Diffeomorphism Invariance and Non-relativistic Holography

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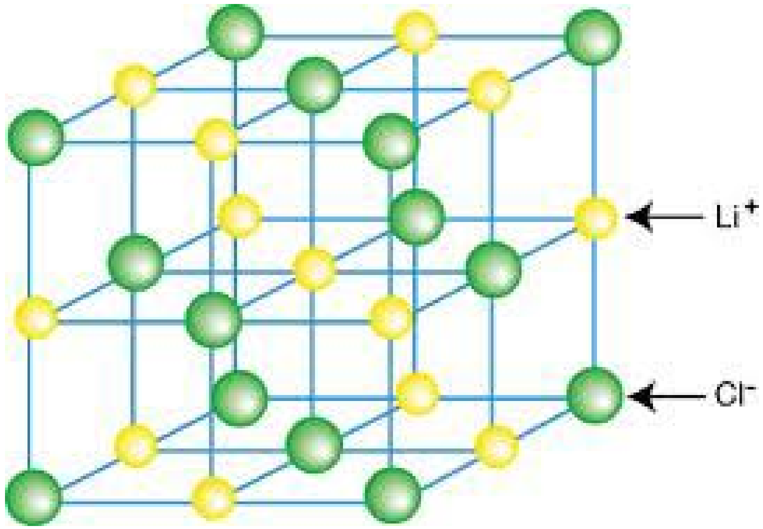
Why NR Holography?



Not useful!!

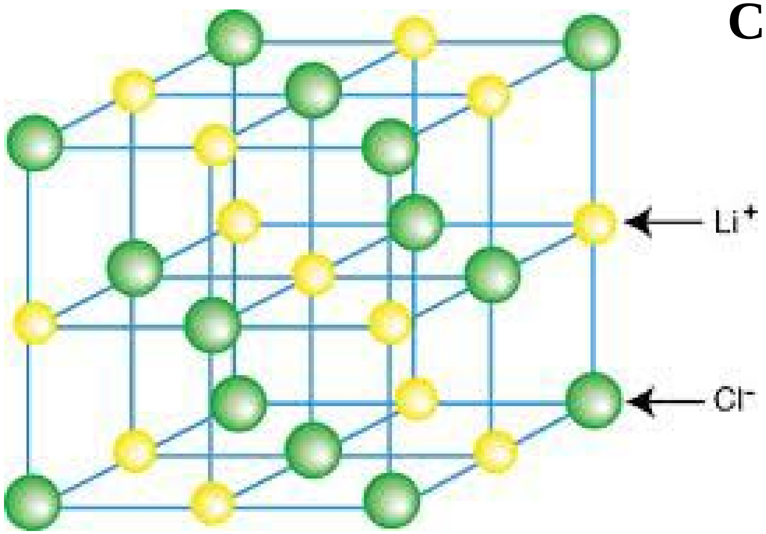
In nature we know the right description for solids is a relativistic QFT!

$$L=L_{QED}+L_{QCD}$$

Condensed Matter Physics=

- study state with finite baryon and lepton number
- analyze low energy fluctuations

Why NR Holography?



Condensed Matter Physics=

$$H = \sum_{Nuclei,A} \frac{P_A{}^2}{m_A} + \sum_{electron,i} \frac{p_i{}^2}{m_e} - \sum\nolimits_{A,i} \frac{e^2}{|x_i - x_A|} + \sum\nolimits_{i \neq j} \frac{e^2}{|x_i - x_j|}$$

Much better.

Can we find holographic duals that directly describe the non-relativistic low energy theory?

Key difference: Lorentz $\rightarrow$ Gallilei

Prelude: Symmetries in QFT

Gauge versus Global

Prelude: Symmetries in QFT

Gauge versus Global

Gauge symmetry:

- not really a symmetry
- redundancy of description
- all physical observables gauge invariant
- Example: Seiberg Duality.

Prelude: Symmetries in QFT

Gauge versus Global

Gauge symmetry:

not really symmetry
redundancy of description

Global symmetry:

- true symmetry of observables
- physical quantities furnish representation
- implies conservation laws
- Example: translations $\longrightarrow$ momentum

Prelude: Symmetries in QFT

Gauge versus Global

Gauge symmetry:

not really symmetry
redundancy of description

Global symmetry:

Conservation laws
constrains observables

Spurionic global symmetry:

- Lagrangian only invariant if couplings transform
- Contains "true" global symmetries as subgroup
- **Constrains low energy effective action**
- No new conservation laws

Prelude: Symmetries in QFT

Spurionic global symmetry:

Example: Massive Dirac Fermion.

$$\mathcal{L} = \bar{\psi}(i\partial_\mu \gamma^\mu - M)\psi$$

Massless theory invariant under chiral rotations:

$$\psi \to e^{-i\phi\gamma_5/2}\psi$$

Symmetry of massive theory if mass transforms:

$$M \to e^{i\phi} M$$

Fixes quark mass dependences of chiral Lagrangian!

Diffeomorphism in GR

GR is built around diffeomorphism invariance

$$x^{\mu} \to \tilde{x}^{\mu} = x^{\mu} + \xi^{\mu}$$

$$\delta g_{\mu\nu} = \xi^{\lambda}\partial_{\lambda}g_{\mu\nu} + g_{\lambda\nu}\partial_{\mu}\xi^{\lambda} + g_{\mu\lambda}\partial_{\nu}\xi^{\lambda}$$

This is a gauge symmetry.

"Quantum Gravity has no local observables."

Diffeomorphism in GR

In GR diffeomorphisms are gauge invariance

Exception: Diffeomorphisms that do not vanish at infinity = global symmetry.

Observables of quantum gravity in:

asymptotically flat space $\leftrightarrow$ **S-matrix**
asymptotically hyperbolic space $\leftrightarrow$ **boundary correlation functions**

Diffeomorphism in QFT

For relativistic QFTs on curved backgrounds

$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu$$

$$\delta g_{\mu\nu} = \xi^\lambda \partial_\lambda g_{\mu\nu} + g_{\lambda\nu}\partial_\mu \xi^\lambda + g_{\mu\lambda}\partial_\nu \xi^\lambda$$

Is a spurionic global symmetry!

not gauged!

Local observables do exist!

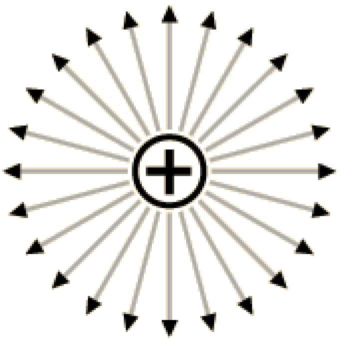
Diffeomorphism in QFT $\leftarrow$ **spurionic global symmetry**

metric $g_{\mu\nu}$: set of coupling constants
(5 for each spacetime point)

"coupling constants" transform non-trivially under our global symmetry (spurions)

Diffeomorphism in QFT

You can change coordinates to analyze questions in a field theory!



electric field
of a point charge

Cartesian:

$$ds^2 = dx^2 + dy^2 + dz^2 \qquad \Phi = ?$$

Spherical:

$$ds^2 = dr^2 + r^2\,(d\theta^2 + sin^2\theta\; d\varphi^2)$$

$$\Phi = 1/(4\,\pi\,r)$$

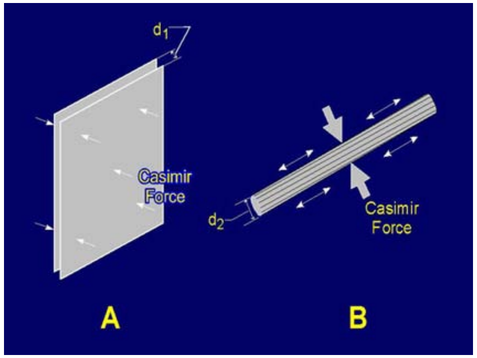
Diffeomorphisms as Spurions:

Two important applications:

1) Low energy effective action constrained by spurionic symmetry!

Example:

$$ds^2 = dx^2 + dy^2 + dz^2$$



Diffeomorphisms as Spurions:

Two important applications:

2) For a given set of couplings (e.g for
a given background metric) the subset of
the diffeomorphisms that leaves these particular
couplings invariant corresponds to the
true global symmetries (conserved charges)

Example:

- Flat space: $g_{\mu\nu} = \eta_{\mu\nu}$
- Subset of diffs leaving this invariant:

Translations
Boosts
Rotations

Implies conservation of energy, momentum, …

Recap:

In a relativistic QFT diffeomorphisms acting on the background metric are a **global** symmetry.

Contains "standard" symmetries as special cases (leaving a given metric invariant).

But this is a genuinely more powerful symmetry (constrains L_{eff})

Diffeomorphisms in NR QFT

(Son & Wingate, Hoyos & Son)

$$S = \int dt\, d^d x\, \sqrt{g}\, \mathcal{L} = \int dt\, d^d x\, \sqrt{g} \left[\frac{i}{2} \psi^\dagger \overleftrightarrow{\partial}_t \psi - A_0 \psi^\dagger \psi - \frac{g^{ij}}{2m} (\partial_i \psi^\dagger - i A_i \psi^\dagger)(\partial_j \psi + i A_j \psi) \right]$$

Free non-relativistic field theory
(many-particle Schrödinger equation)

Boson or Fermion

Background spatial metric, E&B fields

Expect: Spatial Diffeomorphism invariance!

Symmetries of free NR fields:

Actually, this system is invariant even under time dependent spatial diffeomorphisms.

$$\delta A_0 = -\dot{\alpha} + \xi^k \partial_k A_0 + A_k \dot{\xi}^k,$$
$$\delta A_i = -\partial_i \alpha + \xi^k \partial_k A_i + A_k \partial_i \xi^k - m g_{ik} \dot{\xi}^k,$$
$$\delta g_{ij} = \xi^k \partial_k g_{ij} + g_{ik} \partial_j \xi^k + g_{kj} \partial_i \xi^k.$$

$\vec{\xi}(\vec{x}, t) \quad \alpha(\vec{x}, t)$

parameterize global spurionic symmetries

The trivial background

What subgroup leaves "trivial" background invariant?

$$g_{ij} = \delta_{ij}, \vec{A} = A_0 = 0$$

$$\xi^i = c^i$$ **(Translations)**

$$\xi^i = \omega_{ij}\, x^j$$ **(Rotations)**

$$\vec{\xi}(\vec{x}, t) = \vec{v}t, \qquad \alpha(\vec{x}, t) = -m\,\vec{v} \cdot \vec{x}.$$

(Galilean Boosts)

needs time dependent diffeomorphism!

Interactions.

Many interaction terms compatible with these symmetries can be added. This includes:

- Coulomb interactions

  (e.g. Quantum Hall Systems or other strongly correlated electrons)

- Short Range 2-particle interactions

  (e.g. "Unitary Fermi Gas" = Fermions with infinite scaterring length)

Relativistic Origin:

For free boson we can get symmetries via scaling limit from free relativistic field:

$$S = -\int d^d x dt \sqrt{-g}\frac{1}{2}\left(g^{\mu\nu}\mathcal{D}_\mu\phi^\dagger\mathcal{D}_\nu\phi + c^2m^2e^{2\sigma}\phi^\dagger\phi\right)$$

$$\mathcal{D}_\mu\phi \equiv \partial_\mu\phi - \imath C_\mu\phi$$

Set chemical potential equal to rest mass:

$$C_\mu = -\partial_\mu\Lambda = \delta_{\mu t}mc^2$$

Particles have zero free energy.
Anti-particles have free energy 2 mc^2 and decouple.

and take the c to infinity limit!

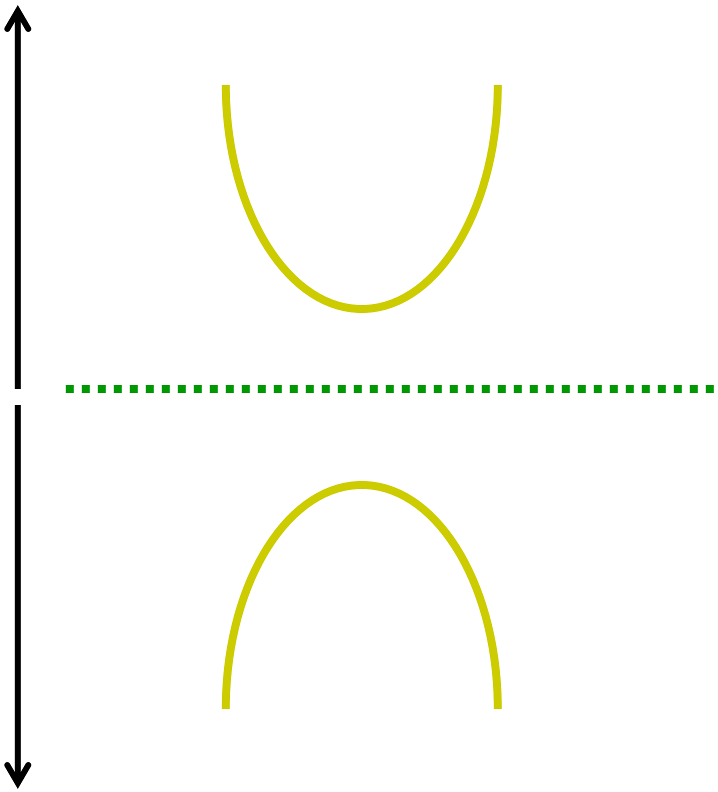
Diffs and Gauge Symmetry descend.

free NR field theory

Relativistic Origin - Illustration

Energy for particles (charge = +1)

$$E = \sqrt{(cp)^2 + (mc^2)^2}$$



chemical potential=0

Energy for antiparticles (charge = -1)

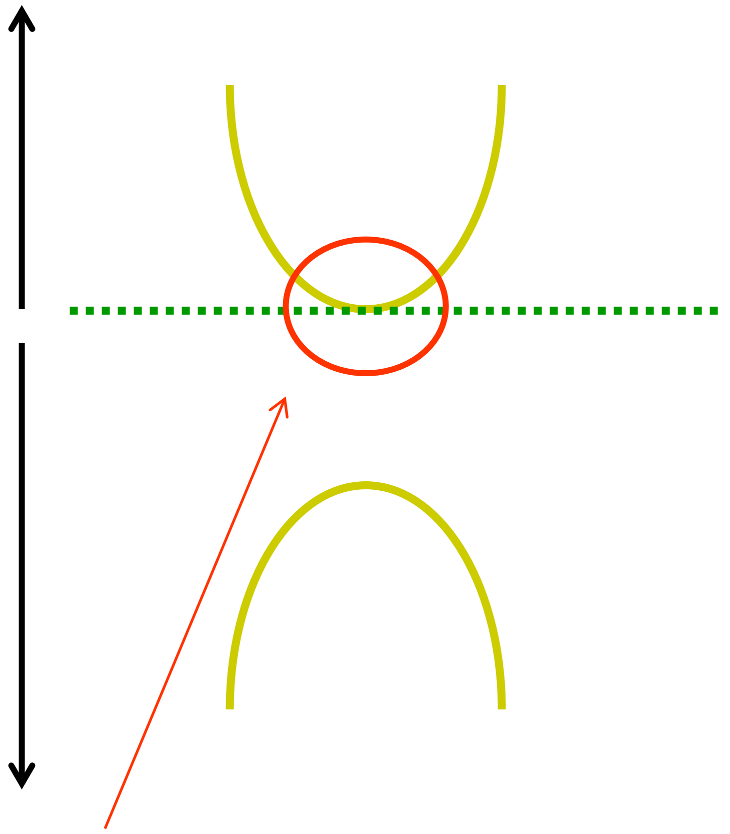
$$E = \sqrt{(cp)^2 + (mc^2)^2}$$

Relativistic spectrum

Relativistic Origin - Illustration

Energy for particles (charge = +1)

$$E \approx \frac{p^2}{2m} + \cdots$$



chemical potential= rest energy

Energy for antiparticles (charge = -1)

$$E \approx 2\,mc^2 + \cdots$$

large c limit

Non-relativistic spectrum

Applications:

Hoyos, Son: In any quantum Hall system (gapped!). Low energy effective action only depends on metric (take flat) and E&B

$$j^i = \frac{\nu}{2\pi}\epsilon^{ij}E_j - \frac{1}{B}\left[\frac{\kappa}{4\pi} - m\epsilon''(B)\right]\epsilon^{ij}\partial_j(\nabla\cdot\mathbf{E}) + \cdots$$

(Hall current)

$$\Delta\tilde{T}_{ij} = -\eta_{\rm H}(\epsilon_{ik}\delta_{jl} + \epsilon_{jk}\delta_{il})V_{kl}\ , \qquad V_{kl} = \tfrac{1}{2}(\partial_k v_l + \partial_l v_k)$$

(Hall viscosity)

Applications: (Hoyos, Son)

(Hall current)

$$j^i = \frac{\nu}{2\pi}\epsilon^{ij}E_j - \frac{1}{B}\left[\frac{\kappa}{4\pi} - m\epsilon''(B)\right]\epsilon^{ij}\partial_j(\boldsymbol{\nabla}\cdot\mathbf{E}) + \cdots$$

Filling fraction. Characteristic Property of given Quantum Hall State. Input in low energy theory.

Applications: (Hoyos, Son)

(Hall current)

$$j^i = \frac{\nu}{2\pi}\epsilon^{ij}E_j - \frac{1}{B}\left[\frac{\kappa}{4\pi} - m\epsilon''(B)\right]\epsilon^{ij}\partial_j(\boldsymbol{\nabla}\cdot\mathbf{E}) + \cdots$$

Wen-Zee shift. Gives change in filling fraction when given QH state is put on the sphere. Known quantity for all the Laughlin states. Input into low energy theory.

Input into low energy theory:

ν

Applications: (Hoyos, Son)

(Hall current)

$$j^i = \frac{\nu}{2\pi}\epsilon^{ij}E_j - \frac{1}{B}\left[\frac{\kappa}{4\pi} - m\epsilon''(B)\right]\epsilon^{ij}\partial_j(\boldsymbol{\nabla}\cdot\mathbf{E}) + \cdots$$

Energy density as function of external magnetic field. Thermodynamic Property. Can be measured/caculated independently. Input into low energy theory.

Input into low energy theory:

ν, κ

Applications: (Hoyos, Son)

(Hall current)

$$j^i = \frac{\nu}{2\pi}\epsilon^{ij}E_j - \frac{1}{B}\left[\frac{\kappa}{4\pi} - m\epsilon''(B)\right]\epsilon^{ij}\partial_j(\boldsymbol{\nabla}\cdot\mathbf{E}) + \cdots$$

$$\eta^a = \kappa B/4\pi$$

Input into low energy theory:
ν, κ, ε(B)

(Hall viscosity= prediction!)

(agrees with earlier result by Read and Rezayi.).

Applications: (Hoyos, Son)

(Hall current)

$$j^i = \frac{\nu}{2\pi}\epsilon^{ij}E_j - \frac{1}{B}\underbrace{\left[\frac{\kappa}{4\pi} - m\epsilon''(B)\right]}\epsilon^{ij}\partial_j(\boldsymbol{\nabla}\cdot\mathbf{E}) + \cdots$$

PREDICTION! Leading correction to Hall conductivity in response to a slowly (spatially) varying external magnetic field completely fixed by spurionic global symmetry. Not previously known.

$$\eta^a = \kappa B/4\pi$$

Input into low energy theory:
ν, κ, ε(B)

(Hall viscosity)

Recap:

- Time dependent spatial diffeomorphisms together with background gauge trafos are global spurionic symmetry for a large class of NR QFTs.

- Put strong constraints on low energy effective action.

Additional Symmetries of NR QFT

One additional symmetry these NR QFTs all share is time translations.

$$t \to t + const.$$

Unlike in the relativistic case, this is not automatically included in diffeomorphisms.

Additional Symmetries of NR QFT

Free NR QFTs actually have a larger symmetry: time reparametrizations.

$$t \to t + f(t).$$

(Clearly contains time translations as special case.)

This is also a global, spurionic symmetry:

$$\begin{aligned} \delta A_0 &= f\dot{A}_0 + \dot{f} A_0, \\ \delta A_i &= f\dot{A}_i, \\ \delta g_{ij} &= f\dot{g}_{ij} - \dot{f} g_{ij}. \end{aligned}$$

Why is this called "conformal"?

Ask again: What subgroup leaves "trivial" background invariant?

$$f(t) = \lambda\, t \qquad\qquad \xi^i = -\frac{\lambda}{2} x^i$$

z=2: dynamical critical exponent. **Scale Transformation.**

Why is this called "conformal"?

Ask again: What subgroup leaves "trivial" background invariant?

$$f = t^2, \quad \xi^i = t x^i, \quad \alpha = -\frac{1}{2} m \vec{x}^2.$$

Special Conformal Transformation.

For z=2 algebra closes with scale and conformal.

"Schrödinger Symmetry"

Interacting NR CFTs

Unlike for the case of NR diffs it is much harder to construct interactions that preserve the full NR conformal invariance, but there are known examples:

Unitary Fermi Gas

Applications:

Son, Wingate:

In unitary Fermi gas hydrodynamic transport coefficient appearing at second order in the derivative expansion severely constrained by spurionic global symmetry.

Recap: Defining Symmetries

A large class of generic NR QFTs has the following symmetries:

- U(1) gauge invariance
- time dependent, spatial diffeomorphisms
- time translations or time reparametrizations.

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↑

All together are referred to as "NR Covariance"

Recap: Defining Symmetries

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Foliation preserving diffeomorphisms.
(Fdiffs)

Relativistic Diffs in holography

Holography: Gravity in asymptotically AdS space has dual description in terms of boundary field theory.

Evidence: Symmetries match!
Global Symmetry: e.g. $SO(4,2)$

For all symmetries to match the bulk has to respect the full global (spurionic) diffeomorphism invariance of the QFT.

Bulk diffeomorphisms

Bulk diffs are a gauge symmetry! Redundancy.

$\longrightarrow$ **gauge fix!**

"Normal" (=Fefferman Graham) form:

$$ds^2 = \frac{dr^2 + g_{\mu\nu}(x,r)dx^\mu dx^\nu}{r^2}$$

$$g_{\mu\nu}(x,r) = g^0{}_{\mu\nu}(x) + r^2\, g^2{}_{\mu\nu}(x) + \cdots$$

field theory metric.

Global diffeomorphism

This fixes the diffeomorphisms that vanish at the ($r \to 0$) boundary.

Diffeomorphisms that do not vanish at $r=0$ are not part of the gauge group but a global symmetry

$$\xi^{\mu}(x,r) = \xi^{\mu}(x)$$

These manifestly act on the boundary metric in agreement with the field theory.

NR holography:

Our lesson learned from relativistic holography:

Spurionic global diffeomorphism symmetry of the boundary QFT appears as

radially independent diffeomorphsims

in the bulk theory.

NR holography:

Conjecture:

A generic NR CFT is dual to a bulk gravitational theory built around

Foliation Preserving Diffeomorphisms

(and an additional U(1) gauge symmetry)

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Conjecture:

A generic NR CFT is dual to a bulk gravitational theory built around

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= Horava Gravity coupled to Maxwell field.

Horava Gravity

"One less gauge symmetry = one more D.O.F."

(FDiffs do not include temporal diff.)

One way of writing Horava gravity: **(Blas, Pujolas, Sibiriyakov)**

GR + a scalar field Φ. $\longleftarrow$ **khronon field.**
background for Φ
picks preferred time direction.

unitary gauge: $\langle\Phi\rangle = c^2\, t$

$\uparrow$

fixes temporal diffs.

Khronon action:

$$S = \frac{1}{16\pi G_N}\int dt d^d x dr \sqrt{-\tilde{G}}\left(\tilde{R} - 2\Lambda\right) + S_{khronon}(\lambda, \alpha)$$

$$\Big\downarrow \quad \textbf{unitary gauge}$$

$$\left.\begin{aligned}
S &= \int dt\, d^d x\, dr\, \left(\mathcal{L}_{kin} - \mathcal{L}_V\right). \\
\mathcal{L}_{kin} &= \frac{1}{16\pi G_N}\sqrt{G}N\left[K_{IJ}K^{IJ} - \lambda K^2\right] \\
-\mathcal{L}_V &= \frac{1}{16\pi G_N}\sqrt{G}N\left[R - 2\Lambda + \alpha\frac{(\nabla_I N)(\nabla^I N)}{N^2}\right]
\end{aligned}\right\} \textbf{Horava Gravity}$$

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ADM Form of metric.

$$G_{MN} = \begin{pmatrix} -N^2 + \frac{N_I N^I}{c^2}, & \frac{N_I}{c} \\ \frac{N_I}{c}, & G_{IJ} \end{pmatrix}$$

Shift

Lapse

Spatial Metric

Extrinsic Curvature of constant time slice

$$K_{IJ} = \frac{1}{2N}(\dot{G}_{IJ} - \nabla_I N_J - \nabla_J N_I)$$

Khronon action:

$$S = \frac{1}{16\pi G_N}\int dt d^d x dr \sqrt{-\tilde{G}}\left(\tilde{R} - 2\Lambda\right) + S_{khronon}(\lambda, \alpha)$$

$\downarrow$ unitary gauge

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λ=1, α=0:
Action of Einstein Gravity

But still a different theory!
Different gauge invariance

Can no longer gauge away
g_{rt}
in Fefferman-Graham coords

Khronon action:

$$S = \frac{1}{16\pi G_N}\int dt d^d x dr \sqrt{-\tilde{G}}(\tilde{R} - 2\Lambda) + S_{khronon}(\lambda, \alpha)$$

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Khronon fluctuations:

$$\mathcal{L} \sim \alpha(\partial_i\dot{\chi})^2 - (\lambda - 1)(\Delta\chi)^2.$$

need α non-zero.
no kinetic term otherwise

Healthy "extension"
(or: α→0 unhealthy reduction)

Khronon action:

$$S_{khronon}(\lambda,\alpha) \propto \alpha[\cdots] + (\lambda - 1)[\cdots]$$

unitary gauge

$$S = \int dt\, d^d x\, dr\, (\mathcal{L}_{kin} - \mathcal{L}_V)\,.$$

$$\mathcal{L}_{kin} = \frac{1}{16\pi G_N}\sqrt{G}N\left[K_{IJ}K^{IJ} - \lambda K^2\right]$$

$$-\mathcal{L}_V = \frac{1}{16\pi G_N}\sqrt{G}N\left[R - 2\Lambda + \alpha\frac{(\nabla_I N)(\nabla^I N)}{N^2}\right]$$

Probe khronon imprints notion of time!

Probe limit:

$$\alpha, (\lambda - 1) \ll 1$$

khronon does not backreact on metric.

Any solution to Einstein gravity descends to solution of Horava gravity

Higher derivative terms

The actions displayed so far were
"2 derivative only" actions.
Still has 2 new free parameters.

Appropriate when $(M_{pl} R)^3 \sim N^2 \gg 1$

But, unlike Einstein gravity, Horava gravity seems to allow power counting renormalizable UV fixed points!

plus evidence from lattice!

Higher derivative terms

UV scaling dimensions: $t \to \lambda^{z_*}\, t, \qquad x \to \lambda x$

$$\Delta\left(dx^{d+1}\, dt\right) = -d - 1 - z_*$$

Potential term with $d + 1 + z_*$ spatial derivatives is marginal! e.g. $d + 1 = 3,\ z_* = 3$ with R^3 terms

Conservative approach: stick to large N and 2 –derivative effective action.

The khronon and string theory

In khronon formalism Horava gravity
= Einstein gravity + scalar field.

Can we use this to embed NR CFTs and their Horava duals into known AdS/CFT dual pairs?

Problems with scalar khronon

- No U(1) symmetry
- Time-translation invariance requires shift invariant scalar

  but there no exact global symmetries in quantum gravity!

- Subject to clumping instabilities

  unitary gauge: $\langle\Phi\rangle = c^2\, t$

  uniform energy density most likely wants to collapse

Solution: Vector khronon

$A_t = mc^2, A_i = 0$ ← bulk gauge field

still imprints preferred spatial slicing.

- No U(1) symmetry
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Solution: Vector khronon

$A_t = mc^2, A_i = 0$ ← bulk gauge field

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  pure gauge! no energy density! no clumping!

Solution: Vector khronon

$$A_t = mc^2, A_i = 0$$ ← bulk gauge field

Maybe most importantly:
this is exactly what we did on the field theory side – followed by the c to infinity limit.

Note: constant A_t can not be gauged away

$$\delta S = -\int_M j^\mu \partial_\mu \Lambda = -\int_{\partial M} (\Lambda j_\mu) dS^\mu + \int_M \Lambda \partial_\mu j^\mu$$

$$\delta S = -mc^2 Qt\Big|_{t_i}^{t_f} = mc^2 Q(t_i - t_f)$$

It's the chemical potential!
Clearly it has an effect.

Vector Khronon from IIB strings

Benefit: Vector Khronon easily embedded in String Theory!
However, this only gives the probe limit.

N=4 SYM

↓

Compacitfy on circle of radius R
new U(1): shifts along R
mass ~ 1/R

3d theory;
massless degrees of freedom: neutral
charged degrees of freedom = massive

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Compacitfy on circle of radius R
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Take NR limit in this theory!
Set chemical potential = rest energy
Take c to infinity limit!

3d theory;
massless degrees of freedom: neutral
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N=4 SYM

$$ds^2 = \frac{1}{r^2}\left(g_{ij}dx^i dx^j + R^2(d\,\theta + A_t dt)^2\right)$$

compact direction

geometric realization of KK gauge field

3d theory;
massless degrees of freedom: neutral
charged degrees of freedom = massive

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$$ds^2 = \frac{1}{r^2}\left(g_{ij}dx^i dx^j + R^2(d\,\theta + A_t dt)^2\right)$$

R=1/(m c)
A_t=mc^2

Take NR limit in this theory!
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$$ds^2 = \frac{1}{r^2}\left(g_{\mu\nu}dx^\mu dx^\nu + R^2(d\,\theta + A_t dt)^2\right)$$

Take NR limit in this theory!
Set chemical potential = rest energy
Take c to infinity limit!

R=1/(m c)
$A_t = mc^2$
$c \to \infty$

$$ds^2 = \frac{1}{r^2}\left(g_{ij}dx^i dx^j + 2\,\frac{dt\,d\theta}{m}\right)$$

3d theory;
massless degrees of freedom: neutral
charged degrees of freedom = massive

Vector Khronon from IIB strings

$$ds^2 = \frac{1}{r^2}\left(g_{ij}dx^i dx^j + 2\frac{dt\, d\theta}{m}\right)$$

This is the Son; Balasubramanian & Mc Greevy; Goldberger description of a Schrodinger invariant theory in terms of a d+2 relativistic theory in light front!

Basically we performed Seiberg/Sen limit
Lightlike circle = zero radius limit of spatial circle

Embedding in relativistic theory gives Horava gravity in the probe limit.

Generic NR CFT = Horava gravity away from probe limit.

qualitatively different?

Vector Khronon from IIB strings

String Theory embedding also helps construct explicit mapping between boundary sources and bulk fields.

$$
\begin{aligned}
A_t &\equiv v_t + \frac{b_t N^I N_I}{2N^2}, \\
A_i &\equiv v_i + \frac{b_t N_i}{N^2} - \frac{b_i N^I N_I}{2N^2}, \\
g_{ij} &\equiv r^2 \left(G_{ij} - \frac{b_i N_j}{b_t} - \frac{b_j N_i}{b_t} + \frac{b_i b_j N^I N_I}{b_t^2} \right)
\end{aligned}
$$

Vector Khronon from IIB strings

The extra dimension.

String theory embedding gives extra U(1) bulk gauge symmetry from sub-leading temporal diffeomorphisms.

α invariance

- Redudancy in the bulk, not global symmetry
- Can easily be implemented with just one extra scalar, does not need an extra dimension.
- without α-invariance have scale and Gallilean, but not conformal invariance

Tests of the duality:

Calculate Correlation Functions.

- Add additional scalar. Usual potential term:

$$\mathcal{L}_{pot,X} = -\frac{1}{2}\sqrt{G}\left(|D_I X|^2 + m^2|X|^2\right).$$

- But: symmetries allow one derivative kinetic term. Can be constructed using khronon.

$$\mathcal{L}_{kin,X} = iQ\sqrt{G}\, X^* u^\mu D_\mu X + h.c..$$

Tests of the duality:

Calculate Correlation Functions.

- Correlation function follows from usual recipe:

$$\langle OO \rangle \sim (\vec{k}^2 - 2Q\omega)^{2\nu}$$

$$\nu = \sqrt{m^2 + \frac{(d+2)^2}{4}}.$$

- This agrees with the uniquely fixed form of the field theory correlation function!

(Nishida, Son)

Beyond the probe: Black holes

(Janiszewski, in progress)

What is a black hole if there is no more speed limit?

Can we get novel thermodynamics from Horava gravity away from the probe limit?

Recall: Schrodinger geometry gives non-sensical thermodynamics.

Beyond the probe: Black holes

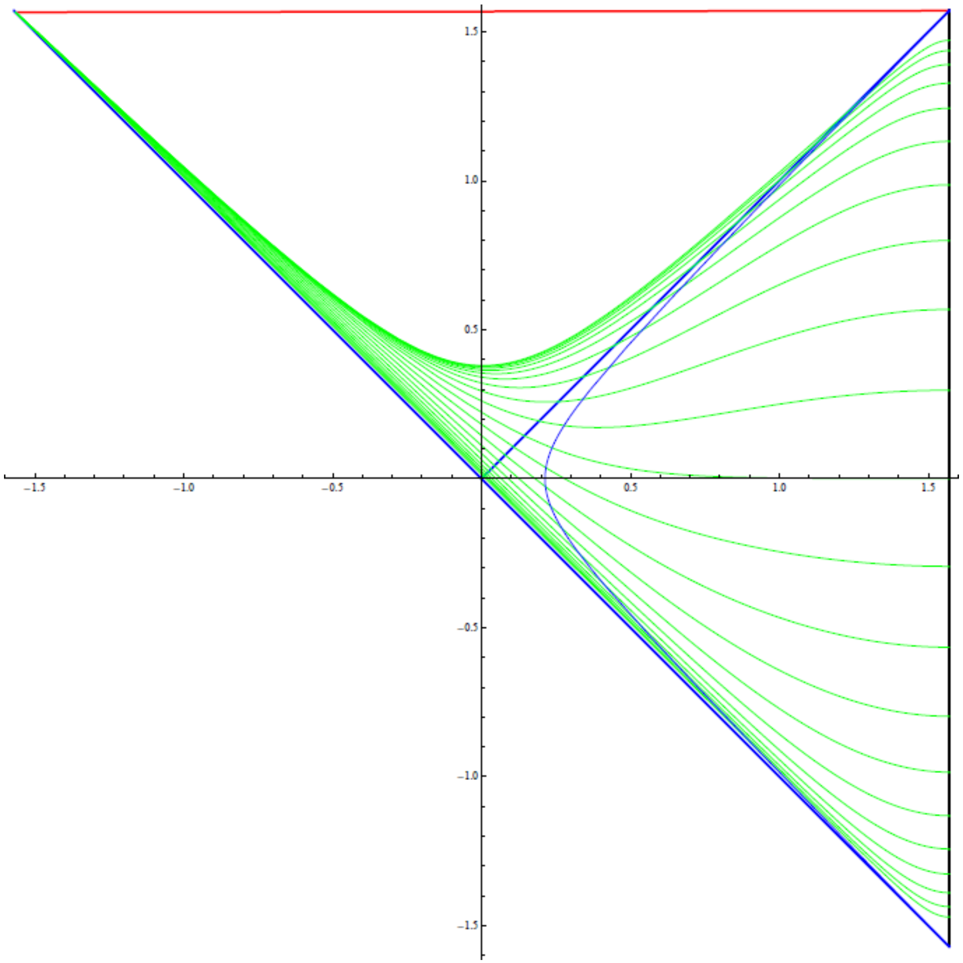
(Janiszewski, in progress)

What is a black hole if there is no more speed limit?

Horava gravity solution =
spacetime + preferred slicing

Universal Horizon: locus beyond which one can not go in finite time; independent of speed.

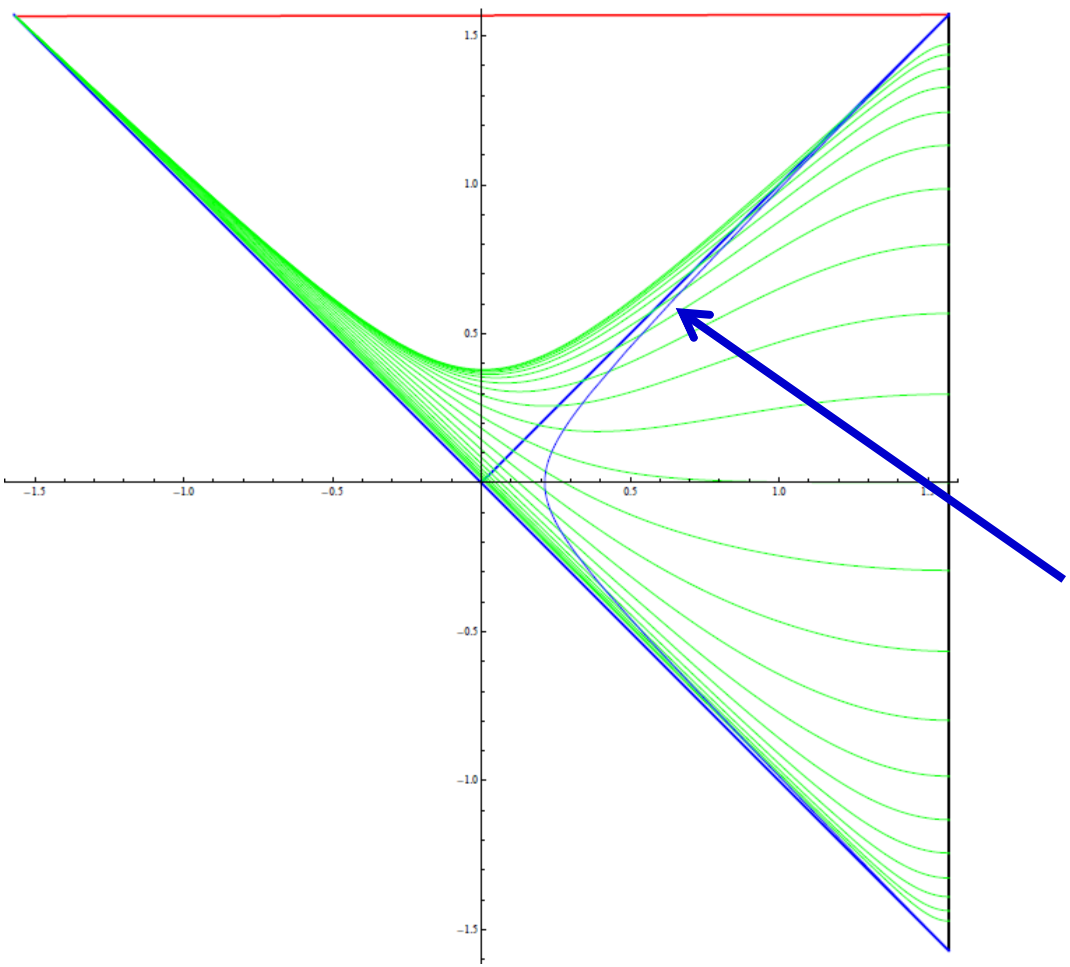
Black holes in Horava gravity



(Janiszewski, in progress)

Horava Gravity Black hole in asymptotic AdS

Black holes in Horava gravity



(Janiszewski, in progress)

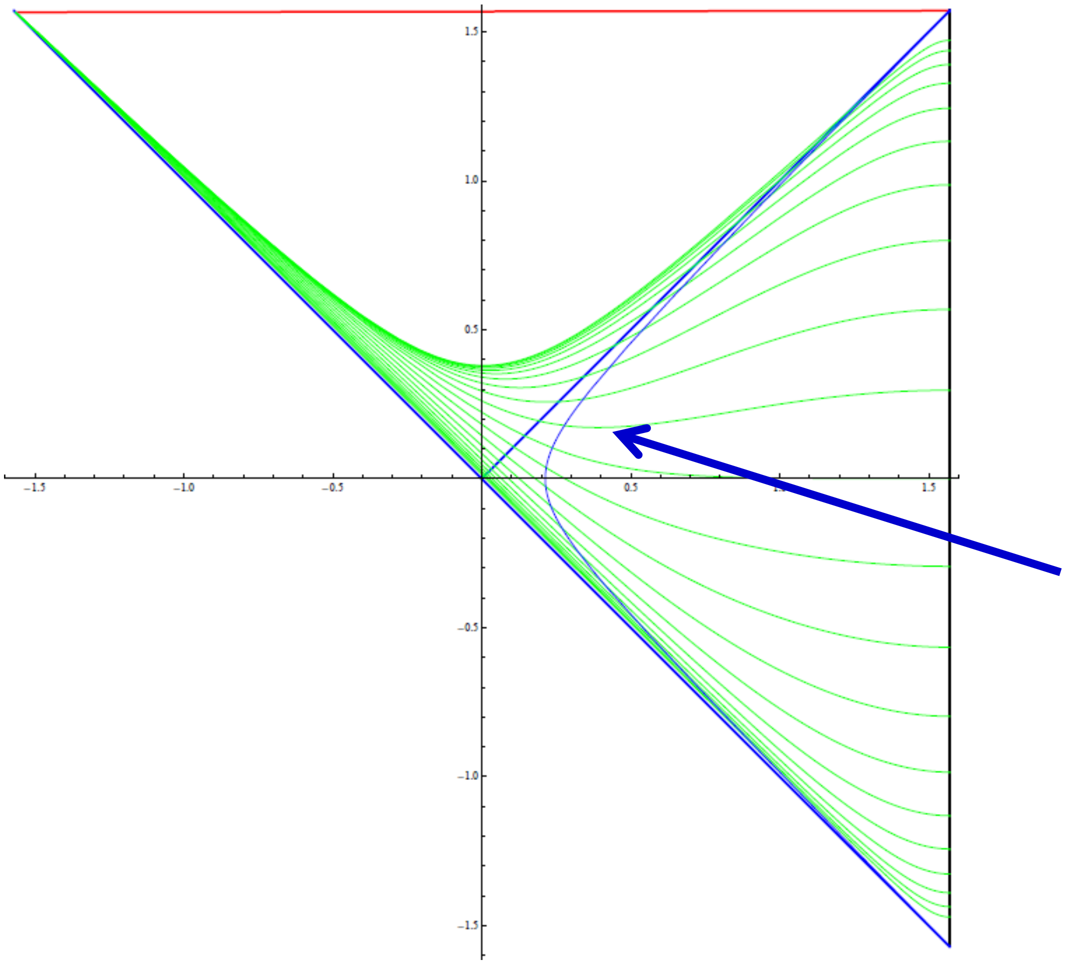
Horava Gravity Black hole in asymptotic AdS

Spacetime geometry itself as in GR black hole.

GR Horizon = place from beyond which the spin-2 graviton moving at the "speed of gravity" can not return.

Black holes in Horava gravity

(Janiszewski, in progress)



Scalar graviton:
long wavelength mode
moves at "speed of sound".

0 < speed of sound < ∞

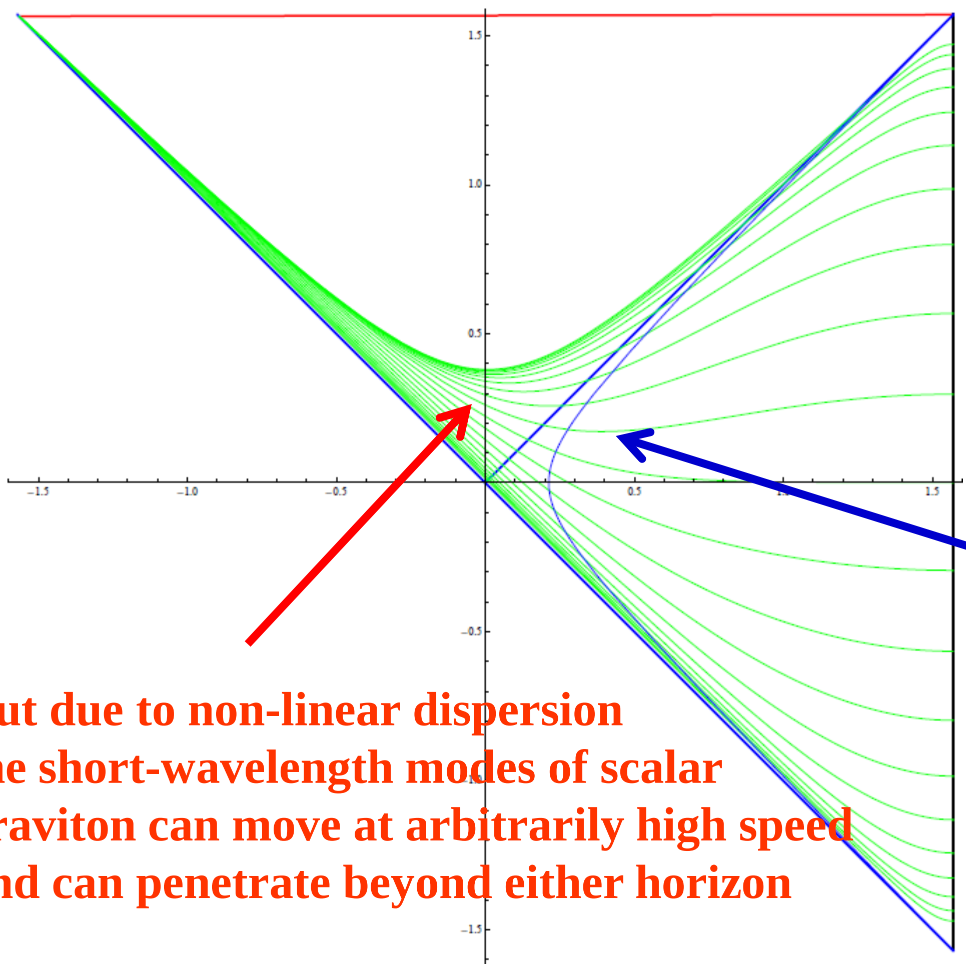
Free parameter of theory.

Here speed of sound < speed of gravity.

Sound horizon outside gravity horizon.

Black holes in Horava gravity

(Janiszewski, in progress)



Scalar graviton:
long wavelength mode
moves at “speed of sound”.

0 < speed of sound < □

Free parameter of theory.

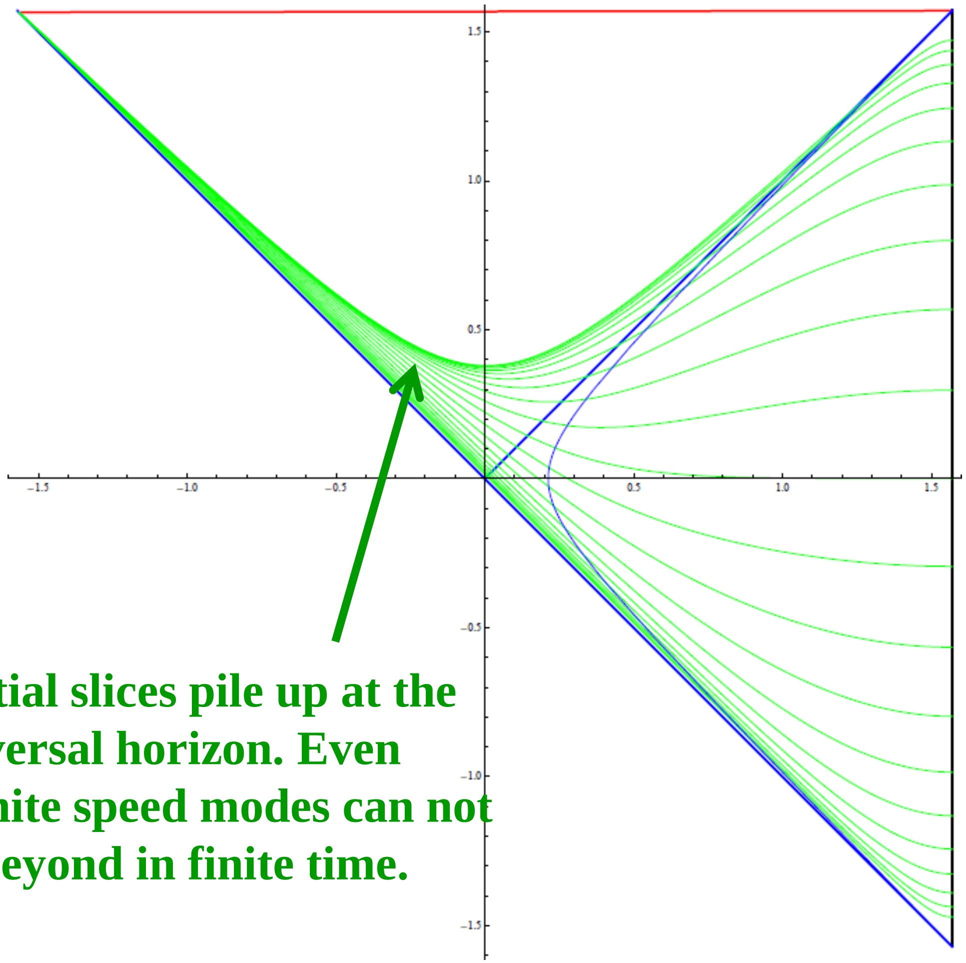
Here speed of sound < speed of gravity.

Sound horizon outside gravity horizon.

But due to non-linear dispersion the short-wavelength modes of scalar graviton can move at arbitrarily high speed and can penetrate beyond either horizon

Black holes in Horava gravity

(Janiszewski, in progress)



Spatial slices pile up at the universal horizon. Even infinite speed modes can not go beyond in finite time.

To complete the solution one needs to find the preferred foliation (the preferred time coordinate) by solving the khronon profile.

Black holes in Horava gravity

(Janiszewski, in progress)

Universal horizon has meaningful thermodynamics.

- Energy/mass from asymptotic metric.
- Temperature from “tunneling” calculation or Euclidean geometry
- Entropy then follows. Gives Bekenstein-Hawking area law with speed of gravity playing the role of the speed of light

To do: Charged Black Holes!

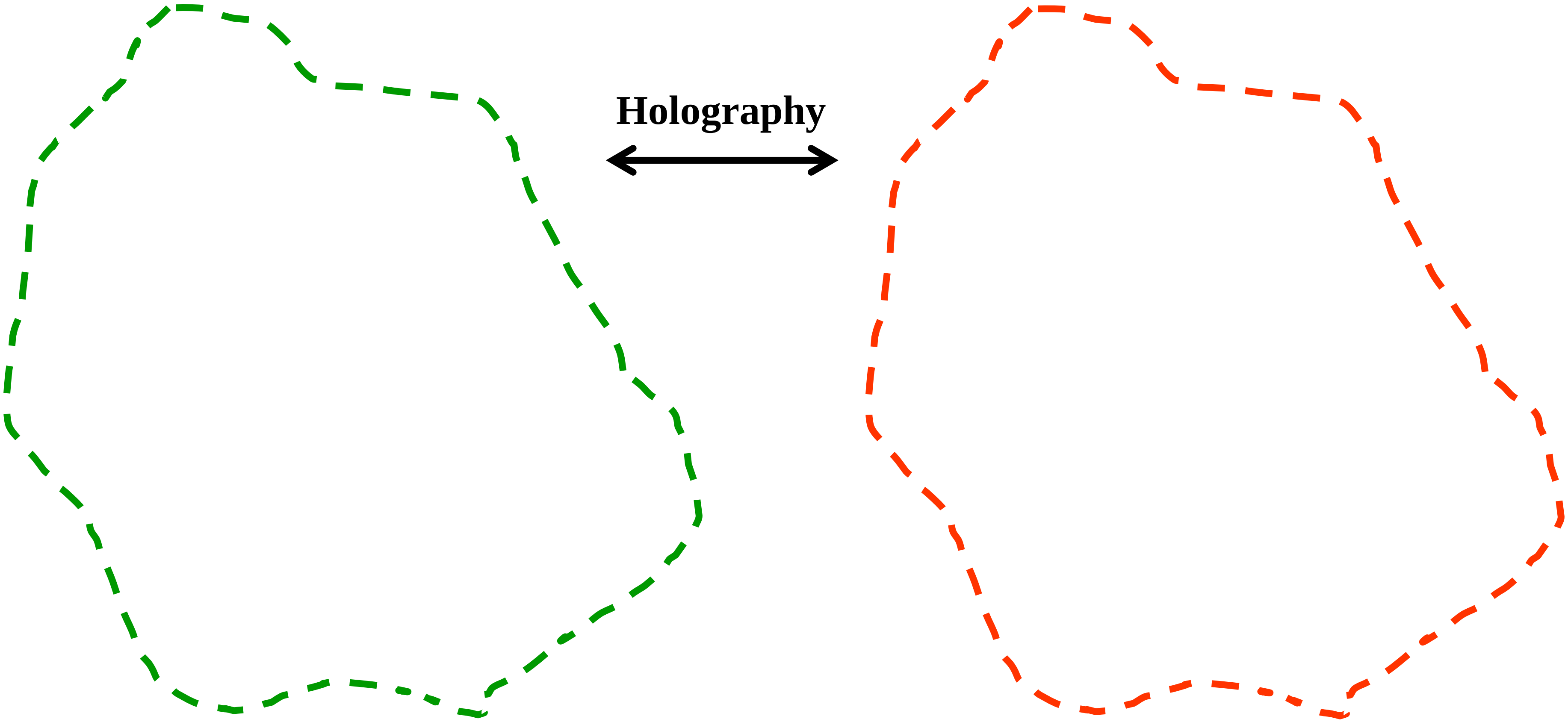
Recap:

- Symmetries suggest that generic NR CFT is dual to Horava Gravity.
- Horava Gravity is believed to be consistent quantum theory with UV fixed point. Duality in principle holds for any N
- For large N we can check that our proposal (equating a large N NR CFT to classical Horava Gravity) gives the correct form of NR CFT 2-pt functions.

Recap:

- Construction can easily be embedded in string theory. However, relativistic parent always gives Horava gravity in the **probe limit**!

Conclusions:



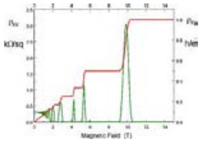
consinstent quantum theories of gravity
(on asymptotically Lifshitz/hyperbolic space)

All Quantum Field Theories

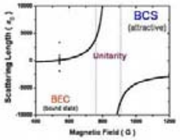
Conclusions:

String Theory

Holography



relativistic QFTs
and their NR deformations
(= probe limit!)



consinstent quantum theories of gravity
(on asymptotically Lifshitz/hyperbolic space)

All Quantum Field Theories