General Relativity and the Cuprates

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1204.0519, 1209.1098
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Signs of a Stranger, Deeper Side to Nature's Building Blocks

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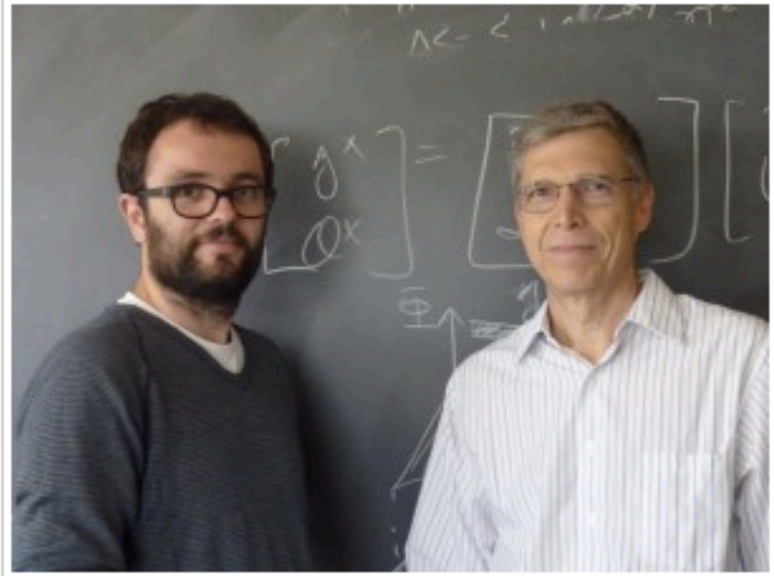
A Superconductive Model

Increasingly over the past decade, studying the black hole equivalents of strongly correlated forms of matter has yielded groundbreaking results, such as a new equation for the viscosity of strongly interacting fluids and a better grasp of interactions between quarks and gluons, which are particles found in the nuclei of atoms.



Gary Horowitz, right, a physics professor at UC-Santa Barbara, and Jorge Santos, a post-doctoral researcher in Horowitz's group, have modeled strange materials called cuprates as peculiarly shaped black holes in higher dimensions. (Photo: Courtesy of Gary Horowitz)

Now, Gary Horowitz, a string theorist at UC-Santa Barbara, and Jorge Santos, a post-doctoral researcher in Horowitz's group, have applied the holographic duality to cuprates. They derived a formula for the conductivity of the metals, which are approximately 2-D, by studying related properties of what may be their counterpart in 3-D: an electrically charged, peculiarly shaped black hole.

The work took numerical virtuosity. In cuprates, a swarm of strongly correlated electrons moves through a fixed lattice of atoms. Modeling the metals with the holographic duality therefore required working the equivalent of a lattice into the structure of the corresponding black hole by giving it a corrugated outer surface, or horizon.

Gauge/gravity duality can reproduce many properties of condensed matter systems, even in the limit where the bulk is described by classical general relativity:

1) Fermi surfaces
2) Non-Fermi liquids
3) Superconducting phase transitions
4) …

It is not clear why it is working so well.

Can one do more than reproduce qualitative features of condensed matter systems?

Can gauge/gravity duality provide a quantitative explanation of some mysterious property of real materials?

We will argue that the answer is yes!

Many previous applications have assumed translational symmetry. But:

Momentum conservation + nonzero charge density => Infinite DC conductivity

Can have effective momentum nonconservation in a probe approximation (Karch, O'Bannon, 2007) or by adding a lattice.

Plan: Calculate the optical conductivity of a simple holographic conductor and superconductor with lattice included.

A perfect lattice still has infinite conductivity. So we work at nonzero T and include dissipation. (Earlier work by: Kachru et al; Maeda et al; Hartnoll and Hofman; Zaanen et al, Siopsis et al, Flauger et al)

Main result: We will find surprising similarities to the optical conductivity of the cuprates.

Simple model of a conductor

Suppose electrons in a metal satisfy

$$m\frac{dv}{dt} = eE - m\frac{v}{\tau}$$

If there are n electrons per unit volume, the current density is J = nev. Letting $E(t) = Ee^{-i\omega t}$, find J = σ E, with

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

where $K=ne^2/m$. This is the Drude model.

$$\mathrm{Re}(\sigma) = \frac{K\tau}{1+(\omega\tau)^2}, \quad \mathrm{Im}(\sigma) = \frac{K\omega\tau^2}{1+(\omega\tau)^2}$$

Note:

(1) For $\omega\tau \gg 1, \quad |\sigma| \approx K/\omega$

(2) In the limit $\tau \to \infty$:

$$\mathrm{Re}(\sigma) \propto \delta(\omega), \quad \mathrm{Im}(\sigma) = K/\omega$$

This can be derived more generally from Kramers-Kronig relation.

Our gravity model

We start with just Einstein-Maxwell theory:

$$S = \int d^4x \sqrt{-g} \left[R + \frac{6}{L^2} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$$

This is the simplest context to describe a conductor. We require the metric to be asymptotically anti-de Sitter (AdS)

$$ds^2 = \frac{-dt^2 + dx^2 + dy^2 + dz^2}{z^2}$$

Want finite temperature: Add black hole

Want finite density: Add charge to the black hole. The asymptotic form of A_t is

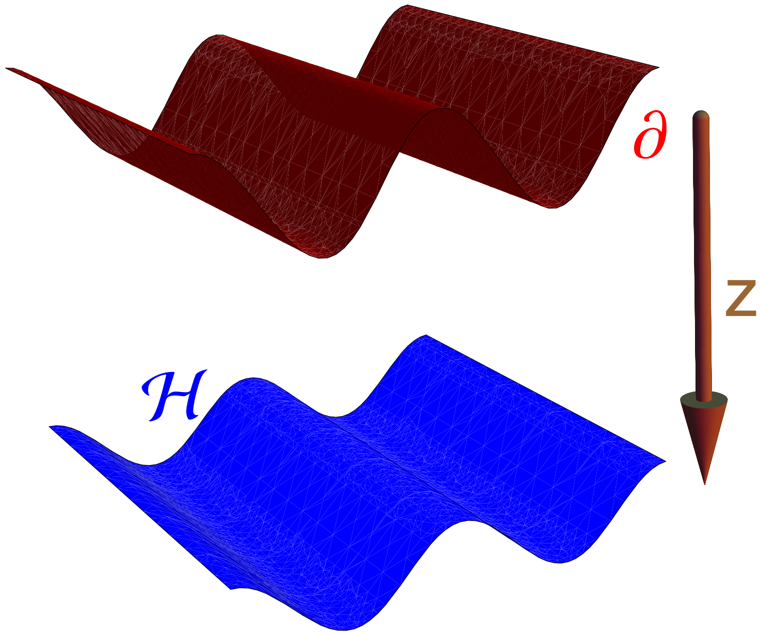
$$A_t = \mu - \rho z + O(z^2)$$

μ is the chemical potential and ρ is the charge density in the dual theory.

Introduce the lattice by making the chemical potential be a periodic function:

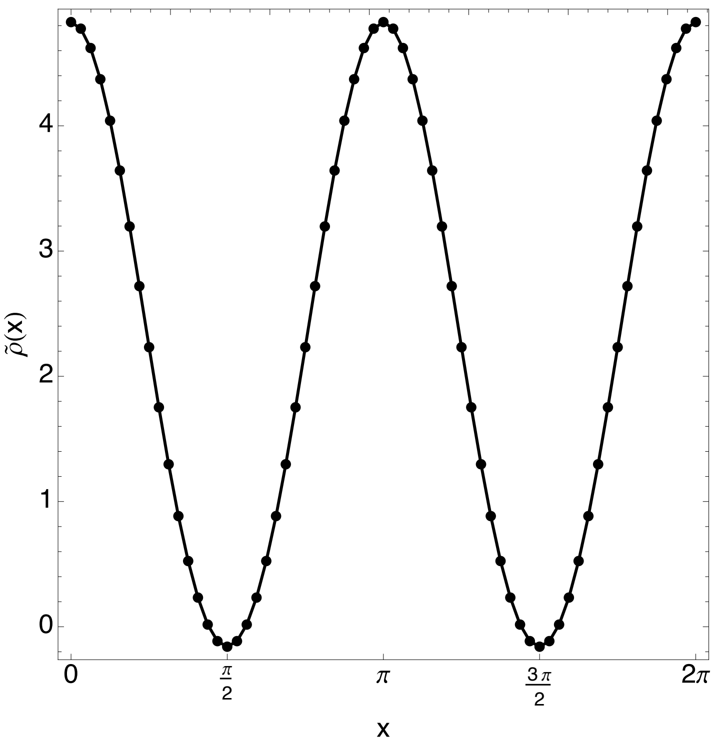
$$\mu(x) = \bar{\mu}\left[1 + A_0 \cos(k_0 x)\right]$$

We numerically find solutions with smooth horizons that are static and translationally invariant in one direction.



Solutions are rippled charged black holes.

Charge density for $A_0 = ½$, $k_0 = 2$, $T/\mu = .055$



Conductivity

To compute the optical conductivity using linear response, we perturb the solution

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + \delta g_{\mu\nu}, \qquad A_\mu = \hat{A}_\mu + \delta A_\mu$$

Boundary conditions:

ingoing waves at the horizon

$\delta g_{\mu\nu}$ normalizable at infinity

$\delta A_t \sim O(z)$, $\delta A_x = e^{-i\omega t}\,[E/i\omega + J\, z + \ldots]$

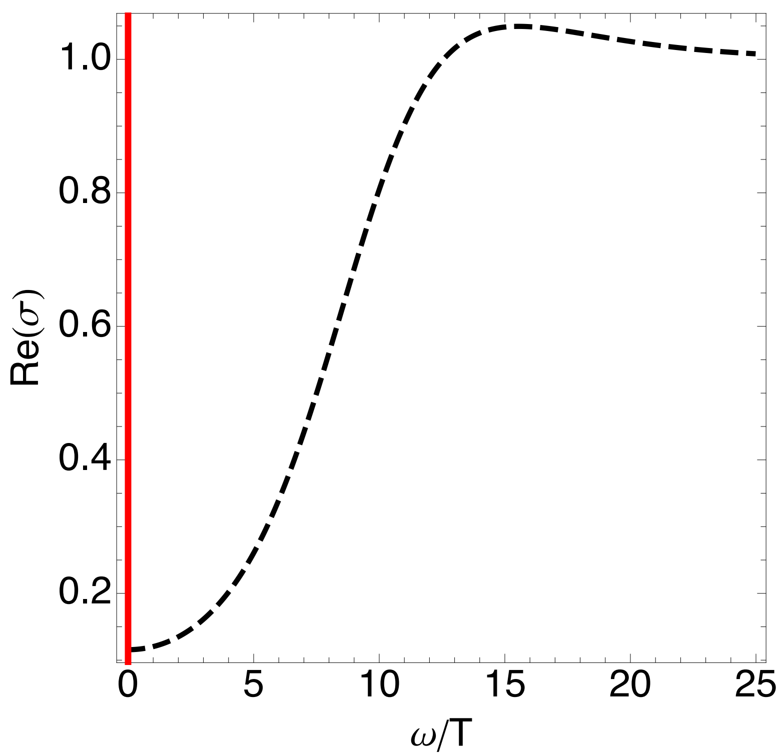
induced current

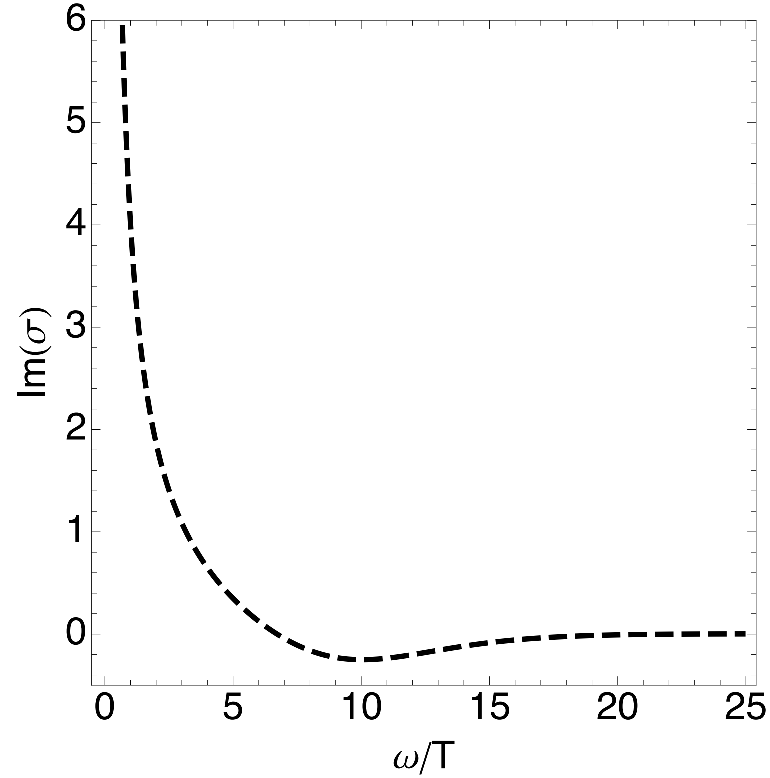
Using Ohm's law, $J = \sigma E$, the optical conductivity is given by

$$\tilde{\sigma}(\omega, x) = \lim_{z \to 0} \frac{\delta F_{zx}(x, z)}{\delta F_{xt}(x, z)}$$

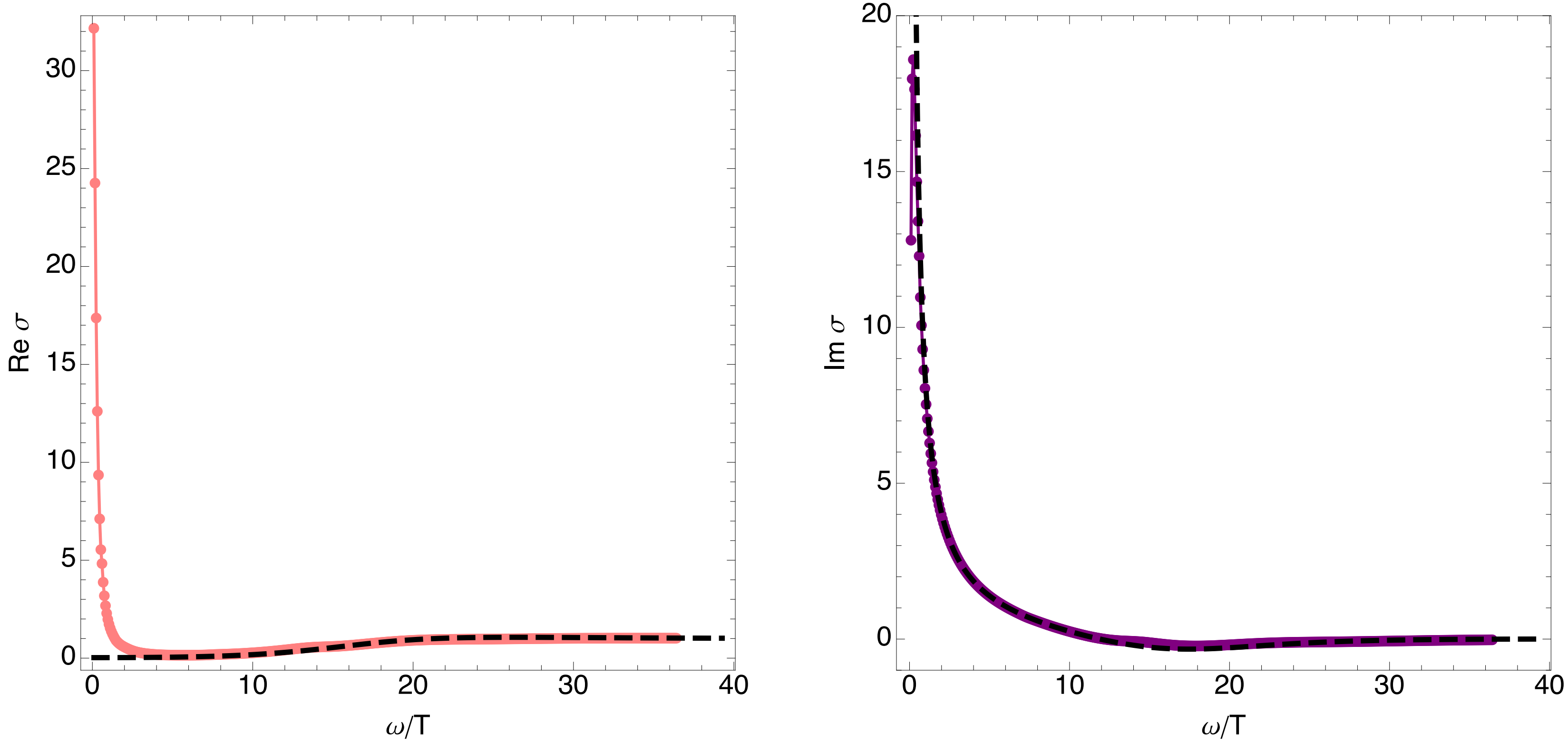
Since we impose a homogeneous electric field, we are interested in the homogeneous part of the conductivity $\sigma(\omega)$.

Review: optical conductivity with no lattice (T/μ = .115)



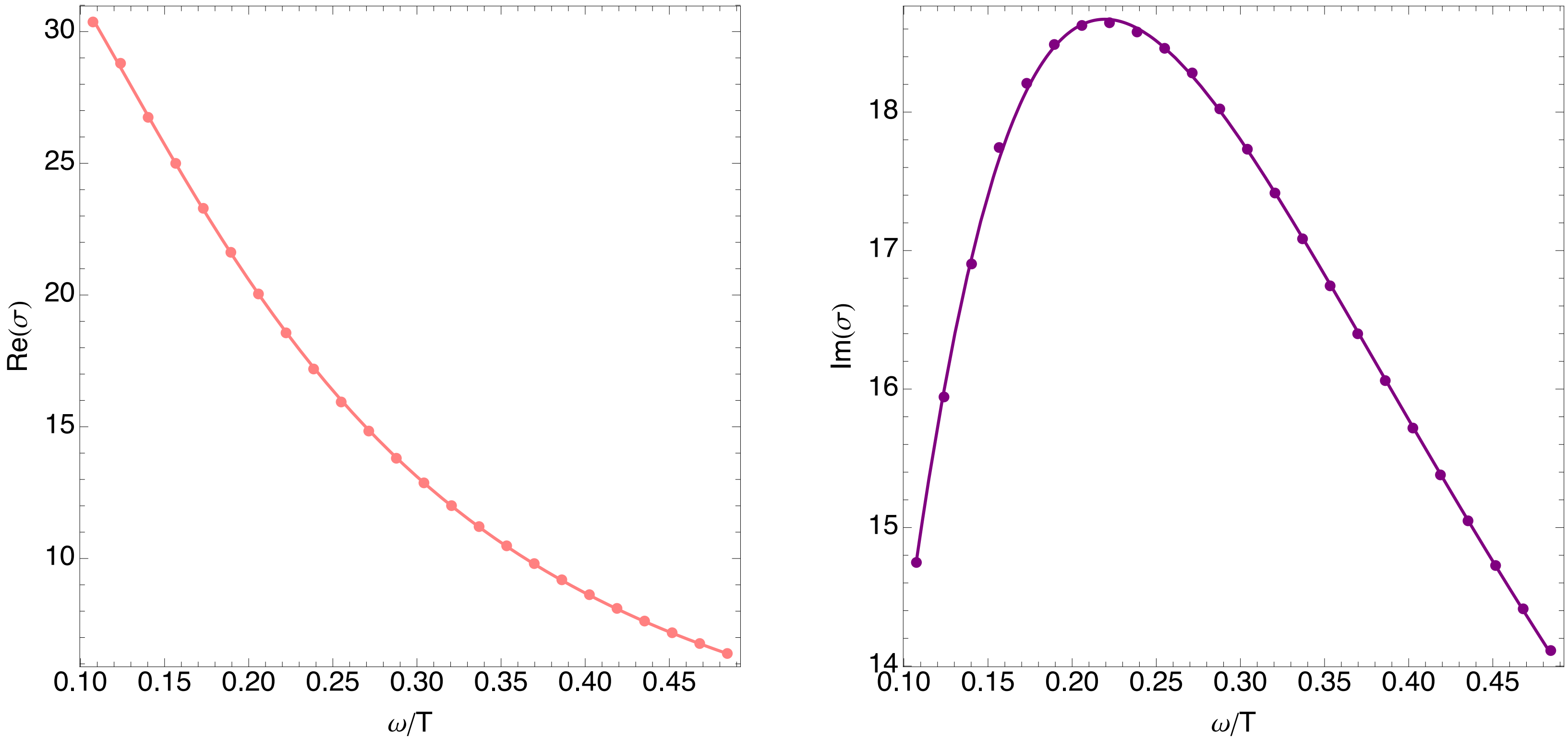


With the lattice, the delta function is smeared out



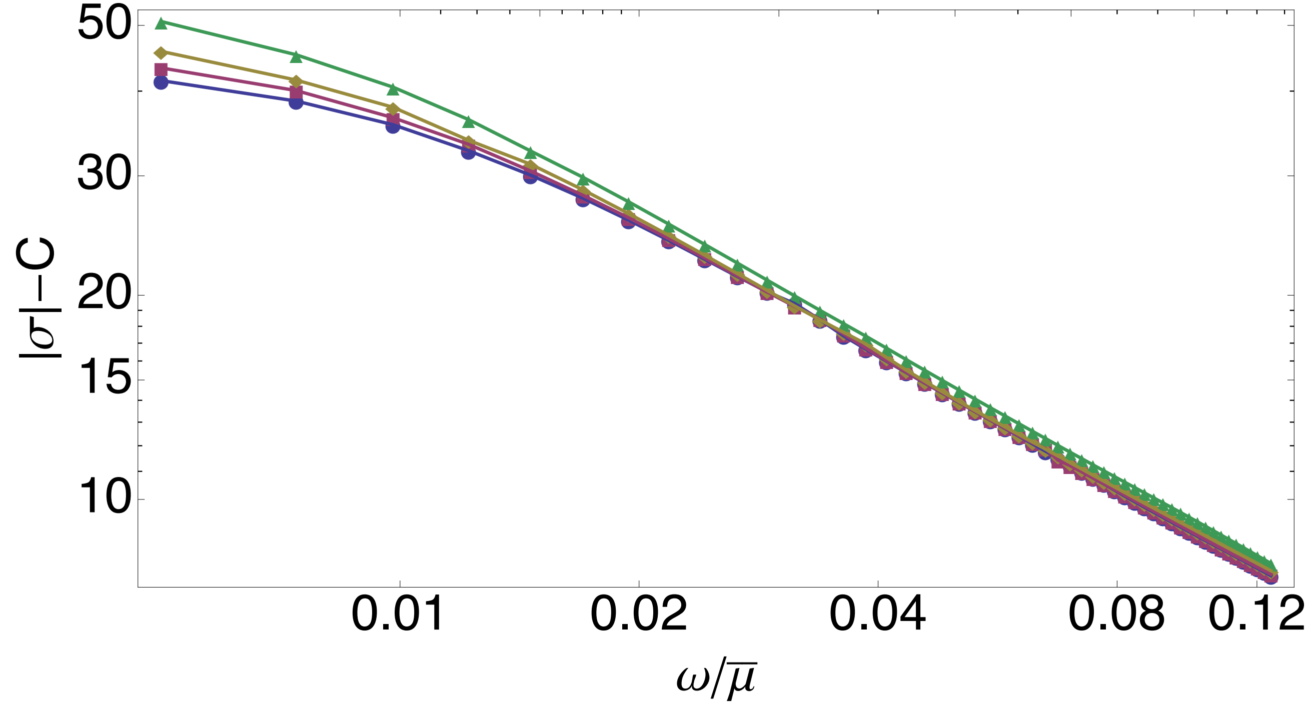
The low frequency conductivity takes the simple Drude form:

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$



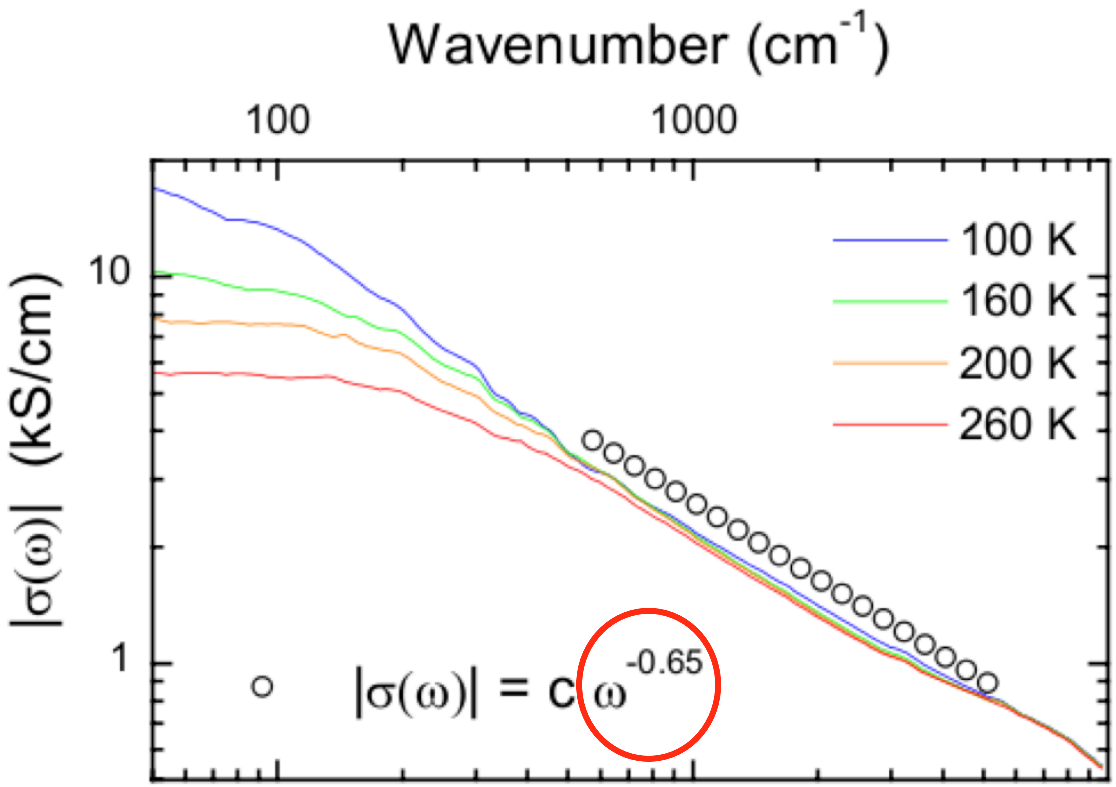
Intermediate frequency shows scaling regime:

$$|\sigma| = \frac{B}{\omega^{2/3}} + C$$



Lines show 4 different temperatures:
.033 < T/μ < .055

Comparison with the cuprates
(van der Marel, et al 2003)



$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

What happens in the superconducting regime?

We now add a charged scalar field to our action:

$$S = \int d^4x \sqrt{-g} \left[R + \frac{6}{L^2} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 2|(\partial - ieA)\Phi|^2 + \frac{4|\Phi|^2}{L^2} \right]$$

Gubser (2008) argued that at low temperatures, charged black holes would have nonzero Φ.

Hartnoll, Herzog, G.H. (2008) showed this was dual to a superconductor (in homogeneous case).

The scalar field has mass $m^2 = -2/L^2$, since for this choice, its asymptotic behavior is simple:

$$\Phi = z\phi_1 + z^2\phi_2 + \mathcal{O}(z^3)$$

This is dual to a dimension 2 charged scalar operator O with source ϕ_1 and $\langle O \rangle = \phi_2$.
We set $\phi_1 = 0$.

For electrically charged solutions with only A_t nonzero, the phase of Φ must be constant.

We keep the same boundary conditions on A_t as before:

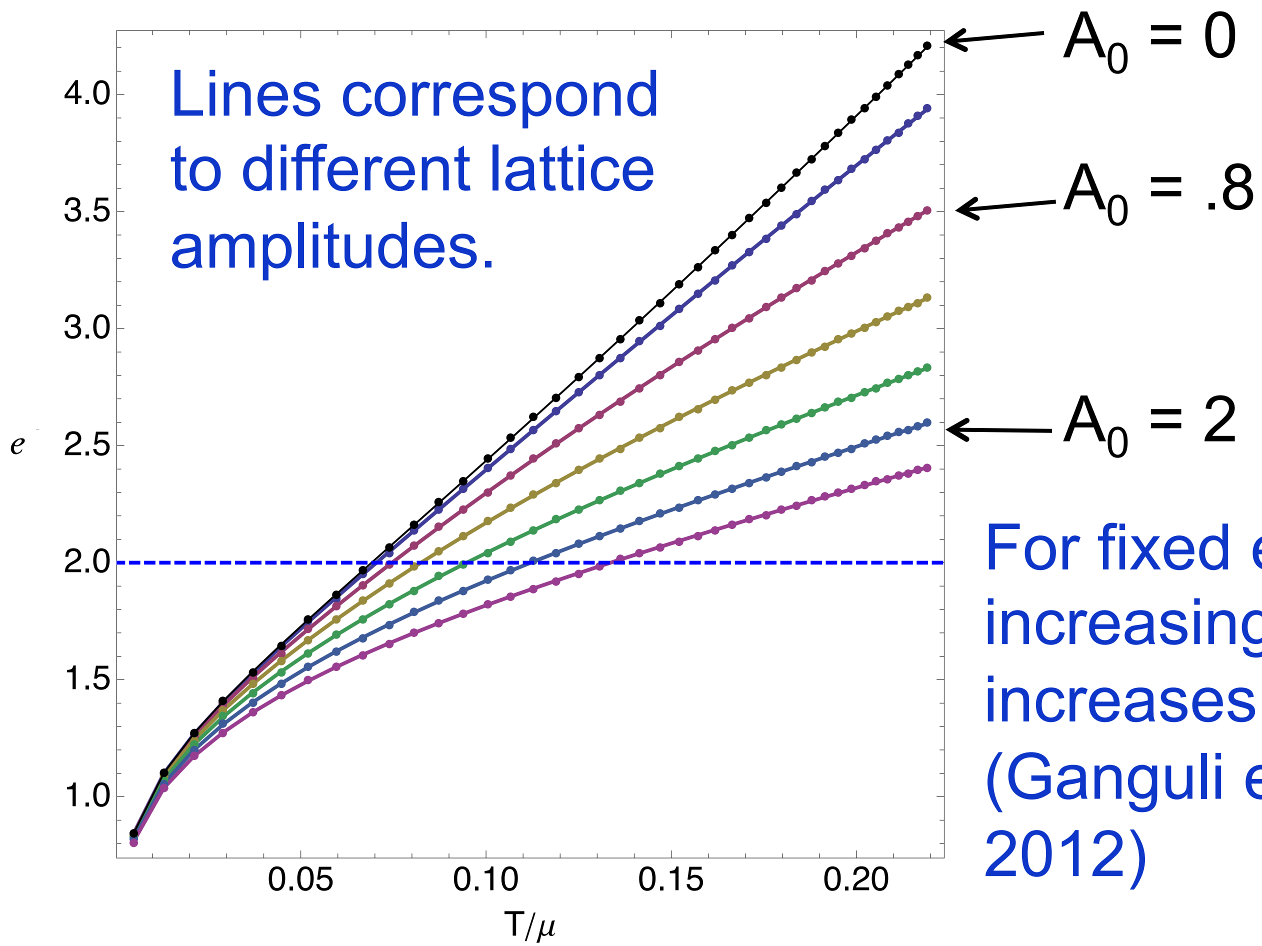
$$\mu(x) = \bar{\mu}\,[1 + A_0 \cos(k_0 x)]$$

Start with previous rippled charged black holes with $\Phi = 0$ and lower T. When do they become unstable?

Onset of instability corresponds to a static normalizable mode of the scalar field.

T_c depends on the charge e of Φ. Larger e makes it easier to condense Φ giving higher T_c.

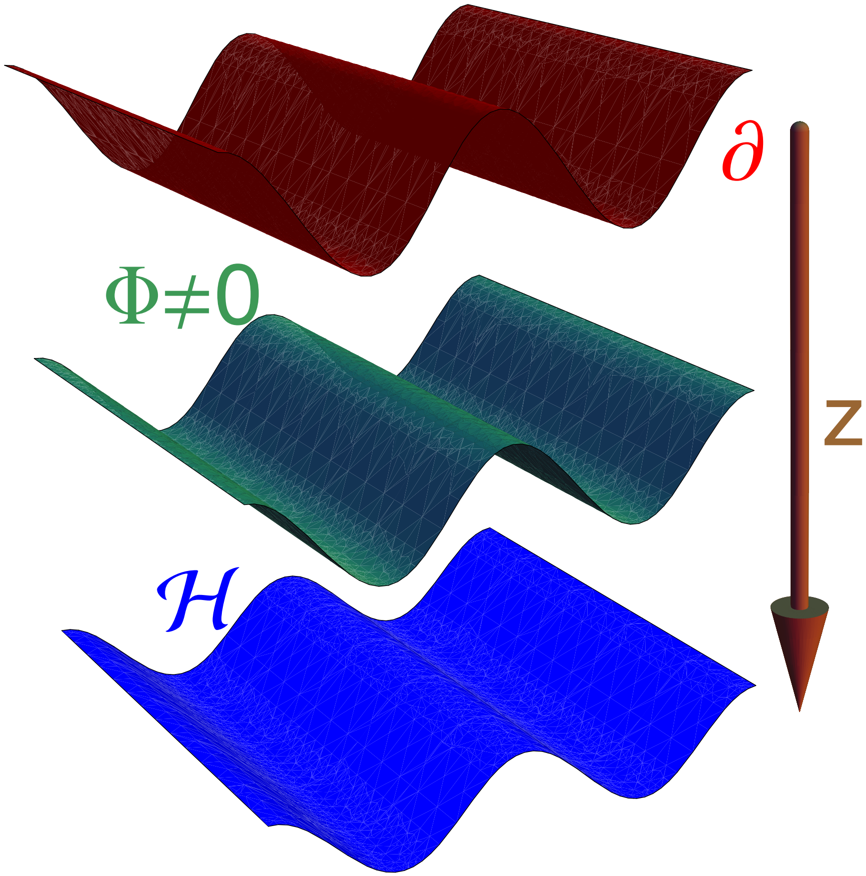
Critical temperature as function of charge



For fixed e, increasing A_0 increases T_c (Ganguli et al, 2012)

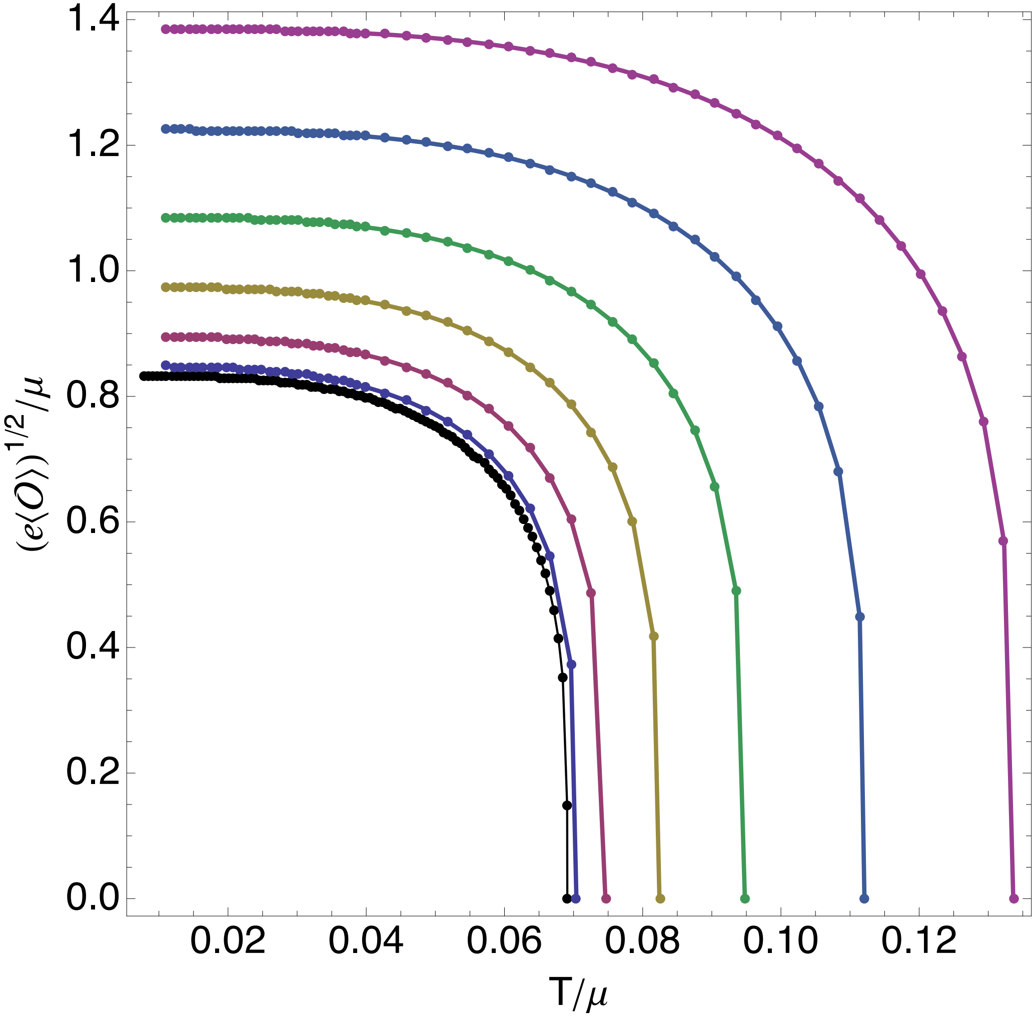
Having found T_c, we now find solutions for $T < T_c$ numerically.

These are hairy, rippled, charged black holes.



From the asymptotic behavior of Φ we read off the condensate as a function of temperature.

Condensate as a function of temperature



Lattice amplitude grows from 0 (inner line) to 2.4 (outer line).

In the homogeneous case, the zero temperature limit is known to take the form

$$ds^2 = r^2(-dt^2 + dx_i dx^i) + \frac{dr^2}{g_0 r^2(-\log r)}$$

and $\Phi = 2(-\log r)^{1/2}$ near r = 0.

With the lattice, the scalar field becomes more homogeneous on the horizon at low T, and S ~ $T^{2.4}$ independent of the lattice amplitude.

We again perturb these black holes as before and compute the conductivity as a function of frequency.

Find that curves at small ω are well fit by adding a pole to the Drude formula

$$\sigma(\omega) = i\frac{\rho_s}{\omega} + \frac{\rho_n \tau}{1 - i\omega\tau}$$

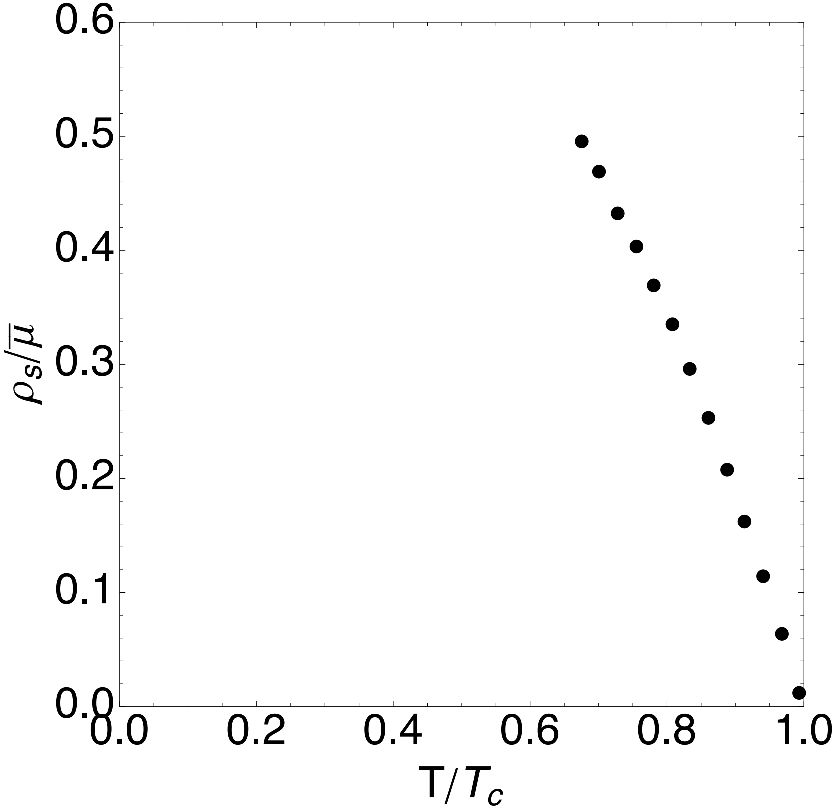
Superfluid component

Normal component

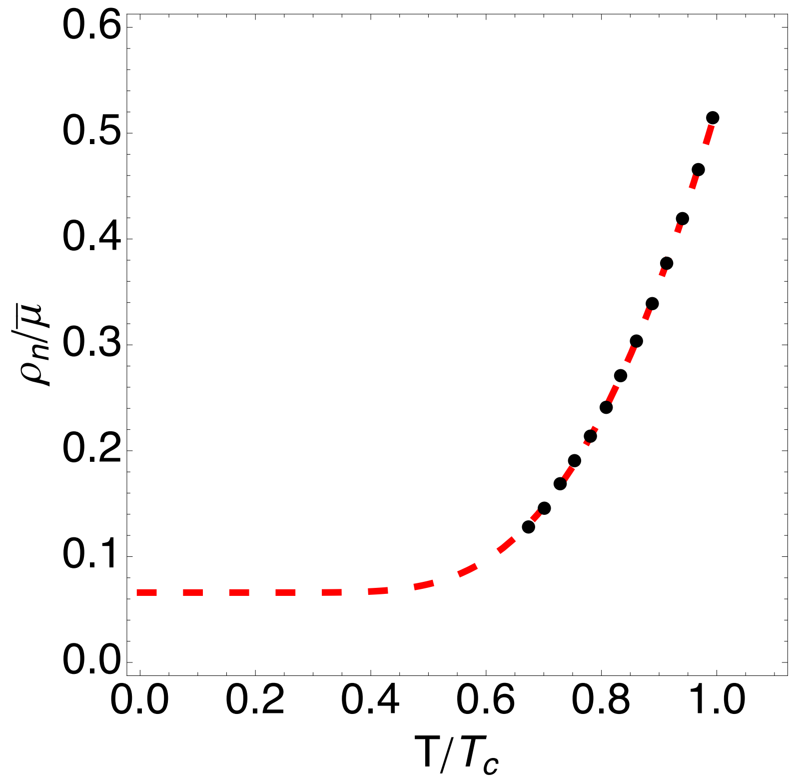
The lattice does not destroy superconductivity

(Siopsis et al, 2012; Iizuka and Maeda, 2012)

Fit to: $\sigma(\omega) = i\frac{\rho_s}{\omega} + \frac{\rho_n \tau}{1 - i\omega\tau}$



superfluid density



normal fluid density

The dashed red line through ρ_n is a fit to:

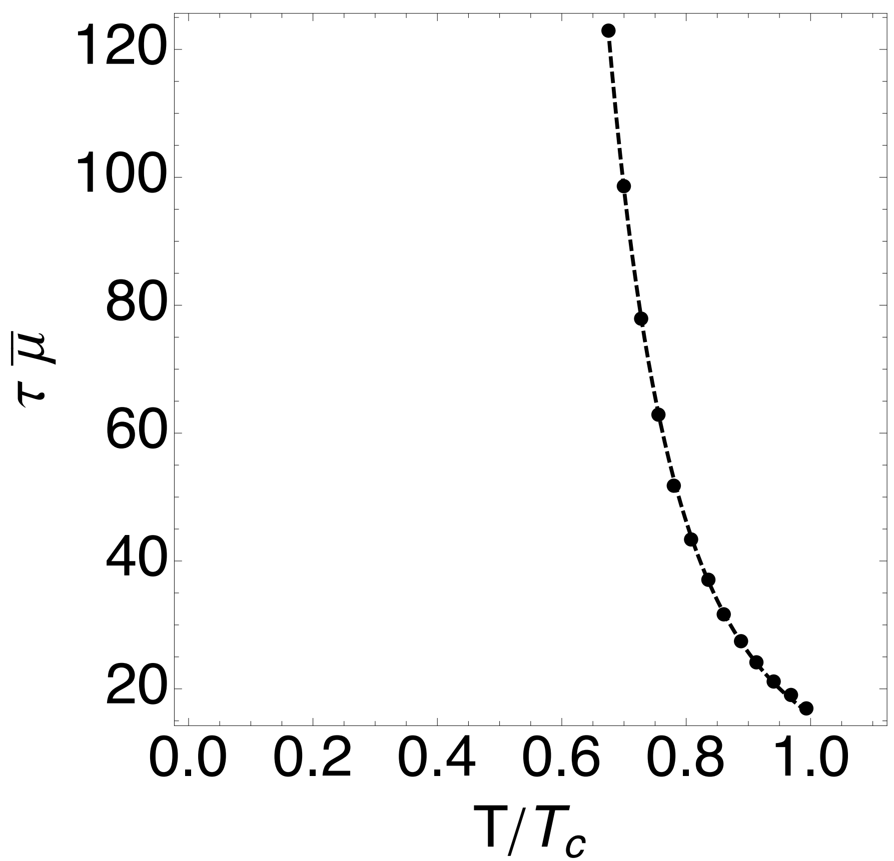
$$\rho_n = a + be^{-\Delta/T}$$

with $\Delta = 4\ T_c$.

This is like BCS with thermally excited quasiparticles but:

(1) The gap Δ is much larger, and comparable to what is seen in the cuprates.
(2) Some of the spectral weight remains uncondensed even at $T = 0$ (this is also true of the cuprates).

The relaxation time rises quickly as the temperature drops:



Line is a fit to

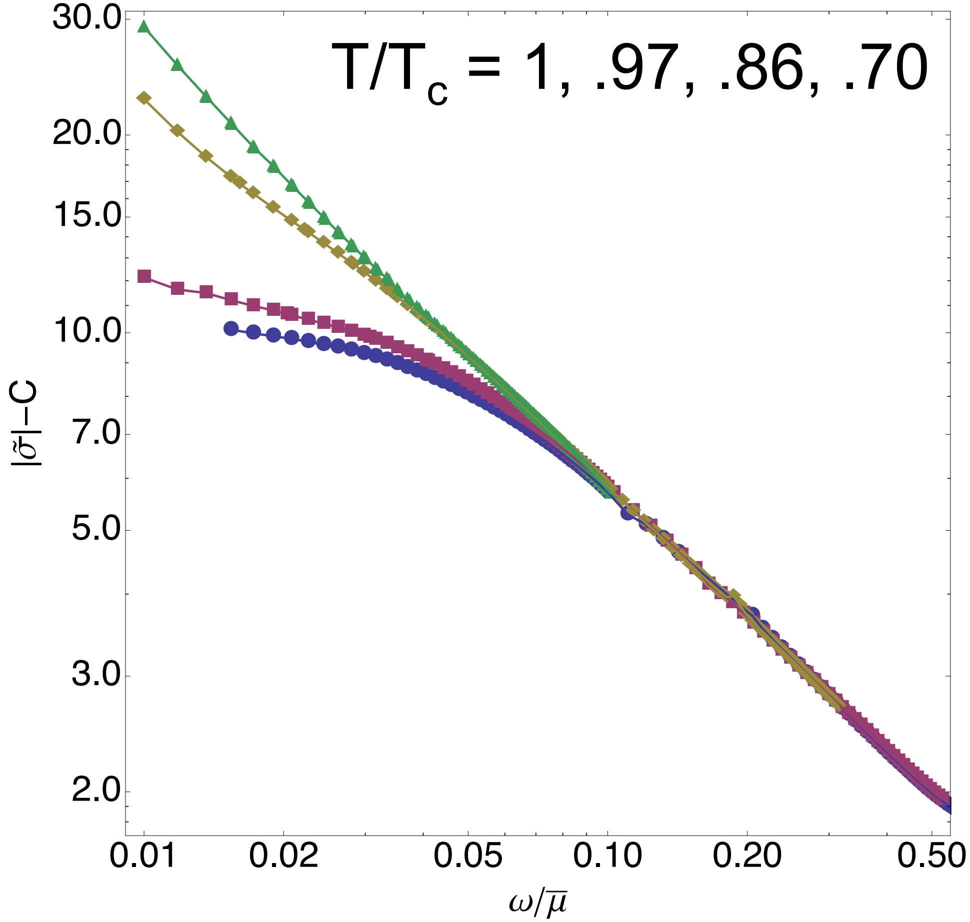
$$\tau = \tau_1 e^{\Delta_1/T}$$

with $\Delta_1 = 4.3\ T_c$

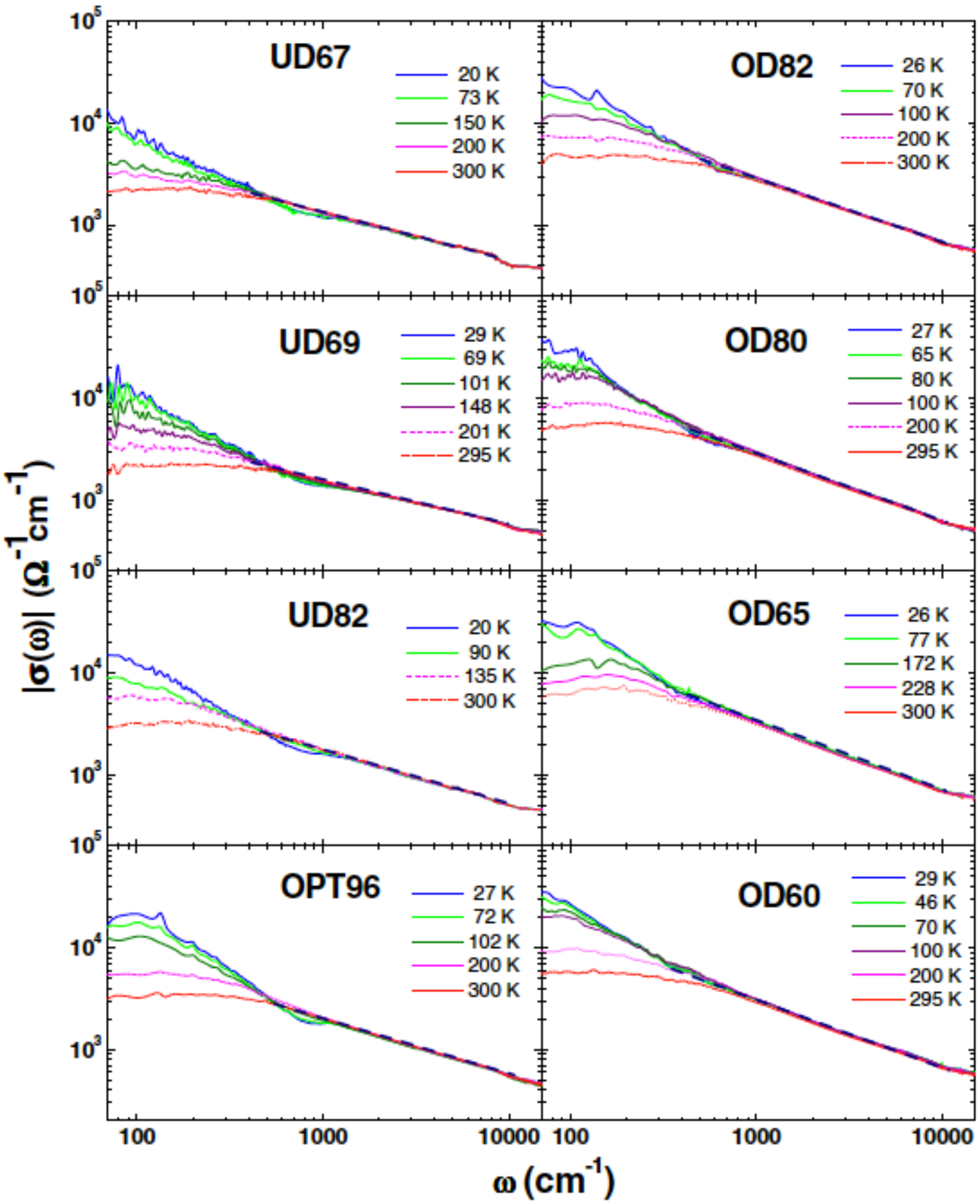
The scattering rate drops rapidly below T_c, another feature of the cuprates.

Intermediate frequency conductivity again shows the same power law:

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$



Coefficient B and exponent 2/3 are independent of T and identical to normal phase.



8 samples of BSCCO with different doping.

Each plot includes $T < T_c$ as well as $T > T_c$.

No change in the power law.

(Data from Timusk et al, 2007.)

The Ferrell-Glover-Tinkham sum rule states:

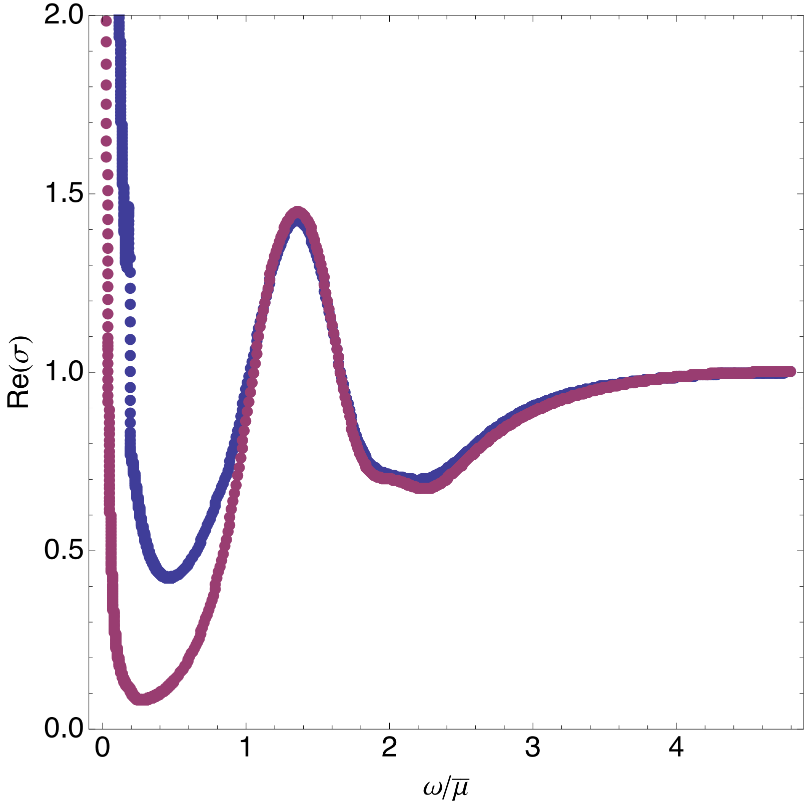
$$\int_{0^+}^{\infty} d\omega \mathrm{Re}[\sigma_N(\omega) - \sigma_S(\omega)] = \frac{\pi}{2}\rho_s$$

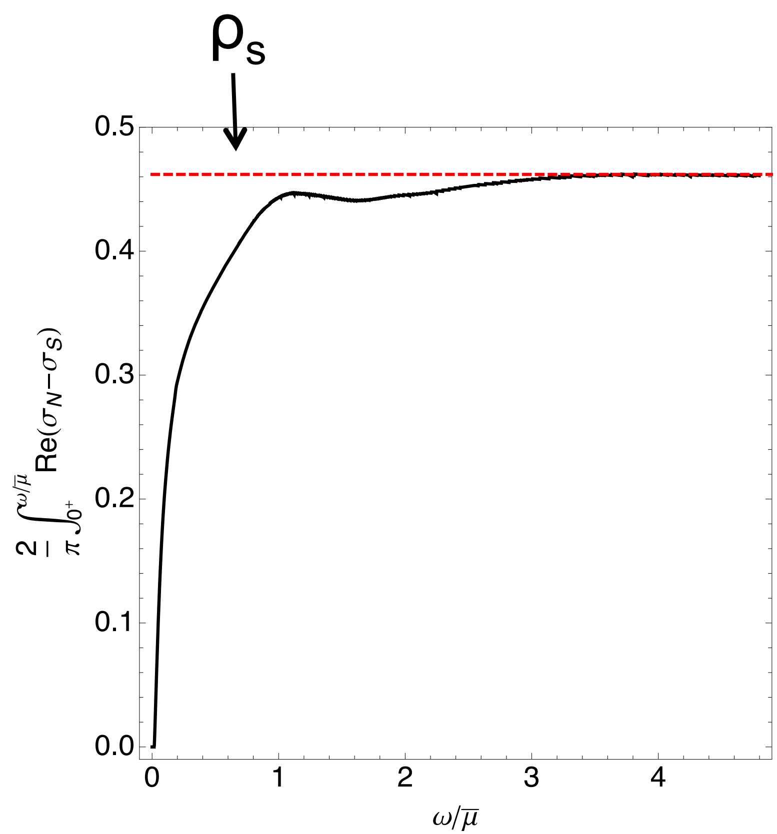
normal phase

superconducting phase

Does this hold in our gravitational model?

Yes





Blue: T = T_c
Red: T = .7 T_c

$$\frac{2}{\pi}\int_{0+}^{\omega} d\tilde{\omega}\,\mathrm{Re}[\sigma_N(\tilde{\omega}) - \sigma_S(\tilde{\omega})]$$

For $T < T_c$, Re[σ] is reduced over a range of ω extending up to the chemical potential.

This is also true for the cuprates, but not for conventional superconductors. In BCS, Re[σ] is reduced over a much smaller range of frequency: $2\Delta = 3.5\, T_c << \mu$.

Resonances

At larger frequencies, the optical conductivity has resonances. In the bulk, this is due to quasinormal modes of the charged black hole.

Quasinormal modes: modes that are ingoing at the horizon and normalizable at infinity. Only exist for a discrete set of complex frequencies.

They correspond to poles in retarded Green's functions (Son and Starinets, 2002).

One can determine the quasinormal mode frequency by fitting

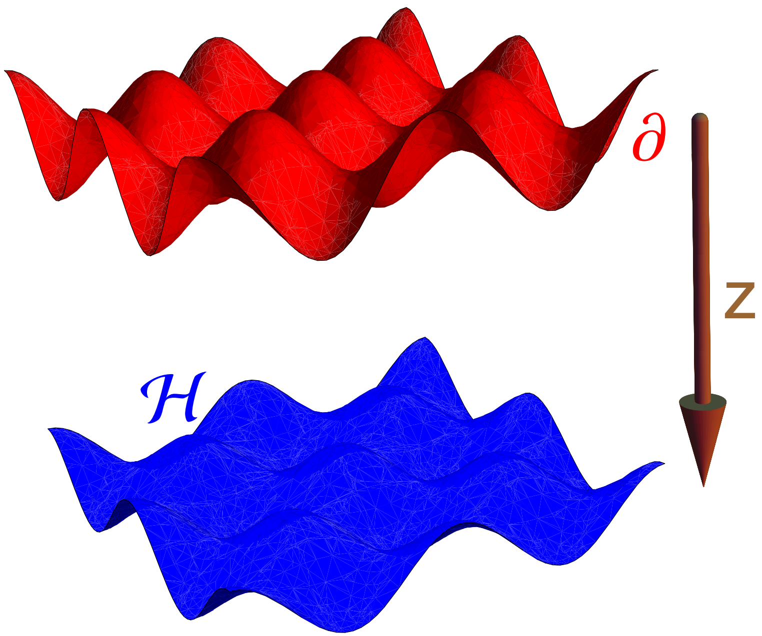
$$\sigma(\omega) = \frac{G^R(\omega)}{i\omega} = \frac{1}{i\omega}\frac{a + b(\omega - \omega_0)}{\omega - \omega_0}$$

One finds:

$$\omega_0/\bar{\mu} = 1.48 - 0.42i$$

Preliminary results on a full 2D lattice ($T > T_c$) show very similar results to 1D lattice.

The optical conductivity in each lattice direction is nearly identical to the 1D results.



Our simple gravity model reproduces many properties of cuprates:

- Drude peak at low frequency
- Power law fall-off $\omega^{-2/3}$ at intermediate ω
- Rapid decrease in scattering rate below T_c
- Gap $2\Delta = 8\ T_c$
- Normal component doesn't vanish at $T = 0$
- Sum rule holds only if one includes frequencies of order chemical potential

But key differences remain

- Our superconductor is s-wave, not d-wave
- Our power law has a constant off-set C
- …