On Spacetime Entanglement

Robert Myers

KIAS-YITP Joint Workshop
"StringTheory, Black Holes & Holography"
Kyoto, July 1 – 5, 2013

(with Bianchi , Smolkin, Pourhasan)
(arXiv:1212.5183; arXiv:1304.2030)

PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

Black Hole Entropy:

- Bekenstein and Hawking: event horizons have entropy!

thermodynamics relativity geometry

$$S_{BH} = \frac{k_B\, c^3}{\hbar}\ \frac{\mathcal{A}}{4G}$$

quantum gravity

- extends to de Sitter horizons and Rindler horizons

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$\boxed{\textbf{quantum} \quad \textbf{gravity}}$

- extends to de Sitter horizons and Rindler horizons
- window into quantum gravity?!?
- quantum gravity provides a fundamental scale

$$\ell_P^2 = 8\pi\, G\ \hbar/c^3$$

Black Hole Entropy:

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$$S_{BH} = 2\pi \ \frac{\mathcal{A}}{\ell_P^2}$$

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Black Hole Entropy:

- Bekenstein and Hawking: event horizons have entropy!

$$S_{BH} = 2\pi\ \frac{\mathcal{A}}{\ell_P^{d-2}}\ +\ \cdots$$

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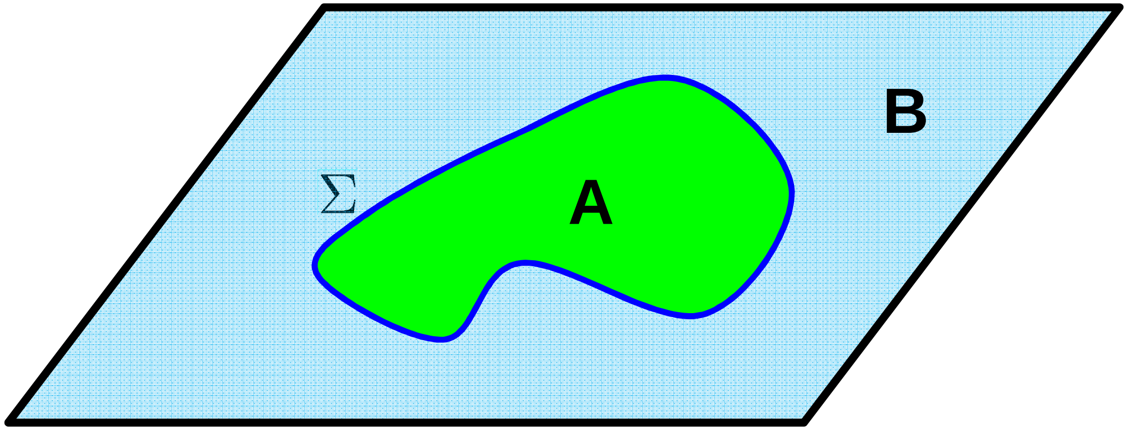
$$\ell_P^{d-2} = 8\pi\, G\ \hbar/c^3$$

Proposal: Geometric Entropy

- in a theory of quantum gravity, for any sufficiently large region with a smooth boundary in a smooth background (eg, flat space), there is an entanglement entropy which takes the form:

$$S_{\mathrm{EE}} = \frac{\mathcal{A}_\Sigma}{4G_N} + \cdots$$



- in QG, short-range quantum entanglement corresponding to area law is a signature of macroscopic spacetime geometry

(Bianchi & Myers)

Proposal: Geometric Entropy

- in a theory of quantum gravity, for any sufficiently large region with a smooth boundary in a smooth background (eg, flat space), there is an entanglement entropy which takes the form:

$$S_{\rm EE} = \frac{\mathcal{A}_\Sigma}{4G_N} + \cdots$$

- evidence comes from several directions:

1. holographic $S_{\rm EE}$ in AdS/CFT correspondence
2. QFT renormalization of G_N
3. induced gravity, eg, Randall-Sundrum 2 model
4. Jacobson's "thermal origin" of gravity
5. spin-foam approach to quantum gravity

(cf. **van Raamsdonk's** "Building up spacetime with quantum entanglement")

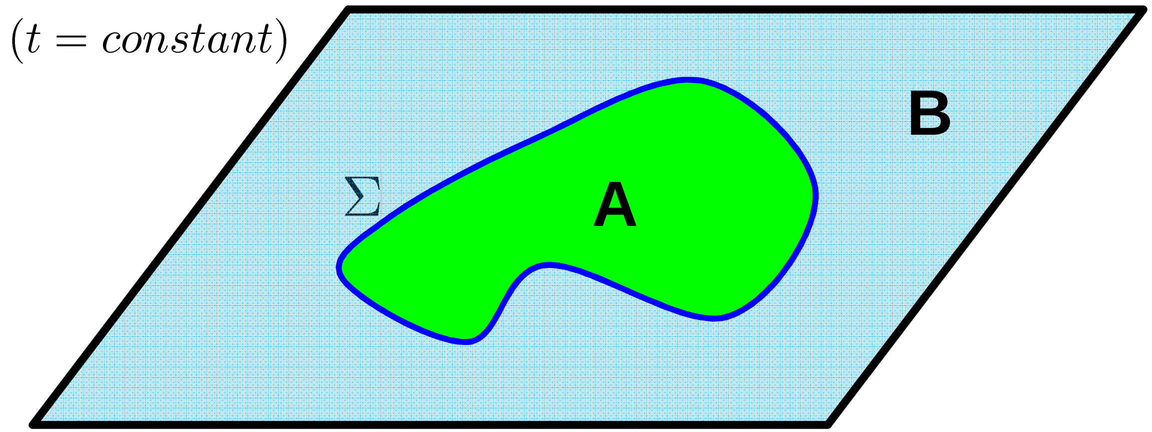
Outline:

1. Introduction and Concluding remarks
2. Entanglement Entropy 1:
   - Basic Definitions
   - Holographic Entanglement Entropy
3. Entanglement Entropy 2: $S_{BH} \simeq S_{EE}$
4. Entanglement Entropy 3:
   Entanglement Hamiltonian and Generalizing S_{BH}
5. Randall-Sundrum calculations:
   Induced Gravity and Holography
6. Conclusions & Outlook

Entanglement Entropy

- general tool; divide quantum system into two parts and use entropy as measure of correlations between subsystems
- in QFT, typically introduce a (smooth) boundary or entangling surface Σ which divides the space into two separate regions
- trace over degrees of freedom in “outside” region
- remaining dof are described by a density matrix ρ_A

→ calculate von Neumann entropy: $S_{EE} = -Tr\left[\rho_A \log \rho_A\right]$



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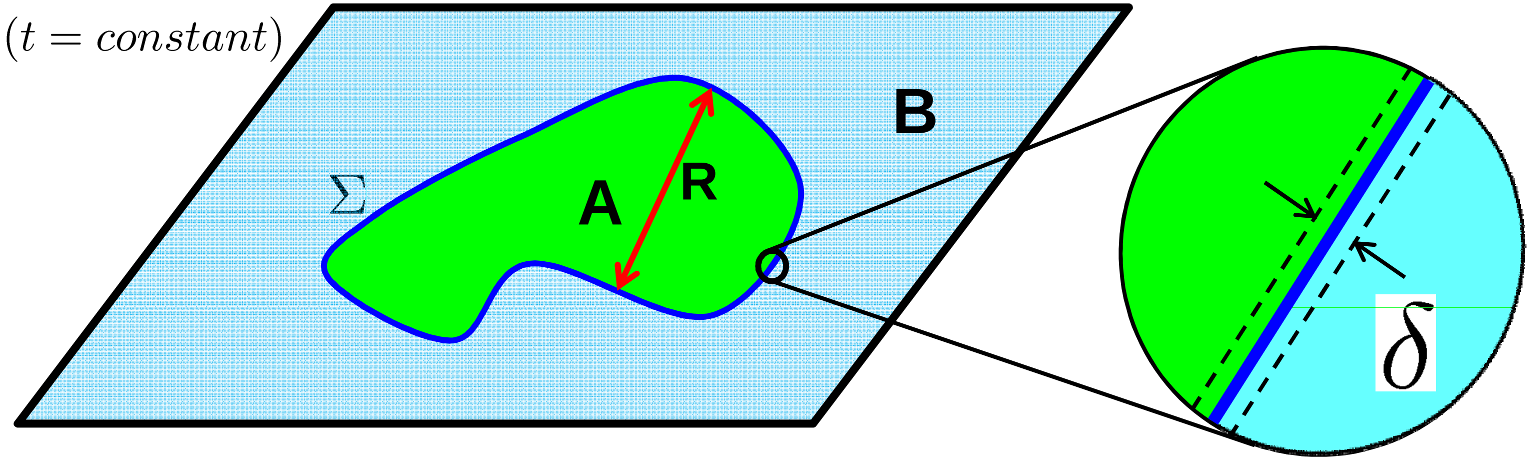

• result is UV divergent!

• must regulate calculation: δ = short-distance cut-off

$$S \;=\; c_0 \frac{R^{d-2}}{\delta^{d-2}} \;+\; c_2 \frac{R^{d-4}}{\delta^{d-4}} \;+\; \cdots$$

d = spacetime dimension

• careful analysis reveals geometric structure, eg, $S = \tilde{c}_0 \dfrac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \cdots$

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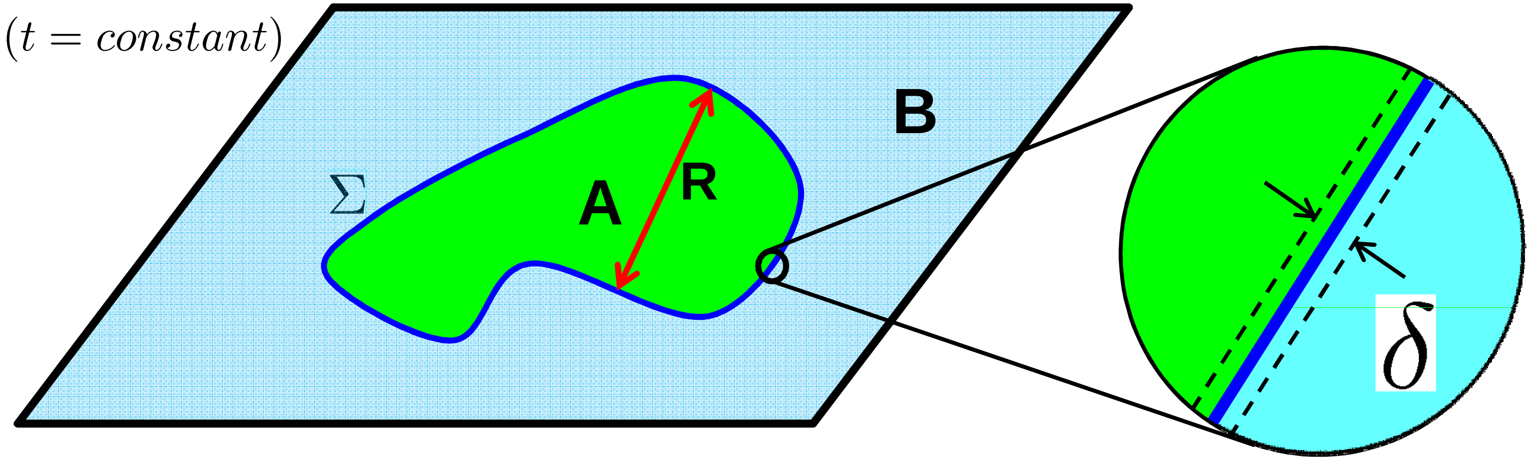

- must regulate calculation: δ = short-distance cut-off

$$S = c_0 \frac{R^{d-2}}{\delta^{d-2}} + c_2 \frac{R^{d-4}}{\delta^{d-4}} + \cdots$$ d = spacetime dimension

- leading coefficients sensitive to details of regulator, eg, $\delta \to 2\delta$
- find universal information characterizing underlying QFT in subleading terms, eg, $S = \cdots + c_d \log(R/\delta) + \cdots$

General comments on **Entanglement Entropy:**

- nonlocal quantity which is (at best) very difficult to measure
  → no (accepted) experimental procedure

- in condensed matter theory: diagnostic to characterize quantum critical points or topological phases (eg, quantum spin fluids)

- in quantum information theory: useful measure of quantum entanglement (a computational resource)

- recently considered in AdS/CFT correspondence

(Ryu & Takayanagi `06)

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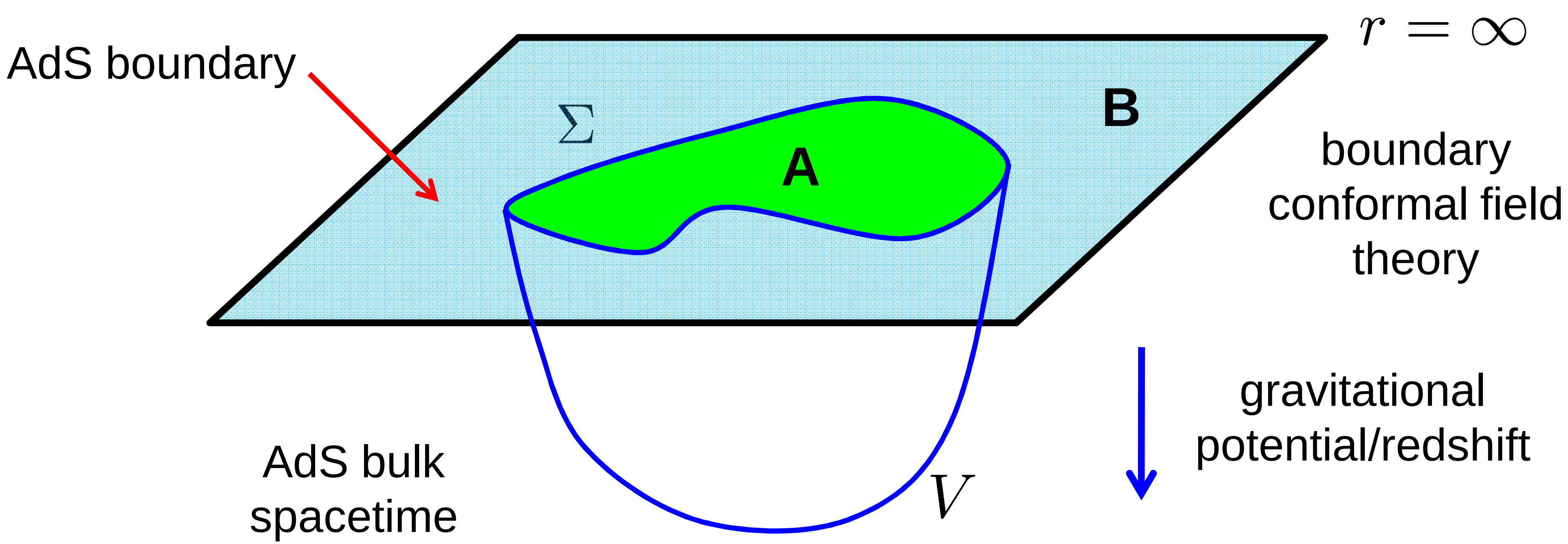
Holographic Entanglement Entropy:



$$S(A) = \underset{\partial V = \Sigma}{\mathrm{ext}} \frac{A_V}{4G_N} = \infty!!$$

- "UV divergence" because area integral extends to $r = \infty$

(Ryu & Takayanagi `06)

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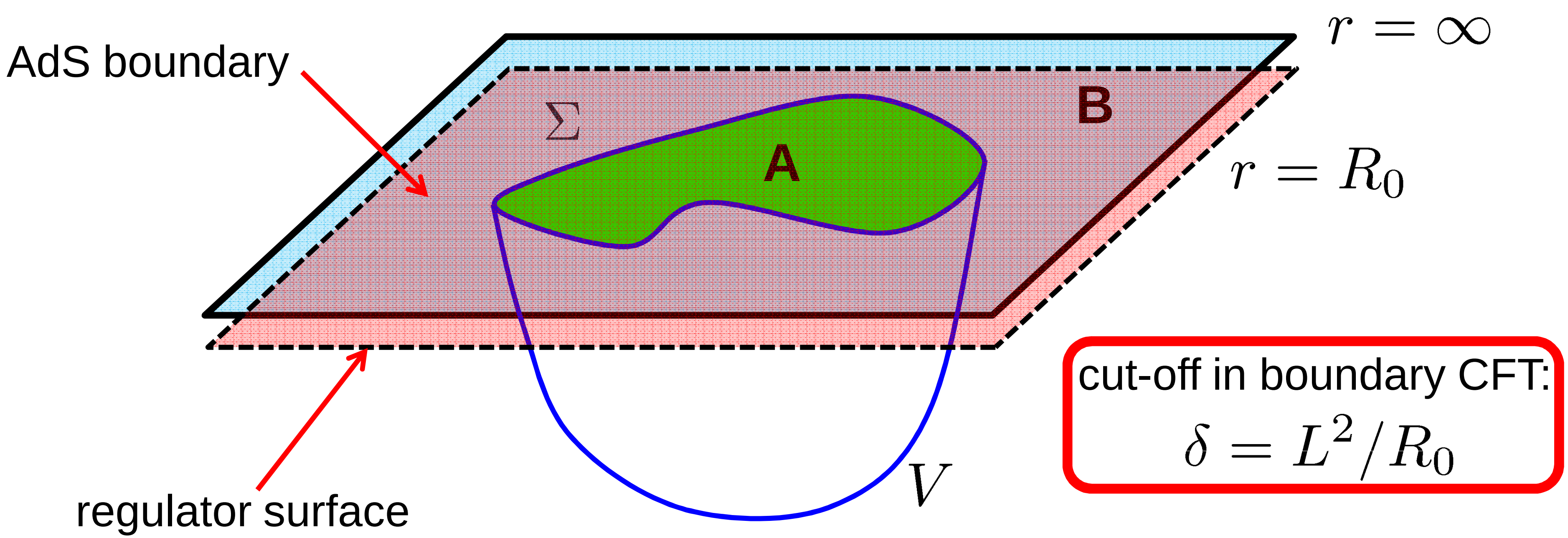


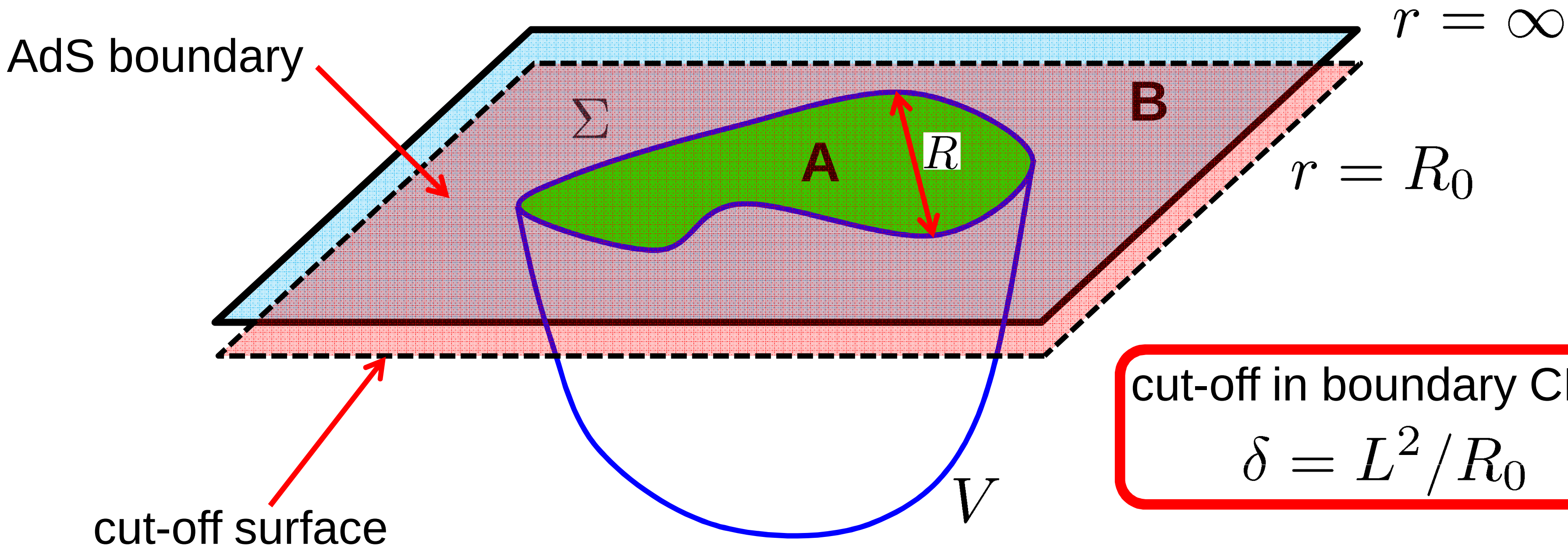
$$S(A) = \operatorname*{ext}_{\partial V = \Sigma} \frac{A_V}{4G_N}$$

- “UV divergence” because area integral extends to $r = \infty$
- introduce regulator surface at large radius: $r = R_0$

$\longrightarrow$ short-distance cut-off in boundary theory: $\delta = L^2/R_0$

(Ryu & Takayanagi `06)

Holographic Entanglement Entropy:



cut-off in boundary CFT:

$$\delta = L^2/R_0$$

• general expression (as desired):

$$S(A) \simeq c_0 (R/\delta)^{d-2} + c_1 (R/\delta)^{d-4} + \cdots +$$ "universal contributions"

• conjecture ⟶ many detailed consistency tests

(Ryu, Takayanagi, Headrick, Hung, Smolkin, Faulkner, . . .)

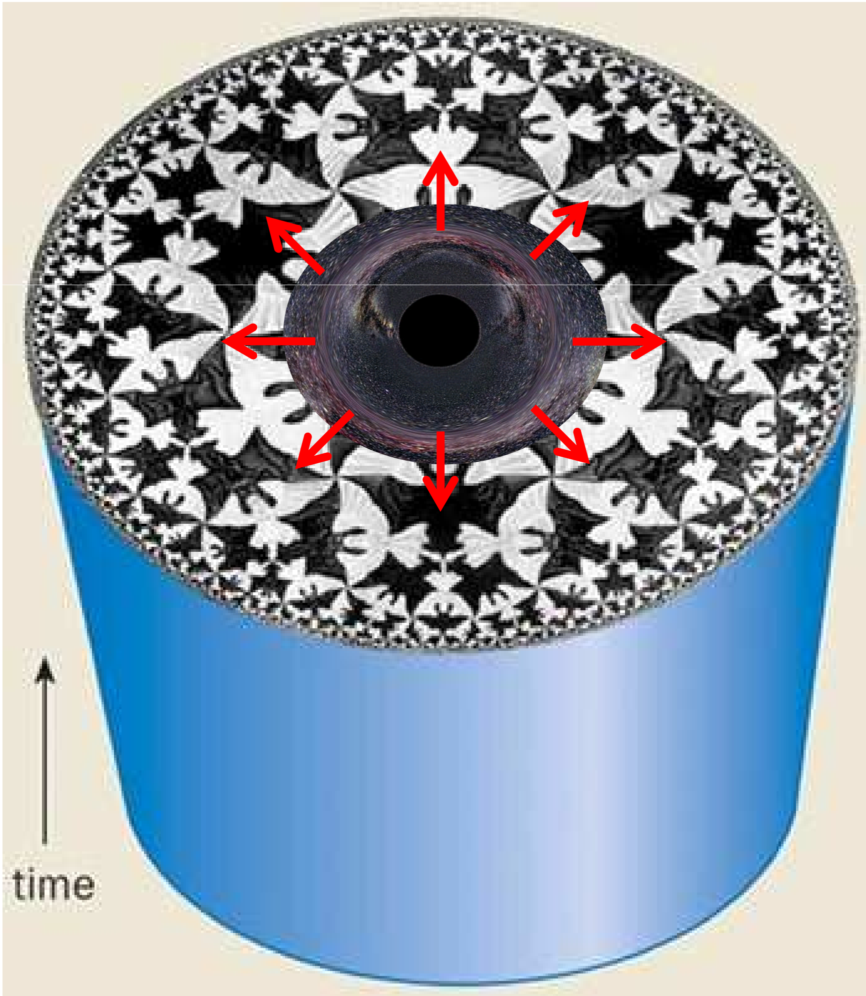
• **proof!!** ⟶ "generalized gravitational entropy"

(Lewkowycz & Maldacena)

Lessons from Holographic EE:

AdS/CFT Dictionary:

Boundary: thermal plasma ⟷ Bulk: black hole

Temperature		Temperature
Energy		Energy
Entropy		Entropy

Lessons from Holographic EE:

(entanglement entropy)$_{\text{boundary}}$

= (entropy associated with extremal surface)$_{\text{bulk}}$

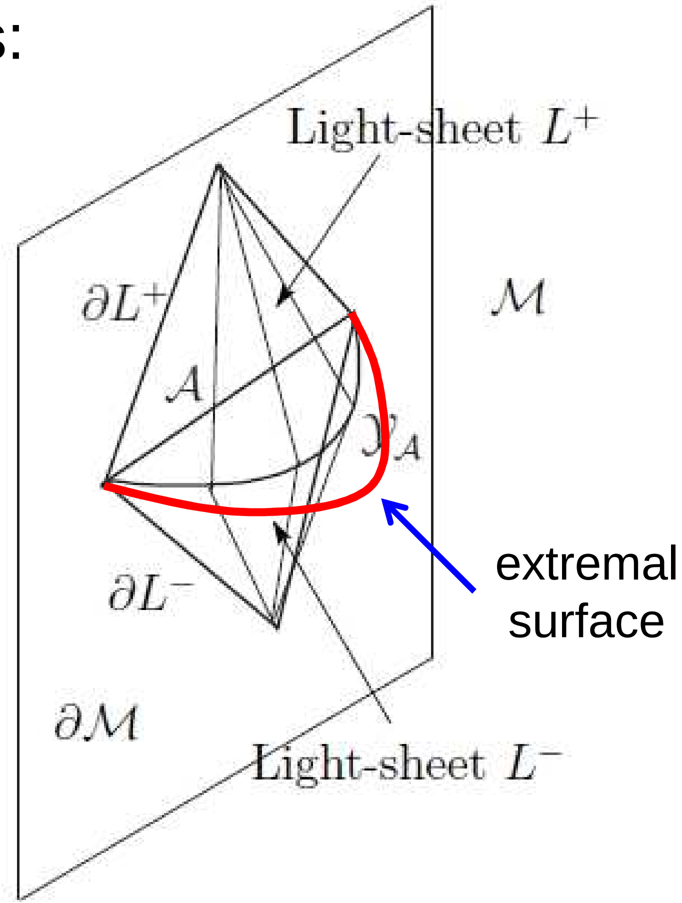
- R&T construction assigns entropy $S_{BH} = \mathcal{A}/(4G_N)$ to bulk regions with “unconventional” boundaries:

not black hole! not horizon!
not causal domain!

What are the rules? 

- **indicates S_{BH} applies more broadly**

⟶ with our proposal, S_{BH} defines S_{EE} in bulk gravity for any surface



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- R&T construction assigns entropy $S_{BH} = \mathcal{A}/(4G_N)$ to bulk regions with "unconventional" boundaries:
- with our proposal, S_{BH} defines S_{EE} in bulk gravity for any surface
- what about extremization?

$\longrightarrow$ needed to make match above (in accord with proof)

- S_{BH} on other surfaces already speculated to give other entropic measures of entanglement in boundary theory

$\longrightarrow$ entanglement between high and low scales

(Balasubramanian, McDermott & van Raamsdonk)

$\longrightarrow$ causal holographic information

(Hubeny & Rangamani; H, R & Tonni; Freivogel & Mosk)

- see Maldacena's talk (Faulkner, Lewkowycz & Maldacena)

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Where did “**Entanglement Entropy**” come from?**:**

- Sorkin ’84: looking for origin of black hole entropy
- recall that leading term obeys “area law”: $S = c_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \cdots$

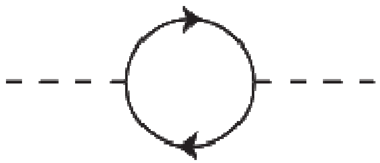
$\longrightarrow$ suggestive of BH formula if $\delta \simeq \ell_P$

(Sorkin `84; Bombelli, Koul, Lee & Sorkin; Srednicki; Frolov & Novikov)

- problem?: leading singularity not universal; regulator dependent
- resolution: this singularity represents contribution of “low energy” d.o.f. which actually renormalizes “bare” area term $S_0 = \mathcal{A}/4G_0$

(Susskind & Uglum)

$$\delta\left(\frac{1}{G}\right) = \frac{4c_0}{\delta^{d-2}} \qquad\qquad S = c_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \cdots$$



both coefficients are regulator dependent
but for a given regulator should match!

BH Entropy ~ Entanglement Entropy

(Demers, Lafrance & Myers)

- massive d=4 scalar: integrating out yields effective metric action

$$e^{iW(g)} = \int \mathcal{D}\phi\; e^{iI(\phi,g)} \quad \text{with} \quad I = -\frac{1}{2}\int d^4x\sqrt{-g}\left[g^{ab}\nabla_a\phi\nabla_b\phi + m^2\phi^2\right]$$

- must regulate to control UV divergences in *W(g)*

$\longrightarrow$ Pauli-Villars fields: $-\frac{1}{2}\int d^4x\sqrt{-g}\sum_{i=1}^{5}\left[g^{ab}\nabla_a\phi_i\nabla_b\phi_i + m^2\phi_i^2\right]$

$\phi_{1,2}$: anti-commuting, $m_{1,2}^2 = m^2 + \mu^2$, $\phi_{3,4}$: commuting, $m_{3,4}^2 = m^2 + 3\mu^2$,

ϕ_5 : anti-commuting, $m_5^2 = m^2 + 4\mu^2$

UV regulator scale

- effective Einstein term: $W \simeq \frac{1}{16\pi}\int d^4x\sqrt{-g}\,R\,\frac{B}{12\pi}$ with

quadratic divergence

$$B = \mu^2\left[2\ln\frac{3\mu^2+m^2}{\mu^2+m^2} + 4\ln\frac{3\mu^2+m^2}{4\mu^2+m^2}\right] + m^2\left[\ln\frac{m^2}{4\mu^2+m^2} + 2\ln\frac{3\mu^2+m^2}{\mu^2+m^2}\right]$$

- renormalization of Newton's constant: $\delta\left(\frac{1}{G}\right) = \frac{B}{12\pi}$

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- renormalization of Newton's constant: $\delta\left(\frac{1}{G}\right)=\frac{B}{12\pi}$
- scalar field contribution to BH entropy: $S \simeq \frac{\mathcal{A}}{4}\frac{B}{12\pi}=\frac{\mathcal{A}}{4}\,\delta\left(\frac{1}{G}\right)$
- extends to log divergence; matches curvature correction to S_{Wald}

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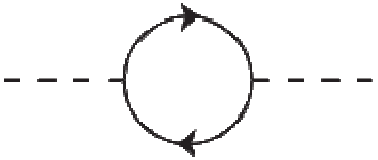

both coefficients are regulator dependent
but for a given regulator should match!

- "a beautiful idea killed by ugly calculations"??

- seemed matching was not always working??
- numerical factors resolved; extra boundary terms interpeted

(Fursaev, Solodukhin, Miele, Iellici, Moretti, Donnelly, Wall . . .)

(Cooperman & Luty)

→ matching of area term works for any QFT (s=0,1/2, 1, 3/2) to all orders in perturbation theory for any Killing horizon

- technical difficulties for spin-2 graviton
- ♥ **results apply for Rindler horizon in flat space**

- some conceptual issues may remain (Jacobson & Satz; Solodukhin)

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(Susskind & Uglum)

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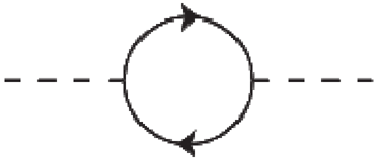

both coefficients are regulator dependent
but for a given regulator should match!

- “a beautiful killed by ugly facts”??

- result “unsatisfying”: $S_{BH} = S_0 + S_{EE} = \frac{\mathcal{A}}{4G_0} + \frac{\mathcal{A}}{4}\,\delta\left(\frac{1}{G}\right) + \cdots$

where did bare term come from?

- consider “induced gravity”: $\frac{1}{G_0} = 0$ (Jacobson; Frolov, Fursaev & Solodukhin)
- formally “off-shell” method is precisely calculation of S_{EE}

(Susskind & Uglum; Callan & Wilczek; Myers & Sinha: extends to S_{wald})

- challenge: understand microscopic d.o.f. of quantum gravity

$\longrightarrow$ AdS/CFT: eternal black hole (or any Killing horizon)

(Maldacena; van Raamsdonk et al; Casini, Huerta & Myers)

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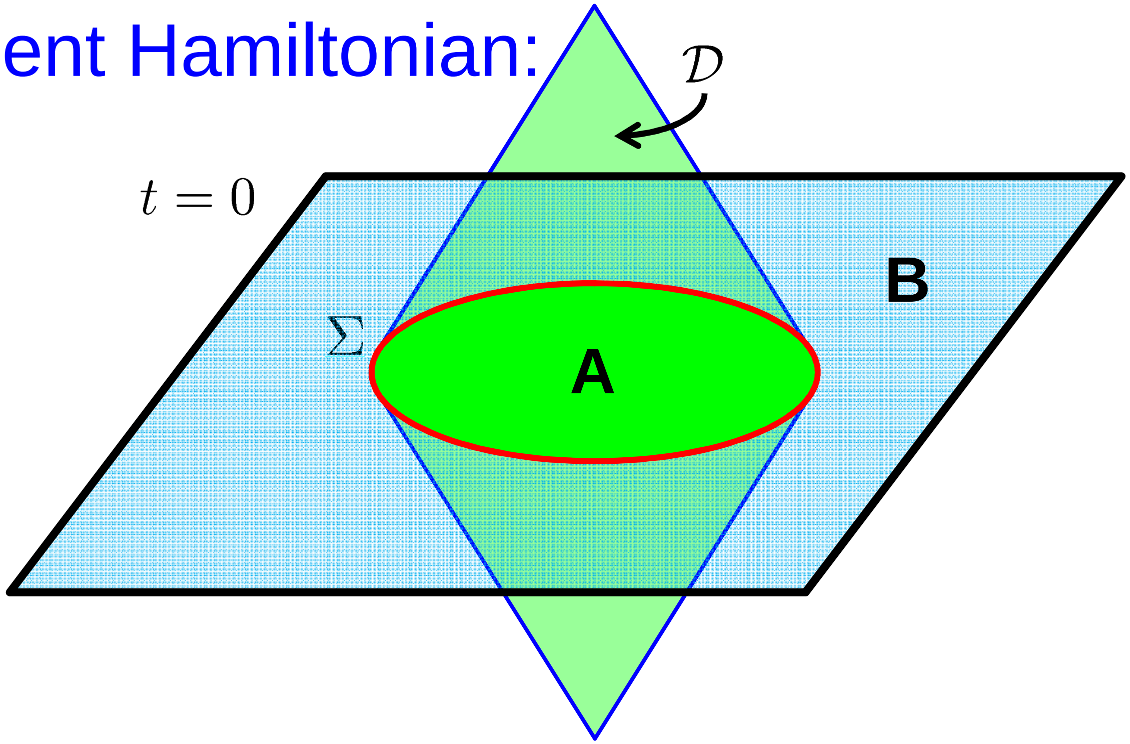
Entanglement Hamiltonian:

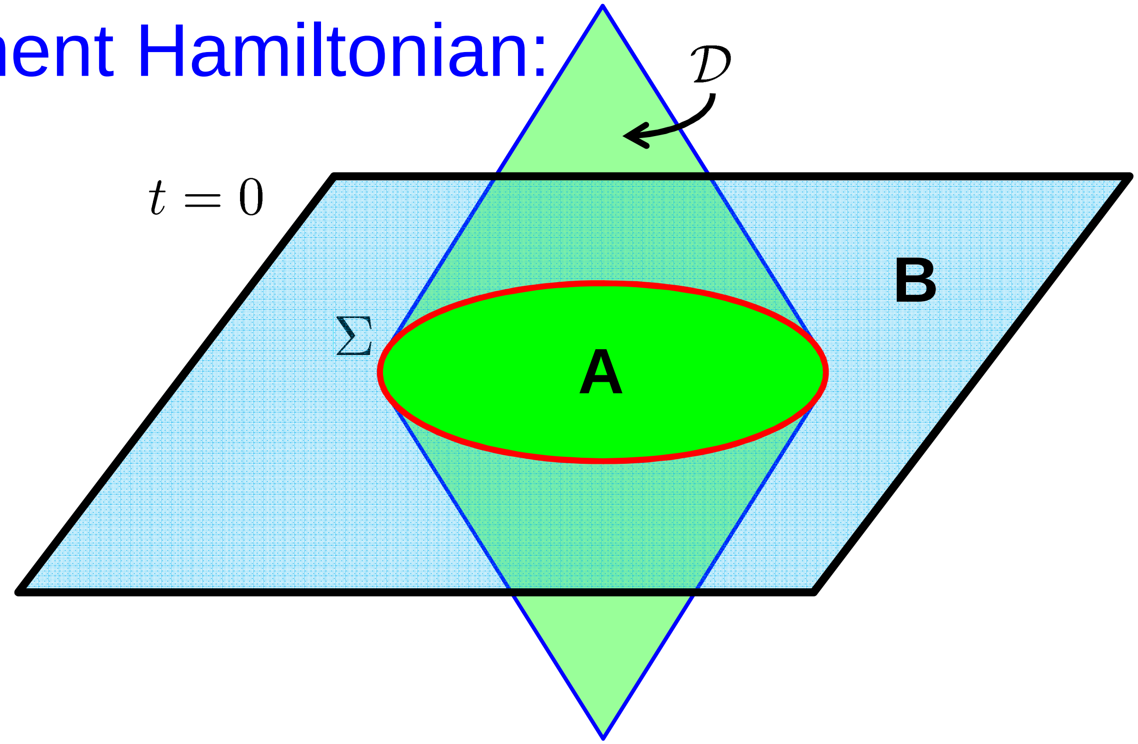


- first step in calculation of S_{EE} is to determine ρ_A
- ρ_A reproduces standard correlators, eg, if global vacuum:

$$\mathrm{Tr}(\rho_A\,\phi(x)\phi(y)) = \langle 0|\phi(x)\phi(y)|0\rangle$$

- by causality, ρ_A describes physics throughout causal domain $\mathcal{D}$

Entanglement Hamiltonian:



• hermitian and positive semi-definite, hence $\rho_A = e^{-H}$

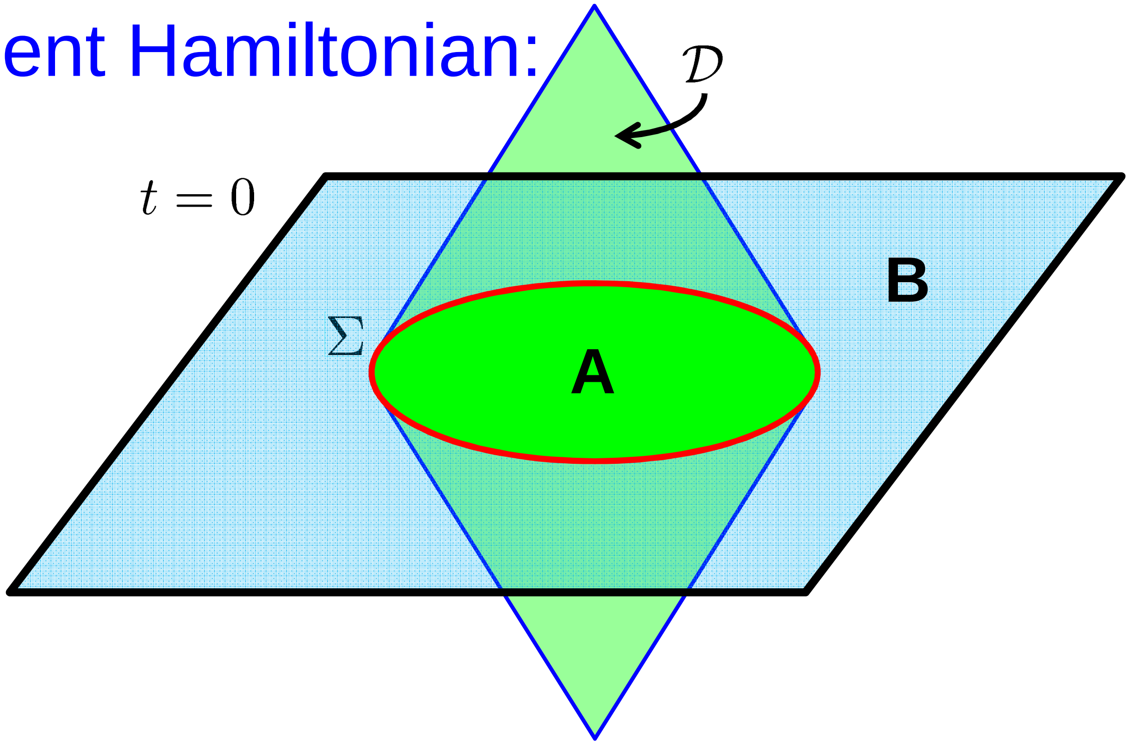
$\longrightarrow$ H = modular or **entanglement Hamiltonian**

• **formally** can consider evolution by $U(s) = \rho_A^{is} = e^{-iHs}$

• unfortunately H is **nonlocal** and flow is nonlocal/**not geometric**

$$H = \int d^{d-1}x\, \gamma_1^{\mu\nu}(x)\, T_{\mu\nu} + \int d^{d-1}x \int d^{d-1}y\, \gamma_2^{\mu\nu;\rho\sigma}(x,y)\, T_{\mu\nu} T_{\rho\sigma} + \cdots$$

Entanglement Hamiltonian:



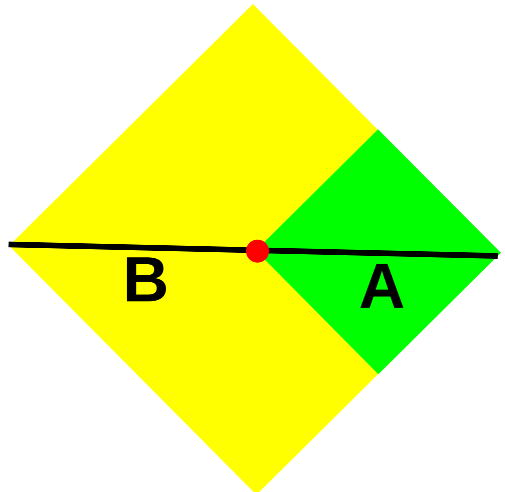
- hermitian and positive semi-definite, hence $\rho_A = e^{-H}$

$\longrightarrow$ H = modular or **entanglement Hamiltonian**

- H explicitly known only in limited examples
- most famous example: Rindler wedge

$H =$ boost generator

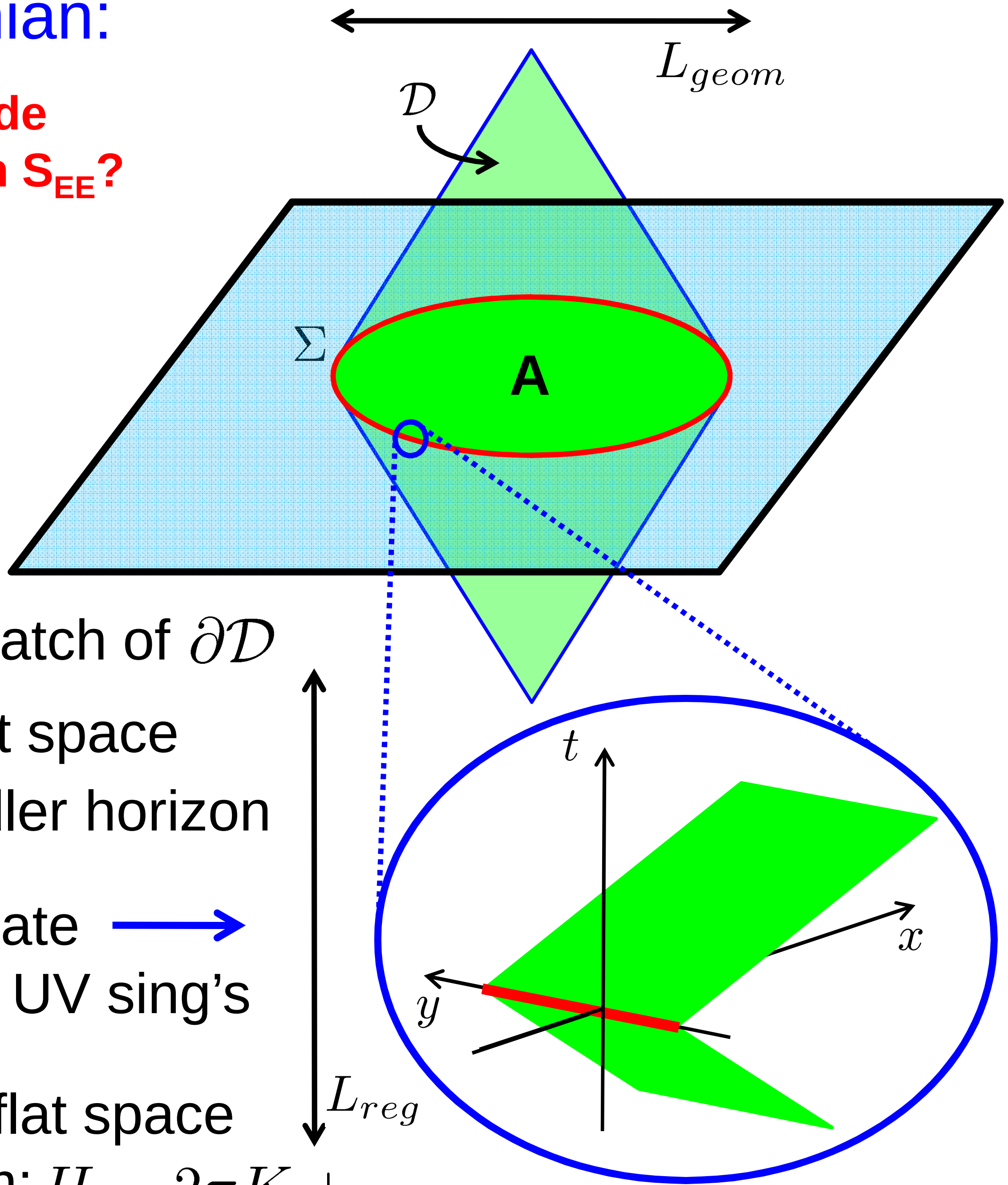
$$= 2\pi K = -2\pi \int_{A(x>0)} d^{d-2}y\, dx\ [x\, T_{00}]$$



Entanglement Hamiltonian:

Can this formalism provide insight for “area law” term in S_{EE}?

$$L_{geo}L_{geom} \gg \delta \gg \delta$$



- zoom in on infinitesimal patch of $\partial\mathcal{D}$
  - → region looks like flat space
  - → $\partial\mathcal{D}$ looks like Rindler horizon
- assume Hadamard-like state → correlators have standard UV sing’s
- UV part of ρ_A same as in flat space
- H must have Rindler term: $H = 2\pi K + \cdots$

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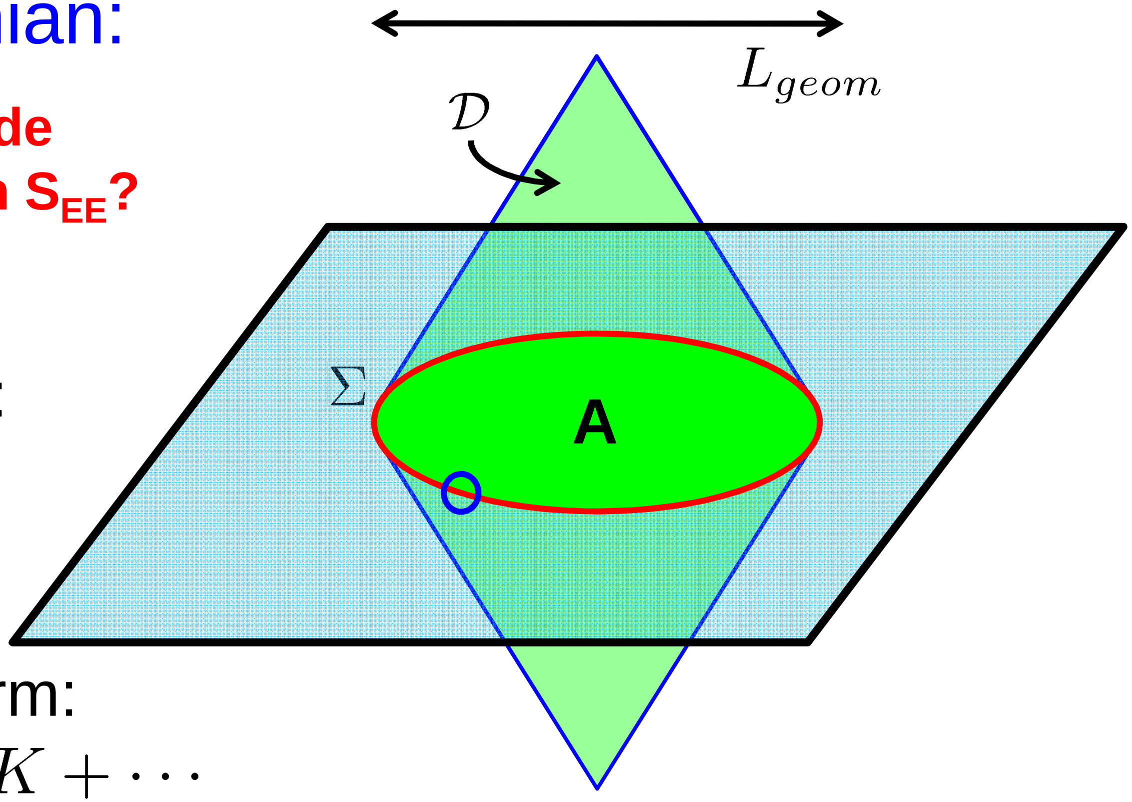


for each infinitesimal patch:

- UV part of ρ_A must be same as in flat space
- H must have Rindler term:

$$H = 2\pi K + \cdots$$

⟶ Rindler H yields area law; hence $\delta S_{EE} = c_0\, \delta\mathcal{A}_\Sigma/\delta^{d-2} + \cdots$

⟶ hence S_{EE} must contain divergent area law contribution!

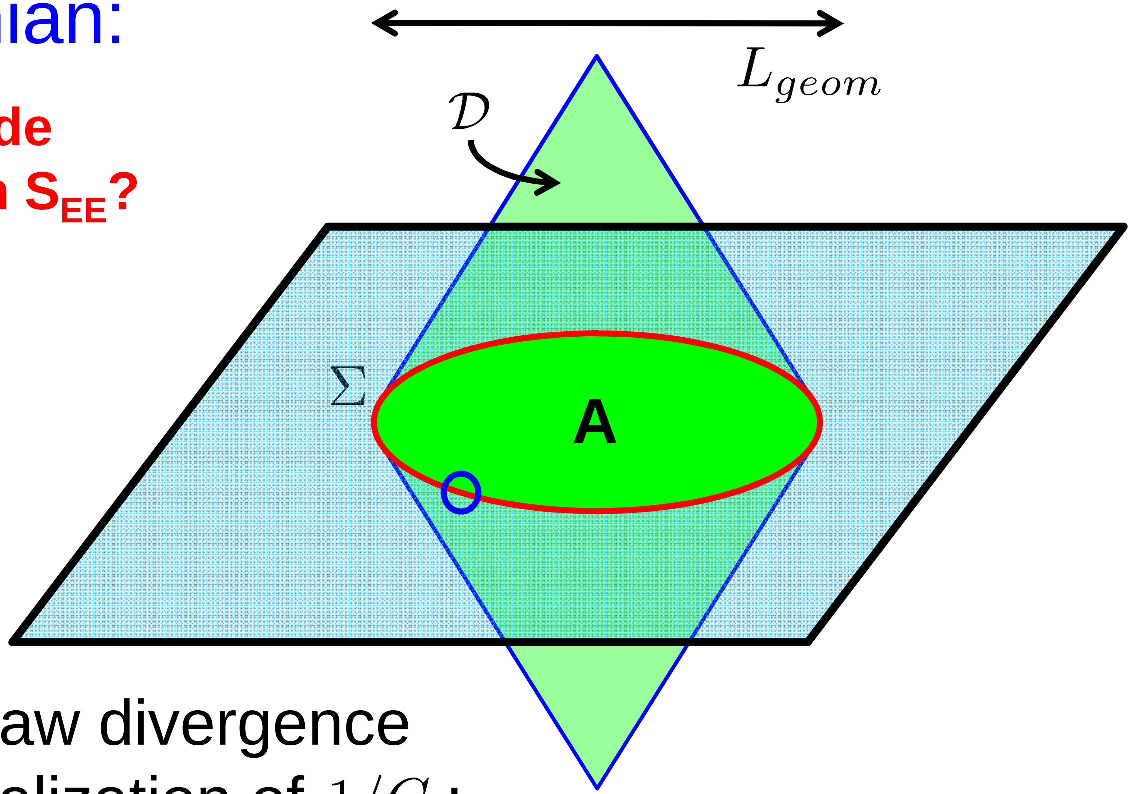
- invoke Cooperman & Luty: area law divergence matches precisely renormalization of $1/G$:

$$S_{EE} = \frac{\mathcal{A}}{4}\,\delta\left(\frac{1}{G}\right) + \cdots$$

for any large region of smooth geometry!!

Entanglement Hamiltonian:

Can this formalism provide insight for "area law" term in S_{EE}?



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for any large region of smooth geometry!!

- consistency check of new proposal but

where did bare term come from?

Entanglement Hamiltonian:

where did bare term come from?

- formally can apply standard geometric arguments in Rindler patches, analogous to the "off-shell" calc's for black holes
- entanglement Hamiltonian not "mysterious" with Killing symmetry

$$H = 2\pi \int_\Sigma dV^\mu \, T_{\mu\nu} \, k^\nu$$

eg, Killing horizons: Rindler, de Sitter, stationary black holes, . . .

(see also: Wong, Klich, Pando Zayas & Vaman)

- carry QFT discussion over to geometric discussion:
  - zooming in on entangling surface restores "rotational" symmetry
  - can apply standard geometric arguments to find bndry terms

$\longrightarrow$ spherical entangling surface in flat space

(Balasubramanian, Czech, Chowdhury & de Boer)

$$I = \int d^dx \sqrt{g} \left[\frac{R}{16\pi G_d} + \cdots \right] \quad \Longrightarrow \quad S_{EE} = \frac{\mathcal{A}_\Sigma}{4G_d} + \cdots$$

Entanglement Hamiltonian:

where did bare term come from?

- consider gravitational action with higher order corrections:

$$I = \int d^dx\sqrt{g}\left[\frac{R}{16\pi G_d} + \frac{\alpha_1}{2\pi}R^2 + \frac{\alpha_2}{2\pi}R_{ij}R^{ij} + \frac{\alpha_3}{2\pi}C_{ijkl}C^{ijkl} + \cdots\right]$$

- apply new technology: **"Distributional Geometry of Squashed Cones"**

(Fursaev, Patrushev & Solodukhin)

$$S_{EE} = \frac{\mathcal{A}_\Sigma}{4G_d} + 4\alpha_1\int_\Sigma d^{d-2}y\sqrt{h}R + 2\alpha_2\int_\Sigma d^{d-2}y\sqrt{h}\left[2R^{ij}\,\tilde{g}^\perp_{ij} \underline{- K^iK_i}\right]$$
$$+ 4\kappa_2\int_\Sigma d^{d-2}y\sqrt{h}\left[h^{ac}h^{bd}C_{abcd} \underline{- K^i_{ab}K_i{}^{ab} + \frac{1}{d-2}K^iK_i}\right] + \cdots$$

- compare to Wald entropy for such higher curvature actions:

$$S_{Wald} = -2\pi\int_\Sigma d^{d-2}y\sqrt{h}\,\frac{\partial\mathcal{L}}{\partial R^{\mu\nu}{}_{\rho\sigma}}\,\hat{\epsilon}^{\mu\nu}\hat{\epsilon}_{\rho\sigma}$$

$\longrightarrow$ in general, S_{EE} and S_{Wald} differ (beyond area law) by extrinsic curvature terms but will agree on stationary event horizon

Entanglement Hamiltonian:

where did bare term come from?

- **challenge:** understand “bare term” from perspective of microscopic d.o.f. of quantum gravity

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5) Randall-Sundrum 2 model

- AdS/CFT cut-off surface becomes a physical brane

$\longrightarrow$ graviton zero-mode becomes normalizable

$\longrightarrow$ D=d+1 AdS gravity (with cut-off) + brane matter

d-dim. CFT (with cut-off) + d-dim. gravity + brane matter

- induced gravity: “boundary divergences” become effective action

$$I_{ind} = 2\sum_{n=0}^{\lfloor d/2 \rfloor} I^{(n)} + 2I_{fin} + I_{brane} \longleftarrow \text{on-shell}$$

$$= 2\int d^dx\sqrt{\tilde{g}}\left(\cancel{\frac{c_0}{\delta^d}} + \underbrace{\frac{c_1}{\delta^{d-2}}R + \frac{c_2}{\delta^{d-4}}R^2 + \cdots}_{\text{boundary gravity}}\right) + 2I_{fin} + I_{brane}$$

cancel with brane tension

CFT correlators

5) Randall-Sundrum 2 model

- AdS/CFT cut-off surface becomes a physical brane

$\longrightarrow$ graviton zero-mode becomes normalizable

$\longrightarrow$ D=d+1 AdS gravity (with cut-off) + brane matter

$\Longleftrightarrow$

d-dim. CFT (with cut-off) + d-dim. gravity + brane matter

- induced gravity: "boundary divergences" become effective action
- AdS scale, L = short-distance cut-off δ in CFT
- fundamental parameters: cut-off scale δ in CFT

CFT central charge (# dof) $C_T \quad (\gg 1)$

$$\frac{1}{(G_N)_{bdry}} = \frac{8\pi}{\ell_{P,bdry}^{d-2}} \simeq \frac{C_T}{\delta^{d-2}} \; ; \; \frac{1}{(G_N)_{bulk}} = \frac{8\pi}{\ell_{P,bulk}^{d-1}} \simeq \frac{C_T}{\delta^{d-1}} \quad (\delta \gg \ell_{P,b})$$

- **only leading contributions in an expansion of large C_T !!**

5) Randall-Sundrum 2 model

(Fursaev; Emparan)

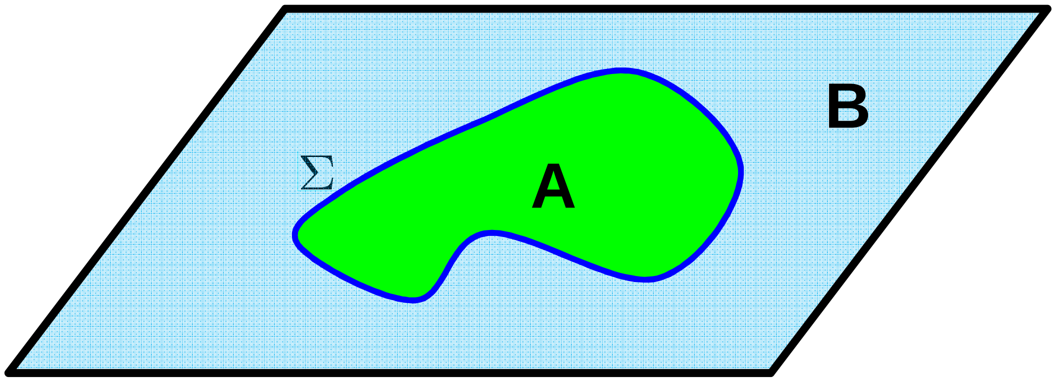
• entanglement entropy calculated with R&T prescription

⟶ for BH's on brane, horizon entropy = entanglement entropy

(see also: Hawking, Maldacena & Strominger; Iwashita et al)

• entanglement entropy for any macroscopic region is finite in a smooth boundary geometry: (Pourhasan, Smolkin & RM)

$$S(A) = \frac{\mathcal{A}_\Sigma}{4\,(G_N)_{bdry}} + \cdots \quad \textbf{!!!}$$



⟶ **brane regulates all entanglement entropies!!**

5) Randall-Sundrum 2 model

(Pourhasan, Smolkin & RM)

- consider boundary gravitational action to higher orders:

$$I = \int d^dx\sqrt{\tilde{g}}\left[\frac{R}{16\pi G_d} + \frac{\kappa_1}{2\pi}\left(R_{ij}R^{ij} - \frac{d}{4(d-1)}R^2\right) + \frac{\kappa_2}{2\pi}C_{ijkl}C^{ijkl} + \cdots\right]$$

using models with Einstein and GB gravity in bulk

- careful analysis of asymptotic geometry yields

$$\begin{aligned} S_{EE} \quad = \quad & \frac{\mathcal{A}_\Sigma}{4G_d} + \kappa_1 \int_\Sigma d^{d-2}y\sqrt{\tilde{h}}\left[2R^{ij}\,\tilde{g}^\perp_{ij} - \frac{d}{d-1}R - K^iK_i\right] \\ & + 4\kappa_2\int_\Sigma d^{d-2}y\sqrt{\tilde{h}}\left[\tilde{h}^{ac}\tilde{h}^{bd}C_{abcd} - K^i_{ab}K_i{}^{ab} + \frac{1}{d-2}K^iK_i\right] + \cdots \end{aligned}$$

- complete agreement with previous geometric calculation!!

Entanglement Hamiltonian:

where did bare term come from?

- consider gravitational action with higher order corrections:

$$I = \int d^dx\sqrt{g}\left[\frac{R}{16\pi G_d} + \frac{\alpha_1}{2\pi}R^2 + \frac{\alpha_2}{2\pi}R_{ij}R^{ij} + \frac{\alpha_3}{2\pi}C_{ijkl}C^{ijkl} + \cdots\right]$$

- apply new technology: **"Distributional Geometry of Squashed Cones"**

(Fursaev, Patrushev & Solodukhin)

$$\begin{aligned} S_{EE} \;&=\; \frac{\mathcal{A}_\Sigma}{4G_d} + 4\alpha_1\int_\Sigma d^{d-2}y\sqrt{h}R + 2\alpha_2\int_\Sigma d^{d-2}y\sqrt{h}\left[2R^{ij}\,\tilde{g}^{\perp}_{ij}\,\underline{-K^iK_i}\right] \\ &\quad + 4\kappa_2\int_\Sigma d^{d-2}y\sqrt{h}\left[h^{ac}h^{bd}C_{abcd}\,\underline{-K^i_{ab}K_i{}^{ab} + \frac{1}{d-2}K^iK_i}\right] + \cdots \end{aligned}$$

- compare to Wald entropy for such higher curvature actions:

$$S_{Wald} = -2\pi\int_\Sigma d^{d-2}y\sqrt{h}\,\frac{\partial\mathcal{L}}{\partial R^{\mu\nu}{}_{\rho\sigma}}\,\hat{\epsilon}^{\mu\nu}\,\hat{\epsilon}_{\rho\sigma}$$

⟶ in general, S_{EE} and S_{Wald} differ (beyond area law) by extrinsic curvature terms but will agree on stationary event horizon

5) Randall-Sundrum 2 model

(Pourhasan, Smolkin & RM)

- consider boundary gravitational action to higher orders:

$$I = \int d^dx \sqrt{\tilde{g}} \left[\frac{R}{16\pi G_d} + \frac{\kappa_1}{2\pi} \left(R_{ij}R^{ij} - \frac{d}{4(d-1)} R^2 \right) + \frac{\kappa_2}{2\pi} C_{ijkl}C^{ijkl} + \cdots \right]$$

using models with Einstein and GB gravity in bulk

- careful analysis of asymptotic geometry yields

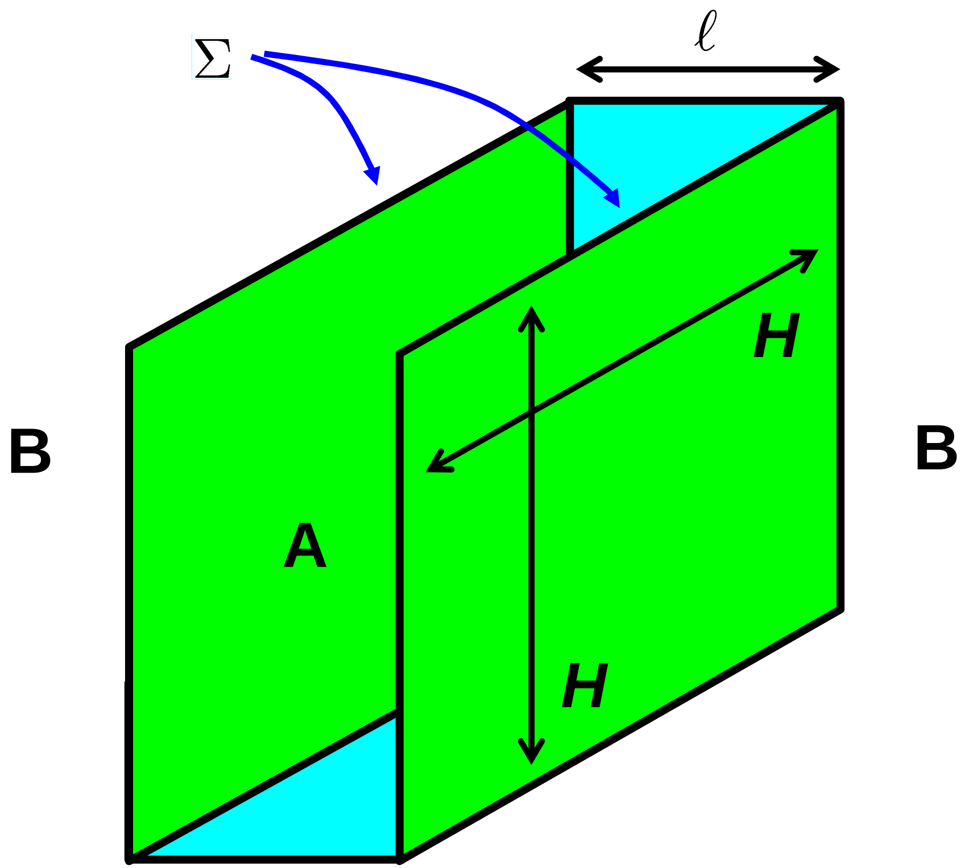
$$\begin{aligned} S_{EE} \;\; = \;\; & \frac{\mathcal{A}_\Sigma}{4G_d} + \kappa_1 \int_\Sigma d^{d-2}y \sqrt{\tilde{h}} \left[2R^{ij}\, \tilde{g}^{\perp}_{ij} - \frac{d}{d-1} R \underline{- K^i K_i} \right] \\ & + 4\kappa_2 \int_\Sigma d^{d-2}y \sqrt{\tilde{h}} \left[\tilde{h}^{ac}\tilde{h}^{bd} C_{abcd} \underline{- K^i_{\ ab} K_i^{\ ab} + \frac{1}{d-2} K^i K_i} \right] + \cdots \end{aligned}$$

- complete agreement with previous geometric calculation!!
- again, S_{EE} and S_{Wald} agree up to extrinsic curvature terms
- supports idea that new results calculate entanglement entropy

5) Randall-Sundrum 2 model

(with Pourhasan & Smolkin)

- consider entanglement entropy of "slab" geometry in flat space:



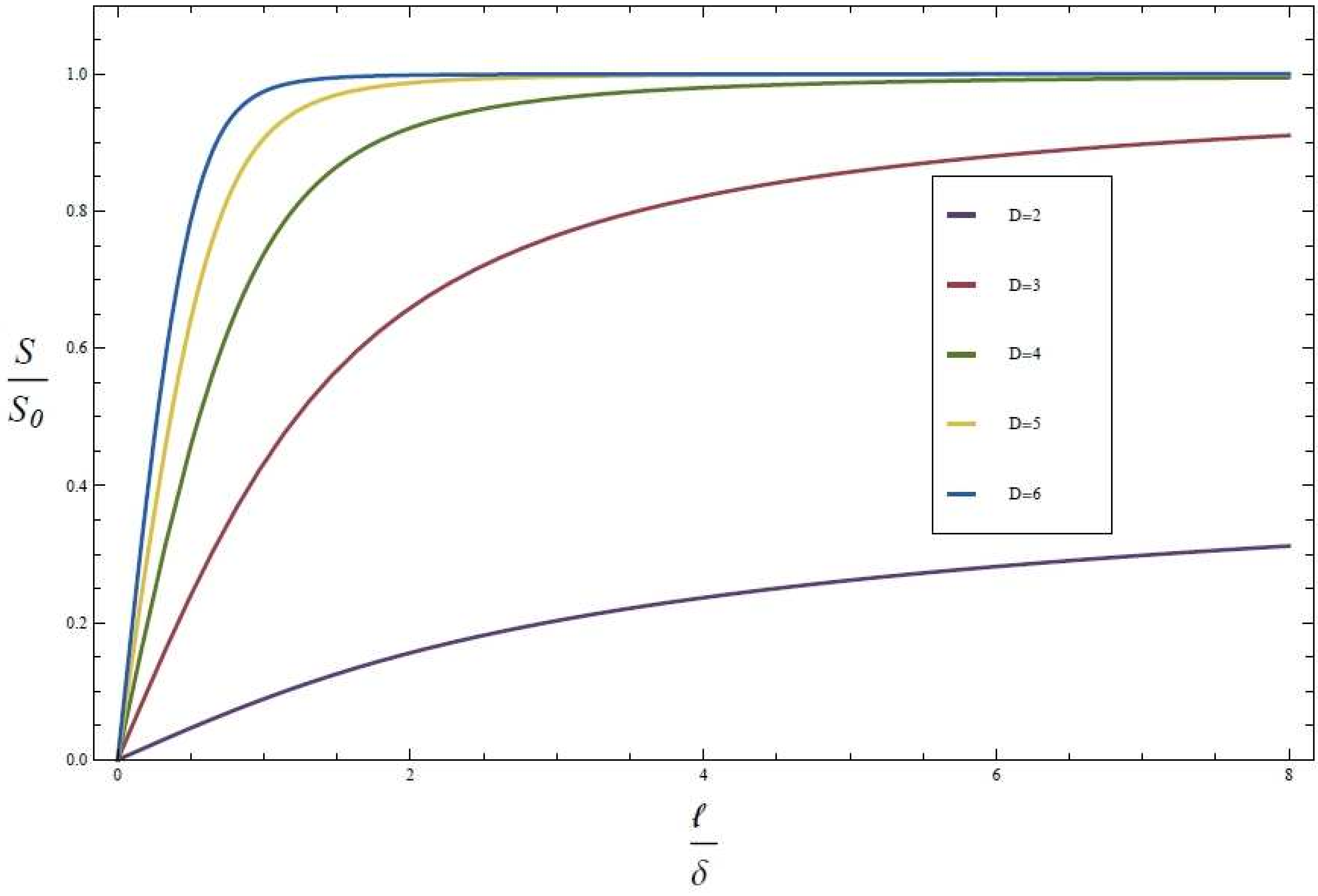
$$S(A) = \frac{4\pi\, H^{d-2}}{\ell_{P,bdry}^{d-2}} + \cdots$$

- test corrections
- define: $C(\ell) \equiv \ell\, S'(\ell)$
- Lorentz inv, unitarity & subadditivity: $C'(\ell) < 0$

(Casini & Huerta; Myers & Singh)

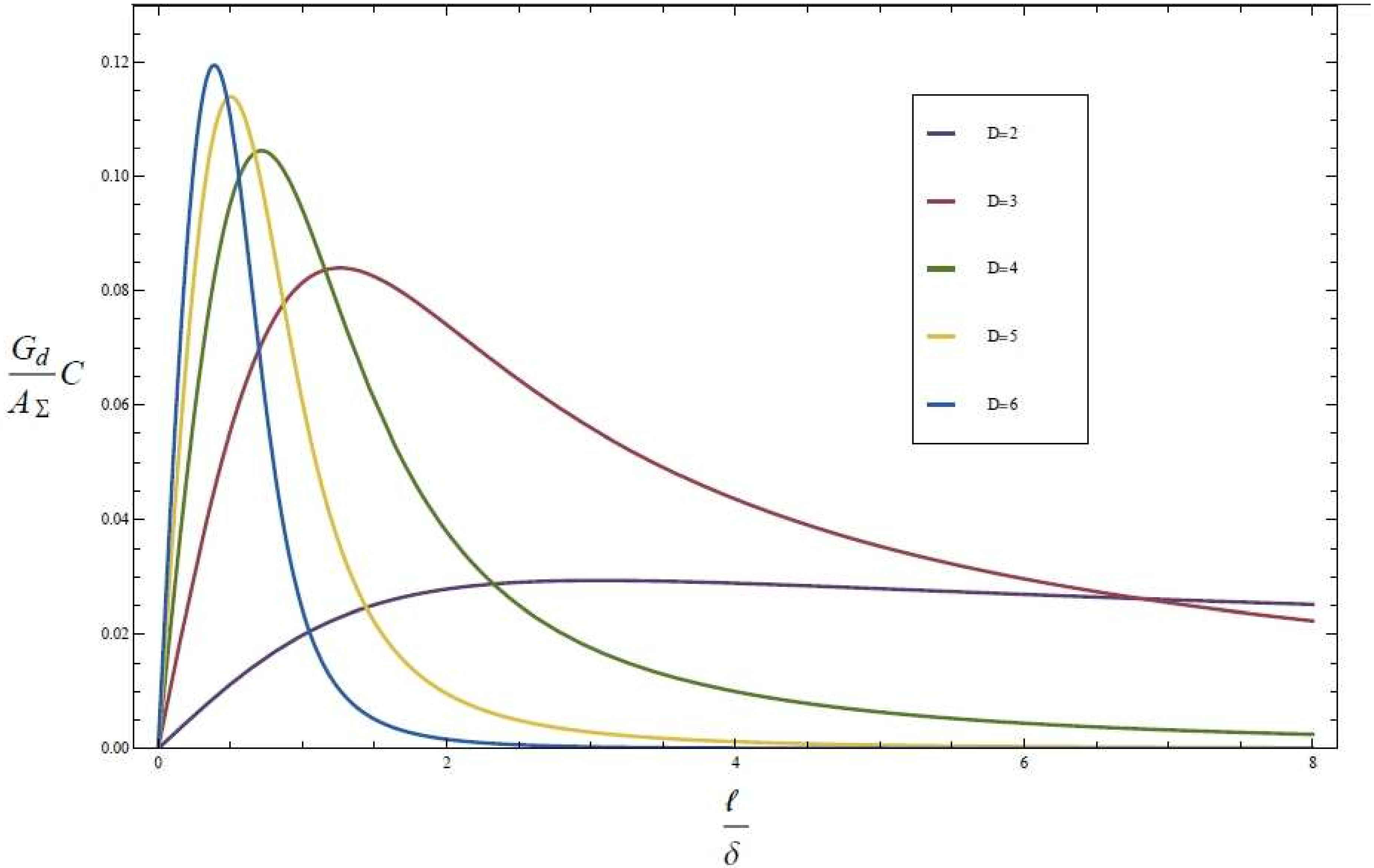
5) Randall-Sundrum 2 model

(with Pourhasan & Smolkin)



5) Randall-Sundrum 2 model

(with Pourhasan & Smolkin)



5) Randall-Sundrum 2 model

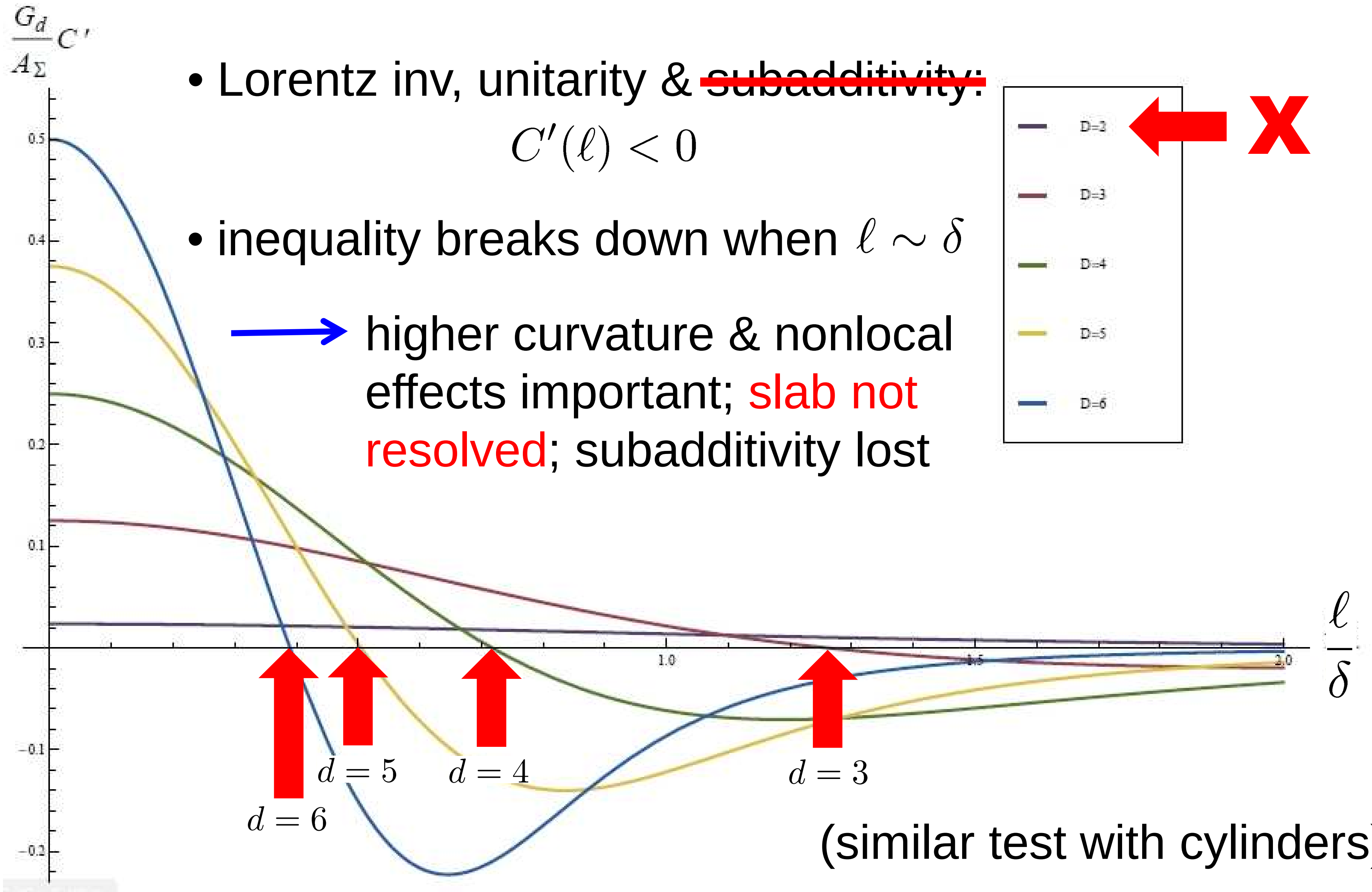
(with Pourhasan & Smolkin)

- Lorentz inv, unitarity & ~~subadditivity:~~

$$C'(\ell) < 0$$

- inequality breaks down when $\ell \sim \delta$

→ higher curvature & nonlocal effects important; slab not resolved; subadditivity lost



(similar test with cylinders)

Outline:

1. Introduction and Concluding remarks

2. Entanglement Entropy 1:
   - Basic Definitions
   - Holographic Entanglement Entropy

3. Entanglement Entropy 2: $S_{BH} \simeq S_{EE}$

4. Entanglement Entropy 3:
   Entanglement Hamiltonian and Generalizing S_{BH}

5. Randall-Sundrum calculations:
   Induced Gravity and Holography

6. Conclusions & Outlook

Conclusions:

- proposal: in quantum gravity, for any sufficiently large region in a smooth background, there is a finite entanglement entropy which takes the form:

$$S_{\rm EE} = \frac{\mathcal{A}_\Sigma}{4G_N} + \cdots$$

- five lines of evidence: 1) holographic entanglement entropy, 2) QFT renormalization of $1/G_0$, 3) induced gravity models, 4) Jacobson's arguments and 5) spin-foam models

FAQ: 1) Why should I care?

After all entanglement entropy is not a measurable quantity?

- **not yet!** it remains an interesting question to find physical processes are governed by entanglement entropy

→ eg, production of charged black holes in bkgd field;

(Garfinkle, Giddings & Strominger)

Renyi entropy & tunneling between spin chain states

(Abanin & Demler)

- compare to quantum many body physics: (Popescu, Short & Winter)

generic states do **not** satisfy "area law" but low energy states do

→ locality of the underlying Hamiltonian restricts the entanglement of the microscopic constituents

→ tensor network program

Lesson(s) for quantum gravity?

FAQ: 2) Why should I care?

"Smooth curvatures are a signature of macroscopic spacetime"
seems a simpler/better/more intuitive slogan.

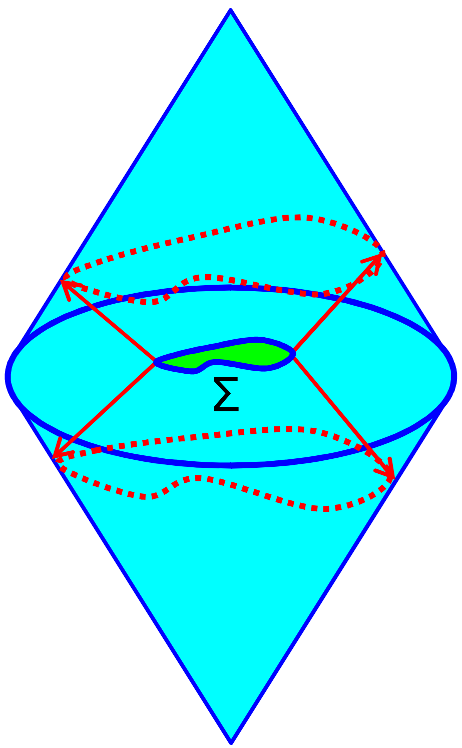
- this proposal relates (semi)classical geometry directly to a property of the underlying quantum description

FAQ: 3) Why should I care?

The area of a finite region can not possibly be an observable in quantum gravity!

• question of observables in QG has a long history; do not have a solution here but suggestion towards a construction

→ in spacetime with boundary, use light sheets to connect entangling surface to boundary and define Σ with corresponding boundary data



FAQ: **4)** Is this the same thing as L&M's "Generalized Gravitational Entropy"?

- not at present; recall the important role of boundary data and extremal surfaces in the "GGE" discussion

- seems like it should be related but must reformulate the "boundary" data in terms of the chosen entangling surface (see Appendix C in L&M)

FAQ: **5)** Is there a relation between this proposal and M&S's *"ER = EPR"* idea?

- sure thing, definitely not and maybe????

- note that $S_{EE}=S_{BH}$ refers to short-range entanglement of "QG degrees of freedom" ("glue holding the spacetime together")

- in contrast, ER=EPR seems primarily to refer to long-range entanglement of widely separated "ordinary quanta"

- seems more like "virtual qubits" of Verlinde^2 ??

FAQ: **6)** Is there a relation between this proposal and Yasha Neiman's "imaginary part of the gravity action"?

- probably; go read: **arXiv:1301.7041**, **arXiv:1303.4752**, **arXiv:1305.2207**

Conclusions:

- proposal: in quantum gravity, for any sufficiently large region in a smooth background, there is a finite entanglement entropy which takes the form:

$$S_{\mathrm{EE}} = \frac{\mathcal{A}_\Sigma}{4G_N} + \cdots$$

- future directions:
  - ► find interesting string framework to calculate
  - ► find connection to “generalized gravitational entropy”
  - ► better understand higher curvature corrections
  - ► does entropy have an operational meaning?
  - ► further develop spin-foam calculations
  - ► avoid being scrambled by any firewalls!