Determining the Equation of State of Dense Matter from Neutron Star Mass and Radius Observations

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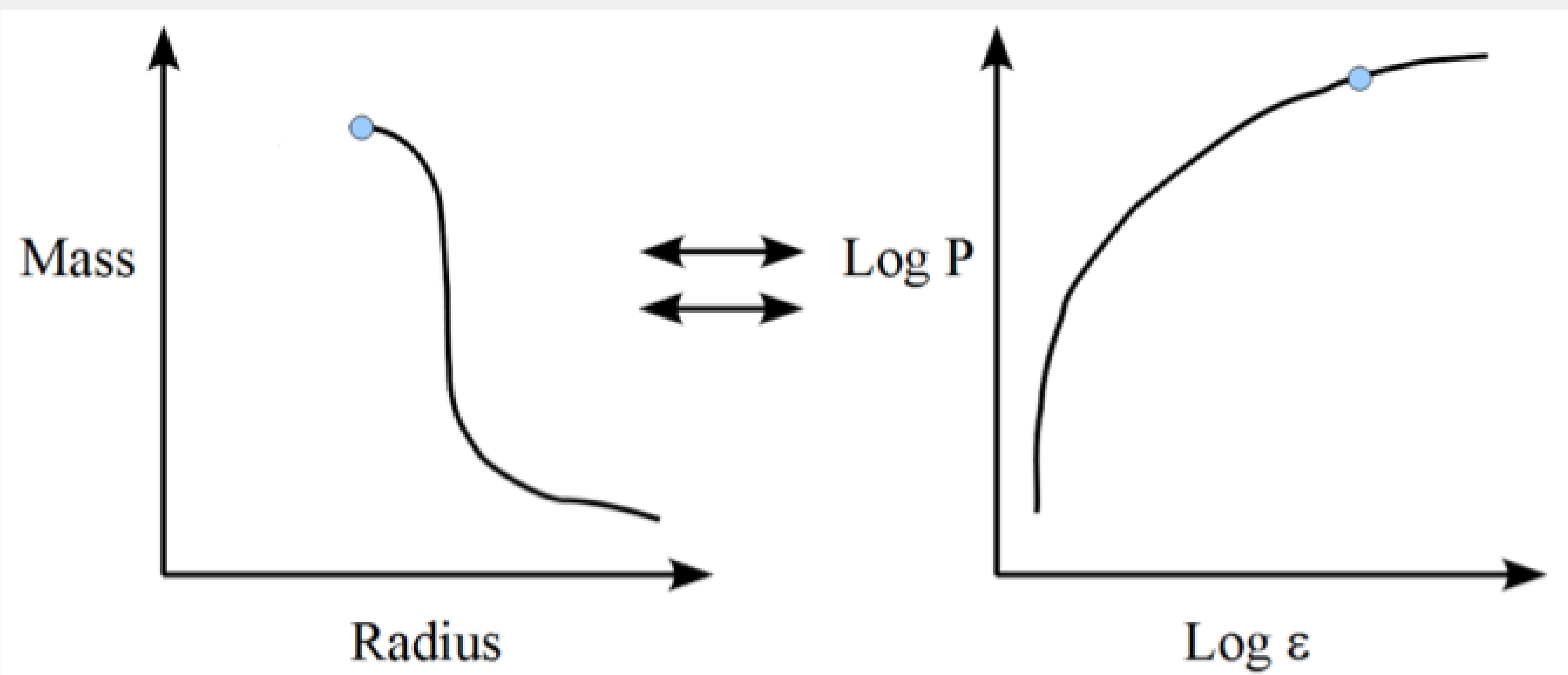
Outline

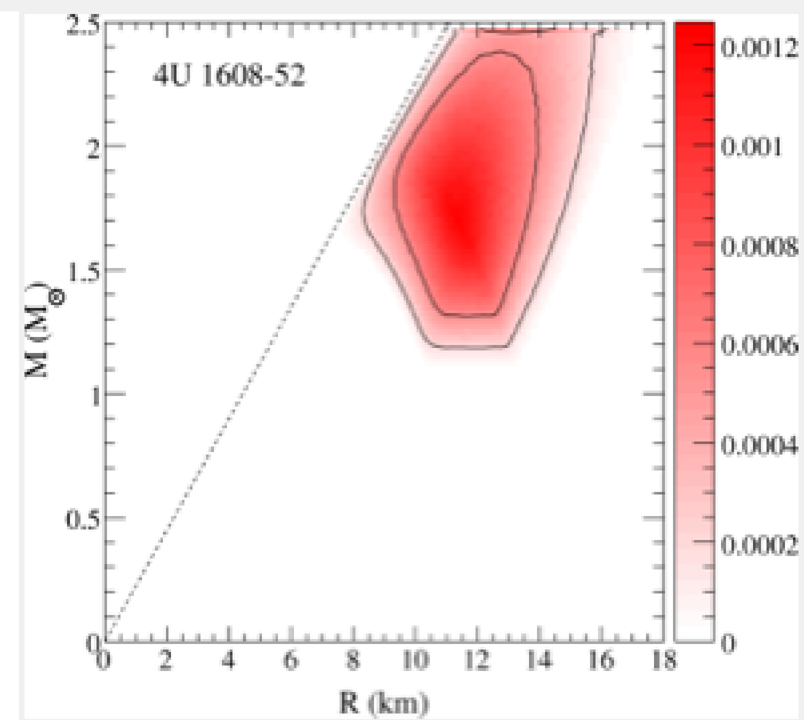
- Basic neutron star questions:
  - What is the (nearly) universal $M - R$ curve?
  - What is the radius of a 1.4 $M_\odot$ neutron star?
- Fundamental nuclear physics questions:
  - What is the nuclear symmetry energy?
  - What is the three-neutron force?
  - What is the nature of dense matter?
- How do you make these connections?
- Model selection
- New EOSs for core-collapse and mergers

Neutron Star Masses and Radii and the EOS

- Unlike planets, neutron stars (to better than 10%) all lie on one universal mass-radius curve
- Except for "strange quark stars"
- Rotation is a <10% effect
- A strong enough magnetic field can also deform the star

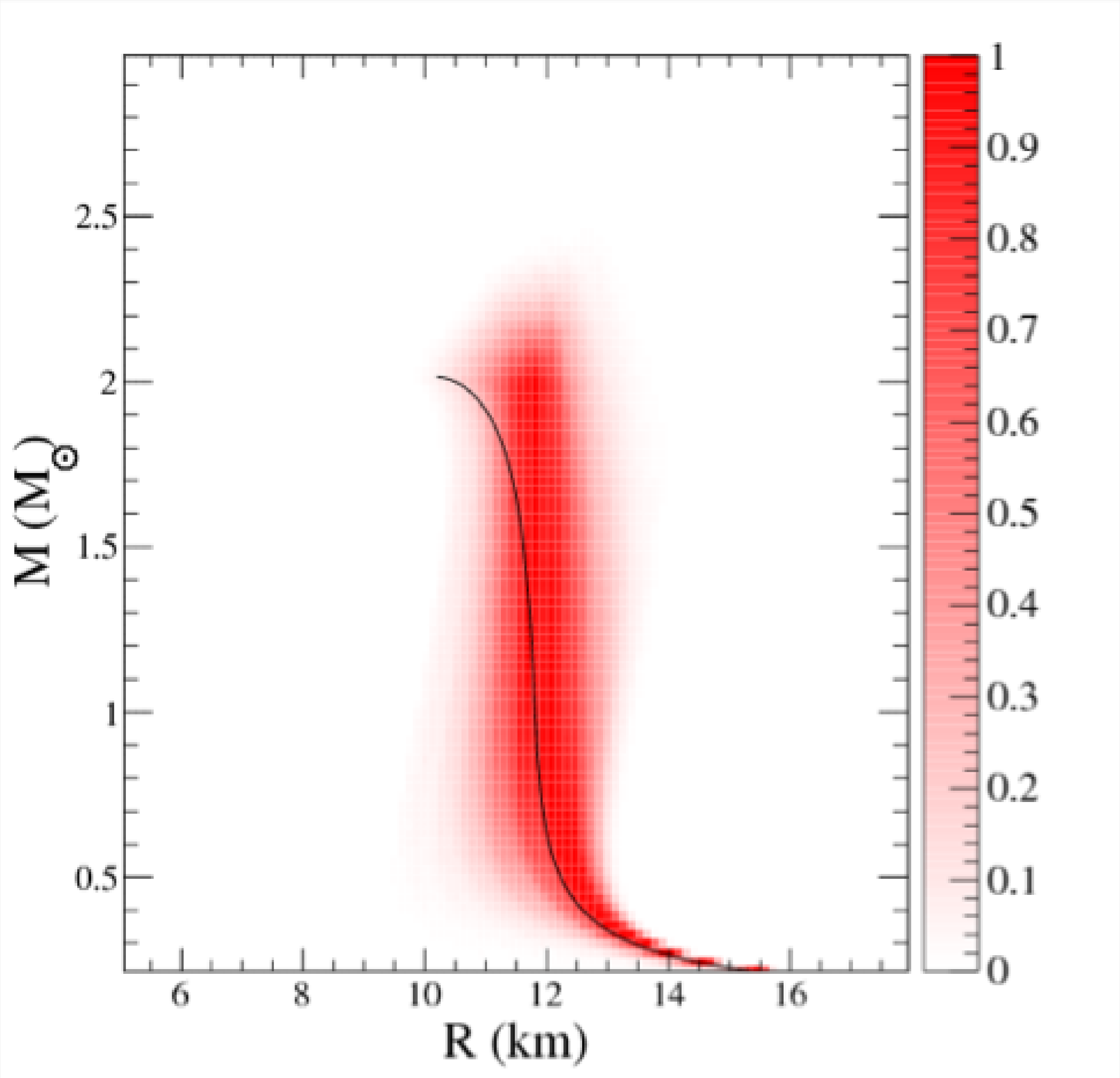
- Until recently, neutron star radii constrained to 8-15 km
  Lattimer and Prakash (2007)
- Recent measurement of two $2\ M_\odot$ neutron stars
  Demorest et al. (2010), Antoniadis et al. (2013)
- Convert X-ray photons into $\mathcal{P}(R, M)$





Steiner, Lattimer, and Brown (2010)

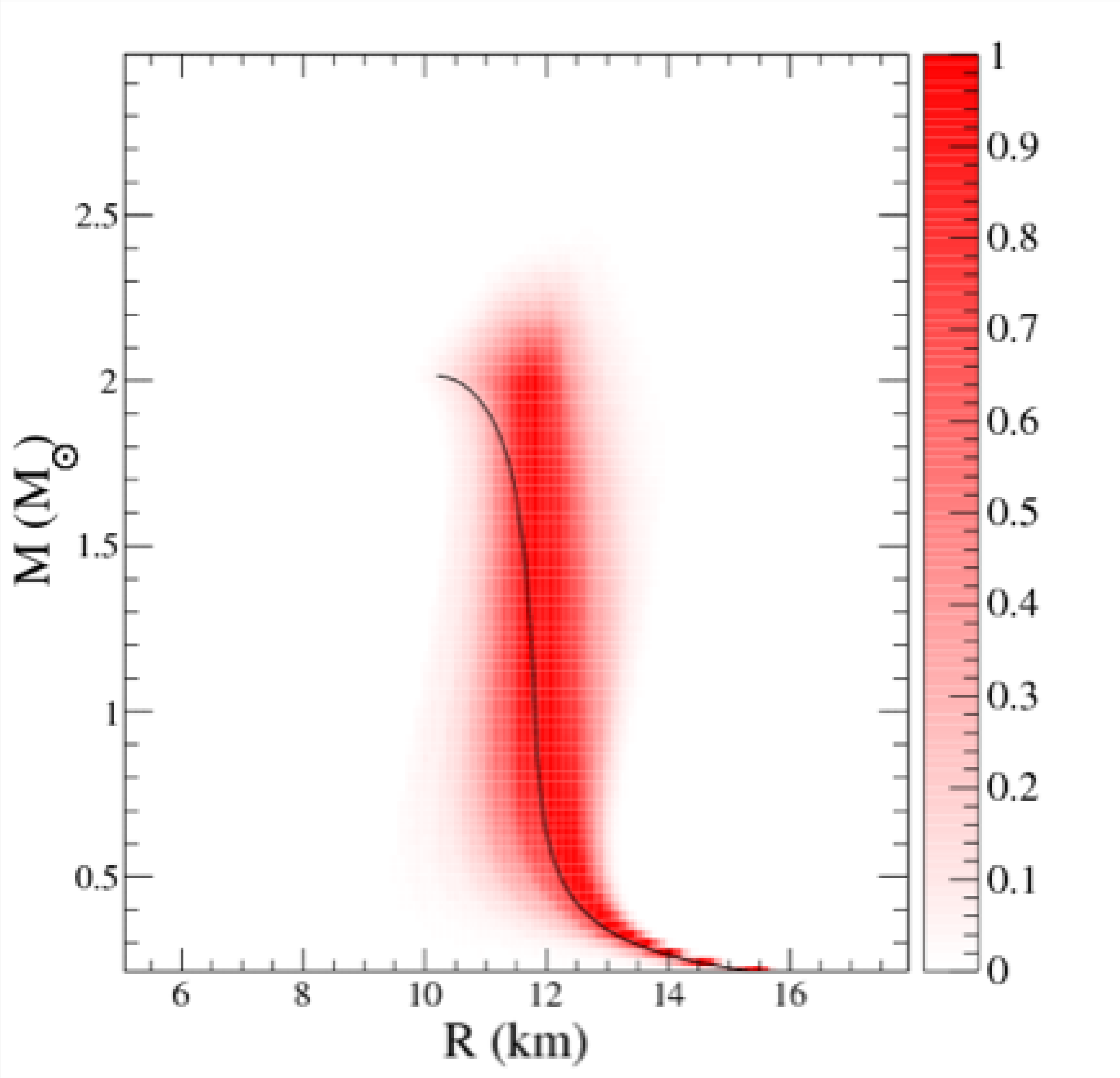
The Geometry of M-R curves



- Neither $M(R)$ nor $R(M)$ need to be functions (but $M(P_c)$ and $R(P_c)$ are)
  even though R(M) is continuous and differentiable
- In the language of χ^2 fitting: c.f. Deming or orthogonal regression and total least squares
  no unique solution in the general case
- Minimize distance between data and the curve (instead of vertical displacement)
  defining a distance is nontrivial
- Formally an underconstrained problem
  cannot divide chi-squared by the number of degrees of freedom
- Unless one performs a parameterization, $M - R$ or the EOS
- However: (R, M) space is difficult to translate to (ε, P) space
  not even a homeomorphism

Bayesian Analysis

How do we get the EOS from several $\mathcal{P}(R, M)$'s?



- Over/under-constrained subspaces (Low vs. high densities)

- Bayesian analysis proven successful
  Lepage et al. (2002) and Schindler and Phillips (2009)

- Why is the M-R curve so vertical?
  Why do all neutron stars in the universe have nearly the same radius?
  Causality + 2 solar mass NS + 1 QLMXB

- Many standard frequentist methods assume something about the shape of the likelihood function near the maximum

- This fails in this case: the best fit not same as "typical" M-R curve
  Posterior maximum mass distribution is strongly skewed

- Naive covariance analysis unrelated to typical M-R curve for high masses
  Just an example of how that method can fail

Radius Measurements in qLMXBs

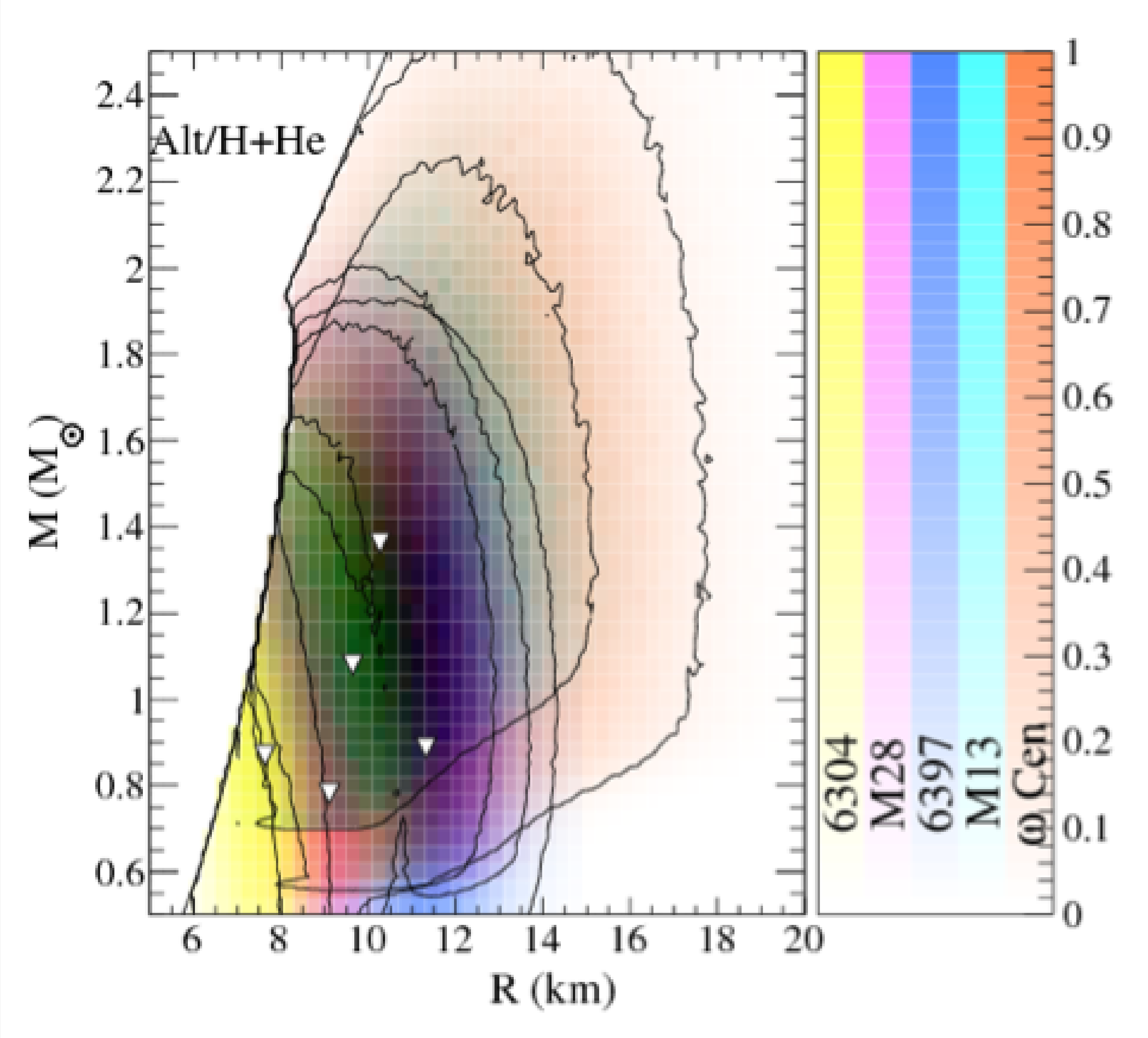
Quiescent LMXBs

- Measure flux of photons and their energy distribution
- Know distance if in a globular cluster
- Implies radius measurement

$$F \propto T_{\rm eff}^4 \left(\frac{R_\infty}{D} \right)^2$$

i.e. Rutledge et al. (1999)

- Need information about the atmosphere, including composition
- Also need X-ray absorption and absolute flux calibration
- Inevitably give small radii for some low-mass stars



Lattimer and Steiner (2013)

- Rotation, anisotropy, and magnetic fields may also be important

Photospheric Radius Expansion X-ray Bursts

- X-ray bursts sufficiently strong to blow off the outer layers - radiate at the Eddington limit
- Flux peaks, then temperature reaches a maximum, "touchdown"

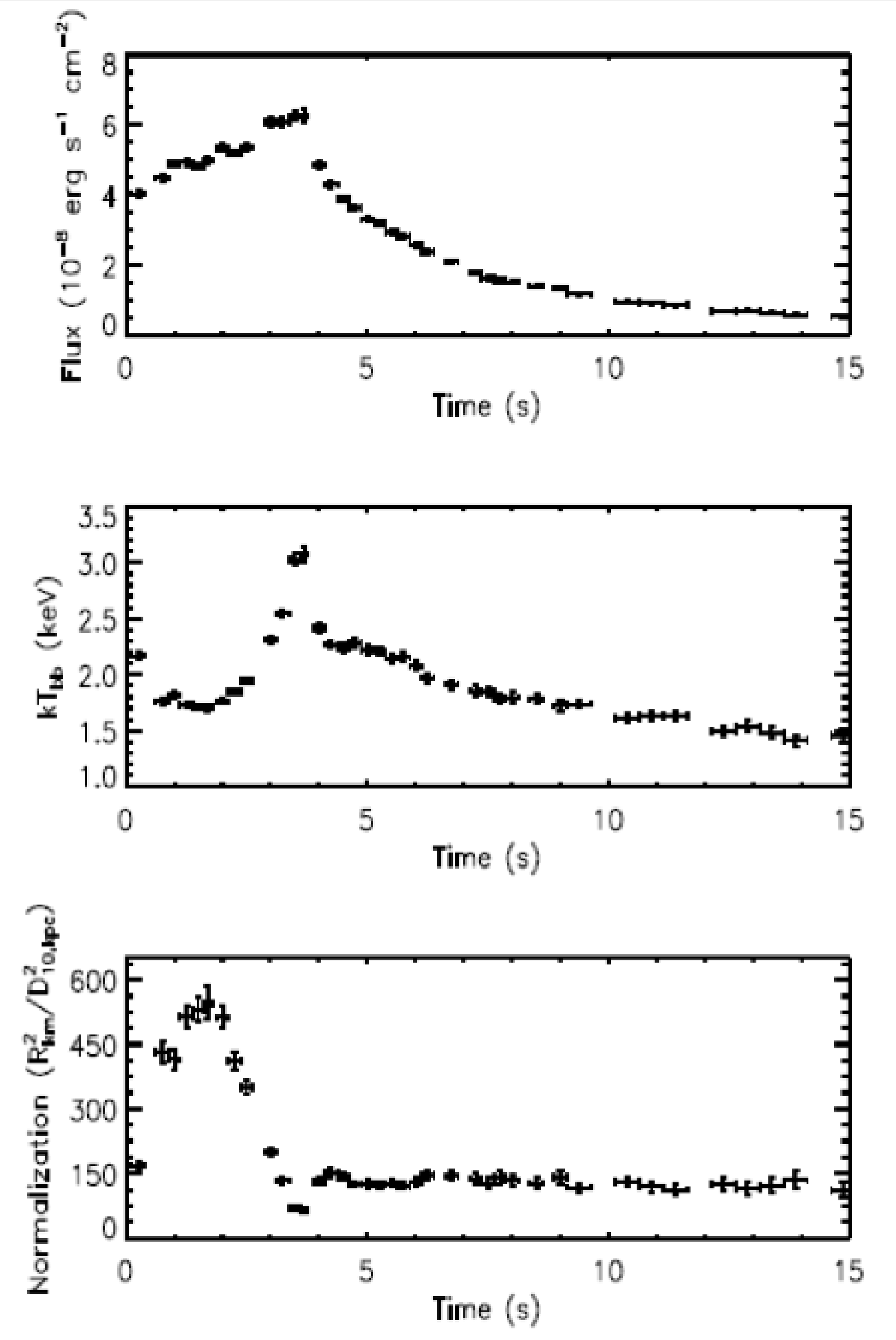
$$F_{TD} = \frac{GMc}{\kappa D^2} \sqrt{1 - 2\beta(r_{ph})}$$

- Normalization during the tail of the burst:

$$\frac{F_\infty}{\sigma T^4_{bb,\infty}} = f_c^{-4} \left(\frac{R}{D}\right)^2 (1-2\beta)^{-1}$$

- If we have the distance, two constraints for mass and radius
- Dimensionless parameter

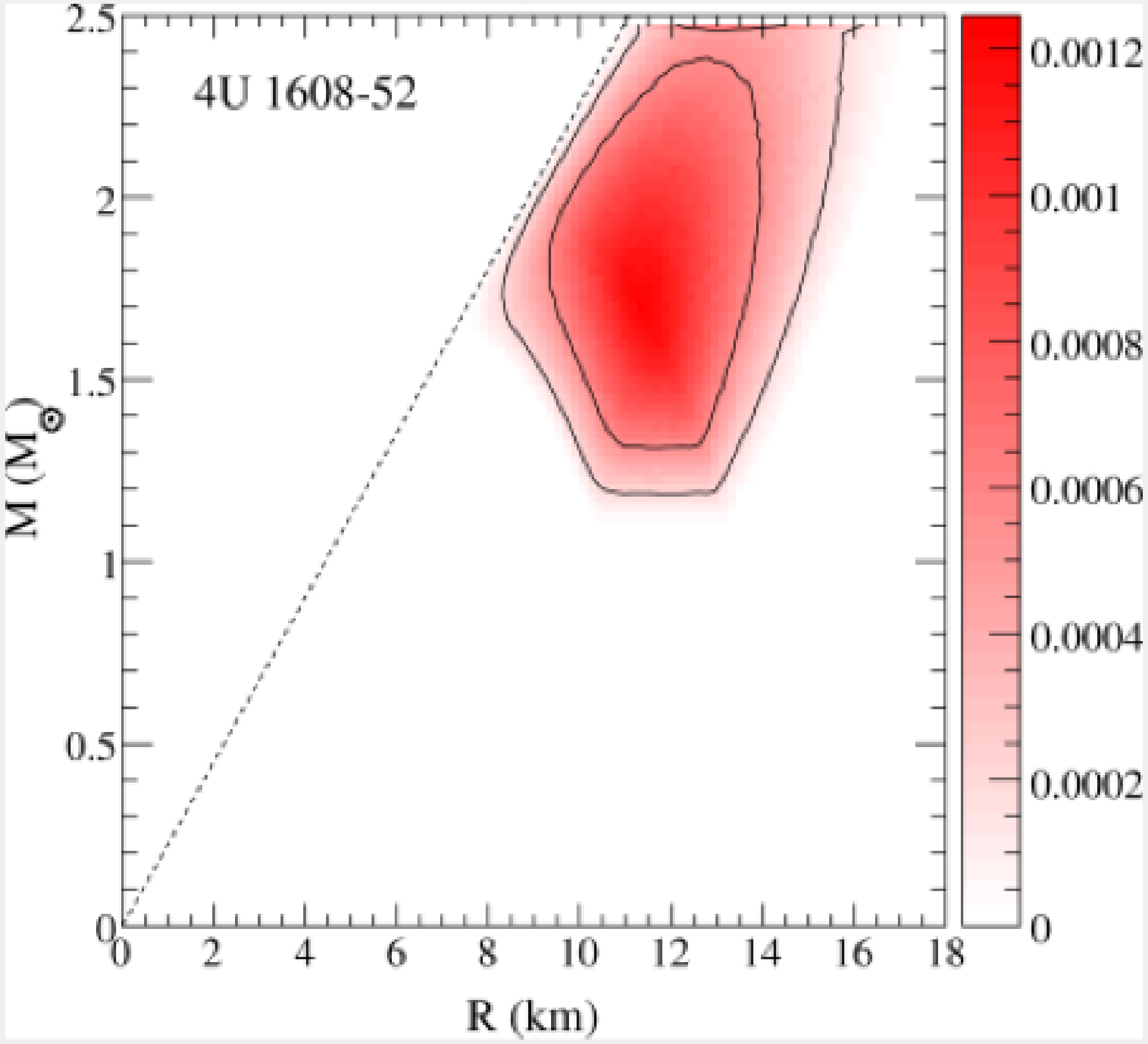
$$\alpha \equiv \frac{F_{TD}\kappa D}{\sqrt{A}c^3 f_c^2}$$



Ozel et al. (2010)

Photospheric Radius Expansion X-ray Bursts

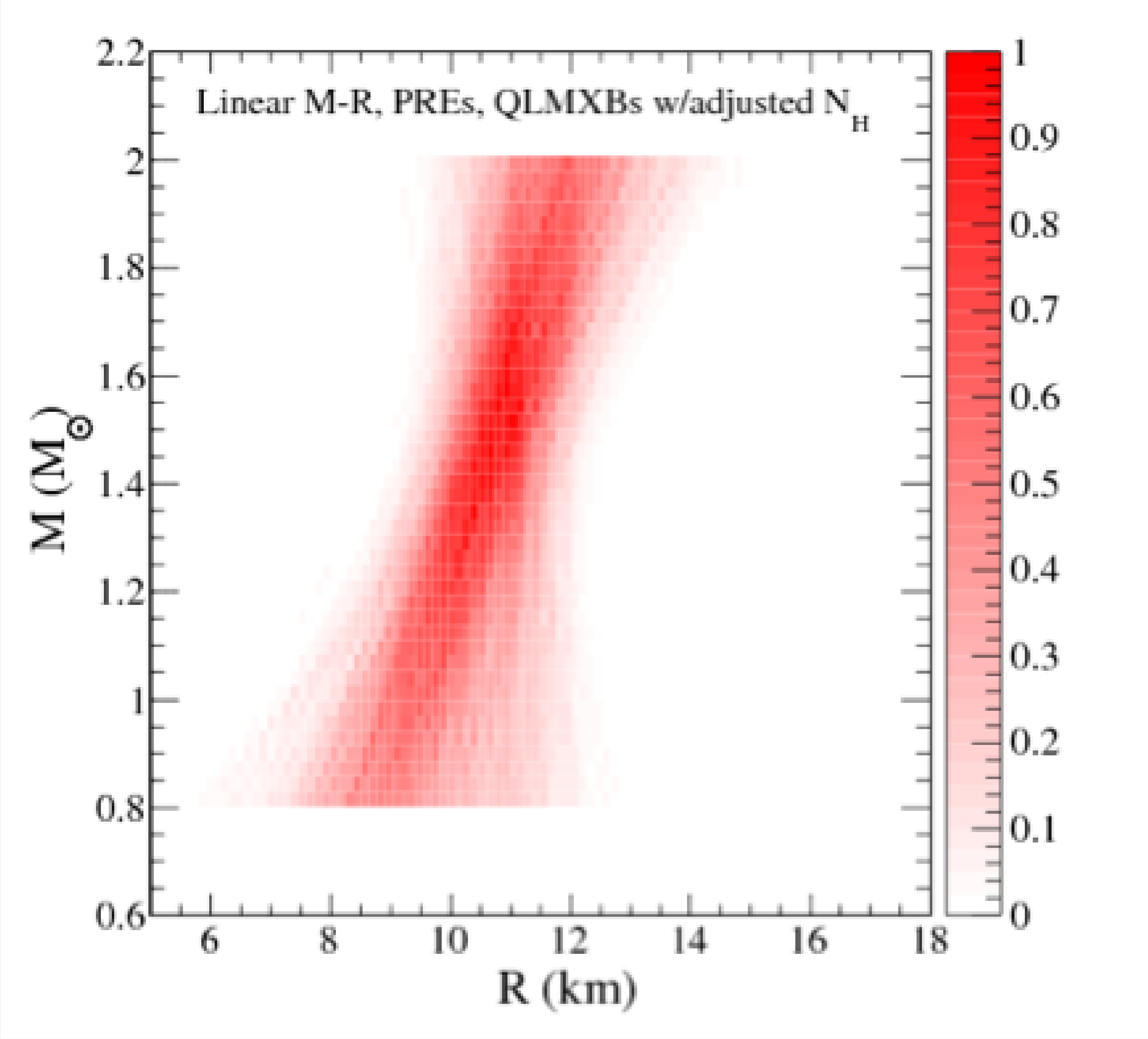
- Several potential systematic uncertainties
- All the complications of qLMXBs
- plus requires assumptions about time-dependence



Steiner et al. (2010)

Minimal Nuclear Physics Models

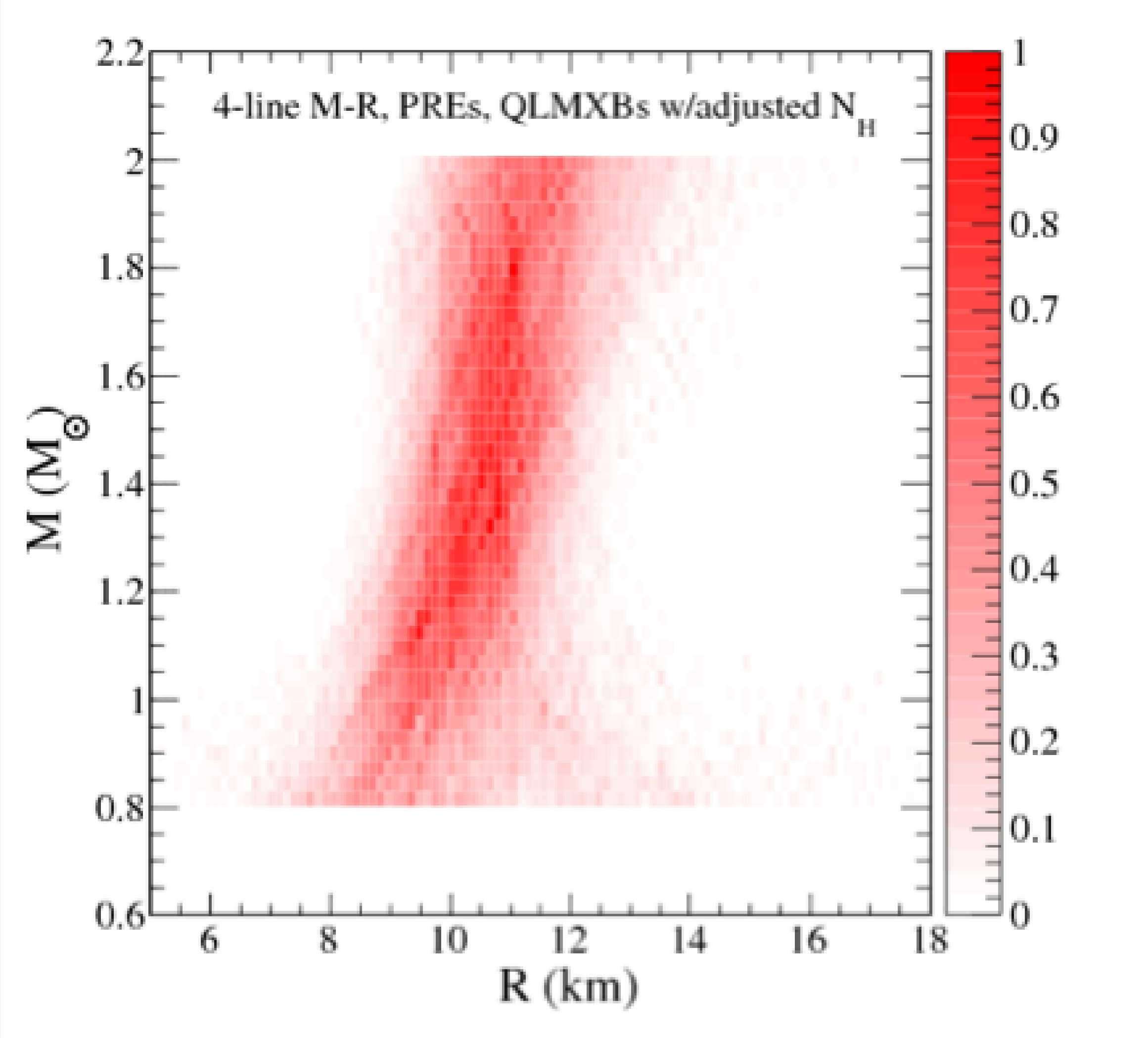
What if we directly parameterize the $M - R$ curve?



Linear model

Lattimer and Steiner (2013)

- Maybe the closest thing to a "model-independent" result
- Consistent with a vertical $M - R$ line at the 2 σ level



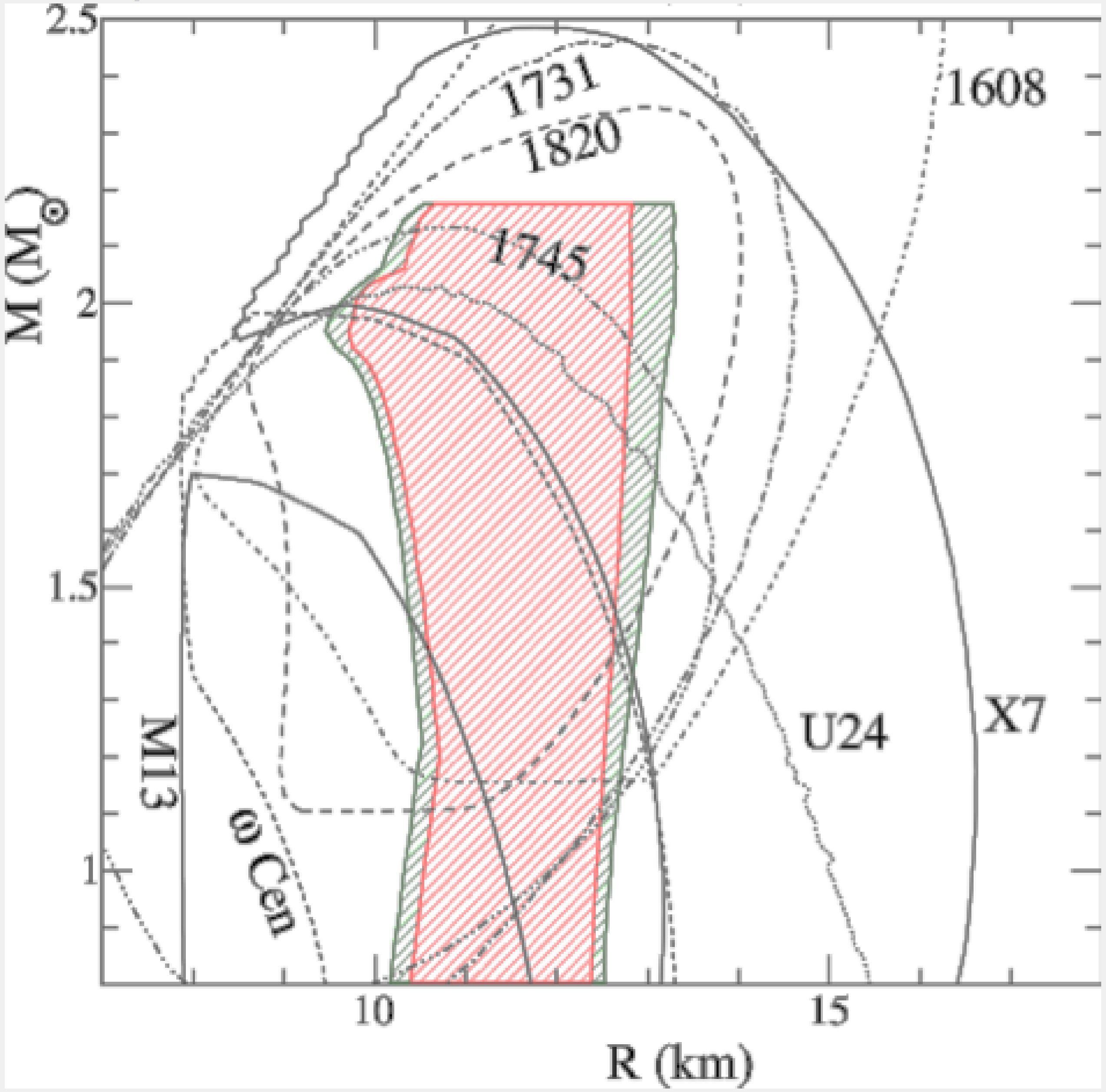
Four-line segments (8 parameters)

Lattimer and Steiner (2013)

- Some of these $M - R$ curves may be unphysical
- Observations suggest positive slope for most masses

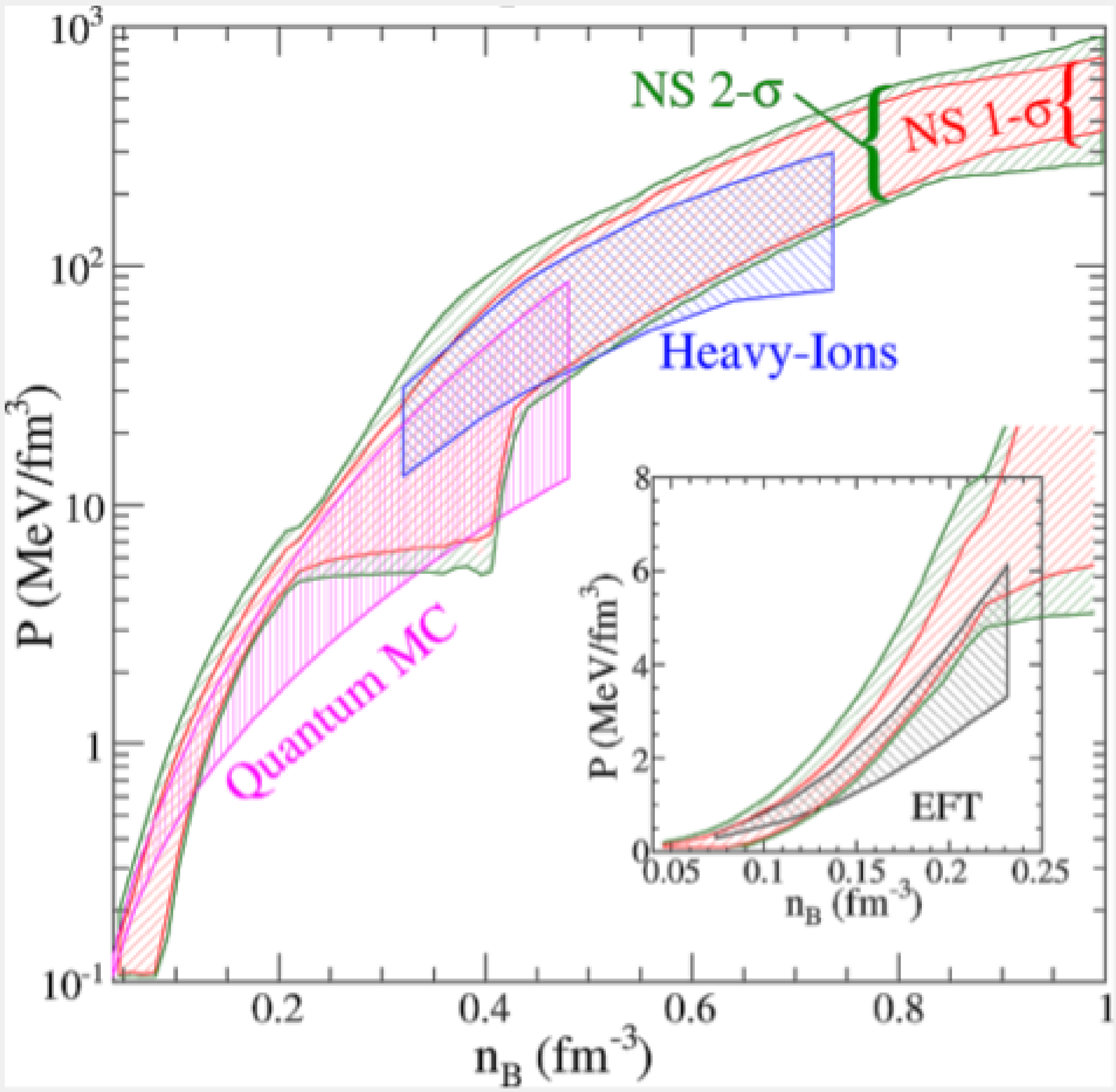
The M-R curve and the EOS of dense matter

Now parameterize the EOS:



Steiner, Lattimer, and Brown (2013)

- Choose several different models, for every observable, find the region which encloses all ranges
- We find concordance between nuclear physics data and astronomical observations



Steiner, Lattimer, and Brown (2013)

- Can determine pressure, but not composition
- Future: novel combinations of several observations with models and careful assessment of uncertainties

The M-R curve and the EOS of dense matter

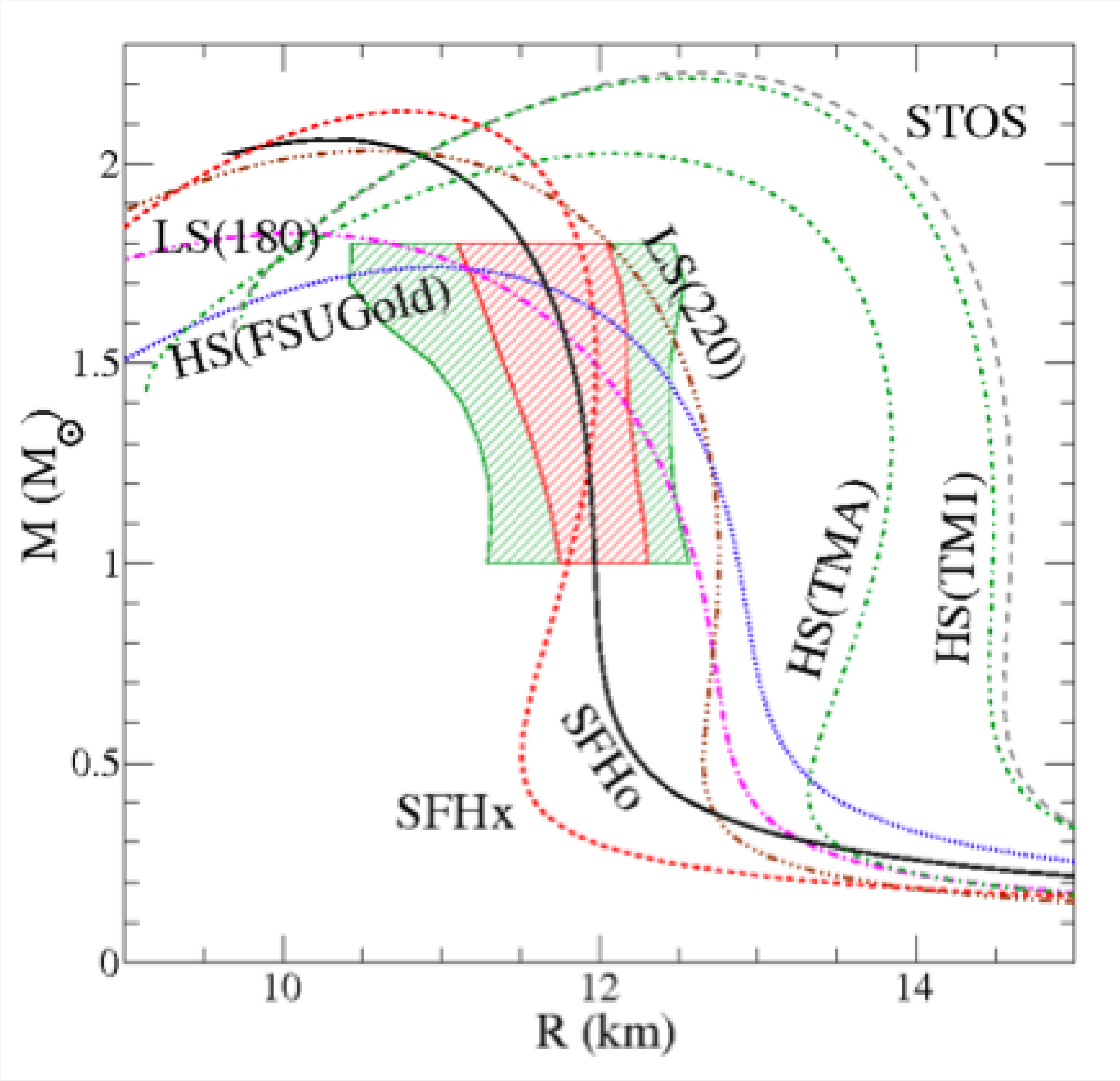
EOS Model	Data modifications	$R_{95\%>}$	$R_{68\%>}$ (km)	$R_{68\%<}$	$R_{95\%<}$
Variations in the EOS model					
A	-	11.18	11.49	12.07	12.33
B	-	11.23	11.53	12.17	12.45
C	-	10.63	10.88	11.45	11.83
D	-	11.44	11.69	12.27	12.54
Variations in the data interpretation					
A	I	11.82	12.07	12.62	12.89
A	II	10.42	10.58	11.09	11.61
A	III	10.74	10.93	11.46	11.72
A	IV	10.87	11.19	11.81	12.13
A	V	10.94	11.25	11.88	12.22
A	VI	11.23	11.56	12.23	12.49
Global limits		10.42	10.58	12.62	12.89

Steiner, Lattimer, and Brown (2013)

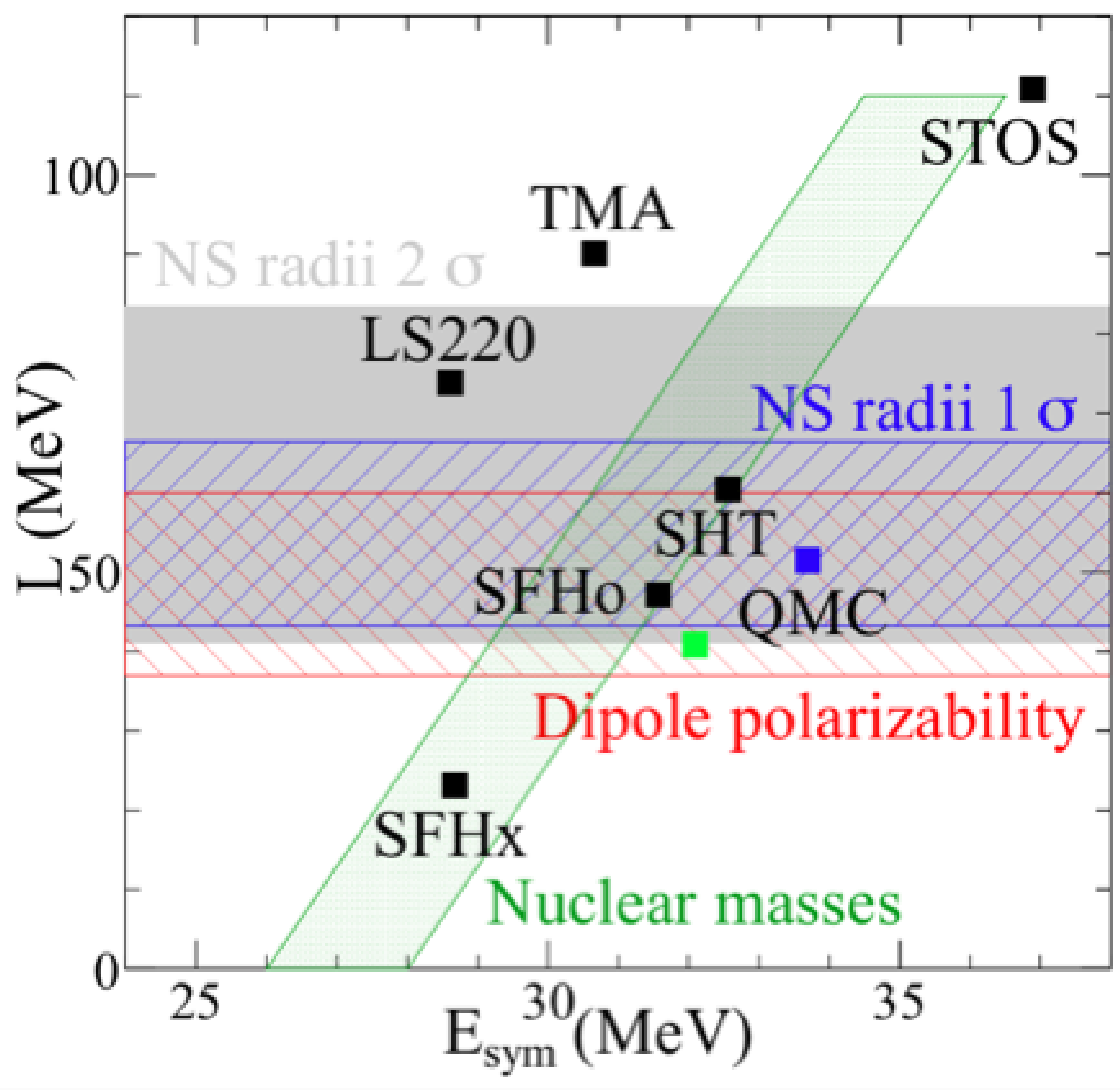
- Critical component: trying different EOS parameterizations and different interpretations of the data

- Modest attempt to address systematic uncertainties

Supernova EOS and the Symmetry Energy



Steiner, Hempel, and Fischer (2013)



Based on Steiner, Hempel, and Fischer (2013)

- Limited number of supernova EOSs which satisfy $M - R$ constraints and the $S - L$ correlation
- Current EOS uncertainties too small to explain explosion
- Many simulation properties are weakly correlated with the symmetry energy

Summary

- Currently available neutron star mass and radius observations constrain the universal neutron star $M - R$ curve
  - Neutron star radii are likely between 10.4 and 13 km
- Constrain the nucleon-nucleon interaction and QCD.
  - 35 MeV $< L <$ 80 MeV
- Must attempt to address systematic uncertainties
- New EOS tables which respect neutron star observations
- Tension between large masses, small radii, and stiff EOSs
- More observations are needed
- ...in the mean time, statistical methods can help us connect experiment and observations