KIAS-YITP joint workshop 2013.7.5

Dynamical Instability of Lovelock AdS Black Holes

T.Takahashi & J.S., ``Stability of Lovelock black holes under tensor perturbations,'' Phys. Rev. D79, 104025 (2009).
T.Takahashi & J.S., ``Instability of small Lovelock black holes in even-dimensions,'' Phys. Rev. D80, 104021 (2009).
T.Takahashi & J.S., ``Catastrophic instability of small Lovelock black holes,'' PTP124 (2010) 711-729.
T.Takahashi & J.S., ``Master equations for gravitational perturbations of static Lovelock black holes in higher dimensions,'' PTP124 (2010) 911-924.
T.Takahashi & J.S., ``Pathologies in Lovelock AdS black branes,'' CQG 29 (2012) 035008.

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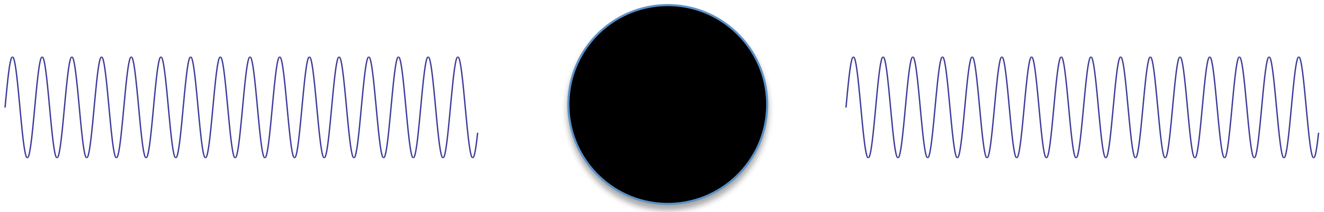
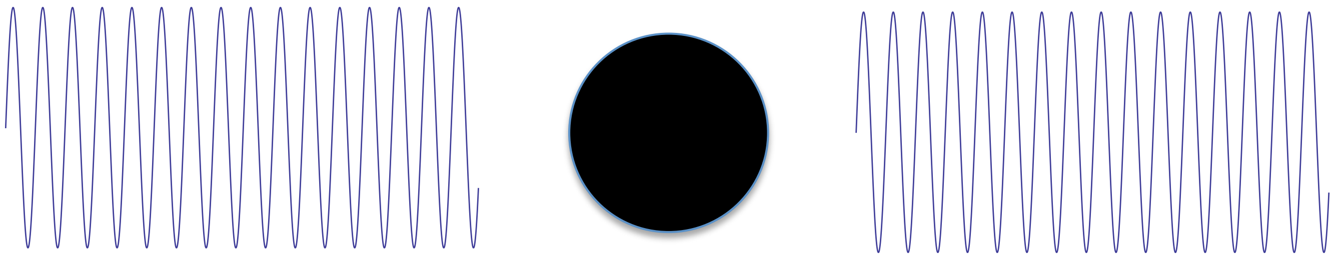
(Kobe University from October 1st, 2013)

Introduction

There are a lot of black holes in the universe.



If BH is dynamically unstable ⇩ the amplitude of perturbations grow in time



$$\propto e^{\omega t}$$

This does not happen for S-BH in 4-dimensional Einstein gravity.

Introduction --- continued

- The dimension of spacetime might be more than 4.
- In higher dimensions, Lovelock gravity is a natural generalization of Einstein gravity
- AdS BH is important for AdS/CFT
- Stability of BH is relevant to the physics of CFT
- Therefore, it is worth for studying

  the stability of AdS BH in Lovelock gravity

The stability analysis of BH in higher dimensional Einstein theory
was done by Kodama & Ishibashi 2003

Takahashi & J.S. (2009-2012) extended their analysis
to the higher dimensional Lovelock theory

We have shown that

there exists the instability analogous to the gradient instability
found by Kawai & J.S 1998 in Gauss-Bonnet cosmology.

$$c_s^2 < 0$$

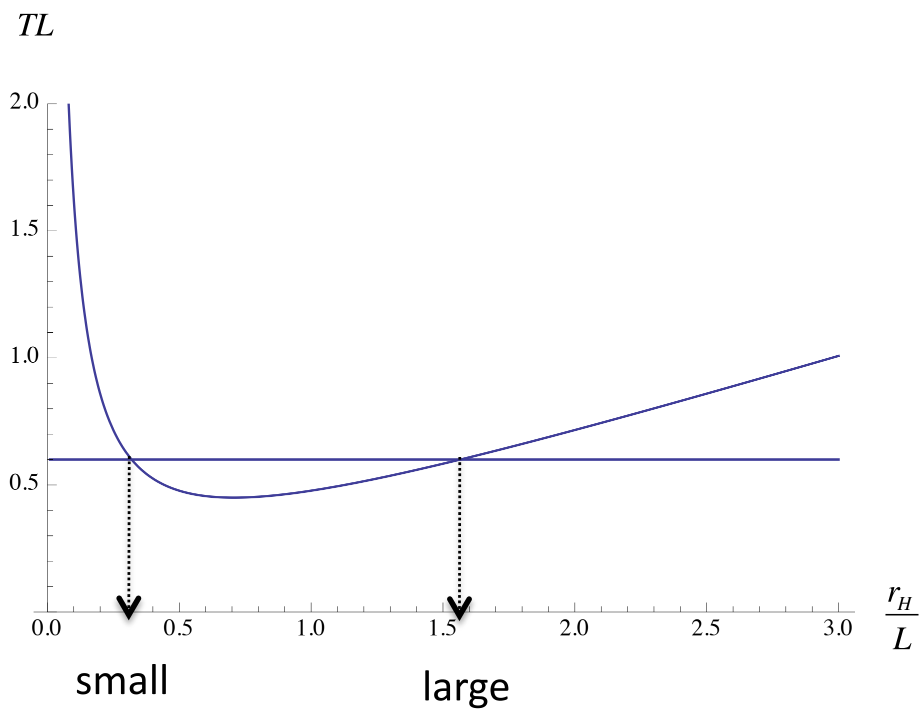
Small and Large BHs

4-d AdS-BH in Einstein gravity

$$ds^2 = -\left(\frac{r^2}{L^2} + 1 - \frac{r_0^4}{L^2 r^2}\right)dt^2 + \left(\frac{r^2}{L^2} + 1 - \frac{r_0^4}{L^2 r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Hawking temperature

$$T = \frac{2r_H^2 + L^2}{2\pi r_H L^2} \qquad \frac{r_H^2}{L^2} + 1 - \frac{r_0^4}{L^2 r_H^2} = 0$$



Large BH $\quad \frac{r_H}{L} \gg 1$

$$ds^2 = -\left(\frac{r^2}{L^2} - \frac{r_0^4}{L^2 r^2}\right)dt^2 + \left(\frac{r^2}{L^2} - \frac{r_0^4}{L^2 r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Small BH $\quad \frac{r_H}{L} \ll 1$

$$ds^2 = -\left(1 - \frac{r_0^4}{L^2 r^2}\right)dt^2 + \left(1 - \frac{r_0^4}{L^2 r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

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Lovelock Gravity

Lovelock 1971

Lovelock gravity

$$L = -2\Lambda + \sum_{m=1}^{k} \frac{a_m}{m} L_m \qquad a_1 = 1 \qquad k = \left[\frac{D-1}{2}\right]$$

: integer Ex. k=2 for D=6

$$L_m = \frac{1}{2^m} \delta^{\lambda_1\sigma_1\lambda_2\sigma_2\cdots\lambda_m\sigma_m}_{\rho_1\kappa_1\rho_2\kappa_2\cdots\rho_m\kappa_m} R_{\lambda_1\sigma_1}{}^{\rho_1\kappa_1} R_{\lambda_2\sigma_2}{}^{\rho_2\kappa_2} \cdots R_{\lambda_m\sigma_m}{}^{\rho_m\kappa_m}$$

$$\delta^{\lambda_1\lambda_2\cdots\lambda_m}_{\rho_1\rho_2\cdots\rho_m} = \det \begin{vmatrix} \delta^{\lambda_1}_{\rho_1} & \delta^{\lambda_1}_{\rho_2} & \cdots & \delta^{\lambda_1}_{\rho_m} \\ \delta^{\lambda_2}_{\rho_1} & \delta^{\lambda_2}_{\rho_2} & \cdots & \delta^{\lambda_2}_{\rho_m} \\ \vdots & \vdots & \ddots & \vdots \\ \delta^{\lambda_m}_{\rho_1} & \delta^{\lambda_m}_{\rho_2} & \cdots & \delta^{\lambda_m}_{\rho_m} \end{vmatrix}$$

$$L_1 = \frac{1}{2} \delta^{\lambda_1\sigma_1}_{\rho_1\kappa_1} R_{\lambda_1\sigma_1}{}^{\rho_1\kappa_1} = R$$

Einstein-Hilbert action

$$L_2 = \frac{1}{2^2} \delta^{\lambda_1\sigma_1\lambda_2\sigma_2}_{\rho_1\kappa_1\rho_2\kappa_2} R_{\lambda_1\sigma_1}{}^{\rho_1\kappa_1} R_{\lambda_2\sigma_2}{}^{\rho_2\kappa_2} = R_{\lambda\sigma}{}^{\rho\kappa} R_{\rho\kappa}{}^{\lambda\sigma} - 4 R^{\lambda\rho} R_{\lambda\rho} + R^2$$

Gauss-Bonnet action

Lovelock action

$$S = \int \sqrt{-g}\, L\, d^D x$$

Equations of motion is the second order

$$\Lambda \delta^{\mu}_{\nu} - \sum_{m=1}^{k} \frac{1}{2^{m+1}} \frac{a_m}{m} \delta^{\mu\lambda_1\sigma_1\cdots\lambda_m\sigma_m}_{\nu\rho_1\kappa_1\cdots\rho_m\kappa_m} R_{\lambda_1\sigma_1}{}^{\rho_1\kappa_1} \cdots R_{\lambda_m\sigma_m}{}^{\rho_m\kappa_m} = 0$$

Lovelock Black Holes

Wheeler 1986

D-dimensional BH

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 \underbrace{\gamma_{ij}\, dx^i dx^j}_{\text{const. curvature space}} \qquad \kappa = 1,\ 0,\ -1$$

$n = D - 2$ dimensions

Curvature components

$$R_{tr}{}^{tr} = -\frac{f''}{2} \qquad R_{ti}{}^{tj} = R_{ri}{}^{rj} = -\frac{f'}{2r}\delta_i^j \qquad R_{ij}{}^{kl} = \left(\frac{\kappa - f}{r^2}\right)\left(\delta_i^k \delta_j^l - \delta_i^l \delta_j^k\right)$$

Using the transformation $f(r) = \kappa - r^2 \psi(r)$ and substituting the above results into equations of motion, we obtain

$$\frac{d}{dr}\left[r^{n+1} W[\psi] \right] = 0 \qquad W[\psi] = \sum_{m=2}^{k} \frac{\alpha_m}{m} \psi^m + \psi - \frac{2\Lambda}{n(n+1)} \qquad \alpha_m = a_m \prod_{p=1}^{2m-2} (n-p)$$

which gives

$$W[\psi] = \sum_{m=2}^{k} \frac{\alpha_m}{m} \psi^m + \psi + 1 = \frac{\mu}{r^{n+1}} \qquad \frac{1}{\ell^2} = -\frac{2\Lambda}{n(n+1)} = 1 \qquad M = \frac{2\mu \pi^{(n+1)/2}}{\Gamma((n+1)/2)}$$

⬇

$$\psi(r)$$

Perturbed Lovelock black holes

Takahashi & Soda 2009

Tensor Perturbations

$$\delta g_{ab}=0,\quad \delta g_{ai}=0,\quad \delta g_{ij}=\frac{r}{\sqrt{T'}}\Psi(r)e^{-i\omega t}h_{ij}(x^i)$$

$$T(r)\equiv r^{n-1}\partial_\psi W[\psi]$$

$$\nabla^k\nabla_k h_{ij}=-\gamma_t h_{ij}\,,\quad \nabla^i h_{ij}=0\,,\quad \gamma^{ij}h_{ij}=0$$

$\gamma_t=\ell(\ell+n-1)-2\,,$ for $\kappa=1$ positive real for others

$$\frac{dr_*}{dr}=\frac{1}{f}$$

$$\left[-\frac{d^2}{dr_*^2}+V_t(r)\right]\Psi_t(r)=\omega^2\Psi_t(r)$$

$$V_t(r)=\frac{2\kappa+\gamma_t}{n-2}\frac{f}{r}\frac{d\log T'}{dr}+\frac{f}{r\sqrt{T'}}\frac{d}{dr}\left(f\frac{d\left(r\sqrt{T'}\right)}{dr}\right)$$

In order not to have ghost, we have to impose $T'>0$

Vector Perturbations

$$\delta g_{\mu\nu}=\begin{pmatrix}0 & 0 & vV_i e^{-i\omega t}\\ 0 & 0 & wV_i e^{-i\omega t}\\ \text{sym} & \text{sym} & 0\end{pmatrix}$$

$$\nabla^k\nabla_k V_i=-\gamma_v V_i\,,\quad \nabla^i V_i=0$$

$\gamma_v=\ell(\ell+n-1)-1\,,$ for $\kappa=1$ non-negative real for others

$$i\omega\frac{T'}{f}v+(fT'w)'=0 \qquad \Psi_v=\frac{f}{r}\sqrt{T'}\,w$$

$$\left[-\frac{d^2}{dr_*^2}+V_v(r)\right]\Psi_v(r)=\omega^2\Psi_v(r)$$

$$V_v(r)=\left(\frac{\gamma_v}{n-1}-\kappa\right)\frac{f}{r}\frac{d\log T}{dr}+f\,r\sqrt{T'}\frac{d}{dr}\left(f\frac{d}{dr}\frac{1}{r\sqrt{T'}}\right)$$

Perturbed Lovelock black holes --- continued

Scalar perturbations

$$\delta g_{\mu\nu} = \begin{pmatrix} f\bar{H}\,Y\,e^{-i\omega t} & H_1 Y\,e^{-i\omega t} & 0 \\ \text{sym} & \dfrac{H}{f} Y\,e^{-i\omega t} & 0 \\ \text{sym} & \text{sym} & r^2 K Y\,e^{-i\omega t}\,\gamma_{ij} \end{pmatrix}$$

$$\nabla^k \nabla_k Y = -\gamma_s Y$$

$$\gamma_s = \ell(\ell+n-1)\,, \quad \text{for } \kappa = 1 \quad \text{positive real for others}$$

$$\bar{H} = H + \frac{rT''}{T'} K \qquad H_1 = -i\omega \frac{r}{f}\left(N\Psi_s + K\right) \qquad N = \frac{A}{r\sqrt{T'}} \qquad A = -2nf + 2\gamma_s + nrf'$$

$$K = -\frac{2}{A}\left[nrf\left(N\Psi_s\right)' + \left(\gamma_s + nrf\frac{T'}{T}\right) N\Psi_s \right]$$

$$H = -\frac{\gamma_s N}{nf}\Psi_s + rK' - \frac{A}{2nf}K$$

$$\left[-\frac{d^2}{dr_*^2} + V_s(r) \right] \Psi_s(r) = \omega^2 \Psi_s(r)$$

$$V_s(r) = 2\gamma_s f \frac{(rNT)'}{nr^2 NT} - \frac{f}{N}(fN')' - \frac{f}{T}(fT')' + 2f^2\frac{N'^2}{N^2} + 2f^2\frac{T'^2}{T^2} + 2f^2\frac{N'T'}{NT}$$

Dynamical Stability Criterions

Master equations

$$\left[-\frac{d^2}{dr_*^2}+V_i(r)\right]\Psi_i(r)=\omega^2\Psi_i(r)$$

Since $\delta g_{\mu\nu}\propto e^{-i\omega t}$, if $\omega^2<0$, black holes are unstable.

It is useful to rewrite it as

$$\omega^2=\langle\Psi|\left[-\frac{d^2}{dr_*^2}+V\right]|\Psi\rangle=\int\left[|D\Psi|^2+V|\Psi|^2\right]dr_*$$

If V>0, black holes are stable. If not, we can deform the potential.

S-deformation Kodama&Ishibashi 2003

$$\langle\varphi|\left[-\frac{d^2}{dr_*^2}+V\right]|\varphi\rangle=\int\left[|D\varphi|^2+\tilde{V}|\varphi|^2\right]dr_* \qquad D=\frac{d}{dr_*}+S \qquad \tilde{V}=V+f\frac{dS}{dr}-S^2$$

If $\tilde{V}>0$, black holes are stable. If not, there may be instability.

Instability Criterions---tensor & vector

Tensor Perturbations

$$S = -f\frac{d}{dr}\log\left(r\sqrt{T'}\right) \qquad \langle\varphi|\left[-\frac{d^2}{dr_*^2}+V\right]|\varphi\rangle = \int |D\Psi|^2 dr_* + (2\kappa+\gamma_t)\int_{r_H}^{\infty}\frac{|\varphi|^2}{(n-2)r}\frac{d\log T'}{dr}dr$$

For arbitrary φ $\quad \dfrac{\langle\varphi|\left[-\frac{d^2}{dr_*^2}+V\right]|\varphi\rangle}{\langle\varphi|\varphi\rangle|} \geq \omega_0^2$

If $T''<0$ in some region, we can choose φ so that $\dfrac{\langle\varphi|\left[-\frac{d^2}{dr_*^2}+V\right]|\varphi\rangle}{\langle\varphi|\varphi\rangle|} < 0$ for large γ_t

In this case, the black hole is unstable

Vector Perturbations

$$S = -f\frac{d}{dr}\log\left(\frac{1}{r\sqrt{T'}}\right) \qquad \langle\varphi|\left[-\frac{d^2}{dr_*^2}+V\right]|\varphi\rangle = \int |D\Psi|^2 dr_* + \left(\frac{\gamma_v}{n-1}-\kappa\right)\int_{r_H}^{\infty}|\varphi|^2\frac{fT'}{rT}dr$$

Vector mode is always dynamically stable if T’>0.

Instability Criterions---scalar

Scalar Perturbations

$$S = f\frac{N'}{N} + f\frac{T'}{T}$$

$$\langle\varphi|\left[-\frac{d^2}{dr_*^2}+V\right]|\varphi\rangle = \int|D\Psi|^2\,dr_* + \int_{r_H}^{\infty}|\varphi|^2\,\tilde{V}\,dr$$

$$\tilde{V} = 2\gamma_s f\frac{(rNT)'}{nr^2NT} = \frac{2\gamma_s f}{nr}\left[\frac{2(\gamma_s - n\kappa)}{2(\gamma_s - n\kappa)+\frac{n(n+1)\mu}{T}}\frac{T'}{T} - \frac{1}{2}\frac{T''}{T'}\right] < \frac{\gamma_s f}{nrTT'}\left[2T'^2 - TT''\right]$$

$$2T'^2 - TT'' < 0$$ The black hole is unstable

Algebraic criterion

$$W[\psi]=\sum_{m=2}^{k}\frac{\alpha_m}{m}\psi^m+\psi-\frac{2\Lambda}{n(n+1)}=\frac{\mu}{r^{n+1}} \qquad \partial_\psi W[\psi]\psi' = -(n+1)\frac{\mu}{r^{n+2}} = -(n+1)\frac{W[\psi]}{r}$$

$$T(r)\equiv r^{n-1}\partial_\psi W[\psi]$$

$$\begin{cases} T'(r)=\dfrac{r^{n-2}}{\partial_\psi W}\left[(n-1)\left(\partial_\psi W\right)^2-(n+1)W\,\partial_\psi^2 W\right]\equiv\dfrac{r^{n-2}}{\partial_\psi W}K[\psi] \\ T''(r)=\dfrac{r^{n-3}}{\left(\partial_\psi W\right)^3}\left[(n-1)(n-2)\left(\partial_\psi W\right)^4-(n+1)(n-4)W\left(\partial_\psi W\right)^2\partial_\psi^2 W+(n+1)^2W^2\left\{\partial_\psi W\,\partial_\psi^3 W-\left(\partial_\psi^2 W\right)^2\right\}\right]\equiv\dfrac{r^{n-3}}{\left(\partial_\psi W\right)^3}L[\psi] \\ 2T'^2-TT''=\dfrac{r^{2n-4}}{\left(\partial_\psi W\right)^2}\left[n(n-1)\left(\partial_\psi W\right)^4-3n(n+1)W\left(\partial_\psi W\right)^2\partial_\psi^2 W+(n+1)^2W^2\left\{3\left(\partial_\psi^2 W\right)^2-\partial_\psi W\,\partial_\psi^3 W\right\}\right]\equiv\dfrac{r^{2n-4}}{\left(\partial_\psi W\right)^2}M[\psi] \end{cases}$$

Ghost instability $\quad K[\psi]=(n-1)\left(\partial_\psi W\right)^2-(n+1)W\partial_\psi^2 W\le 0$

Tensor instability $\quad L[\psi]=(n-1)(n-2)\left(\partial_\psi W\right)^4-(n+1)(n-4)W\left(\partial_\psi W\right)^2\partial_\psi^2 W+(n+1)^2W^2\left\{\partial_\psi W\partial_\psi^3 W-\left(\partial_\psi^2 W\right)^2\right\}<0$

Scalar instability $\quad M[\psi]=n(n-1)\left(\partial_\psi W\right)^4-3n(n+1)W\left(\partial_\psi W\right)^2\partial_\psi^2 W+(n+1)^2W^2\left\{3\left(\partial_\psi^2 W\right)^2-\partial_\psi W\partial_\psi^3 W\right\}<0$

Instability of small black holes

Black Hole Solutions

D-dimensional AdS BH

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\gamma_{ij}\,dx^i dx^j \qquad f(r) = \kappa - r^2\psi(r) \qquad W[\psi] = \sum_{m=2}^{k}\frac{\alpha_m}{m}\psi^m + \psi + 1 = \frac{\mu}{r^{n+1}}$$

Ex. $D = 4, n = 2,\ k = \left[\frac{4-1}{2}\right] = 1$

$$W[\psi] = \psi + 1 = \frac{\mu}{r^3}$$

$$f = \kappa - r^2\psi = \kappa + r^2 - \frac{\mu}{r}$$

Ex. $D = 5,\ n = 3,\ k = 2$

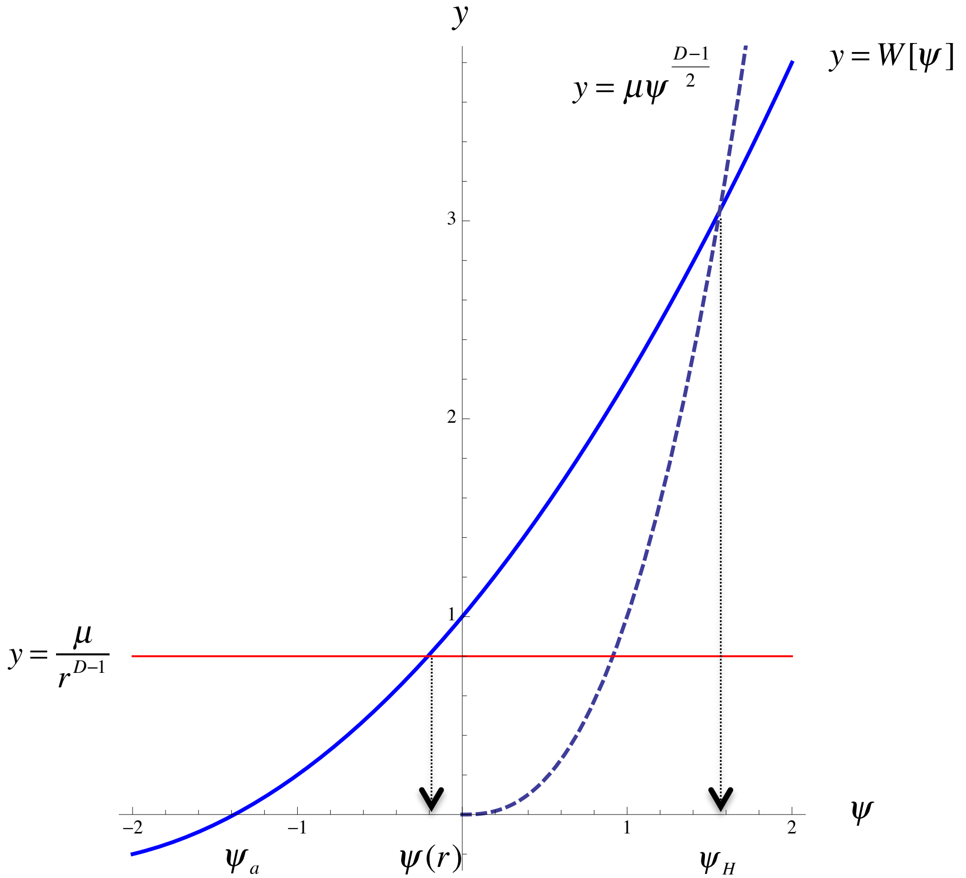
$$W[\psi] = \frac{\alpha_2}{2}\psi^2 + \psi + 1 = \frac{\mu}{r^4} \qquad \psi = \frac{-1 \pm \sqrt{1 - 2\alpha_2 + \frac{2\alpha_2\mu}{r^4}}}{\alpha_2}$$

The asymptotically AdS Gauss-Bonnet BH solution

$$f(r) = 1 + \frac{r^2}{\alpha_2}\left[1 - \sqrt{1 - 2\alpha_2 + \frac{2\alpha_2\mu}{r^4}}\right]$$

Graphical Method

$W[\psi] = \frac{\mu}{r^{D-1}}$ can be solved graphically.



Each r determines the red line.
The function $\psi(r)$ is given as the intersection of the red line and the blue curve.

$$ds^2 = -\left(1 - r^2\psi(r)\right)dt^2 + \frac{dr^2}{1 - r^2\psi(r)} + r^2\gamma_{ij}\,dx^i dx^j$$

horizon

$$\psi_H = \frac{1}{r_H^2}$$

$$W[\psi_H] = \frac{\mu}{r_H^{D-1}} = \mu\psi_H^{\frac{D-1}{2}}$$

the asymptotic infinity

$r \to \infty$ $\quad W[\psi_a] = 0$

the largest negative root

$$R^{\mu\nu\sigma\rho}R_{\mu\nu\sigma\rho} = \left(\partial_r^2 r^2\psi\right)^2 + 2(D-2)\frac{\left(\partial_r^2 r^2\psi\right)^2}{r^2} + 2(D-2)(D-3)\psi^2$$

$$\partial_r\psi = -(D-1)\frac{W}{r\,\partial_\psi W}$$

⇒ There should not be extrema in the interval $[\psi_a, \psi_H]$

Stability Analysis

4-d AdS BH is stable

$$W[\psi]=\psi+1 \;\Rightarrow\; K[\psi]=\left(\partial_\psi W\right)^2-3W\,\partial_\psi^2 W=1 \qquad L[\psi]=0 \qquad M[\psi]=2\left(\partial_\psi W\right)^4=2$$

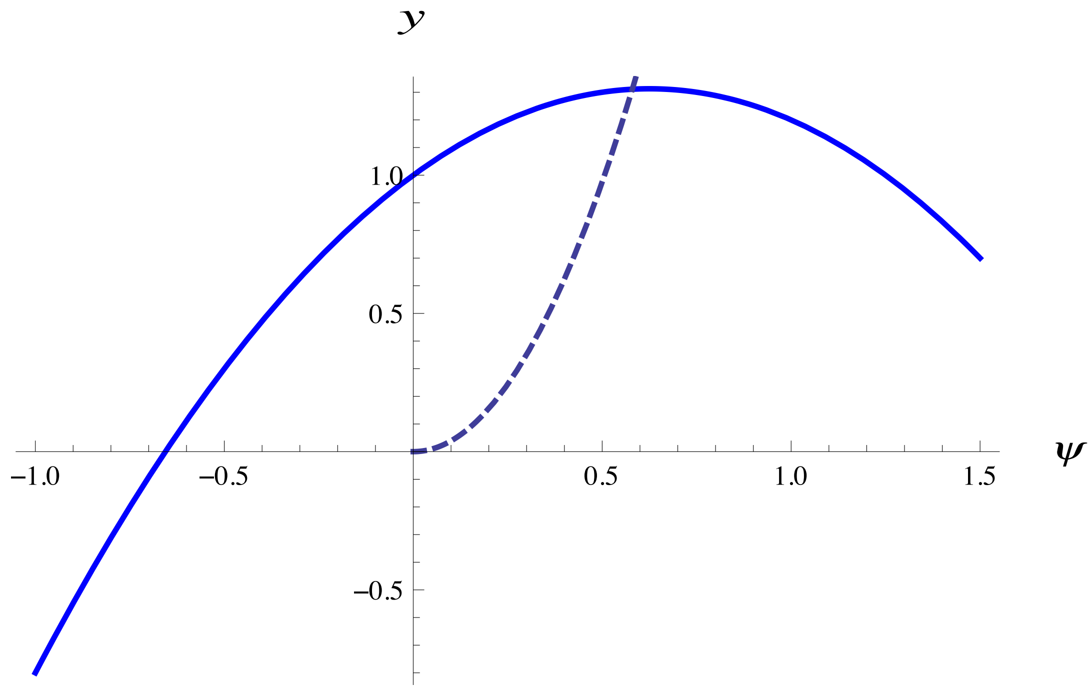
5-d AdS BH

$$W[\psi]=\frac{\alpha_2}{2}\psi^2+\psi+1 \qquad \frac{\alpha_2}{2}\psi_a^2+\psi_a+1=0$$

$$W[\psi_H]=\frac{\alpha_2}{2}\psi_H^2+\psi_H+1=\mu\psi_H^2 \qquad 2\mu>\alpha_2 \qquad \psi_H=\frac{1+\sqrt{1+4\mu-2\alpha_2}}{2\mu-\alpha_2}$$

$$\psi_H=\frac{1}{r_H^2}$$

negative α_2 does not allow small BH



Stability Analysis --- continued

We need to investigate $\alpha_2 > 0$

$$K[\psi] = 2(\alpha_2\psi+1)^2 - 4\alpha_2\left(\frac{\alpha_2}{2}\psi^2+\psi+1\right) = 2(1-2\alpha_2) \quad \Rightarrow \quad \alpha_2 < \frac{1}{2}$$

$$L[\psi] = 2(\alpha_2\psi+1)^4 + 4\alpha_2(\alpha_2\psi+1)^2\left(\frac{\alpha_2}{2}\psi^2+\psi+1\right) - 16\alpha_2^2\left(\frac{\alpha_2}{2}\psi^2+\psi+1\right)^2$$

$$= 2(1-2\alpha_2)(1+4\alpha_2+6\alpha_2\psi+3\alpha_2^2\psi^2) > 0$$

$$M[\psi] = 6(\alpha_2\psi+1)^4 - 36\alpha_2(\alpha_2\psi+1)^2\left(\frac{\alpha_2}{2}\psi^2+\psi+1\right) + 48\alpha_2^2\left(\frac{\alpha_2}{2}\psi^2+\psi+1\right)^2$$

$$= 6(1-2\alpha_2)(1-4\alpha_2-2\alpha_2\psi-\alpha_2^2\psi^2)$$

For sufficiently small black holes $2\mu-\alpha_2 \to 0$ we have $\psi_H = \dfrac{1+\sqrt{1+4\mu-2\alpha_2}}{2\mu-\alpha_2} \to \infty$

Hence, $M[\psi]<0$, namely , there exists the instability in scalar perturbations.

Stability analysis --- continued

6-d AdS BH $\alpha_2 > 0$

$$W[\psi] = \frac{\alpha_2}{2}\psi^2 + \psi + 1 = \frac{\mu}{r^5} \qquad W[\psi_H] = \frac{\alpha_2}{2}\psi_H^2 + \psi_H + 1 = \mu\psi_H^{\frac{5}{2}} \qquad as \quad \mu \to 0 \quad \text{the horizon goes as} \quad \psi_H \to \infty$$

$$K[\psi] = 3(\alpha_2\psi + 1)^2 - 15\alpha_2\left(\frac{\alpha_2}{2}\psi^2 + \psi + 1\right) = \frac{1}{2}\left(5 - 10\alpha_2 + (1+\alpha_2\psi)^2\right)$$

$\alpha_2 < \frac{1}{2}$ guarantees the positivity of K

$$M[\psi] = 12(\alpha_2\psi + 1)^4 - 60a_2(\alpha_2\psi + 1)^2\left(\frac{\alpha_2}{2}\psi^2 + \psi + 1\right) + 75\alpha_2^2\left(\frac{\alpha_2}{2}\psi^2 + \psi + 1\right)^2$$
$$= \frac{3}{4}\left(4 - 10\alpha_2 - 2\alpha_2\psi - \alpha_2^2\psi^2\right)^2 \geq 0$$

Scalar modes are always stable!

$$L[\psi] = 6(\alpha_2\psi + 1)^4 - 25\alpha_2^2\left(\frac{\alpha_2}{2}\psi^2 + \psi + 1\right)^2$$
$$= 6 - 25\alpha_2^2 + \left(24\alpha_2 - 50\alpha_2^2\right)\psi + \left(-25\alpha_2^3 + 11\alpha_2^2\right)\psi^2 - \alpha_2^3\psi^3 - \frac{\alpha_2^4}{4}\psi^4$$

For large ψ, tensor modes are unstable.

Therefore, for sufficiently small black holes in GB gravity, there exists the instability.

Dotti & Gleisler 2005

More general results

Takahashi & Soda 2009, 2010

In even dimensions, we obtain

$$L[\psi] = (D-3)(D-4) + \cdots - \frac{2(D-4)}{(D-2)^2}\alpha_k^4 \psi^{2(D-4)}$$

L becomes negative for small black holes.

In odd dimensions, we obtain

$$K[\psi] = (D-2)(D-3) + \cdots + 4\frac{(D-5)\alpha_{k-1}^2 - 2(D-3)\alpha_k \alpha_{k-2}}{(D-3)(D-5)}\psi^{D-5}$$

$$M[\psi] = (D-2)(D-3) + \cdots - 6\alpha_k^2 \frac{(D-5)\alpha_{k-1}^2 - 2(D-3)\alpha_k \alpha_{k-2}}{(D-3)(D-5)}\psi^{2(D-4)}$$

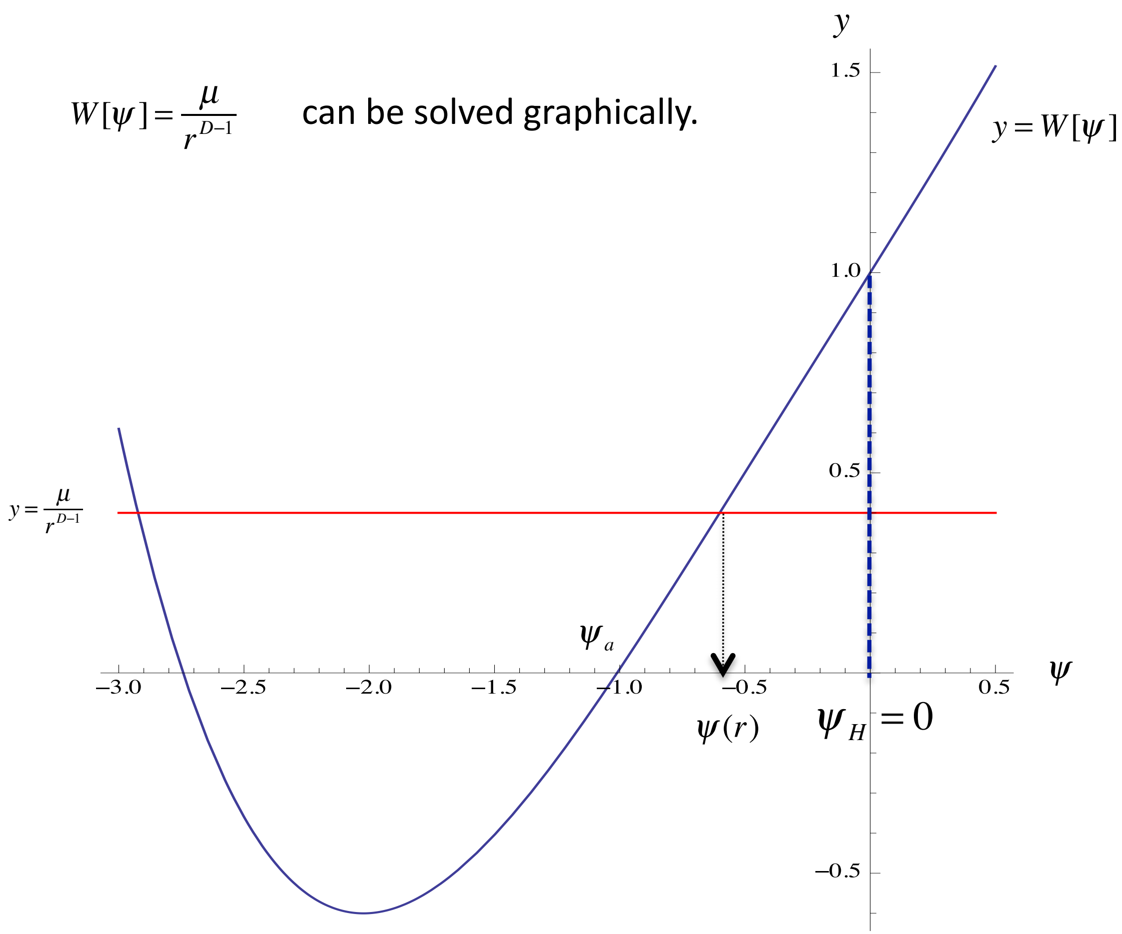
Either K or M becomes negative for small black holes.

Therefore, we can say small AdS black holes are generically unstable.

Stability analysis of large black holes

Graphical Method

$W[\psi] = \frac{\mu}{r^{D-1}}$ can be solved graphically.



Large black holes can be well approximated by planar black holes $\kappa = 0$

$$ds^2 = r^2\psi(r)dt^2 + \frac{dr^2}{-r^2\psi(r)} + r^2\gamma_{ij}\,dx^i dx^j$$

In this case, the horizon is located at

$$\psi_H = 0$$

the asymptotic infinity

$r \to \infty$ $\quad W[\psi_a] = 0$

the largest negative root

$$R^{\mu\nu\sigma\rho}R_{\mu\nu\sigma\rho} = \left(\partial_r^2 r^2\psi\right)^2 + 2(D-2)\frac{\left(\partial_r^2 r^2\psi\right)^2}{r^2} + 2(D-2)(D-3)\psi^2$$

$$\partial_r\psi = -(D-1)\frac{W}{r\,\partial_\psi W}$$

⇨ There should not be extrema in the interval $[\psi_a, 0]$

Stability analysis

Takahashi & Soda 2012

5-d AdS BH $W[\psi]=\frac{\alpha_2}{2}\psi^2+\psi+1$

$$K[\psi]=2(\alpha_2\psi+1)^2-4\alpha_2\left(\frac{\alpha_2}{2}\psi^2+\psi+1\right)=2(1-2\alpha_2) \quad \Rightarrow \quad \alpha_2<\frac{1}{2}$$

$$L[\psi]=2(\alpha_2\psi+1)^4+4\alpha_2(\alpha_2\psi+1)^2\left(\frac{\alpha_2}{2}\psi^2+\psi+1\right)-16\alpha_2^2\left(\frac{\alpha_2}{2}\psi^2+\psi+1\right)^2$$

$$=2(1-2\alpha_2)(1+4\alpha_2+6\alpha_2\psi+3\alpha_2^2\psi^2) \quad \Rightarrow \quad -\frac{1}{4}\le\alpha_2$$

$$\therefore \quad -\frac{1}{4}\le\alpha_2\le\frac{1}{4}$$

$$M[\psi]=6(\alpha_2\psi+1)^4-36\alpha_2(\alpha_2\psi+1)^2\left(\frac{\alpha_2}{2}\psi^2+\psi+1\right)+48\alpha_2^2\left(\frac{\alpha_2}{2}\psi^2+\psi+1\right)^2$$

$$=6(1-2\alpha_2)(1-4\alpha_2-2\alpha_2\psi-\alpha_2^2\psi^2) \quad \Rightarrow \quad \alpha_2\le\frac{1}{4}$$

6-d AdS BH

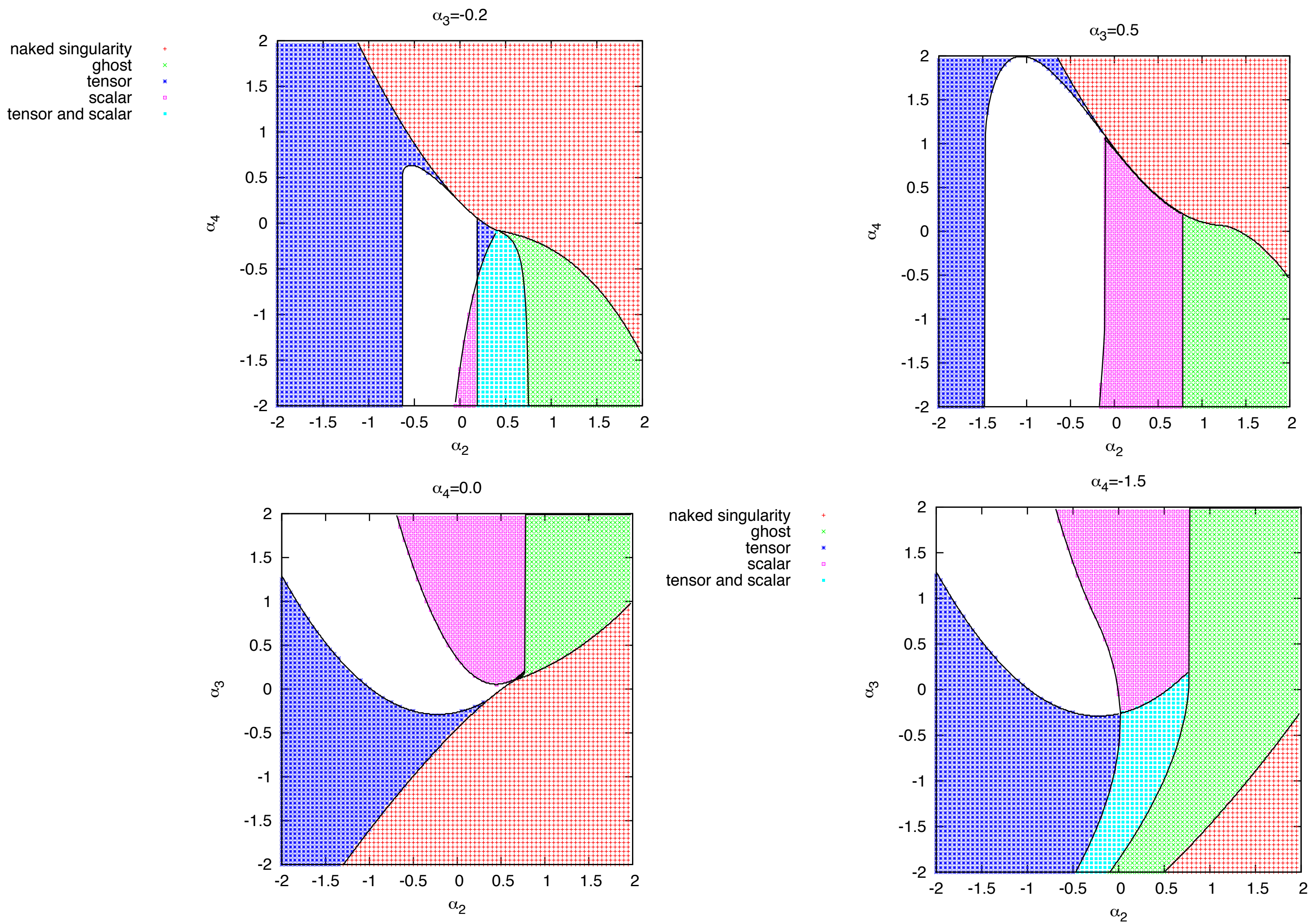
The similar analysis gives $\therefore \; -\frac{\sqrt{6}}{5}\le\alpha_2\le\frac{\sqrt{6}}{5}$

In the case of large black holes, models are constrained by the stability analysis.

Numerical analysis

Ex. 10D

$$W[\psi]=\frac{\alpha_4}{4}\psi^2+\frac{\alpha_3}{3}\psi^2+\frac{\alpha_2}{2}\psi^2+\psi+1$$



Thermodynamical stability?

- We found the dynamical instability of Lovelock black holes
- We could not find corresponding thermodynamical instability

Dynamical instability $\neq$ thermodynamical instability

Summary

- We have obtained master equations for general perturbations of spherical black holes in Lovelock gravity

- Small AdS black holes are dynamically unstable

- Large AdS black holes are unstable for some models

  This should have implications for AdS/CFT correspondence.