Universal quantum dynamics near the two-dimensional Superfluid-to-Insulator transition

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Supported by NSF, NIM DARPA, NNSFC, CAS, NKBRSFC, AFOSR/DoD MURI

Collaborators

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3D XY universality

- **Field theory: (2+1)-Dimensional *relativistic* O(2) model**

$$S = \int_0^{1/T} dt \int d^2 r \left[\left|\partial_t \Psi\right|^2 - c^2 \left|\nabla \Psi\right|^2 - V(\Psi) \right]$$

$$V(\Psi) = \lambda \left|\Psi\right|^2 + u\left(\left|\Psi\right|^2\right)^2$$

- **Complex field Ψ**

- **Quantum critical point : Superfluid-to-Insulator transition**

Mean field scenario and Higgs mode

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u(|\Psi|^2)^2$$

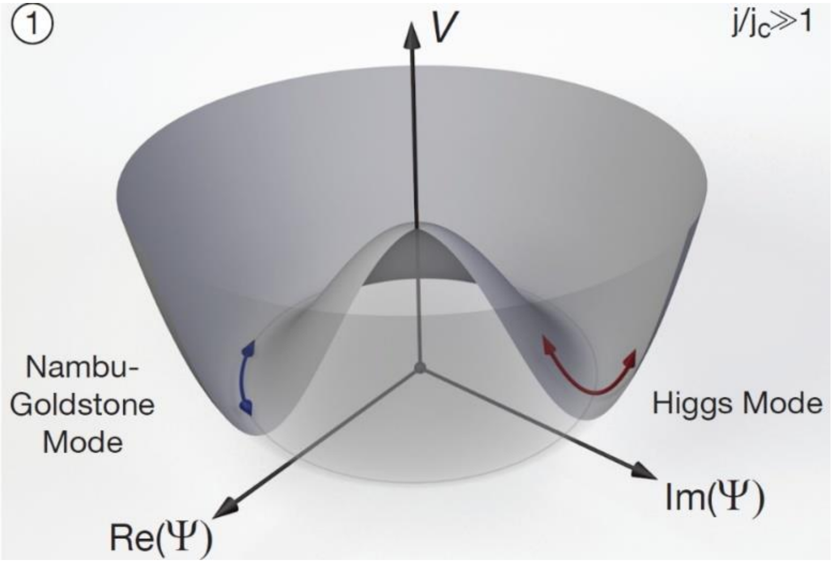
- **Ordered phase (Superfluid phase)**

- **Two decoupled excitation modes**

  Nambu-Goldstone mode

  fluctuation of phase
  massless (gapless)

  Massive Goldstone mode (Higgs mode)
  fluctuation of amplitude
  massive (gapped)



Why 2D?

D=3 Asymptotically exact mean-field → Higgs mode is well-defined.

P. Merchant, B. Normand, K. W. Kramer, M. Boehm, D. F. McMorrow, and Ch. Ruegg
Nature Physics 10, 373-379 (2014)
Ch. Ruegg, *et.al*, Phys. Rev. Lett. 100, 205701 (2008)
U. Bissbort, *et.al*, Phys. Rev. Lett. 106, 205303 (2011)

D=2 Destroyed by strong decay into two Goldstone modes.

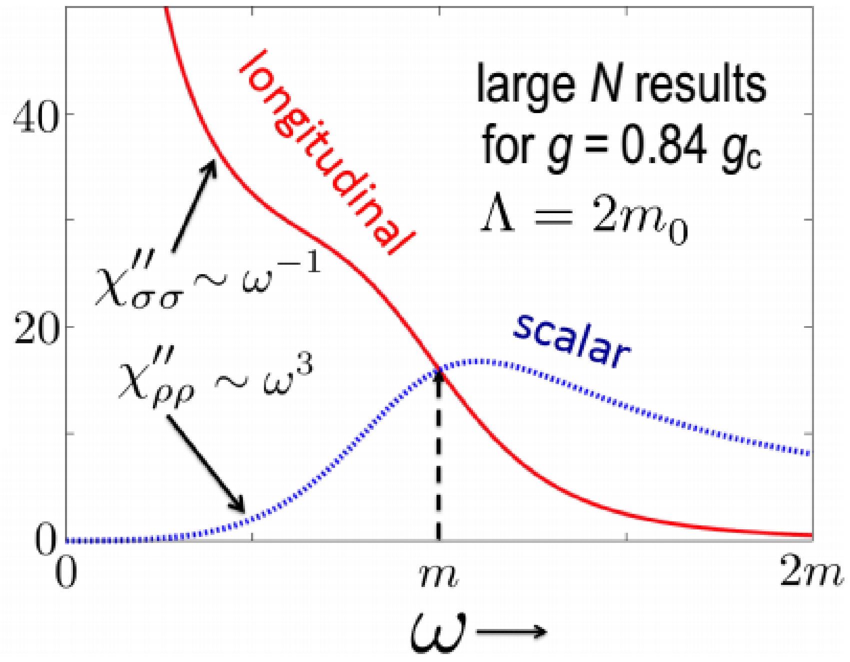
Chubukov, Sachdev, Ye '93
Altman, Auerbach '02
Zwerger '04

Just in case the theory is wrong, the type of the probe matters!

Longitudinal: $S(\omega)=\langle\psi(0)\,\psi(t)\rangle_\omega$

Scalar: $S(\omega)=\left\langle|\psi(0)|^2\;|\psi(t)|^2\right\rangle_\omega$
is the best candidate

Chubukov, Sachdev, Ye '93
Podolsky, Auerbach, Arovas '11



What to expect for the scalar response function?

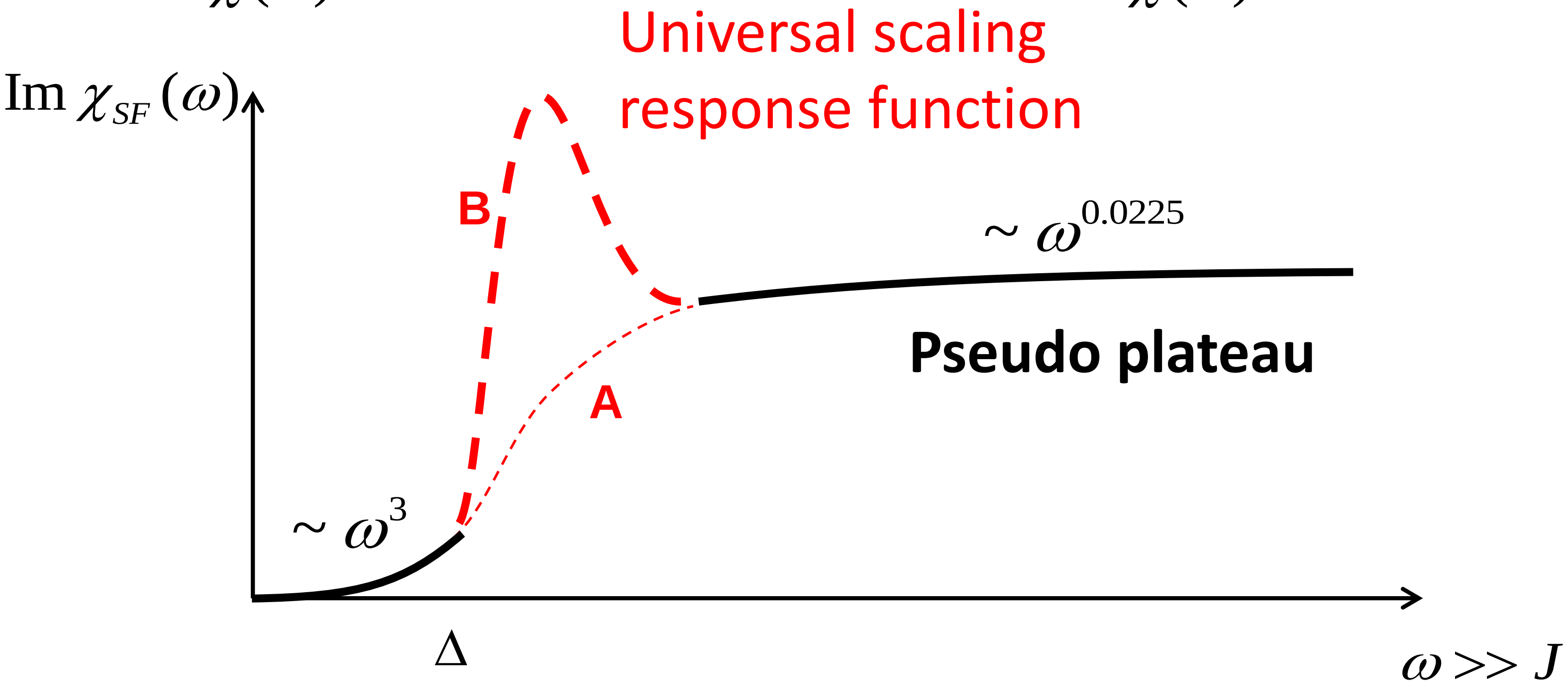
$$\mathrm{Im}\,\chi(\omega) \propto \Delta^{3-2/\nu}\,\Phi(\omega/\Delta)$$

$$\Delta \propto (1-U/U_c)^{\nu}, \quad \nu = 0.6717$$

Chubukov, Sachdev, Ye '93
Sachdev '99

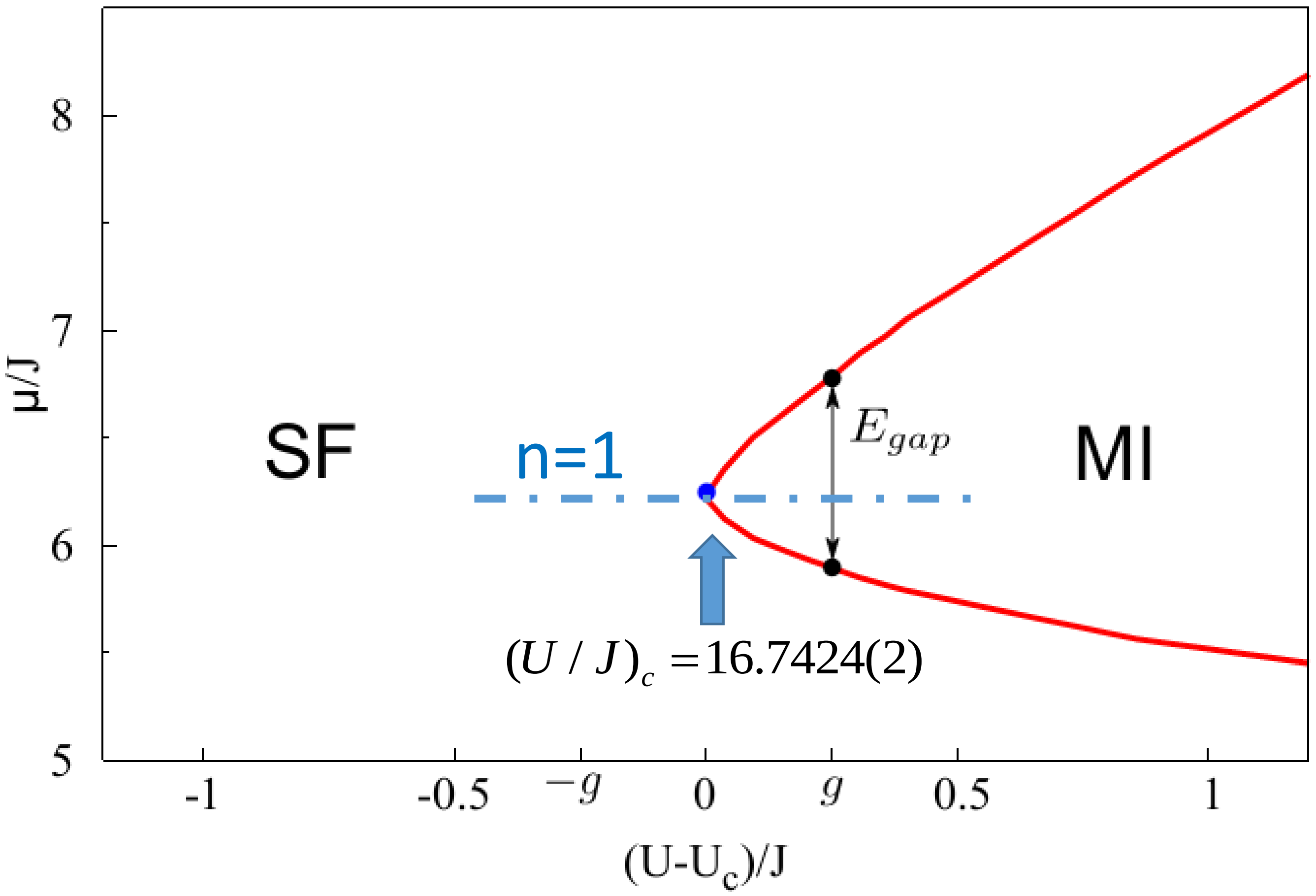
$\omega \sim \Delta$ → $\mathrm{Im}\,\chi(\omega) \propto \omega^{3}$

$\omega >> \Delta$ → $\mathrm{Im}\,\chi(\omega) \propto \omega^{3-2/\nu} = \omega^{0.0225}$



Bose Hubbard model: a testbed

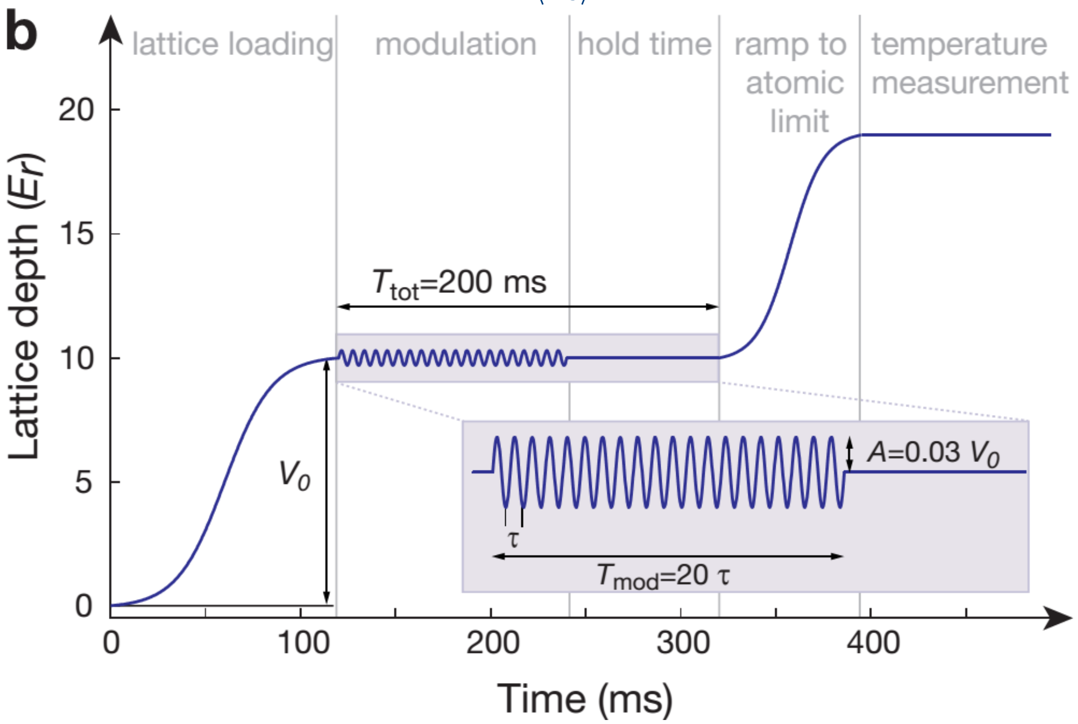
$$\hat{H}_{BH} = -J\sum_{<i,j>} a_i^{\dagger} a_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - (\mu - V_i)\sum_i \hat{n}_i$$



Optical Lattice Emulator for BH model

$$\hat{H} = \hat{H}_{BH} + \frac{\delta J}{J}\hat{K}\, e^{i\omega t} \quad , \quad \hat{K} = -J\sum_{\langle i,j\rangle} b_i^{\dagger}\, b_j$$

Linear response for small $\delta J / J$



Endres, Fukuhara, Pekker, Cheneau, Schau Gross, Demler, Kuhr, Bloch, Nature '12

$$\chi(t) = i\left\langle\left[\hat{K}(t),\hat{K}(0)\right]\right\rangle\Theta(t) \rightarrow$$ **Energy dissipation rate** $\propto \omega\,\mathrm{Im}\,\chi(\omega)$

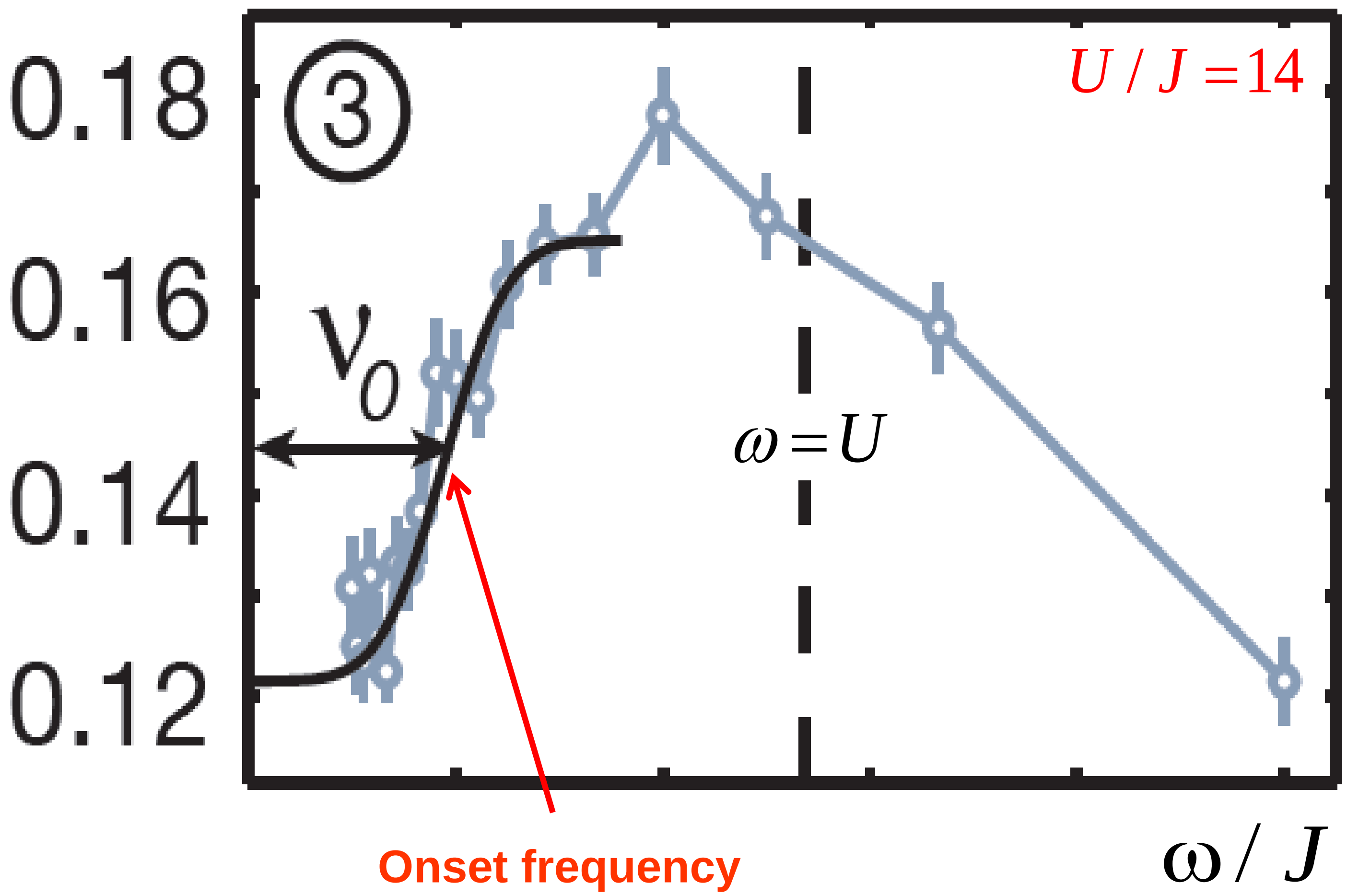
Total energy absorbed: $\Delta E \propto \omega\,\mathrm{Im}\,\chi(\omega)\left(2\pi M/\omega\right) \propto \mathrm{Im}\,\chi(\omega)$

Temperature increase: $\Delta T = \Delta E / C(T)$

Recent experiment @ Munich: Onset of quantum critical response

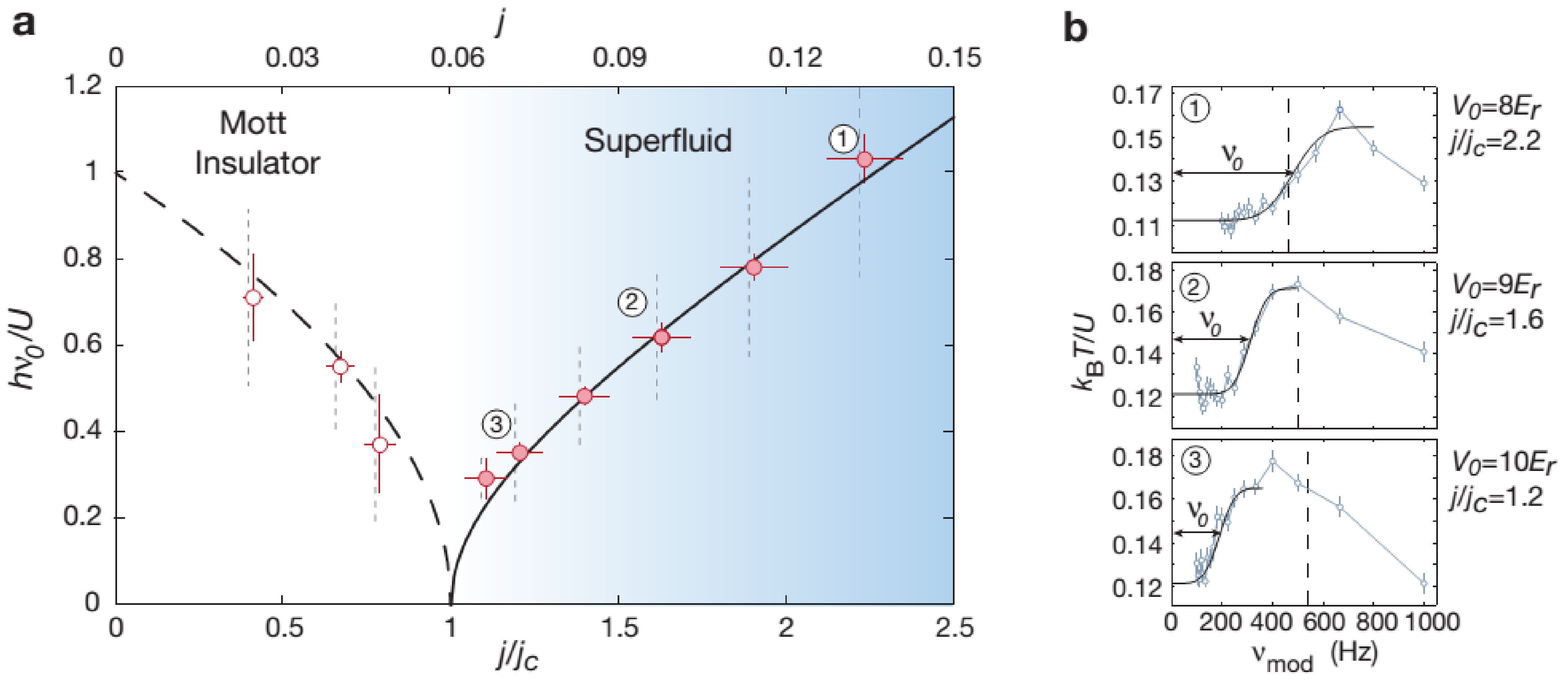
$\Delta T / U \propto \mathrm{Im}\,\chi(\omega)$

Endres, Fukuhara, Pekker, Cheneau, Schau
Gross, Demler, Kuhr, Bloch, Nature '12



Nice scaling behavior, but where is Higgs resonance peak?

$$\nu_0 \propto \Delta \propto \left(1 - J/J_c\right)^{\nu}$$



Time for WORM !

Monte Carlo probe: Kink-kink correlation

Kinetic energy :

$$\beta K_{MC} = -\sum_{kinks}$$

MC measurement:
(Matsubara frequency)

$$K_{MC}(i\omega_n) = \int_0^{\beta} d\tau\, e^{i\omega_n \tau} K_{MC}(\tau) = -\sum_{k=kinks} e^{i\omega_n \tau_k}$$

kink-kink correlation function

$$C(i\omega_n) = \left\langle \left| K_{MC}(i\omega_n) \right|^2 \right\rangle + \left\langle K \right\rangle$$

Analytical continuation:
(real frequency)

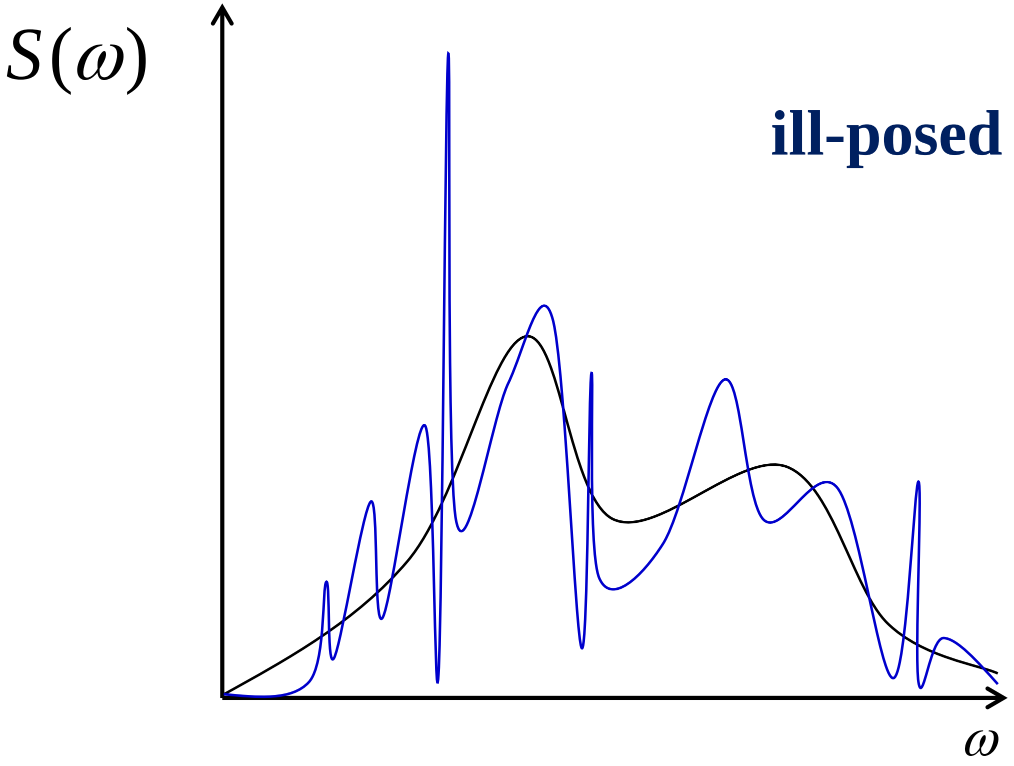
$$C(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left\langle K(t)K(0) \right\rangle e^{-i\omega t} dt$$

But how to do the analytical continuation in numeric sense?

Numerical analytical-continuation method

$$C(\tau) = \frac{1}{2\pi}\int_0^\infty e^{-\omega\tau} S(\omega)\, d\omega$$

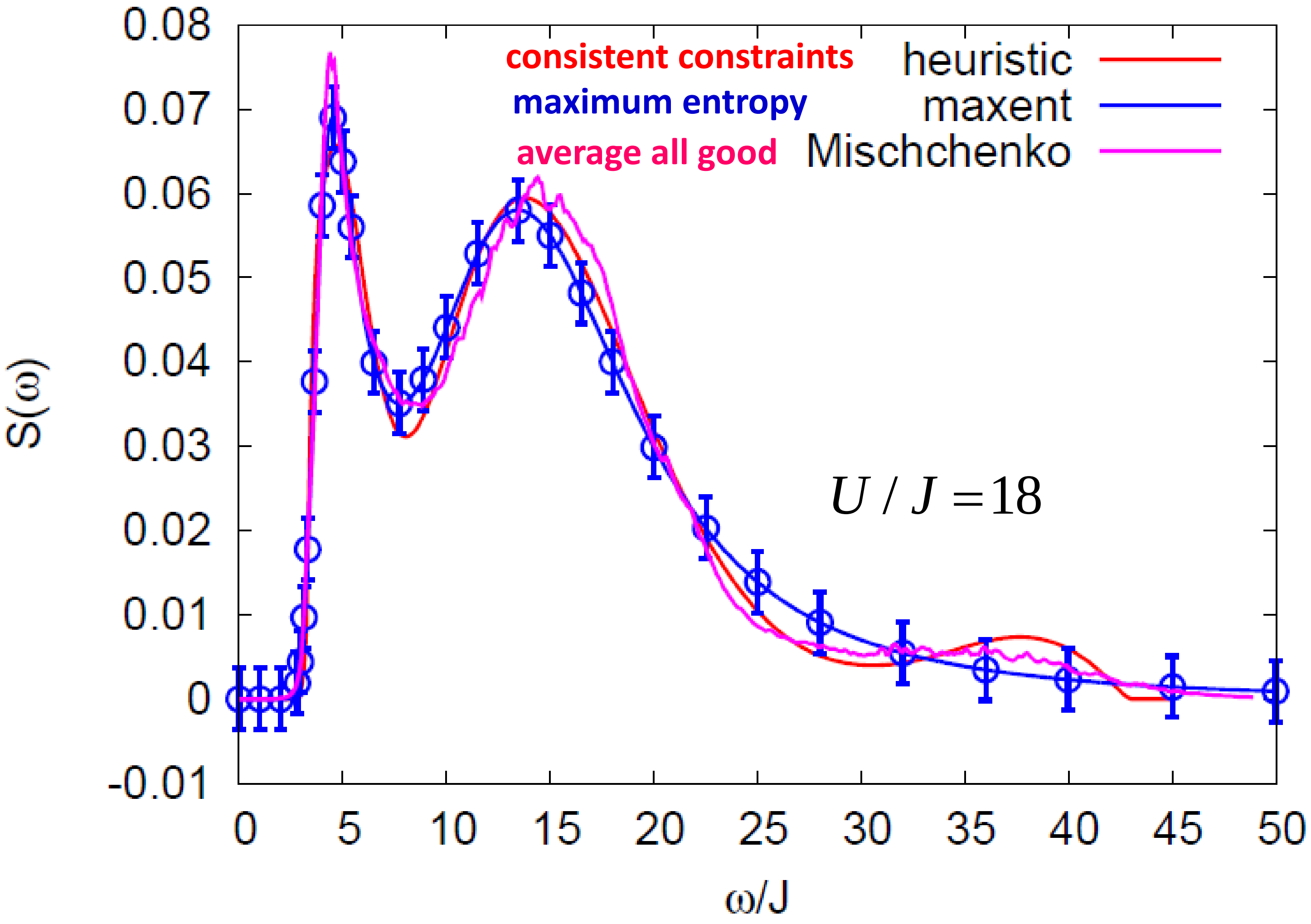
$$S(\omega) = 2\,\mathrm{Im}\,\chi(\omega)\,/\,(1-\mathrm{e}^{-\beta\omega})$$



ill-posed problem

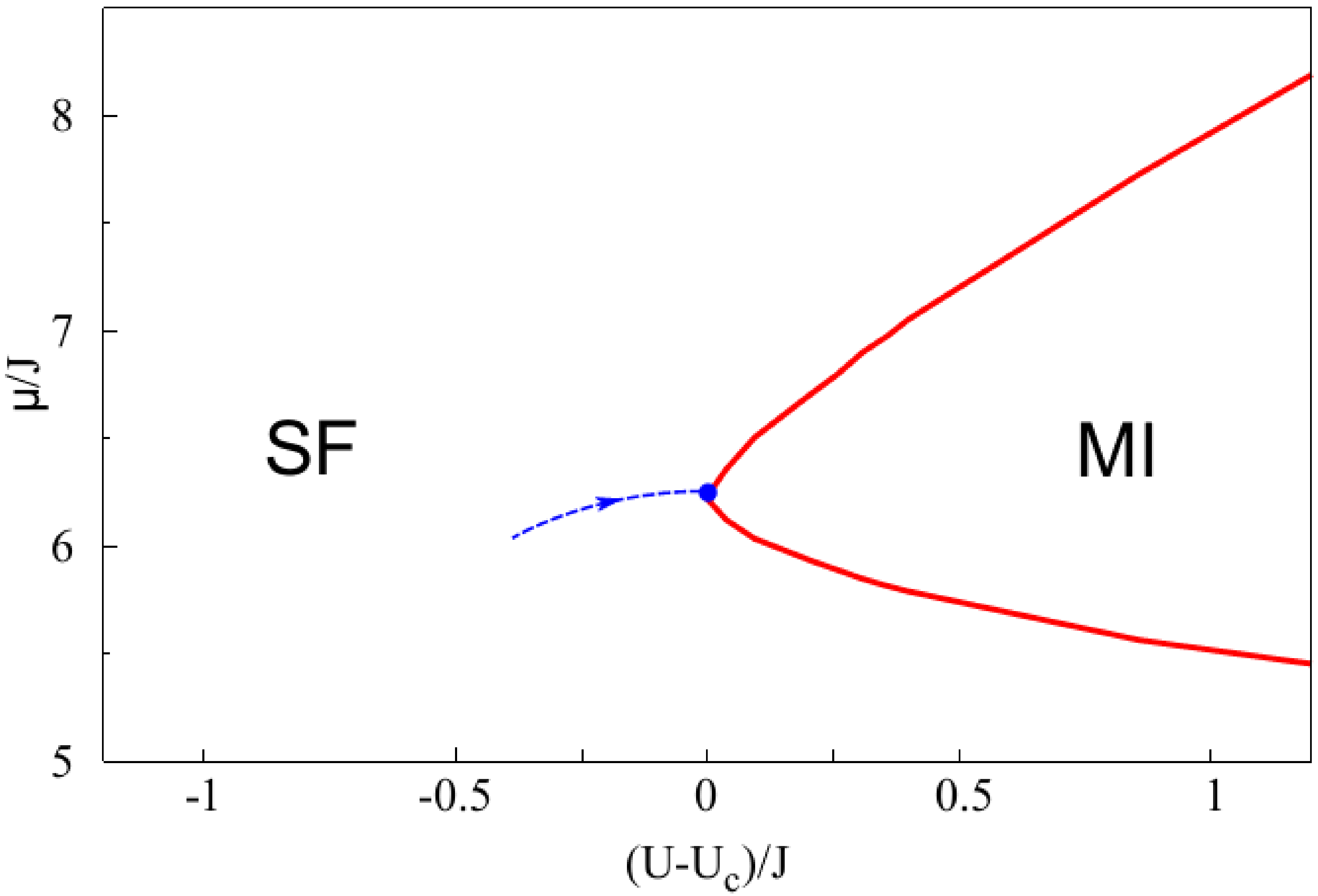
Hopefully, we know what we are doing …

Reliablity of Analytical continuation

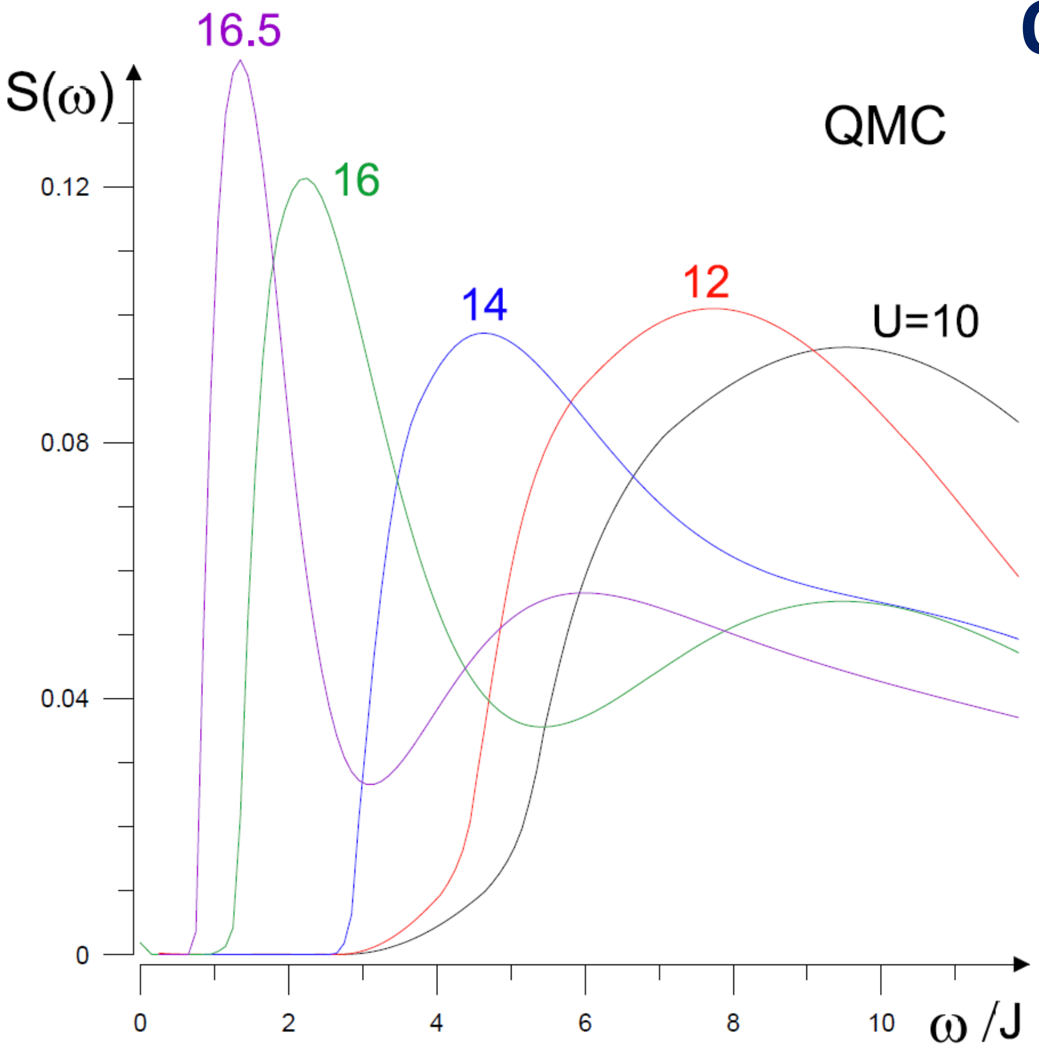


L. Pollet and N. Prokof'ev, PRL 109, 010401 (2012).

In Superfluid



In Superfluid



Oops, Higgs reasonance is there!

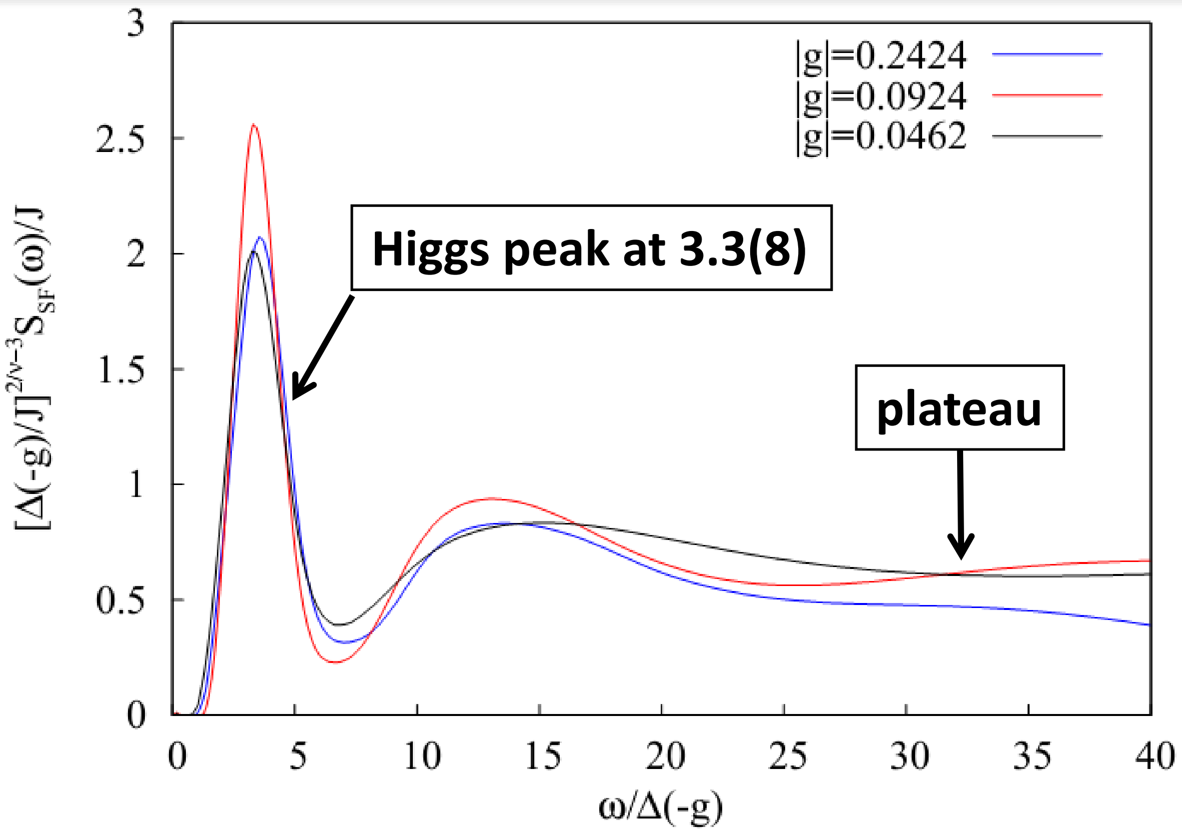
$$(U/J)_c = 16.7424(2)$$

There is a resonance at low frequency which

- **emerges at** $U \geq 14$
- **softens as** $U \to U_C$
- **gets more narrow as** $U \to U_C$

Pollet and Prokof'ev, PRL 109, 010401 (2012).

In Superfluid



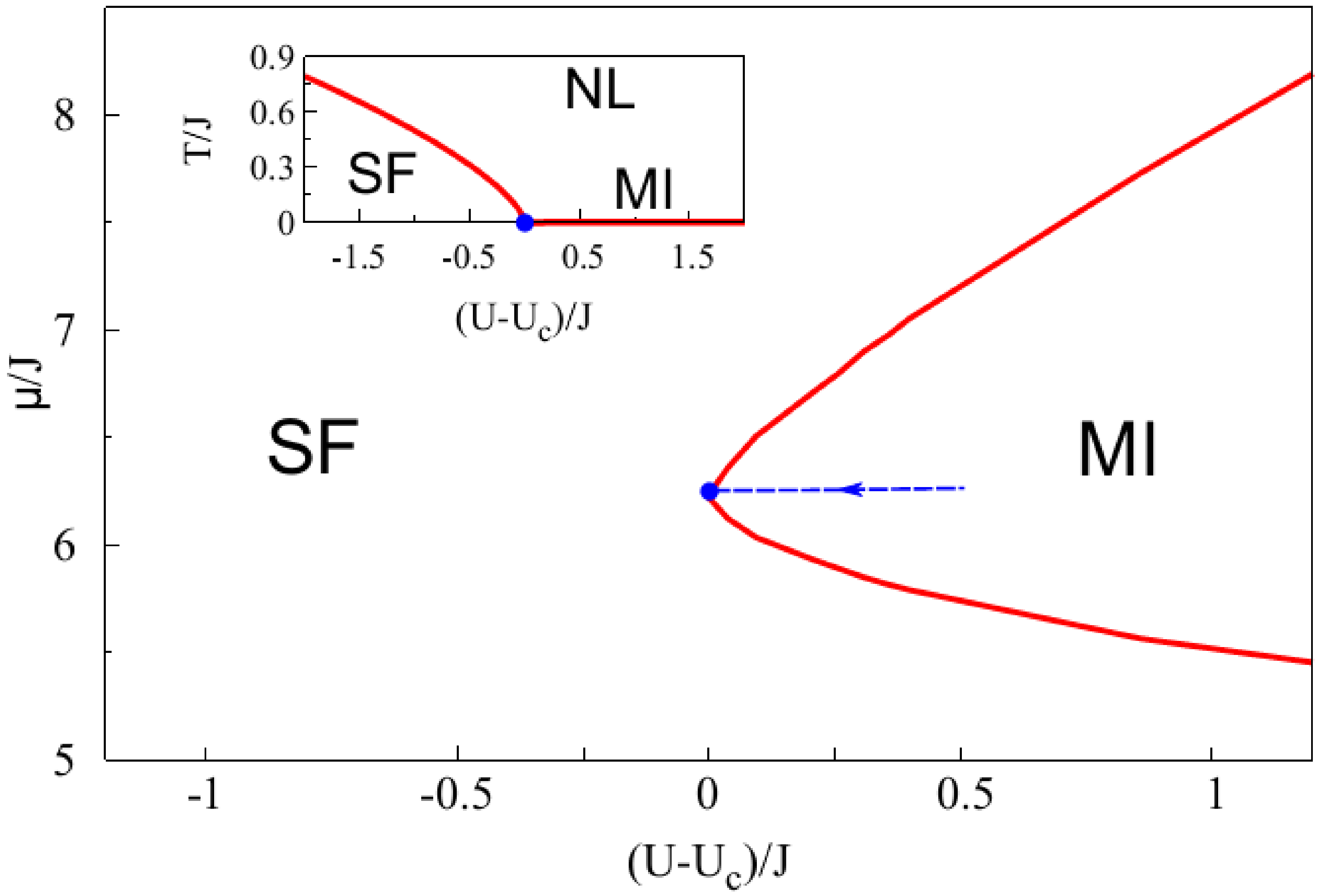
$$g = \frac{U - U_c}{J}$$

Chen, Liu, Deng, Pollet, and Prokof’ev, PRL 110, 170403 (2013)

✯Nice Collapse: universal response function!

✯Clear existence of plateau—a must physics condition

In Mott Insulator



Universal scaling spectral function

Chubukov, Sachdev, Ye '93

Sachdev '99

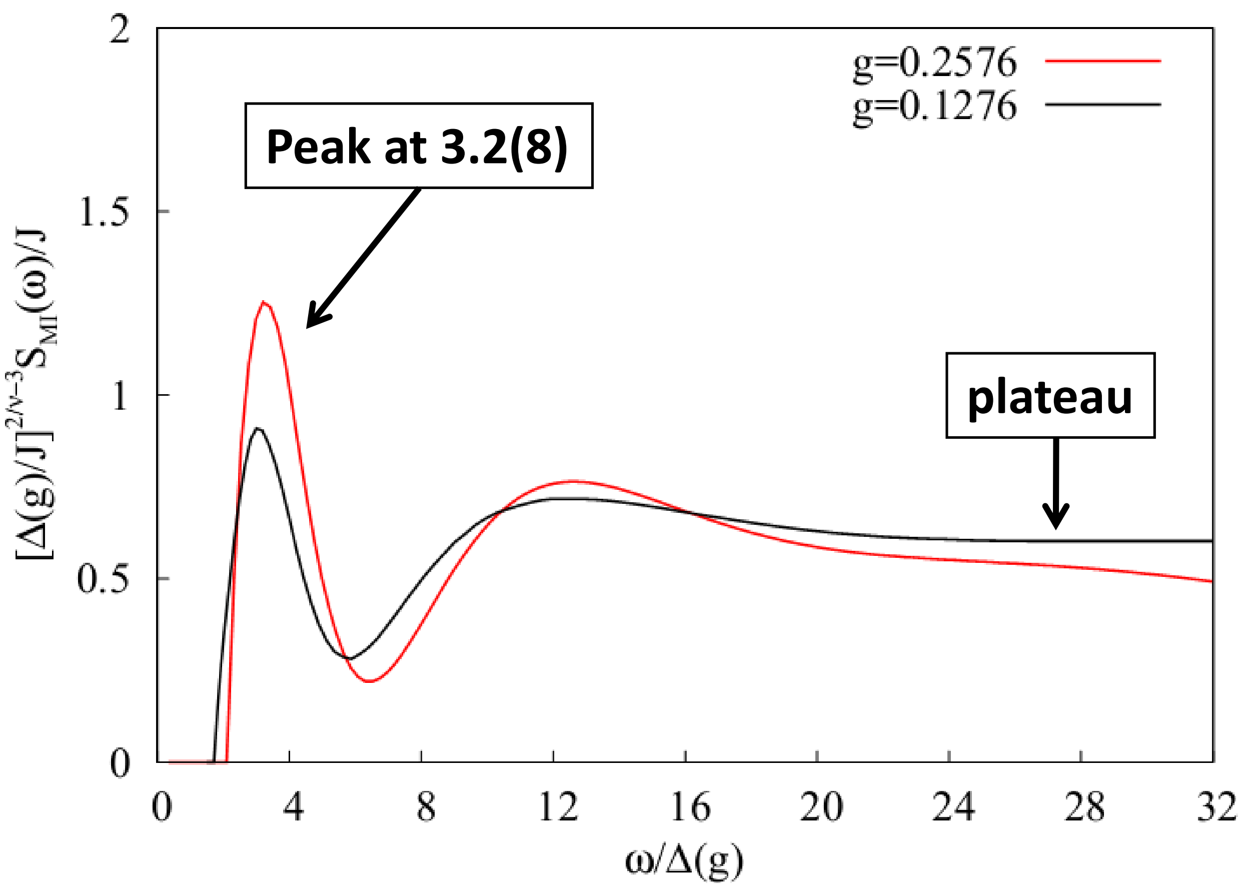
- **Similar form for SF and MI phases:**

$$S_{SF,MI}(\omega) \propto \Delta^{3-2/\nu} \Phi_{SF,MI}(\omega/\Delta), \nu = 0.6717$$

$g = \frac{U - U_c}{J}$	Superfluid $\Delta(-g) = E_{gap}/2$	MI $\Delta(g) = E_{gap}/2$
$\omega << \Delta$	$\sim \omega^3$	$\sim \Theta(\omega - 2\Delta)$
$\omega >> \Delta$	$\sim \omega^{0.0225}$	$\sim \omega^{0.0225}$

A plateau!

In Mott Insulator

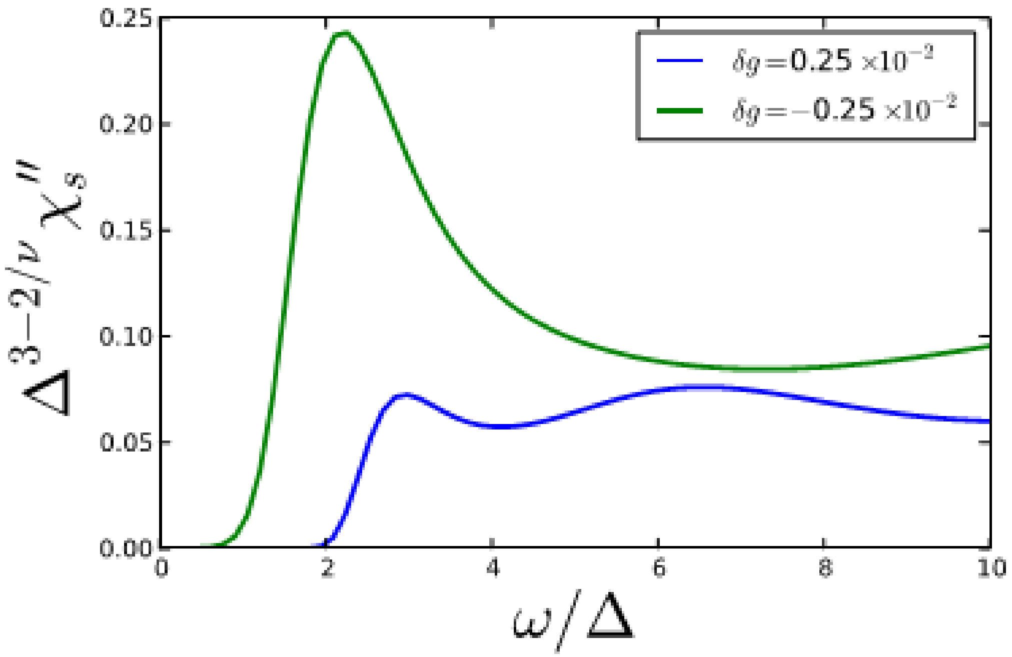


$$g = \frac{U - U_c}{J}$$

Chen, Liu, Deng, Pollet, and Prokof'ev, PRL 110, 170403 (2013)

- ✭ **What is the physics of the resonance peak?**
- ✭ **Looks so much as in the superfluid phase!**

Higgs mode in Relativistic O(2) model



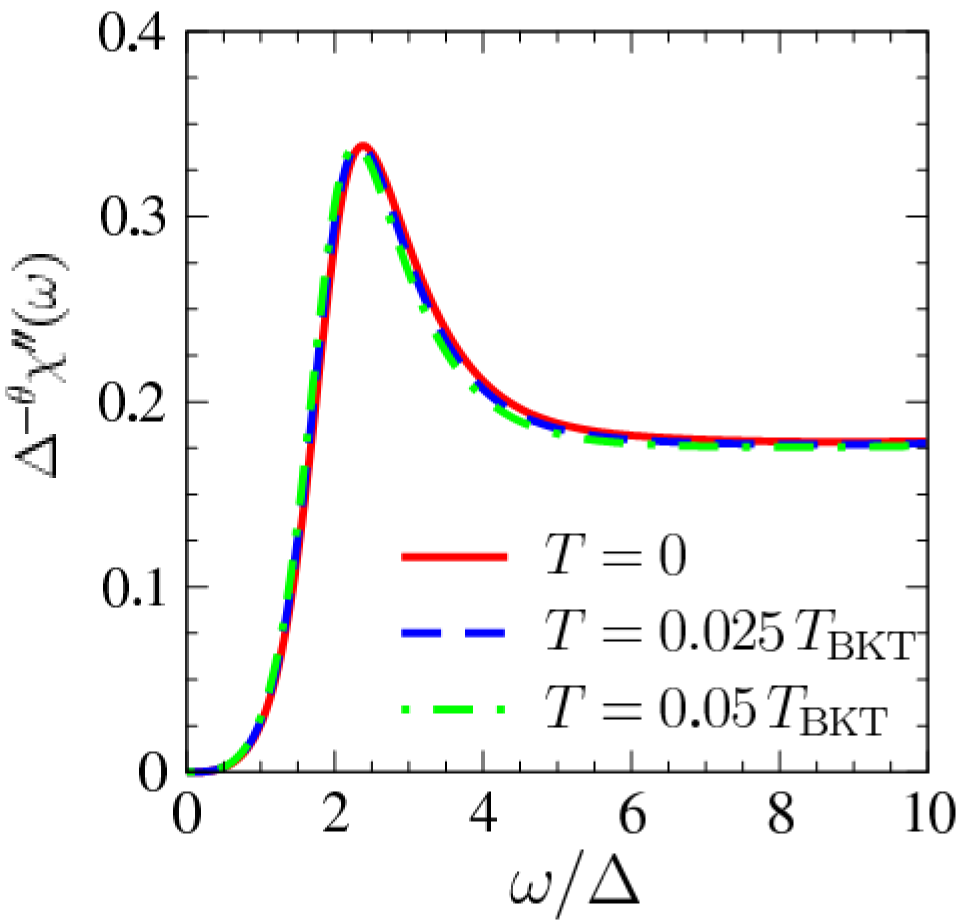
Discrete version of relativistic O(2) model

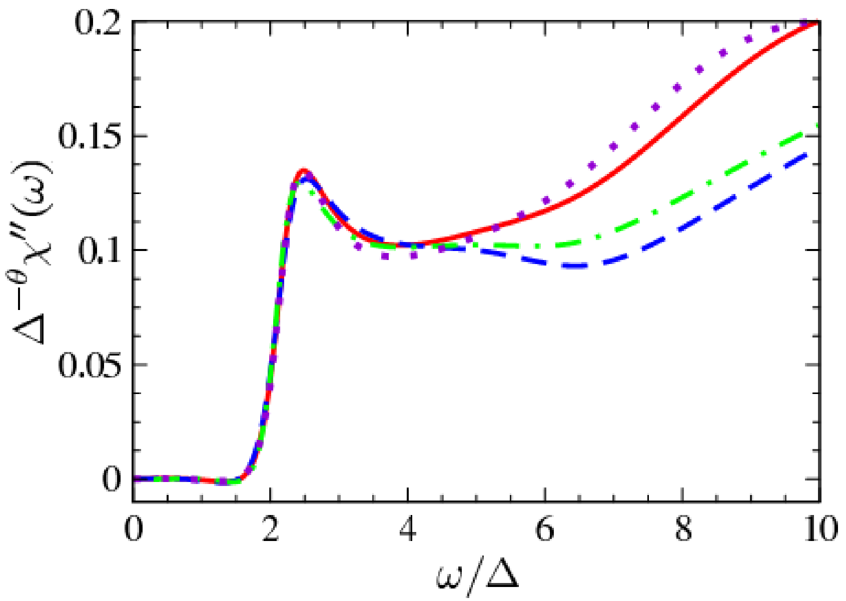
$$\chi(\omega) = \left\langle \left|\psi(t)\right|^2 \left|\psi(0)\right|^2 \right\rangle_\omega$$

$$\omega_H / \Delta \approx 2.1(3)$$

S. Gazit, D. Podolsky, A. Auerbach, PRL 110, 140401 (2013)
S. Gazit, D. Podolsky, A. Auerbach, and D.P. Arovas, Phys. Rev. B 88, 235108

Nonperturbative renormalization group study





$$\chi(\omega) = \left\langle \left|\psi(t)\right|^2 \left|\psi(0)\right|^2 \right\rangle_\omega$$

$$\omega_H / \Delta \approx 2.4$$

A. Rancon, N. Dupuis, Phys. Rev. B 89, 180501(2013)

Numerics vs Experiment

With a harmonic trap term: $V_i = \frac{1}{2}\alpha\omega^2 r_i^2$ in $\hat{H}_{BH}$

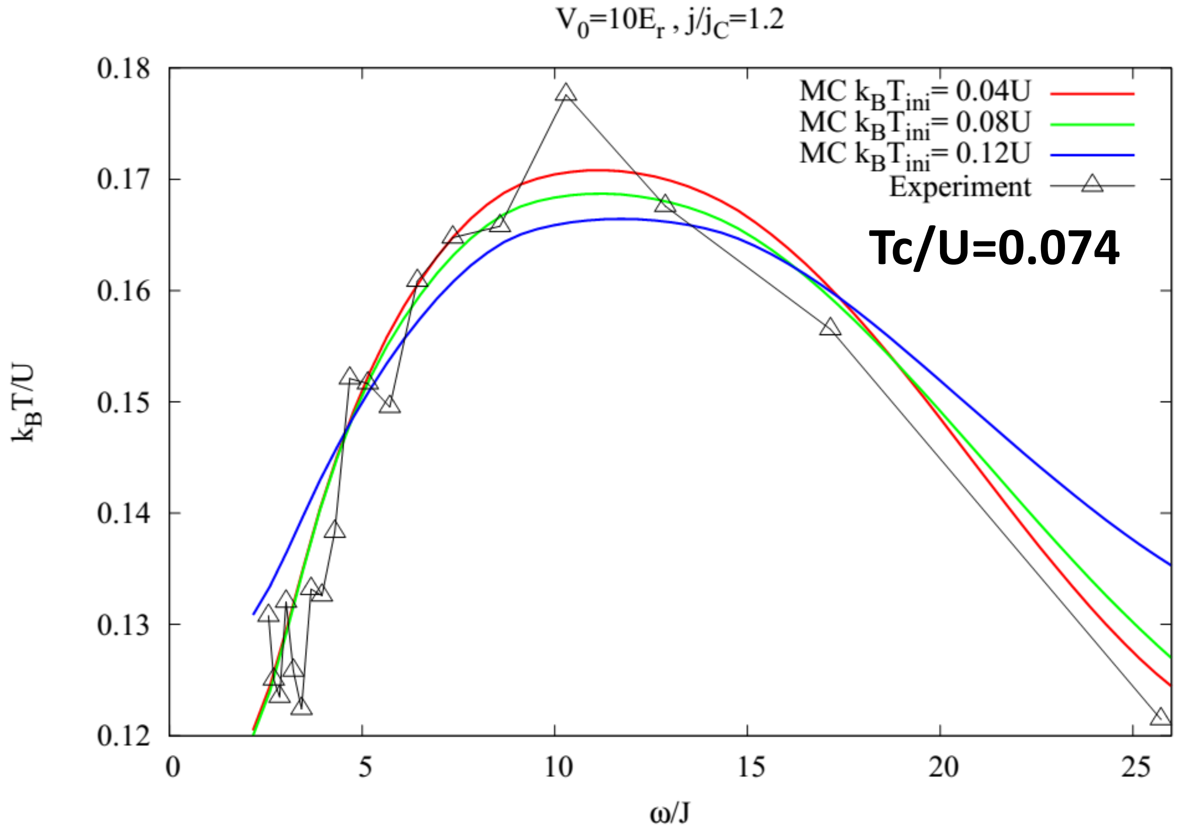
$$T_0 = 0.12U$$

$$\dot{E} = \frac{1}{2}\omega\, S(\omega, T_i)\left(\frac{\delta J}{J}\right)^2$$

$$\dot{T}_i = (\dot{E} + Q)/C(T_i) = f(T_i, \omega)$$

$$T_{i+1} = T_i + f(T_i, \omega)\cdot dt$$

$$T_{SF}$$

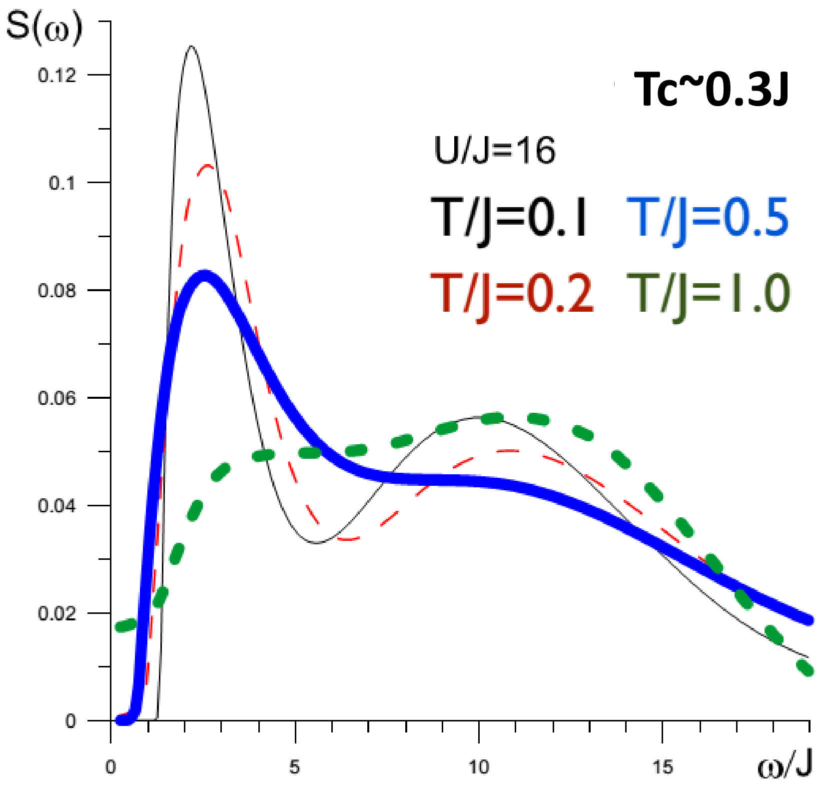


K. Chen et.al, unpublished

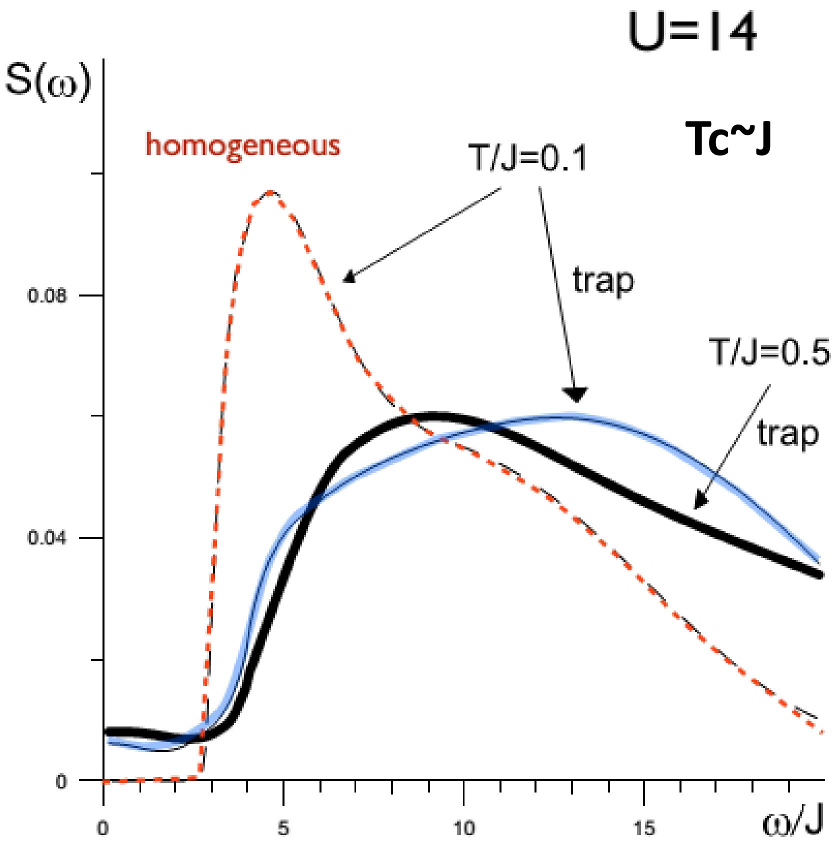
Consistent with experimental result!

Why no resonance peak?

Temperature effect

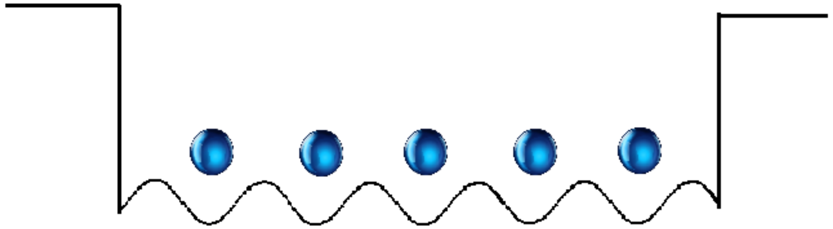


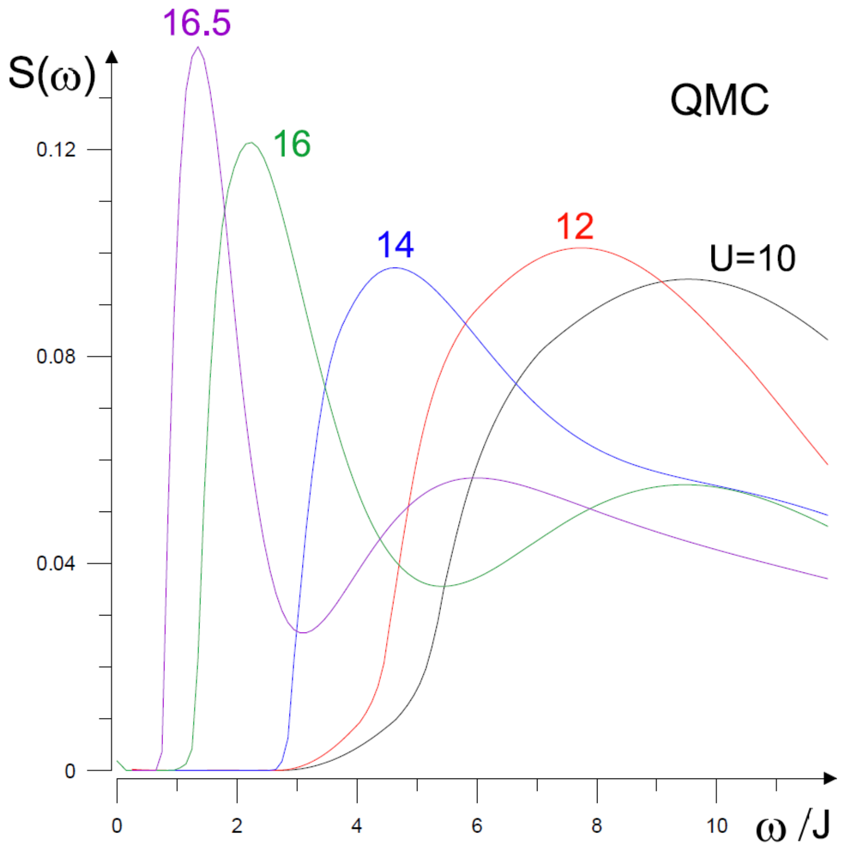
Trap effect



Pollet and Prokof'ev, PRL 109, 010401 (2012).

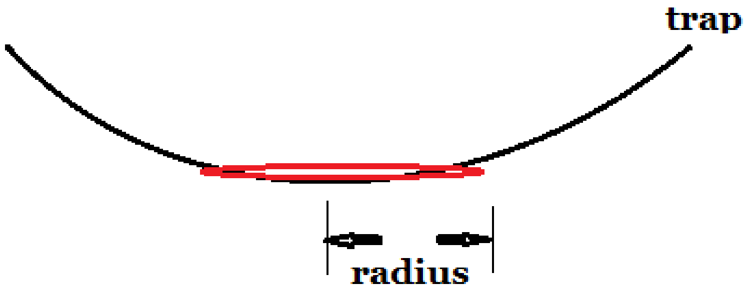
How to reproduce resonance peak?

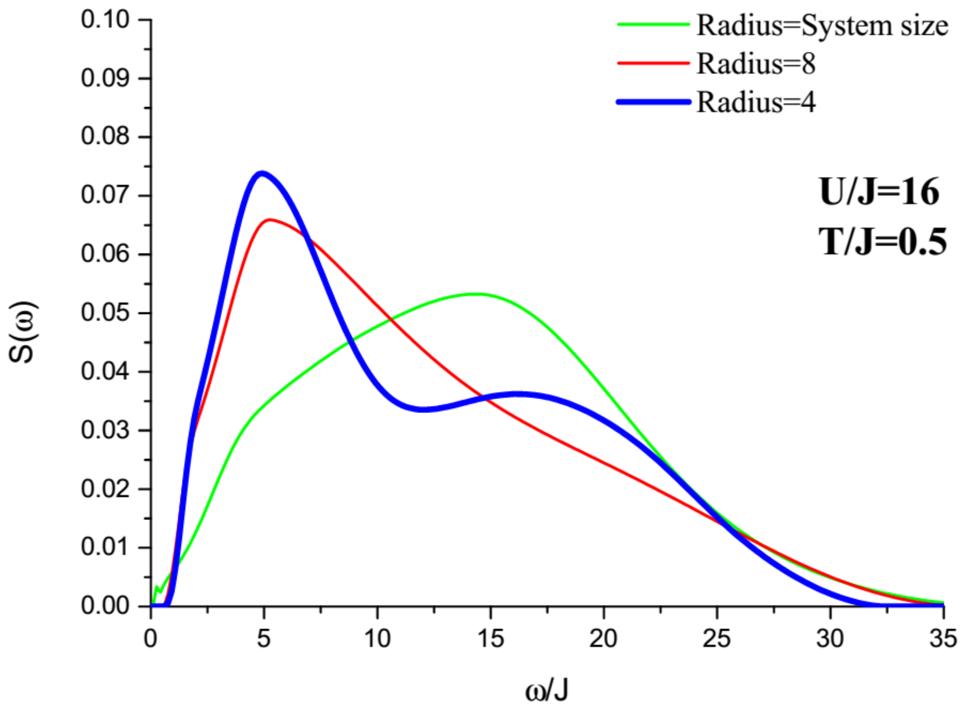




Pollet and Prokof'ev, PRL 109, 010401 (2012).

OR





K. Chen et.al, unpublished

Summary

- Higgs(-like) mode exists in SF and MI.

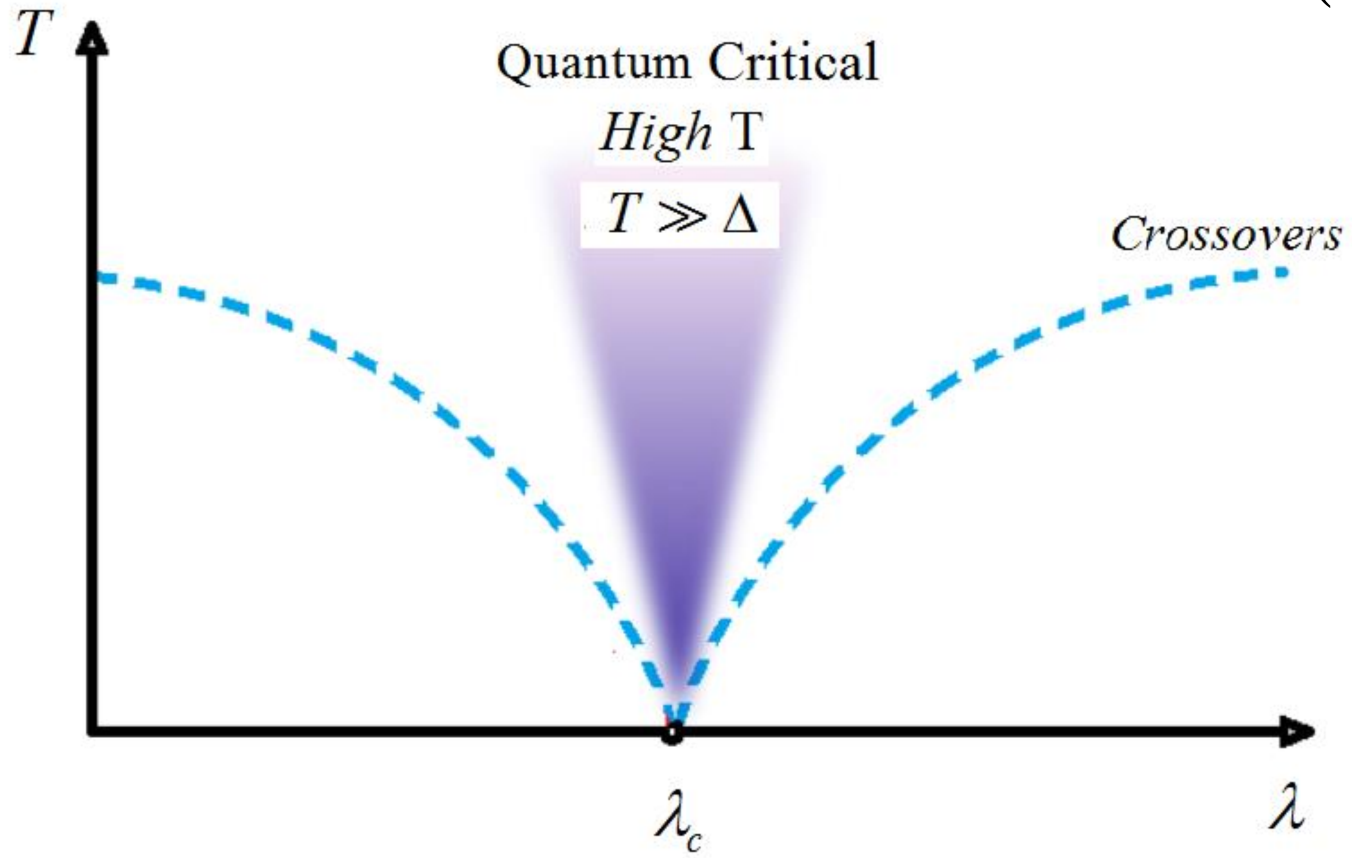
- Universal spectral functions:
  - Sharp, low-frequency resonance
  - Damped second oscillation
  - Saturation to quantum critical plateau

- A direct observation of Higgs resonance peak in experiment still need to be done.

When dynamics meets quantum criticality

High-T regime: $T > 0, \lambda \to \lambda_c$

- **Only one energy scale T !**
- **Scaling of any dynamic quantity:** $S(\omega) \propto \mathrm{F}\left(\omega / T, T \to 0\right)$

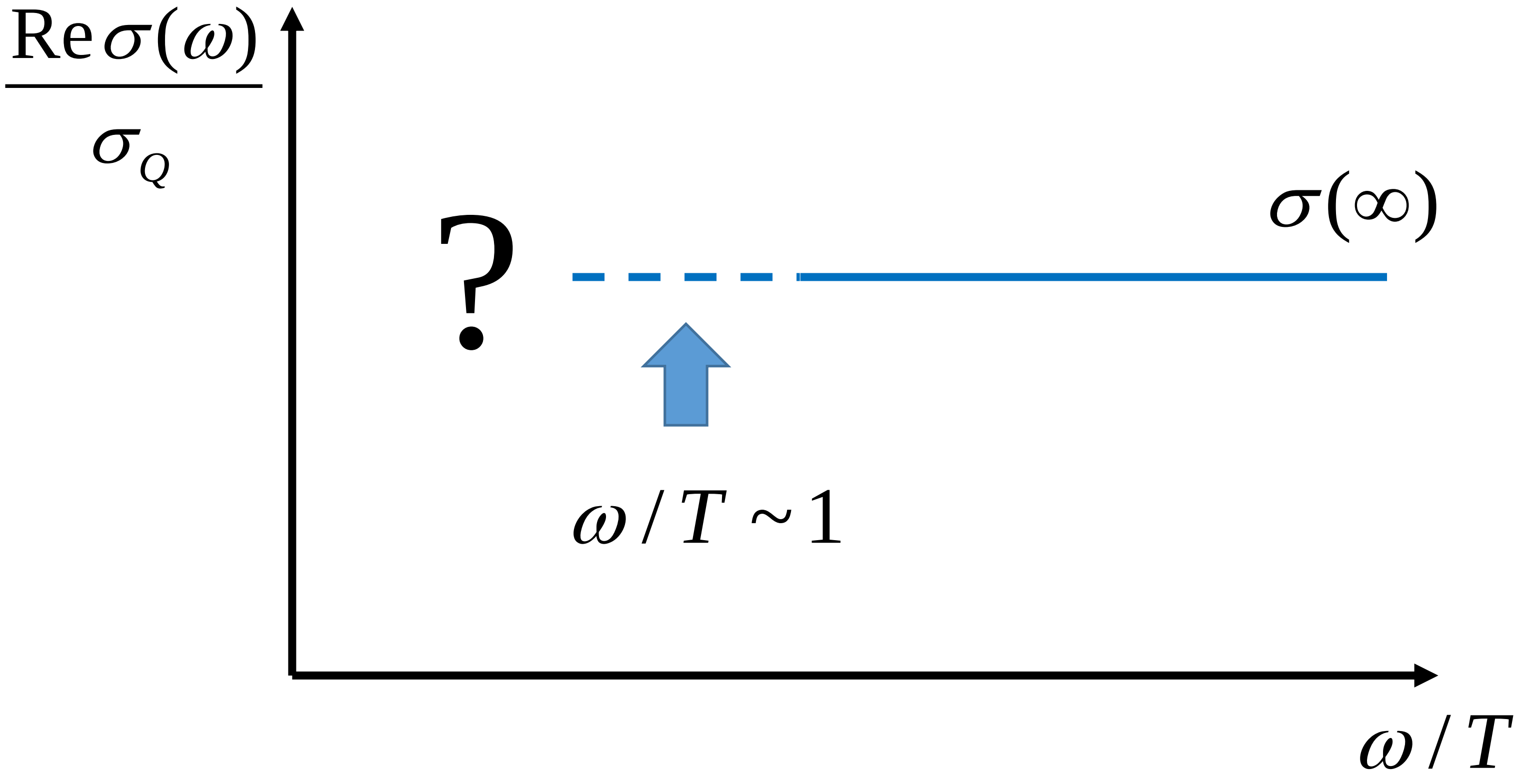


Universal Conductivity in high-T regime

For a two-dimensional quantum critical system

$$\sigma(\omega) = \sigma_Q \Sigma\left(\omega / T, T \to 0\right) \quad \text{or} \quad \sigma(i\omega_n) = \sigma_Q \Sigma\left(\omega_n / T, T \to 0\right)$$

K. Damle and S. Sachdev, Phys. Rev. B56, 8714 (1997)



Holographic insight

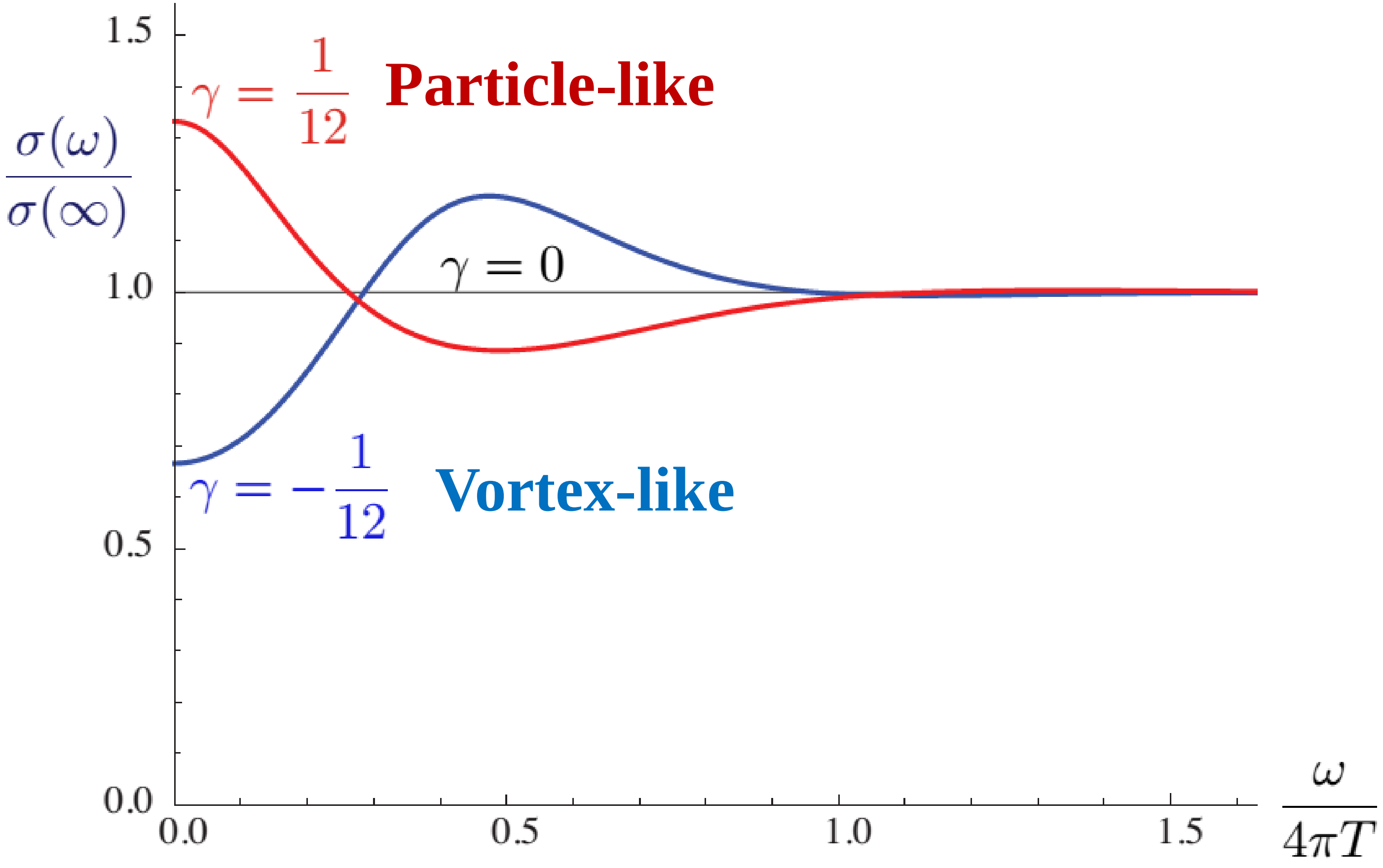
A holographic theory (AdS4) suggests:
The universal conductivity curve is controlled by two parameters:

$$\sigma(0) \text{ and } \sigma(\infty)$$

So define

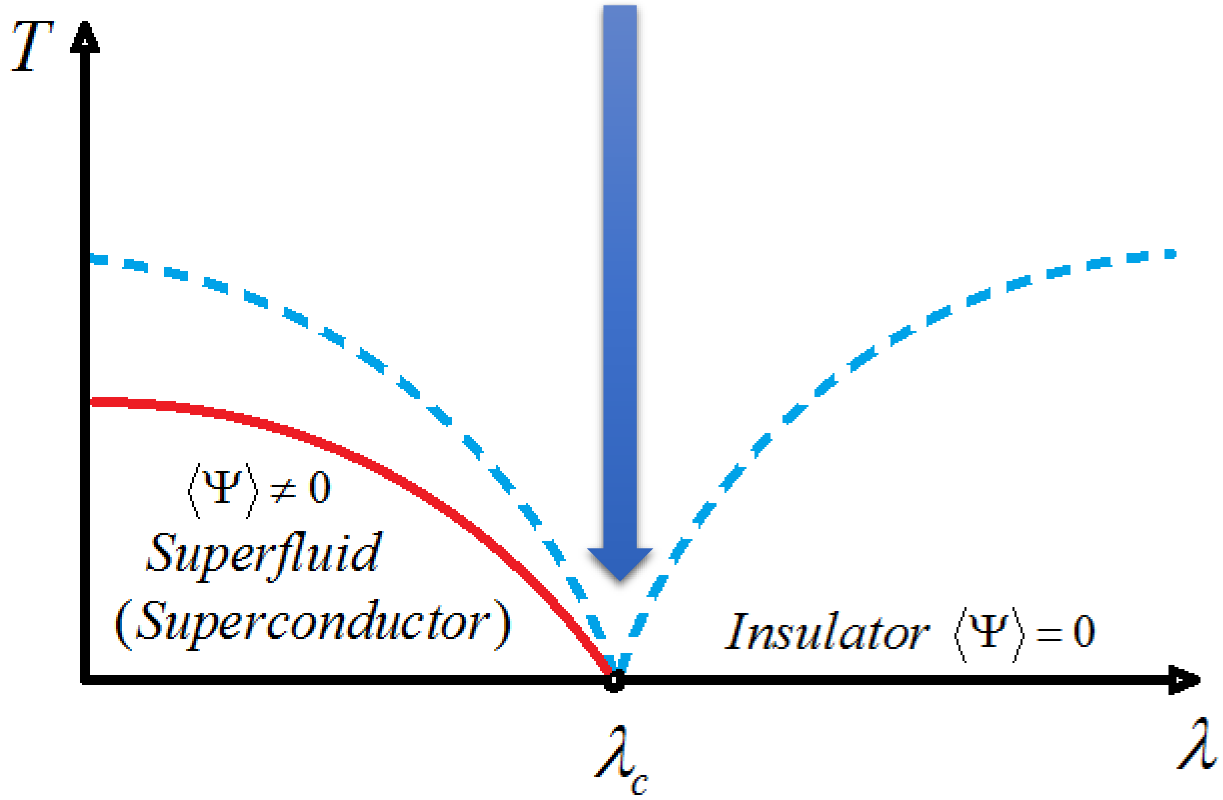
$$\gamma = \frac{\sigma(0) - \sigma(\infty)}{4\sigma(\infty)}$$

$$= \begin{cases} < 0 & \text{vortex-like} \\ > 0 & \text{particle-like} \end{cases}$$



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Could it describe the simplest CFT3 --relativistic O(2) QCP?



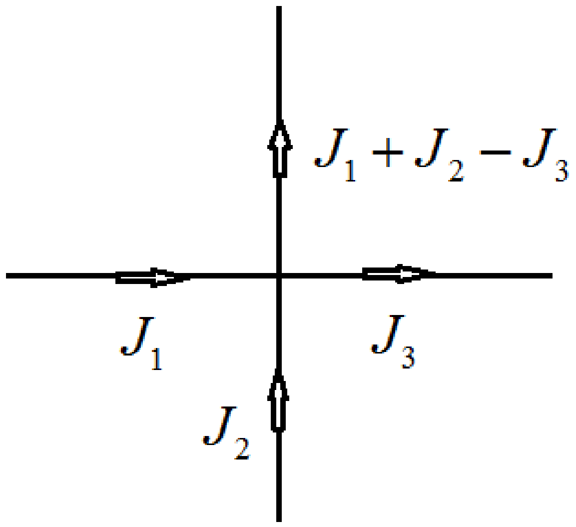
Models characterize 3D XY universality

- **Bose Hubbard model at quantum critical point**

$$\hat{H}_{BH} = -J\sum_{<i,j>} b_i^\dagger b_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - \mu\sum_i \hat{n}_i$$

- **J current model (Villain Model, classical) at critical point**

$$H = \frac{1}{2K}\sum_{<ij>}^{\nabla \mathbf{J}=0} J^2_{<ij>} \qquad J_{\langle i,j\rangle} \in (-\infty, \infty)$$



- **square lattice**

- **Monte Carlo with worm algorithm**

Measurement of conductivity

- **Kubo formula in Matsubara frequency:**

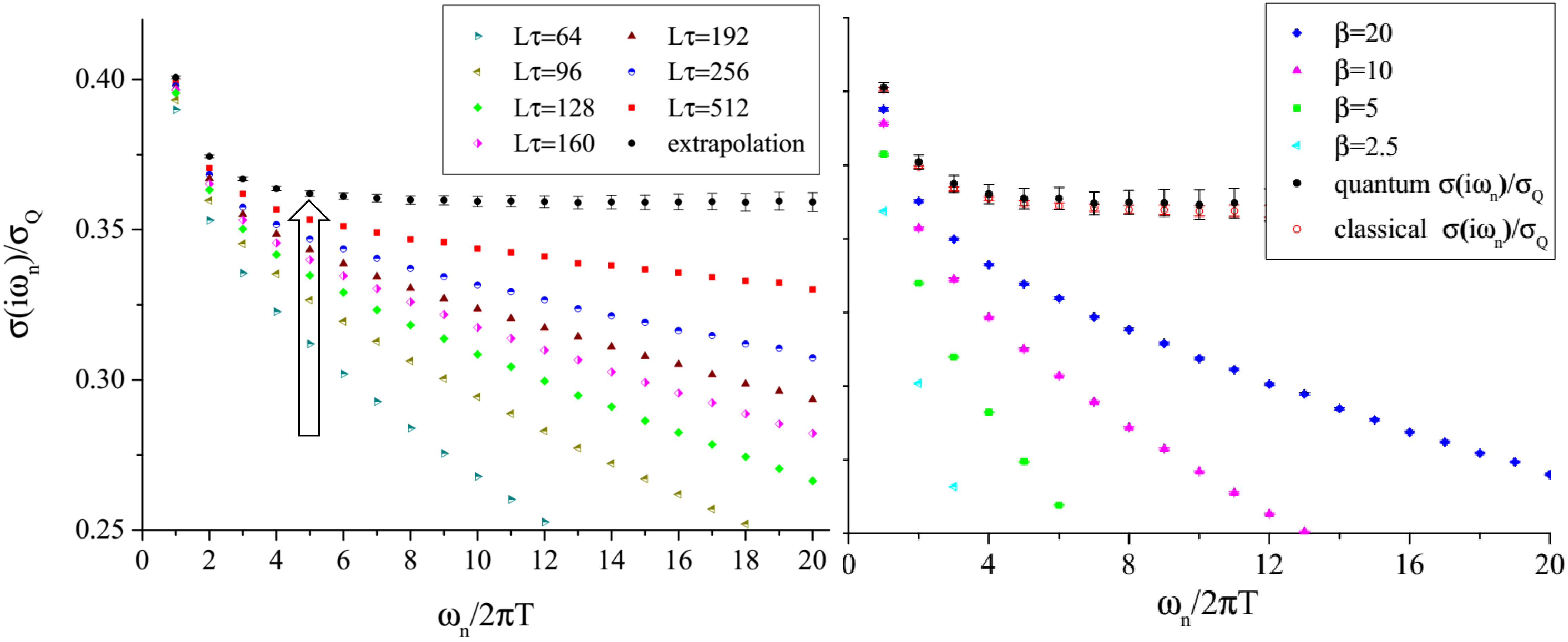
$$\sigma(i\omega_n) \propto \frac{\left\langle -\varepsilon_{kinect} \right\rangle - \left\langle j(\tau) j(0) \right\rangle_{i\omega_n}}{\omega_n}, \quad \omega_n = 2\pi nT, \quad (n = 1, 2, 3 \cdots)$$

- **Analytical continuation(** $\sigma(i\omega_n) \xrightarrow{i\omega_n \to \omega} \sigma(\omega)$ **)**

$$\sigma(i\omega_n) = \frac{2}{\pi} \int_0^\infty \frac{\omega_n}{\omega^2 + \omega_n^2} Re\sigma(\omega) d\omega$$

- **ill-defined problem**
- **Don't expect too much for** $\omega / T < \omega_1 / T = 2\pi$

J current model vs BH model



Extroplate all curves to T=0 with finite temperature correction hypothesis:

$$\sigma(i\omega_n / T, T) = \sigma(i\omega_n / T, T \rightarrow 0) + A_n T^{0.85}$$

Main results

- **Universal conductivity at large frequency:**

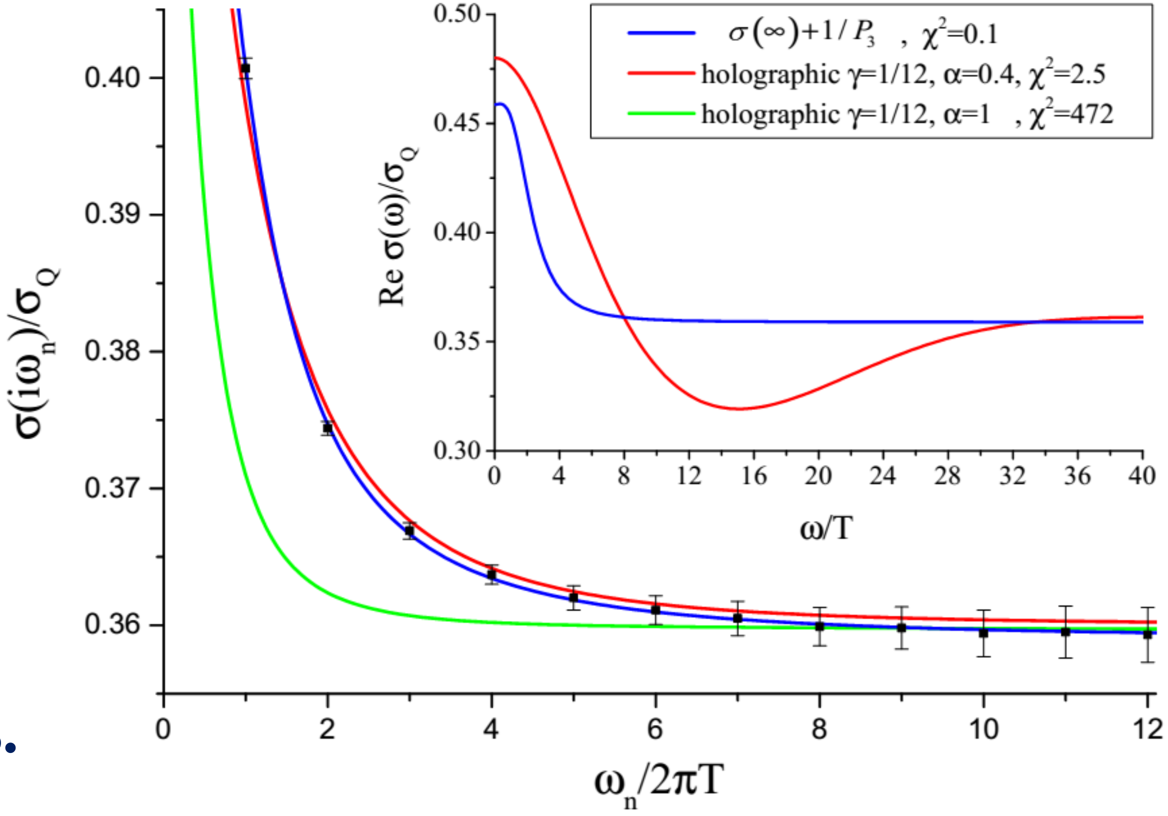
$$\sigma(\infty) = 0.359(4)$$

- **Particle-like transport.**

- **The holographic result can not fit our data unless we are allowed to rescale the temperature of black hole by a factor ~0.4.**

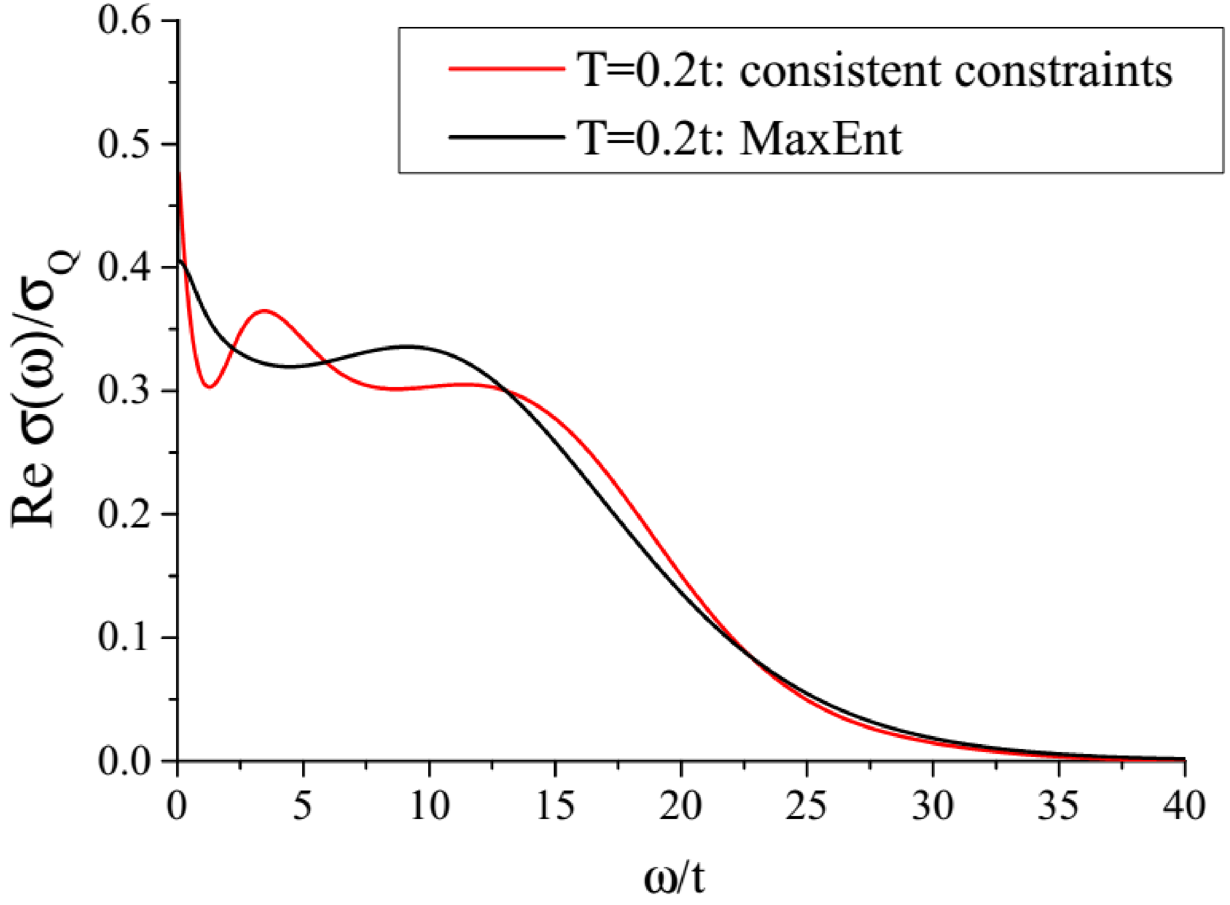
- **Low frequency part of $\sigma(\omega / T)$ is unstable against different analytical continuation procedures.**

S. Gazit et.al. Phys. Rev. B 88, 235108 (2013).
W. Witczak-Krempa et.al. arXiv:1309.2941 (2013).
M. Swanson et.al. arXiv. 1310.1073 (2013).



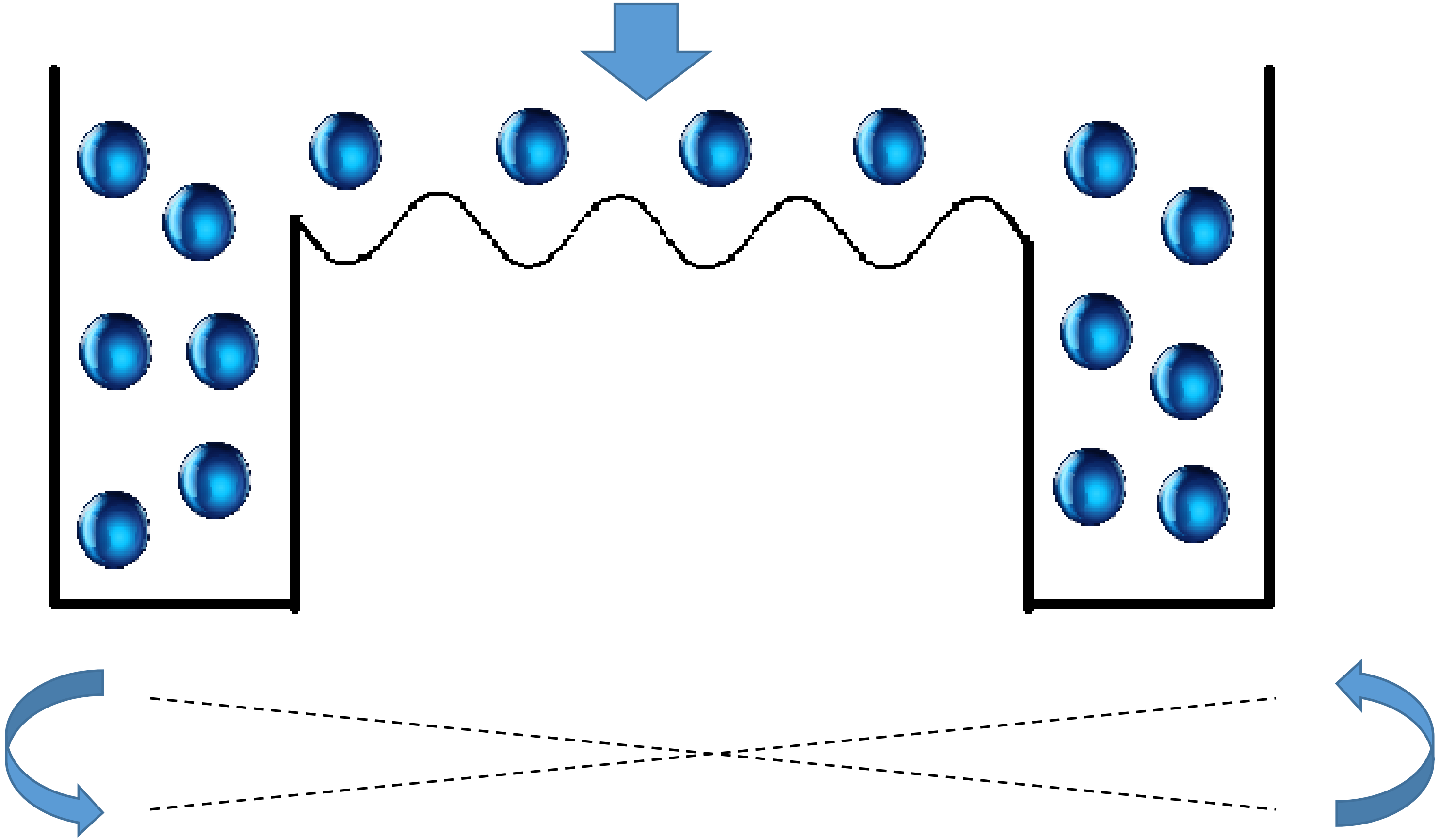
Experimental accessible conductivity

The plateau value $\sigma(\infty)$ can also be recovered at experimental accessible temperature



Ultracold atoms meet quantum gravity

Lower the temperature to $T/t<1\sim2$



“Sway” the lattice with frequency ω

Conclusion

Phys. Rev. Lett. 110, 170403 (2013). Editors' suggestion

- **Universal conductivity at large frequency:**

$$\sigma(\infty) = 0.359(4)$$

- **Particle-like transport is found.**

- **The holographic result can not fit our data unless we are allowed to rescale the temperature of black hole by a factor ~0.4.**