

# Non-spherical collapse & Early Thermalization,

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@ KIAS-YITP, 2013.07.05

[1302.1277](#) with Eunseok Oh,

[hep-th/0511199](#) with Shuryak, Zahed

# Announced title was Thermal Mass and Plasmino

Based on

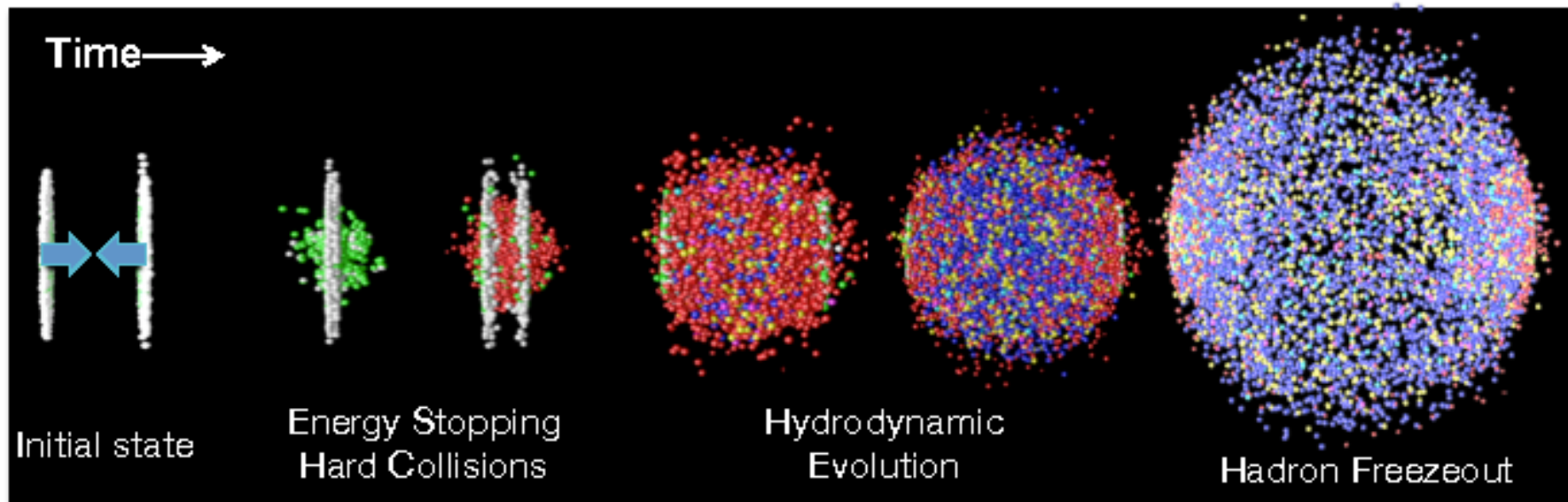
1205.3377; 1305.1446

with Yunseok Seo + Yang Zhou

I will talk about this as second topic.

# Early thermalization.

- Is one of the puzzles in RHIC : Heavy ion collision fireball equilibrate just in passing time  $t \sim 1 \text{ fm}/c$



This is one of the evidence that  
QGP in RHIC exp. is sQGP!

Other evidences for sQGP:

small  $\eta/s$ , elliptic collective flow,  
Jet quenching

Why strong? Asymptotic freedom + Rapid  
particle creation.

Initial  $E=200\text{GeV}/\text{particle}$ ,  $T=200\text{MeV}$ .



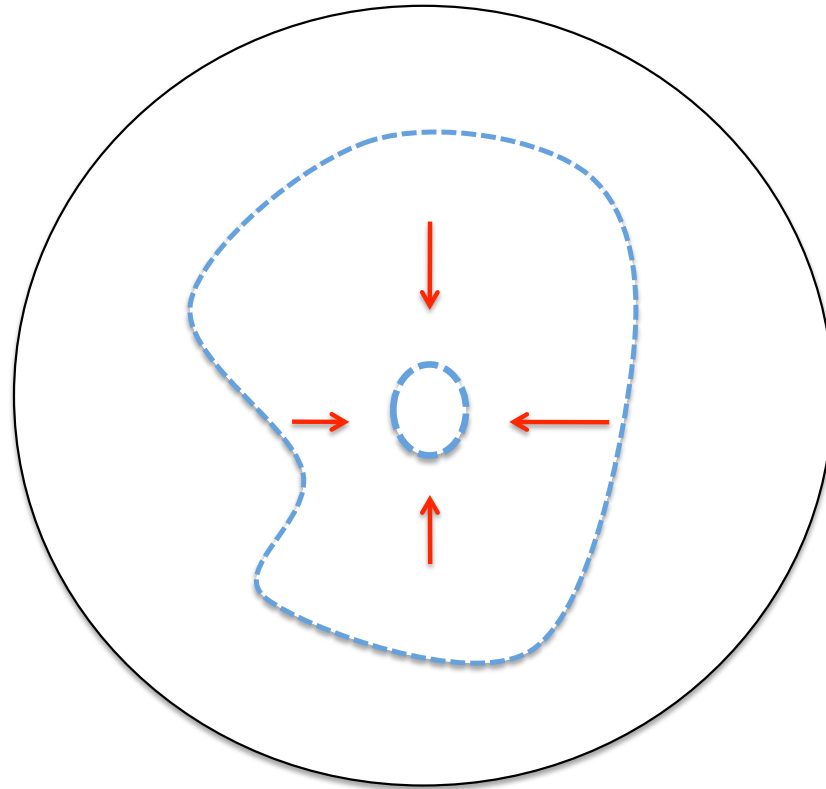
# In gravity dual,

- Thermalized state = BH, so  
Early thermalization means, Easy BH formation.  
So Our question will be:  
Can a generic collapse in AdS give a BH  
in ONE dynamical time?
- many works are done!  
Aharony, Bucher, Chesler,...,Gubser, Minwalla,  
.... Yaffe, Zayas  
My work with Shuryak and Zahed ('05) [hep-th/0511199](https://arxiv.org/abs/hep-th/0511199)

## Claim and conclusion

- Any shape of dust particle distribution will collapse into Black hole in a time less than the one Falling time.
- This is the mechanism of Early Thermalization. (review of 0511199)

# In AdS space



# In flat space,

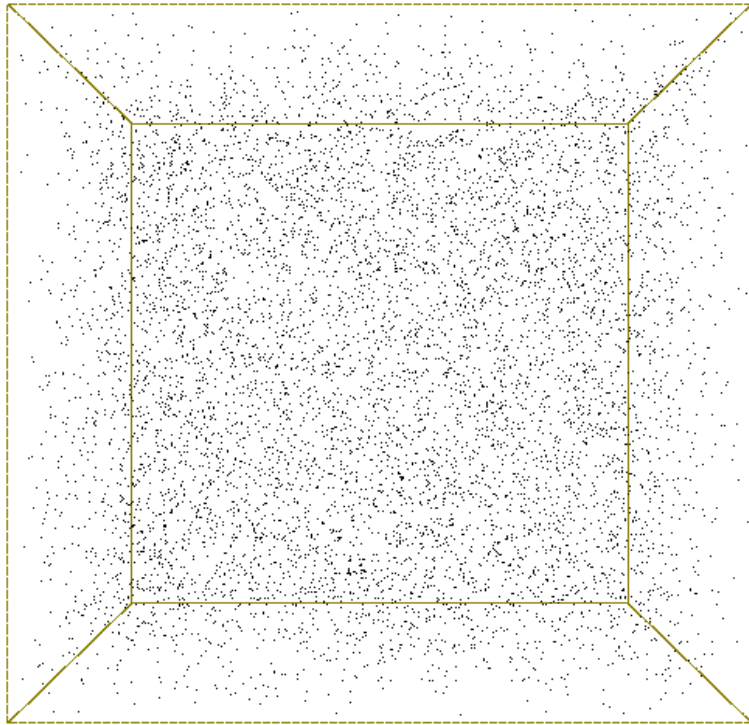


Figure 1: The system of particles after initialization.

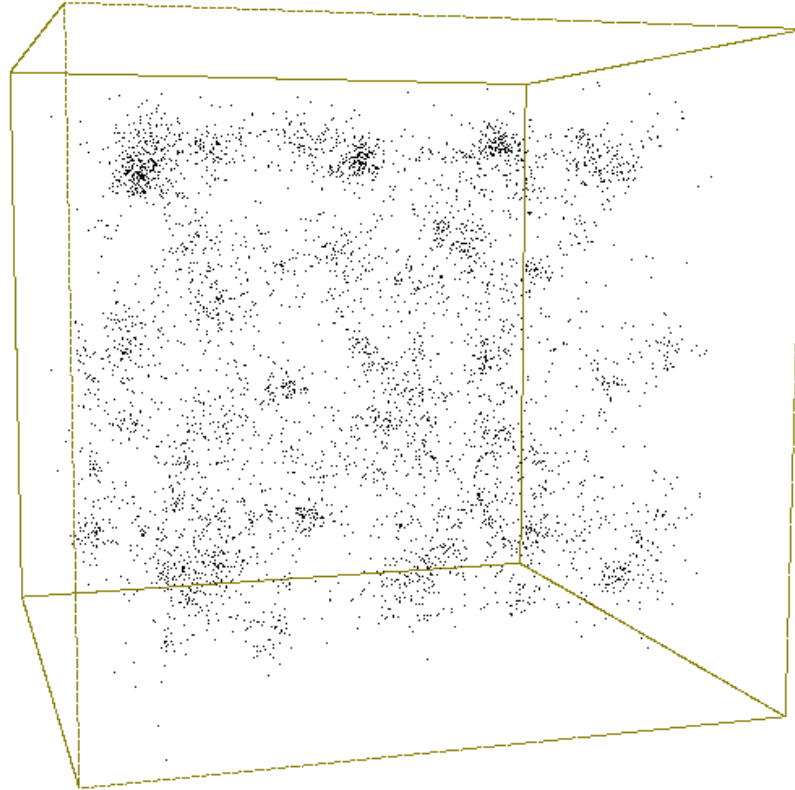


Figure 2: The system of particles after the simulation has run for 2.5 days nonstop.

# Then Why in ads? 1302.1277

- Consider a free fall of a massive particle in ads.

$$ds^2 = -(1 + r^2/R^2)c^2 dt^2 + r^2 d\Omega^2 + \frac{dr^2}{1 + r^2/R^2}$$

$$S = -m \int \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} dt, \text{ with } \dot{x} = \frac{dx}{dt}.$$

$$\dot{r}^2 + (m/E)^2 (1 + r^2)^2 (1 + r^2 - E^2/m^2) = 0$$

The equation of motion turns out to be integrable.

With change of variable

$$r = \frac{v}{\sqrt{1 - v^2}}$$

The equation of motion become SHO:

$$\dot{v}^2 + v^2 = v_c^2 := 1 - (m/E)^2$$

# Solution

$$r = \frac{v_c \cos t}{\sqrt{1 - v_c^2 \cos^2 t}}. \quad E = \frac{m}{\sqrt{1 - v_c^2}}$$

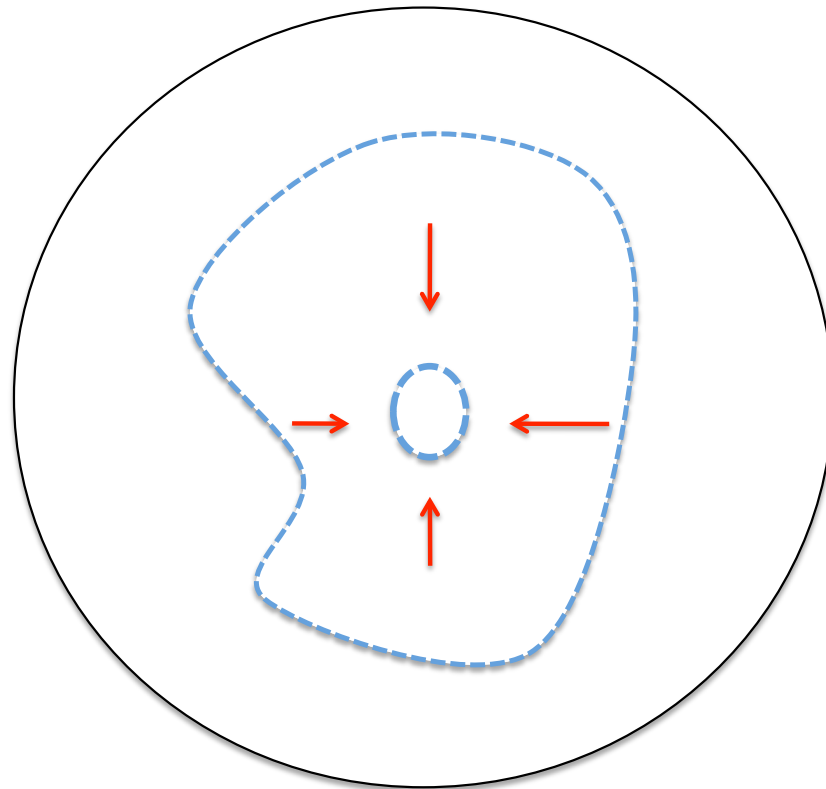
$r_0$  is the initial radial position.  $v_c$  is the velocity at the center.

Falling time is the same **independent of the initial condition** of particle.

$$T_{fall} = \frac{\pi R}{2c}.$$

# This means what I claim.

Generic collapse will give a black hole, provided that they start from static configuration.





# Status on BH making in AdS see Ishibashi's talk.

1 scalar field collapse → turbulent instability

If one can wait long, OK

But if you need to form BH in one dynamical time,

→ even spherical collapse is not easy, not to mentioning the generic shell.

Is this a contradiction to what I said?

No. I used dust.

Wave is hard to localize! This in particle QM is **uncertainly principle**.

- **2. Spherical initial condition** → even in the case one get BH in one falling (remember BIG shell), such fine tuned Initial Condition is not of no use for **RHIC exp.**

## II. Is this the Mechanism of early thermalization?

- Q1. What/Why dust ?
- Q2. Why fall?
- Q3. Why non-interacting dust?
- Q4. Poincare patch?
- Q5. What is the initial velocity effect?
- Q6. What happen if we add interaction?
- Q7. Any prediction?  
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- Q8. Where is entropy generation?
- Q9. back reaction of gravity?
- Q10. Is this mechanism universal?
- Q11. What is wrong with Shock wave?  
.....etc.....

# Q1. What/Why dust?

- The holographic image of the created particles is the dust in the bulk.

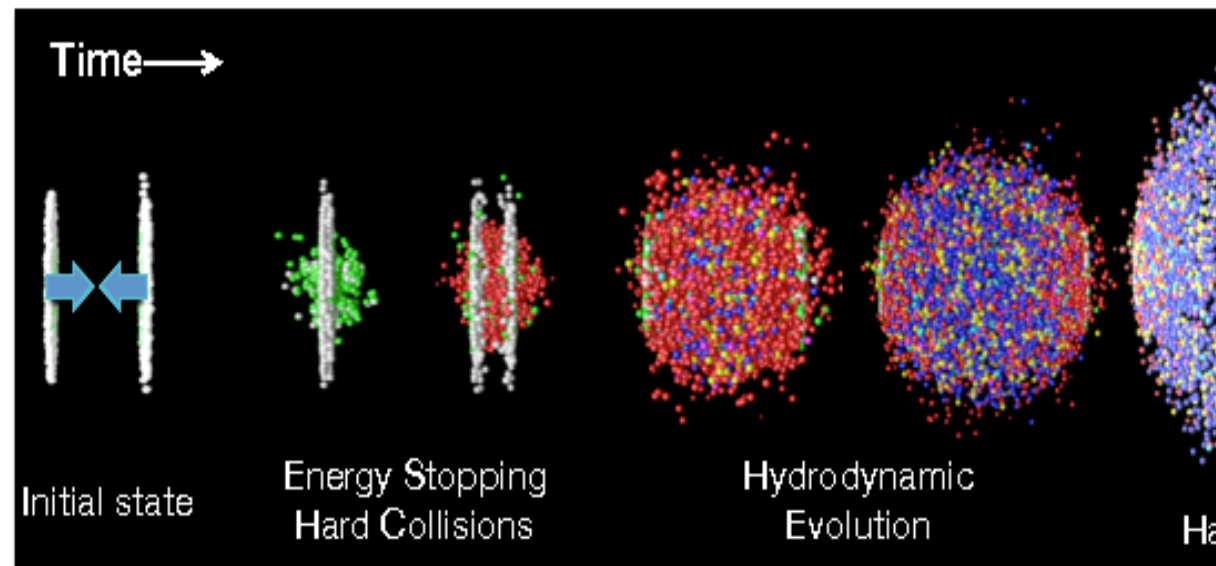
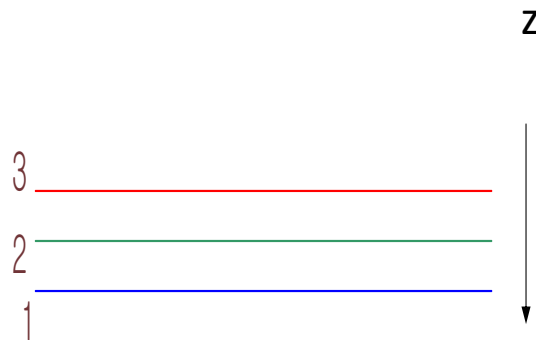
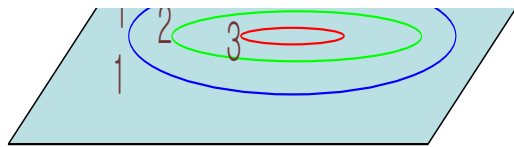
Since they are NOT in a coherent state or Bose condensation state, it is better not to be described as a non-trivial field configuration of scalar or any tensor.

- More proper discussion should involve Particle creation mechanism in string theory. Open string scattering can pinch off closed string.
- But we start from the creation of 5000 particles.

## Q2. Why Fall ?

### 1. It equilibrates after some Expansion

- According to UV/IR relation,  
Expansion @bdry is dual to the Falling in bulk.



# Why Fall? continued

2. Because holographic image of a created particle is at  $r > 0$ .

Polchinski+Strassler ([hep-th/0109174](https://arxiv.org/abs/hep-th/0109174))

Maximum contribution to scattering amplitude is from a definite height. )

$$r_{scat} \sim R^2 p.$$

Shuryak, SS, Zahed ([hep-th/0511199](https://arxiv.org/abs/hep-th/0511199) )

Look at Maximum probability to find created particle of given 4 momentum to get similar result.

$$P(z) = \sqrt{g} |\phi(z)|^2. \quad \phi(z) = z^2 I_\nu(kz) \quad \text{for } k^2 > 0.$$

## Q3. Why non-interacting dust?

- All the gluon mediated interactions are transformed away as background gravity  
→ dusts are free particle in a ads background.
- But non-gluonic interactions should be remained and will be discussed later.

## Q4. Poincare Patch?

- Motion not periodic. Falling time is infinite.

$$r = \frac{\epsilon R}{\sqrt{(\epsilon t/R)^2 + 1}} = R^2/t - (R/t)^3/2\epsilon^2 + \mathcal{O}(t^{-5}),$$

- Initial condition dependence rapidly disappear.  $\rightarrow$  Synchronization effect is still there.
- Formation of trapped surface in “finite time” seems “obvious”.

## Q5. Effect of initial velocity

- Holographic Image at its creation moment does not have radial velocity.
- Work in Poincare patch.

$$\frac{mr^2}{\sqrt{r^2(1 - \dot{x}^2) - \dot{r}^2/r^2}} = E, \quad \frac{mr^2\dot{x}}{\sqrt{r^2(1 - \dot{x}^2) - \dot{r}^2/r^2}} = p, \quad V = p/E,$$
$$r = \frac{\epsilon(1 - V^2)}{\sqrt{1 + (\epsilon(1 - V^2)t/2)^2}}.$$

Remarkably the large time behavior of the radial position is independent of

all of the initial conditions  $m, E, p$ .



# Q6. Non-gluonoc Interaction effect

- Newtonian potential in AdS.

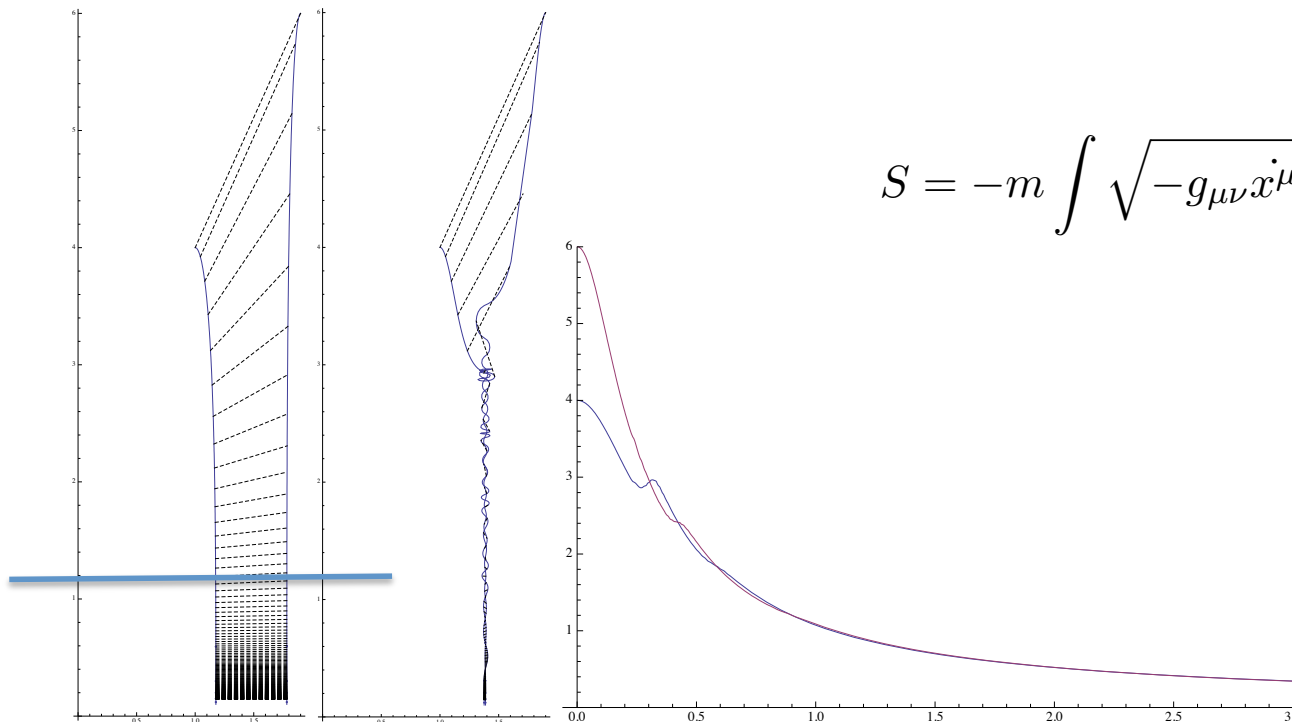
$$\begin{aligned} V(\{x_i, y_i\}) &= \int \int d^5x d^5y J(x) G(x, y) J(y), \\ &= G_N \sum_{i < j} \int dt \frac{(x_{i0} x_{j0})^{2\Delta}}{(|x_i(t) - x_j(t)|^2 + |x_{i0}(t) - x_{j0}(t)|^2)^{\Delta-1/2}} \end{aligned}$$

from  $G \sim \left( \frac{1}{u(2+u)} \right)^\Delta$  with  $u = \frac{(x-y)^M (x-y)_M}{2x_0 y_0}$

$$ds^2 = \frac{1}{x_0^2} (dx_0^2 + dx^\mu dx_\mu), \text{ with } x_0 = 1/r$$

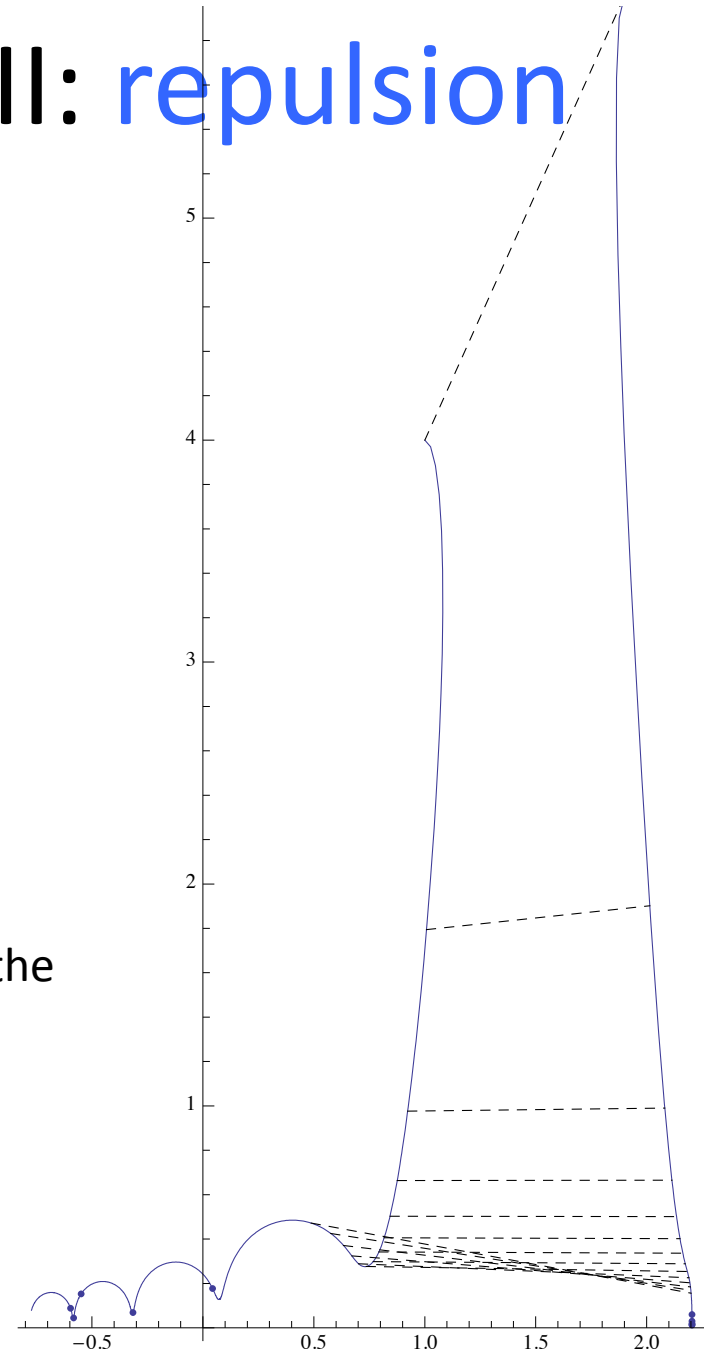
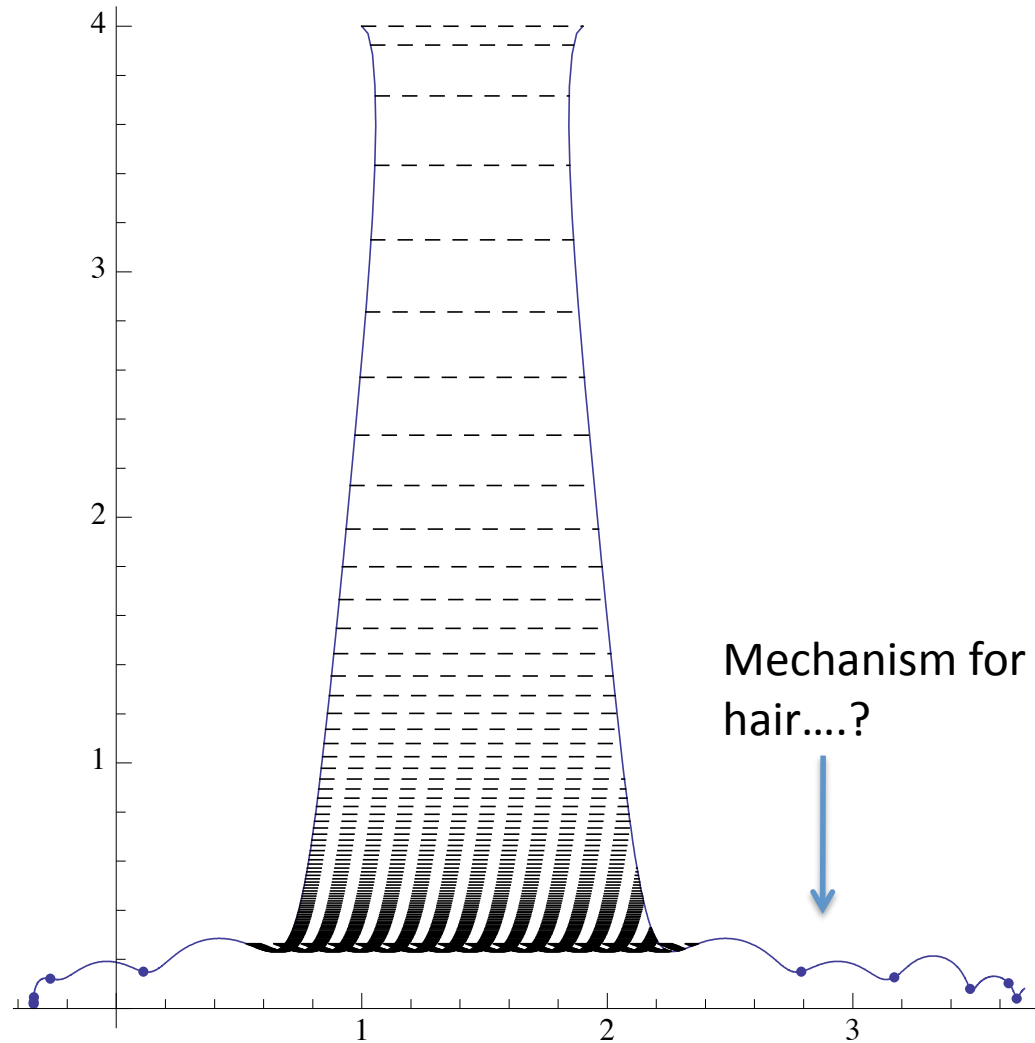
# Q6. Interaction (non-gluonic) effect I:

- Interaction is not so important if **attractive**.



$$S = -m \int \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} dt, +V(\{x_i, y_i\})$$

# Interaction effect II: repulsion



# Q7. Prediction

- Thermalization time < One Dynamical time,

$$T_{fall} = \frac{\pi R}{2c}$$

- Thermalization: Soft first and Hard later.

(for pure gluon int. in the large N.)

higher energy particles arrive at the apparent horizon later.

$$t_{Thermalization} = \sqrt{1/r_H^2 - 1/r_o^2}$$

Future projects.

- Q8. Where is entropy generation?
- Q9. back reaction of gravity?
- Q10. Is this mechanism universal for other background?
- Q11. What is wrong with Shock wave?  
.....etc.....

# Topic II: Thermal Mass and Plasmino for strongly interacting Fermions

With  
Yunseok Seo, Yang Zhou<sup>a,b</sup>

- Massless particle in finite temperature gets mass  $\sim gT$  : Thermal mass.
- There are Three scales in weak coupling:  
 $T \gg gT \gg g^2 T$
- Q: What will happen to strong coupling?
- Ans: No thermal mass in strong coupling.  
[See also Maldacena's recent work]

## Fermion In hot Medium: $T \gg m$ ,

$$S(\omega, \mathbf{p}) = \frac{1}{\omega \gamma_0 - \mathbf{p} \boldsymbol{\gamma} - \Sigma(\omega, \mathbf{p})}$$

In Hard Thermal loop approximation

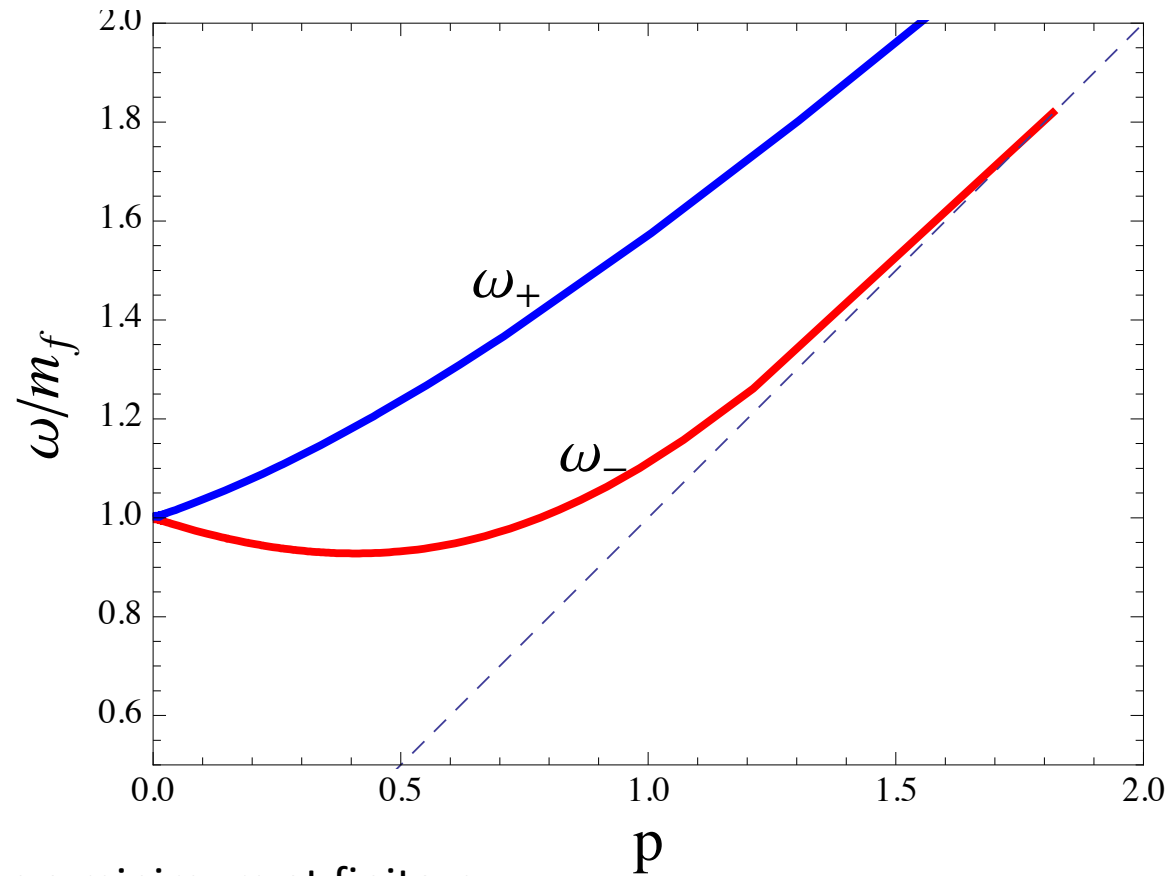
1. Thermal mass( Klimov '82, Weldon '83)

$$m_T = \frac{gT}{\sqrt{6}} \quad m_f^2 = \frac{1}{8} g^2 C_F \left( T^2 + \frac{\mu^2}{\pi^2} \right)$$

2. Plasmino : New collective mode ([Braaten, Pisarski '89](#) )



# Plasmino



- The plasmino mode has a minimum at finite  $p$ .

$$p \ll m_f : \quad \omega_{\pm}(p) \simeq m_f \pm \frac{1}{3}p + \frac{1}{3m_f}p^2 + \dots ,$$

$$p \gg m_f : \quad \omega_{\pm}(p) \simeq p .$$

# Importance

- Van Hove singularity.  
Density of state is enhanced in low dim.

$$\begin{aligned}\rho(\omega) &= \sum_n \int \frac{d^3k}{(2\pi)^3} \delta(\omega - \omega_n(k)) \\ &= \sum_n \int \frac{dS}{(2\pi)^3} \frac{1}{|\nabla_k \omega_n(k)|}\end{aligned}$$

New mechanism of SC.....

Enhanced dilepton production .....(Thoma ph/0008218)

# However

- Resummation needs ladder approximation, not justified for strong coupling.

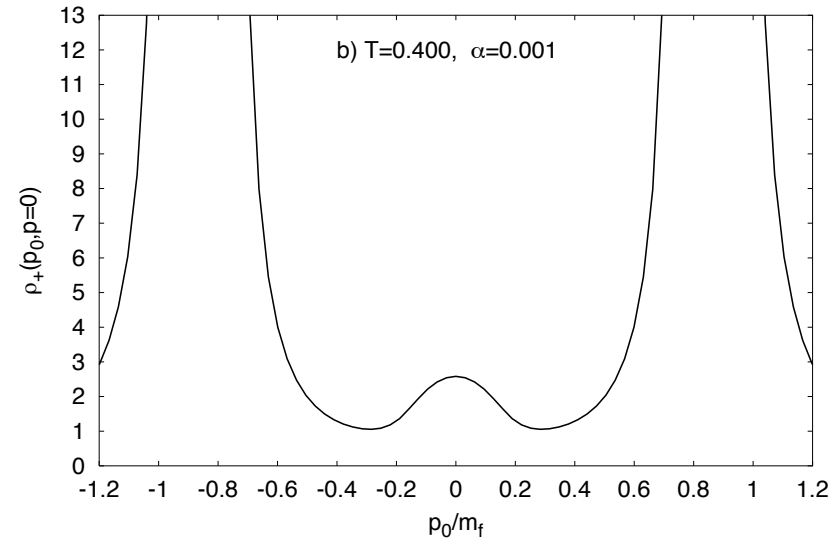
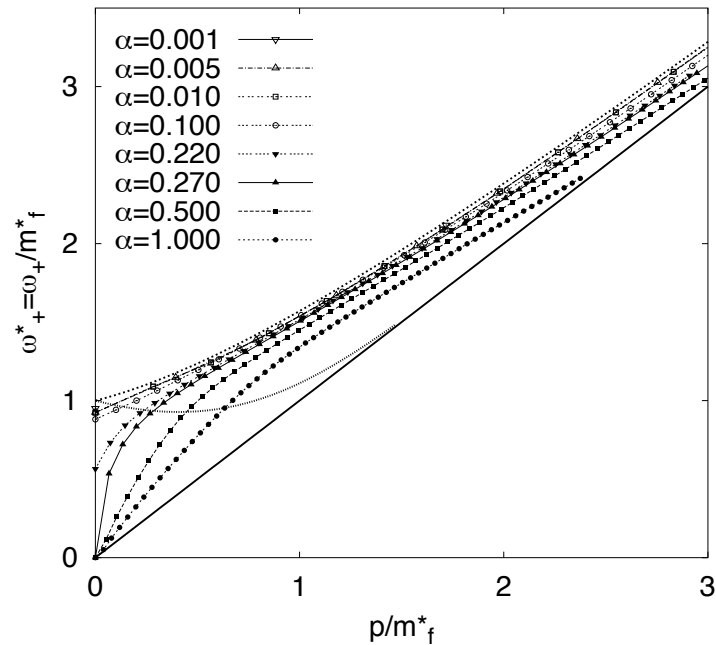
It is not clear whether plasmino continues to exist in the strong coupling limit.

& .....

# Evidence

An interesting Numerical study suggested  $m_T=0$ .

ArXiv: 1111.0117 , Nakkagawa et.al.



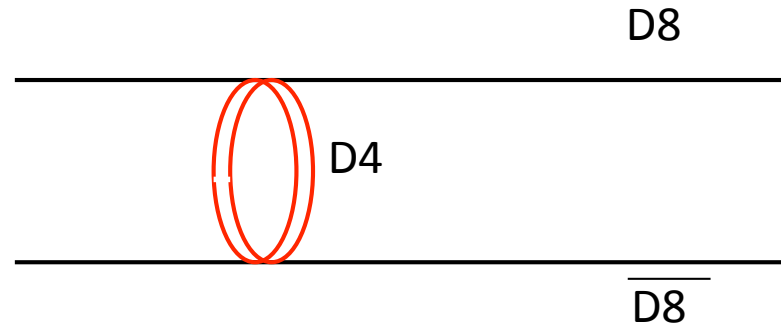
However, this work is also based on SD idea.

# Set up

- Use D4/D8/D8bar : SS model :  
Confinement(cf): by solitonic bg.  
Deconfinement (dcf): Black hole bg.
- Chiral Symmetry breaking: Joined D8/D8bar
- Density/chemical potential: U(1) gauge field  
(sourced by the strings emanating from horizon  
of the BH or compact D4 (baryon vertex).)

# The D4-D8-D8 System

Sakai, Sugimoto



0 1 2 3 4 5 6 7 8 9

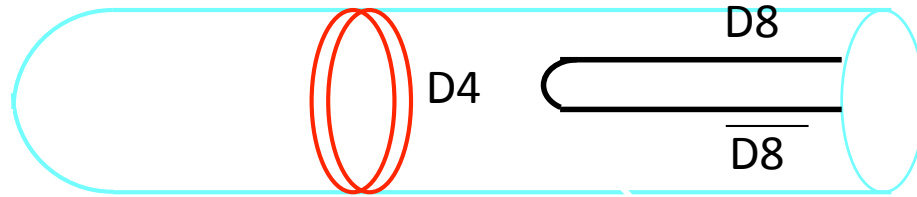
D4 x x x x x

D8 x x x x x x x x x x

# The D4-D8-D8 System

Sakai, Sugimoto;

Aharony, Sonnenschein, Yankielowicz

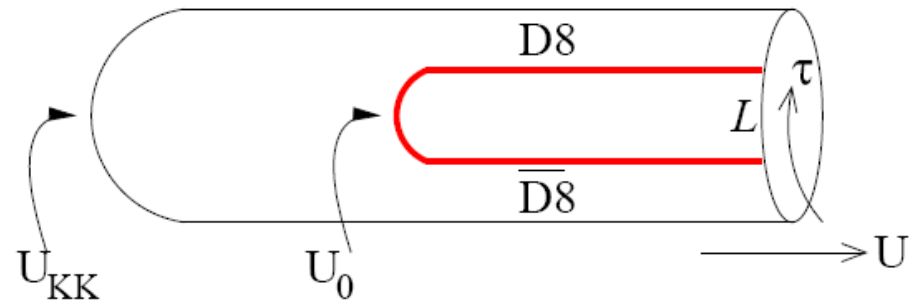


0 1 2 3 4 5 6 7 8 9

D4 x x x x x

D8 x x x x x x x x x x

# D4 brane geometry



$$ds^2 = \left(\frac{U}{R}\right)^{3/2} (\eta_{\mu\nu} dx^\mu dx^\nu - f(U) d\tau^2) - \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$f(U) = 1 - \frac{U_{KK}^3}{U^3} \quad R^3 = \pi g_s N l_s^3$$

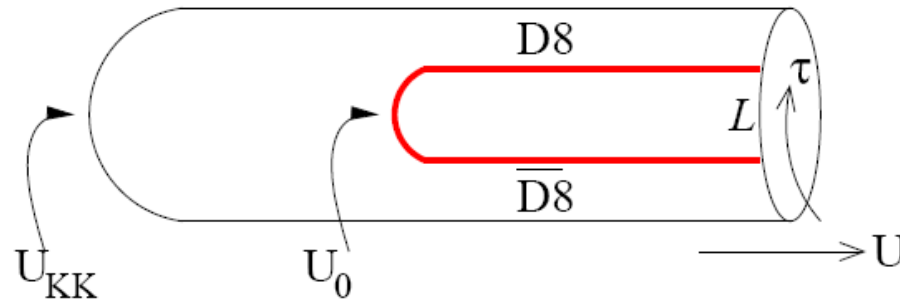
$\tau$  period:  $4\pi R^{3/2} / (3U_{KK}^{1/2})$



# Probe brane limit

$$N_f \ll N_c$$

Karch, Katz



D8-brane action

$$S_{DBI} = -T \int d^9 x e^{-\phi} \sqrt{\det g_{MN}} \quad e^{\phi} = g_s \left( \frac{U}{R} \right)^{3/4}$$

Stationary Solution:

$$f(U) + \left( \frac{R}{U} \right)^3 \frac{U'(\tau)^2}{f(U)} = \frac{U^8 f(U)^2}{U_0^8 f(U_0)}$$

$$ds^2 = \left( \frac{U}{R} \right)^{3/2} \eta_{\mu\nu} dx^\mu dx^\nu - \left( \frac{R}{U} \right)^{3/2} U^2 d\Omega_4^2 - \left( \frac{R}{U} \right)^{3/2} \left[ \frac{1}{f(U)} + \left( \frac{U}{R} \right)^3 \frac{f(U)}{U'(\tau)^2} \right] dU^2$$

# Fermion on D8

- Fermion = mode of D4-D8 string  
= bi-fundamental field  $\psi_i^a$
- When D4 is replaced by a gravity, color index is interpreted as “averaged over”  
so that D8 fermions are color averaged quarks.
- Here only 1 flavor.
- Remark: NOT a “bulk” fermion, No ads/cft.

# Fermion action and eq. of M

- Ignore S4: D8 becomes effectively 5d with one dimension compactified.  $\rightarrow$  3+1 d theory.

$$S = \int d^5x \sqrt{-g} i (\bar{\psi} \Gamma^M D_M \psi - m_5 \bar{\psi} \psi) ,$$

$$D_M = \partial_M + \frac{1}{4} \omega_{abM} \Gamma^{ab} - iq A_M .$$

$$\psi = (-g g^{rr})^{-1/4} e^{-i\omega t + ik_i x^i} \Psi$$

$$\sqrt{g_{ii}/g_{rr}} (\Gamma^r \partial_r - m_5 \sqrt{g_{rr}}) \Psi + i K_\mu \Gamma^\mu \Psi = 0 ,$$

$$K_\mu = (-v(r), k_i) \text{ and } v(r) = \sqrt{-g_{ii}/g_{tt}} (\omega + qa_0) .$$

# Def. of Green function

$$\Psi = (\Phi_1, \Phi_2) = (y_1, z_1, y_2, z_2)^T$$

$$G_1(r) := y_1(r)/z_1(r) \text{ and } G_2(r) := y_2(r)/z_2(r)$$

Then,

$$\begin{aligned} & \sqrt{\frac{g_{ii}}{g_{rr}}} \partial_r G_\alpha + 2m_5 \sqrt{g_{ii}} G_\alpha \\ & = (-1)^\alpha k + v(r) + ((-1)^{\alpha-1} k + v(r)) G_\alpha^2 . \end{aligned}$$

Finally,

$$G_{1,2}^R = \lim_{\epsilon \rightarrow 0} e^{-\frac{1}{2} m_5 R r^{1/4}} G_{1,2}(r)|_{r=1/\epsilon}$$

For retarded green fct, we need Boundary condition:

# IR Boundary condition:

## 1. deconfining case

- BC ← horizon regularity
- Retarded Green function:

$$G_{1,2}(r_0) = i ,$$

$$v(r) = (\omega + qa_0(r)) / \sqrt{f}$$
$$a_0(r) = \mu + \int_{\infty}^r d\hat{r} \left( \frac{D'^2}{\hat{r}^5 + D'^2} \right)^{1/2}$$

$$\mu = \frac{m}{q} + \int_{r_0}^{\infty} a'_0(r) dr.$$

# IR Boundary condition:

## 2. confining case

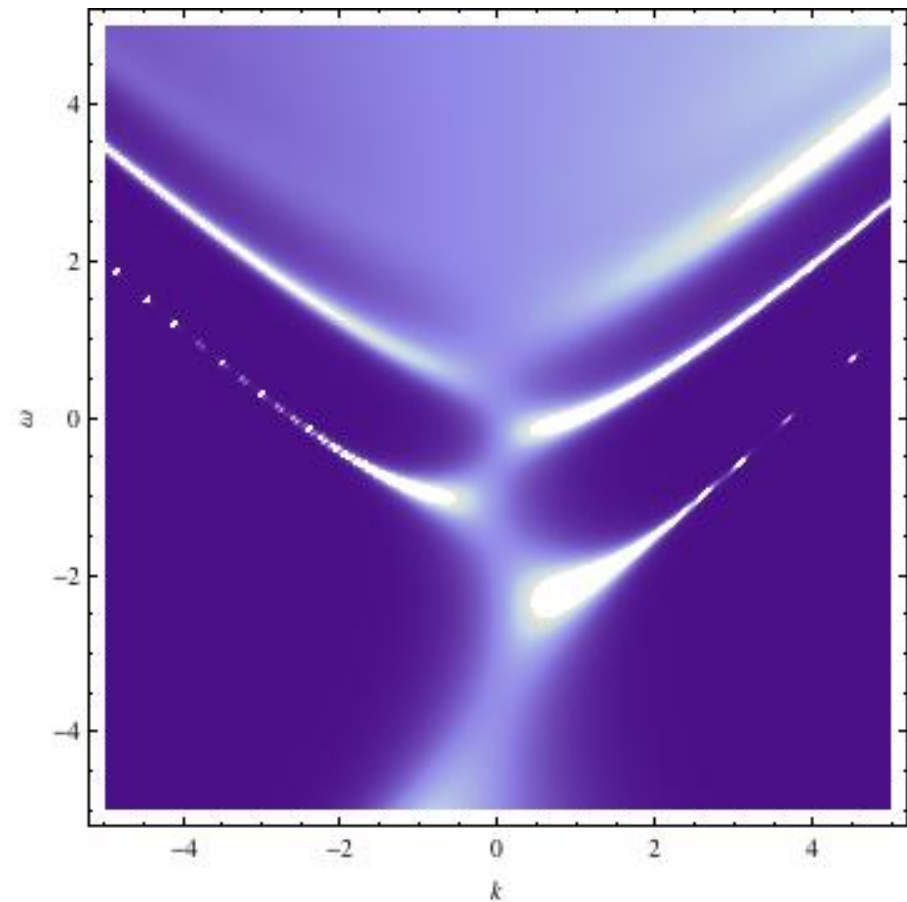
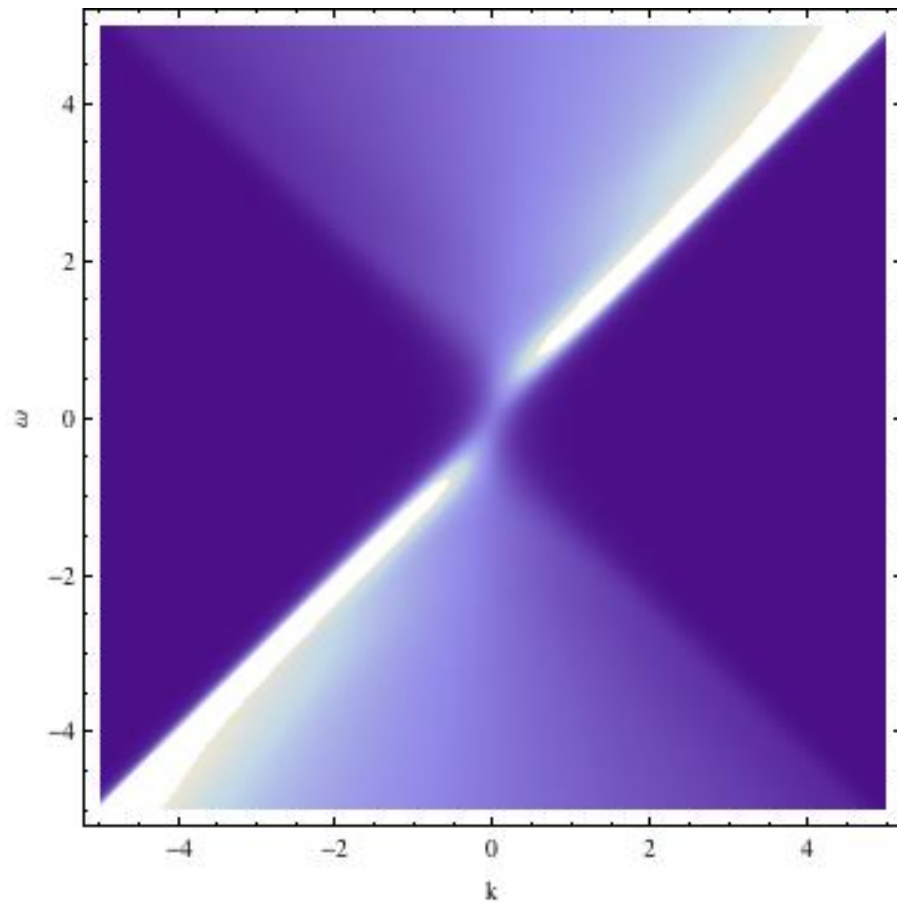
- For retarded(advanced) green fct

$$G_\alpha(r_0) = \frac{-mR + \sqrt{m^2 R^2 + k^2 - \hat{\omega}^2}}{(-1)^\alpha k - \hat{\omega}},$$

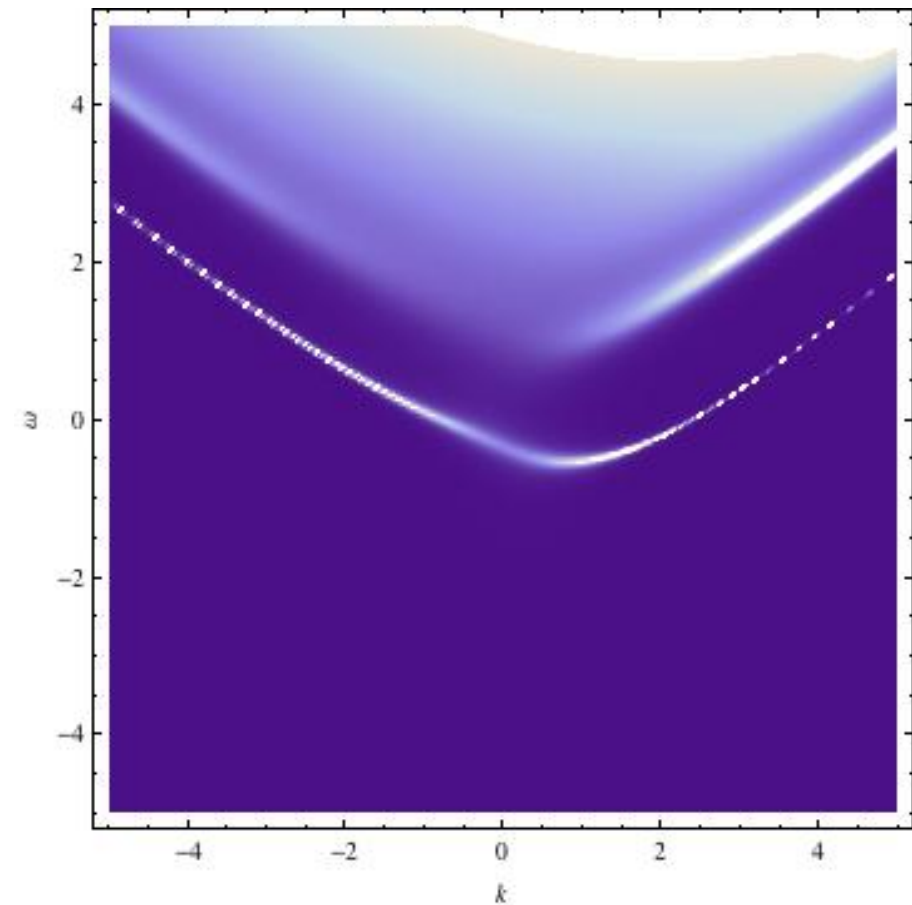
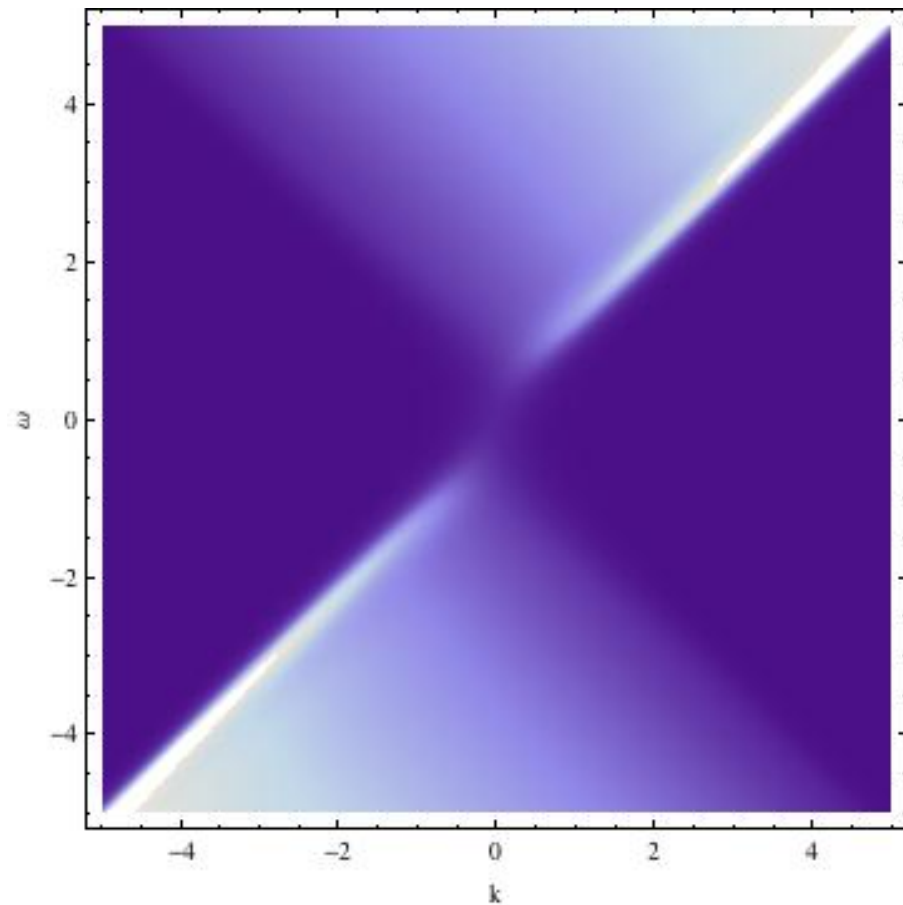
where  $\hat{\omega} = \omega + m_*$  and  $m := m_5 r_0^{3/4}$

$$\omega \rightarrow \omega + i\epsilon \quad (\omega \rightarrow \omega - i\epsilon).$$

Deconfining case, massless fermion.  
Vanishing thermal mass: 0 or non-0 density



# Deconfining case, massive fermion. 0 / non-0 density



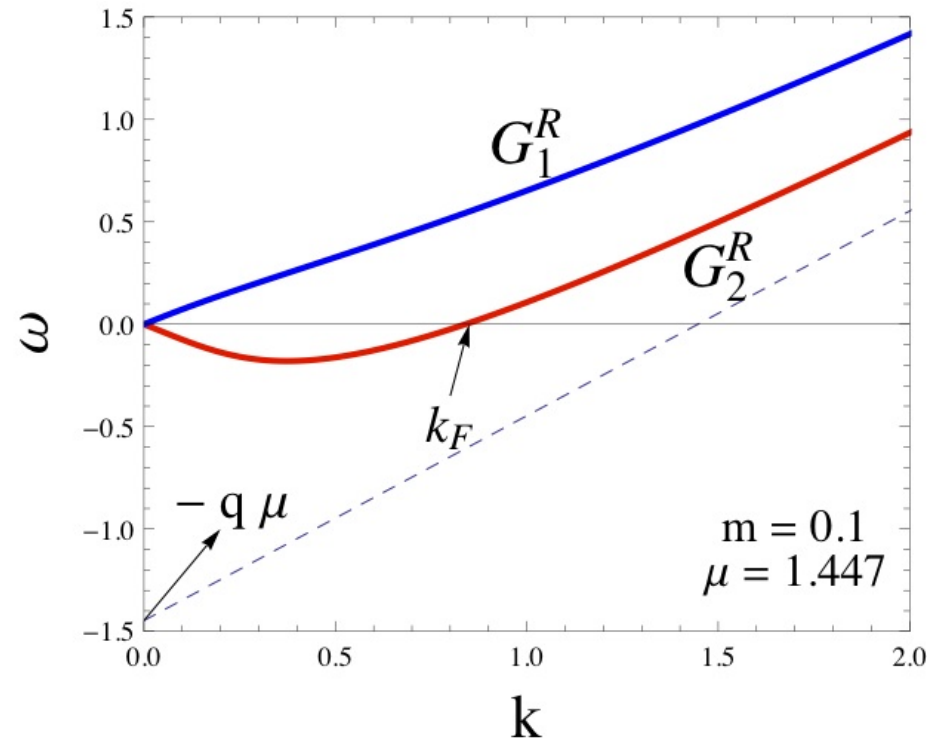


# Confining case

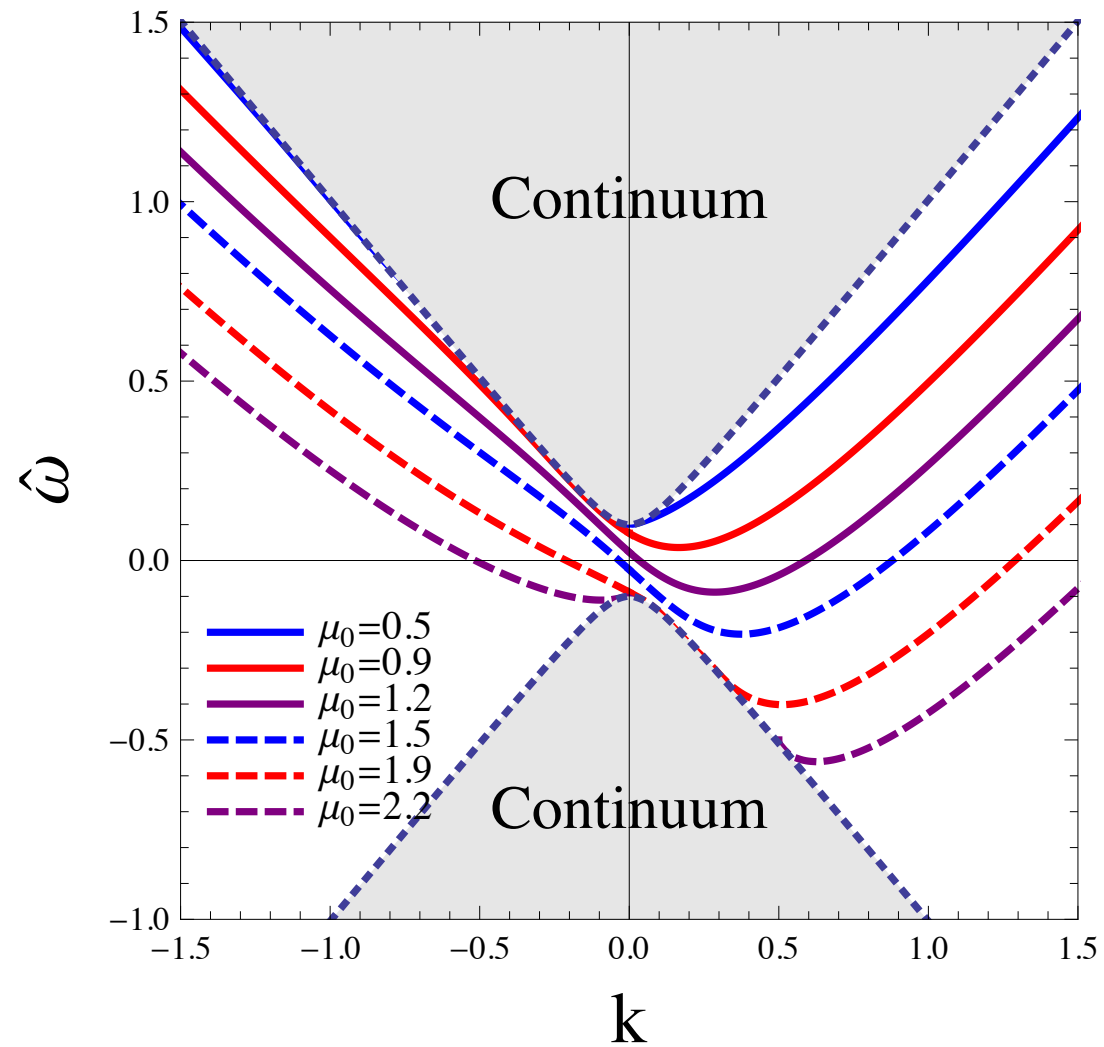
There is Plasmino only for large but not too large chemical potential.

$$\mu_1 \leq \mu \leq \mu_2.$$

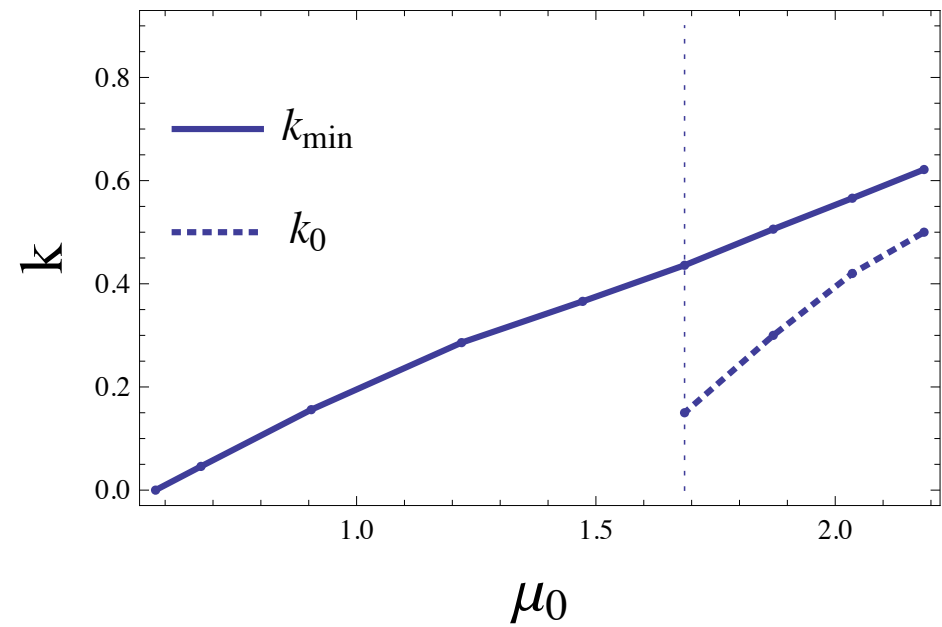
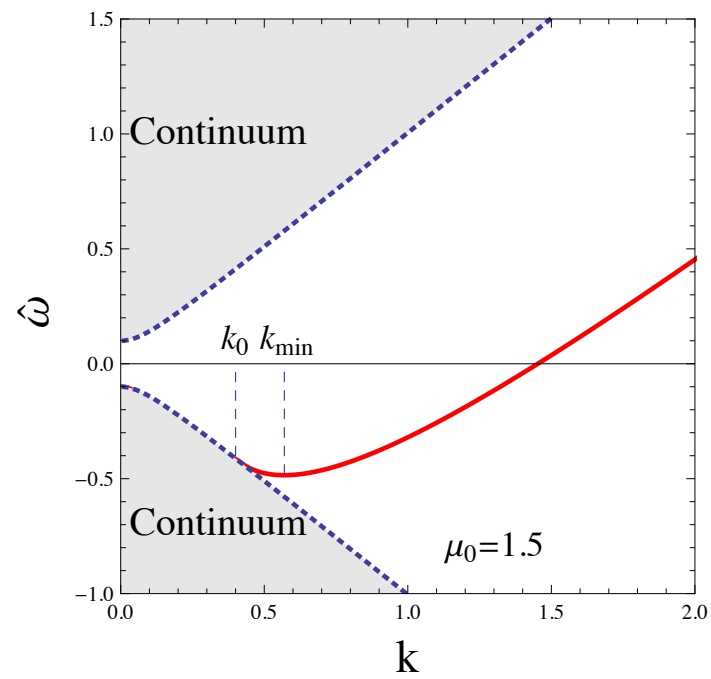
- Extreme high density behavior is very complex and rich and will not be presented here.



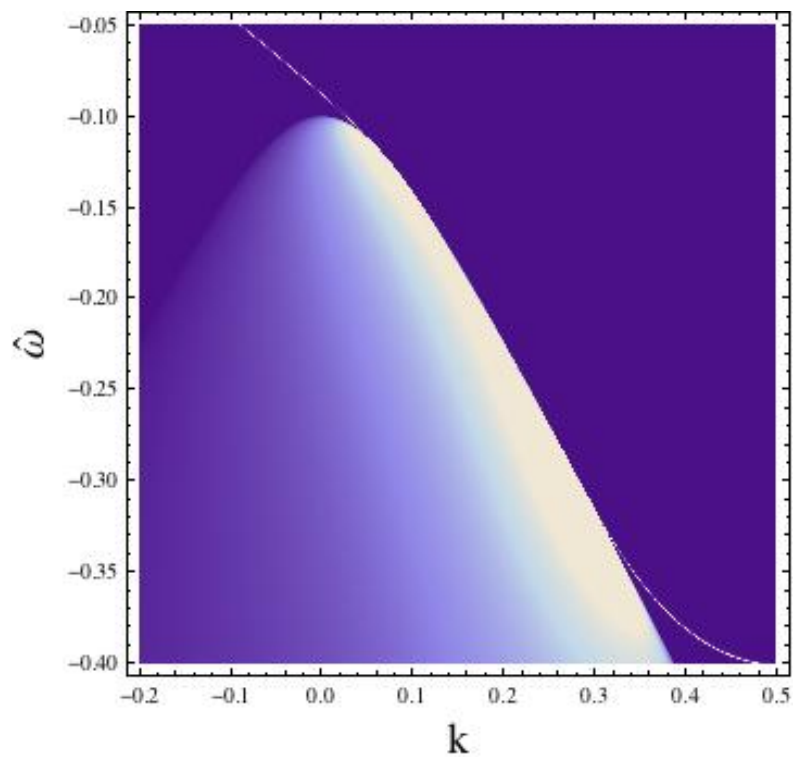
# Confinging case



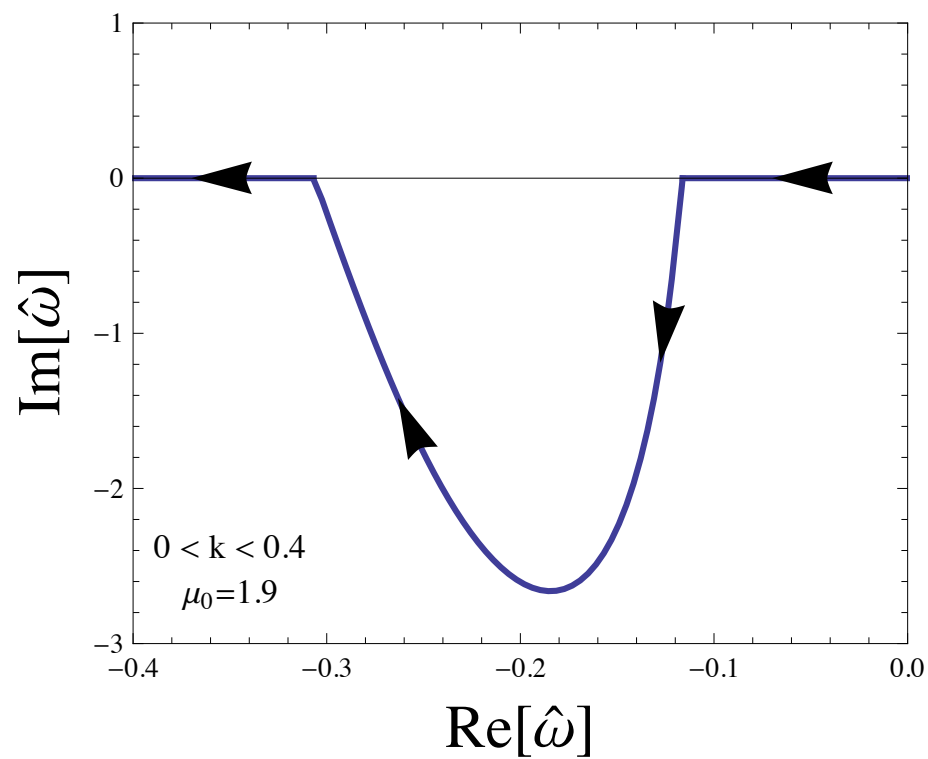
# Confining case



# Spectral function



(a)



(b)

# Density dependence of plasmino slope at $k=0$ .

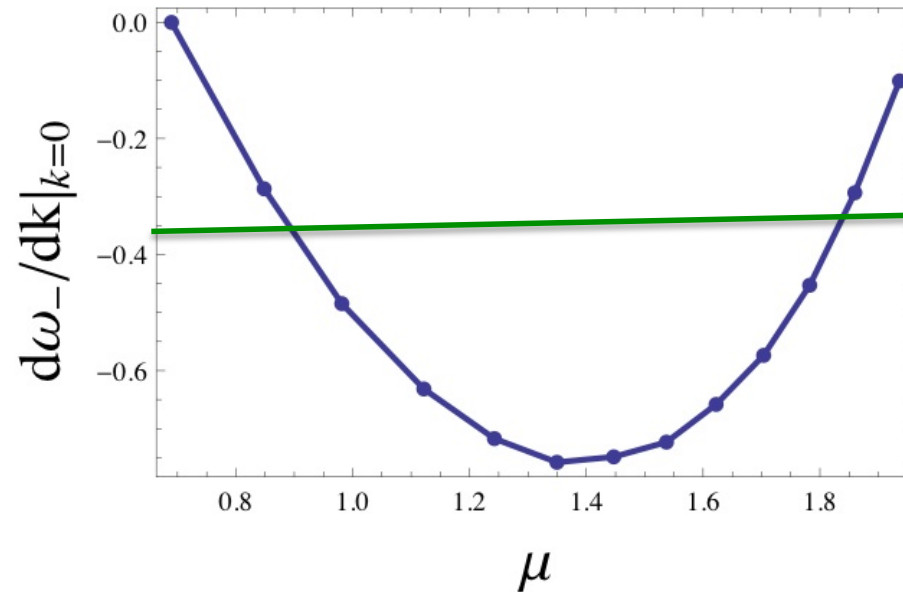


FIG. 3.  $\mu$  dependence of  $\alpha$ . The curve is plotted only in the density window where there is plasmino, namely  $\mu_1 \leq \mu \leq \mu_2$ .

Cf: HTL

$$p \ll m_f : \quad \omega_{\pm}(p) \simeq \pm \frac{1}{3}p ,$$

$$p \gg m_f : \quad \omega_{\pm}(p) \simeq p .$$

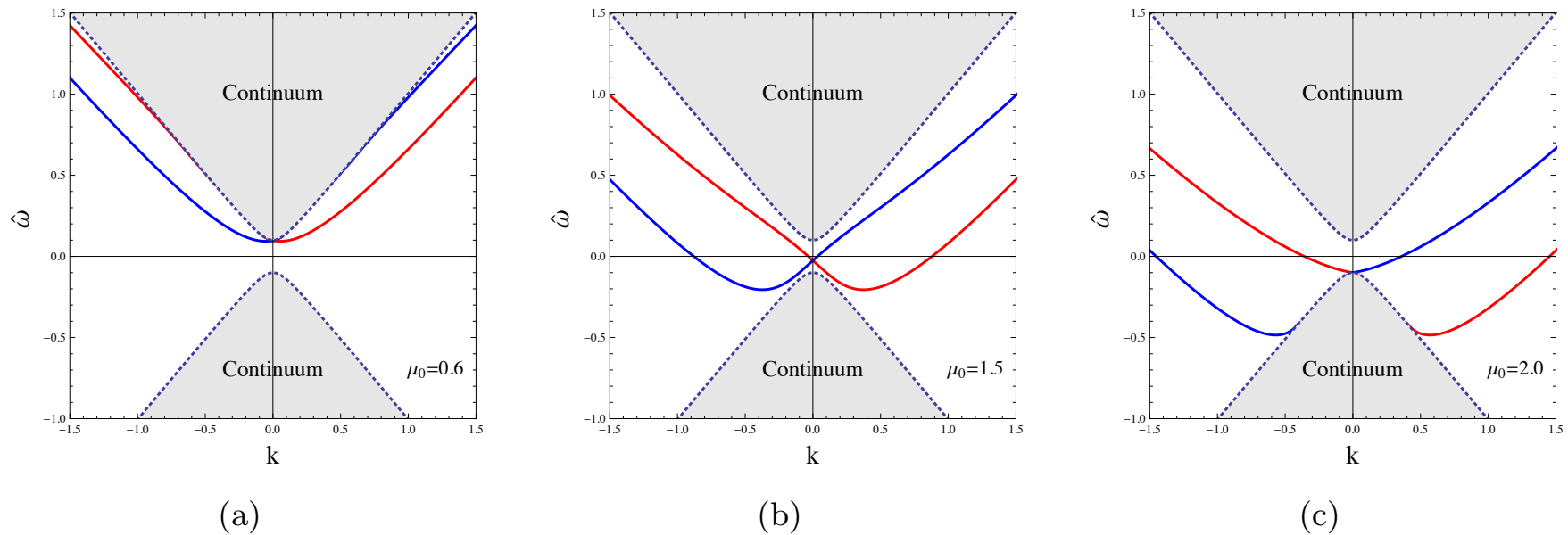
# Plasmino as Dual Rashiba effect Herzog et.al (1204.1518)

- Bulk field can couple to fermion spin

$$H_{\pm} = \frac{k^2}{2m_{eff}(r)} + \alpha E(r) \times \sigma \cdot k + \dots,$$

- What is the field theory Dual?  
Ans Density generated plasmino

# Density and plasmino



**Figure 8.** Density dependence of dispersion curves for  $G_1^R$  and  $G_2^R$ . Blue solid line denotes  $G_1^R$  and red line denotes  $G_2^R$ .

# Summary

Parameter		Top down		Bottom up	
		Confining	Deconfining	Confining	Deconfining
Fermion mass	$= 0$				
	$> 0$		⊙		⊙
Chemical potential	$< \mu_c$				
	$> \mu_c$	⊙	⊙	⊙	⊙



# Conclusion .

- Plasmino is present only in the presence of density and mass.
- Plasmino exist only for a window of density in Confining case.
- Thermal mass is 0, in deconfined case even at zero mass limit → non-fermi liquid.
- High Density seems to restore the fermi liquid character.

Thank you

# Review hep-th/0511199

- Poincare coordinate:  
Radial motion is Harmonic oscillator in proper time.
- So each particle has the same period in proper time  $\pi/2$  independent of the initial height.
- However, different particle has different proper time.  $\pi/2$  in proper time is infinite real time.

$$t = R/\epsilon \cdot \tan(\tau/R)$$

# However

- Initial condition dependence rapidly disappear. → we called it “synchronization”.

$$r = \frac{\epsilon R}{\sqrt{(\epsilon t/R)^2 + 1}} = R^2/t - (R/t)^3/2\epsilon^2 + \mathcal{O}(t^{-5}),$$

# Dictionary in Poincare Patch

$$p = r_0 \frac{mv_x}{\sqrt{1 - v_x^2}}$$

$$E = r_0 \frac{m}{\sqrt{1 - v_x^2}}$$

This is polchinski-strassler ansatz.

Notice that  $p$ ,  $E$  are conserved one.  $v_x$  are language of Bulk.  
The radial velocity goes to 0 unlike the global coordinate.  
It is clear that  $r_0$  is the parameter which sets the energy scale.

# Q11. Shock wave approach.

- Chesler+ Yaffe, Gubser,....
- Good in mimicking the boosted beam, which look like pancake, collision.
- Intuitively, shock wave originally was used to make black hole/brane in real space rather than ads space by t'Hooft, Giddings, ...
- Particle creation does not do any role.