Chern-Simons-Matter theory and Its Holographic Dual

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Introduction

- Quantum gravity is interesting.
- By definition, it's anything that combines quantum mechanics with gravity.
- For various reasons, people keep wanting to find theories of gravity other than string theory.
- This is not a good idea.

Outline

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CSM-Vasiliev Proposal

- The proposal began with a remarkable classical theory.
- For many years, theorists had investigated the possibility of a relativistic dynamical theory of massless fields of spin greater than 2.
- At the quantum level, these theories were long ago strongly constrained by a theorem due to Weinberg and subsequent extensions:
- No relativistic interacting theories in Minkowski space in $D \geq 4$.
- Even at the classical level, such theories have not been constructed.

CSM-Vasiliev Proposal

- In 1987, Fradkin and Vasiliev constructed a consistent classical theory for such fields an anti-de Sitter space.
- The nonzero cosmological constant is crucial:
- Couplings to background curvature blur the distinction between massless and massive fields.
- The theory represents only an equation of motion and not an action.
- Not clear whether or not this is just a technicality or a sign of some larger issue.
- Here I shall focus exclusively on the four-dimensional Vasiliev theory.

CSM-Vasiliev Proposal

- The theory is formulated from the start as a theory in several extra dimensions.
- These dimensions are not Lorentz invariant with the visible four dimensions; Vasiliev is not a Kaluza Klein theory in the usual sense.
- The fields are functions not only of the spacetime coordinates x^μ but also of auxiliary dimensions $y^\alpha, \bar{y}^{\dot{\alpha}}$ and $z^\alpha, \bar{z}^{\dot{\alpha}}$.
- These dimensions are noncommutative dimensions, with the coordinates obeying the commutators

$$[y^1, y^2] = [z^1, z^2] = +2$$
$$[\bar{y}^1, \bar{y}^2] = [\bar{z}^1, \bar{z}^2] = -2$$
$$[y^\alpha, \bar{y}^{\dot{\alpha}}] = [z^\alpha, \bar{z}^{\dot{\alpha}}] = [y^\alpha, z^\beta] = [y^\alpha, \bar{z}^{\dot{\alpha}}] = [\bar{y}^{\dot{\alpha}}, z^\alpha] = [\bar{y}^{\dot{\alpha}}, \bar{z}^{\dot{\beta}}] = 0\ .$$

CSM-Vasiliev Proposal

- The dynamical objects of the theory are two fields $\hat{\mathcal{A}}$ and B.
- The field B is a scalar.
- The field $\hat{\mathcal{A}}$ is a one-form, but with components only in the x^μ and $z^\alpha, \bar{z}^{\dot{\alpha}}$ directions, but not in the $y^\alpha, \bar{y}^{\dot{\alpha}}$ directions.
- In terms of these fields the equation of motion is

$$d_x \hat{\mathcal{A}} + \hat{\mathcal{A}} * \hat{\mathcal{A}} = f_*(B * K) dz^2 + \bar{f}_*(B * \bar{K}) \bar{z}^2,$$
$$d_x B + \hat{\mathcal{A}} * B - B * \pi(\hat{\mathcal{A}}) = 0.$$

- Here:
  - f is an arbitrary* function of one variable
  - $dz^2 \equiv \frac{1}{2} \epsilon_{\alpha\beta} dz^\alpha \wedge dz^\beta$ and similarly for $d\bar{z}^2$.
  - the symbol $*$ is the noncommutative product.
  - d_X is the exterior derivative on the x^μ directions.
  - The functions $K, \bar{K}$ are $K \equiv e^{\epsilon_{\alpha\beta} z^\alpha y^\beta}$ and $\bar{K} \equiv e^{\epsilon_{\dot{\alpha}\dot{\beta}} \bar{z}^{\dot{\alpha}} y^{\dot{\beta}}}$.
  - The operation π is a reflection of the internal coordinates.

CSM-Vasiliev Proposal

- The equations of motion are invariant under a gauge symmetry parametrized by a single scalar function $\epsilon(x, y, \bar{y}, z, \bar{z})$.
- The infinitesimal transformation is:

$$\delta\hat{\mathcal{A}} = d\epsilon + [\hat{\mathcal{A}}, \epsilon], \qquad \delta B = -\big(\epsilon * B - B * \pi(\epsilon).\big)$$

CSM-Vasiliev Proposal

- To find the semiclassical spectrum, start by finding a classical solution to the EOM:

$$B_{(0)} = 0 \ ,$$
$$\hat{\mathcal{A}}_{(0)} = \frac{1}{2} z_\alpha dz^\alpha + \frac{1}{2} \bar{z}_{\dot\alpha} d\bar{z}^{\dot\alpha} + \left(y \, , \bar{y}\right) e^\mu \, \Gamma_\mu \begin{pmatrix} y \\ \bar{y} \end{pmatrix}$$

- Here
  - Γ^μ are gamma matrices of $SO(4)$
  - $e^\mu(x)$ is a vierbein for a locally AdS_4 metric on x-space of Ricci scalar curvature $\texttt{Ric} = -4$.
- This is not the only maximally symmetric solution of the theory, even up to gauge transformation, but it is the simplest.

CSM-Vasiliev Proposal

- To find the spectrum of the theory, expand around the classical vacuum solution $B, \hat{\mathcal{A}}$, truncating at terms linear in fluctuations.
- Write:

$$B = B_{(0)} + \sqrt{\hbar}\,\delta B$$
$$\hat{\mathcal{A}} = \hat{\mathcal{A}}_{(0)} + \sqrt{\hbar}\,\delta\hat{\mathcal{A}}$$

  where $\hbar$ is taken here only as a formal parameter to count the number of fluctuations in a given term.
- At order $\sqrt{\hbar}$ the EOM read:

$$d_x\delta\hat{\mathcal{A}} + \hat{\mathcal{A}}_{(0)} * \delta\hat{\mathcal{A}} + \delta\hat{\mathcal{A}} * \hat{\mathcal{A}}_{(0)} = f'(0)\,(\delta B * K)dz^2 + \bar{f}'(0)\,(\delta B * \bar{K})d\bar{z}^2,$$
$$d_x\delta B + \hat{\mathcal{A}}_{(0)} * \delta B - \delta B * \pi(\hat{\mathcal{A}}_{(0)}) = 0.$$

CSM-Vasiliev Proposal

- Decomposing into modes on the internal space, the linearized solutions of the Vasiliev system mod gauge transformations are a set of massless fields on the AdS_4 background, with exactly one of each even spin.
- In AdS or any curved space for that matter, the concept of "masslessness" is slightly fuzzy due to the curvature couplings.
- The more precise statement is that each field precisely saturates the Breitenlohner-Friedman bound for its particular spin.
- Another precise characterization of the value of the mass is that each field with nonzero spin lies at the value of the mass such that the number of independent helicity states is reduced from $2s + 1$ to 2, just as for massless fields in Minkowski space.

CSM-Vasiliev Proposal

- This description of the spectrum is suggestive when translated into the language of the dual conformal field theory living on the boundary via the AdS_4/CFT_3 correspondence.
- Each massless field in the bulk corresponds to a conserved current of the same spin s, with operator dimension $\Delta = s + 1$.
- This operator spectrum tells us important things about what the conformal field theory is.

CSM-Vasiliev Proposal

- The first thing you notice is, that's a lot of conserved, higher-spin currents!
- Most well-studied conformal field theories such as $\mathcal{N} = 4$ super-Yang-Mills have no conserved currents beyond the stress tensor, at spin 2.
- This corresponds holographically to the property that conventional gravity solutions in the bulk do not have massless fields of spin greater than 2.

CSM-Vasiliev Proposal

- However there are very familiar CFT with higher-spin conserved currents, namely those based on free fields.
- For a set of free complex scalar fields ϕ^A, there exists for each spin s exactly one primary operator bilinear in the fields and invariant under the $SU(N)$ global symmetry

$$\mathcal{O}^{(\mu_1 \cdots \mu_s)} \equiv \bar{\phi}^A \partial^{\mu_1} \cdots \partial^{\mu_s} \phi^A + \text{total derivatives}$$

that is symmetric and traceless in its indices $\mu_1, \cdots \mu_s$, as well as conserved:

$$\partial_{\mu_s} \mathcal{O}^{(\mu_1 \cdots \mu_s)} = 0 \ .$$

- In other words, the field content of the Vasiliev theory in the bulk can be identified with the $SU(N)$-singlet sector of a free field theory!

CSM-Vasiliev Proposal

- Now, this doesn't yet sound like a full proposal for a duality, because we would need to construct a CFT whose operator spectrum contains only the singlet sector of the free field theory, and nothing more.
- Since local operators correspond to states on S^2, this doesn't sound like a very local operation.
- How do you project a free theory to only its singlet sector?

CSM-Vasiliev Proposal

- On the one hand, at the level of the local operator spectrum, you should always be able to project to a singlet sector of whatever symmetry you like.
- Since the OPE of two singlets yields another singlet, the OPE of the singlet sector closes associatively if the OPE in the parent theory does.
- On the other hand, there are robust reasons you should not be able to do this in a local field theory.
- At this point the Vasiliev advocates might want to disagree strenuously.
- They will say – quite accurately – that there does exist an explicitly local, explicitly causal, explicitly quantum mechanical, and explicitly modular invariant construction of the singlet sector in terms of Chern-Simons-matter theory.

CSM-Vasiliev Proposal

- Something like this:

$$Z_{\text{singlet}} = Z_{\text{CSM}} = \lim_{k\to\infty} \int \frac{\mathbf{D}A\mathbf{D}\phi}{\mathbf{D}\Omega} \exp\left(-S_k^{\text{CSM}}\right) \ .$$

  where S_k^{CSM} is the standard Chern-Simons-matter action with level k and a single scalar in the fundamental.
- Indeed, on $S^2 \times S^1$, this action does in fact do the job. The gauge constraint relates the flux of the gauge field to the charge density with a factor of k, so the integrated flux equals $\frac{1}{k}$ times the gauge charge:

$$\hat{F}_{12} = \frac{1}{k}\,\hat{J}^0 \ ,$$

  so the gauge charge with respect to any generator of the gauge group must be a multiple of k, by Dirac quantization. As $k \to \infty$, we recover the singlet sector.

CSM-Vasiliev Proposal

- This is the core idea of the proposal for a dual of Vasiliev theory.
- On some level, the proposal works perfectly.
- The CS construction successfully produces a local CFT whose local operator spectrum matches the perturbative excitation spectrum of Vasiliev theory.

Non-Evidence for CSM-Vasiliev Duality

- The proposal has many apparent successes beyond the spectrum.
- The three-point functions of arbitrary currents have been matched with cubic bul interactions of the Vasiliev fields.
- Also, properties of interacting conformal deformations (the Wilson-Fisher critical point) of the theory have been matched with those of Vasiliev theory in AdS_4 with a modified boundary condition breaking some of the higher-spin gauge symmetry.
- However we shall see that all the existing apparent evidence is not actual evidence in favor of Vasiliev gravity as an well-defined dynamical theory, but rather completely dictated by symmetry.

Non-Evidence for CSM-Vasiliev Duality

- A theorem due to Maldacena and Zhiboedov (2011) shows that the structure constants of the operator product expansion (OPE) of conserved higher-spin currents is the same in any 3-dimensional CFT.
- This sounds good for CSM-Vasiliev duality but on further reflection it clearly isn't:
- Says that the existing tests are not in fact tests at all, but rather something that must be true of any bulk theory containing massless higher-spin gauge fields.

Non-Evidence for CSM-Vasiliev Duality

- The tests of correlation functions that have been done bear neither on existence nor uniqueness of "Vasiliev theory":
- There need not exist a minimal CFT containing only higher-spin currents in the local operator spectrum.

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- There need not exist a minimal CFT containing only higher-spin currents in the local operator spectrum. In fact there does not.
- Furthermore, a non-minimal extension containing the currents as a subsector need not be unique. In fact it is not.
- Nor does the MZ theorem determine other basic observables of the theory, such as the partition function on S^3 (the F-coefficient) or the spectrum on spatial slices other than S^2.
- We shall see signs of the breakdown of "Vasiliev theory" in these observables.

No-Go Theorem for the Minimal Theory

- Now we will give a simple argument showing that the theory with the minimal spectrum does not exist, in the following sense.
- We will show there exists no local conformal field theory whose spectrum is equal to the singlet sector of the free theory on arbitrary spatial slices.
- Any consistent local field theory should be quantizable on a manifold of arbitrary topology (modulo a subtletly I will ignore for the moment because it isn't relevant). Therefore we can also ask whether the minimal theory would have a on $T^3 = S^1 \times T^2$, for instance.
- We answer this in the negative.

No-Go Theorem for the Minimal Theory

- Start with the partition function of the free parent theory:

$$Z_{\text{parent}} = \sum_{\text{all E}} 1 \cdot \exp\left(-\beta\, E\right) = \text{tr}\left(\exp\left(-\beta H\right)\right) \,.$$

- Note the positive integer coeffiicient of every exponential. This just means that the multiplicity of every energy level is a positive integer.
- This form simply follows from quantum mechanics and unitarity in particular.

No-Go Theorem for the Minimal Theory

- In the "projected" theory, we have

$$Z_{\text{projected}} = \sum_{\text{singlets}} 1 \cdot \exp\left(-\beta\, E\right) = \text{tr}(\mathbf{P} \exp\left(-\beta H\right)) \ .$$

- This can be rewritten as:

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- This can be rewritten as:

$$Z_{\text{projected}} = \sum_{\text{singlets}} 1 \cdot \exp\left(-\beta\, E\right) = \frac{1}{|G|} \sum_{g} \mathrm{tr}(\mathbf{g} \exp\left(-\beta H\right)) \ .$$

- Note that this is an average, rather than a sum over G.

No-Go Theorem for the Minimal Theory

- Computing the same partition function at $\beta = R_1 = R$ and $R_2 = \beta'$, we find
-

$$Z_{\text{projected}} = \frac{1}{|G|} \sum_{\text{twist by g}} \exp\left(-\beta' E'_{twisted\ by\ g}\right)$$

If we took this prescription, this would be a disaster. Not only is the partition function not the same as in the original channel at $\beta' \to \beta$, there is simply no quantum mechanics in the x^2 channel at all.

No-Go Theorem for the Minimal Theory

- The coefifcients of the exponentials are not positive integers, and therefore there is simply no Hilbert space at all. This is because we have to average rather than sum over the group G to implement the singlet projection in the original x^0 channel.
- We conclude that the theory projected onto the singlet sector is simply inconsistent – it can never be a unitary, local quantum field theory.

No-Go Theorem for the Minimal Theory

- One possible attitude is to take the Chern-Simons-matter theory to define a theory "as close to minimal" as possible.
- This at least has the virtue of having the correct local operator spectrum, because as we have shown, it has the correct partition function on S^2 spatial slices.
- The local operator spectrum then follows from the state-operator correspondence.
- However, the partition function on $S^2 \times S^1$ is a bit beside the point: That's not where we needed to use the Chern-Simons construction in the first place.
- We'd like to know what happens on other three-manifolds.

No-Go Theorem for the Minimal Theory

- Hint:

No-Go Theorem for the Minimal Theory

- Hint:
  Nothing good.

No-Go Theorem for the Minimal Theory

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  Nothing good.

Spectrum on T^2 Spatial Slices

- Let's reason it out.
- You can see right away that there's trouble when you consider perturbing the system with a relevant operator – a mass m for the scalars.
- If you consider the partition function for any familiar three-manifold other than $S^2 \times S^1$, you will find a divergent free energy.
- This is easy to see because at energies below m, the scalars decouple and we are left with pure Chern-Simons at level k.
- The partition function is well known for pure CS on many 3-manifolds – including all of the form $S^1 \times \Sigma_g$ where Σ_g is a Riemann surface of genus g.

Spectrum on T^2 Spatial Slices

- The Hilbert space of pure CS with $G = SU(N)$ and level k on Σ_g spatial slices is the space of conformal blocks of the WZW model on $SU(N)$ at level k.
- For $g = 1$ the spatial slice is a torus, and the number of conformal blocks is

$$n_{\rm CS} = Z_{\rm CS} = \exp\left(F_{\rm CS}\right)$$
$$F_{\rm CS} \simeq N\ln(k) - \ln(N!) + O(1/k) \ ,$$

  at large k and fixed N, according to the Verlinde formula.
- The large-k asymptotics can be computed directly by the semiclassical method, just using the dimension of the moduli space of flat connections and the identification $\hbar = \frac{1}{k}$.

Spectrum on T^2 Spatial Slices

- This divergence is physical and cannot be gotten rid of. It comes from an actual entropy – a degeneracy of states that goes as

$$n_{\rm CS} \simeq k^{(g-1)N^2}/N! \ .$$

- At large M, corrections to the low-lying spectrum are exponentially suppressed, as $\exp\left(-M\,R_{\rm torus}\right)$. Even for moderately large M, the huge Chern-Simons degeneracy is unbroken.
- Ultimately we are interested in vanishing M, and we have performed an explicit computation of the large-k spectrum that shows the picture is not changed qualitatively.

Spectrum on T^2 Spatial Slices

- We begin by integrating out the nonzero modes of the gauge field and scalars.
- We also fix the gauge $A_0 = 0$ and impose a singlet constraint.
- This leaves us with an effective quantum mechanical system with Lagrangian

$$L = R^2 [\ \frac{k}{8\pi}\ \mathrm{tr}(A_1 \frac{dA_2}{dt}) + \frac{d\phi^{A*}}{dt}\frac{d\phi^{A}}{dt} - \phi^* \phi\ (A_1^2 + A_2^2)\ ]$$

where $\phi, \phi^*, A_{1,2}$ are zero modes of the scalars and gauge fields, and R is the radius of the square torus.

Spectrum on T^2 Spatial Slices

- The Hamiltonian can be written as,

$$H = \sum_{i=1}^{N} \pi_i^* \pi_i + \frac{1}{2} \sum_{a,b=1}^{N^2-1} M^{ab}(\phi)(P^a P^b + \hbar^2 Q^a Q^b)$$

where

$$P^a = A_1^a \,, \qquad Q^a = \frac{k}{16\pi} A_2^a$$

$$\hbar = \frac{16\pi}{k}$$

$$M^{ab}(\phi) = \phi^\dagger \{T^a, T^b\} \phi$$

Also, (ϕ^{A*}, π_A) and (ϕ^A, π_A^*) are canonically conjugate.

Spectrum on T^2 Spatial Slices

- Define the creation and annihilation operators in the gauge sector as,

$$\beta^a = \frac{1}{\sqrt{2\hbar}}(P^a - i\hbar Q^a),\ \beta^{a\dagger} = \frac{1}{\sqrt{2\hbar}}(P^a + i\hbar Q^a)$$

which satisfy the commutation relation,

$$[\beta^a, \beta^{b\dagger}] = \delta^{ab}$$

- Creation and annihilation operators in the scalar sector are

$$\alpha_i = \frac{1}{\sqrt{2\omega}}(\pi_i - i\omega\phi_i),\ \alpha_i^\dagger = \frac{1}{\sqrt{2\omega}}(\pi_i^* + i\omega\phi_i^*)$$

with $\omega^2 = \frac{N\hbar}{2} \sim \frac{N}{k} = \lambda$.

Spectrum on T^2 Spatial Slices

- We define,

$$H_0 = \omega(\alpha_i^\dagger \alpha_i + \bar{\alpha}_i^\dagger \bar{\alpha}_i) + \frac{\hbar}{2\omega}\beta^{a\dagger}\beta^a + N\omega$$

and

$$V = -\frac{\hbar}{2\omega}\left[\bar{\alpha}_i\{T^a, T^b\}_{ij}\alpha_j + \alpha_i^\dagger\{T^a, T^b\}_{ij}\bar{\alpha}_j^\dagger\right]\beta^{a\dagger}\beta^b = -\frac{\hbar}{2\omega}\tilde{V}$$

$$V_1 = \frac{\hbar}{2\omega}\left[\bar{\alpha}_j^\dagger\{T^a, T^b\}_{ij}\bar{\alpha}_i + \alpha_i^\dagger\{T^a, T^b\}_{ij}\alpha_j\right]\beta^{a\dagger}\beta^b$$

The total Hamiltonian H can be written as,

$$H = H_0 + V + V_1$$

which has the nice property that $[H_0, V_1] = 0$.

Spectrum on T^2 Spatial Slices

- At fixed N the quantum mechanical perturbation theory in $1/k$ is a classic degenerate perturbation theory problem, with a continuum of unperturbed states at zero energy coming from the flat directions of the scalars and Wilson lines.
- Expected: Energies of order $O(\sqrt{\hbar}) = O(1/\sqrt{k})$ upon diagonalization of the perturbing Hamiltonian V_1.
- Diagonalizing the perturbing Hamiltonian exactly is hard and gets harder as N gets large.
- At very large N, the problem simplifies again.

Spectrum on T^2 Spatial Slices

- In the large-N limit the energy of the state $Tr\beta^\dagger\beta^\dagger|\Omega>$can be written as,

$$\Delta = \frac{\hbar}{\omega}\left(1 + 0 - \frac{1}{2} + 0 + \frac{a_4}{N} + 0 + \frac{a_6}{N^2} + 0 + ...\right)$$

where a_4 and a_6 are $O(1)$ numbers. This expression justifies our treatment of the potential V as perturbation in the large-N limit. So in the large-N limit the leading term in the gap is

$$\Delta = \frac{\hbar}{2\omega} \sim \frac{\sqrt{\lambda}}{N}$$

- As expected, the energy spectrum goes as $1/\sqrt{k}$, and at large k the entropy diverges in the massless theory, just as in the massive theory.

Spectrum on T^2 Spatial Slices

- This divergent entropy is not an artifact of the massive perturbation. We have shown that the massless theory on T^2 spatial slices has the same divergent free energy due to low energy states coming partially from the Chern-Simons sector.
- Let me now make some comments on this divergence.
- For Vasiliev theory, it really is as bad as you think.
- The theory does not and cannot capture this divergent free energy – it simply doesn't have the right degrees of freedom!

Spectrum on T^2 Spatial Slices

- In the unperturbed theory there is no sense in which these states "factorize out" of the system.
- The mixing is strong enough not to factorize but weak enough that the divergent free energy persists. There is an infinite degeneracy of light, non-decoupled states in the theory!

Spectrum on T^2 Spatial Slices

- These states are not captured by the Vasiliev theory in any useful sense. They are certainly not solitons – they are too light for that.
- Nor are they any kind of bound states of the Vasiliev particles, because the latter have energies of order 1 in AdS units.
- Though the Vasiliev theory does admirably at reproducing amplitudes in the CFT for certain boundary geometries, it seems that the status of the Vasiliev theory can be that of an auxiliary theory that is useful when certain topological excitations of the theory can be integrated out.

Spectrum on T^2 Spatial Slices

- As a general candidate for a dual to large-k CSM theory, Vasiliev is in disagreement with the bulk parametrically.
- There has also been proposed that there is a deformation of the Vasiliev theory that is dual to the theory at nonzero 't Hooft coupling.

Spectrum on T^2 Spatial Slices

- This doesn't seem to work well either. The deformation breaks parity but otherwise affects amplitudes only perturbatively as the 't Hooft coupling goes to zero.
- But such smooth behavior (in k) cannot capture the divergent free energy that appears in this limit. It's quite clear why – the divergence is associated not with deformed interactions but with an infinitely dense spectrum of light states. This simply can't be reproduced by a vertex proportional to the 't Hooft coupling.

The Big Picture and Large-N Scaling

- From the Verlinde formua, the entropy of the Chern-Simons theory on surfaces of genus g is

$$\ln(Z) \simeq (g-1)(N^2-1)\ \ln(k) + O(k^0)\ .$$

The Big Picture and Large-N Scaling

- To understand this formula, we can use semiclassical analysis to determine the leading large-k behavior of the number of states.

The Big Picture and Large-N Scaling

- For a compact phase space, the number of quantum states is given, for small Planck constant $\hbar$, to the volume of phase space in units of $\hbar$:

$$n_{\mathrm{states}} = (\mathrm{const.}) \cdot \frac{\mathtt{Vol}_{\mathrm{phase\ space}}}{\hbar^{\frac{\mathtt{Dim.}}{2}}} \left[1 + O(\hbar)\right] ,$$

The Big Picture and Large-N Scaling

- For Chern-Simons theory in canonical quantization, the phase space is the moduli space $\mathcal{M}_{G,g}$ of flat G-connections on the spatial slice Σ_g, and the Planck constant $\hbar$ is proportional to $\frac{1}{k}$.

The Big Picture and Large-N Scaling

- The volume of the moduli space of flat connections is k-independent, and its dimension is

$$\texttt{Dim.}(\mathcal{M}_{G,g}) = (2g-2)\,\mathrm{Dim.}(G)\ .$$

The Big Picture and Large-N Scaling

- Therefore the number of quantum states, in the large-k limit, is

$$Z = n_{\mathrm{states}} = (\mathrm{const.}) \cdot k^{\frac{1}{2}\mathtt{Dim.}(\mathcal{M}_{G,g})} \left[1 + O(k^{-1})\right]$$

and the entropy is

$$\ln(Z) = (g-1)\,(N^2-1)\,\ln(k) + O(k^0)\ .$$

The Big Picture and Large-N Scaling

- The coefficient of the $\ln(k)$ term does not depend on the numerical, k-independent factor in the volume of $\mathcal{M}_{G,g}$, only on its volume. This order N^2 entropy overwhelms the entropy of the matter. This $N^2 \ln(k)$ divergence of the entropy is striking, because it is larger than any gravitational contribution to the entropy, which would scale at most as $\frac{1}{G_N} = N$.

Degrees Of Freedom

- We want to emphasize that the divergent entropy at large k is not attributable to the nonpositive scalar curvature of the boundary in the case where the boundary is $S^1 \times \Sigma_g, \; g \geq 1$.

Degrees Of Freedom

- It is known that CFT partition functions on such geometries need not be convergent, and the corresponding bulk instabilities have been studied in some cases. .

Degrees Of Freedom

- However the large-k divergence of the entropy in CSM theory cannot be an artifact of vanishing or negative scalar curvature, as the instability is not present in some cases where the entropy is nonetheless still logarithmically divergent with k.

Degrees Of Freedom

- In the case of the Wilson-Fisher fixed point, for instance, the unstable direction of the scalars is always stabilized independently of k, by the quartic interaction.

Degrees Of Freedom

- In the case of the free scalar or the critical Wilson-Fisher scalar, the partition function on S^3 is stabilized by the conformal coupling but still displays a $\ln(k)$ divergence in the free energy ,

$$F = -\ln(Z_{S^3}) \simeq +\frac{N(N-1)}{2}\ln(k) + O(k^0) \ .$$

Degrees Of Freedom

- This comes entirely from the Chern-Simons sector, as the conformal coupling of the scalars allows them to contribute only terms analytic in k.

Degrees Of Freedom

- The value of $F = -\ln(Z_{S^3})$ for various conformal and superconformal field theories in three dimensions has been an object of much recent study , particularly the investigation of the hypothesis that F is a measure of the number of degrees of freedom of the system that decreases along renormalization group flows, analogously to the c coefficient in two dimensions or the a coefficient in four dimensions.

Degrees Of Freedom

- Since the work of Casini *et al.* in 2011, know that there exists an equivalence between entanglement entropy in a 3-dimensional CFT and its free energy on S^3 . With this interpretation, we see again that there are of order $N^2 \ln(k)$ degrees of freedom in the Chern-Simons-matter system, attributable to the topological sector.

Light States In ABJM Theory

There have been proposals to derive Vasiliev gravity as a limit of the ABJ theory . For Chern-Simons-matter theories with ultraviolet-complete string duals, this same large-k divergence on a torus is natural when interpreted in light of string- and M- theory.

Light States In ABJM Theory

- We can for instance compactify the ABJM model on T^2 rather than S^2 spatial slices, and ask what the holographic duality predicts, qualitatively, for the entropy.

Light States In ABJM Theory

- Without doing a fully controlled calculation, we simply observe that the total entropy of the AdS should be approximately extensive in the radial direction, and that the entropy at every point in the radial direction is divergent in the limit $k \to \infty$ with N large but fixed.

Light States In ABJM Theory

- At any point in the radial direction, there are new states due to the topology that become light at large k, corresponding to membranes that wrap the Hopf fiber of the S^7/Z_k, and one direction of the longitudinal T^2.

Light States In ABJM Theory

- At large N these states are still very heavy, but at fixed N, however large, the proper energy of these states, at any fixed point in the radius, goes to zero at large k, because the size of the Hopf fiber is $1/k$ in 11-dimensional Planck units.

Light States In ABJM Theory

- The fixed-N, infinite-k entropy contributed by any point in the radial direction diverges, and this is visible in every duality frame. In the type IIA duality frame, the Hopf fiber is invisible, having been turned into the M-direction, but the AdS radius in string units is inversely proportional to k, at fixed N.

Light States In ABJM Theory

- Therefore fundamental strings wrapping a cycle of the longitudinal torus become light, and make a divergent contribution to the entropy.

Light States In ABJM Theory

- As the longitudinal torus shrinks further towards the infrared, we T-dual to type IIB and the T-dual radius decompactifies. In this duality frame, there is a divergent entropy due simply to the decompactification of the emergent T-dual dimension.

Light States In ABJM Theory

- We could also ask what is the entropy of N M2-branes wrapped on T^2 and probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity in M-theory, without taking the near-horizon limit or taking the back-reaction into account.

Light States In ABJM Theory

- This is a different approximation, but also illuminating because we see again a naturally emerging divergent entropy at large k. Reducing on the T^2 from M-theory to type IIB, we transform the M2-branes into N particles each carrying one unit of Kaluza-Klein momentum on the T-dual direction.

Light States In ABJM Theory

- Even restricting ourselves to normalizable states that saturate the BPS bound in this framework, we see an entropy that diverges at large k.

Light States In ABJM Theory

- Each of these particles can occupy any of k massless twisted sectors of the orbifold, and still saturate the BPS bound for a Kaluza-Klein momentum unit.

Light States In ABJM Theory

- Since each of N interchangeable particles can inhabit one of k possible states, the total degeneracy of such quantum states gives a contribution to the partition function of

$$\Delta Z \gtrsim k^N / N! \; ,$$

because the symmetry factor by which one divides is no more than $N!$.

Light States In ABJM Theory

- This corresponds to a contribution to the entropy of

$$\Delta \mathrm{ln}(Z) \gtrsim N\ \mathrm{ln}(k) - \mathrm{ln}(N!) \simeq N\ \mathrm{ln}(\lambda^{-1})\ ,$$

which is remarkably similar to the Chern-Simons degeneracy:

$$\mathrm{ln}(Z) \simeq (N-1)\ \mathrm{ln}(k) - \mathrm{ln}((N-1)!) + O(k^{-1})$$

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- This counting is most likely an underestimate. Though interactions between particles may in principle lift some of these BPS vacua, a massive perturbation lifting the flat directions allows us to reduce to Chern-Simons theory in the unhiggsed vacuum and compute the supersymmetric index.
- This classical vacuum alone contributes to the index with the full degeneracy of the pure Chern-Simons system on the torus for $U(N) \times U(N)$ at level k.

N^2 Entropy

- The N^2 scaling of the partition functions on S^3 and $S^1 \times \Sigma_g$ with $g \geq 2$ indicates difficulties for the interpretation of the CSM theory in terms of Vasiliev gravity.

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- The N^2 scaling of the partition functions on S^3 and $S^1 \times \Sigma_g$ with $g \geq 2$ indicates difficulties for the interpretation of the CSM theory in terms of Vasiliev gravity.
- The four-dimensional Newton constant G_N as inferred from stress tensor correlators is of order $1/N^1$ in units of the AdS scale, rather than $1/N^2$, so the order N^2 entropy cannot be attributed to a gravitational effect like a horizon entropy if L_{AdS}/N is indeed the true Newton constant of the theory.

N^2 Entropy

- In terms of the proposal to complete Vasiliev gravity in terms of an open-closed topological string theory , the N^2 scaling of the entropy is an indication that the graviton should reside in the closed string, rather than open string sector, of such a theory, in accordance with the principle that it is the gravitational force that must always carry the largest entropy and weakest interaction of any sector of a quantum gravitational theory.

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- Reconciling this with the identification $G_N \propto 1/N$ apparently dictated by stress tensor correlation functions is a challenge for any proposal for a gravitational dual in terms of a quantized "Vasiliev gravity".

RG Flow

- Understanding the renormalization group flow of the theory to pure Chern-Simons theory may be useful for understanding the holographic dynamics of CSM theory, including the order N^2 entropy and the $\ln(k)$ divergence. For many 3-manifolds, the holographic dual to pure Chern-Simons theory is understood in terms of the topological string , including cases where an order N^2 free energy is present.
- For the case of S^3 for example, there is a well-controlled dual in terms of the topological string on the resolved conifold, where the singular behavior of the $k \to \infty$ limit arises from the vanishing of the complexified Kähler parameter of the blown-up $\mathbb{CP}_1$ base of the resolved conifold, leading to unsuppressed contributions of worldsheet instantons.

RG Flow

- The massive RG flow should simply be a classical solution to Vasiliev theory.
- This should be a powerful clue as to the correct dual description in the bulk.
- In this dual, certain properties we expect for the CSM theory should lift simply from the topological string description to the UV.
- For instance, the level-rank duality is manifest as a flop transition. This should lift to the Bose-Fermi duality we heard about yesterday from Minwalla.

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- Similar divergence in Einstein gravity coming from complex saddle points.

Alternative?

- If there is no unique "Vasiliev theory" then what is the correct dual?
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- The dual should be approximated by the dual of pure Chern-Simons.
- On S^3, the dual is the topological A-model on the conifold, with a perturbation.
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- Thank you.