

Aspects of Exotic branes

Masaki Shigemori

(YITP/Hakubi Center, Kyoto U)

CIAS-YITP Workshop 2013

“String Theory, Black Holes and Holography”

July 2, 2013

With I. Bena, J. de Boer, S. Giusto, D. Myerson & N. Warner,
1004.2521, 1107.2650, 1110.2781, 1209.6056, 1307.xxxx, 1308.xxxx

What are
exotic branes?

Duality in string theory

- ▶ Maps various branes into one another

$$T: Dp \leftrightarrow D(p + 1), F1 \leftrightarrow P, NS5 \leftrightarrow KKM, \dots$$

$$S: F1 \leftrightarrow D1, NS5 \leftrightarrow D5, \dots$$



“U-duality”

- ▶ Enhances in lower dims.
 - ▶ Torus compactification to D dims

→ U-duality group:

$$E_{k(k)}(\mathbb{Z}), \quad k = 11 - D$$

[Cremmer+Julia, & others] [Hull+Townsend]

D	k	U-duality group G
10A	1	1
10B	1	$SL(2, \mathbb{Z})$
9	2	$SL(2, \mathbb{Z}) \times \mathbb{Z}_2$
8	3	$SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$
7	4	$SL(5, \mathbb{Z})$
6	5	$O(5,5, \mathbb{Z})$
5	6	$E_{6(6)}(\mathbb{Z})$
4	7	$E_{7(7)}(\mathbb{Z})$
3	8	$E_{8(8)}(\mathbb{Z})$

Duality & codim-2 objects

- ▶ U-duality on codim-2 objects produces **exotic branes**

Known codim-2
object

10D/11D branes
wrapped on
internal torus



U-duality

Exotic branes

10D/11D origin
not obvious

can have tension $\sim g_s^{-3}, g_s^{-4}$

[Elitzur+Giveon+Kutasov+Rabinovici]

[Blau+O'Loughlin] [Hull]

[Obers+Pioline+Rabinovici], all 1997

Example: particles in 3D

- ▶ M on T^8 or Type II on $T^7 \rightarrow$ 3D theory
 - \rightarrow U-duality group $E_{8(8)}(\mathbb{Z})$
 - \rightarrow Scalars $\mathcal{V} \in E_{8(8)}(\mathbb{Z}) \setminus E_{8(8)}(\mathbb{R}) / SO(16)$
- ▶ Particle multiplet:
 - \rightarrow Start from a point-like object
 - e.g. D7(3456789) wrapped on T^7
 - \rightarrow Take T- and S-dualities to get other states

Particle multiplet in 3D

Type IIA	P (7), F1 (7), D0 (1), D2 (21), D4 (35), D6 (7), NS5 (21), KKM (42), 5_2^2 (21), 0_3^7 (1), 2_3^5 (21), 4_3^2 (35), 6_3^1 (7), $0_4^{(1,6)}$ (7), 1_4^6 (7)
Type IIB	P (7), F1 (7), D1 (7), D3 (35), D5 (21), D7 (1), NS5 (21), KKM (42), 5_2^2 (21), 1_3^6 (7), 3_3^4 (35), 5_3^2 (21), 7_3 (1), $0_4^{(1,6)}$ (7), 1_4^6 (7)
M-theory	P (8), M2 (28), M5 (56), KKM (56), 5^3 (56), 2^6 (28), $0^{(1,7)}$ (8)

► Notation for exotic states

$$b_n^c : M = \frac{R^b (R^c)^2}{g_s^n} \qquad b_n^{(d,c)} : M = \frac{R^b (R^c)^2 (R^d)^3}{g_s^n}$$

Example: $5_2^2(34567,89) : M = \frac{R_3 \cdots R_7 (R_8 R_9)^2}{g_s^2 l_s^8}$

Duality rules

- ▶ Duality rules can be read off from:

$$T_y: R_y \rightarrow \frac{l_2^2}{R_y}, \quad g_s \rightarrow \frac{l_s}{R_y} g_s \qquad S: g_s \rightarrow \frac{1}{g_s}, \quad l_s \rightarrow g_s^{1/2} l_s$$

- ▶ Example:

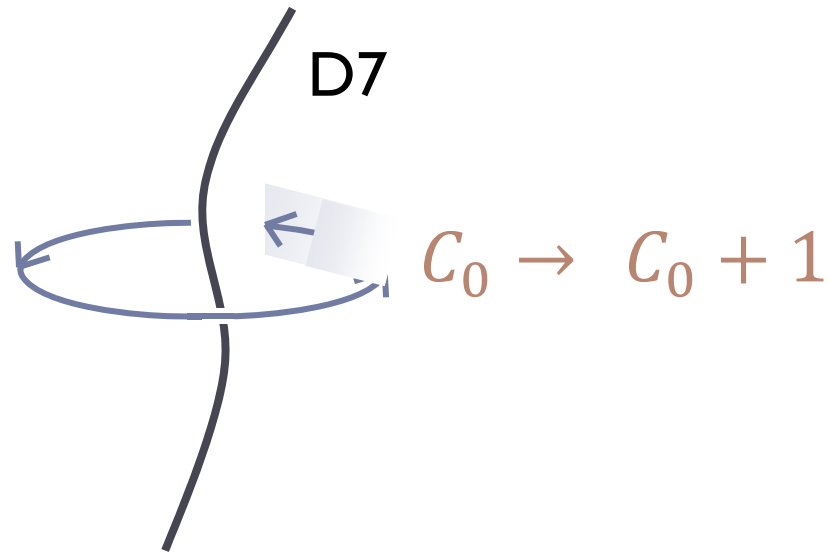
$$\text{NS5}(34567) \xrightarrow{T_8} \text{KKM}(34567,8) \xrightarrow{T_9} 5_2^2(34567,89)$$

$$M = \frac{R_3 \cdots R_7}{g_s^2 l_s^6} \xrightarrow{T_8} \frac{R_3 \cdots R_7}{(l_s/R_8)^2 l_s^6} = \frac{R_3 \cdots R_7 R_8^2}{g_s^2 l_s^8} : 5_2^1 = \text{KKM}$$

$$\xrightarrow{T_9} \frac{R_3 \cdots R_7 R_8^2}{(l_s/R_9)^2 l_s^8} = \frac{R_3 \cdots R_7 (R_8 R_9)^2}{g_s^2 l_s^{10}} : 5_2^2$$

Exotic branes as U-folds

D7-brane

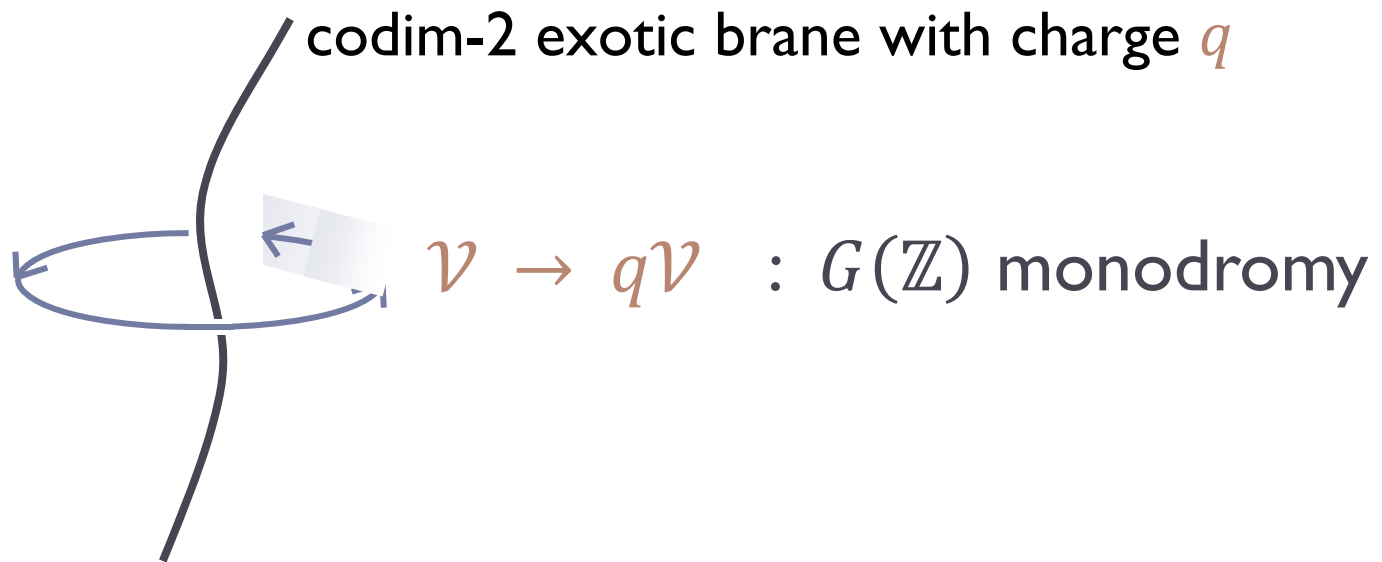


More generally...

- ▶ C_0 : one of moduli scalars \mathcal{V}
- ▶ Shift sym: part of duality group $G(\mathbb{Z})$

Exotic brane = U-fold

- ▶ U -duality group: $G(\mathbb{Z})$
- ▶ Moduli scalars: \mathcal{V}
- ▶ Charge: $q \in G(\mathbb{Z})$

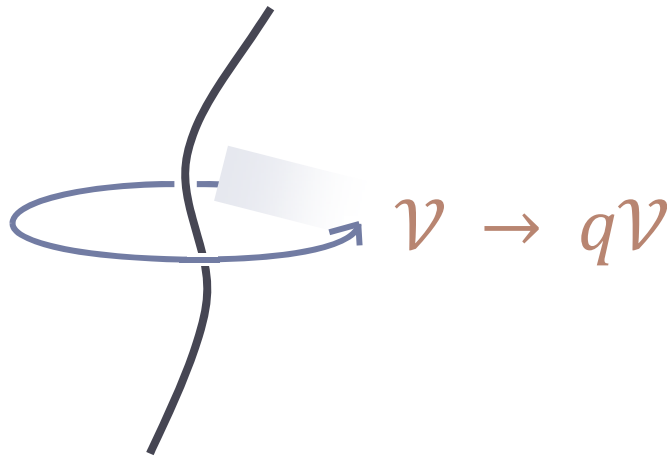


Exotic brane as non-geom bg.

[de Boer+MS 2010, 2012]

- ▶ Moduli \mathcal{V} :
internal components of metric and other fields
- ▶ U-duality mixes them

$$\mathcal{V} \ni G_{ij} \quad B_{ij} \quad \Phi \quad C \quad C_{ij} \quad C_{ijkl}$$



metric not single-valued;
It has monodromy

➔ **non-geometric
background (“U-fold”)**

[Greene+Shapere+Vafa+Yau] [Vafa], [Kumar+Vafa] [Liu+Minasian] [Hellerman+McGreevy+Williams] ...
[Hull] [Dabholkar+Hull] [Flournoy+Wecht+Williams] [Kachru+Schultz+Tripathy+Trivedi]
[McOrist+Morrison+Sethi] ...

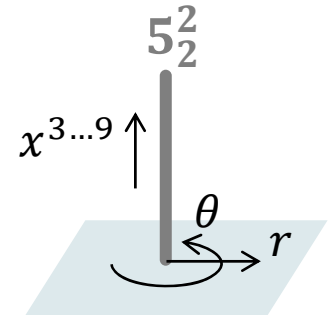
Sugra solution for 5_2^2

[de Boer+MS '10, '12]

$5_2^2(56789,34)$ metric:

$$ds^2 = -dt^2 + H(dr^2 + r^2d\theta^2) + HK^{-1}dx_{34}^2 + dx_{56789}^2$$

$$B_{34} = -K^{-1}\theta\sigma, \quad e^{2\Phi} = HK^{-1}, \quad K \equiv H^2 + \sigma^2\theta^2$$



$$H(r) = h + \sigma \log\left(\frac{\mu}{r}\right)$$

$$\sigma = \frac{R_3 R_4}{2\pi\alpha'}$$

Cf. [Blau+O'Loughlin 1997]: 6₃¹

► T-fold structure:

$$\theta = 0 : G_{33} = G_{44} = H^{-1}$$

$$\theta = 2\pi : G_{33} = G_{44} = \frac{H}{H^2 + (2\pi\sigma)^2}$$

→ x_3 - x_4 torus size doesn't come back to itself

What are they
good for?

Are they relevant?

- ▶ Codim-2: not well-defined as stand-alone object
 - ▶ Log behavior

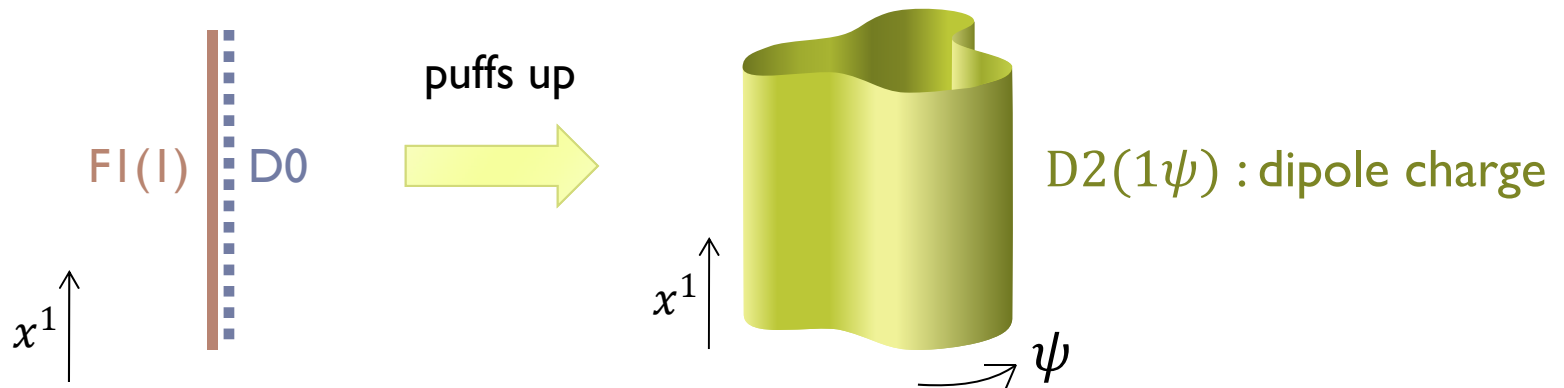
$$V \sim \frac{1}{r^{d-2}} \xrightarrow{d=2} V \sim \log\left(\frac{\mu}{r}\right)$$

- ▶ Resolution:
 - Superpotision (cf. F-theory 7-branes)
 - Configs. with higher codims.

Supertube effect

- ▶ Supertube effect
= spontaneous polarization phenomenon

[Mateos+Townsend 2001]



Dualizing supertubes

$$D0 + F1(1) \rightarrow D2(1\psi)$$

$$\Downarrow T_{234}$$

$$D3(234) + F1(1) \rightarrow D5(1234\psi)$$

$$\Downarrow S$$

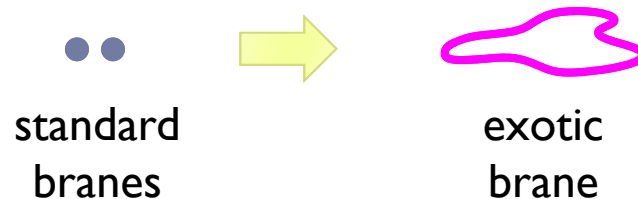
$$D3(234) + D1(1) \rightarrow NS5(1234\psi)$$

$$\Downarrow T_{256}$$

$$D4(3456) + D4(1256) \rightarrow 5_2^2(1234\psi; 56)$$

Exotic supertubes [de Boer+MS '10, '12]

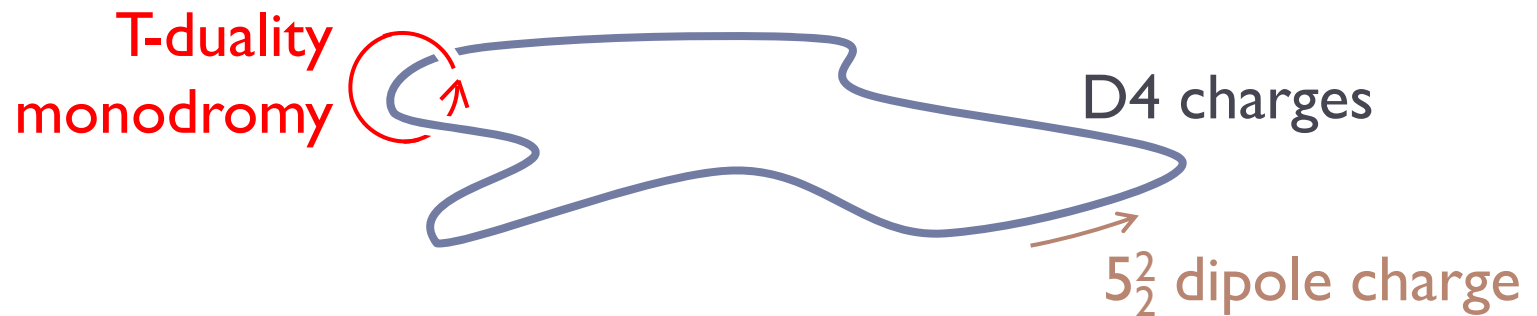
- ▶ Ordinary branes can puff up to produce **exotic dipole** charges



- No log divergence (codim > 2 at large distance)
- Exotic branes relevant to non-exotic physics;
More common than previously thought!

Sugra sol'n for exotic supertube

$$D4(6789) + D4(4589) \rightarrow 5_2^2(4567\psi, 89)$$



D4(6789)+D4(4589) → 5₂² (4567ψ, 89)

$$ds^2 = -\frac{1}{\sqrt{f_1 f_2}} (dt - A)^2 + \sqrt{f_1 f_5} dx_{123}^2 + \sqrt{\frac{f_1}{f_2}} dx_{45}^2 + \sqrt{\frac{f_2}{f_1}} dx_{67}^2 + \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2} dx_{89}^2,$$

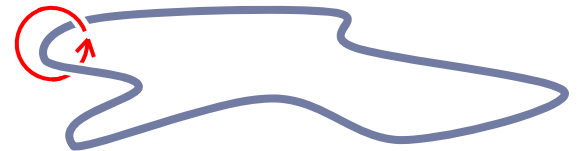
f_i, A : sourced along curve

$$f_1 = 1 + \frac{Q_1}{L} \int_0^L \frac{dv}{|\vec{x} - \vec{F}(v)|}, \quad f_2 = 1 + \frac{Q_1}{L} \int_0^L \frac{|\dot{\vec{F}}(v)|^2}{|\vec{x} - \vec{F}(v)|} dv, \quad A_i = -\frac{Q_1}{L} \int_0^L \frac{\dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|}$$

$$d\beta_I = *_3 df_I, \quad d\gamma = *_3 dA$$

- ▶ β_i, γ have monodromy around curve

$$\beta_I \rightarrow \beta_I - 2Q_I, \quad \gamma \rightarrow \gamma - 2q,$$



- ▶ Asymptotically flat 4D

T-fold structure just as before

- ▶ Non-geometric microstates (cf. [Sen])

$$D4(6789)+D4(4589)\rightarrow 5_2^2 (4567\psi, 89)$$

Other fields:

$$e^{2\Phi} = \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2}, \quad B_{89}^{(2)} = \frac{\gamma}{f_1 f_2 + \gamma^2}, \quad C^{(3)} = -\gamma\rho + \sigma$$

$$\rho = (f_2^{-1} + dt - A) \wedge dx^4 \wedge dx^5 + (f_1^{-1} + dt - A) \wedge dx^6 \wedge dx^7$$

$$\sigma = (\beta_1 - \gamma dt) \wedge dx^4 \wedge dx^5 + (\beta_2 - \gamma dt) \wedge dx^6 \wedge dx^7$$

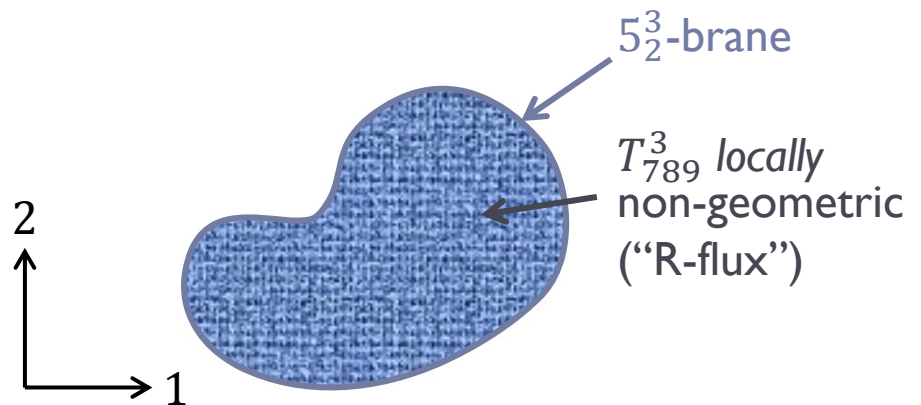
Comments

► Other small-codim objects [Bergshoeff, Riccioni, ...]

□ codim-2: exotic / defect branes

□ codim-1: domain walls E.g. [Haßler+Lüst 2013]

□ codim-0: space filling



$$\begin{aligned} & D5(34789) + D5(56789) \\ & \rightarrow 5_2^3(3456\psi; 789) \end{aligned}$$

Double bubbling & black holes

Double bubbling (1)


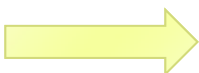
▶ 5D BH system

M2(56), M2(78), M2(9A)

: Well studied for microstate geometry

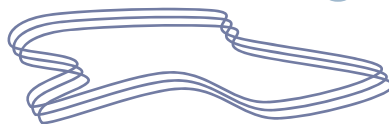
[Mathur] [Bena+Warner] [Berglund+Gimon+Levi]
[de Boer+El-Showk+Messamah+Van de Bleeken]

▶ Possible supertube transitions:

M2(56)	bubble	M5(789A ψ)	bubble	$5^3(789A\phi, 56\psi)$
M2(78)		M5(568A ψ)		$5^3(569A\phi, 78\psi)$
M2(9A)		M5(5678 ψ)		$5^3(5678\phi, 9A\psi)$



cf. black ring



non-geometric

Double bubbling (2)

▶ Standard 4D BH system (MSW)

D0, D4(6789), D4(4589), D4(4567)

: Well studied for microstate counting

▶ Possible supertube transitions:



	D4(6789)	bubble	NS5(6789 ψ)	$5\frac{1}{2}$ (6789,45 ψ)
D0	D4(4589)		NS5(4589 ψ)	$5\frac{1}{2}$ (4589,67 ψ)
	D4(4567)		NS5(4567 ψ)	$5\frac{1}{2}$ (4567,89 ψ)

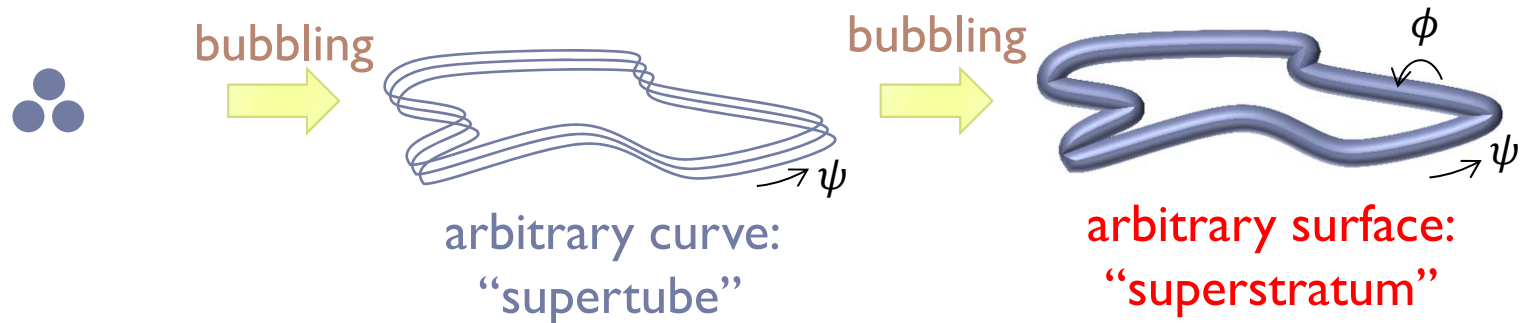


non-geometric

More exotic charges?

Exotic branes & BH microstates

[de Boer+MS 1004.2521] [Bena+de Boer+MS+Warner 1107.2650]
[Bena+Giusto+MS+Warner 1110.2781]



- ▶ Bubbling can in principle occur multiple times, producing all possible branes, including exotic ones
- ▶ *Generic BH microstates involve exotic branes*
- ▶ Still work in progress...

Charge as monodromy

[de Boer+MS 1209.6056]

Charge as monodromy (1)

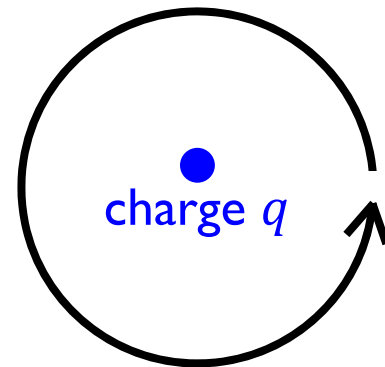
▶ How do we define charge = monodromy?

- Moduli:

$$\mathcal{V} \in G(\mathbb{Z}) \backslash G(\mathbb{R}) / H$$

- Monodromy:

$$\mathcal{V} \rightarrow q\mathcal{V}, \quad q \in G(\mathbb{Z})$$



→ Need more precise data

Charge as monodromy (2)

▶ Defining data

▶ Base point P and value of moduli there, $\mathcal{V}(P)$

▶ Path γ

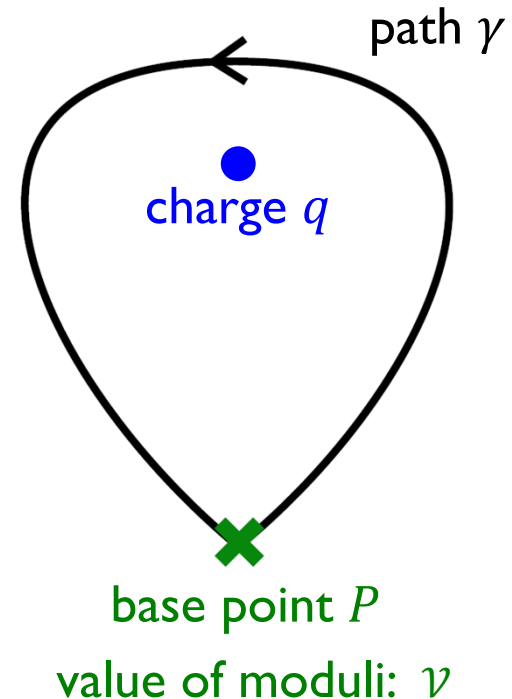
▶ Monodromy q :

$$\mathcal{V} \rightarrow q\mathcal{V}$$

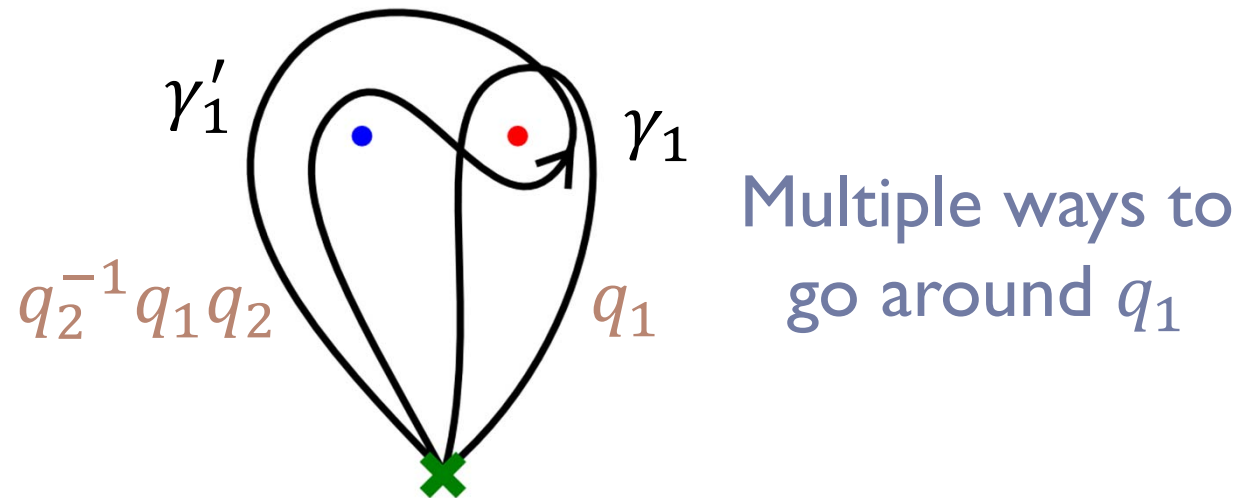
▶ Other moduli values

▶ Moduli value $\tilde{\mathcal{V}} = U\mathcal{V}$

$$\rightarrow \tilde{\mathcal{V}} \rightarrow \tilde{q}\tilde{\mathcal{V}}, \quad \tilde{q} \equiv UqU^{-1}$$

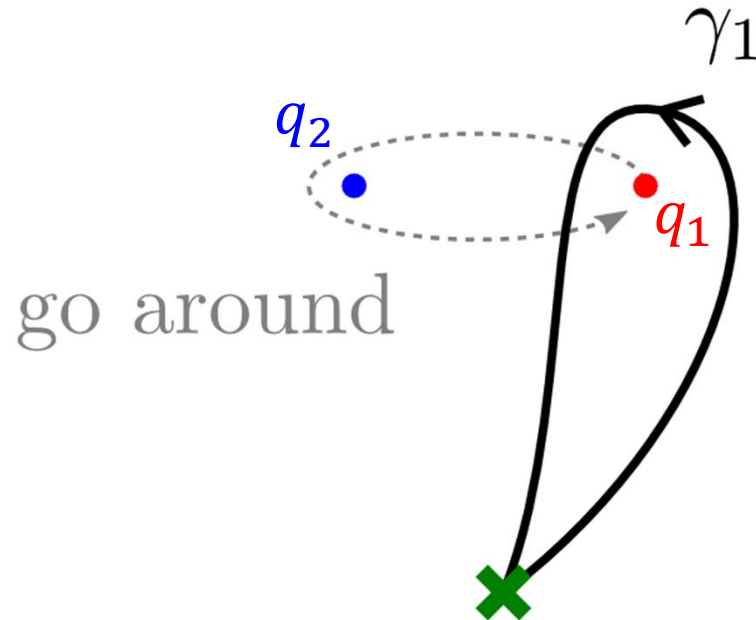


Charge and defining paths



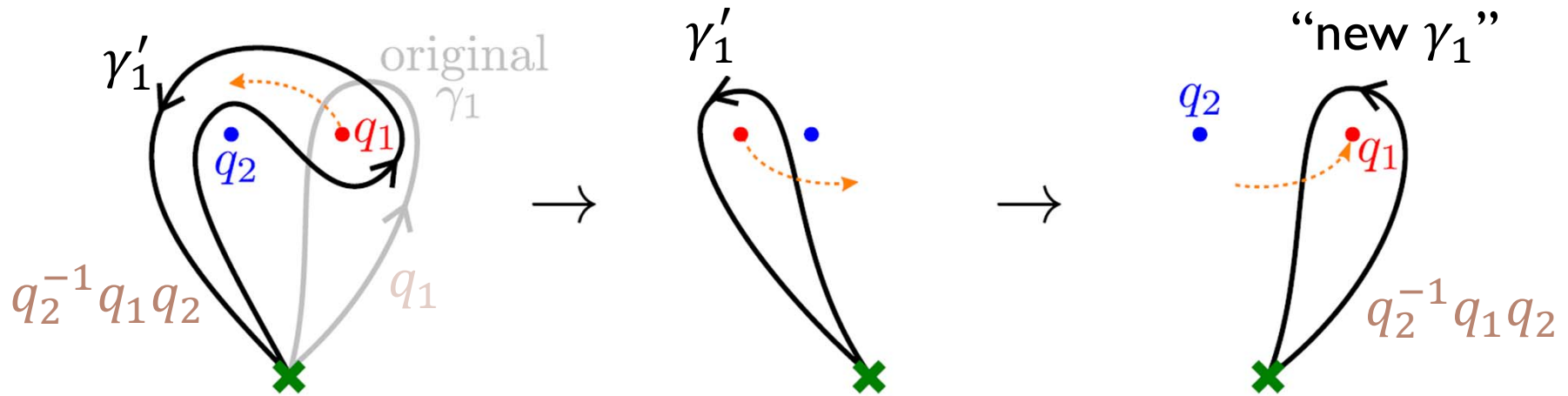
- ▶ Crucial to fix path to define charge
- ▶ Homotopically different paths define different charges related to each other by U -duality

Moving charge around another (1)



Q: What happens to monodromy,
if q_1 is moved around q_2 ?

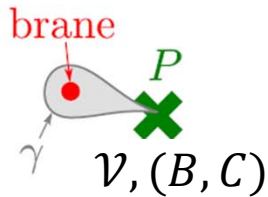
Moving charge around another (2)



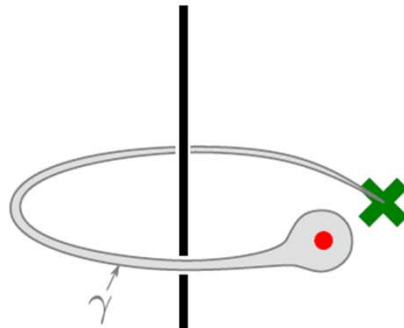
- ▶ After moving q_1 around, γ_1' becomes "new γ_1 "
- ▶ Looks as if charge jumps every time q_1 goes around
- ▶ No problem if we stick to one definition of charge

Lower charges

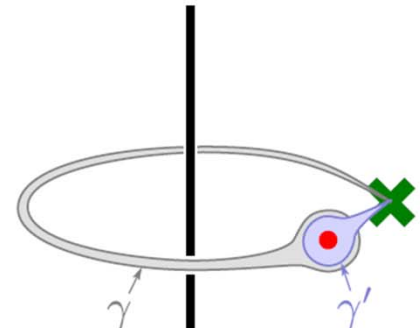
exotic brane



(a)



(b)



(c)

- ▶ Moduli \mathcal{V} and forms (B, C) are multi-valued
- ▶ Charge defined in relation to base point P
- ▶ Different paths define different charges related by U -duality

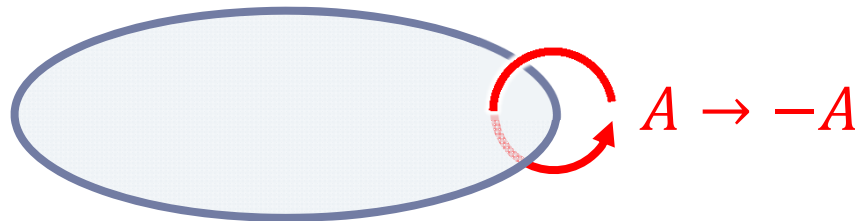
Alice string

Alice string

[A. Schwarz, 1982]

[Kikuchi+Okada+Sakatani | 205.5549]

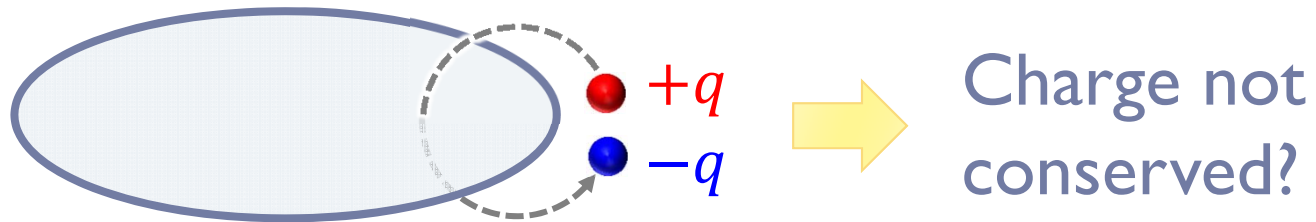
- ▶ Gauge theory with discrete gauge symmetry
 - ▶ E.g. $U(1)$ theory with \mathbb{Z}_2 : $A_\mu = -A_\mu$
- ▶ Vortex solution with \mathbb{Z}_2 twist



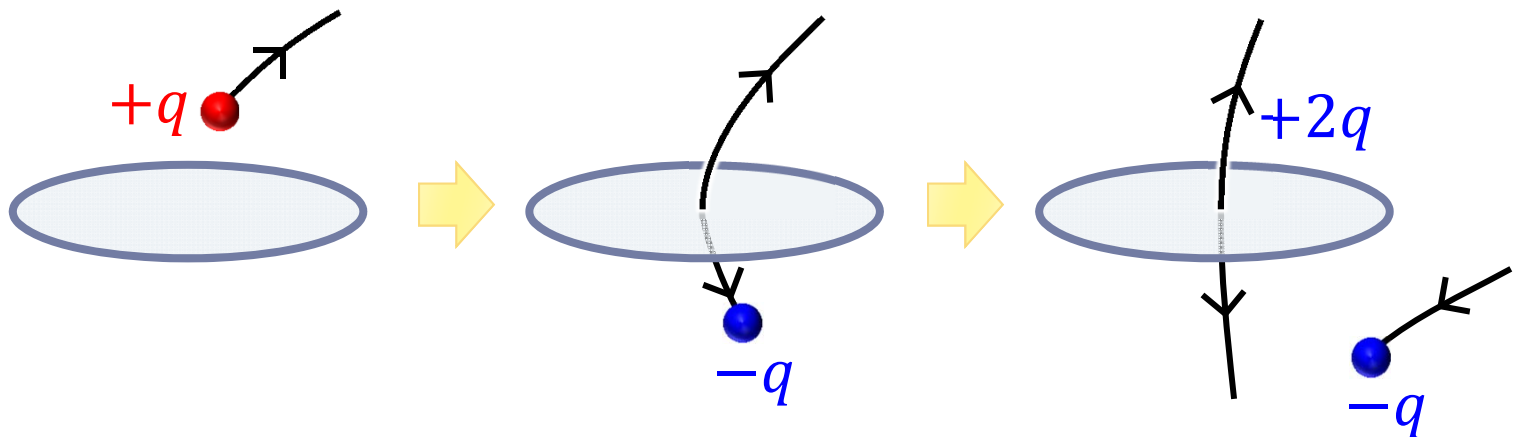
“Alice string”

Alice string & charge conservation

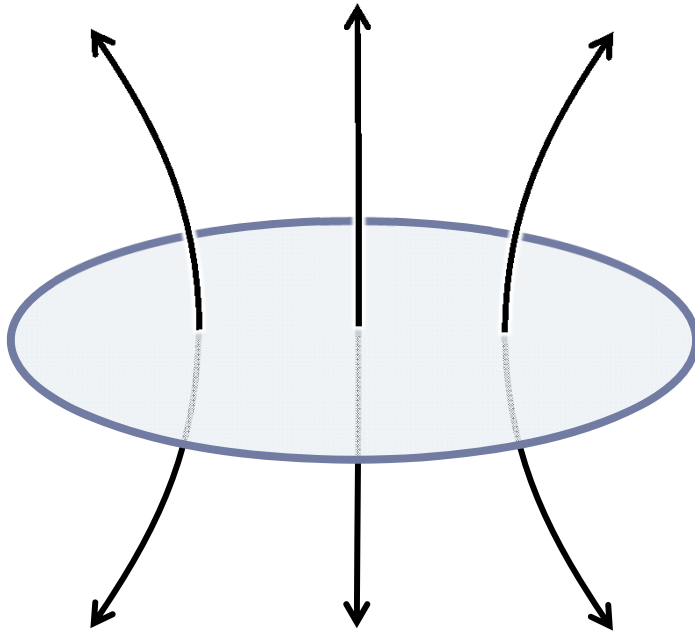
- ▶ Move charge through Alice string loop



- ▶ Charge left behind at the Alice string

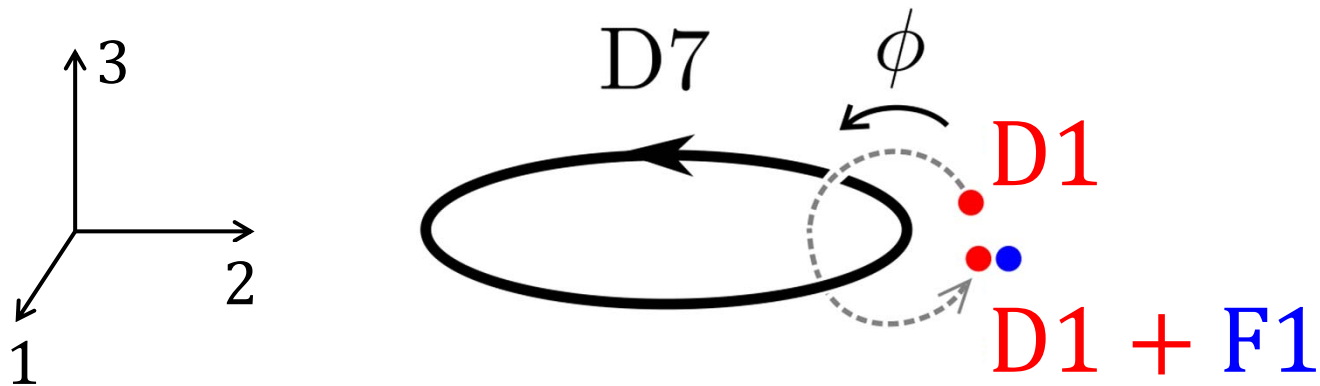


Cheshire charge



Charge without source

D7 as Alice string

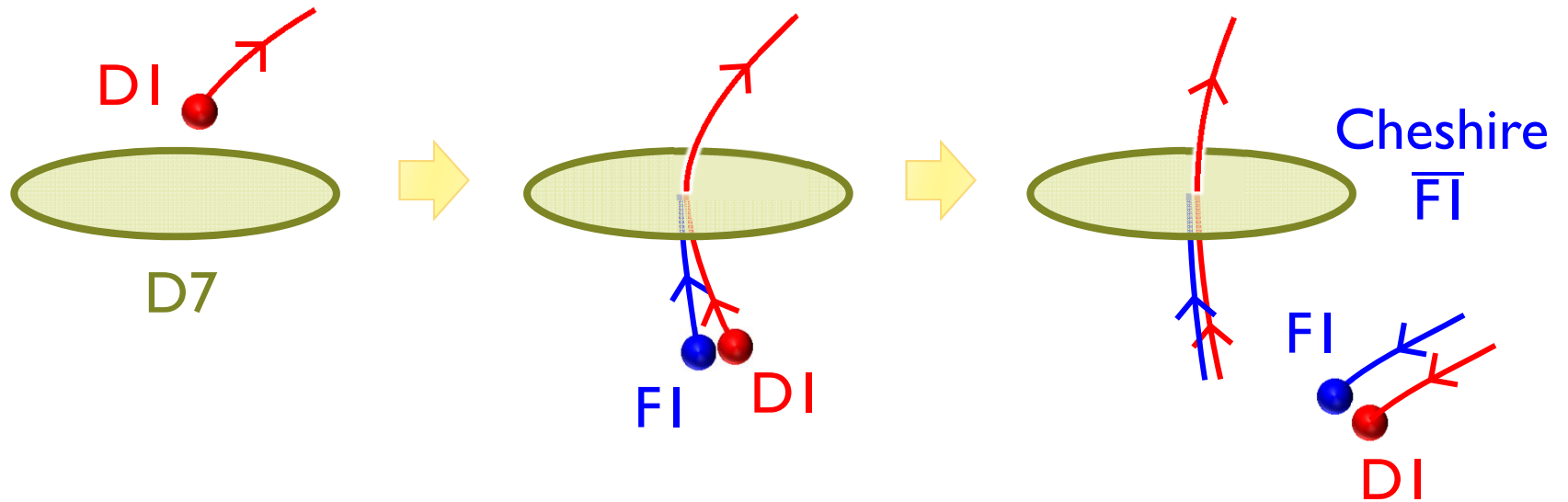


Going around $D7(\psi_{456789})$:

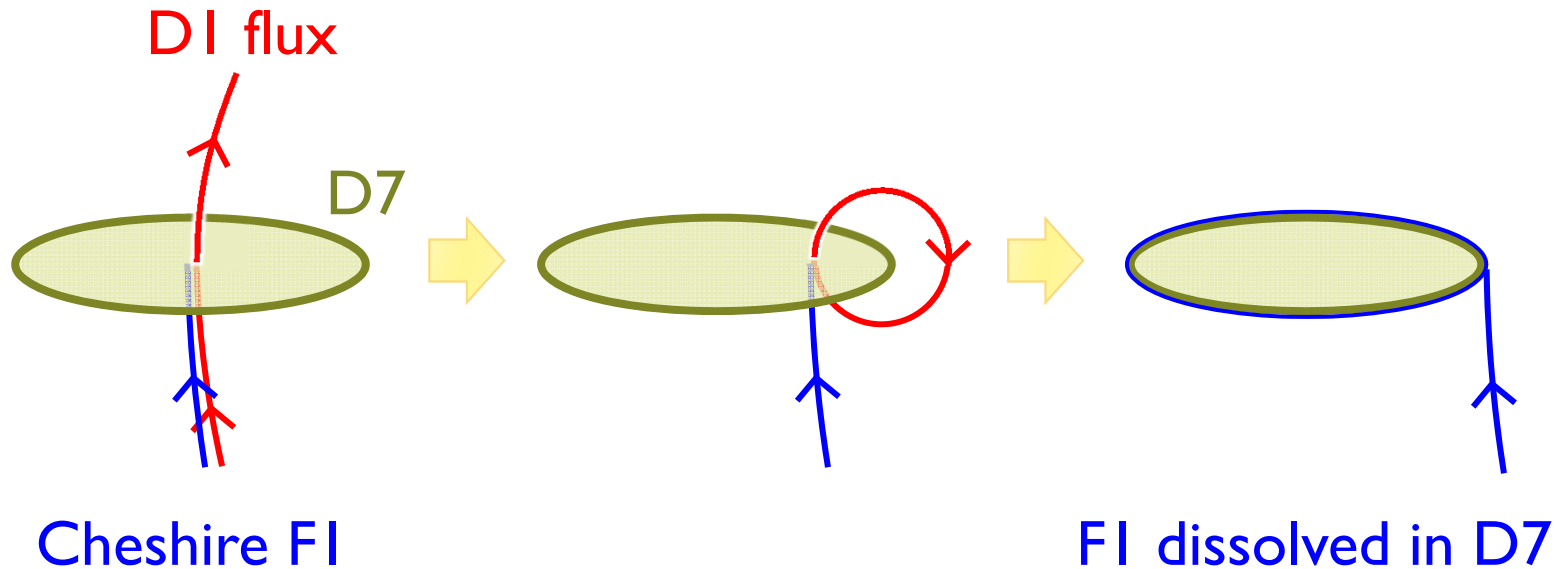
- ▶ Monodromy: $C_0 \rightarrow C_0 + 1$
- ▶ $D1(4) \rightarrow D1(4) + F1(4)$

* Ring D7 stable as supertube $D5(56789) + F1(4)$

Stringy Cheshire charge



Cheshire charge & supertube



➔ Can grow/reduce D5+FI \rightarrow D7 supertube by moving D1 through it

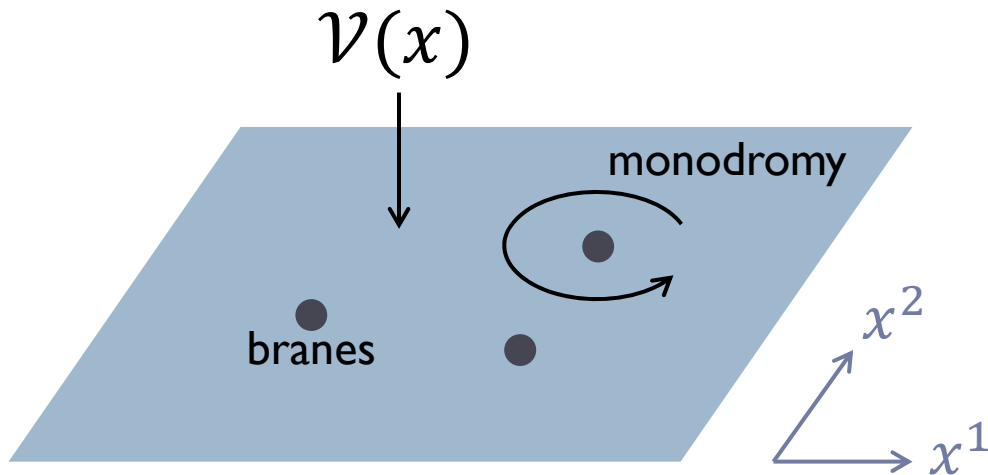
Classification

[de Boer+Mayerson+MS 1307.xxxx]

Classification

“How many” exotic branes are there?

- ▶ Classify configurations up to U -duality group G
- ▶ 3D: codim-2 branes are point-like
- ▶ All fields packaged in moduli matrix $\mathcal{V}(x) \in G/H$



An exercise

Simple example: $G = SL(2), H = SO(2)$

- ▶ 10D IIB sugra, ignore $x^{3\dots 9}$
- ▶ Moduli

$$\tau = \tau_1 + i\tau_2 = C_0 + ie^{-\Phi}$$

$$\mathcal{V} = \begin{pmatrix} \sqrt{\tau_2} & \tau_1/\sqrt{\tau_2} \\ 0 & 1/\sqrt{\tau_2} \end{pmatrix} \in SL(2)/SO(2)$$

Want to classify configurations of τ or \mathcal{V}

Know the answer: 7-branes of F-theory

Susy solutions

[Greene+Shapere+Vafa+Yau 1990]

[Bergshoeff+Hartong+Ortin+Roest 2006]

► Susy transformation

$$\mathcal{V}^{-1}d\mathcal{V} = \underbrace{P}_{sl(2)} + \underbrace{Q}_{so(2)} \in sl(2)$$

$sl(2) \ominus so(2)$ $so(2)$
non-compact, physical compact, gauge

$$\delta\chi \sim P_\mu \Gamma^\mu \epsilon, \quad \delta\psi_\mu \sim (D_\mu + P_\mu)\epsilon$$

► Susy solution

$$ds^2 = -dt^2 + \tau_2 |f|^2 dzd\bar{z} + dx_{3\dots 9}^2, \quad \tau = \tau(z), \quad f = f(z)$$

$$P_z = \frac{\tau'(z)}{2\tau_2} \tilde{Y}, \quad P_{\bar{z}} = \frac{\bar{\tau}'(\bar{z})}{2\tau_2} \tilde{X}, \quad Q = \frac{d\tau_1}{2\tau_2} \tilde{H}$$

↑ nilpotent ↑ nilpotent

$$X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad [X, Y] = H, \quad [H, X] = 2X, \quad [H, Y] = -2Y$$

$$\tilde{X} = \frac{1}{2}(X + Y + iH), \quad \tilde{Y} = \frac{1}{2}(X + Y - iH), \quad \tilde{H} = i(X - Y)$$

The real thing: $E_{8,8}/SO(16)$

Type II on T^7 / M on T^8

- ▶ $d = 3, \mathcal{N} = 16$ sugra [Marcus-Schwarz 1983]
- ▶ U-duality group: $G = E_{8,8}, H = SO(16)$
- ▶ Moduli matrix: $\mathcal{V} \in E_{8,8}/SO(16)$

Susy solution:

$$ds^2 = -dt^2 + e^{U(z,\bar{z})} dzd\bar{z}$$

$$\delta\chi \sim \gamma^\mu \epsilon^I \Gamma_{AA}^I P_\mu^A = 0$$

$$\mathcal{V}^{-1} d\mathcal{V} = P + Q$$

$$\delta\psi_\mu^I \sim (D_\mu + P_\mu) \epsilon^I = 0$$

Strategy

$\delta\chi \sim \gamma^\mu \epsilon^I \Gamma_{AA}^I P_\mu^A = 0$: difficult to solve in general



Embed $SL(2)$ into $E_{8,8}$:

$$P_Z \sim X, \quad P_{\bar{Z}} \sim Y, \quad Q \sim H$$

$$[X, Y] = H, \quad [H, X] = 2X, \quad [H, Y] = -2Y$$



Classify nilpotent elements of $\mathfrak{g} = \mathfrak{e}_{8,8}$
up to conjugation by U -duality $E_{8,8}$

Namely, classify *nilpotent orbits* of $E_{8,8}$

(* For single-center case, can almost show nilpotency without susy)

Nilpotent orbits of $E_{8,8}$

- ▶ 115 nilpotent orbits
- ▶ Representatives X, Y, H explicitly known [Djokovic 2000]
- ▶ Can compute #susy preserved

C#	C BC lbl	wtd diag	dim	$\mathbb{R}\#$	\mathbb{R} label	inv	odr	susy	representative for X
1	A_1	00000001	58	1	A_1	64	3	16	E_{120}
2	$2A_1$	10000000	92	2	$2A_1$	44	3	8	$E_{97} + E_{120}$
3	$3A_1$	00000010	112	3	$3A_1$	40	4	4	$E_{74} + E_{104} + E_{118}$
4	A_2	00000002	114	4	$(4A_1)''$	70	5	4	$E_8 + E_{74} + E_{104} + E_{118}$
				5	$(4A_1)''$	64	5	0	$E_8 + E_{74} + E_{104} - E_{118}$
					A_2		5	0	$\sqrt{2}(E_8 + E_{119})$
5	$4A_1$	01000000	128	6	A_2	32	4	2	$E_{69} + E_{91} + E_{106} + E_{114}$
6	$A_2 + A_1$	10000001	136	7	$5A_1$	38	5	2	$E_{47} + E_{81} + E_{97} + E_{100} + E_{110}$
				8	$5A_1$	32	5	0	$E_{47} - E_{81} + E_{97} + E_{100} + E_{110}$
					$A_2 + A_1$		5	0	$\sqrt{2}(E_{47} + E_{112}) + E_{97}$
7	$A_2 + 2A_1$	00000100	146	9	$6A_1$	38	5	2	$E_{61} - E_{73} - E_{84} + E_{97} + E_{98} + E_{99}$
				10	$6A_1$	26	5	0	$E_{61} + E_{73} - E_{84} + E_{97} + E_{98} + E_{99}$
					$A_2 + 2A_1$		5	0	$\sqrt{2}(E_{73} + E_{102}) + E_{61} + E_{97}$
8	A_3	10000002	148	11	A_3	32	7	0	$\sqrt{3}(E_8 + E_{74}) + 2E_{97}$
9	$A_2 + 3A_1$	00100000	154	12	$7A_1$	50	5	2	$E_{44} + E_{71} - E_{83} + E_{89} + E_{90} + E_{91} - E_{92}$
				13	$7A_1$	26	5	0	$E_{44} + E_{71} + E_{83} + E_{89} + E_{90} + E_{91} - E_{92}$
					$A_2 + 3A_1$		5	0	$\sqrt{2}(E_{83} + E_{95}) + E_{44} + E_{71} + E_{89}$
10	$2A_2$	20000000	156	14	$8A_1$	92	5	2	$E_1 + E_{44} + E_{71} - E_{83} + E_{89} + E_{90} + E_{91} - E_{92}$
				15	$8A_1$	50	5	0	$E_1 + E_{44} + E_{71} + E_{83} + E_{89} + E_{90} + E_{91} - E_{92}$
					$A_2 + 4A_1$		5	0	$\sqrt{2}(E_{83} + E_{95}) + E_1 + E_{44} + E_{71} + E_{89}$
				16	$8A_1$	44	5	0	$E_1 - E_{44} + E_{71} + E_{83} + E_{89} + E_{90} + E_{91} - E_{92}$
					$A_2 + 4A_1$		5	0	$\sqrt{2}(E_{83} + E_{95}) + E_1 - E_{44} + E_{71} + E_{89}$
	$2A_2$		5	0	$\sqrt{2}(E_1 + E_{112}) + \sqrt{2}(E_{44} + E_{96})$				

Susy configs (1)

- ▶ Find representatives in higher dims.
- ▶ Are there configs that *cannot* be represented by standard branes?

I/2 BPS

	3	4	5	6	7	8	9
D4	-	-	-	○	○	○	○

I/4 BPS

	3	4	5	6	7	8	9
D4	-	-	-	○	○	○	○
D4	-	○	○	-	-	○	○

Susy configs (2)

I/8 BPS

	3	4	5	6	7	8	9
D4	-	-	-	○	○	○	○
D4	-	○	○	-	-	○	○
D4	-	○	○	○	○	-	-
D0	-	-	-	-	-	-	-

Susy configs (3)

I/I6 BPS

	3	4	5	6	7	8	9
D4	-	-	-	○	○	○	○
D4	-	○	○	-	-	○	○
D4	-	○	○	○	○	-	-
D0	-	-	-	-	-	-	-
FI	○	-	-	-	-	-	-
KKM	○	⊙	-	○	○	○	○
KKM	○	○	○	⊙	-	○	○
KKM	○	○	○	○	○	⊙	-

Comments

- ▶ All *single-center susy* configs can be represented by standard branes
- ▶ New $\frac{1}{16}$ -BPS config with standard branes??
- ▶ *Non-susy single-center* configs involve exotic branes
- ▶ *Multi-center* configs involve exotic branes
- ▶ Work in progress toward more complete classification...

Conclusions

Summary

- ▶ Exotic branes are
 - ▶ Non-geometric U -folds
 - ▶ More common than previously thought
 - Relevant for BH physics?
- ▶ Codim-2 nature leads to non-trivial interplay between charges and monodromies
- ▶ Classification of codim-2 branes
 - ~ classification of nilpotent orbits

Future directions

- ▶ Non-Abelian vortices, Alice electrodynamics
- ▶ Spacetimes with exotic monodromies
[Chiodaroli+D'Hoker+Guo+Gutperle] [Martucci+Morales+Pacifici]
- ▶ Doubled geometry, double (extended) field theory, non-geometric compactification
[Hull]... [Hohm] [Zwiebach][Hillmann][Bermann+Perry]...
[Aldazabal+][Grana+Marquess][Berman+][Andriot+]...
- ▶ Worldsheet description [Kimura+Sasaki]
- ▶ BH microstates

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