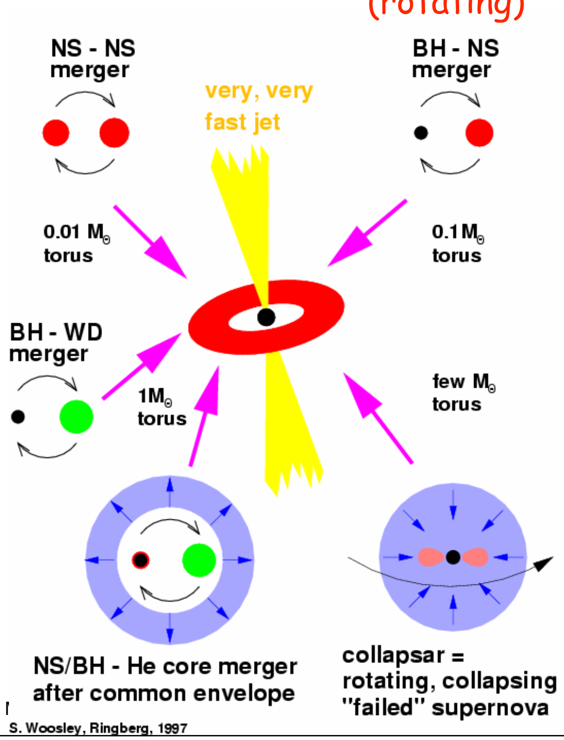
From Engine to Afterglow: Moving mesh “Lagrangian” Hydrodynamics with TESS

Andrew MacFadyen (New York University)

Paul Duffell (NYU), Hendrik van Eerten (MPE)

Duffell&AM (2011,2012,2012)

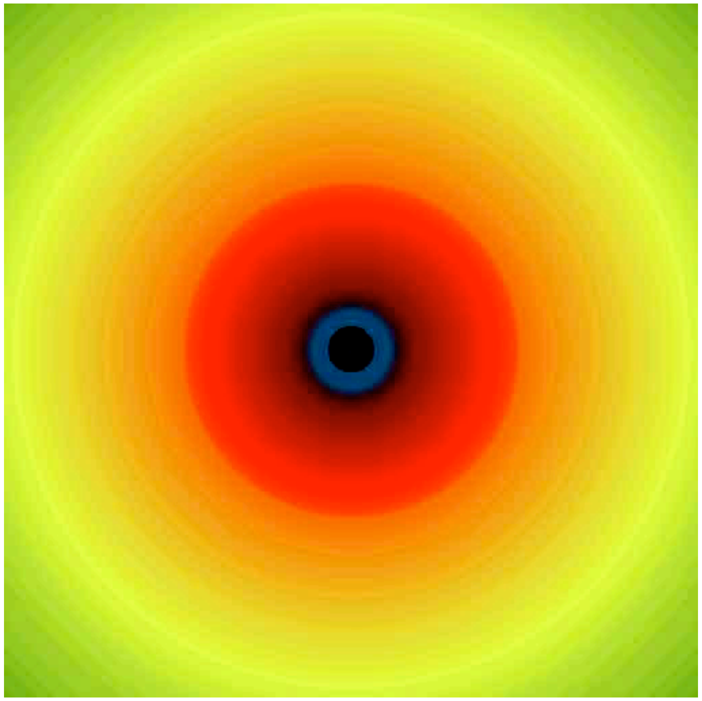
Hyper-accreting black hole or high field neutron star (rotating)



GRB photons are made far away from engine.

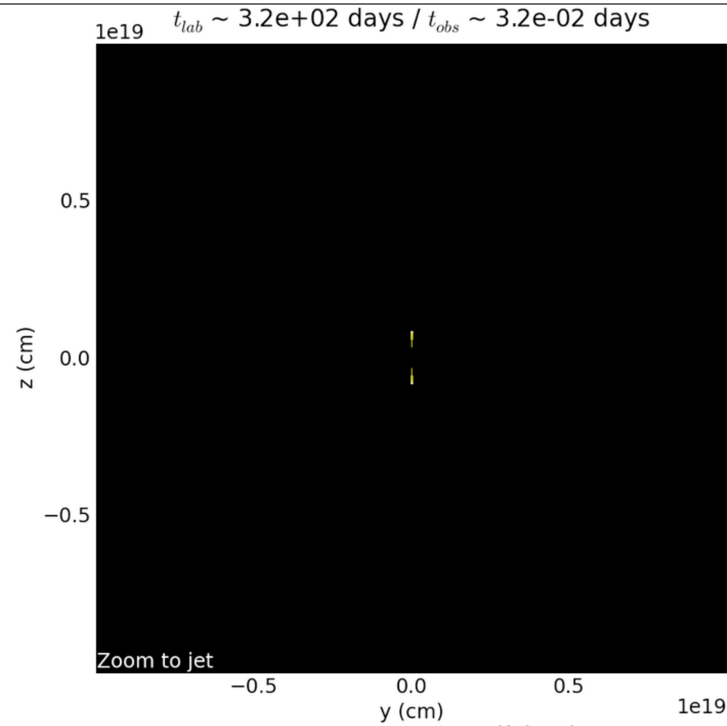
Can't observe engine directly with light. (neutrinos, gravitational waves?)

Electromagnetic process or neutrino annihilation to tap power of central compact object.



AMR
jet+wind

AM&Zhang
(2009)



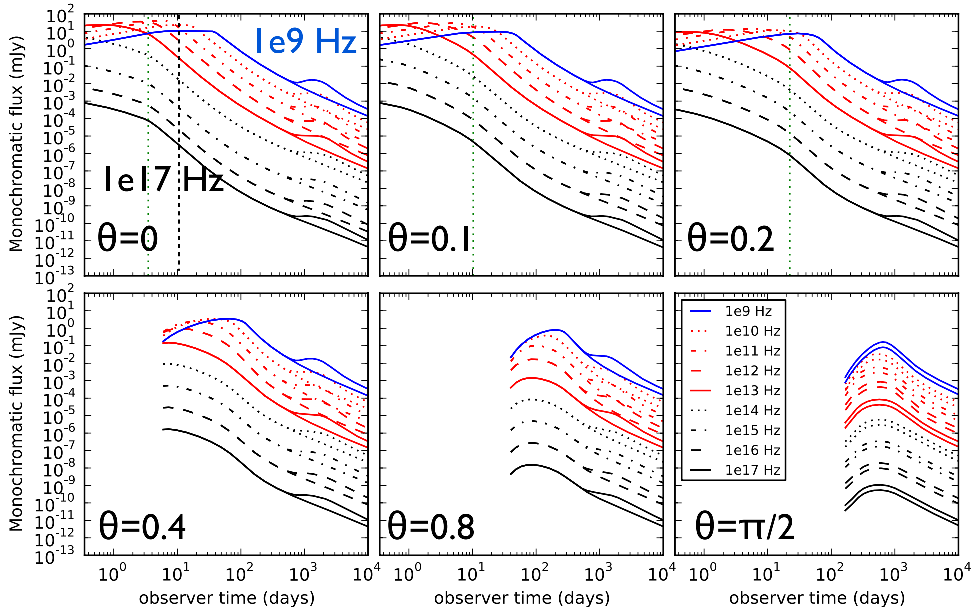
Ej = 2e52
θj=0.05
n=1cm^-3

van Eerten & AM (2011)

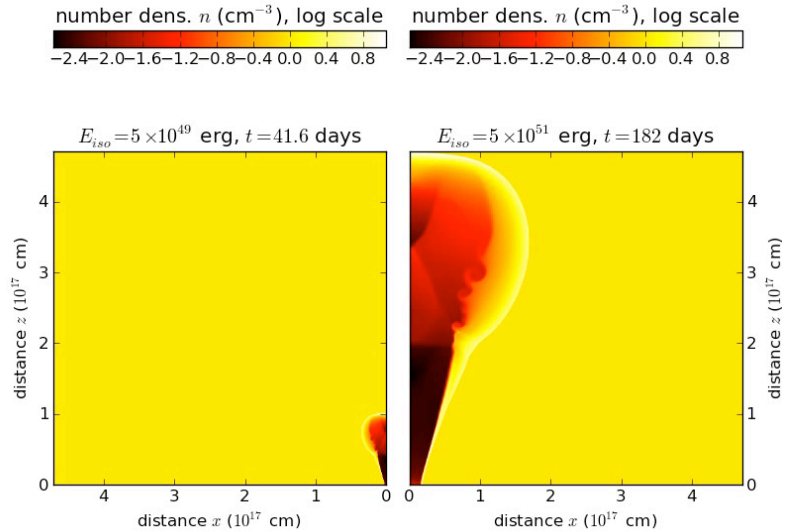
A. MacFadyen (NYU)

Off-Axis Light Curves

van Eerten, Zhang & AM (ApJ, 2010)

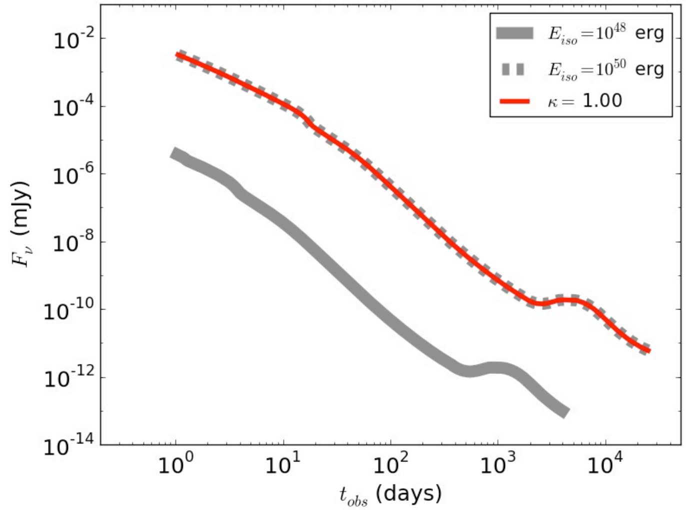


Scaling of jet dynamics



$$E'_{iso} = \kappa E_{iso}, \quad n' = \lambda n, \quad t' = (\kappa/\lambda)^{1/3} t, \quad r' = (\kappa/\lambda)^{1/3} r$$

Scaling of light curves



$$E'_{iso} = \kappa E_{iso}, \quad n' = \lambda n,$$

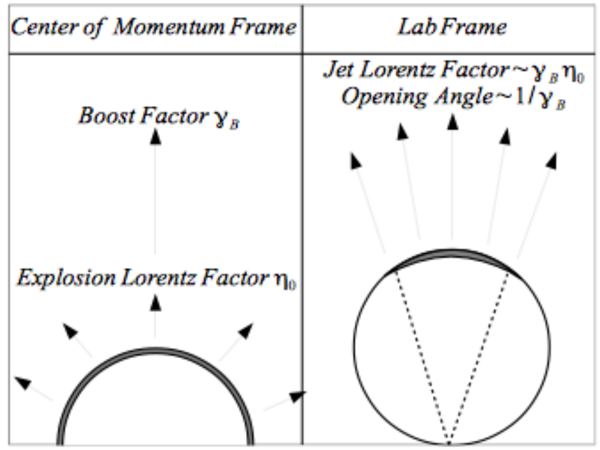
$$t'_{obs} = (\kappa/\lambda)^{1/3} t_{obs}, \quad F'_{optical} = \kappa \lambda^{(1+p)/4} F_{optical}$$

http://cosmo.nyu.edu/afterglowlibrary/

van Eerten & AM (2010,2011)

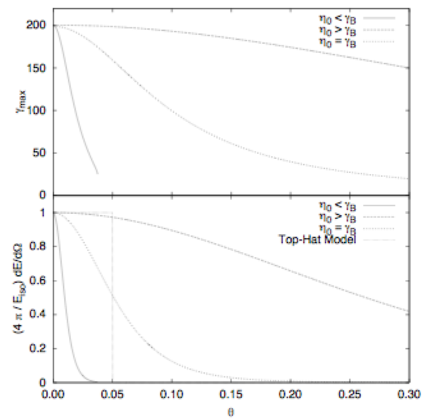
Supported by NASA 09-ATP-0190

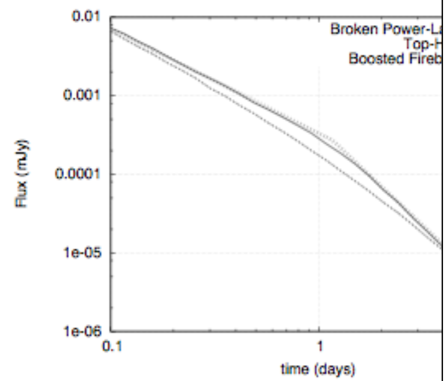
Boosted Fireball



A. MacFadyen (NYU)

Boosted Fireball





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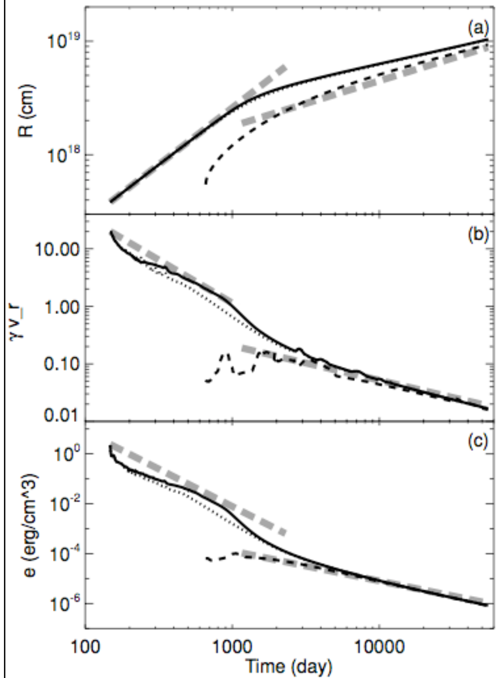
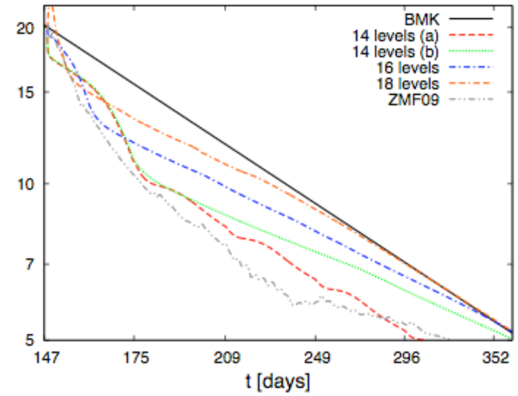
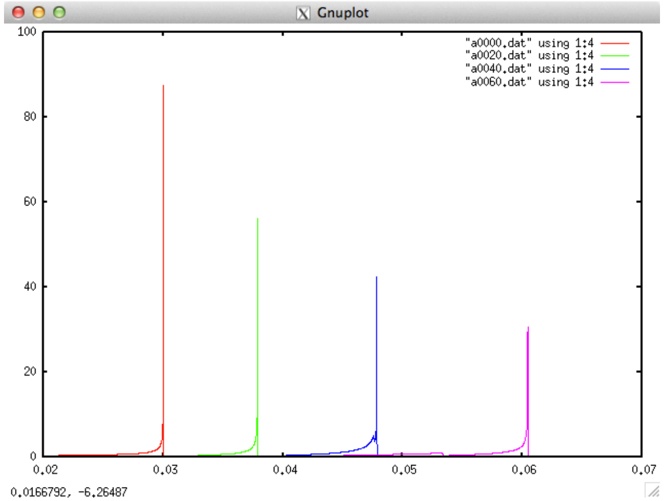


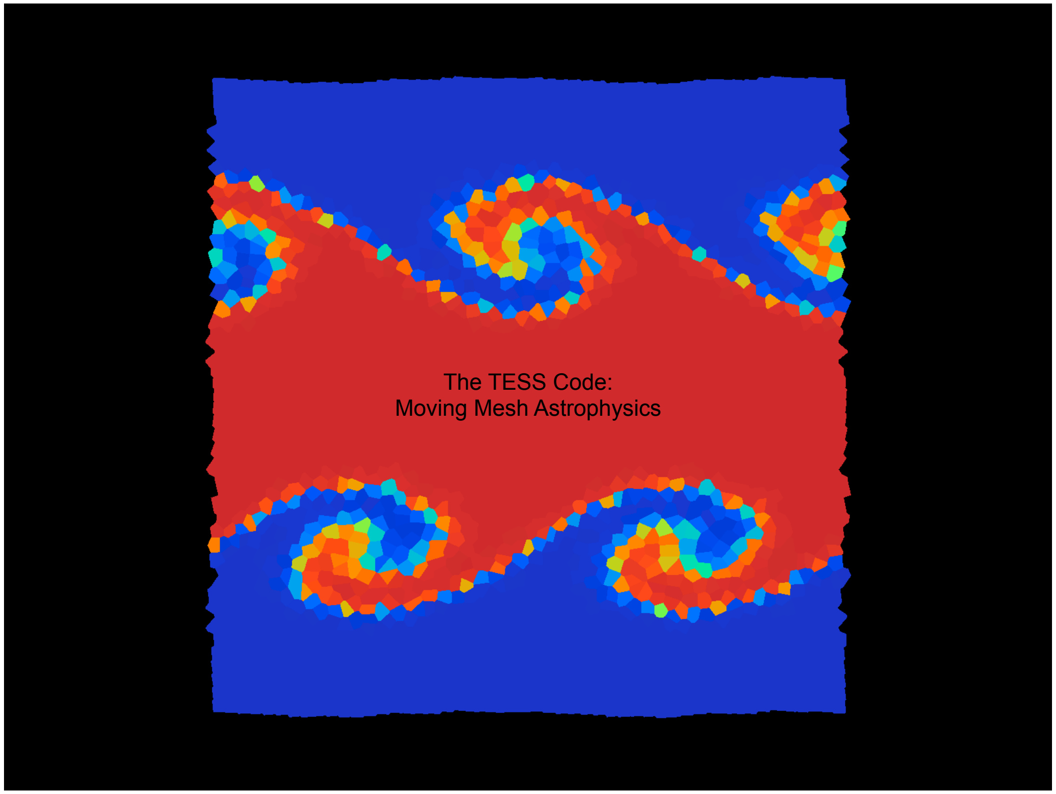
Zhang & AM (2009)

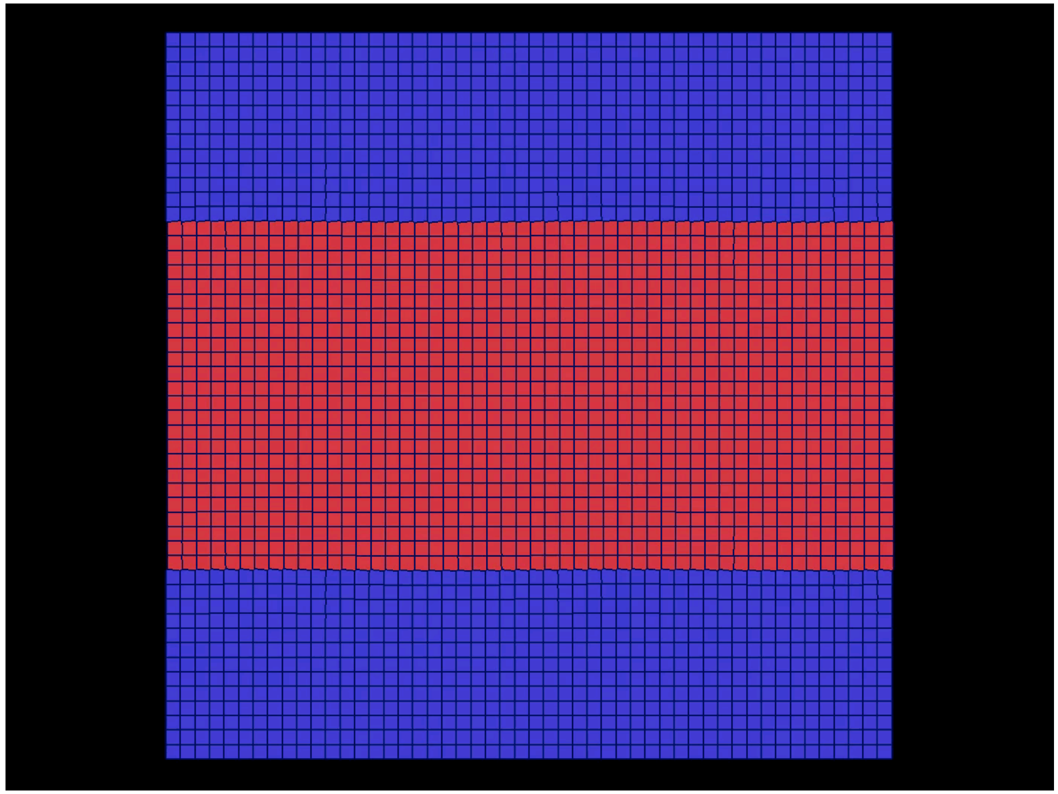


DeColle+ (2012)

TESS: Blandford-McKee




The TESS Code:
Moving Mesh Astrophysics



Numerical Methods for Solving Conservation Laws

Euler's Equations

$$\frac{\partial}{\partial t}(\rho)+\nabla\cdot(\rho\vec{v})=0$$

$$\frac{\partial}{\partial t}(\rho v_i)+\nabla\cdot(\rho v_i\vec{v})+\vec{\nabla}P=0$$

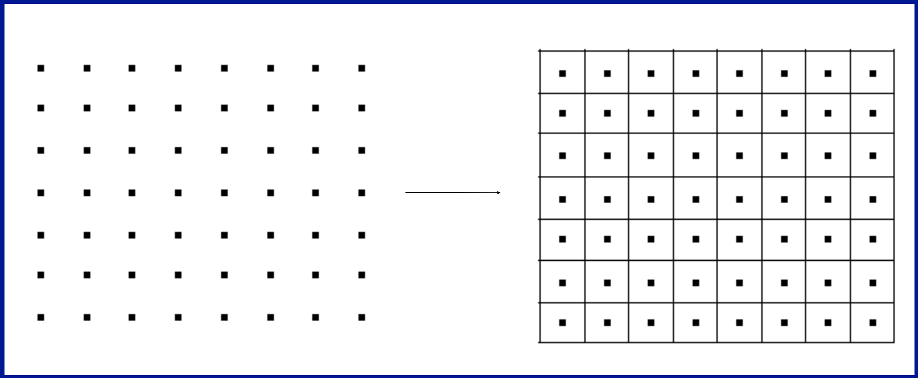
$$\frac{\partial}{\partial t}(\frac{1}{2}\rho v^2+\epsilon)+\nabla\cdot((\frac{1}{2}\rho v^2+\epsilon+P)\vec{v})=0$$

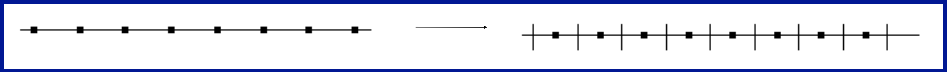
In General,

$$\frac{\partial}{\partial t}(U)+\nabla\cdot(\vec{F})=0$$

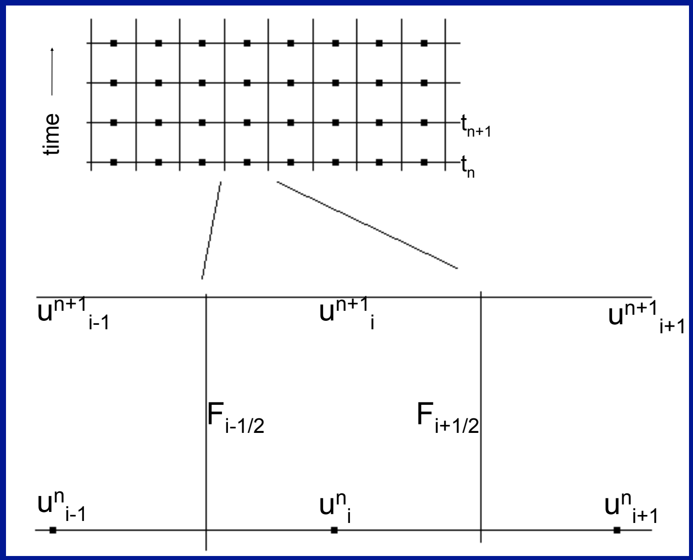
$$\partial_\mu Q^\mu=0$$

Numerical Methods for Solving Conservation Laws





Numerical Methods for Solving Conservation Laws



Numerical Methods for Solving Conservation Laws

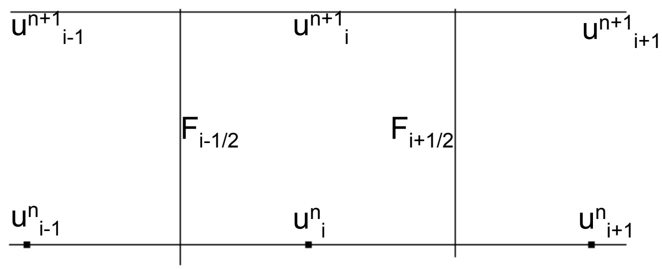
$$\frac{\partial u}{\partial t}+\frac{\partial F}{\partial x}=0$$

$$\int dx\,dt\,\frac{\partial u}{\partial t}+\int dx\,dt\,\frac{\partial F}{\partial x}=0$$

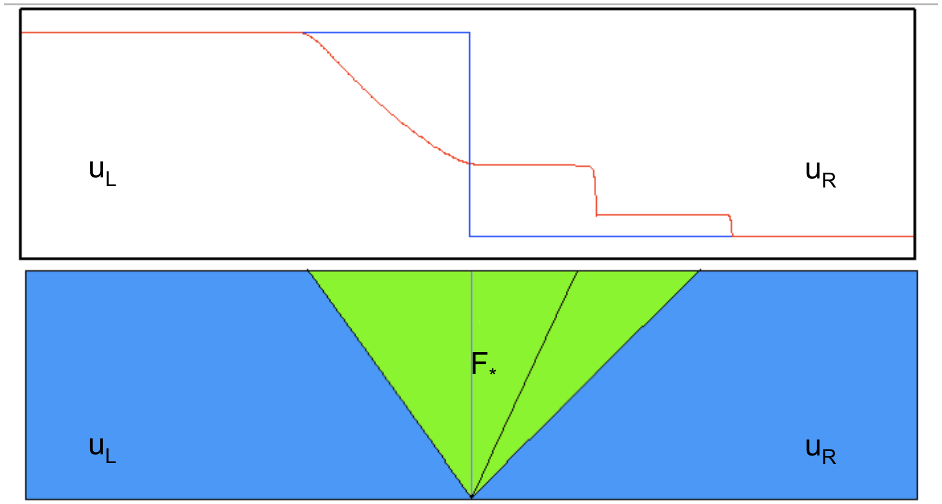
$$\left[\int dx\,u\right]_n^{n+1}+\left[\int dt\,F\right]_{i-1/2}^{i+1/2}=0$$

$$\Delta x\,(u_i^{n+1}-u_i^n)+\Delta t\,(F_{i+1/2}-F_{i-1/2})=0$$

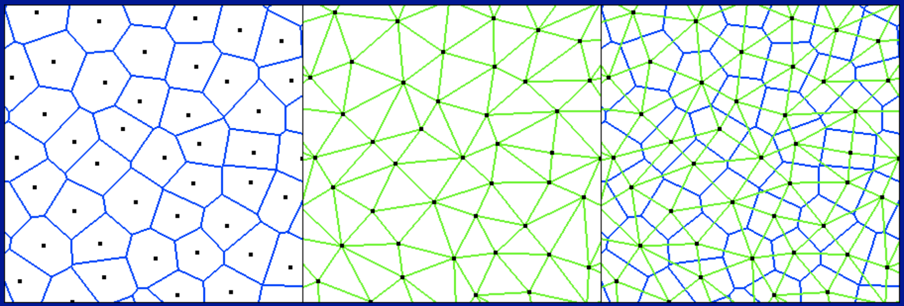
$$u_i^{n+1}=u_i^n-\frac{\Delta t}{\Delta x}(F_{i+1/2}-F_{i-1/2})$$



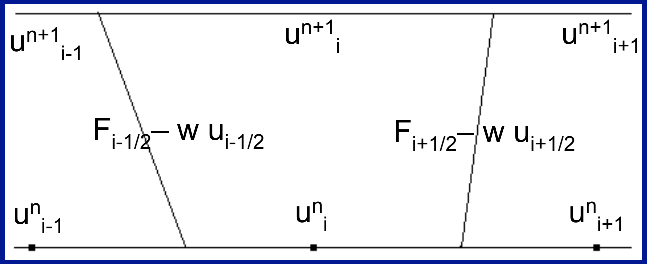
Numerical Methods for Solving Conservation Laws



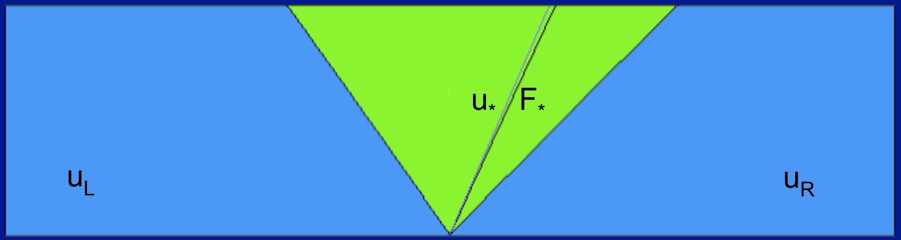
TESS: Lagrangian Hydrodynamics using a Dynamic Voronoi Mesh



TESS: Lagrangian Hydrodynamics using a Dynamic Voronoi Mesh



$$\vec{F}_* \to \vec{F}_* - \vec{w}\,u_*$$



Relativistic Hydrodynamics

$$\partial_\mu(\rho u^\mu)=0$$

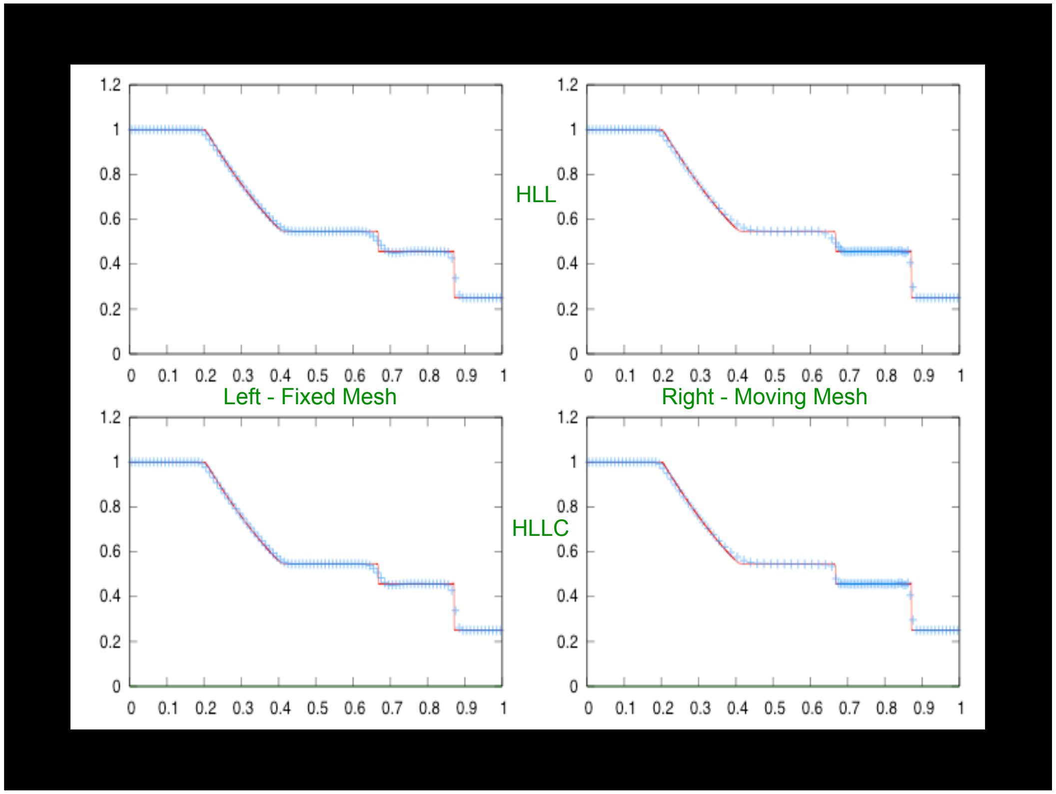
$$\partial_\mu((\rho+\epsilon+P)u^\mu u^\nu+P g^{\mu\nu})=0$$

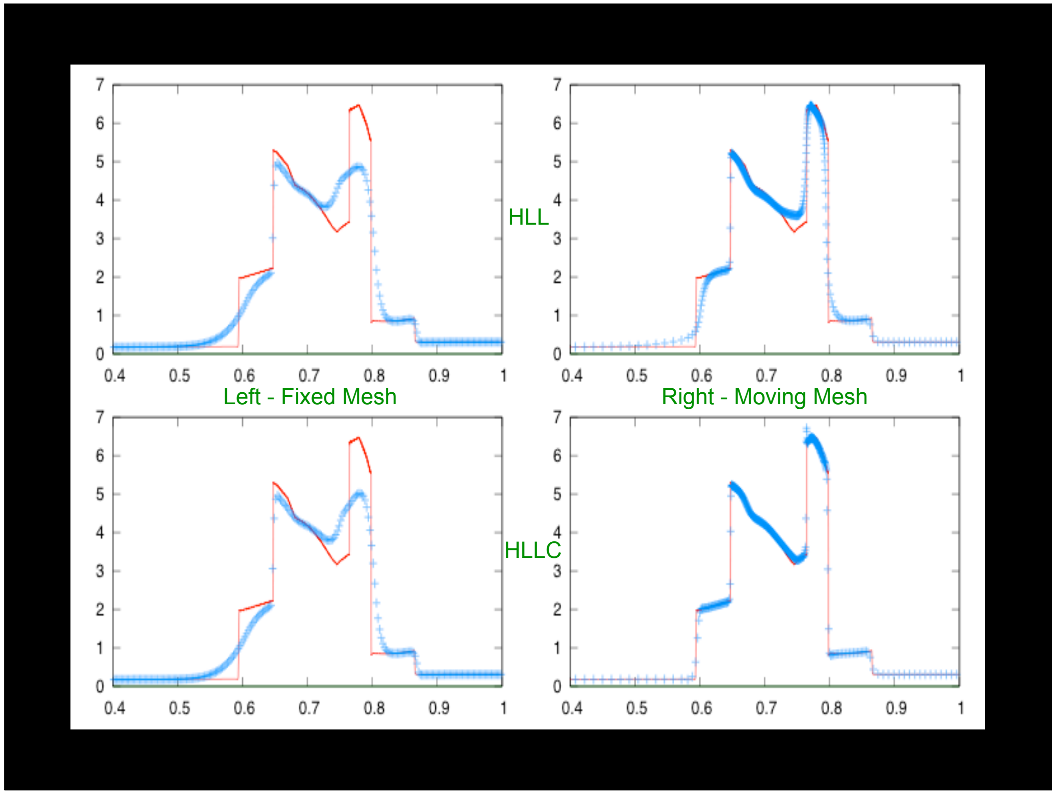
$$D=\rho u^0$$

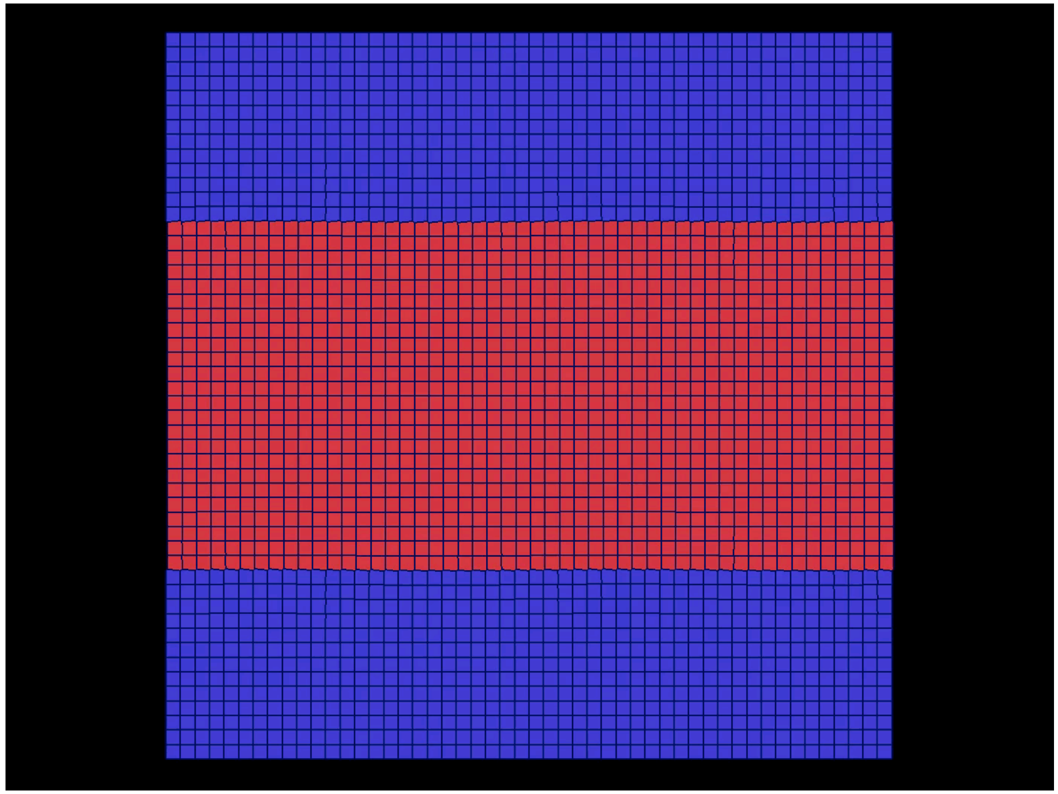
$$S_i=(\rho+\epsilon+P)u^0 u^i$$

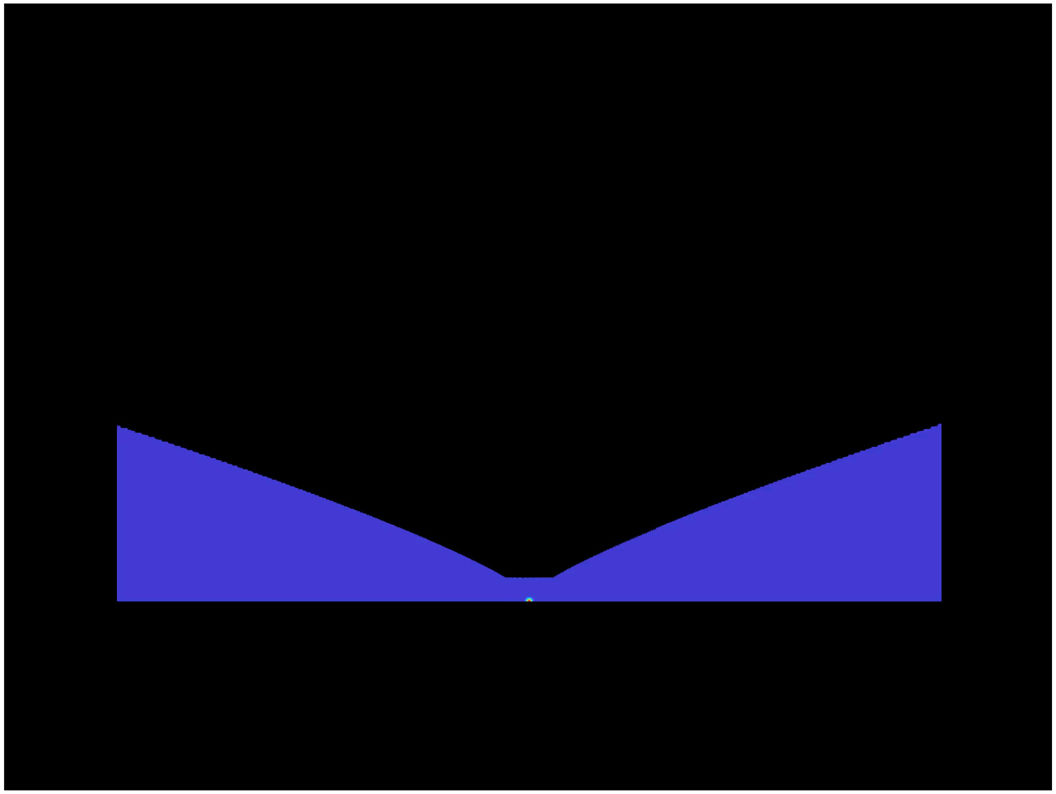
$$E=(\rho+\epsilon+P)u^0 u^0-P$$

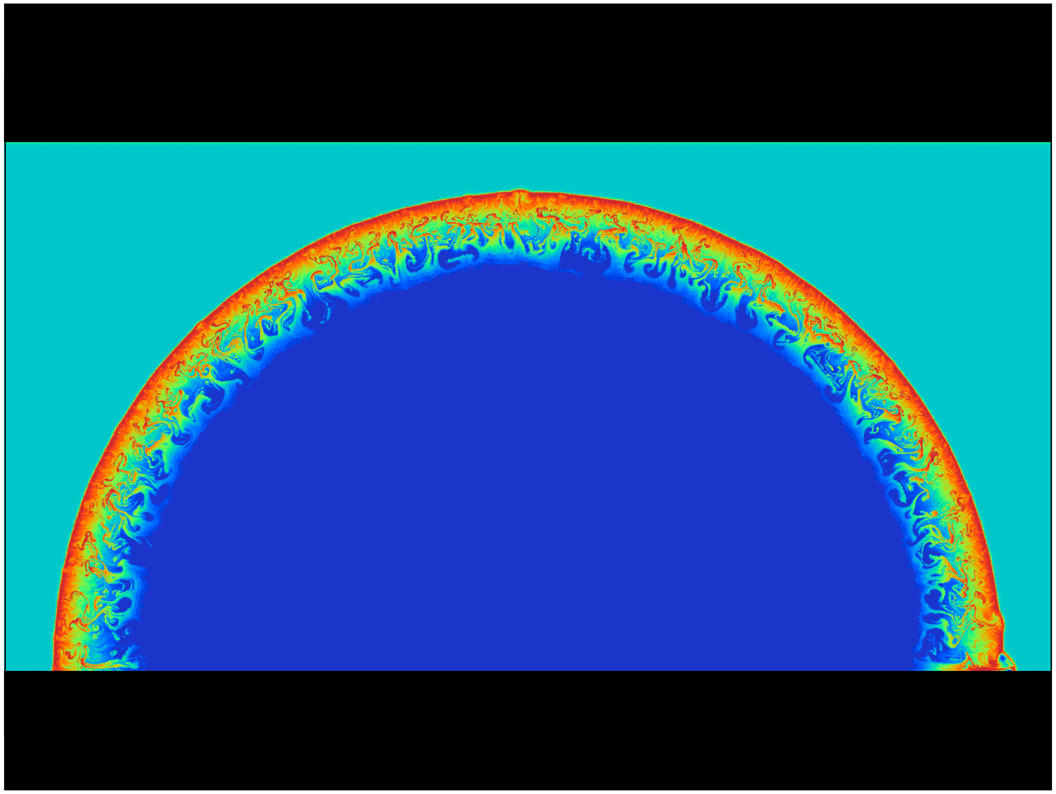
The Voronoi mesh doesn't have to take into account relativity
(it's okay to have faces that move superluminally)

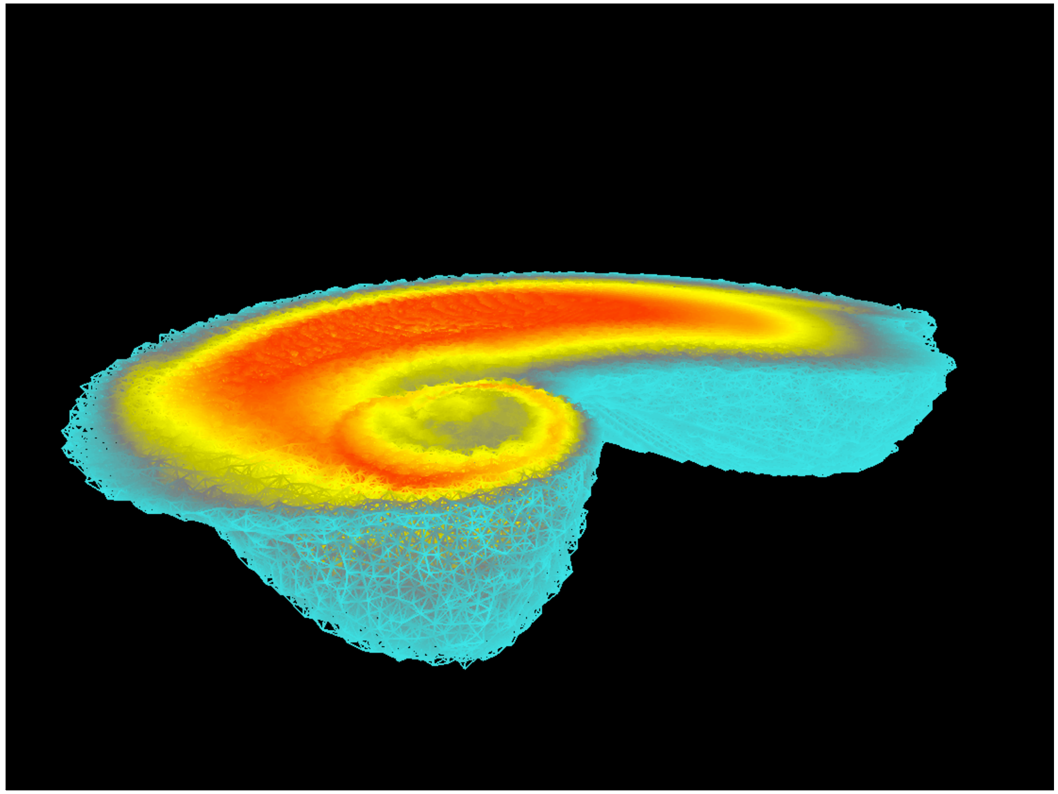

HLL
Left - Fixed Mesh
Right - Moving Mesh
HLLC

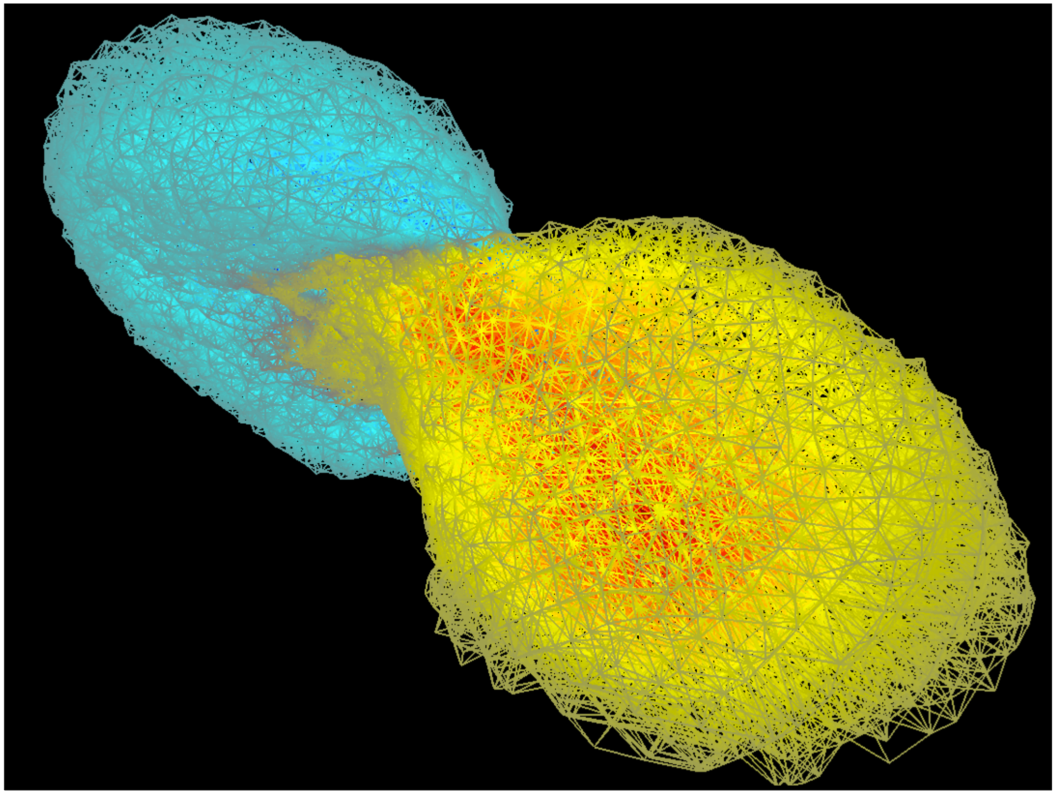

HLL
HLLC
Left - Fixed Mesh
Right - Moving Mesh

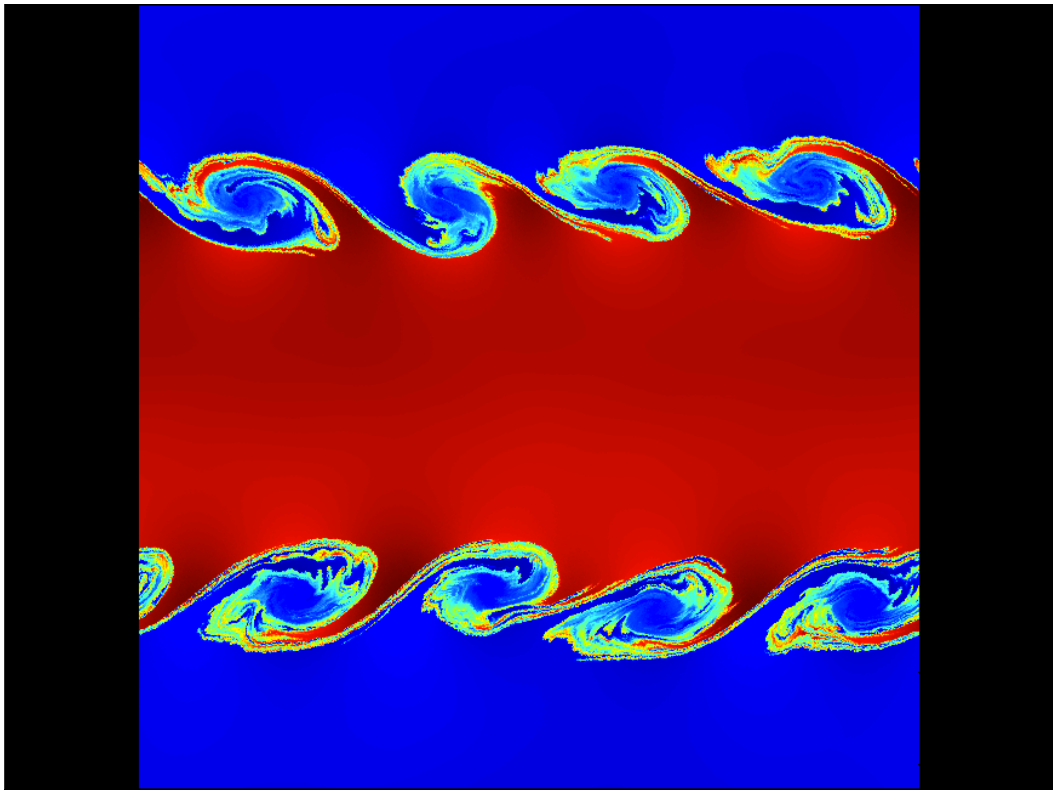


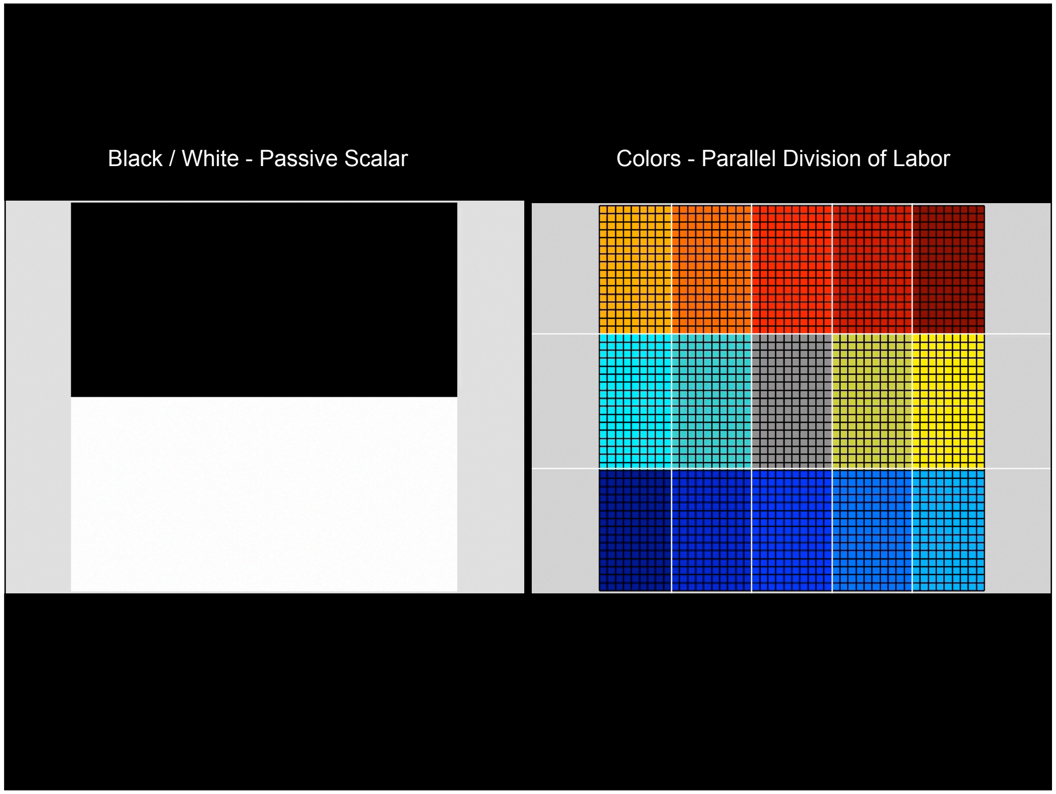


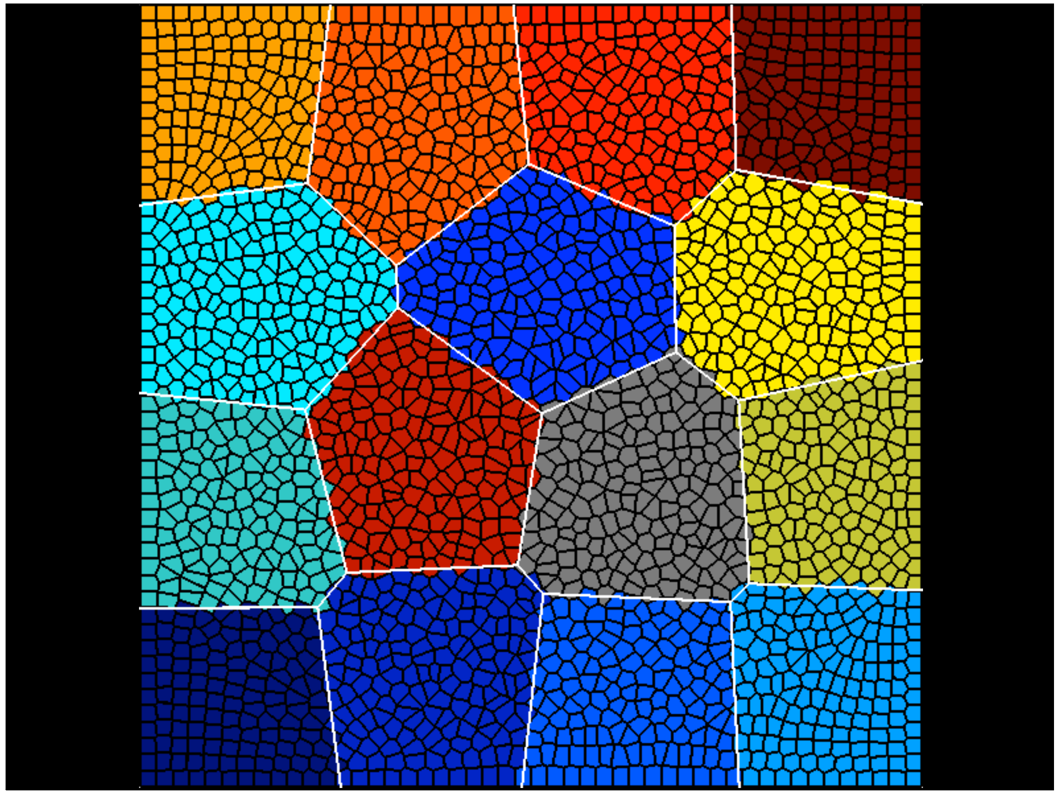


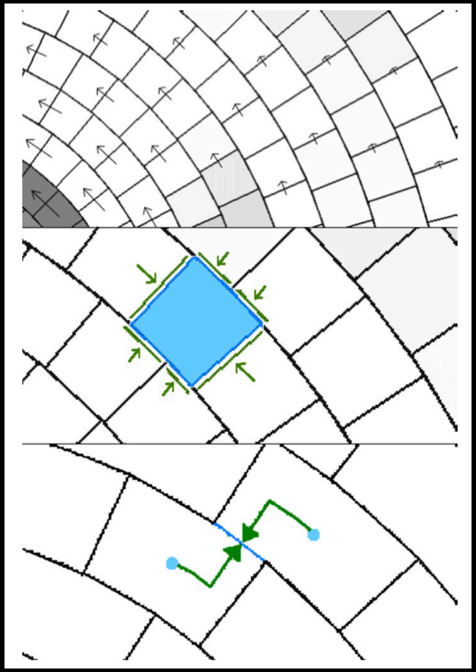


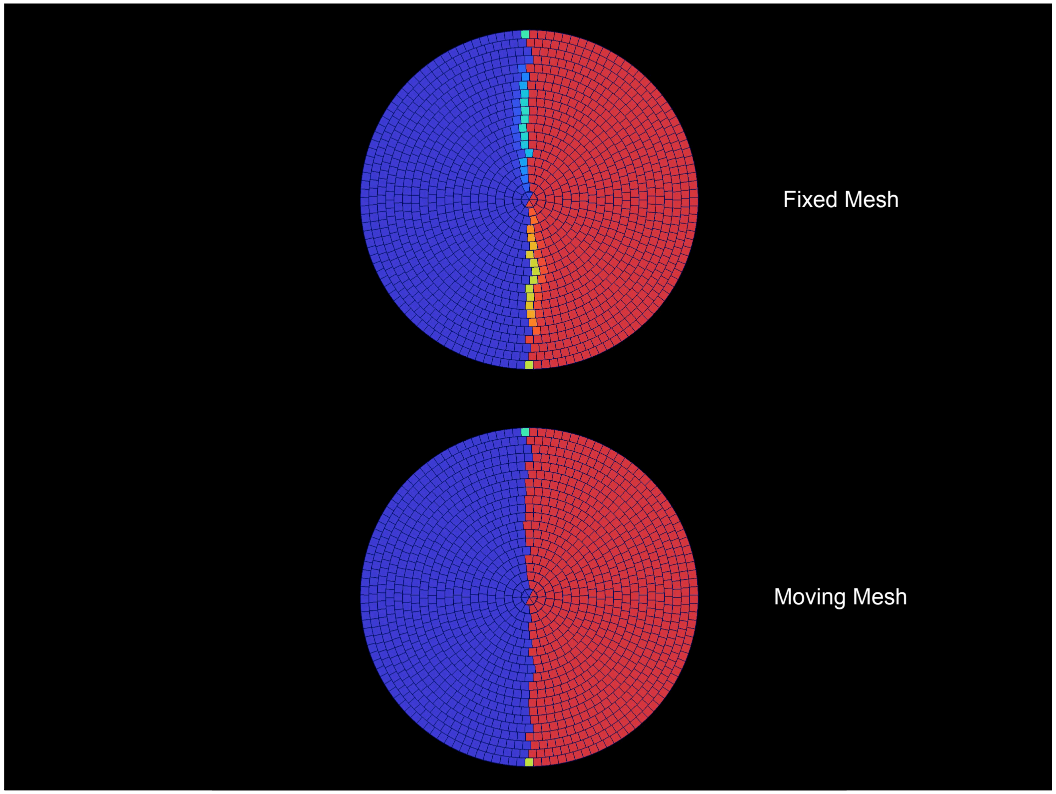


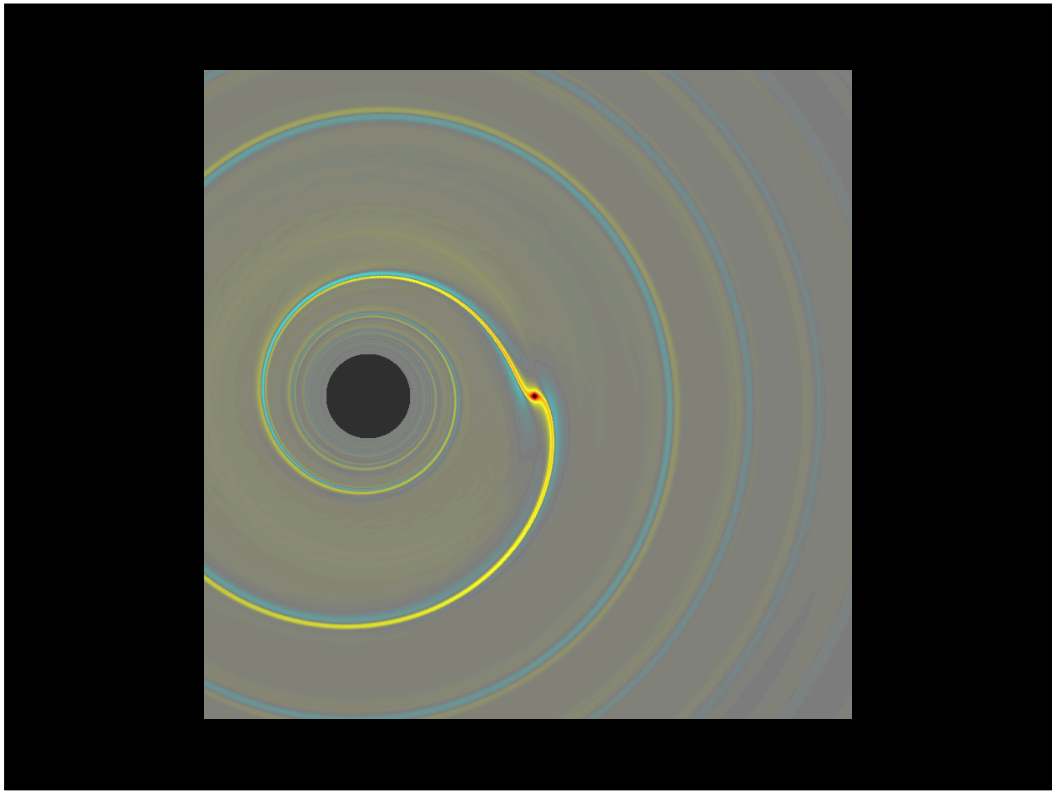


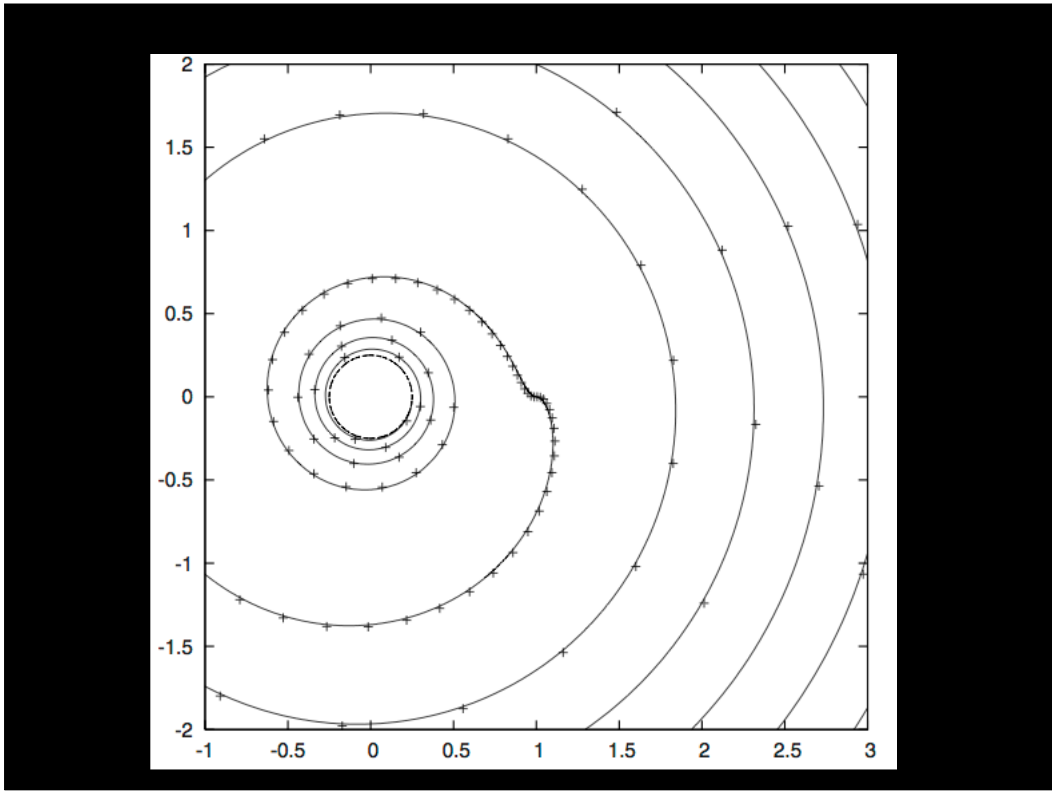

Black / White - Passive Scalar
Colors - Parallel Division of Labor

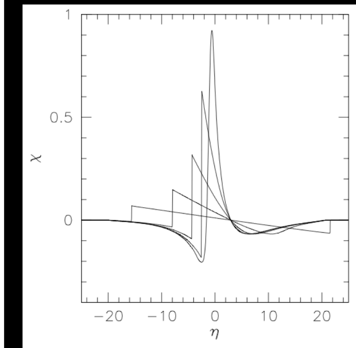




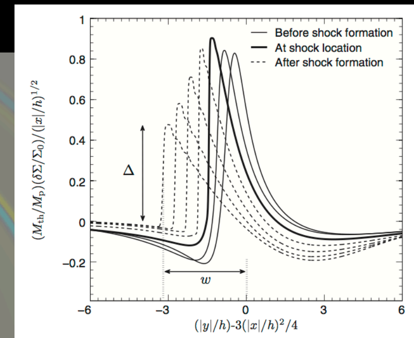

Fixed Mesh
Moving Mesh



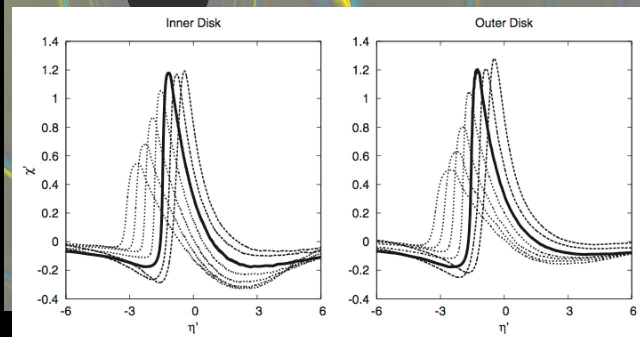

2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
-0.5
0
0.5
1
1.5
2
2.5
3



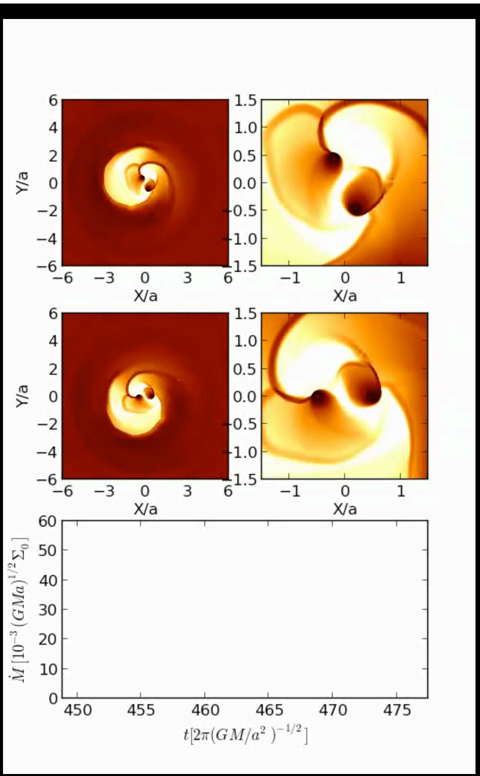
Goodman & Rafikov, 2001



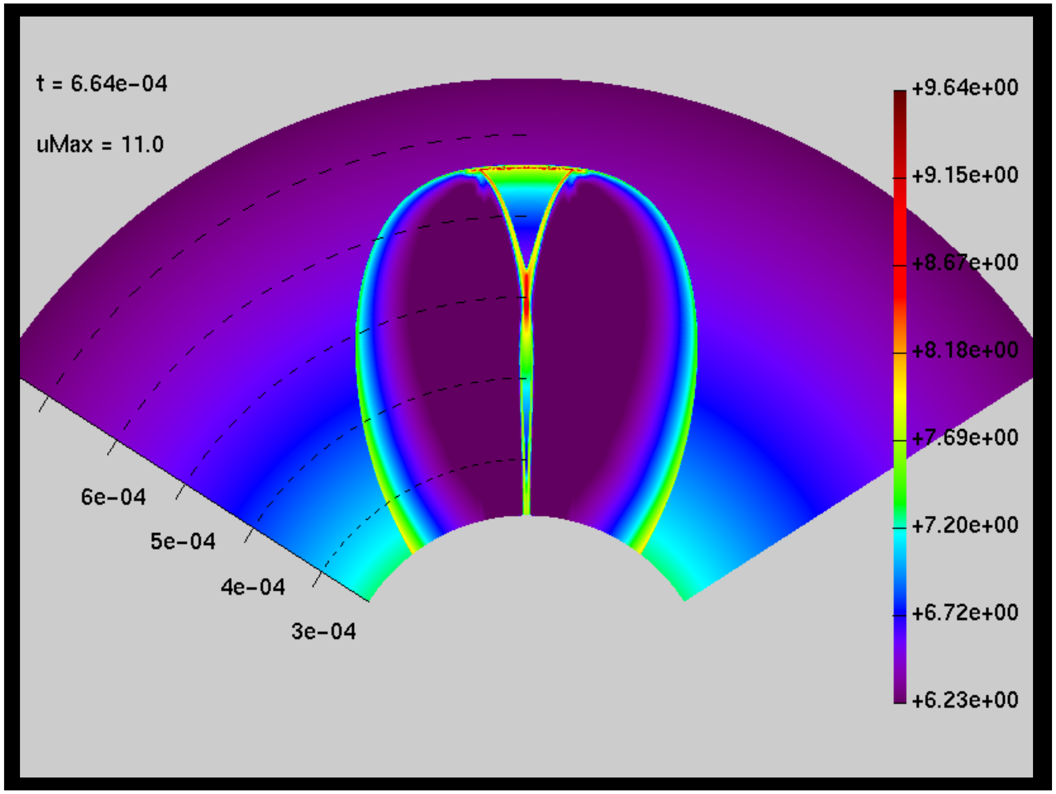
Dong, Rafikov & Stone, 2011

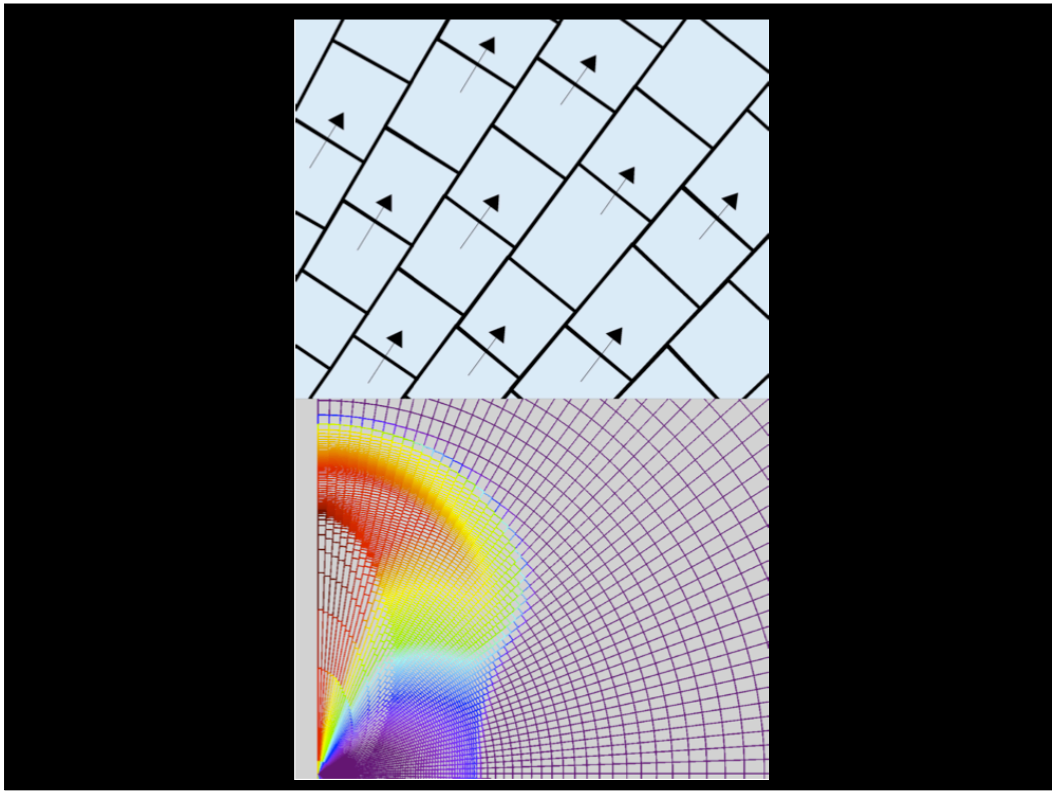


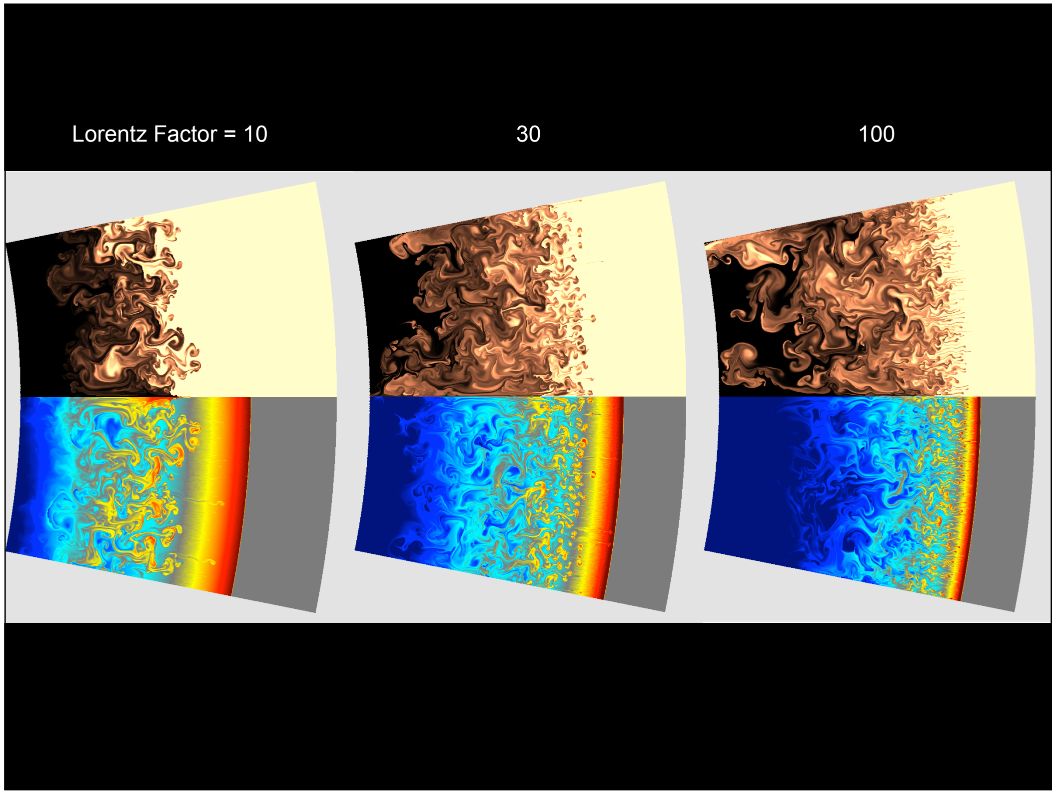
Duffell & MacFadyen, 2012



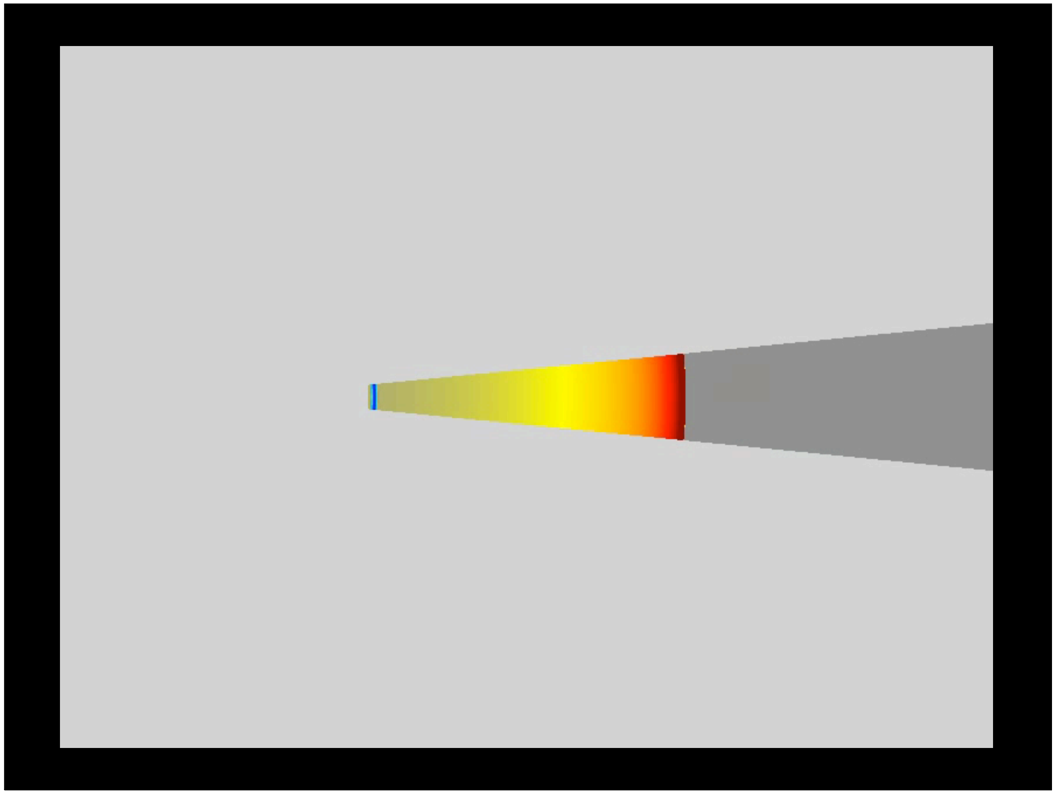
Y/a
X/a
$\dot{M}\,[10^{-3}(GMa)^{1/2}\Sigma_0]$
$t[2\pi(GM/a^2)^{-1/2}]$

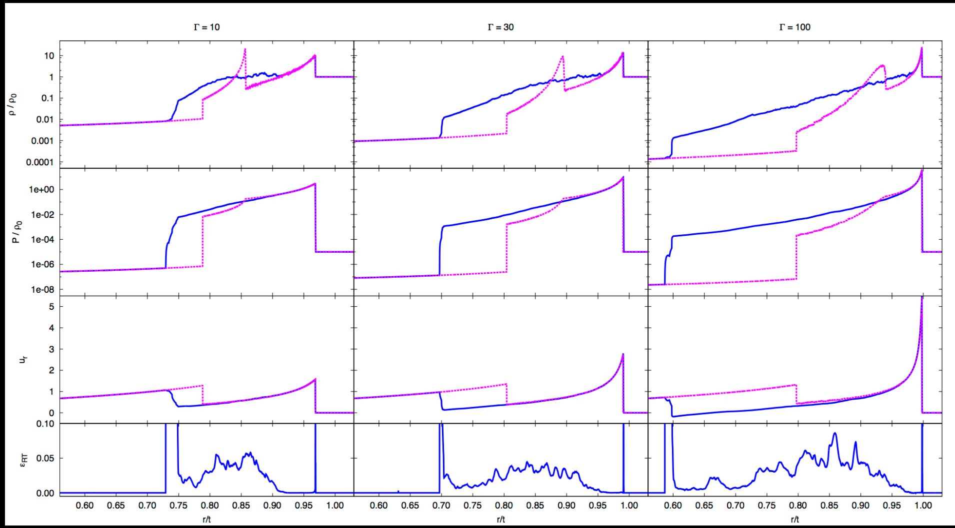

t = 6.64e−04
uMax = 11.0
6e−04
5e−04
4e−04
3e−04
+9.64e+00
+9.15e+00
+8.67e+00
+8.18e+00
+7.69e+00
+7.20e+00
+6.72e+00
+6.23e+00



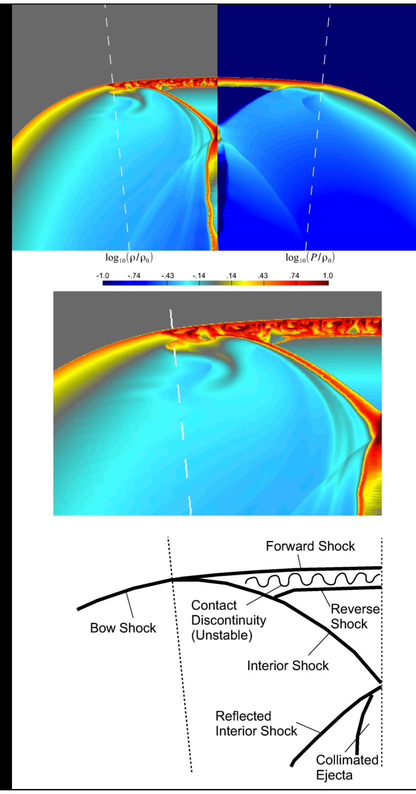


Lorentz Factor = 10
30
100

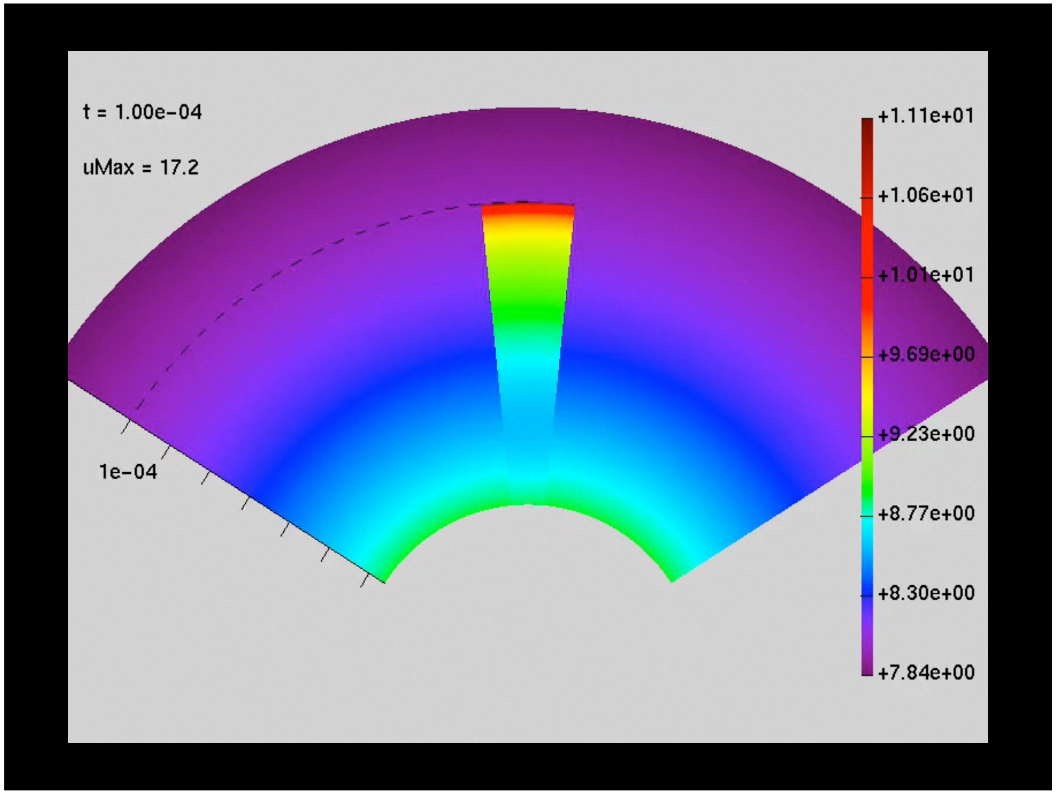




Γ = 10
Γ = 30
Γ = 100
ρ / ρ0
P / ρ0
ur
εRT
r/t



$\log_{10}(\rho/\rho_0)$
$\log_{10}(P/\rho_0)$
-1.0 -.74 -.43 -.14 .14 .43 .74 1.0
Forward Shock
Bow Shock
Contact Discontinuity (Unstable)
Reverse Shock
Interior Shock
Reflected Interior Shock
Collimated Ejecta



t = 1.00e−04
uMax = 17.2
1e−04
+1.11e+01
+1.06e+01
+1.01e+01
+9.69e+00
+9.23e+00
+8.77e+00
+8.30e+00
+7.84e+00