Coefficients of heat and electric conductivity in magnetized neutron star

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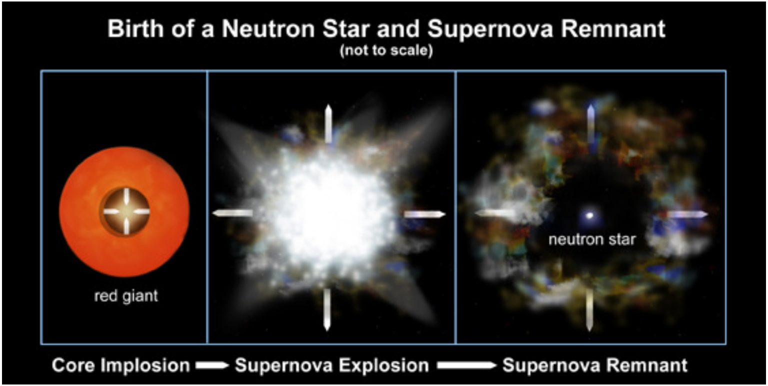
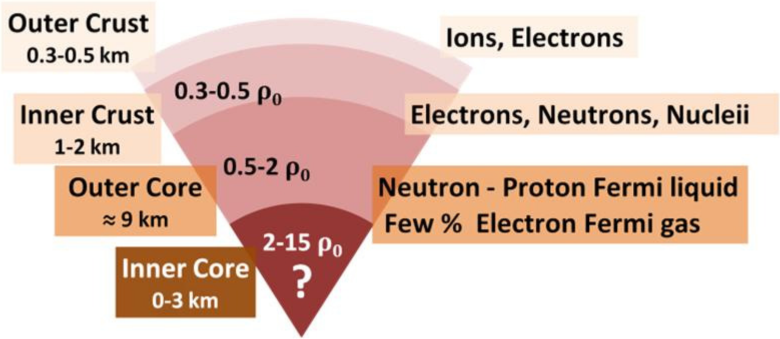
Introduction



It is widely accepted that NSs are born at the final stage of evolution of normal stars of mass $M \geq 8 * M\odot$ in gravitational collapse of their cores. During collapse, the matter of central layers is compressed to nuclear densities and enriched by neutrons. As a result, a compact NS is created of mass $M \sim M\odot$ and radius $R \sim 10km$.

Structure

4 internal regions:



$\rho_0 = 2.8 * 10^{14} g/cm^3$

The cooling

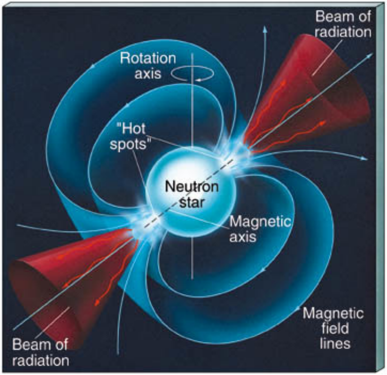
- The cooling of a NS is accompanied by the loss of its thermal energy which is mainly stored in the stellar core. The energy is carried away through two channels:
  1) by neutrino emission from the stellar body,
  2) by heat conduction through the internal stellar layers towards the surface, and further, by thermal emission of photons from the surface.
- The nature of the cooling depends on many parameters of the star: the equation of state of the inner layers, NS mass, magnetic field, chemical composition of the surface layers, and superfluidity of nucleons in the core, etc. Comparison of the cooling theory with observations can impose restrictions on these parameters.

Observations

- It has been understood since Greenstein & Hartke (1983) that in presence of a sufficiently strong magnetic field, $\geq 10^{10}\,G$, the surface temperature of a neutron star (NS) will not be uniform as is expected in the unmagnetized case.
- Page (1995) and Page & Sarmiento (1996) applied the Greenstein & Hartke formula to explain the lightcurves of the isolated thermally emitting NSs PSR 0833-45 (Vela), PSR 0656+14, PSR 0630+178 (Geminga) and PSR 1055-52, considering dipolar field configurations.

Observations

- This result, that the geometry of the magnetic field in the interior of the NS leaves an observable imprint at the surface, potentially allows us to study the internal structure of the magnetic field through modelling of the spectra and pulse profile of thermally emitting NSs.



Formulation of the problem

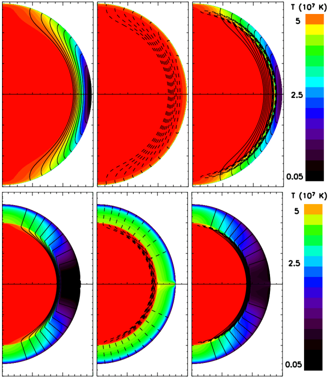
- To analyse temperature distribution on the surface of NS we have to obtain temperature distribution in the crust of NS from the border region to the insulated surface.
- the heat flux density:$q_i = -\lambda_{ik} \dfrac{\partial T}{\partial r_k}$
- the current density $j_i = e n_e \langle \overline{v_i} \rangle$
- It is necessary to consider the tensor properties of the thermal conductivity and electrical conductivity:
  $\lambda_{ik} = \dfrac{5}{2} \dfrac{k^2 T n}{m}((a_0^1 - a_1^1)\delta_{ik} - \epsilon_{ikn} B_n (b_0^1 - b_1^1) + B_i B_k (c_0^1 - c_1^1), i, k = x, y, z$
  $\sigma_{ik} = \dfrac{kTn}{m}(x_0 \delta_{ik} - \epsilon_{ikn} B_n b_0 + B_i B_k z_0), i, k = x, y, z$

Analysis of previous work

- In previous works devoted to the kinetic coefficients in a magnetized neutron star, the following approximation was used for the coefficients of thermal and electric conductivity along and across the magnetic field lines: (Flowers, Itoh 1975), (Yakovlev, Urpin 1980)
- $\dfrac{\lambda_\perp}{\lambda_\parallel} = \dfrac{1}{1+(\omega\tau)^2}.$
- $\dfrac{\sigma \perp}{\sigma \parallel} = \dfrac{1}{1+(\omega\tau)^2}.$
- $\omega = \dfrac{eB}{m_e c}, \quad \tau = \dfrac{3h^3 n_e}{32\pi^2 Z^2 e^4 n_N m_e \Lambda}$

Isotherms

- Simulation of the isotherms in an isolated magnetized neutron star by Perez-Azorin, Miralles, Pons 2005:



- The ratio for the coefficients along and across the magnetic field lines: $\dfrac{\lambda_\perp}{\lambda_\parallel} = \dfrac{1}{1+(\omega\tau)^2}$.

Analysis of previous work

There are a lot of papers, devoted to kinetic coefficients of magnetized plasma in Maxwell's approximation, performed by a number of authors: W. Marshall (1961), Bisnovatyi-Kogan G.S.(1964) и R. Lansdorf(1959). The approximation obtained in these papers for the coefficients along and across the magnetic field lines for nondegenerate plasma is:

$$\frac{\lambda \perp}{\lambda \parallel} = \frac{1}{1 + \frac{17(\omega\tau)^2 + 2.76(\omega\tau)^4}{2.58 + 0.39(\omega\tau)^2}}$$

- $$\frac{\sigma \perp}{\sigma \parallel} = \frac{1}{1 + \frac{11.93(\omega\tau)^2 + 2.07(\omega\tau)^4}{1.93 + 1.06(\omega\tau)^2}},$$
- $$\tau = \frac{3\sqrt{m_e}(kT)^{3/2}}{4\sqrt{2\pi}Z^2 e^4 n_e \Lambda}$$

New approximation

- We assume that for the degenerate case, we can obtain a more complex ratio between the coefficients along and across the magnetic field lines, that it was used in previous papers. For large values of the magnetic field the value of the coefficient in front of $(\omega\tau)$ can be substantial.
- So we need to calculate coefficients of heat and electrical conductivity by solving Boltzmann equation.
- Obtained result can be used for calculation of isotherm geometry on the surface and in the crust of neutron star.

Boltzmann's equation

- $$\frac{\partial f}{\partial t} + c_i \frac{\partial f}{\partial r_i} + \frac{e}{m}(E_i + \frac{1}{c}\epsilon_{ikl} c_k B_l)\frac{\partial f}{\partial c_i} + J = 0. \quad (1)$$
- For collision integral we take into account only the binary collisions: $J = J_{ee} + J_{eN}$,
- $J_{ee} = 2m^3/(2\pi\hbar)^3 \int [f' f_1'(1-f)(1-f_1) - ff_1(1-f')(1-f_1')] \times g_{ee} W_{ee}(\theta, g_{ee}) d\Omega dc_{1i}$,
- $J_{eN} = \int [f' f_N'(1-f) - ff_N(1-f')] \times g_{eN} W_{eN}(\theta, g_{eN}) d\Omega dc_{Ni}$.
- $g_{ee} = \mid c_{1i} - c_{2i} \mid$ is relative velocity of colliding electrons. g_{eN} is relative velocity of colliding electron and nuclei.
- W_{ee}, W_{eN} are differential cross sections for collisions of particles with relative speed g_{ee} or g_{eN}, inclined at an angle θ and lying after a collision in the solid angle $d\Omega$.

solution

- The zero approximation for the distribution function: $f_0 = [1 + \exp \frac{m_e v^2 - 2\mu}{2kT}]^{-1}$.
- The first approximation for the electrons: $f = f_0[1 + \chi(1 - f_0)]$
- The first approximation for the nuclei: $f_N = f_{N0}(1 + \chi_N)$.
- χ and χ_N are linear and admits representation of the solution in the form:
- $\chi = -A_i \frac{\partial \ln T}{\partial r_i} - n_e D_i d_i \frac{G_{5/2}}{G_{3/2}}$,
- $\chi_N = -A_{Ni} \frac{\partial \ln T}{\partial r_i} - n_N D_{Ni} d_i \frac{G_{5/2}}{G_{3/2}}$

solution

- Functions A_I, A_{Ni} и D_I, D_{Ni} determine heat conductivity and diffusion.
- It was shown by Bisnovatyi-Kogan (1964) that in the presence of the magnetic field, we can seek B_i vectors A_i and A_{Ni} in the form :
- $A_i = A^1 v_i + A^2 \epsilon_{ijk} v_j B_k + A^3 B_i (v_j B_j)$,
- $A_{Ni} = A_N^1 v_i + A_N^2 \epsilon_{ijk} v_j B_k + A_N^3 B_i (v_j B_j)$,
- Let's introduce:
- $\xi = A^1 + iBA^2$,
- $\xi_N = A_N^1 + iBA_N^2$

solution

- Generalized equation for ξ and ξ_N can be written in the form:

$$f_0(1-f_0)(u^2-\frac{5G_{5/2}}{2G_{3/2}})u_i=-\frac{i}{3}\frac{em_eB}{\rho_e kTc}u_i(f_{N0}\int\xi_N f_{N0}u_N^2dc_i- \\ -f_0(1-f_0)\int\xi f_0(1-f_0)u_i^2dc_i)+iBf_0(1-f_0)\frac{e\xi}{m_ec}u_i+ \\ +I_{ee}(\xi u_i)+I_{eN}(\xi_{Ni}u_{Ni}) \quad (5)$$

- Solving this equation by polynomial expansion, analogous to Sonine expansion, we obtain the following results

Results: coefficientsof tensor of heat conductivity and electro conductivity

- For degenerate case:
- $$\sigma_{xx} = \sigma_{yy} = \frac{kTn_e}{m} \frac{-\tau(1.06 + 1.1(\omega\tau)^2)}{1 + 6.5(\omega\tau)^2 + 4.1(\omega\tau)^4},$$
  $$\sigma_{xy} = -\sigma_{yx} = \frac{kT}{m} \frac{-\tau(0.2(\omega\tau) + 1.2(\omega\tau)^3)}{1 + 6.5(\omega\tau)^2 + 4.1(\omega\tau)^4},$$
  $$\sigma_{zz} = -1.06\tau \frac{kT}{m}$$
- For degenerate case:
- $$\lambda_{xx} = \lambda_{yy} = \frac{5}{2} \frac{k^2 Tn_e}{m} \frac{\pi^2\tau(0.14 + 0.6(\omega\tau)^2)}{1 + 6.5(\omega\tau)^2 + 4.1(\omega\tau)^4},$$
  $$\lambda_{xy} = -\lambda_{yx} = \frac{5}{2} \frac{k^2 Tn_e}{m} \frac{-\pi^2\tau(0.2(\omega\tau) + 1.2(\omega\tau)^3)}{1 + 6.5(\omega\tau)^2 + 4.1(\omega\tau)^4},$$
  $$\lambda_{zz} = 0.33\pi^2\tau \frac{k^2 n_e T}{m}$$

Relation of coefficients of heat- and electocunductivities across and along magnetic fields lines for degenerate plasma.

- $$\frac{\lambda\perp}{\lambda\parallel}=\frac{1}{1+\frac{0.31(\omega\tau)^2+0.55(\omega\tau)^4}{0.14+0.6(\omega\tau)^2}}.$$
- $$\frac{\sigma\perp}{\sigma\parallel}=\frac{1}{1+\frac{5.6(\omega\tau)^2+4.3(\omega\tau)^4}{1.06+1.1(\omega\tau)^2}}.$$

Conclusion

- In our work we have reconsidered the thermal conductivity of electrons in magnetic field produced by electron - ion scattering. The new coefficients can be used for calculation of temperature distribution on the surface and in the crust of magnetized neutron star. The temperature distribution can be very useful in understanding of geometry of magnetic field the the dense regions of the star.
- There are some ways of elaboration of our results such as consideration of the quantum effects of particle interaction at high magnetic fields.

Thank you for attention!