$.

$\delta = \text{Tr}(\varepsilon)$

$\gamma = J_2(\varepsilon)$

$P = \text{Tr}(\sigma)$

$\tau = J_2(\sigma)$
Salient features:

- Overall dilatant behaviour in the region close to the intruder
- Non linear constitutive law

? Pressure stiffening = > Dilatancy => Shear weakening ?
Conclusion

- Vibrated granular media are suitable tools for probing the vicinity of jamming, (in particular low enough $T_{\text{eff}}$)

- Two distinct crossovers (one dynamical, one structural) converge toward J-point in the limit of low vibration

- Pulling an intruder in vibrated hard discs has allowed us to probe the yield stress of “thermal origin” => Suggest to try in the soft photo-elastic discs to capture the yield stress of “jamming origin”

- Inflating an intruder in soft photo-elastic discs => First indications of intricate interplay between dilatancy and non linear shear law.

Further readings:
- Europhysics Letters, 100, 44005 (2012).
Integrated quantities vs. control parameter $a/R_0$

Compressive part

- Linear with $a/R_0$

Nota Bene: $\text{Tr}(\varepsilon) > 0 \Rightarrow \text{Overall dilatant behaviour.}$
Integrated quantities vs. control parameter $a/R_0$

Shear part

- Non linear behaviour of shear strain => $a/R_0$ does not strictly control strain
- Both shear strain and shear stress are responses and non linear
- The shear work however is quadratic in $a/R_0$ as prescribed by linear elasticity
Radial profiles (azimuthally averaged)

- Compressive part

- Dilatancy strongly localized close to inflating intruder
- Pressure decreases exponentially
Deviation from the $1/r^2$ law, expected from linear elasticity

Some dependence with the packing fraction
Parametric plot shear stress vs. shear strain

In both case, clear evidence for non linear constitutive law.

G_eff increases with the packing fraction, however shear weakening

=> Suggest rather complex non linear interplay between shear and dilatancy