The Supersymmetric M5 Brane Theories on $\mathbb{R}\times\mathbb{CP}^2$

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Hee-Cheol Kim, KM [arXiv:1210.0853]
Hee-Cheol Kim, Seok Kim, Sung-Soo Kim, KM to appear soon.

6d (2,0) Superconformal Theories

- A, D, E type: type IIB on $R^{1+5} \times C^2/\Gamma_{ADE}$
- A_{N-1}, D_N type: N M5 branes, N M5 +OM5
- superconformal symmetry: $OSp(2,6|2) \supset O(2,8) \times Sp(2)_R$
- fields: B, Φ_I, Ψ
- selfdual strength $H=dB={}^*H$, purely quantum $\hbar=1$

- We do not know how to write down the theory for nonabelian case.✓
  - covariant derivative?
- N^3 degrees of freedom✓

- **Can you calculate something of (2,0) theories?**

5d $\mathcal{N}$=2 SYM as the M5 brane theory

- compactification on R^{1+4} x S^1 with radius r
- the lowest KK modes ⇒ 5d SYM
- coupling constant $1/g_{YM}^2 = 4\pi^2/r$
- instanton = quantum of KK modes of unit momentum
- drop KK modes and keep instantons
  - otherwise, it is over-counting
  - there may quantum-gauge-invariance identifying two
  - dyonic instanton index
  - monopole string+ momentum

Hee-Cheol Kim, Seok Ki, E. Koh, KL, Sungjay Lee 2011

Haghighat, Iqbal, Kozcaz, Lockhart, Vafa

- 5d SYM + instantons = ? 6d (2,0) theory
  - 6-loop UV divergence in four-point function

Z. Bern, J. J. Carrasco, L. J. Dixon, M. R. Douglas, H. Johansson, M. von Hippel

More Lessons from 5d SYM

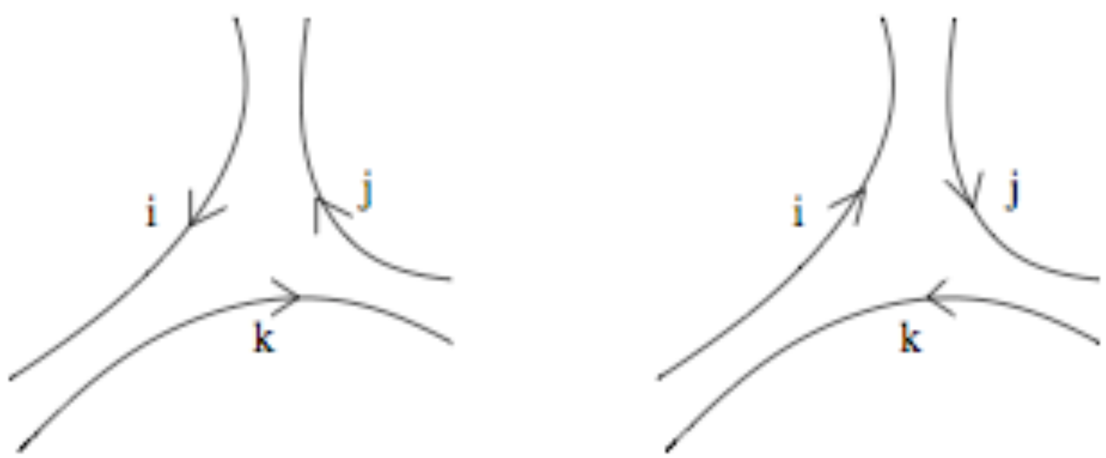
- DLCQ of 6d (2,0) Theory: Nonrelativistic SCFT index

  Berkooz, Rozali, Seiberg 1997,Berkooz, Douglas1996

  Hee-Cheol Kim, Seok Ki, E. Koh, KL, Sungjay Lee:2011

- 1/4 BPS selfdual string junction in the Coulomb phase of 6d (2,0) theory
  - possible solution for N^3 degrees of freedom.
  - The rough entropy calculation in the Coulomb phase seems to work.

KL,Yee:0606150
Bolognesi,KL:1105.5073



BPS Junction Math for A(DE)

Schwarz's talk at Strings 2013

- dimension of A_{N-1}: $d=N^2-1$
- rank of A_{N-1}: $r=N$
- Coxeter number= number of roots/rank: $h=N$
    - Coxeter=Dual Coxeter for simple laced groups
- Anomaly coefficient : $c = dh/3 = N(N^2-1)/3$
- Relation:

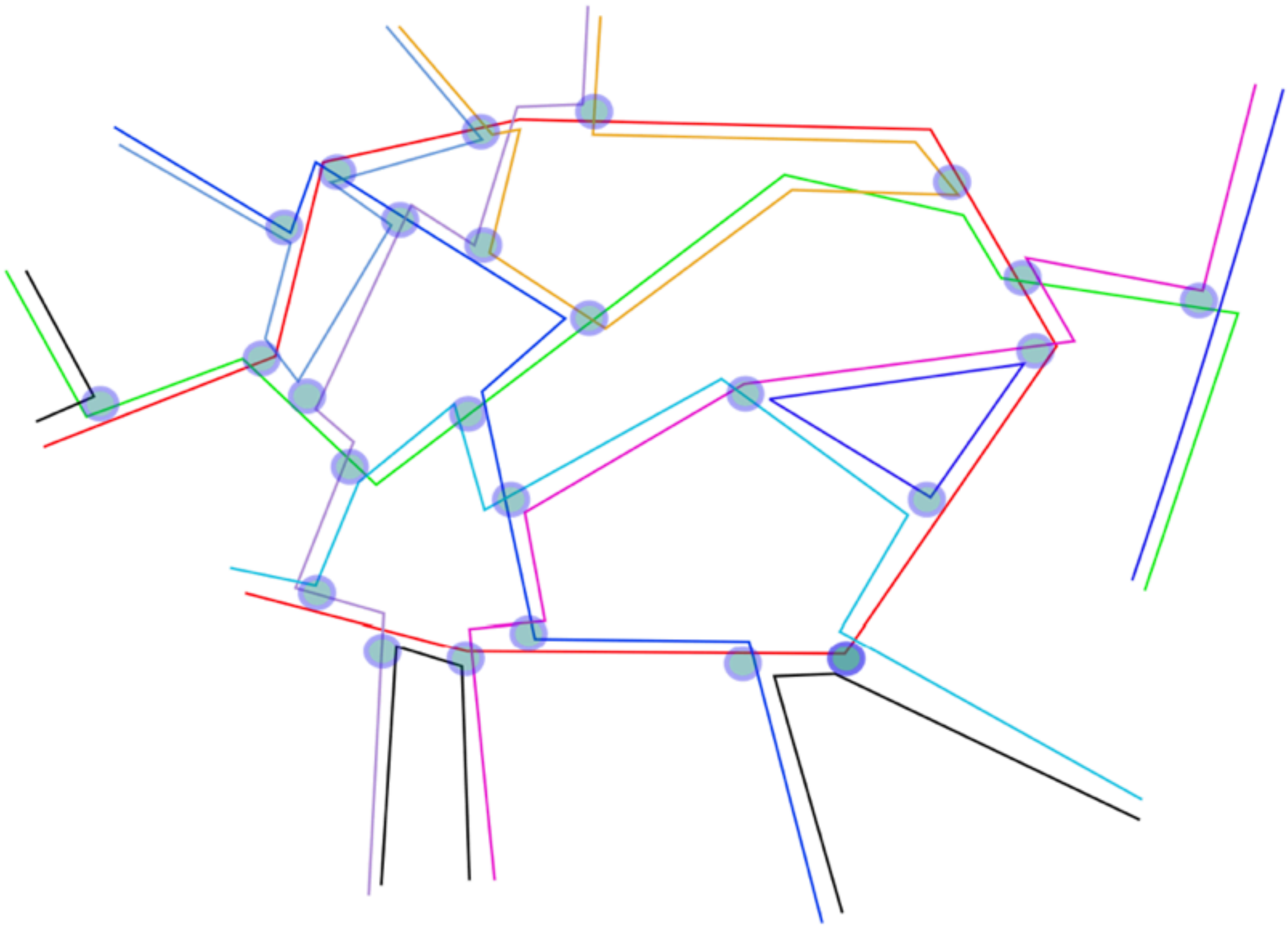
$$c= N(N^2-1)/3= N^2-N + N(N-1)(N-2)/3$$

- # of roots= ij selfdual strings=# of roots: $N(N-1)$
- # of SU(3) imbedding= ijk of BPS (anti)junctions: $N(N-1)(N-2)/3$
- True for ADE algebras

High Temperature in Coulomb Phase

- Micro-canonical
- Massless on N M5 branes: O(N)
- Loops of self-dual strings excitations: $O(N^2)$
- Beyond the Hagedorn temperature
- Webs of junctions and anti-junctions: $O(N^3)$
- ≈ Excitations of webs of tensionless strings in symmetric phase

Nonzero Temperature in Symmetric Phase



Index Function on S^1 x S^5

- Supercharge $$Q^{R_1,R_2}_{j_1,j_2,j_3} \Rightarrow Q = Q^{\frac{1}{2},\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}}, S = Q^\dagger$$

- BPS bound: $$E = j_1 + j_2 + j_3 + 2(R_1 + R_2)$$
- Index function:

$$I = \mathrm{Tr}\left[(-1)^F e^{-\beta'\{Q,S\}} e^{-\beta\left(E - \frac{R_1+R_2}{2} - m(R_1-R_2) + aj_1 + bj_2 + cj_3\right)}\right], \; a+b+c=0$$

- Euclidean Path Integral of (2,0) Theory on S^1x S^5
  - Small S^1 : Partition function on S^5

    Hee-Cheol Kim, Seok Kim 2012, Hee-Cheol Kim, Jooho Kim, Seok Kim 2012, Lockhart, Vafa 2012

    Kallen Zabzine 2012, Hosomich, Seong Terashima2012, Kallen et.al 2012, Imamura 2012

  - S^5= S^1 fiber over CP^2 : Z_K-modding of S^1 fiber with k/K= integers...

$$k \equiv j_1 + j_2 + j_3 + \frac{3}{2}(R_1 + R_2) + \frac{p}{2}(R_1 - R_2), \; p = \text{odd integer}$$

Hee-Cheol Kim, KM [arXiv:1210.0853]

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6d Abelian Theory (Fermion+ Scalar)

- on R x S^5

$$-\frac{i}{2}\bar{\lambda}\Gamma^M\hat{\nabla}_M\lambda-\frac{1}{2}\partial_M\phi_I\partial^M\phi_I-\frac{2}{r^2}\phi_I\phi_I$$

- gamma matrices Γ^M, ρ^a
- Symplectic Majorana $\lambda = -BC\,\lambda^*$, $\varepsilon = BC\,\varepsilon^*$
- Weyl: $\Gamma^7\,\lambda=\lambda$, $\Gamma^7\,\varepsilon = -\varepsilon$
- 32 supersymmetry

$$\begin{aligned}\delta\phi_I &= -\bar{\lambda}\rho_I\epsilon=+\bar{\epsilon}\rho_I\lambda,\\ \delta\lambda &= +\frac{i}{6}H_{MNP}\Gamma^{MNP}\epsilon+i\partial_M\phi_I\Gamma^M\rho_I\epsilon-2\phi_I\rho_I\tilde{\epsilon},\\ \delta\bar{\lambda} &= -\frac{i}{6}H_{MNP}\bar{\epsilon}\Gamma^{MNP}+i\partial_M\phi_I\bar{\epsilon}\Gamma^M\rho_I-2\bar{\tilde{\epsilon}}\rho_I\phi_I.\end{aligned}$$

- additional condition on Killing spinor:

$$\hat{\nabla}_M\epsilon=\frac{i}{2r}\Gamma_M\tilde{\epsilon},\quad \Gamma^M\hat{\nabla}_M\tilde{\epsilon}=2i\epsilon,\qquad \tilde{\epsilon}=\pm\Gamma_0\epsilon.$$

Killing spinors of S^5

- The first 16 Killing spinors case: $\tilde{\epsilon} = +\Gamma_0 \epsilon.$
- two classes depending on the isometry SU(3) of CP^2
- $J = e^1 \wedge e^2 + e^3 \wedge e^4$
- SU(3) singlet: 4+4 $\gamma_{12}\varepsilon = \gamma_{34}\varepsilon = i\,\varepsilon$: $j_1+j_2+j_3=3/2$:

$$\epsilon_+ \sim e^{-\frac{i}{2}t+\frac{3i}{2}y}\epsilon_0^{++},$$

- SU(3) triplet: 12+12: $j_1+j_2+j_3=-1/2$

$$\epsilon_+ \sim e^{-\frac{i}{2}t-\frac{i}{2}y}(\epsilon_1^{+-}, \epsilon_2^{-+}, \epsilon_3^{--}),$$

Dimensional Reduction to CP^2

- Define new variables so some of Killing spinors are y-independent
- New variables with twisting

$$
\begin{aligned}
\epsilon_{old} &= e^{-\frac{y}{4}M_{IJ}\rho_{IJ}}\epsilon_{new},\\
\lambda_{old} &= e^{-\frac{y}{4}M_{IJ}\rho_{IJ}}\lambda_{new},\\
(\phi_1+i\phi_2)_{old} &= e^{+(3+p)iy/2}(\phi_1+\phi_2)_{new}\\
(\phi_4+i\phi_5)_{old} &= e^{+(3-p)iy/2}(\phi_4+i\phi_5)_{new}.
\end{aligned}
$$

$$
M_{12}=-M_{21}=\frac{3+p}{2},\ M_{45}=-M_{54}=\frac{3-p}{2}
$$

$$
p=\cdots,-5,-3,-1,1,3,5,\cdots.
$$

$$
\partial_y\to\partial_y+\frac{3i}{2}(R_1+R_2)+\frac{ip}{2}(R_1-R_2)
$$

- y-independent supersymmetry: $Q=Q^{++}_{---}, S=Q^{--}_{+++}$
- Z_k-modding: k/K= integers

$$
k\equiv j_1+j_2+j_3+\frac{3}{2}(R_1+R_2)+\frac{p}{2}(R_1-R_2),\ p=\text{odd integer}
$$

  - singlet ε_+, ε_- with $j_1+j_2+j_3=\pm3/2$, $R_1+R_2=\pm1$: 2 supersymmetries

$$
\begin{aligned}
\lambda(y)_{old} &\sim e^{-\frac{3(\rho_{12}+\rho_{45})+p(\rho_{12}-\rho_{45})}{2K}\pi}\lambda(y+\frac{2\pi}{K})_{old},\\
(\phi_1+i\phi_2)(y)_{old} &\sim e^{+\frac{(3+p)\pi i}{K}}(\phi_1+i\phi_2)(y+\frac{2\pi}{K})_{old}\\
(\phi_4+i\phi_5)(y)_{old} &\sim e^{+\frac{(3-p)\pi i}{K}}(\phi_4+i\phi_5)(y+\frac{2\pi}{K})_{old}.
\end{aligned}
$$

5d Lagrangian

$$Q = Q^{++}_{---}, S = Q^{--}_{+++}$$

* Lagrangian on R x CP^2 with 2 supersymmetries for any p:

$$\begin{aligned} S \;=\; & \frac{K}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\sqrt{|g|}\;\mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2\sqrt{|g|}}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\Big(A_\rho\partial_\sigma A_\eta-\frac{2i}{3}A_\rho A_\sigma A_\eta\Big) \\ & -\frac{1}{2}D_\mu\phi_I D^\mu\phi_I+\frac{1}{4}[\phi_I,\phi_J]^2-2\phi_I^2-\frac{1}{2}(M_{IJ}\phi_J)^2-i(3-p)[\phi_1,\phi_2]\phi_3-i(3+p)[\phi_4,\phi_5]\phi_3 \\ & -\frac{i}{2}\bar\lambda\gamma^\mu D_\mu\lambda-\frac{i}{2}\bar\lambda\rho_I[\phi_I,\lambda]-\frac{1}{8}\bar\lambda\gamma^{mn}\lambda J_{mn}+\frac{1}{8}\bar\lambda M_{IJ}\rho_{IJ}\lambda\Big], \end{aligned} \tag{2.27}$$

* Supersymmetry Transformation

$$\begin{aligned} &\delta A_\mu = +i\bar\lambda\gamma_\mu\epsilon = -i\bar\epsilon\gamma_\mu\lambda, \quad \delta\phi_I = -\bar\lambda\rho_I\epsilon = \bar\epsilon\rho_I\lambda, \\ &\delta\lambda = +\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon + iD_\mu\phi_I\rho_I\gamma^\mu\epsilon - \frac{i}{2}[\phi_I,\phi_J]\rho_{IJ}\epsilon - 2\phi_I\rho_I\tilde\epsilon - M_{IJ}\phi_I\rho_J\epsilon. \end{aligned}$$

* p=-1/2 : $k = j_1+j_2+j_3+R_1+2R_2$
  * additional supersymmetries: Total 8 supersymmetries

$$Q^{+-}_{-++},\; Q^{+-}_{+-+},\; Q^{+-}_{++-} \quad \text{conjugates}$$

Coupling Constant Quantization

- Instanton number on CP2

$$\nu = \frac{1}{8\pi^2}\int_{\mathrm{CP}^2} \mathrm{Tr}(F\wedge F) = \frac{1}{16\pi^2}\int_{\mathrm{CP}^2} d^4x\sqrt{|g|}\ \mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

- Instantons represents the momentum K or energy K:

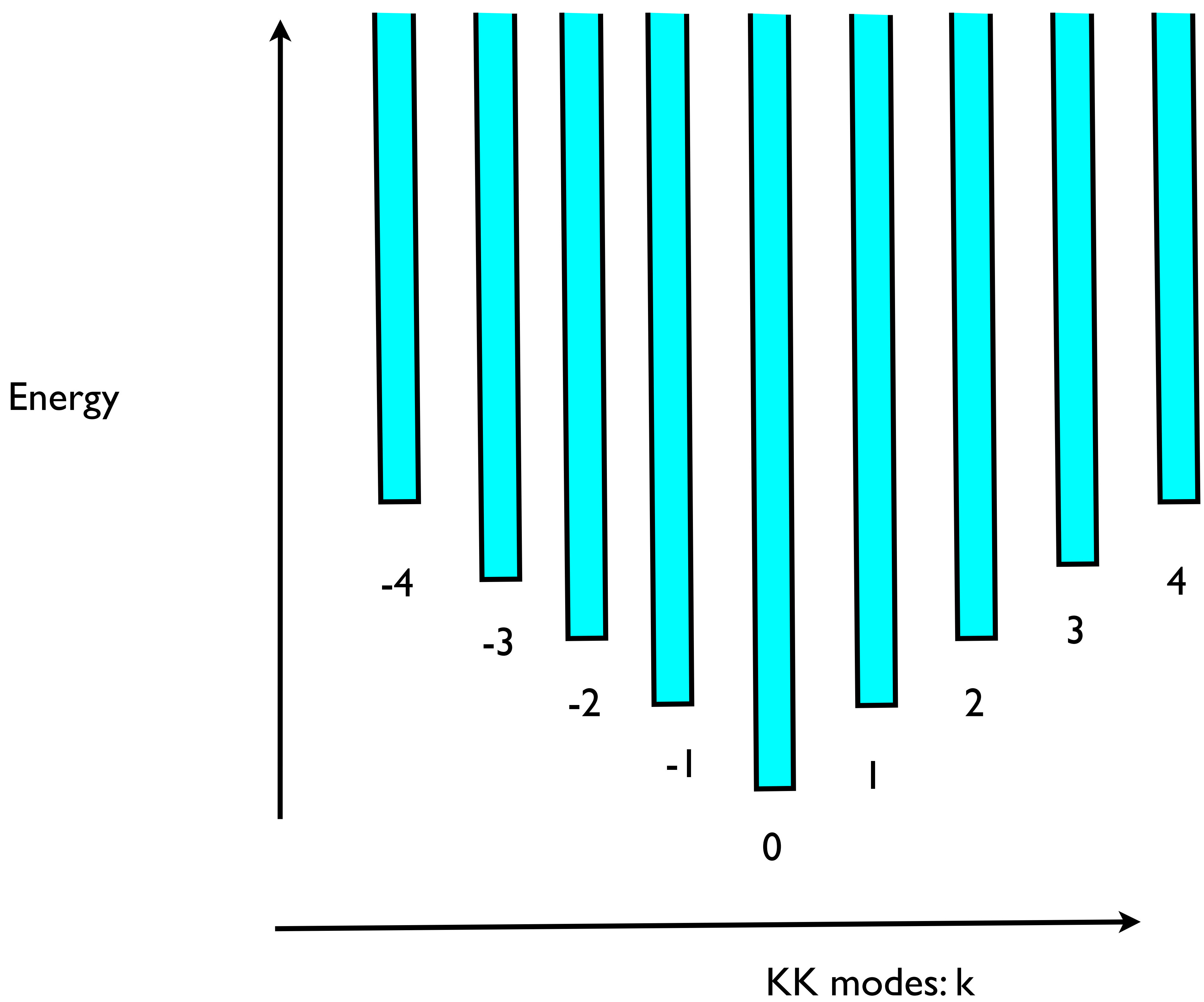
$$\frac{1}{g^2_{YM}} = \frac{K}{4\pi^2 r}$$

- Another approach to quantization: F=2J: 2π flux on a cycle, 1/2 instanton

$$\frac{K}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\ \frac{1}{2}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\partial_\rho A_\sigma A_\eta \Rightarrow K\int dt\ A_0$$

- ‘t Hooft coupling constant: λ= N/K
- Large K => Free Theory

Diluting degrees of freedom with Z_K modding+ Twisting



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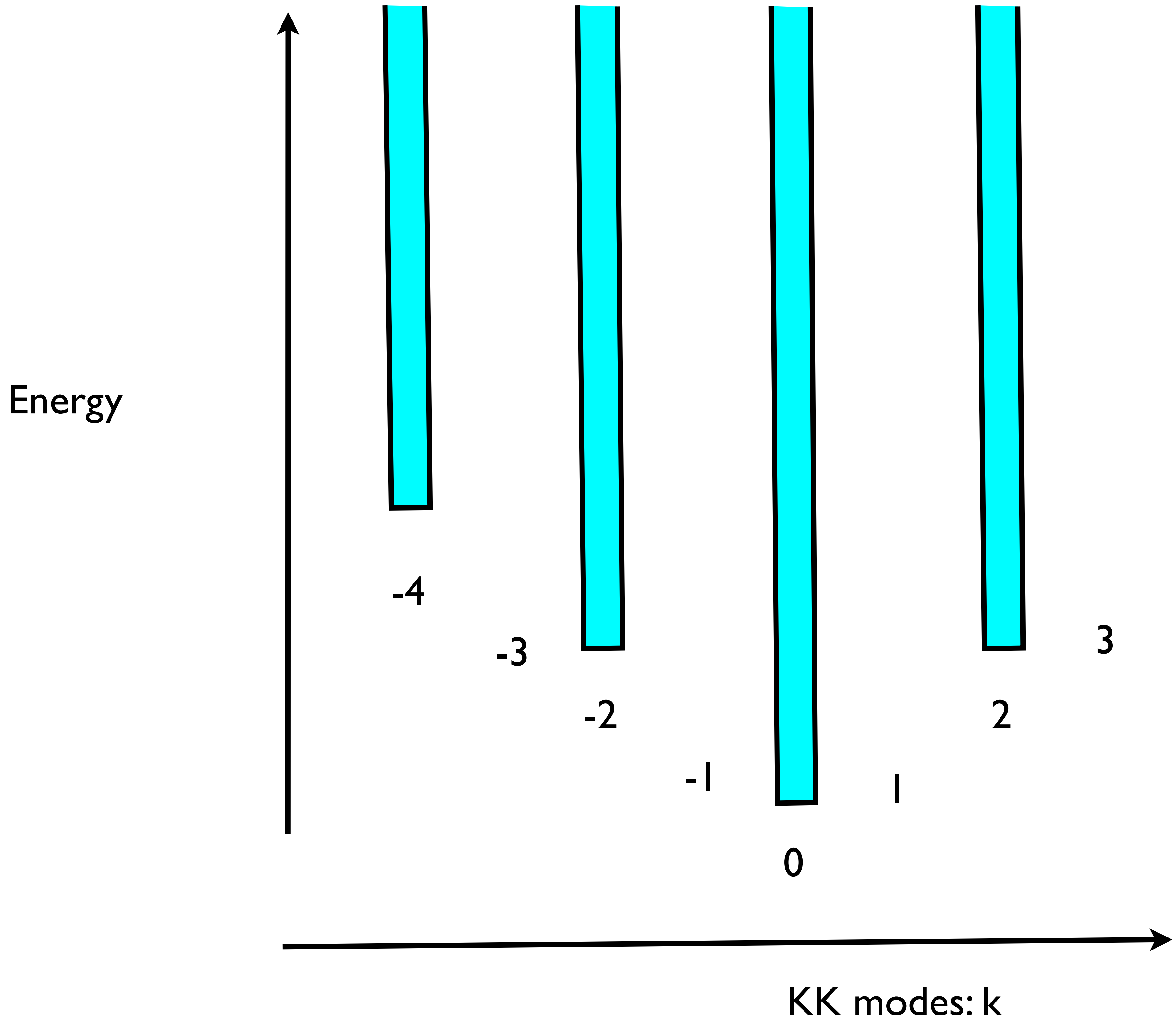


Expected Enhanced Supersymmetries

- Killing spinors with $p=-1/2$, $k = j_1+j_2+j_3+ R_1+ 2R_2$
  - $k=0$: 8 kinds
  - $k= \pm 1$: 14 kinds
  - $k= \pm 2$: 8 kinds
  - $k= \pm 3$: 2 kinds
- # of supersymmetries
  - $K \geqq 4$: 8 supersymmetries
  - $K=3$: 10 supersymmetries
  - $K=2$: 16 supersymmetries
  - $K=1$: 32 supersymmetries

the index function on S^1 x S^5

* 5d SYM on S^5 Hee-Cheol Kim, Seok Kim:1206.6339; Hee-Cheol Kim, Joonho Kim, S.K. 1211.0144
  * S-dual version of the index
  * Vacuum energy:

$$(\epsilon_0)_{index} = \lim_{\beta'\to 0} \mathrm{Tr}\left[(-1)^F \frac{E-R}{2} e^{-\beta'(E-R_1)}\right]$$
$$= \frac{N(N^2-1)}{6} + \frac{N}{24}$$

* See Seok Kim's Talk
* Stationary phase: $D^1=D^2=0$, $F= 2s\, J$, φ + $D=4s$, $s=\mathrm{diag}(s_1,s_2,\ldots,s_N)$
* Path Integral: Off-shell, localization

$$\sum_{s_1,s_2,\cdots s_N=-\infty}^{\infty} \frac{1}{|W_s|} \oint \left[\frac{d\lambda_i}{2\pi}\right] e^{\frac{\beta}{2}\sum_{i=1}^N s_i^2 - i\sum_i s_i\lambda_i} Z^{(1)}_{\text{pert}} Z^{(1)}_{\text{inst}} \cdot Z^{(2)}_{\text{pert}} Z^{(2)}_{\text{inst}} \cdot Z^{(3)}_{\text{pert}} Z^{(3)}_{\text{inst}} \cdot$$

* For K=1, well-confirmed for k ≤ N with N=1,2,3 with the AdS/CFT calculation

Check (Seok Kim's talk)

- E.g. k = N = 3: (all results multiplied by vacuum energy factor & $e^{-3\beta}$) $y_i = e^{-\beta a_i}\ ,\quad y = e^{\beta(m-\frac{1}{2})}$

$$\begin{aligned}
Z_{(2,0,-2)} &= 3\left[y^2(y_1+y_2+y_3)+y(y_1^2+y_2^2+y_3^2)+y^{-1}(y_1+y_2+y_3)-(1+\frac{y_1}{y_2}+\frac{y_2}{y_1}+\cdots)+y^3\right] \\
&\quad +6y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right]+y^3 \\
Z_{(2,-1,-1)}+Z_{(1,1,-2)} &= -2y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right] \\
&\quad -2y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right] \\
&\quad -4y^3-4y^2(y_1+y_2+y_3)-2y\left(y_1^2+y_2^2+y_3^2-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)+2\left(\frac{y_1}{y_2}+\frac{y_2}{y_3}+\cdots\right)-2y^{-1}(y_1+y_2+y_3) \\
Z_{(1,0,-1)} &= y^3+y^2(y_1+y_2+y_3)-y(y_1^{-1}+y_2^{-1}+y_3^{-1})+1
\end{aligned}$$

add all

$$Z_{SUGRA} = 3y^3+2y^2(y_1+y_2+y_3)+y\left(y_1^2+y_2^2+y_3^3-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)-\left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\cdots\right)+y^{-1}(y_1+y_2+y_3)$$

- Non-zero flux states contributing to the index
  - s=(N-1,N-3,…,-(N-1)) = s_0 : SU(N) Weyl vector
  - index vacuum energy: $E_0 = -\frac{N(N^2-1)}{6}$

Strange Vacua

- K=1, F=2sJ background

$$U(2)\ (1,-1)$$
$$U(3)\ (2,0,-2),(2,-1,-1),(1,1,-2),(1,0,-1)$$
$$U(4)\ (3,1,-1,-3),(3,1,-2,-2),(2,2,-1,-3),(3,0,-1,-2),$$
$$(2,1,0,-3),(2,0,0,-2),(2,0,-1,-1),(1,1,0,-2),(1,0,0,-1)$$

- Ground State for Index: K ≤ N (Strong ‘t Hooft coupling λ=N/K)

K	$U(2)$	$U(3)$	$U(4)$	$U(5)$	$U(6)$	$U(7)$	$U(8)$	$U(9)$	$U(N)$
1	-1	-4	-10	-20	-35	-56	-84	-120	$-\frac{N(N^2-1)}{6}$
2	0	-1	-2	-5	-8	-14	-20	-30	
3		0	-1	-2	-3	-6	-9	-12	
4			0	-1	-2	-3	-4	-7	
5				0	-1	-2	-3	-4	
6					0	-1	-2	-3	
7						0	-1	-2	
8							0	-1	
9								0	

Table 1: Vacuum energies divided by K, at general $\mathbb{Z}_K$ (and fluxes)

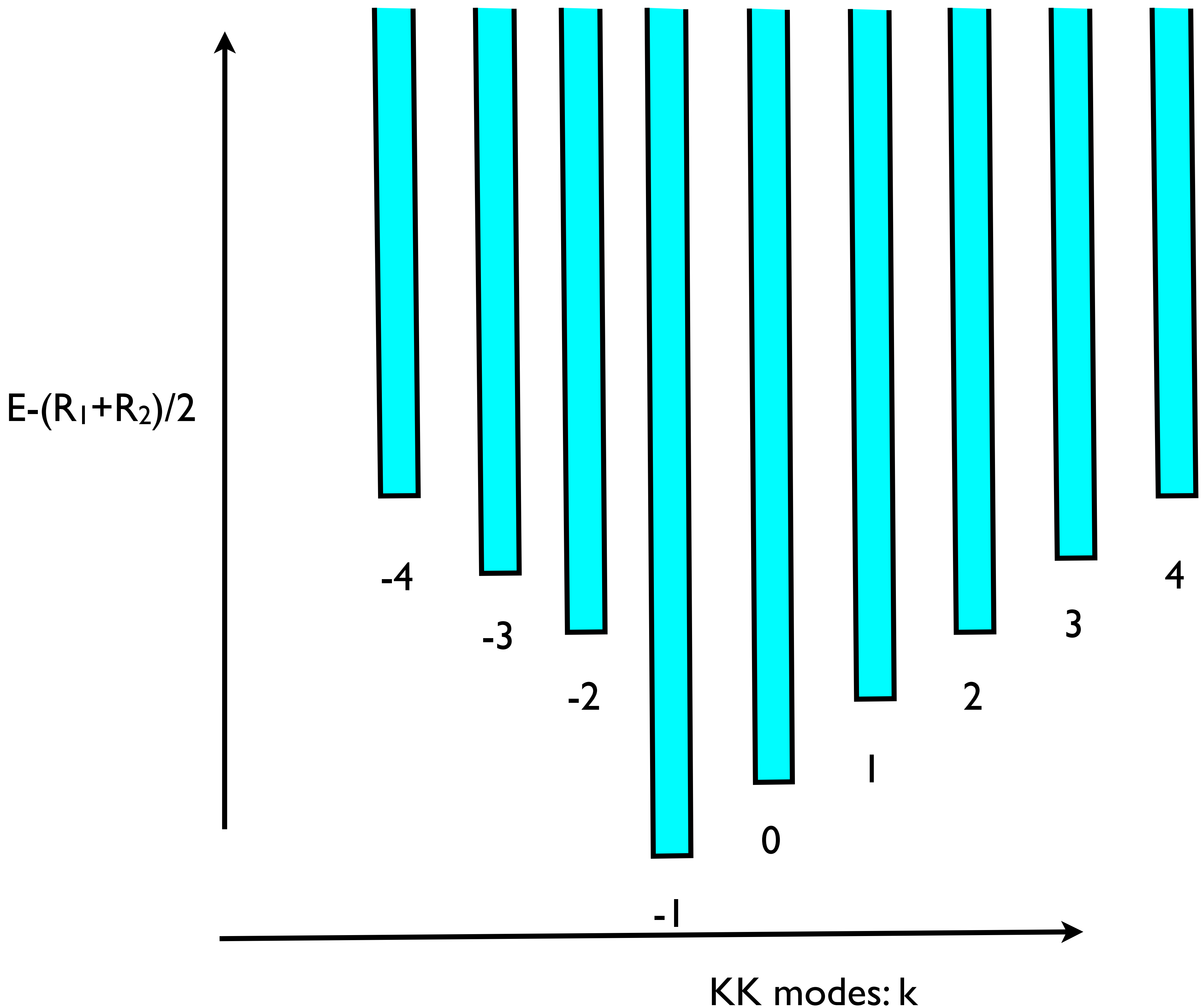
SU(2) Case

* BPS Eq. for Homogeneous Configuration

$$A = V\mathrm{diag}(n, -n), \;\; F = 2J\mathrm{diag}(n, -n),$$

* homogeneous solutions possible only with n=+1,-1
* but gauss law is violated
* for one of the constant bps solutions, the homogeneous fermionic zeromo is possible.
* gauss law can be satisfied with fermionic contribution for K=1
* energetic is more complicated to due to zero-point contribution to the classical one,

Diluting degrees of freedom with Z_K modding



Conclusion

- New 5d supersymmetric theories for M5 are found with discrete coupling constant
- Index Function of 6d A_N (2,0) is obtained.
- Vacuum Structure, …….
- UV finite? How rigid is the theory with eight supersymmetries.
- Enhanced supersymmetry to K=1,2,3?
- Near BPS objects?
- Formulation and Calculation of 6d N=1 SCFT Theories