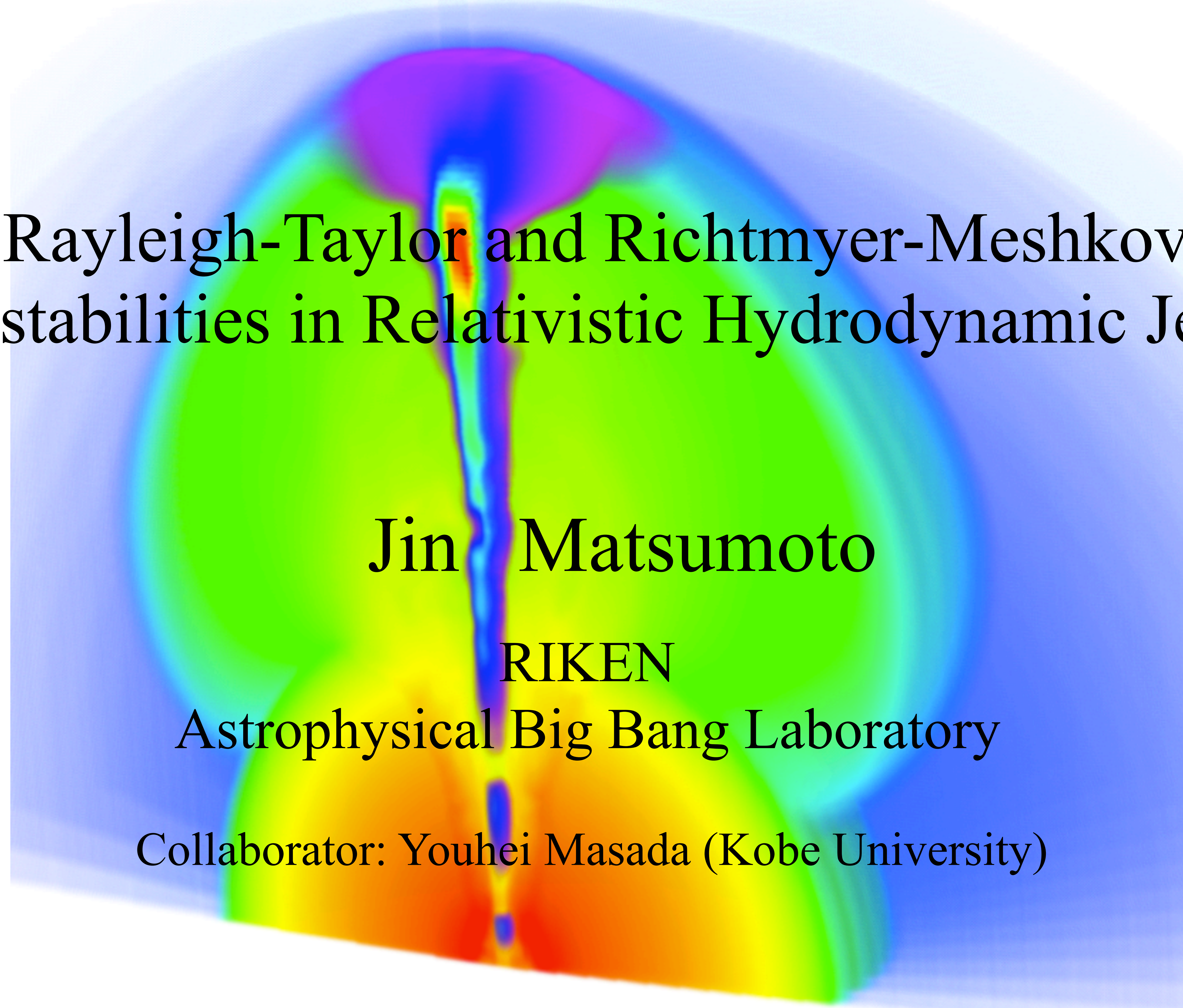

Rayleigh-Taylor and Richtmyer-Meshkov Instabilities in Relativistic Hydrodynamic Jets

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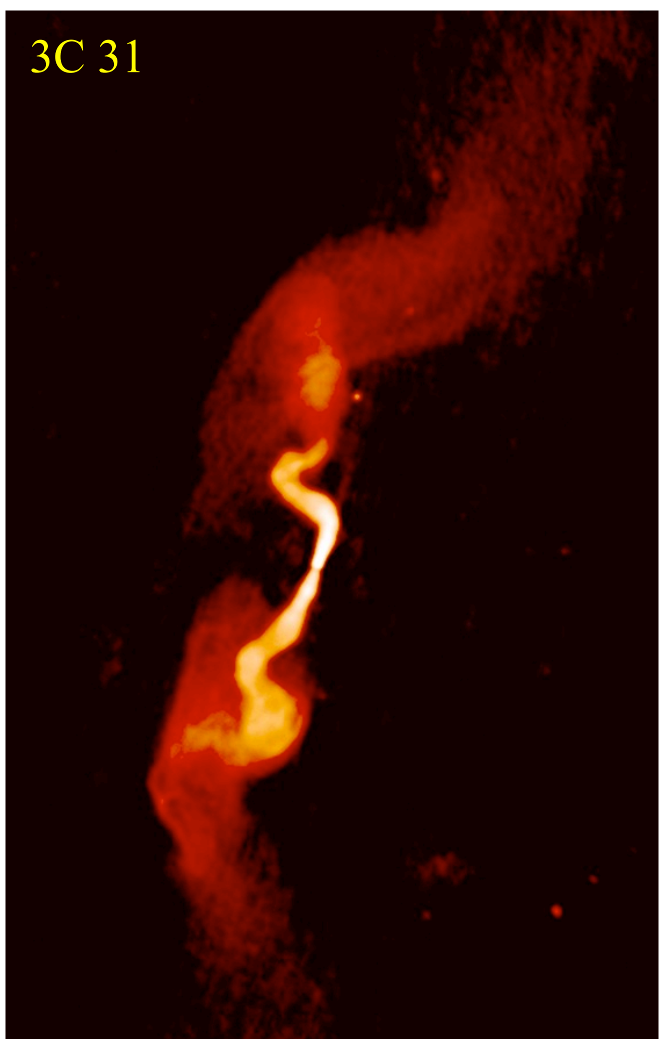
What a relativistic jet?

collimated bipolar outflow from gravitationally bounded object

- active galactic nuclei (AGN) jet: $\gamma \sim 10$
- microquasar jet: $v \sim 0.9c$
- Gamma-ray burst: $\gamma > 100$

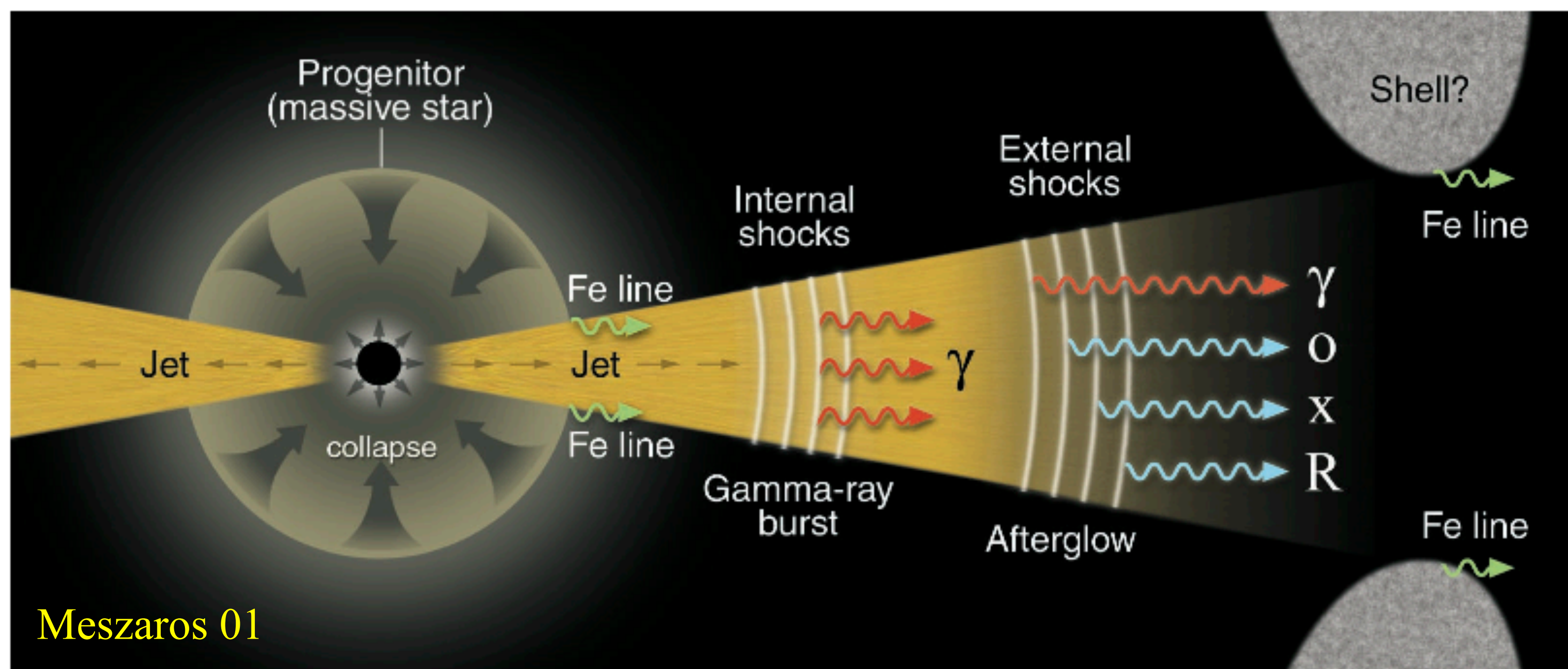
Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

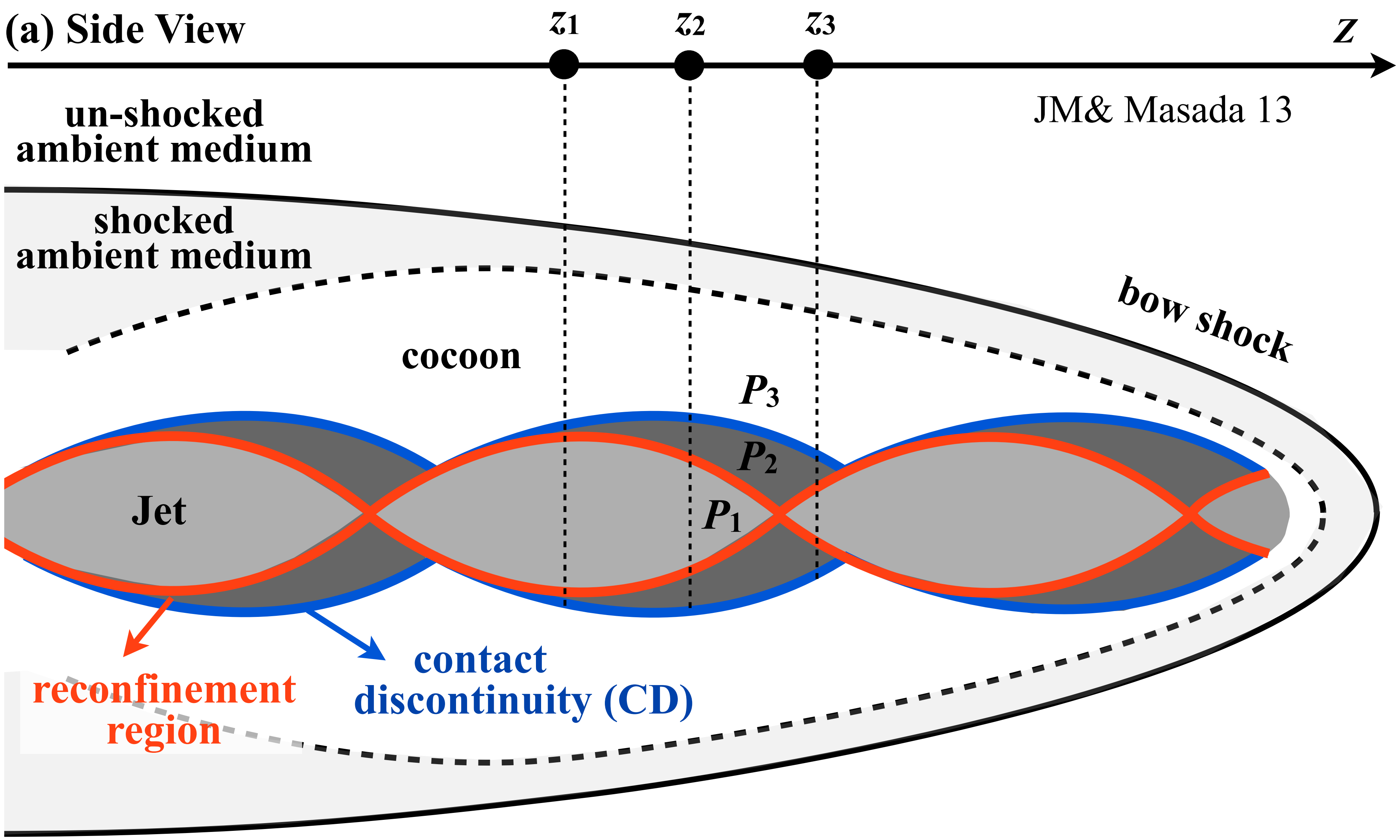


AGN jet

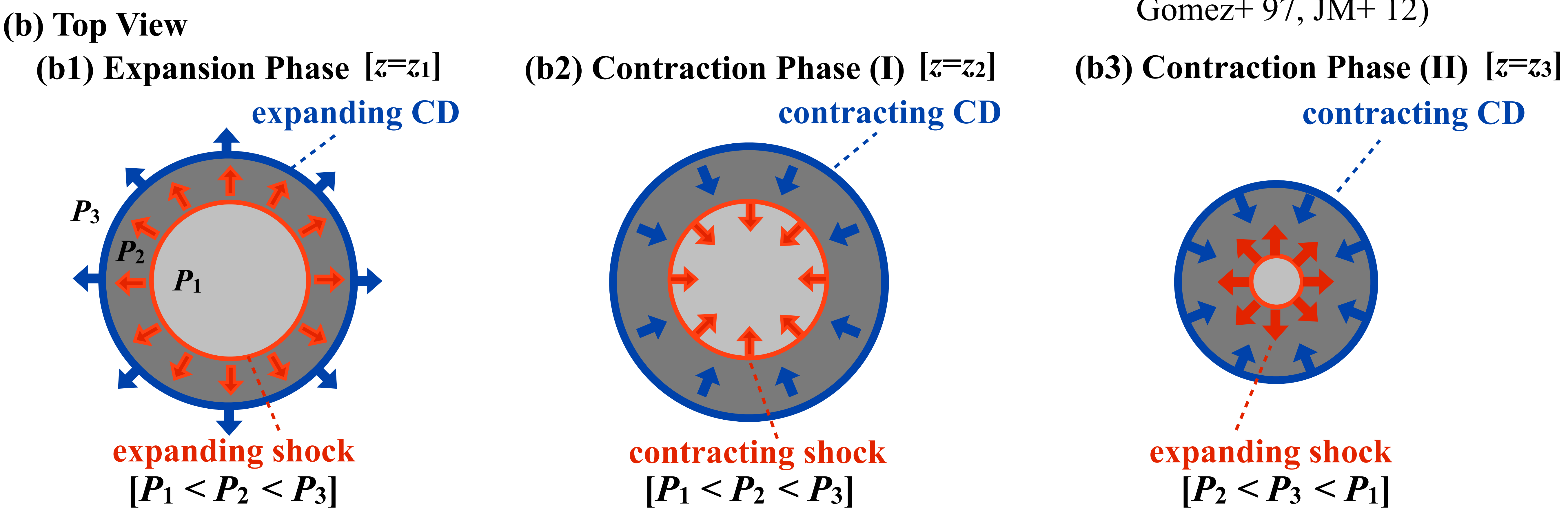
schematic picture of the GRB jet



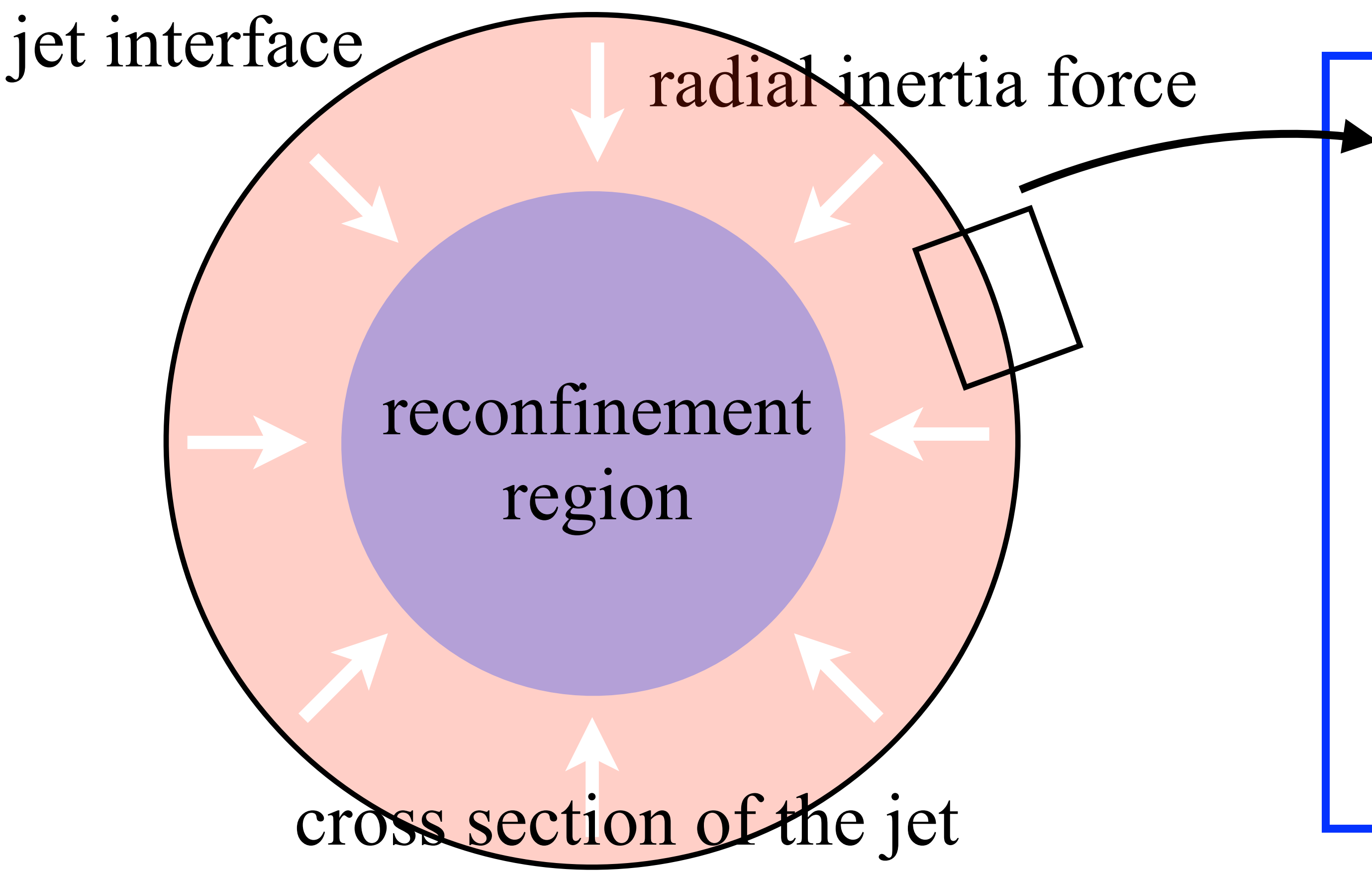
A relativistic jet is considered to be launched form the central engine and propagates the progenitor star.

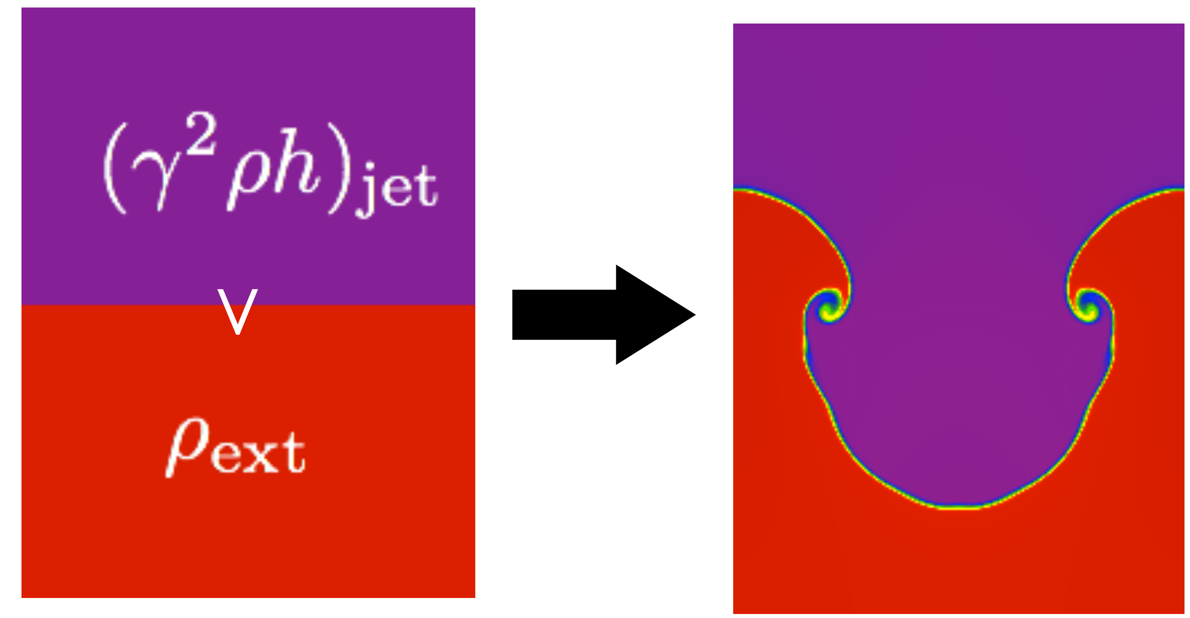


- many numerical works in order to investigate the propagation dynamics of the relativistic jet (e.g., Marti+ 97, Aloy+ 00, Zhang+ 03,04, Mizuta+ 06, Morsony+ 07, Lazzati+ 09, Nagakura+ 11, Lopez-Camara+ 13)
- reconfinement shock (Norman et al. 1982; Sanders 1983)
- radial oscillating motion and repeated excitation of the reconfinement region (e.g., Gomez+ 97, JM+ 12)



Motivation of Our Study





Rayleigh-Taylor instability grows?

To investigate the propagation dynamics and stability of the relativistic jet

- using relativistic hydrodynamic simulations

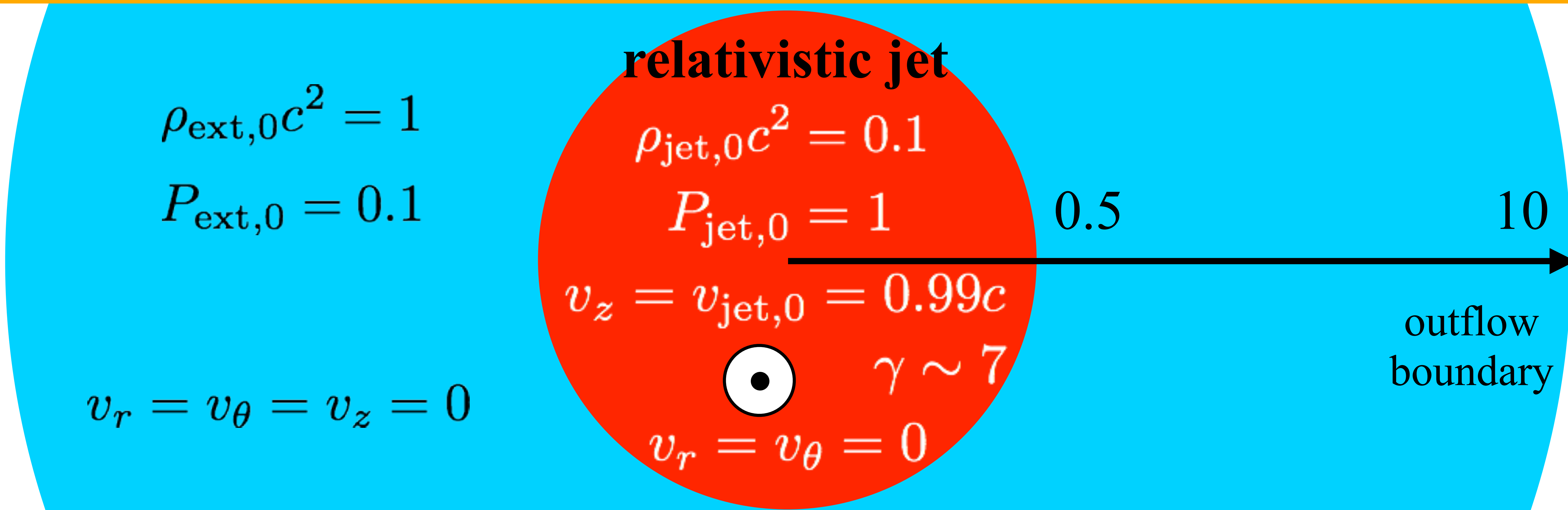
  focus on the transverse structure of the jet

- 2D simulations: evolution of the cross section of the relativistic jet
- 3D simulations: evolution of the cross section of the relativistic jet
- 3D simulation: propagation of the relativistic jet

2D simulations:
evolution of the cross section of the relativistic jet

Matsumoto & Masada, ApJL, **772**, L1 (2013)

Numerical Setting: 2D Toy Model



- cylindrical coordinate: (r - θ plane)
- relativistically hot jet (z-direction)
- ideal gas
- numerical scheme: HLLC (Mignone & Bodo 05)
- uniform grid: $\Delta r = \Delta z = 10/320$, $\Delta\theta = 2\pi/200$

Basic Equations

mass conservation

$$\frac{\partial}{\partial t}(\gamma\rho)+\frac{1}{r}\frac{\partial}{\partial r}(r\gamma\rho v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\gamma\rho v_\theta)=0$$

momentum conservation

$: r$

$$\frac{\partial}{\partial t}(\gamma^2\rho h v_r)+\frac{1}{r}\frac{\partial}{\partial r}(r(\gamma^2\rho h v_r^2+P))+\frac{1}{r}\frac{\partial}{\partial \theta}(\gamma^2\rho h v_r v_\theta)=\frac{P}{r}$$

$: \theta$

$$\frac{\partial}{\partial t}(\gamma^2\rho h v_\theta)+\frac{1}{r}\frac{\partial}{\partial r}(r(\gamma^2\rho h v_\theta v_r))+\frac{1}{r}\frac{\partial}{\partial \theta}(\gamma^2\rho h v_\theta^2+P)=-\frac{\gamma^2\rho h v_r v_\theta}{r}$$

$: z$

$$\frac{\partial}{\partial t}(\gamma^2\rho h v_z)+\frac{1}{r}\frac{\partial}{\partial r}(r(\gamma^2\rho h v_z v_r))+\frac{1}{r}\frac{\partial}{\partial \theta}(\gamma^2\rho h v_z v_\theta)=0$$

energy conservation

$$\frac{\partial}{\partial t}(\gamma^2\rho h-P)+\frac{1}{r}\frac{\partial}{\partial r}(r(\gamma^2\rho h v_r))+\frac{1}{r}\frac{\partial}{\partial \theta}(\gamma^2\rho h v_\theta)=0$$

specific enthalpy

$$\frac{h}{c^2}=1+\frac{\Gamma}{\Gamma-1}\frac{P}{\rho c^2}$$

ratio of specific heats

$$\Gamma=\frac{4}{3}$$

Lorentz factor

$$\gamma=\frac{1}{\sqrt{1-(v_r^2+v_\theta^2+v_z^2)}}$$

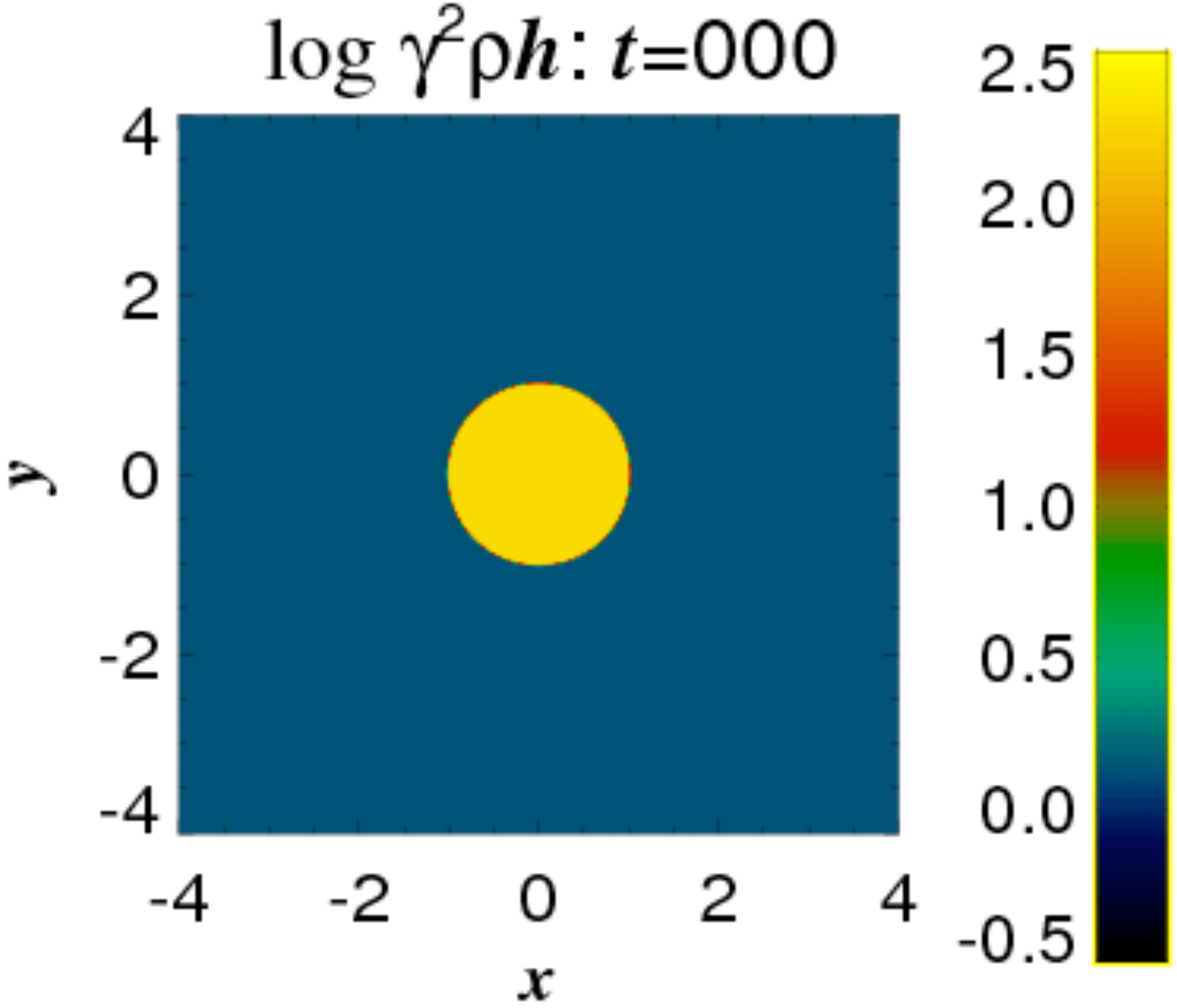
Time Evolution of Jet Cross Section

relativistically hot plasma:

$$\rho_{\rm jet} c^2 \leq P_{\rm jet}$$

effective inertia:

$$\gamma^2 \rho h = \gamma^2 (\rho c^2 + 4P)$$



The amplitude of the corrugated jet interface grows as time passes.

A finger-like structure is a typical outcome of the Rayleigh-Taylor instability.

Richtmyer-Meshkov Instability

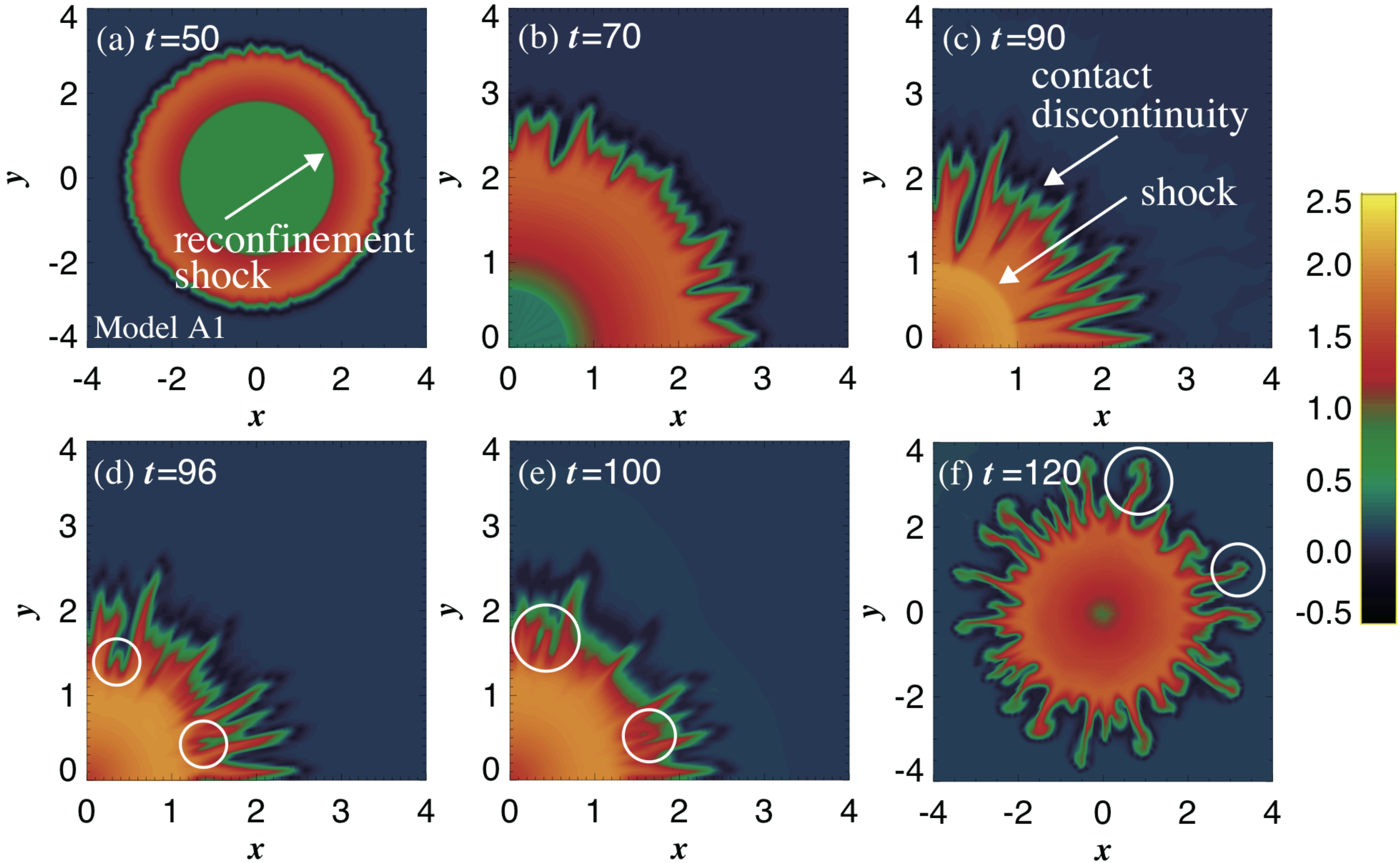
contact discontinuity

- The Richtmyer-Meshkov instability is induced by impulsive acceleration due to shock passage.
- The perturbation amplitude grows linearly in time (Richtmyer 1960)

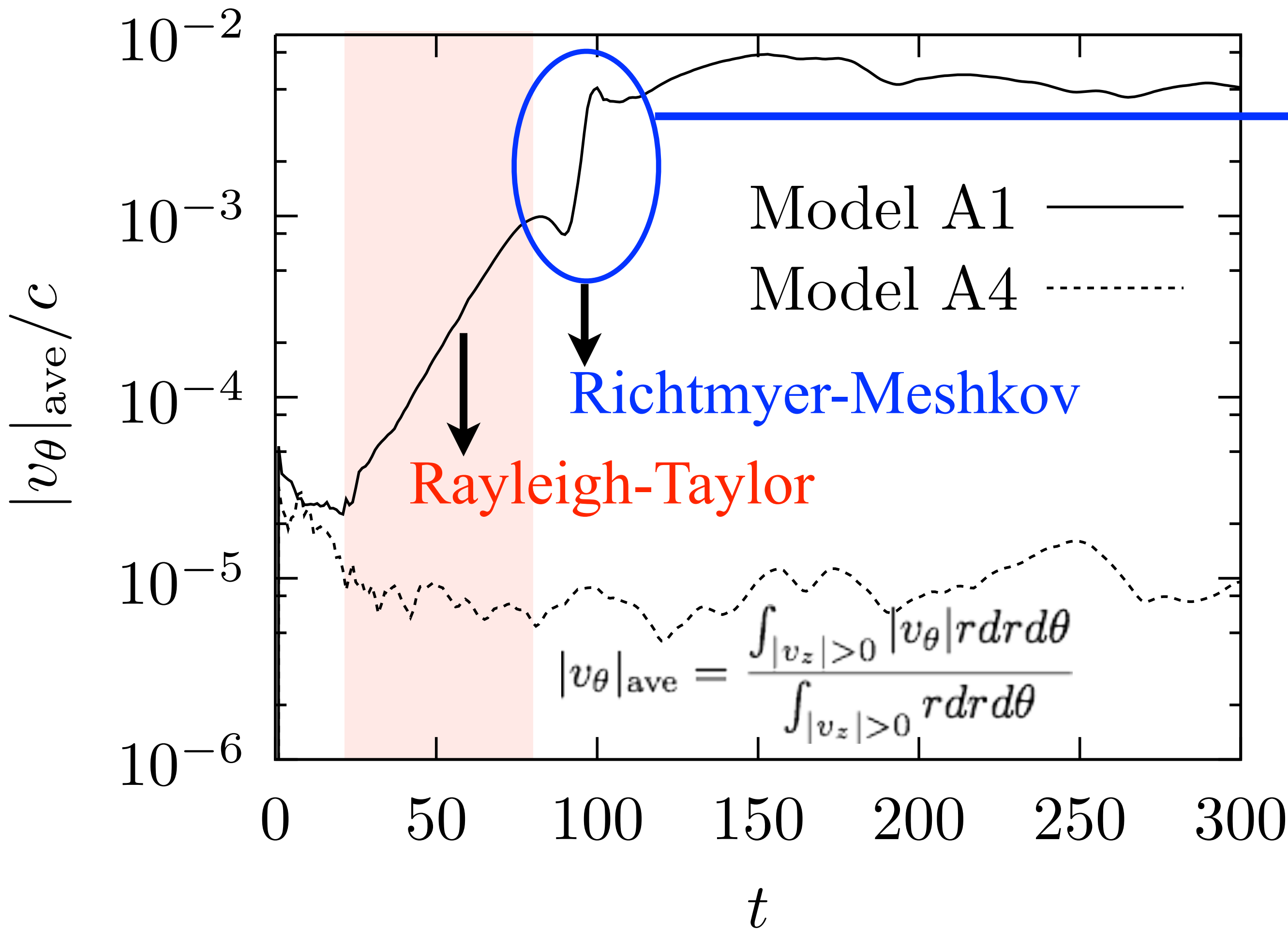
$$\frac{\partial \delta}{\partial t} = k\delta_0^* A^* v^* \quad , \qquad A^* = \frac{\rho_1^* - \rho_2^*}{\rho_1^* + \rho_2^*}$$

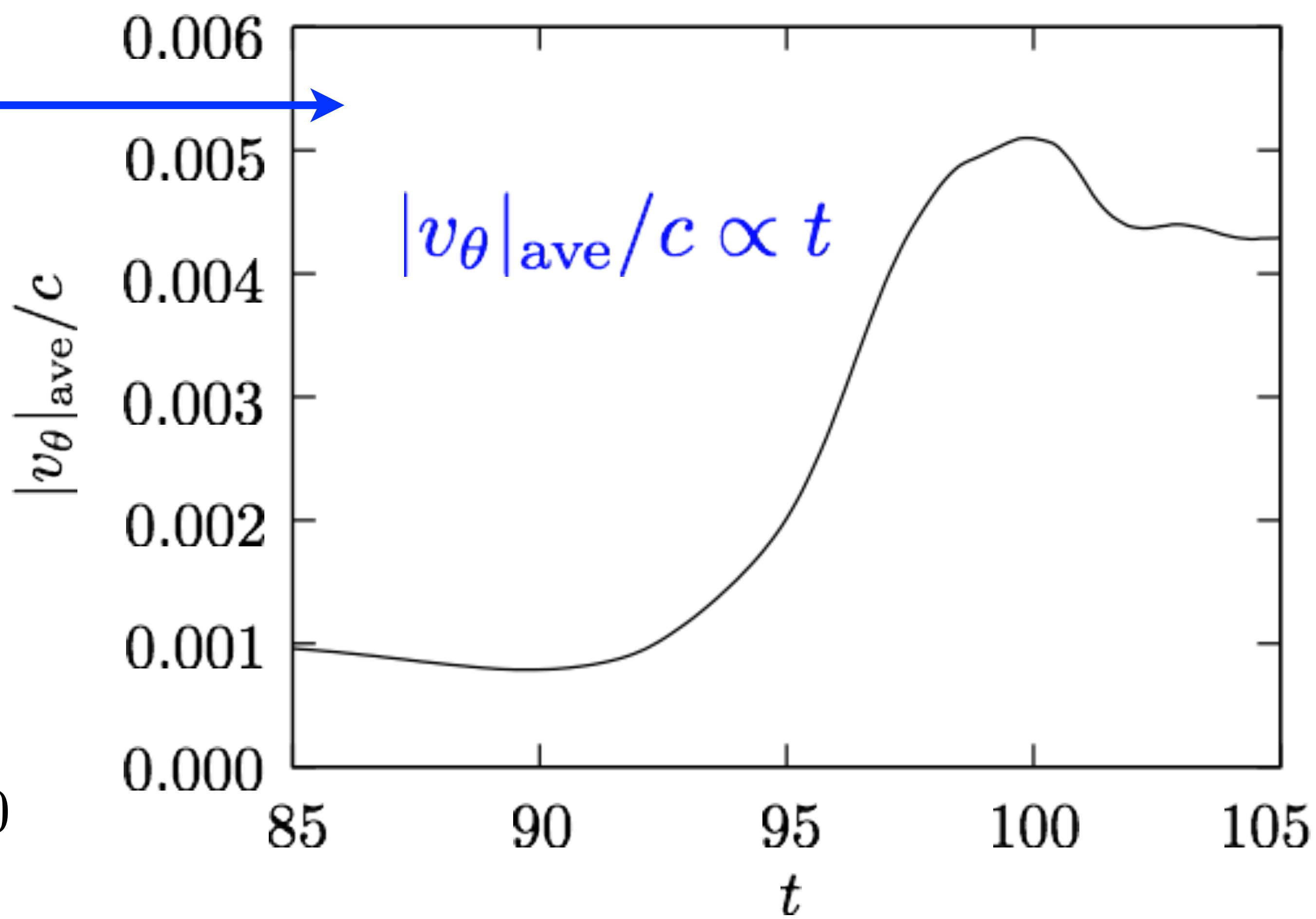
Time Evolution of Jet Cross Section

Effective inertia: $\log \gamma^2 \rho h$



Synergetic Growth of Rayleigh-Taylor and Richtmyer-Meshkov Instabilities





development of the Rayleigh-Taylor instability at the jet interface

$|v_\theta|_{\rm ave}$ increases exponentially.

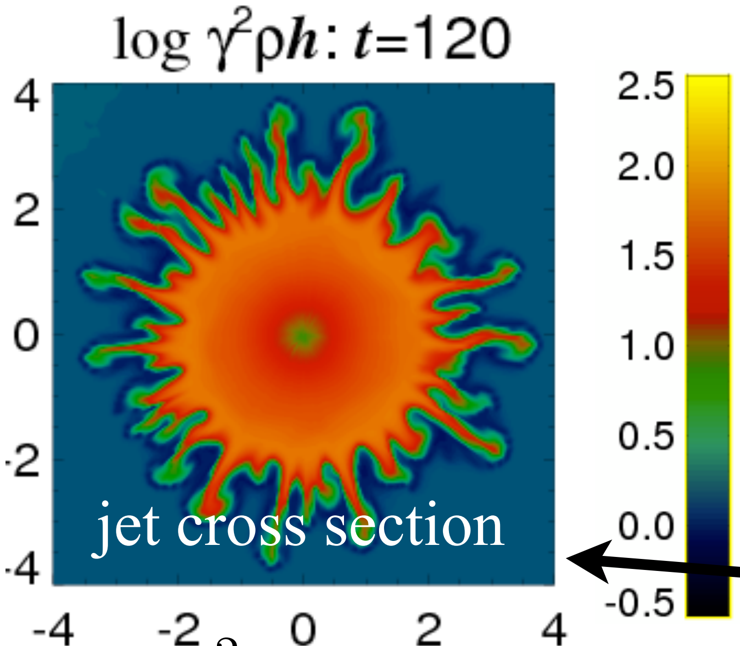
excitation of the Richtmyer-Meshkov instability at the jet interface

$|v_\theta|_{\rm ave}$ grows linearly with time.

The transverse structure of the jet is dramatically deformed by a synergetic growth of the Rayleigh-Taylor and Richtmyer-Meshkov instabilities once the jet-external medium interface is corrugated in the case with the pressure-mismatched jet.

Stability Condition of the Jet

2D simulations of transverse structure of the jet is **excluding** the destabilization effects by the **Kelvin-Helmholtz** mode



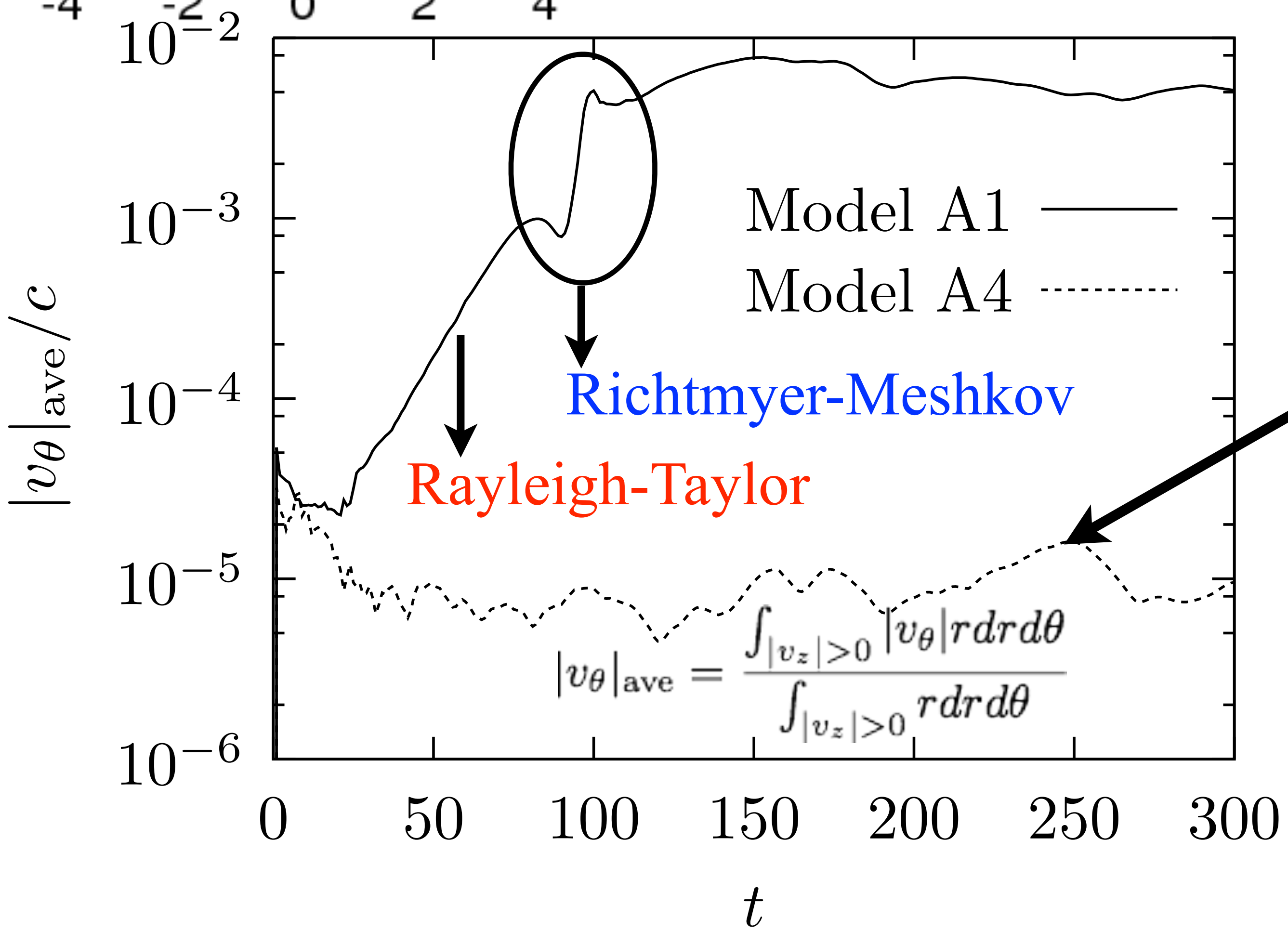
the stability criterion of the jet

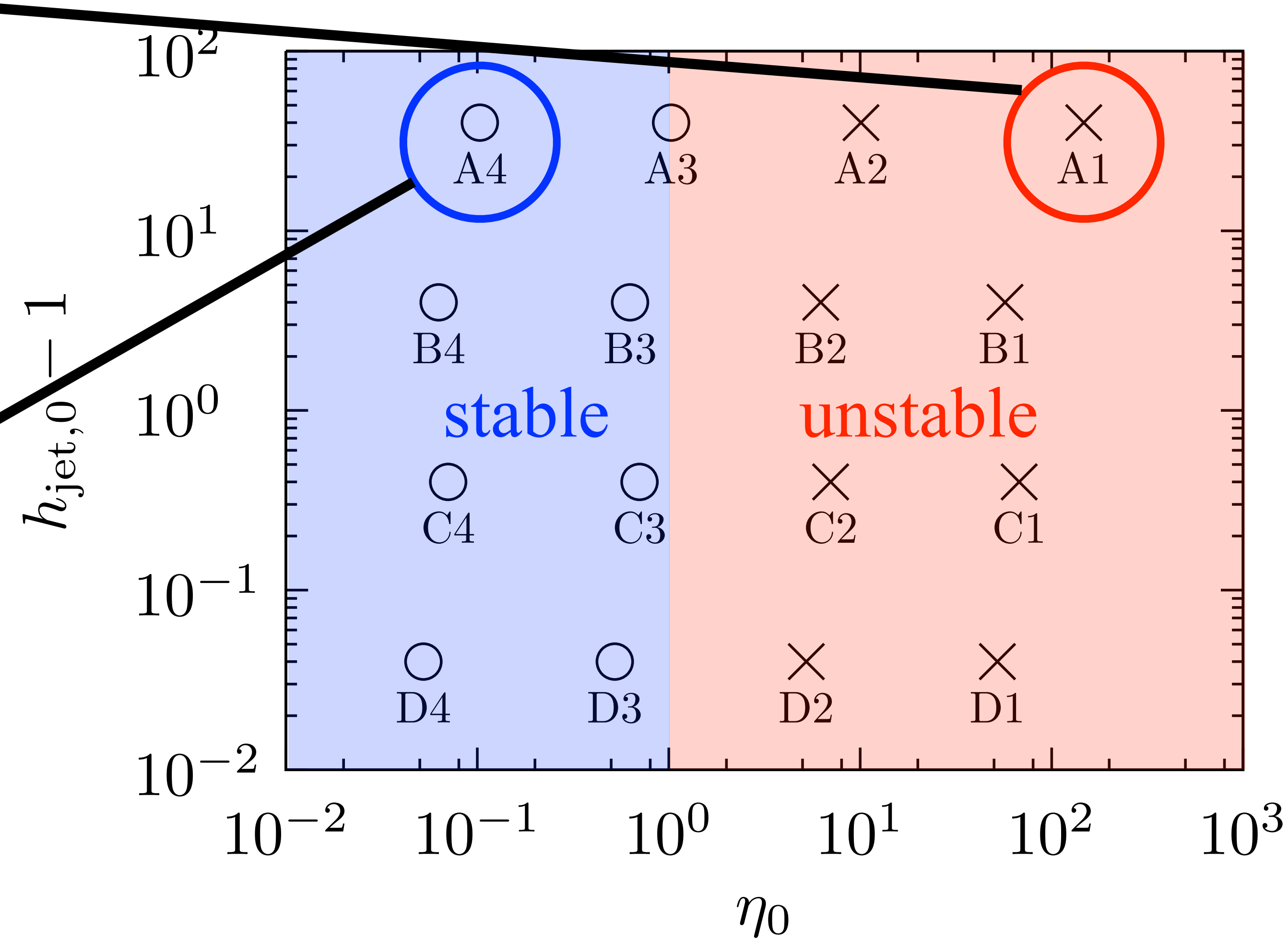
$$\eta_0 = \frac{{\gamma_{\rm jet,0}}^2 \rho_{\rm jet,0} h_{\rm jet,0}}{\rho_{\rm ext,0} h_{\rm ext,0}} > 1$$

fixed

$$\begin{cases} P_{\rm jet,0}/P_{\rm ext,0} = 10 \\ \gamma_{\rm jet} = 7 \end{cases}$$

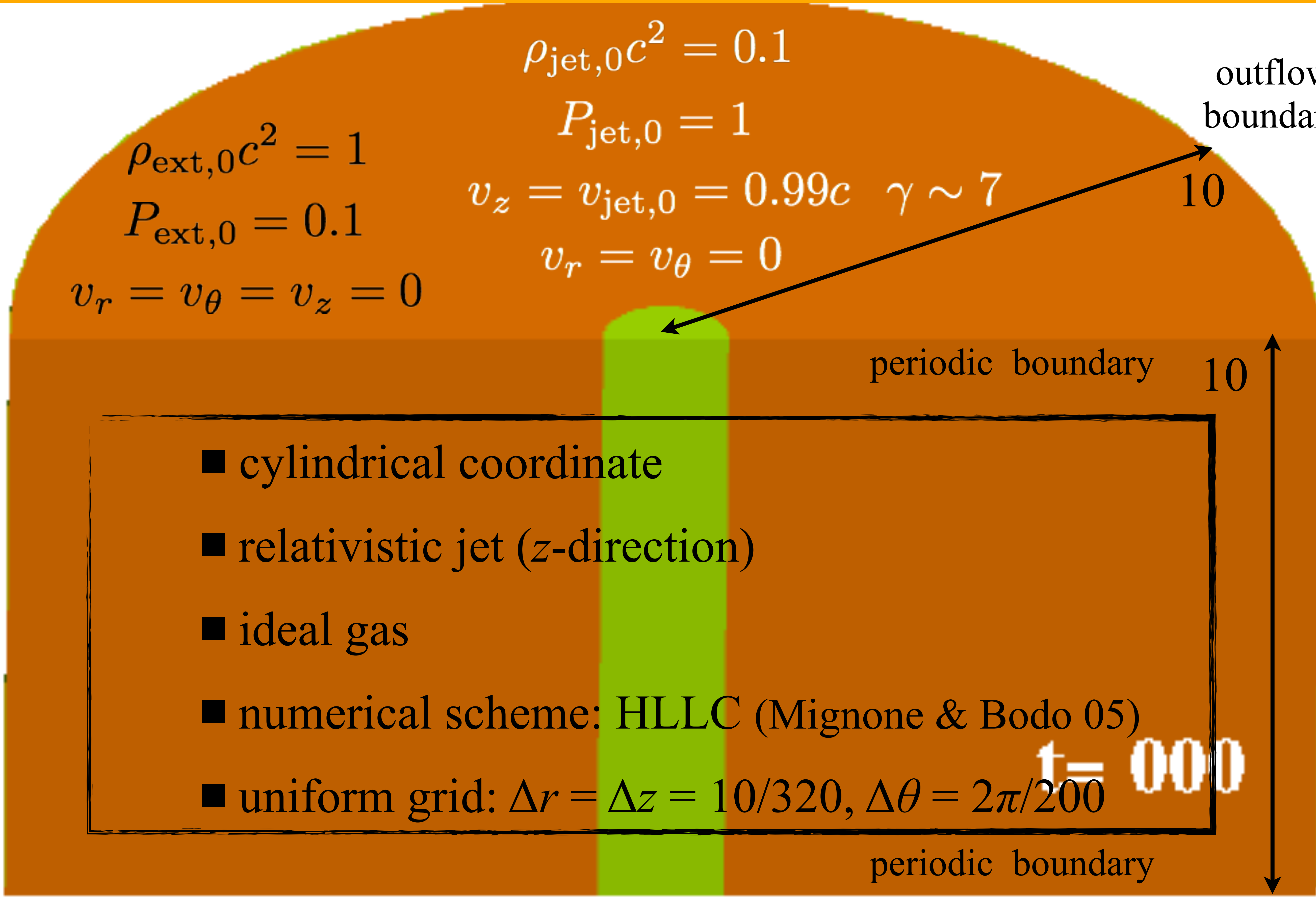
$$h_{\rm jet,0} = 1 + \frac{\Gamma}{\Gamma - 1} \frac{P_{\rm jet,0}}{\rho_{\rm jet,0} c^2}$$





3D simulations:
evolution of the cross section of the relativistic jet

Numerical Setting: 3D Toy Model 1



- cylindrical coordinate
- relativistic jet (*z*-direction)
- ideal gas
- numerical scheme: HLLC (Mignone & Bodo 05)
- uniform grid: $\Delta r = \Delta z = 10/320$, $\Delta\theta = 2\pi/200$

Basic Equations

mass conservation	$$\frac{\partial}{\partial t}(\gamma\rho) + \nabla\cdot(\gamma\rho\mathbf{v}) = 0$$
momentum conservation	$$\frac{\partial}{\partial t}(\gamma^2\rho h\mathbf{v}) + \nabla\cdot(\gamma^2\rho h\mathbf{v}\mathbf{v} + Pc^2\mathbf{I}) = 0$$
energy conservation	$$\frac{\partial}{\partial t}(\gamma^2\rho h - P) + \nabla\cdot(\gamma^2\rho h\mathbf{v}) = 0$$

specific enthalpy

$$\frac{h}{c^2} = 1 + \frac{\Gamma}{\Gamma - 1}\frac{P}{\rho c^2}$$

ratio of specific heats

$$\Gamma = \frac{4}{3}$$

Lorentz factor

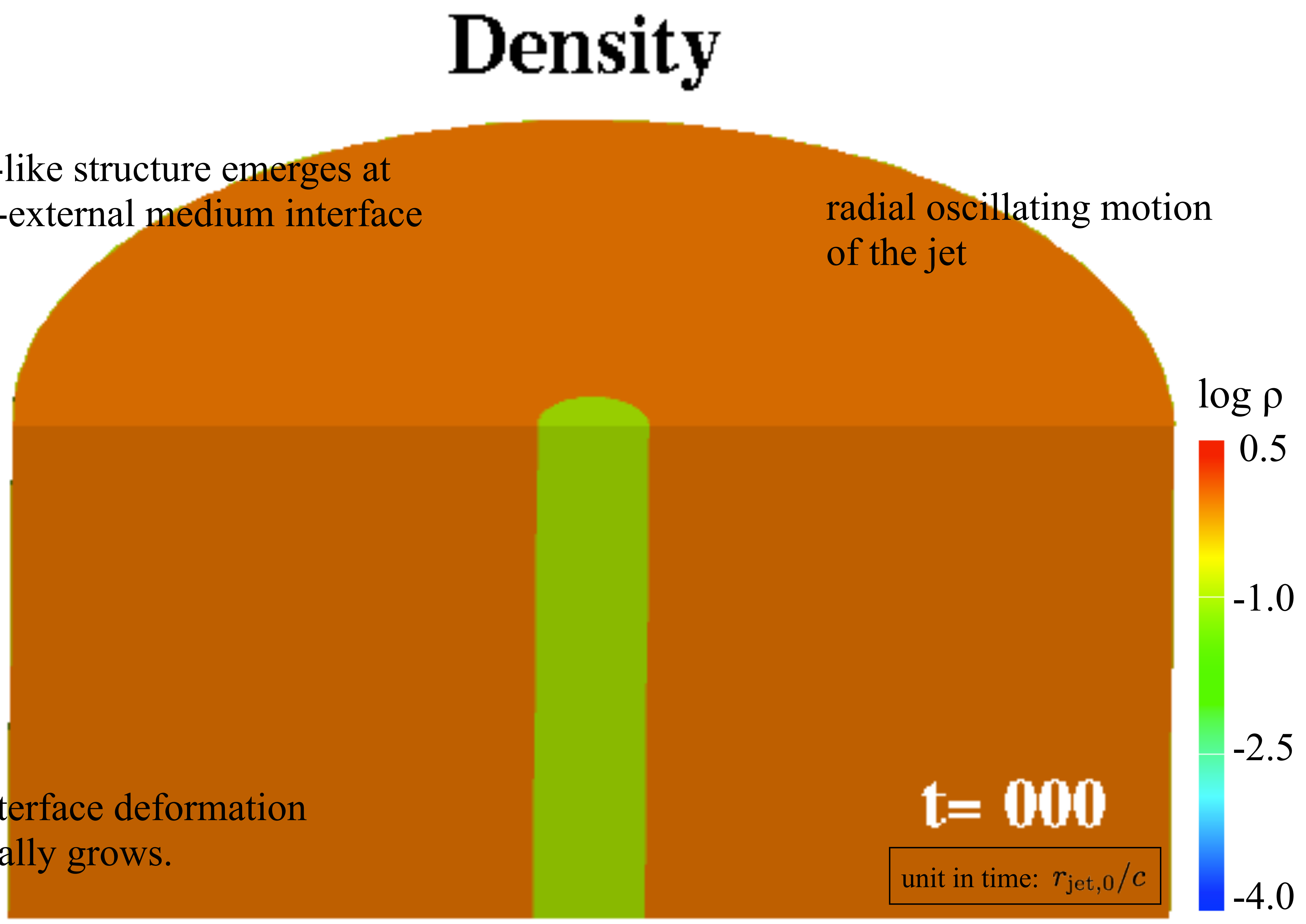
$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Result: Density

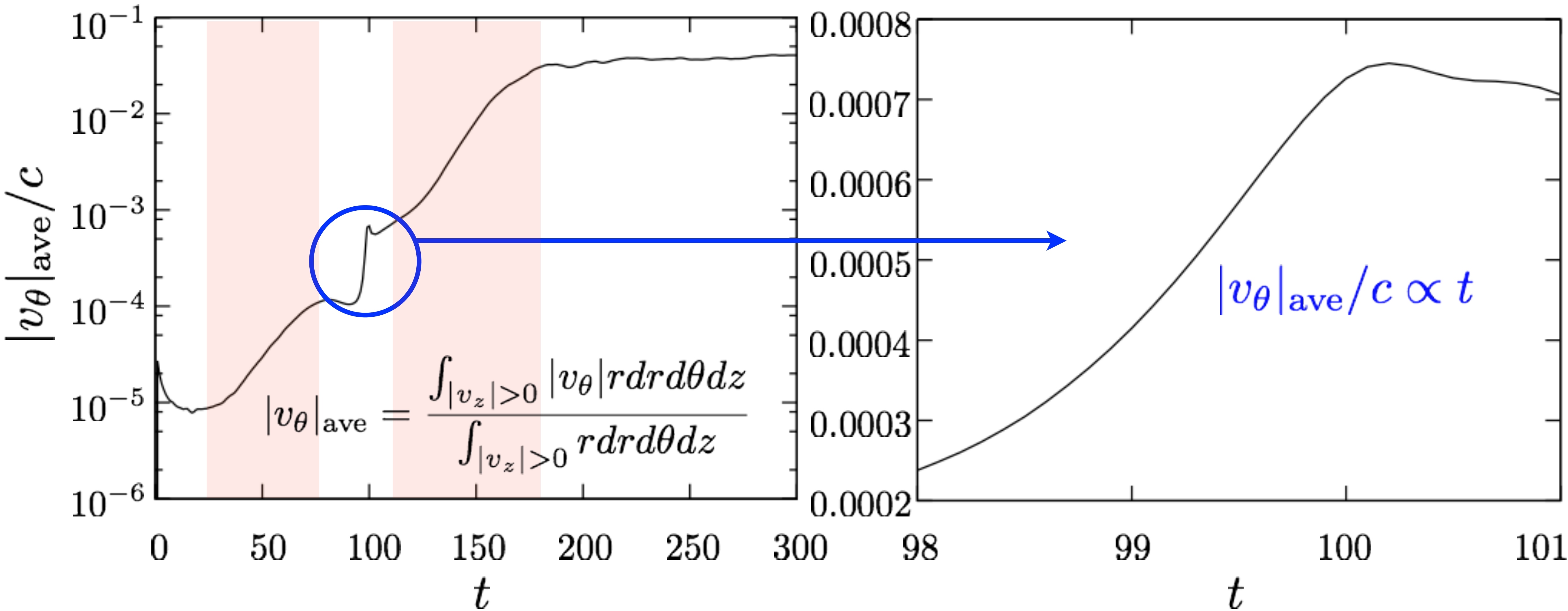
finger-like structure emerges at the jet-external medium interface

radial oscillating motion of the jet

the interface deformation gradually grows.



Synergetic Growth of Rayleigh-Taylor and Richtmyer-Meshkov Instabilities



development of the Rayleigh-Taylor instability at the jet interface

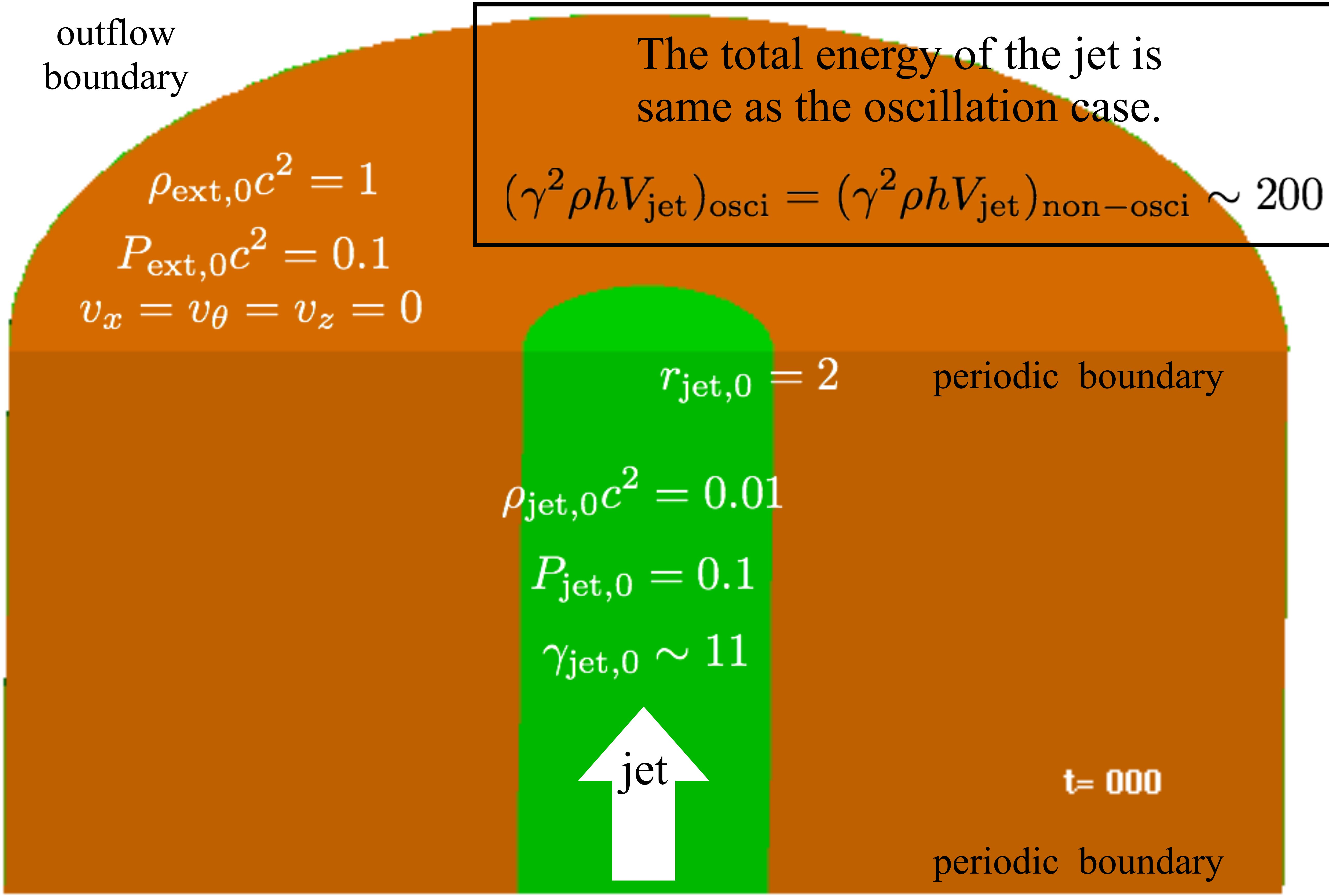
$|v_\theta|_{\rm ave}$ increases exponentially.

excitation of the Richtmyer-Meshkov instability at the jet interface

$|v_\theta|_{\rm ave}$ grows linearly with time.

We can not find the destabilization effect by the Kelvin-Helmholtz mode in the case of the radial oscillation motion of the jet although such effect is not excluded in the settings.

Without Oscillation

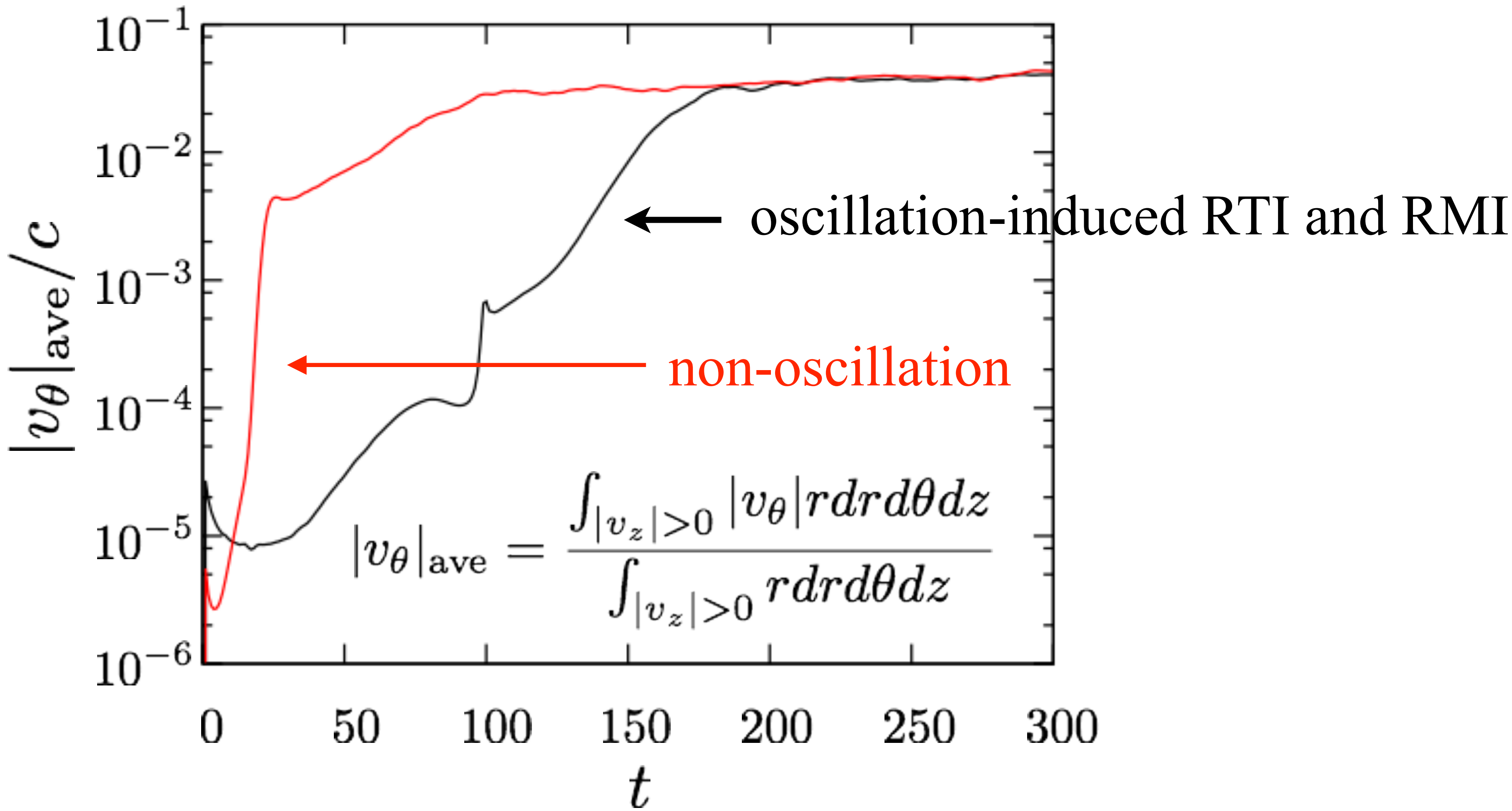


Without Oscillation

Kelvin-Helmholtz instability grows at the jet interface.

The interface deformation gradually grows.

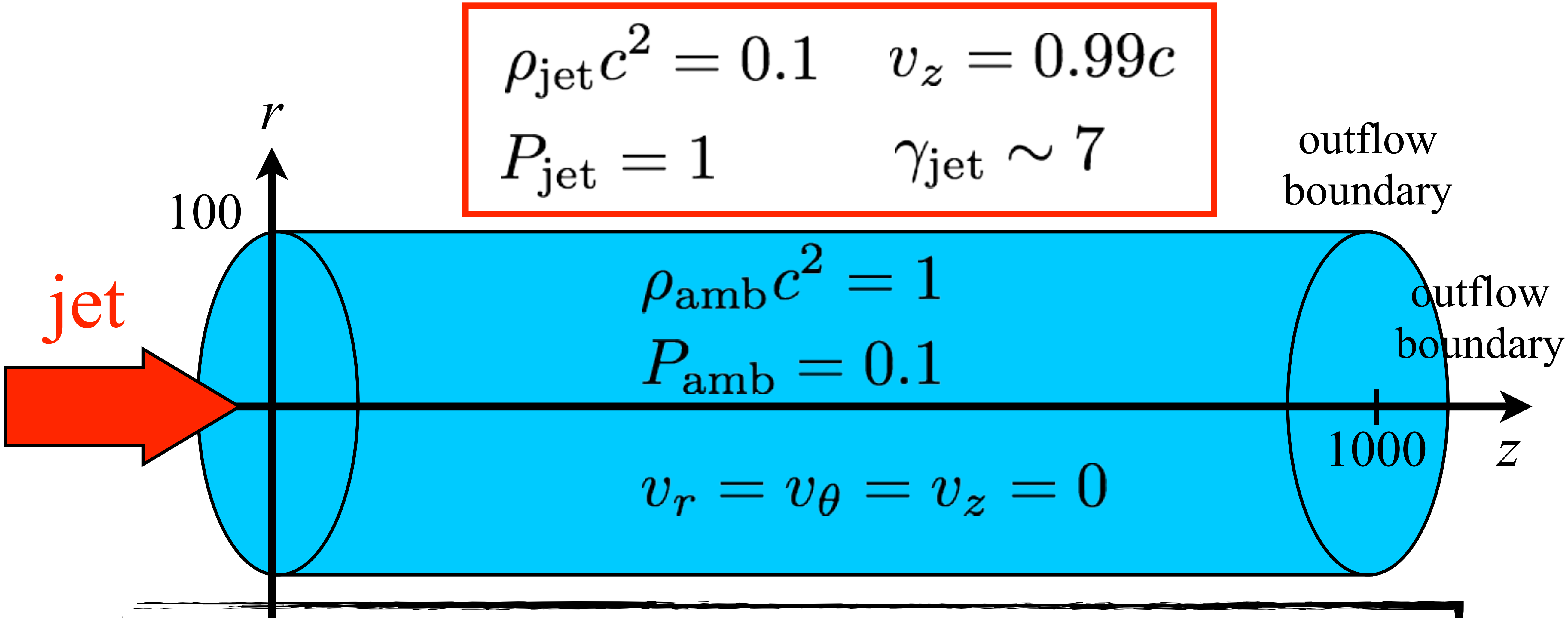
Trigger the Deformation of the Jet



- The growth rate of the volume-averaged azimuthal velocity due to the Kelvin-Helmholtz instability is greater than the oscillation-induced Rayleigh-Taylor and Richtmyer-Meshkov instabilities.
- The synergetic growth of the Rayleigh-Taylor and Richtmyer-Meshkov instabilities trigger the deformation of the radially oscillating jet.

3D simulation:
propagation of the relativistic jet

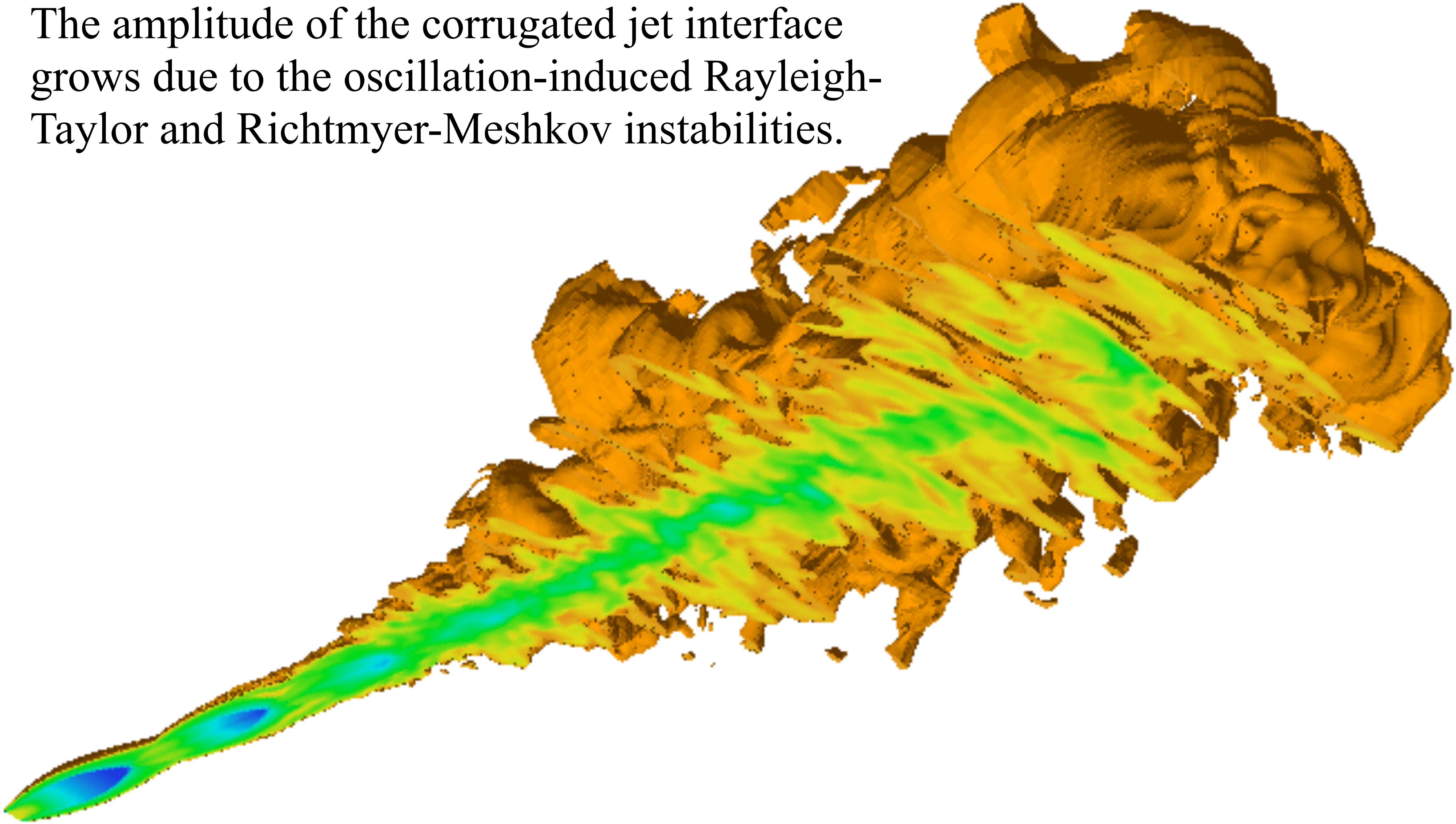
Numerical Setting: 3D Toy Model 2



- cylindrical coordinate
- relativistic jet (z-direction)
- ideal gas
- numerical scheme: HLLC (Mignone & Bodo 05)
- uniform grid: $\Delta r = 0.0666$, $\Delta\theta = 2\pi/160$, Δz=1

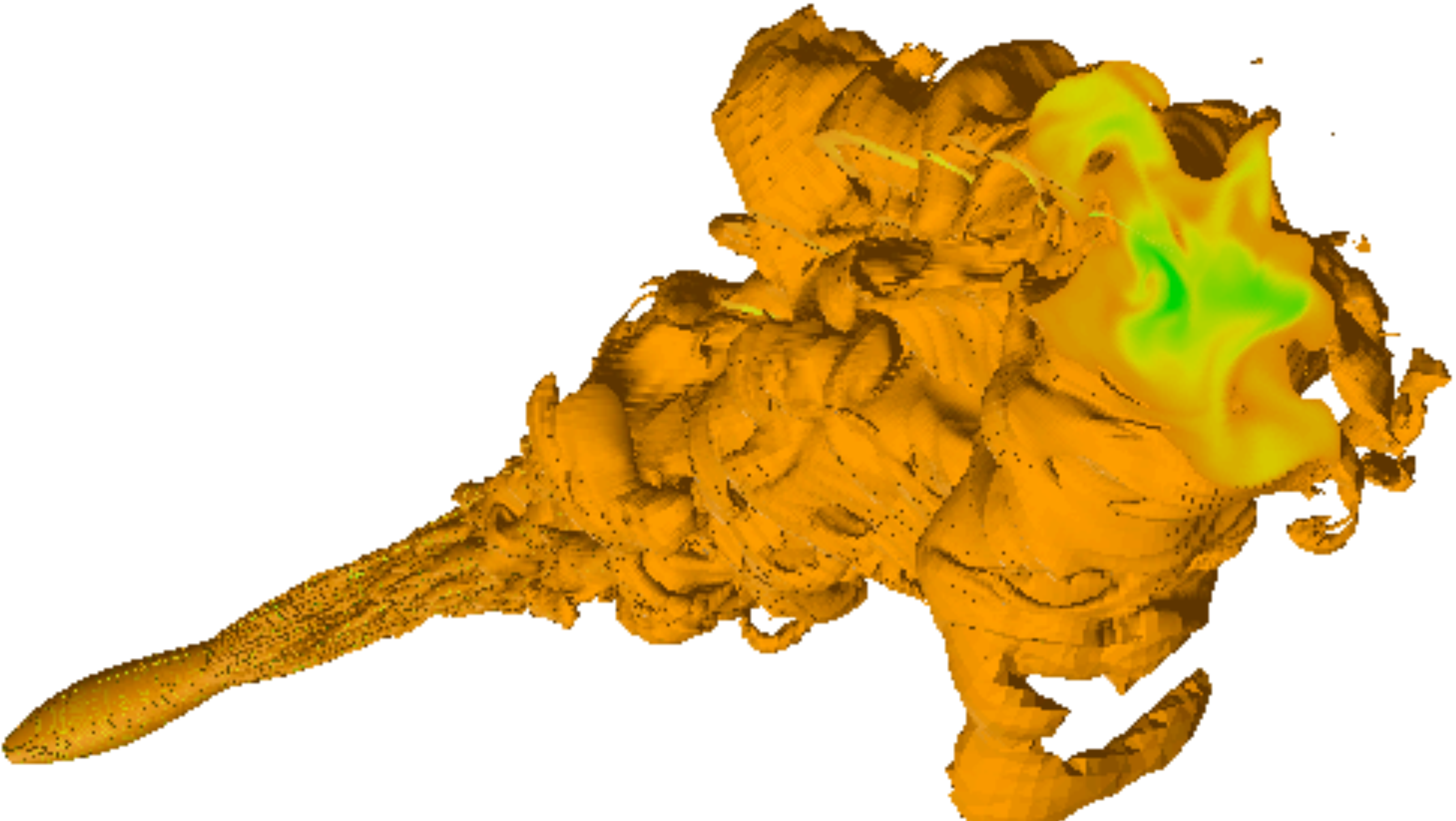
Result: Density

The amplitude of the corrugated jet interface grows due to the oscillation-induced Rayleigh-Taylor and Richtmyer-Meshkov instabilities.



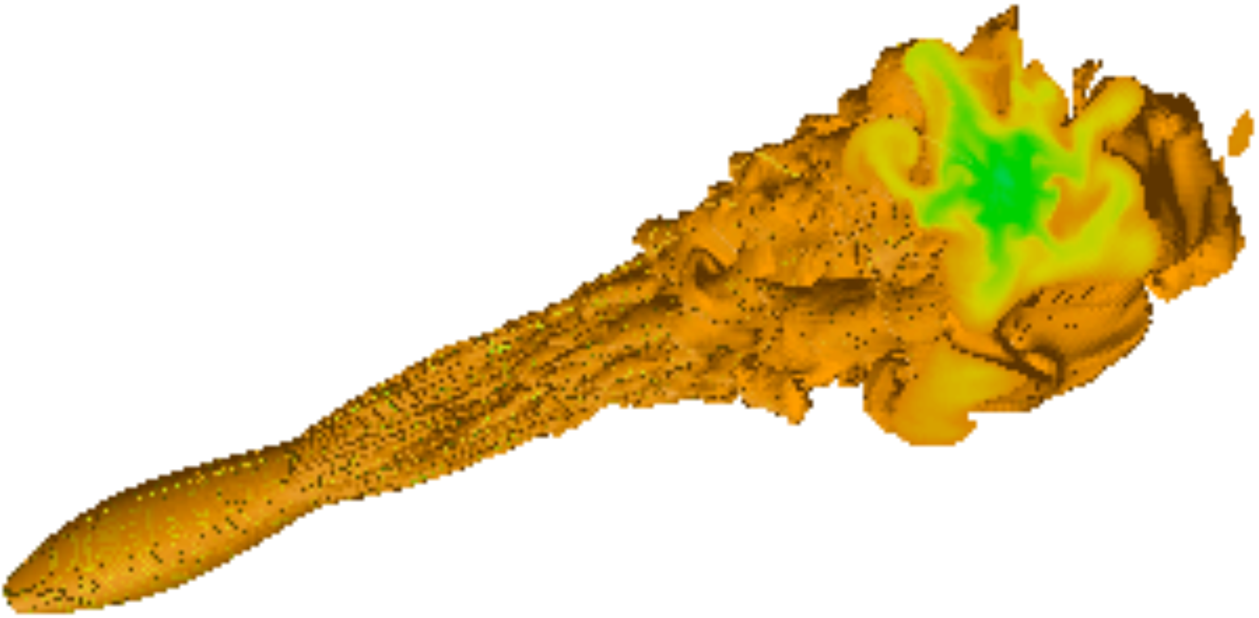
Result: Density

The amplitude of the corrugated jet interface grows due to the oscillation-induced Rayleigh-Taylor and Richtmyer-Meshkov instabilities.



Result: Density

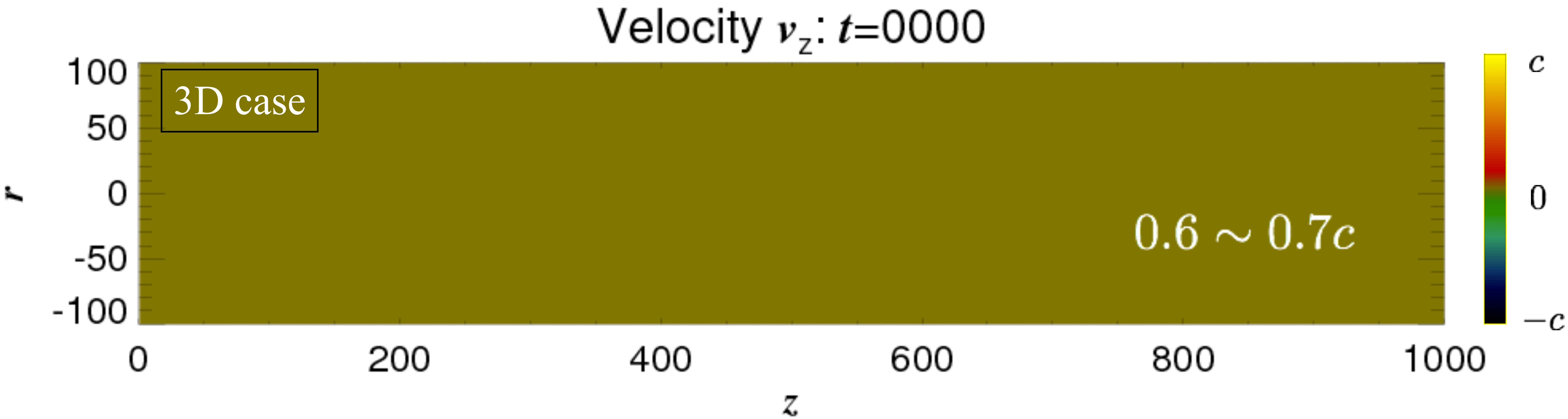
The amplitude of the corrugated jet interface grows due to the oscillation-induced Rayleigh-Taylor and Richtmyer-Meshkov instabilities.

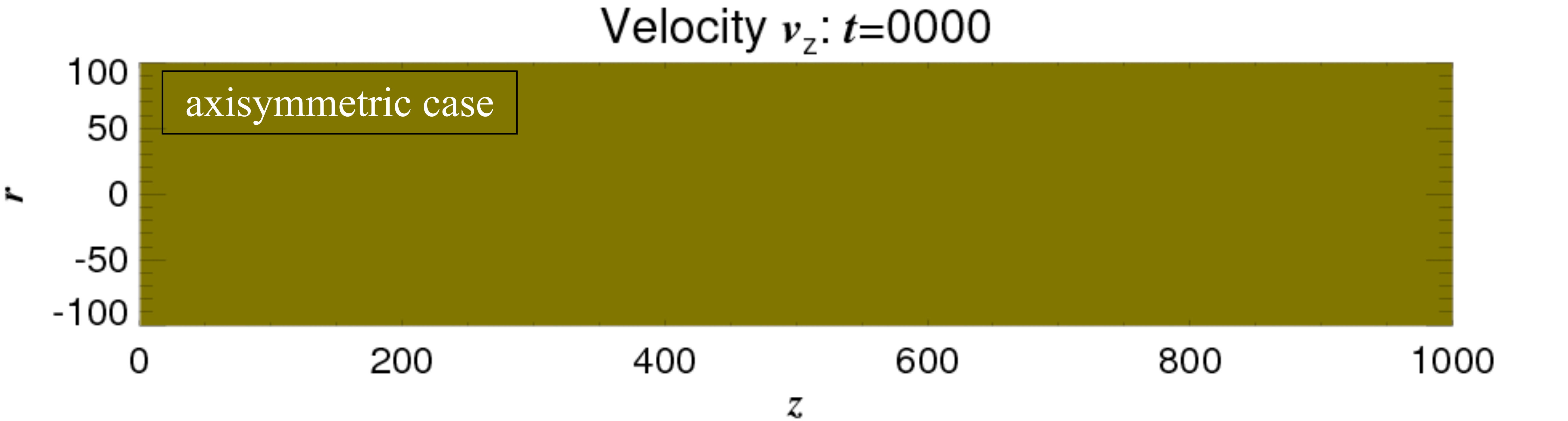


Result: Density

The amplitude of the corrugated jet interface grows due to the oscillation-induced Rayleigh-Taylor and Richtmyer-Meshkov instabilities.

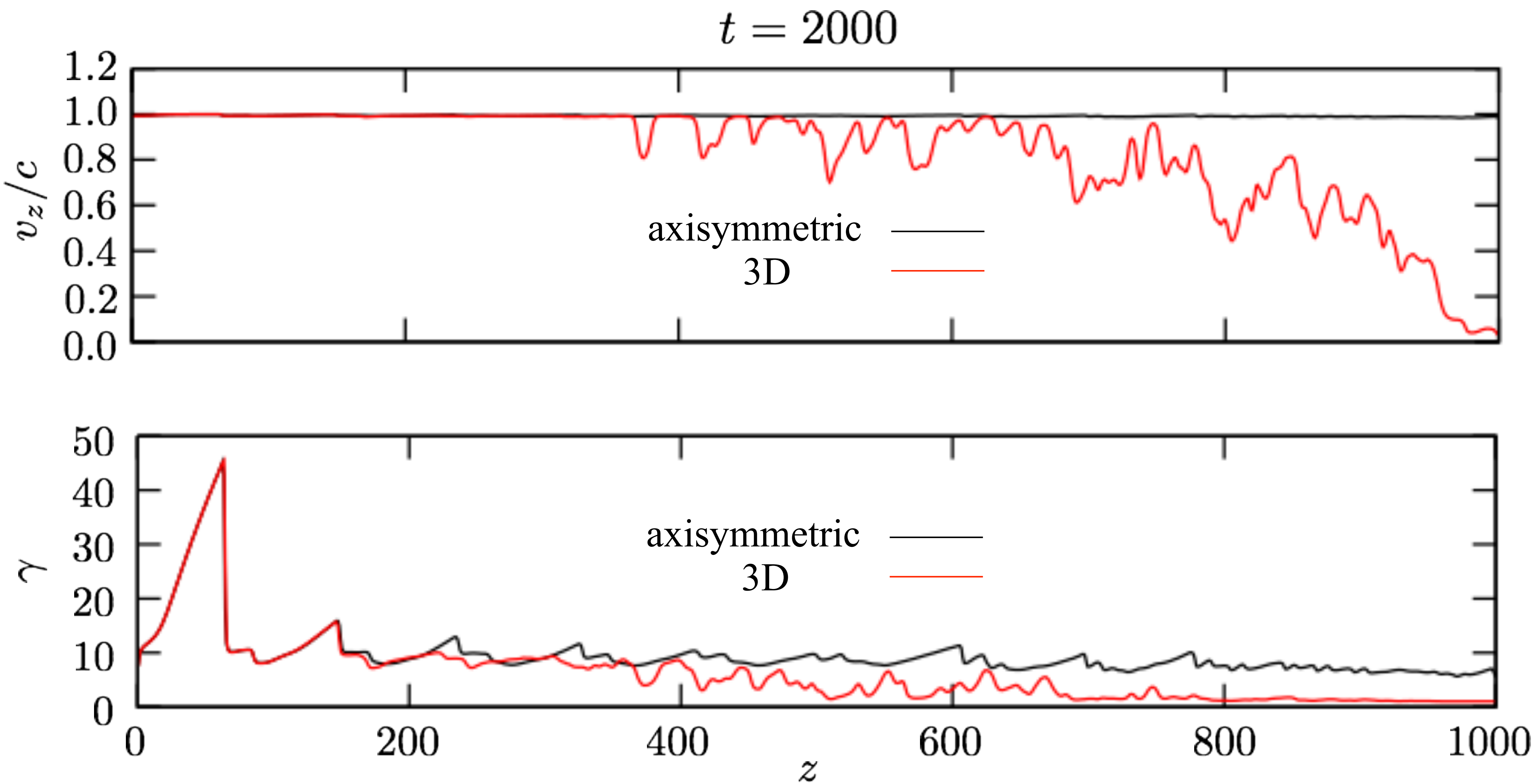
3D vs Axisymmetric





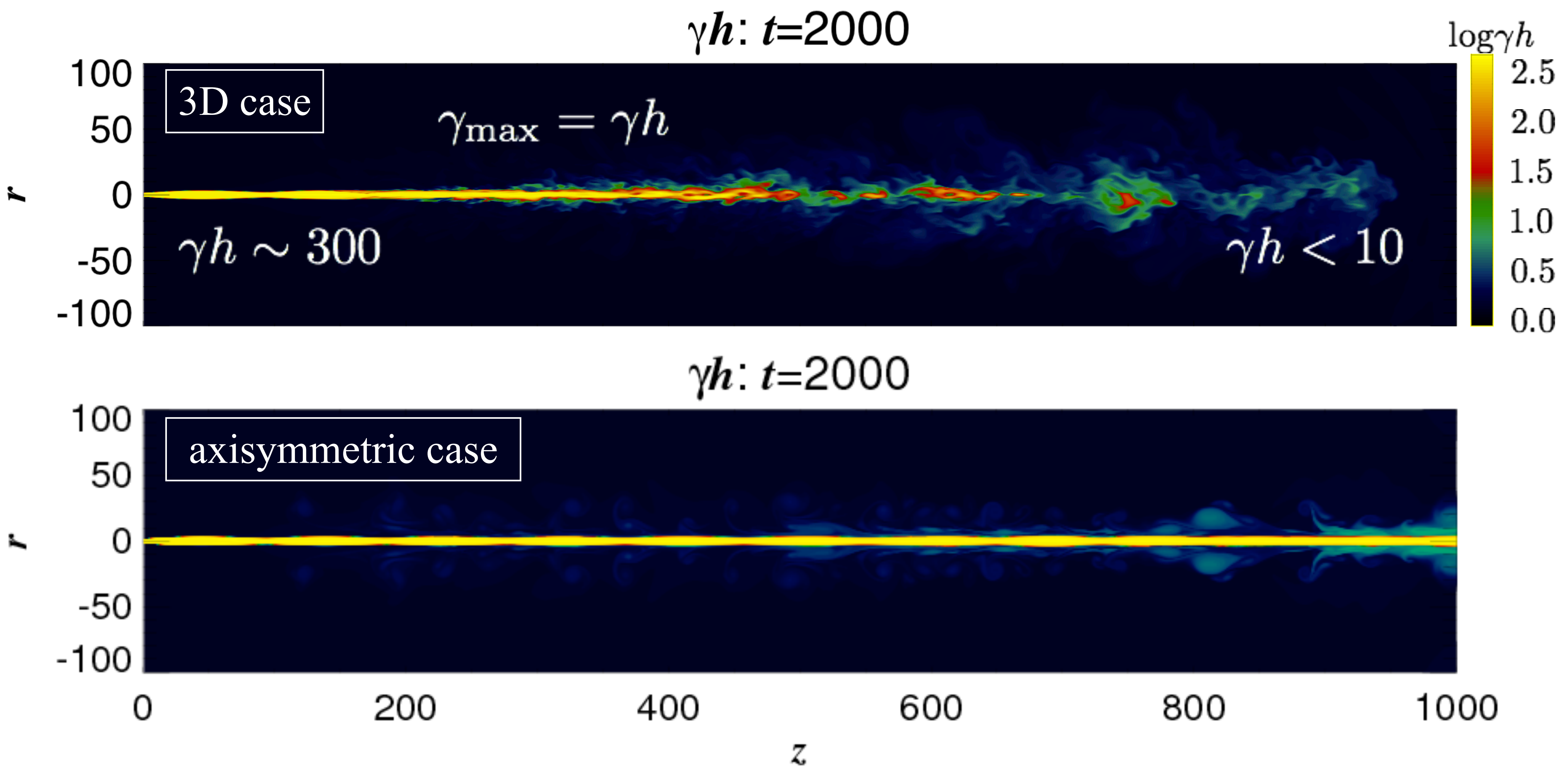
- deceleration of the jet due to the mixing between the jet and surrounding medium in the 3D case.

Deceleration of the jet due to mixing



- deceleration of the jet due to the mixing between the jet and surrounding medium in the 3D case.

Deceleration of the jet due to mixing

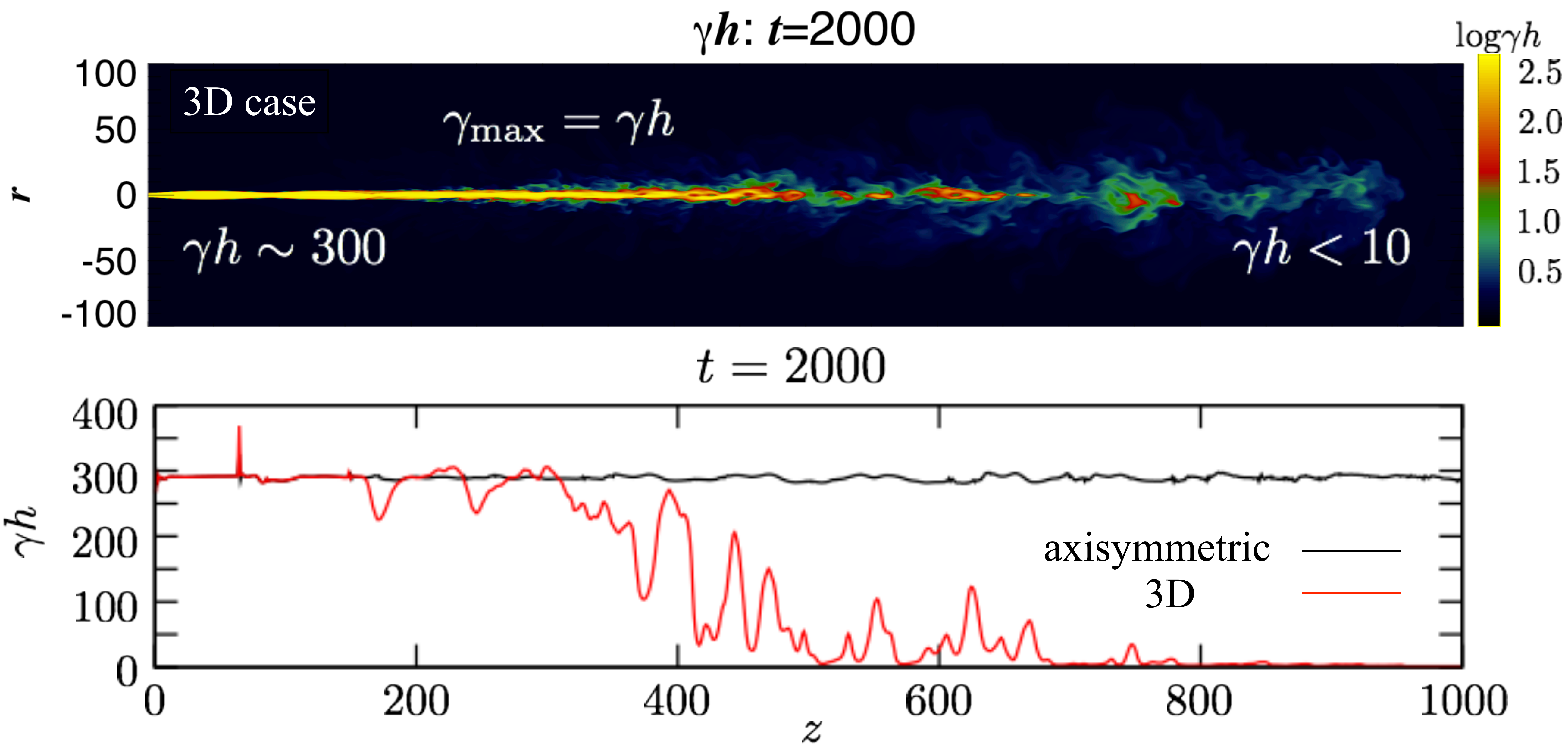


- relativistic Bernoulli equation: $\gamma h \sim$ const.

γh gives the maximum Lorentz factor of the jet after adiabatic expansion.

However, γh drops to ~ 10 due to the mixing in this case.

Deceleration of the jet due to mixing



■ relativistic Bernoulli equation: $\gamma h \sim \text{const.}$

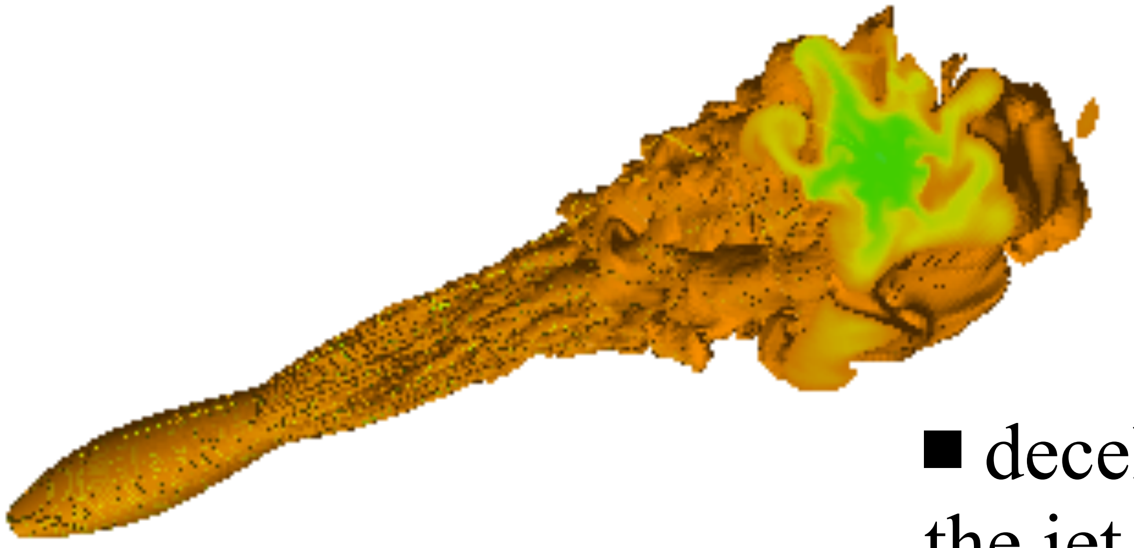
γh gives the maximum Lorentz factor of the jet after adiabatic expansion.

However, γh drops to ~ 10 due to the mixing in this case.

Summary

Propagation dynamics and stability of the relativistically hot jet is studied through 2D and 3D relativistic hydrodynamic simulations.

- A pressure mismatch between the jet and surrounding medium leads to the radial oscillating motion of the jet.



- The jet-ambient medium interface is unstable due to the oscillation-induced
  - Rayleigh-Taylor instability
  - Richtmyer-Meshkov instability
- deceleration of the jet due to the mixing between the jet and surrounding medium

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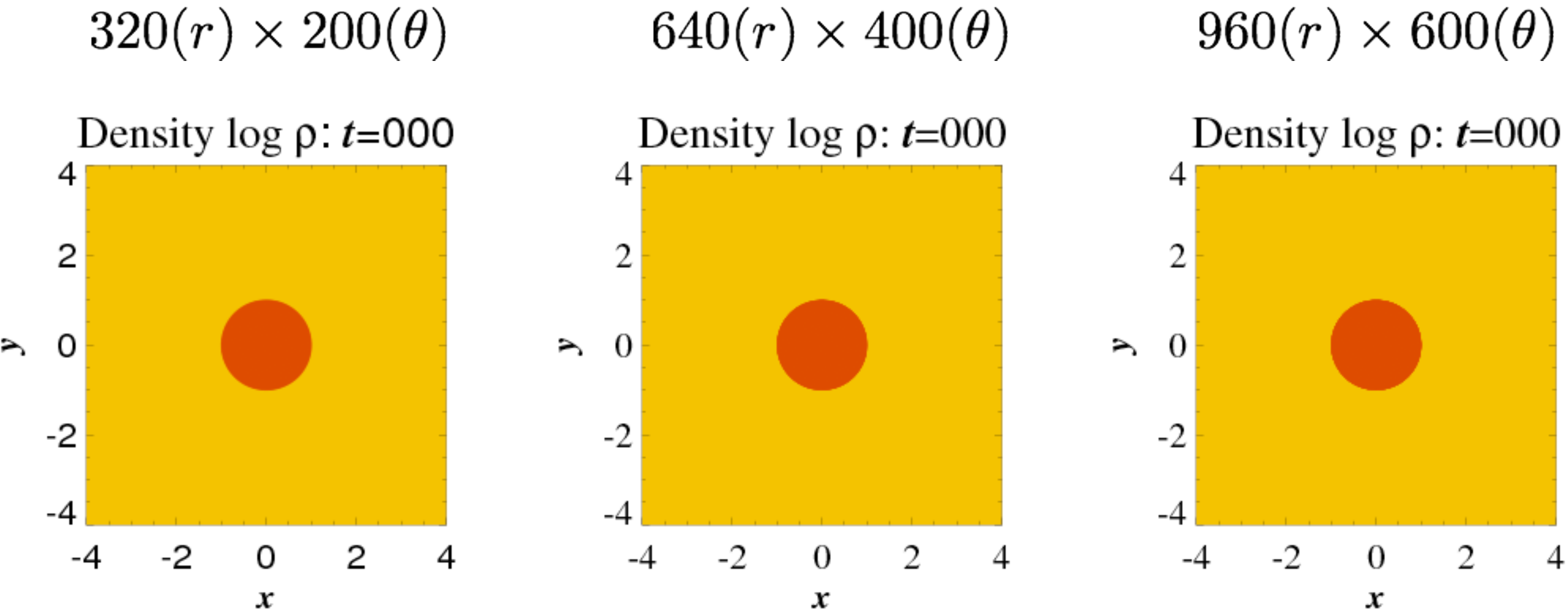
Next Study:

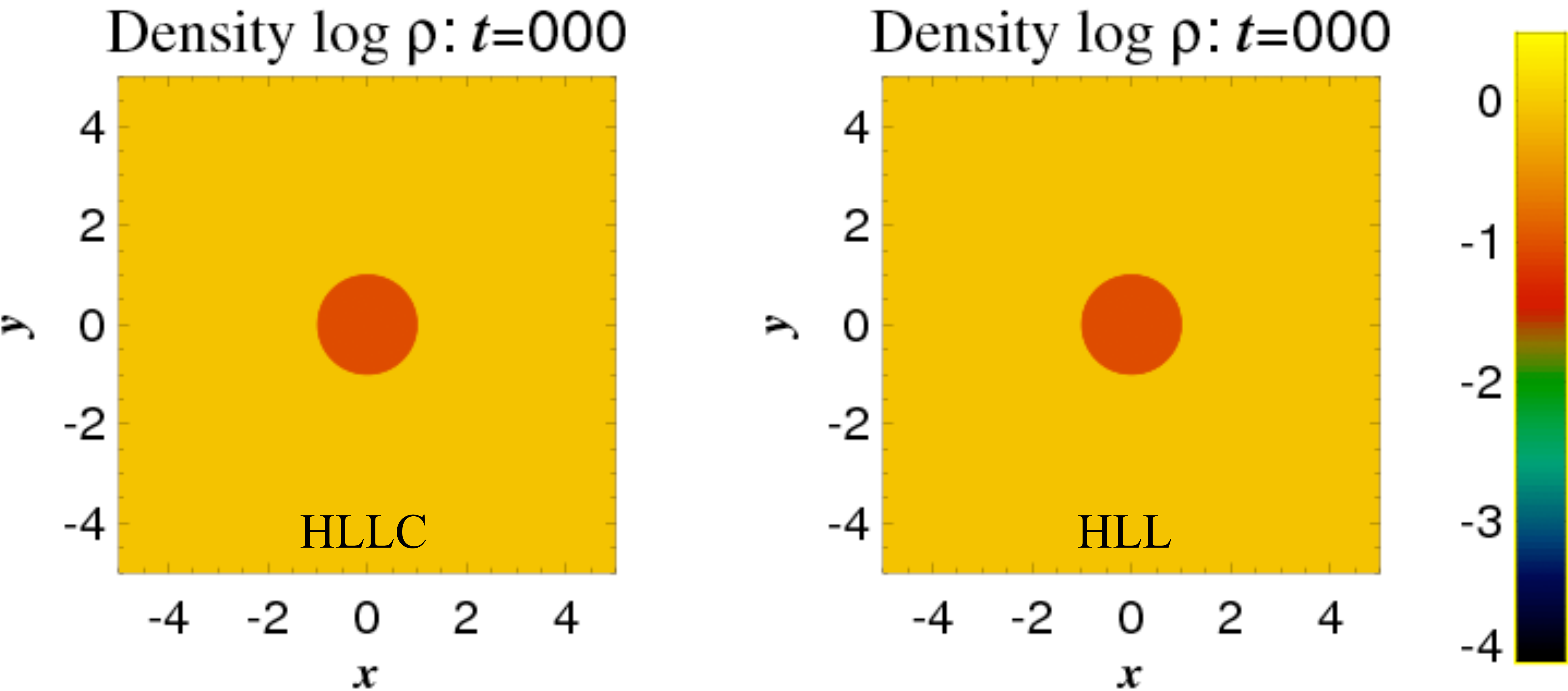
- more realistic situation for relativistic jets such as AGN jets and GRBs
- effect of the magnetic field on the dynamics and stability of the jet

Comparison of Grid Points



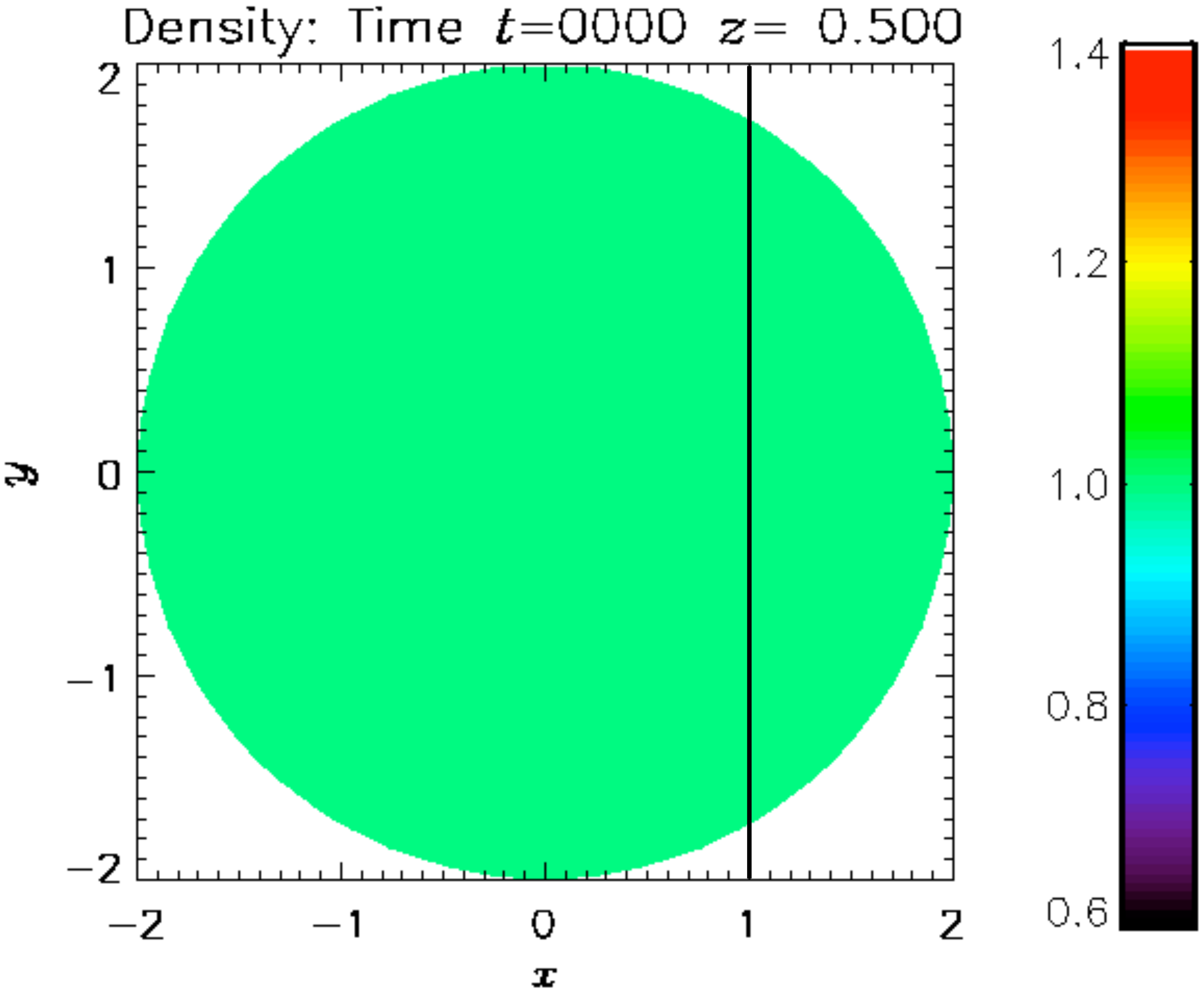
In higher resolution case, you can find smaller structures due to the growth of the Rayleigh-Taylor and Richtmyer-Meshkov instabilities.

Comparison of Numerical Scheme

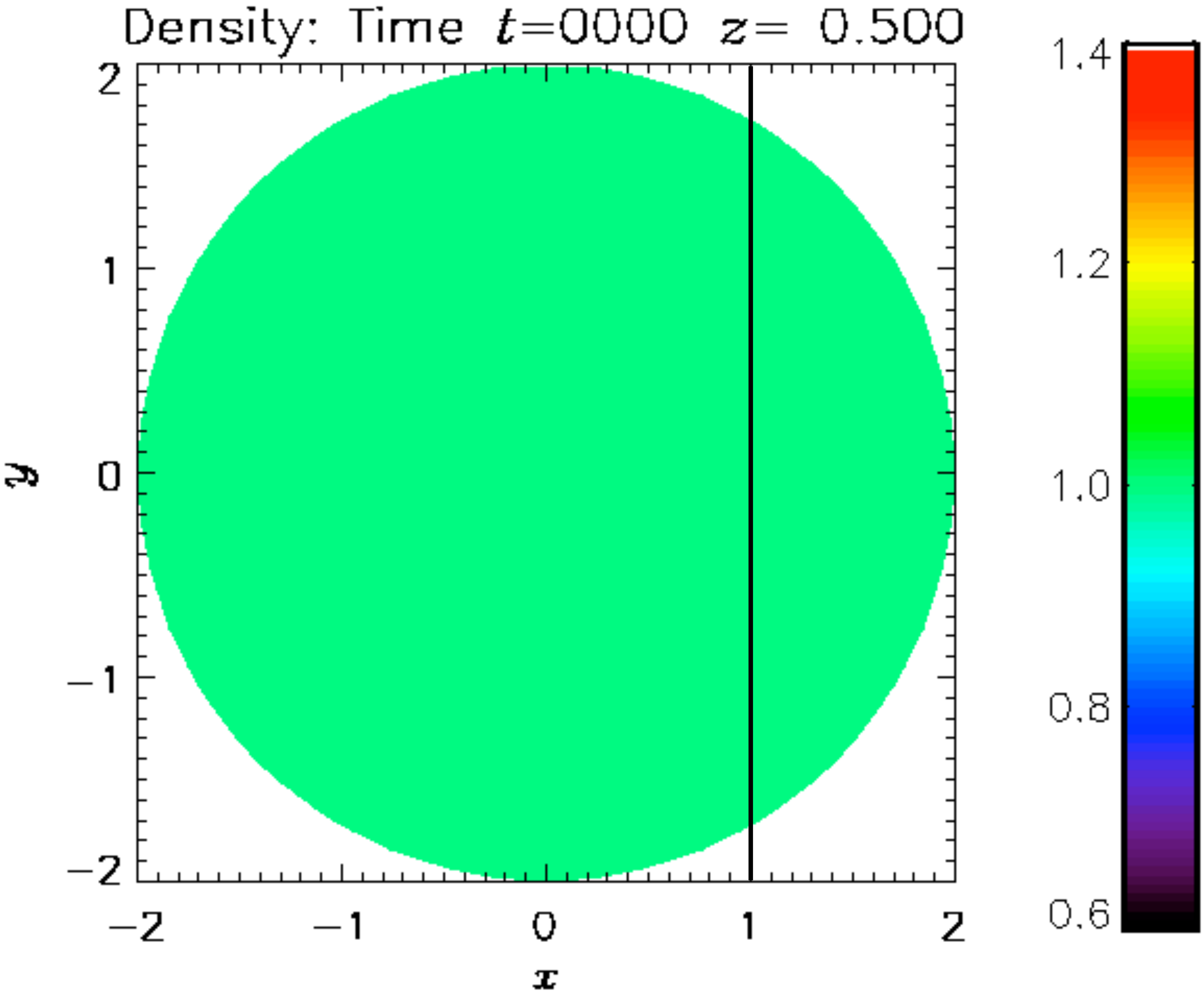


It is not easy to find Rayleigh-Taylor and Richtmyer-Meshkov fingers in the model with HLL scheme although the completely same initial settings and grid spacing (320 x 200 zones r- and \theta directions) are adopted in both models.

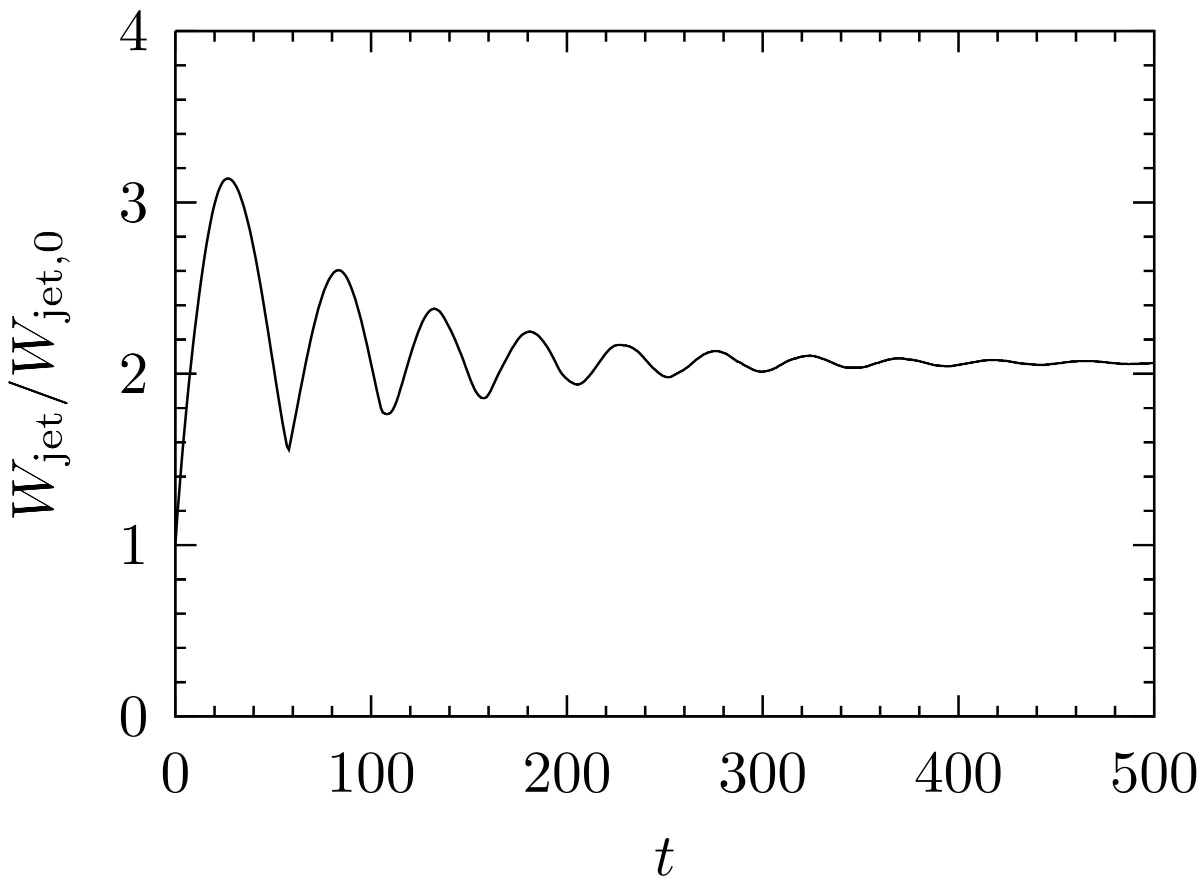
Propagation of Rarefaction Wave through the Origin

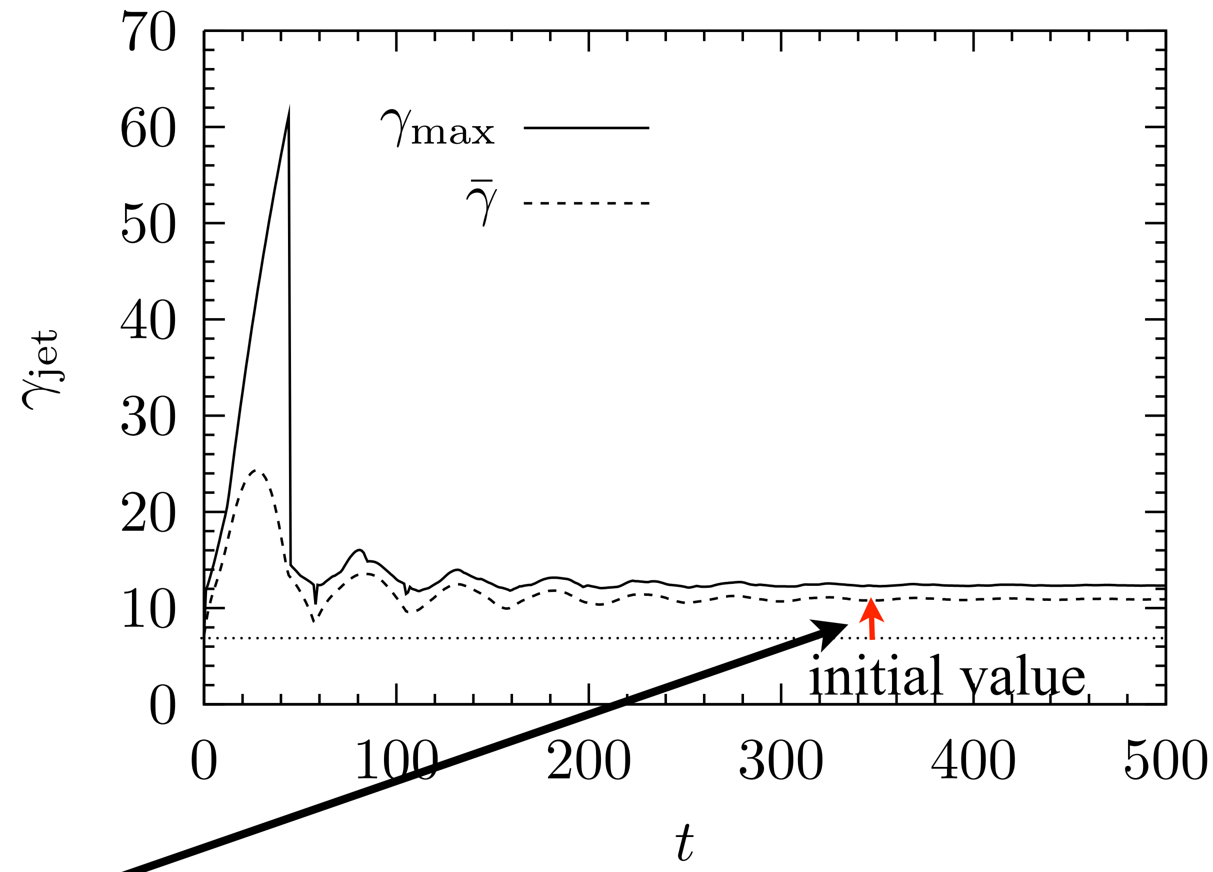


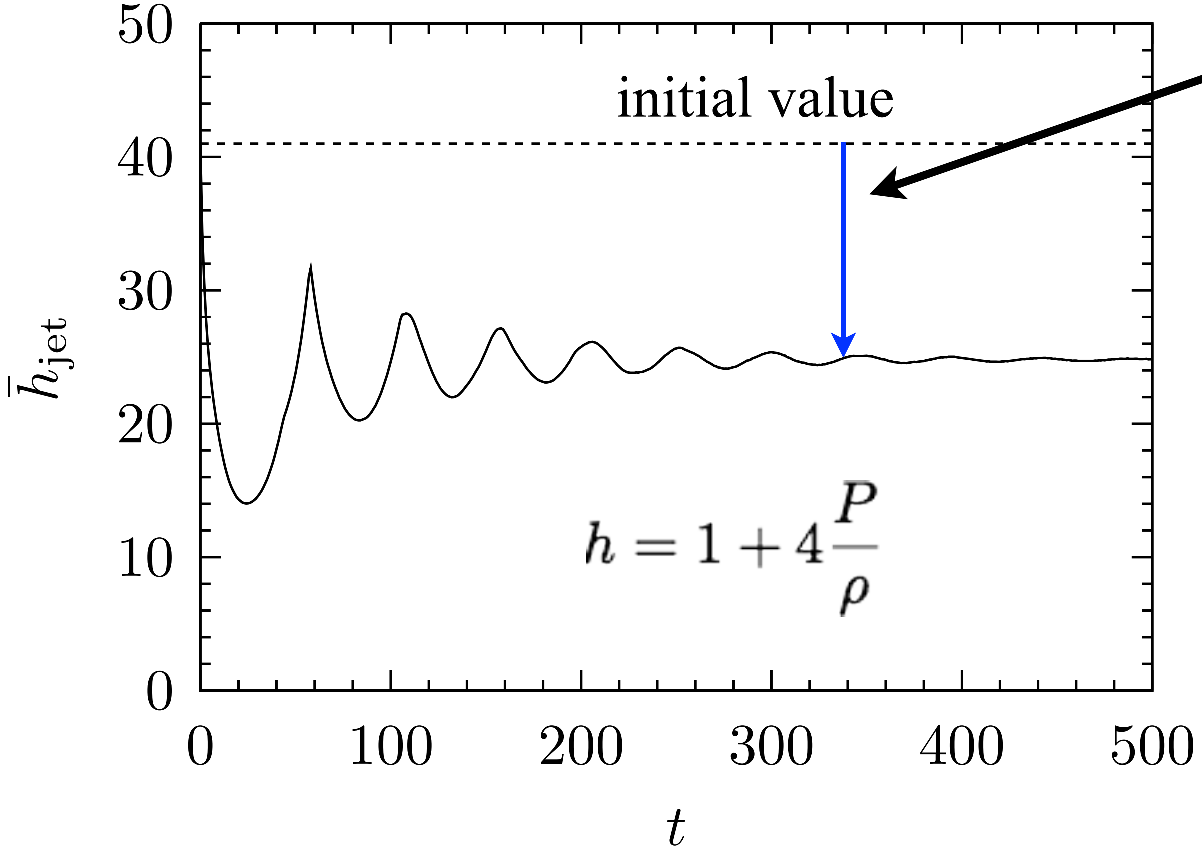
Propagation of Shock Wave through the Origin



Relaxation of Initial non-equilibrium State







- Transition stage (oscillation)

  ➡ Steady state

- Hydrostatic balance
- Energy conversion from thermal energy into bulk motion energy

  relativistic Bernoulli: $\gamma h \sim \text{const.}$

Scaling for the Oscillation Timescale

oscillation timescale: propagation time of the sound wave over the jet width

$$\tau = \sqrt{3}\gamma_{\rm jet} W_{\rm jet}/c$$

total energy conservation neglecting rest mass energy

$${W_{\rm jet}}^2 {\gamma_{\rm jet}}^2 P_{\rm amb,0} = {W_{\rm jet,0}}^2 {\gamma_{\rm jet,0}}^2 P_{\rm jet,0}$$

oscillation timescale

$$\tau = \sqrt{3}\gamma_{\rm jet,0}\left(\frac{W_{\rm jet,0}}{c}\right)\left(\frac{P_{\rm jet,0}}{P_{\rm amb,0}}\right)^{1/2}$$

proportional to the square root of the initial pressure ratio between the jet and ambient medium.