Exploring Seiberg-like dualities in 2+1 dimensions

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Motivation

- Lots of interesting questions in field theory/string theory beg the understanding of the strong coupling behavior
- Dualities provide a useful framework (strong coupling -> weak couping) but difficult to prove
- (Partition function), Superconformal Index are especially useful in 2+1-d, they can probe the strong coupling behavior
- Explore Seiberg-like dualities in 2+1 dimensions
- Continuation of the study of SCFTs in 2+1 dimensions

3d N=2 SUSY gauge theories

- dimensional reduction of N=1 4d gauge theories
- vector multiplet $(A_\mu, \lambda) \to (A_i, \sigma, \lambda)$
- chiral multiplet (ϕ, ψ)
- vector is dual to scalar $\partial_\mu a = \epsilon_{\mu\nu\lambda} F^{\nu\lambda}$ a is periodic
- Coulomb branch of 3d theory is parametrized by the complex scalar $Y = exp(\sigma + ia)$.
- Valid for generic point of moduli space of Coulomb branch where all charged fields are massive
- Most of Coulomb branches are lifted due to (monopole)-instanton effects.
  e.g. $U(N), \ \ \sigma = (\sigma_1, \sigma_2, \cdots \sigma_N)$
  unlifted Coulomb branch are parametrized by
  $V_+ = exp(\sigma_1 + ia_1), V_- = exp(-\sigma_N - ia_N)$ in the convention $\sigma_1 \geq \sigma_2 \geq \cdots \sigma_N$.
- One can define the monopole operator which creates the unit monopole flux around x. This flows in IR to the above Coulomb branch coordinate.

$U(N)$ Aharony duality

- (Aharony 1997) $U(N)$ gauge theory with N_f fundamentals, antifundamentals $Q, \tilde{Q}$ is equivalent to $U(N - N_f)$ with N_f fundamentals $q_a, \tilde{q}_b$, $N_f \times N_f$ meson fields M and additional singlets $V_\pm$ with

$$W = V_+ v_- + V_- v_+ + q\tilde{q}M.$$

Here v_+, v_- are the monopole operators of $U(N - N_f)$.

- For $N_f = N$ the dual theory is described by $V_\pm, M$ with

$$W = V_+ V_- \mathrm{det} M$$

with no gauge group

- Chiral Ring matching

$$Q^a \tilde{Q}_{\bar{b}} \to M^a_{\bar{b}}$$
$$V_\pm \to V_\pm$$

$U(N)$ Aharony duality

- In 1990s we did not have tools to probe this duality
- We now have the superconformal index and the partition function on S^3 to check.
- Similar dualities hold for O, SO, Sp
- One notes that the pattern of the duality is quite similar to 4d Seiberg duality. One might wonder if there's relation between 3d and 4d dualities. This was investigated by Aharony, Razamat, Seiberg, Willett recently and showed that the Aharony dualities can be derived from 4d Seiberg duality, firstly by dimensional reduction and then by suitable deformation.

$$W = Y \to W = \tilde{Y} + Mq\tilde{q}$$

Giveon-Kutasov duality for Chern-Simons matter theory with $U(N)$

- (Giveon, Kutasov 2008)
- Giveon-Kutasov duality can be derived from Aharony duality. (Kapustin, Willet, Yaakov)
- Starting from $U(N)$ with N_f flavors and giving real mass for one flavor one obtains $U(N)$ with $N_f - 1$ flavors with CS level 1. Monopole operators also get massive.
- In the dual side we obtain $U(N - N_f)$ with N_f flavors with CS level -1. The superpotential is $W = Mq\tilde{q}$.
- In general $U(N)_k$ with N_f flavors is dual to $U(N_f + k - N)_{-k}$ with N_f flavors, $W = Mq\tilde{q}$.

3d superconformal index

- $I = Tr(-1)^F e^{\beta' \{Q,S\}} x^{\epsilon + j_3} y_j^{F_j}$
- No dependence on β'
- defined on $S^2 \times S^1$
- Evaluation can be done via localization (S. Kim: Imamura and Yokoyama, arbitrary R-charge)
- It has holomorphic dependence on $m + ir$, combination of the mass and R-charge. Dual theories have the same index with arbitrary R charge consistent with symmetry.

3d superconformal index

$$I(x) = \sum_m \int da\, e^{-S^{(0)}_{CS}} e^{ib_0(a)} y_j^{q_{0j}} x^{\epsilon_0} \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} f_{tot}(e^{ina}, y_j^n, x^n)\right]$$

$$S^{(0)}_{CS} = i \sum_{\rho \in R_\Phi} k\rho(m)\rho(a)$$

$$b_0(a) = -\frac{1}{2} \sum_\Phi \sum_{\rho \in R_\Phi} |\rho(m)|\rho(a)$$

$$y_j^{q_{0j}} = y_i^{\frac{1}{2}\sum_\Phi \sum_{\rho \in R_\Phi} |\rho(m)| F_i(\Phi)}$$

$$\epsilon_0 = \frac{1}{2} \sum_\Phi (1 - \Delta_\Phi) \sum_{\rho \in R_\Phi} |\rho(m)| - \frac{1}{2} \sum_{\alpha \in G} |\alpha(m)|$$

$$f_{chiral}(e^{ia}, y_j, x) = \sum_\Phi \sum_{\rho \in R_\Phi} \left[e^{i\rho(a)} y_j^{F_j} \frac{x^{|\rho(m)| + \Delta_\Phi}}{1 - x^2} - e^{-i\rho(a)} y_j^{-F_j} \frac{x^{|\rho(m)| + 2 - \Delta_\Phi}}{1 - x^2} \right]$$

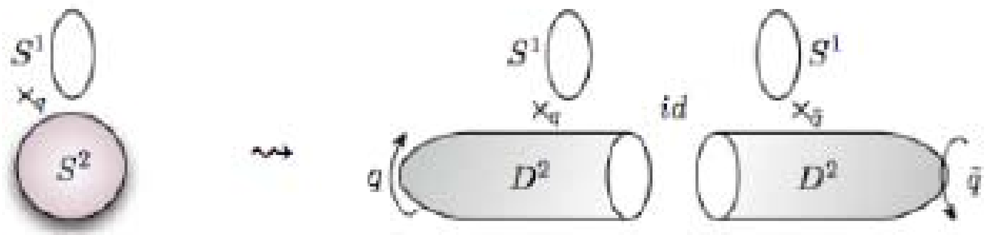
$$f_{vector}(e^{ia}, x) = \sum_{\alpha \in G} \left(-e^{i\alpha(a)} x^{|\alpha(m)|} \right)$$

Typical example

(N_c, N_f)	Electric $U(N_c)$	Magnetic $U(N_f - N_c)$	Index (r is the IR R-charge of Q.)
$(1,1)$	$U(1)$	XYZ	$1 + 2x^{1-r} + x^{2r} + x^{4r} + x^{6r} + x^2\left(-2+2x^{-2r}\right) + x^6\left(2+2x^{-2r}\right) + x^3\left(2x^{-3r}+2x^r\right) + x^5\left(2x^{-5r}-4x^{-r}+4x^r\right) + x^4\left(-3+2x^{-4r}+x^{-2r}-2x^{2r}\right) + \cdots$
$(1,2)$	$U(1)$	$U(1)$	$1 + 4x^{2r} + 9x^{4r} + 16x^{6r} + x^6\left(64+2x^{-6r}-16x^{-2r}\right) + x^4\left(14+2x^{-4r}+4x^{-2r}-16x^{2r}\right) + x^2\left(-8+2x^{-2r}-16x^{2r}-24x^{4r}\right) + \cdots$
$(1,3)$	$U(1)$	$U(2)$	$1 + 2x^{3-3r} + 18x^{5-r} + 9x^{2r} + 36x^{4r} + 100x^{6r} + x^6\left(88+2x^{-6r}-36x^{-2r}\right) + x^4\left(81+9x^{-2r}+153x^{2r}\right) + x^2\left(-18-90x^{2r}-252x^{4r}\right) + \cdots$

Factorization of the superconformal index

- 3d superconformal index has the factorization property. This was discussed in the context of the conformal block of 3d SCFTs. (Beem, Dimofte, Pasquetti 2012)



  - $S^1 \times S^2$ partition function can be thought as the overlap of the wave function

$$Z = \langle 0_q | 0_{\tilde{q}} \rangle$$
$$q = e^{2\pi i}, \ \ \tilde{\tau} = -\tau$$

  - We explicitly work out the factorization of the index by the residue evaluation. (C. Hwang, C Kim, J.P. 2012) Similar factorization was worked out for the partition function on the squashed sphere. (M. Taki 2013)

Factorization of the superconformal index

- The factorized part has the interpretation as the vortex partition function on $R^2 \times S^1$. This was evaluated by H. Kim, J. Kim, S. Kim, K. Lee. Checking the index for Aharony dual is reduced to checking the vortex partition function.
- For $N = 2$ $U(N)$ Chern-Simons theory with N_f fundamentals and $\tilde{N}_f$ antifundamentals, the factorized index is given by

$$\begin{aligned} & I(x,t,\tilde{t},\tau,w,\kappa) \\ =& \frac{1}{N!(N_f-N)!}\sum_{\sigma} I_{1-loop}(x,\sigma(t),\tilde{t},\tau)\times\sum_{\vec{n}=0}^{\infty} Z_{vort}^{\vec{n}}(x,\sigma(t),\tilde{t},\tau;w,\kappa)\times\sum_{\vec{\bar{n}}=0}^{\infty} Z_{vort}^{\vec{\bar{n}}}(x,\sigma(t),\tilde{t},\tau;w^{-1},-\kappa) \end{aligned}$$

$$t,\tilde{t}:\text{flavor symmetry},\quad \tau:\text{axial symmetry},\quad w:\text{topological symmetry}$$

Factorization of the superconformal index

$$
\begin{aligned}
&I_{1-loop}(x, t=e^{iM}, \tilde{t}=e^{i\tilde{M}}, \tau) \\
&= \left(\prod_{\substack{i,j=1\\(i\neq j)}}^{N} 2\sinh\frac{iM_i - iM_j}{2}\right)\left[\prod_{j=1}^{N}\prod_{k=0}^{\infty}\left(\frac{\prod_{a=1(\neq j)}^{N_f} 1 - t_j t_a^{-1} x^{2+2k}}{\prod_{a=1}^{\tilde{N}_f} 1 - t_j \tilde{t}_a \tau^2 x^{2k}}\right)\left(\frac{\prod_{a=1}^{\tilde{N}_f} 1 - t_j^{-1}\tilde{t}_a^{-1}\tau^{-2}x^{2+2k}}{\prod_{a=1(\neq j)}^{N_f} 1 - t_j^{-1} t_a x^{2k}}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&Z_{vort}^{\vec{n}}\left(x=e^{-\gamma}, t=e^{iM}, \tilde{t}=e^{i\tilde{M}}, \tau=e^{i\mu}; w, \kappa\right) \\
&=(-1)^{-[\kappa+(N_f-\tilde{N}_f)/2]\sum n_j} e^{i\kappa\sum_j (M_j n_j + \mu n_j + i\gamma n_j^2)} \\
&\quad \times (-w)^{\sum n_j}\left[\prod_{j=1}^{N}\prod_{k=0}^{n_j-1}\frac{\prod_{a=1}^{\tilde{N}_f} 2\sinh\frac{-i\tilde{M}_a - iM_j - 2i\mu + 2\gamma k}{2}}{\left(\prod_{i=1}^{N} 2\sinh\frac{iM_i - iM_j + 2\gamma(k-n_i)}{2}\right)\left(\prod_{a=N+1}^{N_f} 2\sinh\frac{iM_a - iM_j + 2\gamma(1+k)}{2}\right)}\right]
\end{aligned}
$$

γ : chemical potential for r+2j, $\quad M, \tilde{M}, \mu$: mass parameters, $\quad w$: vortex fugacity

Factorization of the superconformal index

- Since we know the explicit expressions for the index, the index agreement for an Aharony dual pair is equivalent to nontrivial identities for the vortex partition function expression.
- The simplest example is $U(1)$ with $N_f = 1$ flavor and its dual XYZ model whose superpotential is $W = V_+ V_- M$

$$I^{N=N_f=1} = \prod_{l=0}^{\infty} \frac{1-\tau^{-2}x^{2l+2}}{1-\tau^2 x^{2l}} \times Z_{vortex}^{N=N_f=1} \times Z_{anti}^{N=N_f=1}$$

$$I^{M,V_\pm} = \mathrm{PE}[f_M] \times \mathrm{PE}[f_+] \times \mathrm{PE}[f_-] \qquad \mathrm{PE}[f(\cdot)] = \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} f(\cdot^n)\right]$$

$$f_M = \frac{\tau^2 - \tau^{-2}x^2}{1-x^2},$$

$$f_+ + f_- = \frac{\tau^{-1}x - \tau x}{1-x^2}(w + w^{-1}) = f_{V_+} + f_{V_-}$$

Factorization of the superconformal index

- Especially we expect the following identity should hold

$$Z_{vortex}^{N=N_f=1} = \sum_{n=0}^{\infty}(-w)^n \prod_{k=1}^{n} \frac{\tau^{-1}x^{-(k-1)} - \tau x^{k-1}}{x^{-(k-1-n)} - x^{k-1-n}} = \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} w^n \frac{(\tau^{-n} - \tau^n)x^n}{1 - x^{2n}}\right]$$

  - This was proved by Kratternthaler, Spridinov, Vartanov (2011) when they consider the mirror pair
  - One can similarly generate whole classes of nontrivial identities

$SU(N)$ duality

- In 4-d $U(1)$ dynamic is trivial. It does not matter if we consider $SU(N)$ or $U(N)$ duality. However in 3d it does matter.
- If we know $SU(N)$ duality, one can obtain $U(N)$ duality by gauging $U(1)$ global symmetry in both sides. However the inverse process of ungaugung is nontrivial.
- For ungauging, we use the Witten's work on $SL(2, Z)$ transformation of 3d CFTs with $U(1)$ flavor symmetry.
- adding background CS level 1/2
  $T : L \to L + \frac{1}{2} A \wedge dA$
- gauging $U(1)$ one has new $U(1)$ flavor symmetry sometimes called the topological symmetry $J = *dA$.
  $S : L \to L + A_{new} \wedge dA$
- using $S^2 = C$, C being charge conjuagation, gauging $U(1)$ twice one obtains the original theory related by charge conjugation.

$SU(N)$ duality

- The above suggests that if we gauge topological $U(1)_T$ for $U(N)$ dual pair, one can obtain the dual of $SU(N)$. (J.P, K.Park, Aharony, Razamat, Seiberg, Willett)
- Schematically $SU(N) \to_S U(N) \to_{dual} U(N-N_f) \to_S SU(N)\mathrm{dual}$

- Electric theory: $SU(N_c)$ gauge theory(without Chern-Simons term), N_f pairs of fundamental/ anti-fundamental chiral superfields Q^a, $\tilde{Q}_b$(where a, b denote flavor indices).

- Magnetic theory: $U(1) \times U(N_f - N_c)$ gauge theory with the BF coupling

$$A_{U(1)} \wedge d\mathrm{Tr} A_{U(N_f-N_c)} \tag{5.1}$$

, with N_f pairs of fundamental/anti-fundamental chiral superfields q_a, $\tilde{q}^a$ of $U(N_f - N_c)$, $N_f \times N_f$ singlet superfields $(M_j)^a_b$, $j = 0, \ldots, n-1$. We have $v_\pm, V_\pm$ charged under $U(1)$ with charge ± 1. The superpotential is given by

$$W = v_+ V_- + v_- V_+ + Mq\tilde{q} \tag{5.2}$$

where $u_\pm$ is the monopole operator of $U(1)$.

$SU(N)$ duality

- Chiral ring matchings

$$Y \to v_+ v_-$$
$$Q\tilde{Q} \to M$$
$$B \sim Q^{a_1} \cdots Q^{a_{N_c}} \to u_+ \tilde{q}^{a_1} \cdots \tilde{q}^{a_{N_f - N_c}}$$

- dual baryon; due to the BF term unit monopole of $A_{U(1)}$ accompanied by $N_f - N_c$ matters.
- Generating $SU(N)$ dual using S transformation can be applied other cases, e.g., $SU(N)$ with N_f flavors, one adjoint X, with superpotential $W = \mathrm{Tr} X^n$.

- We expect to generate whole new classes of SCFTs related by $SL(2, Z)$

$SU(N)$ Giveon-Kutasov duals

- By introducing the real mass for flavors one can obtain the dual theories for $SU(N)$ with N_f flavors with CS level k.

- Electric theory: $SU(N_c)_k$ gauge theory, N_f pairs of fundamental/ anti-fundamental chiral superfields Q^a, $\tilde{Q}_b$(where a, b denote flavor indices) with Chern-Simons level k.

- Magnetic theory: $U(N_f + k - N_c)$ gauge theory, N_f pairs of fundamental/anti-fundamental chiral superfields q_a, $\tilde{q}^a$ of $U(N_f + k - N_c)$, $N_f \times N_f$ singlet superfields $(M_j)^a_b$, $j = 0, \ldots, n-1$. The superpotential is given by

$$W = Mq\tilde{q}. \tag{6.1}$$

with the Chern-Simons term

$$\tilde{A} \wedge d\tilde{A} - k(A_{N_{\tilde{c}}} \wedge dA_{N_{\tilde{c}}} - \frac{2i}{3} A_{N_{\tilde{c}}} \wedge A_{N_{\tilde{c}}} \wedge A_{N_{\tilde{c}}}) \tag{6.2}$$

where $A_{N_{\tilde{c}}}$ is the $U(N_{\tilde{c}}) = U(N_f + k - N_c)$ gauge field and $\tilde{A} = \mathrm{Tr} A_{N_{\tilde{c}}}$ is the overall $U(1)$ gauge field.

Niarchos duality

- Niarchos duality: 3d analogue of Kutasov-Schwimmer-Seiberg duality, duality with two index tensor matter plus fundamentals

- Electric theory: $U(N_c)_k$ gauge group, N_f pairs of fundamental/anti-fundamental chiral superfields Q^a, $\tilde{Q}_b$(where a, b denote flavor indices), an adjoint superfield X, and the superpotential $W_e = \mathrm{Tr}\, X^{n+1}$.

- Magnetic theory: $U(n(N_f+k)-N_c)_{-k}$ gauge theory, N_f pairs of fundamental/anti-fundamental chiral superfields q_a, $\tilde{q}^a$, $N_f \times N_f$ singlet superfields $(M_j)^a_b$, $j = 1, \cdots, n$, an adjoint superfield Y and a superpotential $W_m = \mathrm{Tr}\, Y^{n+1} + \sum_{j=1}^{n} M_j \tilde{q} Y^{n-j} q$.

- $N_f = 0, k = 1, n = N_c$; $U(N_c)_1$ Chern-Simons theory with an adjoint X and $W = TrX^{N_c+1}$ is dual to a trivial CFT (theory is massive)

Generalized Jafferis-Yin duality

- This can be derived from the generalized Jafferis-Yin duality
- Jafferis-Yin duality; $SU(2)_1$ with an adjoint is dual to the free theory of one chiral multiplet.
- Generalized Jafferis-Yin duality; $U(N_c)_1$ with an adjoint is dual to the theory of N_c free fields
- The special case of the Niarchos corresponds to the generalized Jafferis-Yin perturbed by a superpotential having unique critical point.
- Niarchos and Generalized Jafferis-Yin have Sp/O version.
- One can also confirm nonperturbative truncation of chiral rings if it occurs;
  number of independent generators of TrX^i are
  $min(n-1, N_c)$; mapped to TrY^i

$U(N)$ with adjoint, no CS term

- Consider the $U(N)$ theory with N_f flavors, one adjoint X, $W = \mathrm{Tr}X^{n+1}$.
- One important question is how many independent monopole operators parametrizing Coulomb branch
- Considering the deformation

$$W = \sum_{j=0}^{n} \frac{s_j}{n+1-j} \mathrm{Tr}X^{n+1-j}$$

$$W'(x) = \sum_{j=0}^{n} s_j x^{n-j} \equiv s_0 \prod_{j=1}^{n} (x - a_j)$$

- If all a_j are different, the adjoints get massive and the gauge group is broken to

$$U(N_c) \to U(r_1) \times U(r_2) \times \cdots \times U(r_n)$$

- We have n decoupled $N = 2 U(N)$ theory with N_f flavors
- Thus we need at leat n pairs of monopole operators. It turns out that we have just n pairs of monopole operators

$U(N)$ with adjoint, no CS term

- Independent n pairs of monopole operators can be constructed as follows. Consider the radially quantized theory on $R \times S^2$. Turning on the monopole of unit flux $| \pm 1, 0, \cdots, 0 >$. The guage group is broken to $U(1) \times U(N-1)$ and

$$X = \begin{pmatrix} X_{11} & 0 \\ 0 & X' \end{pmatrix}$$

- Define $v_{i,\pm} = X_1^i | \pm 1, 0, \cdots, 0 >, \;\; i = 0 \cdots n-1$
- Due to the characteristic equation for X, $v_{j,\pm}, j \geq n$ is not independent

$U(N)$ with adjoint, no CS term

- Electric theory: $U(N_c)$ gauge theory(without Chern-Simons term), N_f pairs of fundamental/anti-fundamental chiral superfields Q^a, $\tilde{Q}_b$(where a, b denote flavor indices), an adjoint superfield X, and the superpotential $W_e = \mathrm{Tr}\, X^{n+1}$.

- Magnetic theory: $U(nN_f - N_c)$ gauge theory(without Chern-Simons term), N_f pairs of fundamental/anti-fundamental chiral superfields q_a, $\tilde{q}^a$, $N_f \times N_f$ singlet superfields $(M_j)^a_b$, $j = 0, \ldots, n-1$, $2n$ singlet superfields $v_{0,\pm}, \ldots, v_{n-1,\pm}$, an adjoint superfield Y, and a superpotential $W_m = \mathrm{Tr}\, Y^{n+1} + \sum_{j=0}^{n-1} M_j \tilde{q} Y^{n-1-j} q + \sum_{i=0}^{n-1} (v_{i,+} \tilde{v}_{n-1-i,-} + v_{i,-} \tilde{v}_{n-1-i,+})$.

Conclusions and Future directions

- We explore various Seiberg-like dualities.

  Partition function and the index support such dualities
- The origin of such duality?
- Non-supersymmetric generalization

  (cf. Minwalla's talk)