Dispersion relations of Nambu-Goldstone modes at finite temperature and density

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Based on YH, Phys. Rev. Lett. 110, 091601 (2013), 1203.1494 [hep-th],
Hayata, YH, 1406.6271 [hep-th]

Low energy excitations

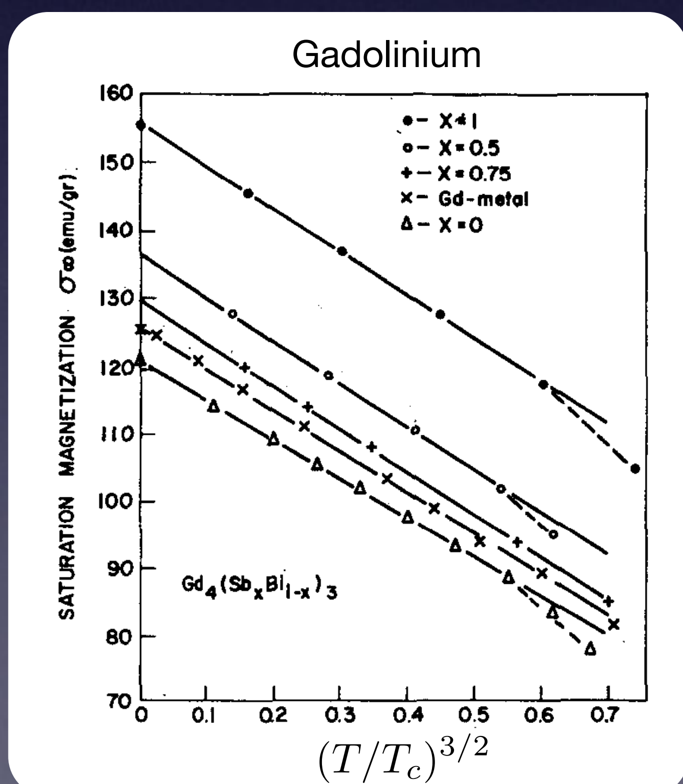
Why important?

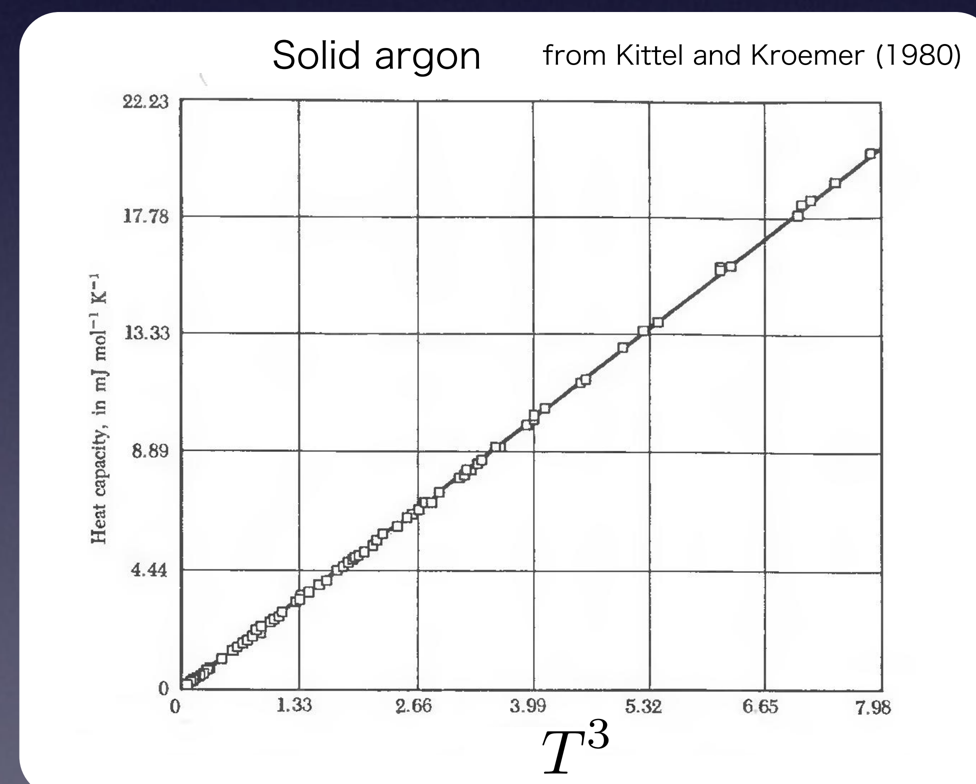
Without knowledge details of systems,

one can predict many things:

dispersion relations, low-energy theorem,...

Bloch $T^{3/2}$ law, Debye T^3 law, ...



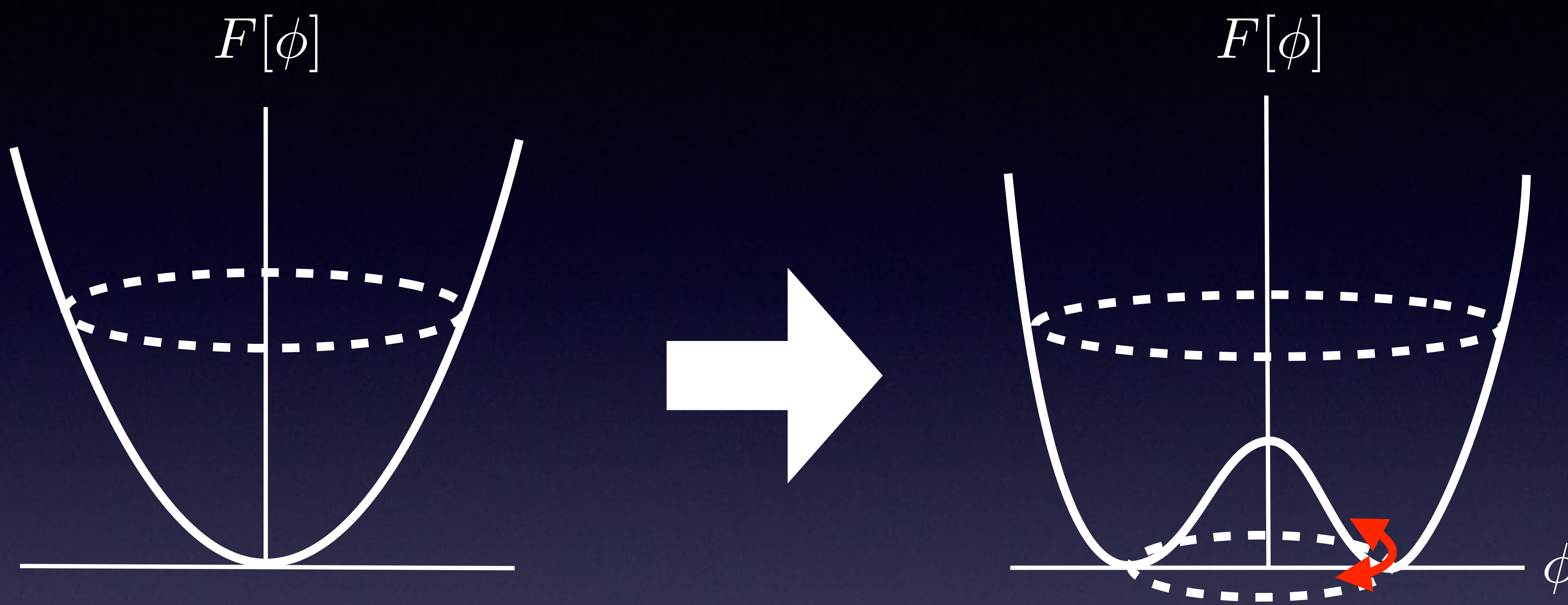


Holtzberg, McGuire, M'ethfessel, Suits, J. Appl. Phys. 35,1033 (1964)

Chiral condensate $$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - \frac{1}{8}\frac{T^2}{f_\pi^2} + \cdots$$

Spontaneous symmetry breaking

For fields

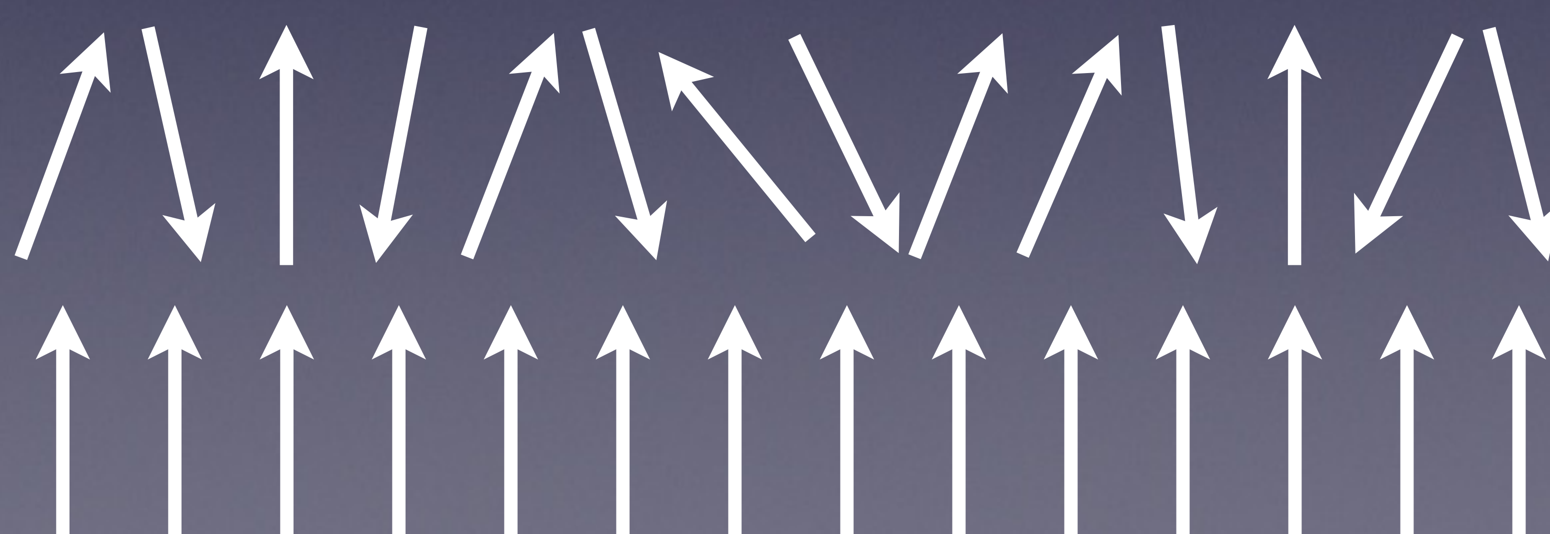


Degeneracy of grand states

For spins

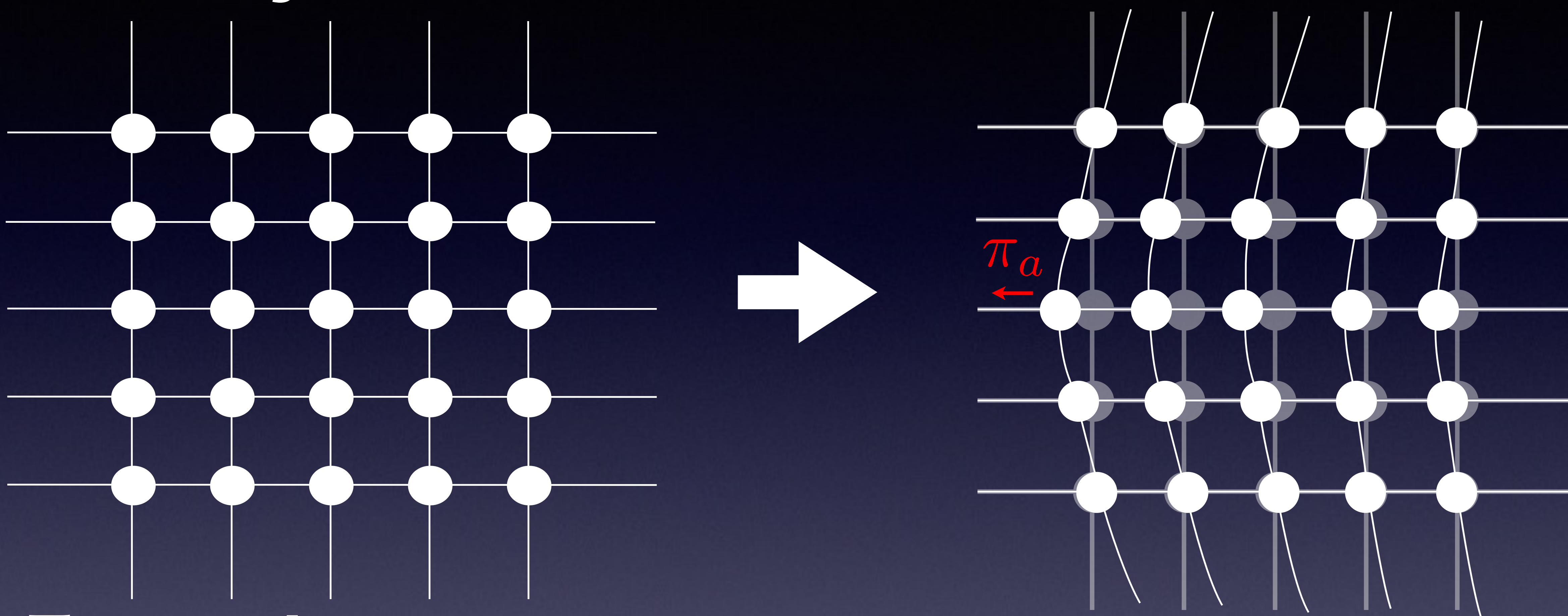
random

aligned

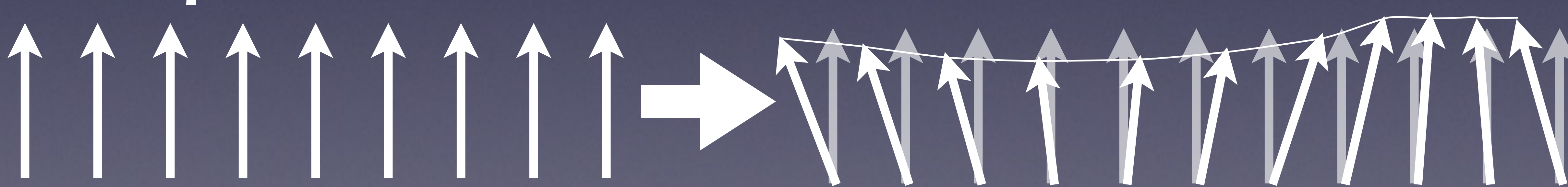


Elasticity

For crystal



For spin



Free energy $F = \dfrac{1}{2}(\partial_i \pi^a)^2 + \cdots$

What is the NG mode?

Nambu('60), Goldstone(61), Nambu, Jona-Lasinio('61)

charge densities are slow:

$$\partial_t n_a(t,\boldsymbol{x}) = -\partial_i j_a^i(t,\boldsymbol{x})$$

ex) in medium $j_a^i = \Gamma\partial_i n_a$

Diffusion equation $\partial_t n_a(t,\boldsymbol{x}) = -\Gamma\partial_i^2 n_a(t,\boldsymbol{x})$

When SSB occurs,

the charge density and the local operator are canonically conjugate

cf. Nambu ('04)

$$\langle[iQ_a, \pi_b(\boldsymbol{x})]\rangle \neq 0$$

$$\Rightarrow \quad \partial_t \pi_a = c n_a$$

$$\partial_t n_a = b\partial_i^2 \pi_a$$

Example in relativistic systems

Approximate symmetry of QCD

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

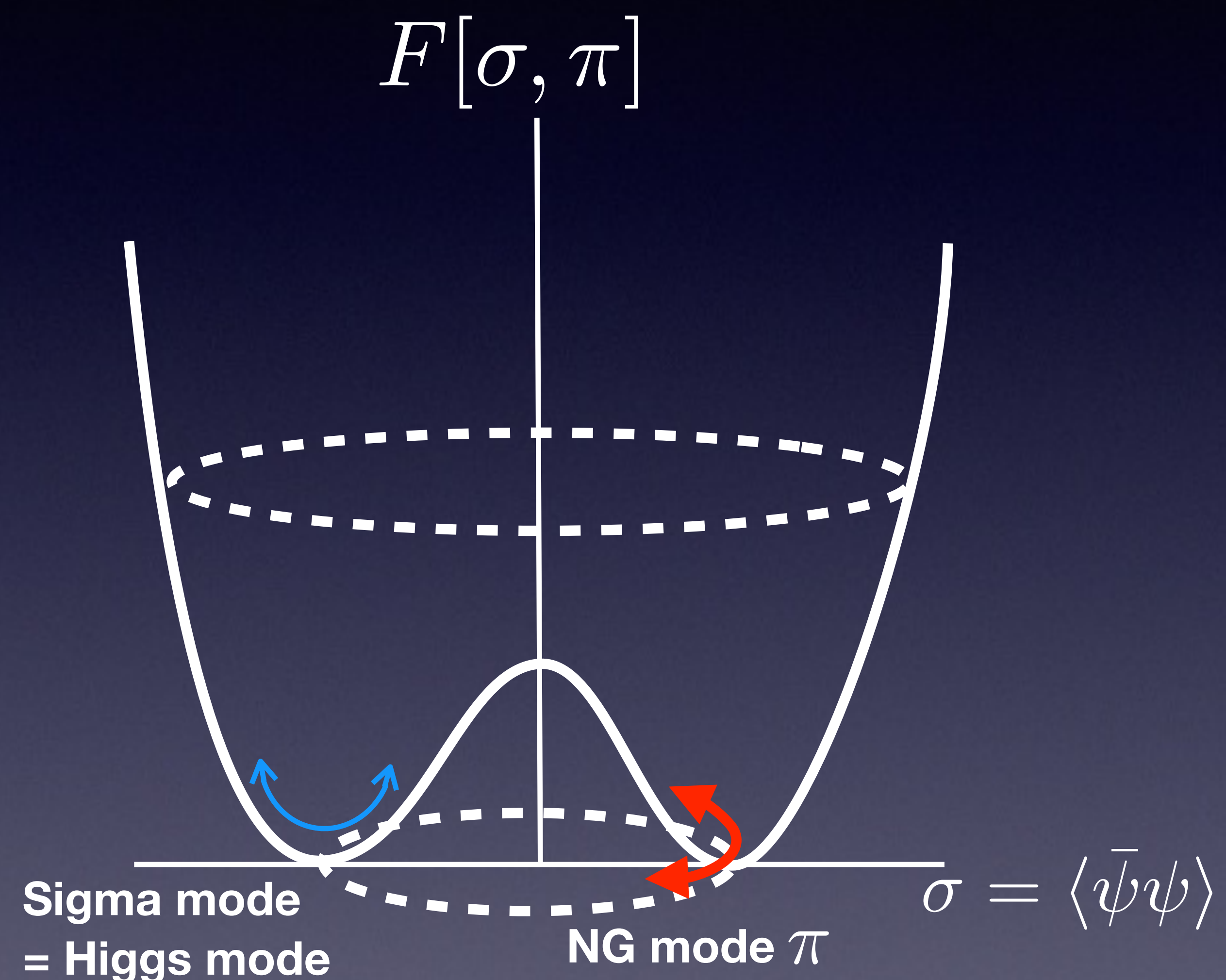
Three broken generators

Three NG modes: Pions

$$\pi^+, \pi^-, \pi^0$$

Dispersion relation

$$\omega = \sqrt{\boldsymbol{k}^2 + m_\pi^2}$$



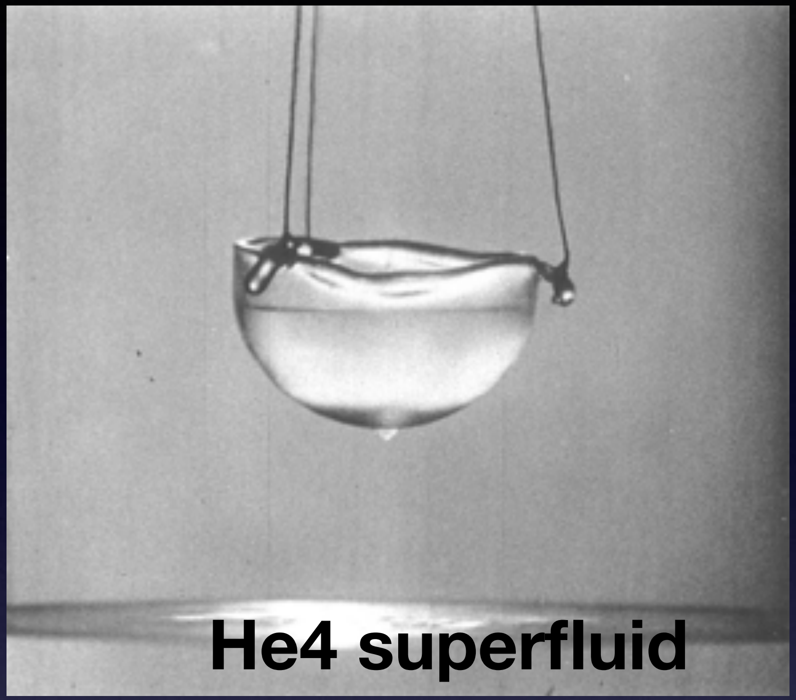
Example of NG modes

Superfluid phonon

broken of number

Broken generator: Q

One phonon $\omega \sim |\boldsymbol{k}|$

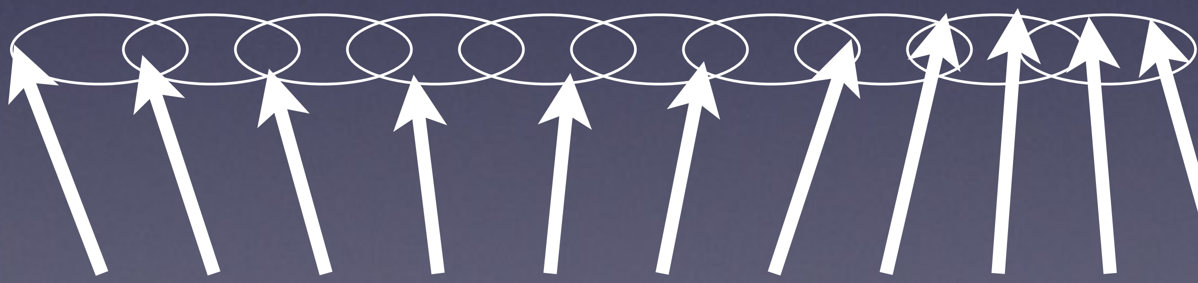


Magnon

Broken of rotation

Two broken generators S_x, S_y

one magnon $\omega \sim \boldsymbol{k}^2$



 # and dispersion are different from relativistic ones

Nambu-Goldstone theorem

Nambu ('60), Goldstone (61), Nambu, Jona-Lasinio ('61)   Goldstone, Salam, Weinberg ('62)

For Lorentz invariant vacuum

Spontaneous breaking of global symmetry

$$\Downarrow$$

# of broken symmetry = # of NG modes

Dispersion relation $\omega = c|\boldsymbol{k}|$

Generalization

Nielsen - Chadha ('76)

$$N_{\text{type-I}} + 2N_{\text{type-II}} \geq N_{\text{BS}}$$

Type-I: $\omega \propto k^{2n+1}$ **Type-II:** $\omega \propto k^{2n}$

Schafer, Son, Stephanov, Toublan, and Verbaarschot ('01)

$$\langle [iQ_a, Q_b] \rangle = 0 \Rightarrow N_{\text{NG}} = N_{\text{BS}}$$

Nambu ('04)

$$\langle [iQ_a, Q_b] \rangle \neq 0 \Rightarrow (Q_a, Q_b)$$

canonical conjugate

Watanabe - Brauner ('11)

$$N_{\text{BS}} - N_{\text{NG}} \leq \frac{1}{2} \text{rank} \langle [iQ_a, Q_b] \rangle$$

Recent progress

Effective Lagrangian method **Watanabe, Murayama ('12)**

Mori's projection operator method **YH ('12)**

- $N_{\mathrm{BS}} - N_{\mathrm{NG}} = \frac{1}{2}\mathrm{rank}\langle[iQ_a, Q_b]\rangle$
- $N_{\text{type-I}} + 2N_{\text{type-II}} = N_{\mathrm{BS}}$
- $N_{\text{type-II}} = \frac{1}{2}\mathrm{rank}\langle[iQ_a, Q_b]\rangle$

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- $N_{\mathrm{BS}} - N_{\mathrm{NG}} = \frac{1}{2}\mathrm{rank}\langle[iQ_a, Q_b]\rangle$
- $N_{\text{type-A}} + 2N_{\text{type-B}} = N_{\mathrm{BS}}$
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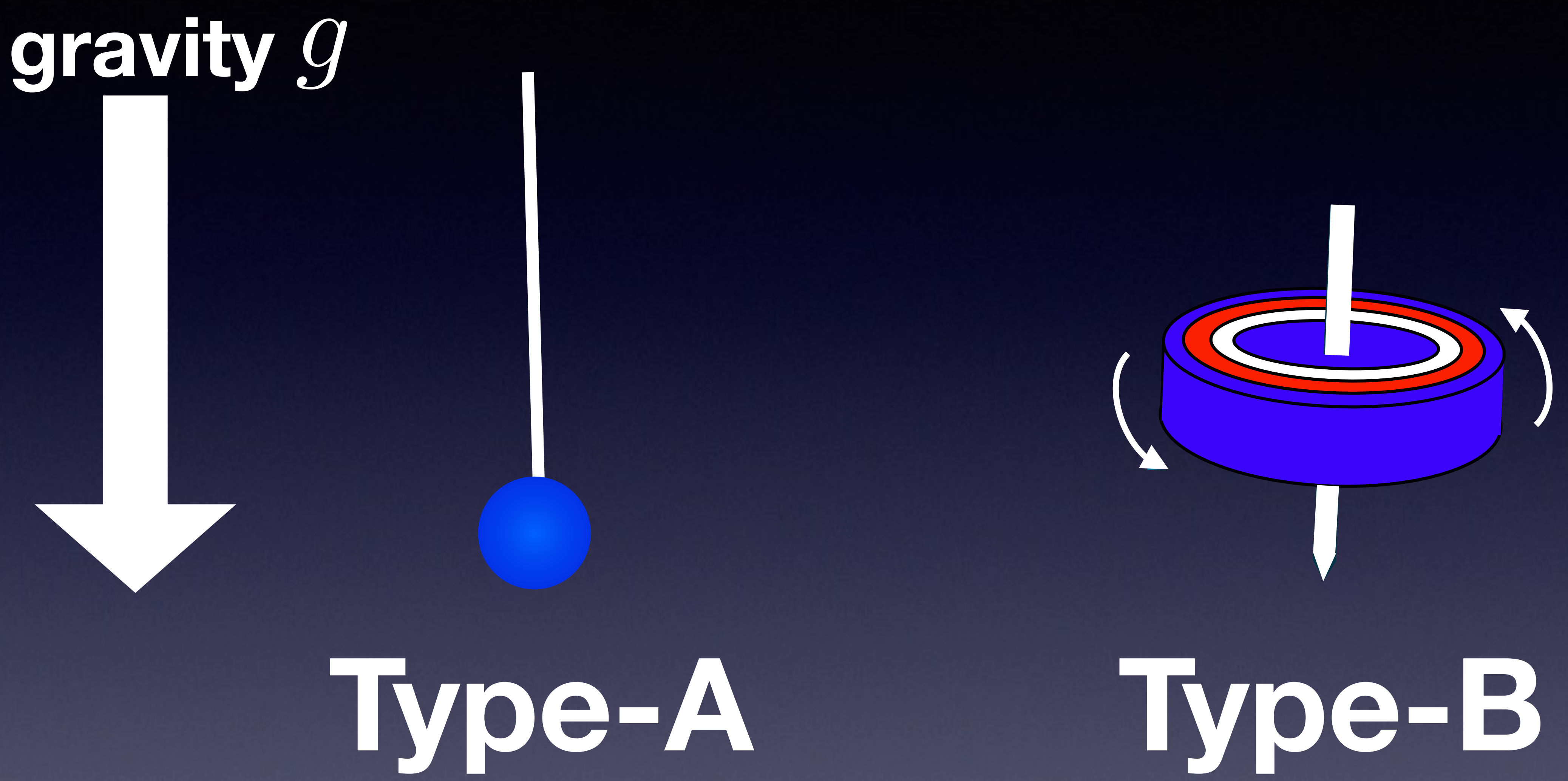
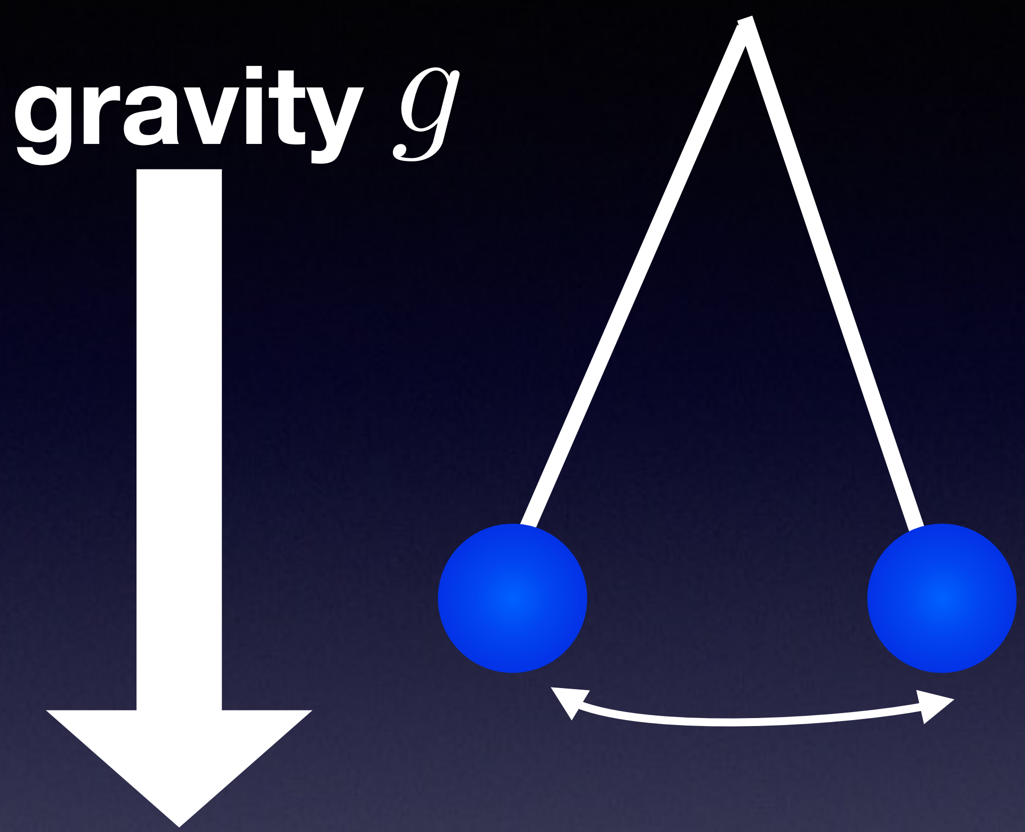
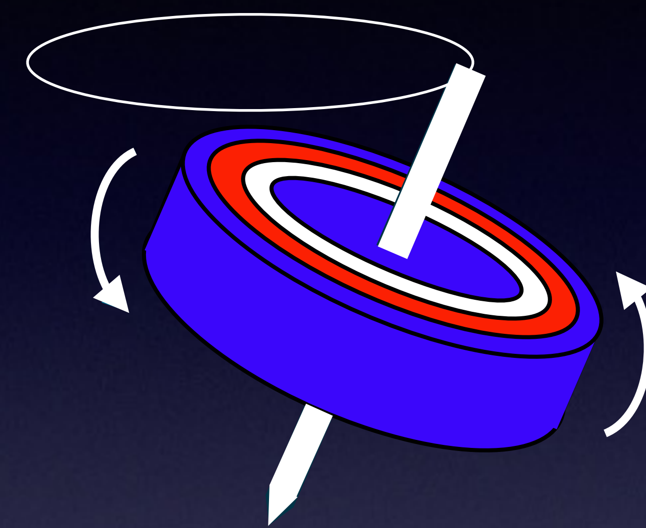
Two type of excitations



Two type of excitations





Type-A

Harmonic oscillation

$$\omega \sim \sqrt{g}$$

Type-B

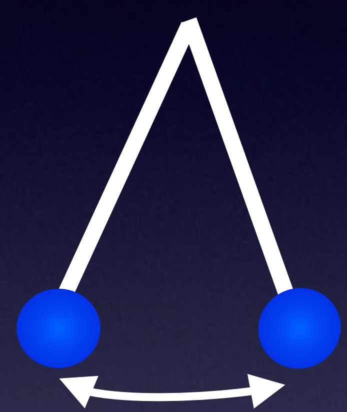
Precession

$$\omega \sim g$$

Recent Progress

Watanabe, Murayama ('12), YH ('12)

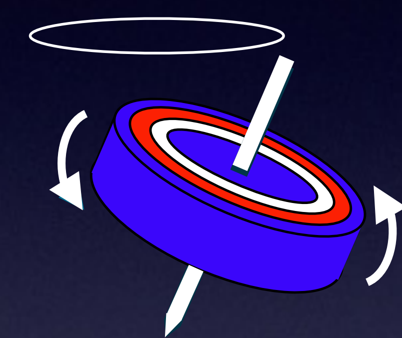
NG modes associated with spontaneous breaking of internal symmetry can be classified by two types:



Type-A

harmonic oscillation

$$N_{\text{type-A}} = N_{\text{BS}} - 2N_{\text{type-B}}$$



Type-B

precession

$$N_{\text{type-B}} = \frac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$$

- $N_{\text{BS}} - N_{\text{NG}} = \dfrac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$

Classification

Expectation value of commutation relations

	Broken charges		Local operators	
	Q_b^A	Q_b^B	ϕ_i^A	ϕ_i^B
Type-A Q_a^A	0	0	$\langle[iQ_a^A, \phi_i^A]\rangle \neq 0$	$\langle[iQ_a^A, \phi_i^B]\rangle \neq 0$
Type-B Q_a^B	0	$\langle[iQ_a^B, Q_b^B]\rangle \neq 0$	0	$\langle[iQ_a^B, \phi_i^B]\rangle \neq 0$

$\langle[iQ_a^B, \phi_i^B]\rangle \neq 0$ ϕ_i^B becomes massive.

YH (12'), Hayata, YH ('14)

"Almost NG modes"

Effective Lagrangian approach: Kapustin ('12), Karasawa, Gongyo ('14)

The number of Type-B pairs

$$N_{\text{type-B}} = \frac{1}{2}\text{rank}\langle[iQ_a^B, Q_b^B]\rangle = \frac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$$

The number of Type-A pairs

$$N_{\text{type-A}} = N_{\text{BS}} - 2N_{\text{type-B}}$$

The number of Type-B fields.

$$\text{rank}\langle[iQ_a, \phi_i]\rangle - N_{\text{type-A}}$$

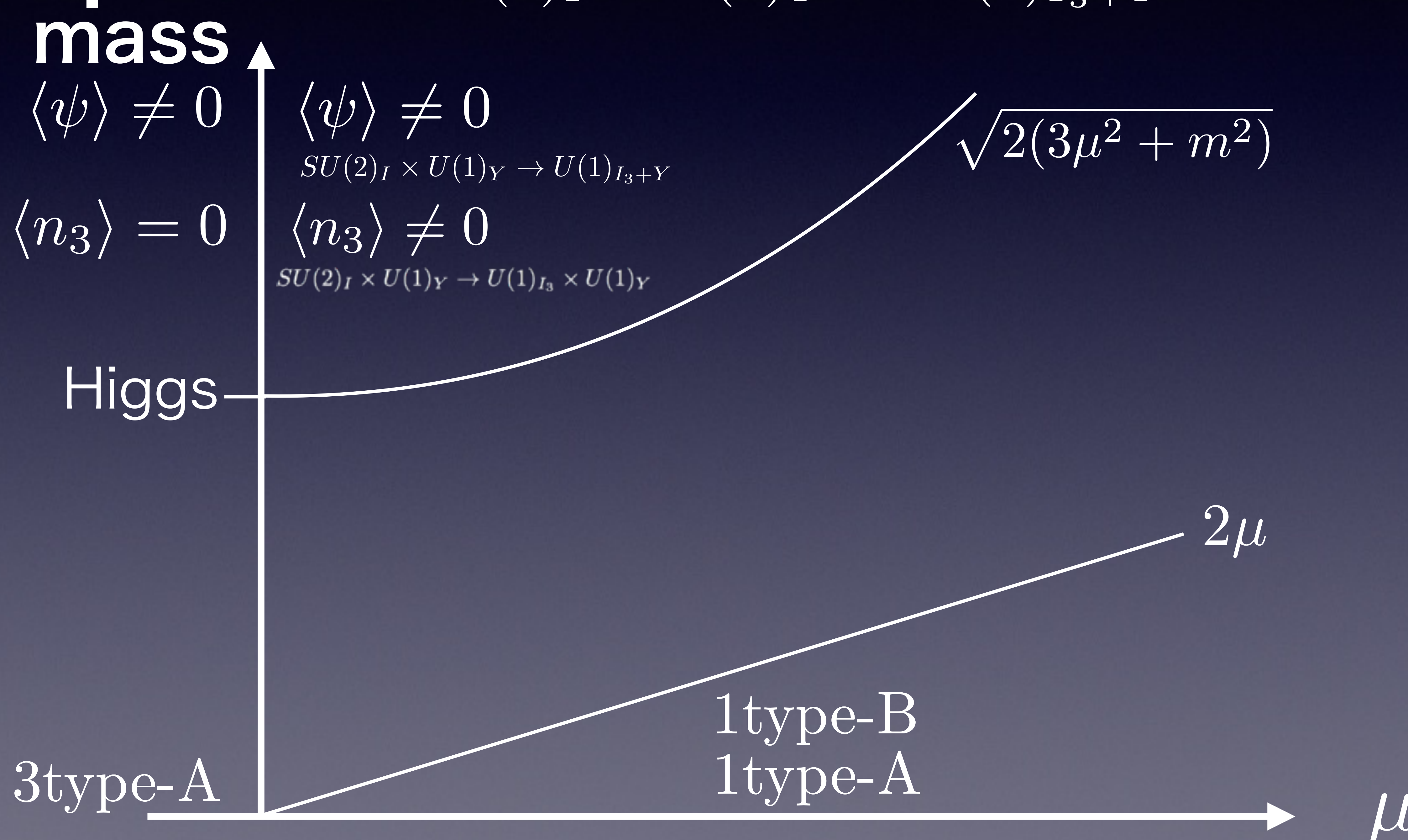
- $N_{\text{type-A}} + 2N_{\text{type-B}} = N_{\text{BS}}$
- $N_{\text{BS}} - N_{\text{NG}} = \frac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$
- $N_{\text{gapped}} = \frac{1}{2}\Big(\text{rank}\langle[iQ_a, \phi_i]\rangle - N_{\text{type-A}}\Big)$

Gapped partners

Ex.) SU(2)xU(1) model

$$\mathcal{L} = (\partial_\mu \varphi)^\dagger (\partial \varphi) + m^2 \varphi^\dagger \varphi - \lambda (\varphi^\dagger \varphi)^2 \quad \varphi = (\chi_1 + i\chi_2, \psi + i\pi)$$

SSB pattern: $SU(2)_I \times U(1)_Y \to U(1)_{I_3+Y}$



Examples of Type-B NG modes

	$N_{\rm BS}$	$N_{\text{type-A}}$	$N_{\text{type-B}}$	$\frac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$	$N_{\text{type-A}} + 2N_{\text{type-B}}$
Spin wave in ferromanget O(3)→O(2)	2	0	1	1	2
NG modes in Kaon condensed CFL SU(2)xSU(1)	3	1	1	1	3
Kelvin waves in vortex translation	2	0	1	1	2
nonrelativistic massive C U(1)x	2	0	1	1	2

$$N_{\text{type-A}} + 2N_{\text{type-B}} = N_{\rm BS} \qquad N_{\rm BS} - N_{\rm NG} = \frac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$$

Question:

What is the relation between type-A (B) and Type-I (II) dispersion relation?

Generalized Langevin equation

formal derivation: Mori ('65)

$$\partial_t A_n = \{A_n, F[A]\}_P - \Gamma_{nm} \frac{\partial F}{\partial A_m} + \xi_n$$

Free energy: $F[A]$

Poisson bracket: $\{A_n, A_m\}_P \equiv -i\langle [A_n, A_m] \rangle$

Dissipation term: Γ_{nm}

Noise term: ξ_n

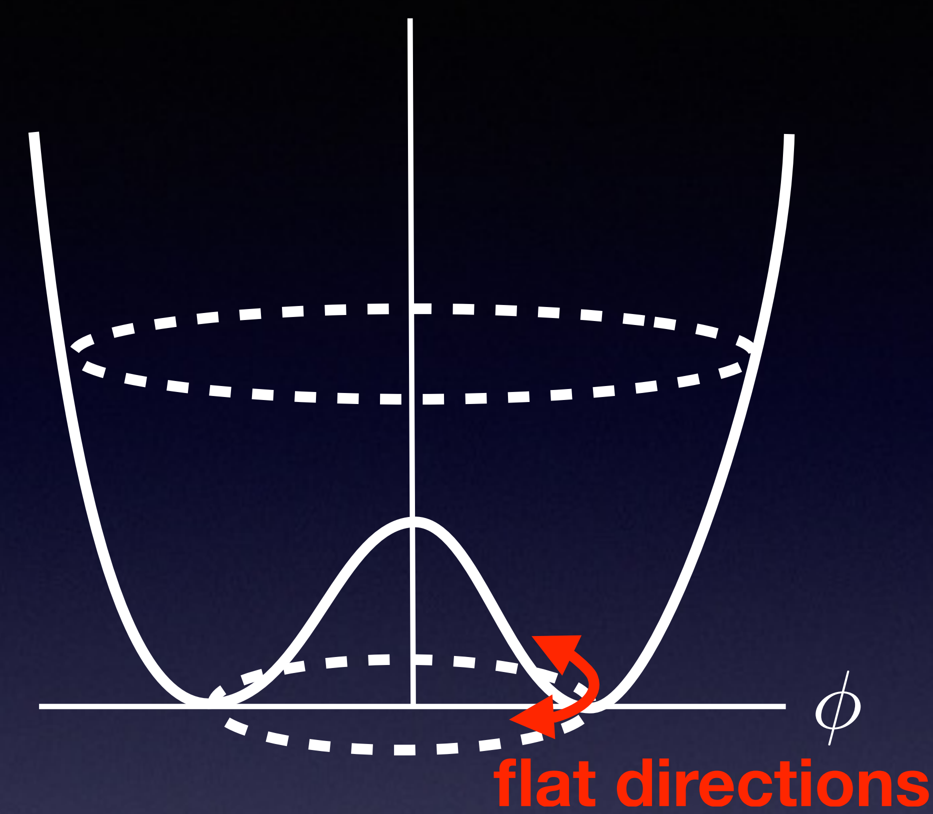
First, we neglect dissipation effect, i.e. $\Gamma=0$.

Type-A NG mode

$$F = \frac{\chi_n^{-1}}{2} n^2 + \frac{\rho}{2} (\nabla \pi)^2 + \cdots$$

$$\langle [iQ, \pi] \rangle = 1, \langle [iQ, n] \rangle = 0$$

with $Q = \int d^3 x n$



Equations of motion

$$\partial_t \phi = \chi_n^{-1} n$$

$$\partial_t n = \rho \nabla^2 \phi$$

$\Rightarrow$

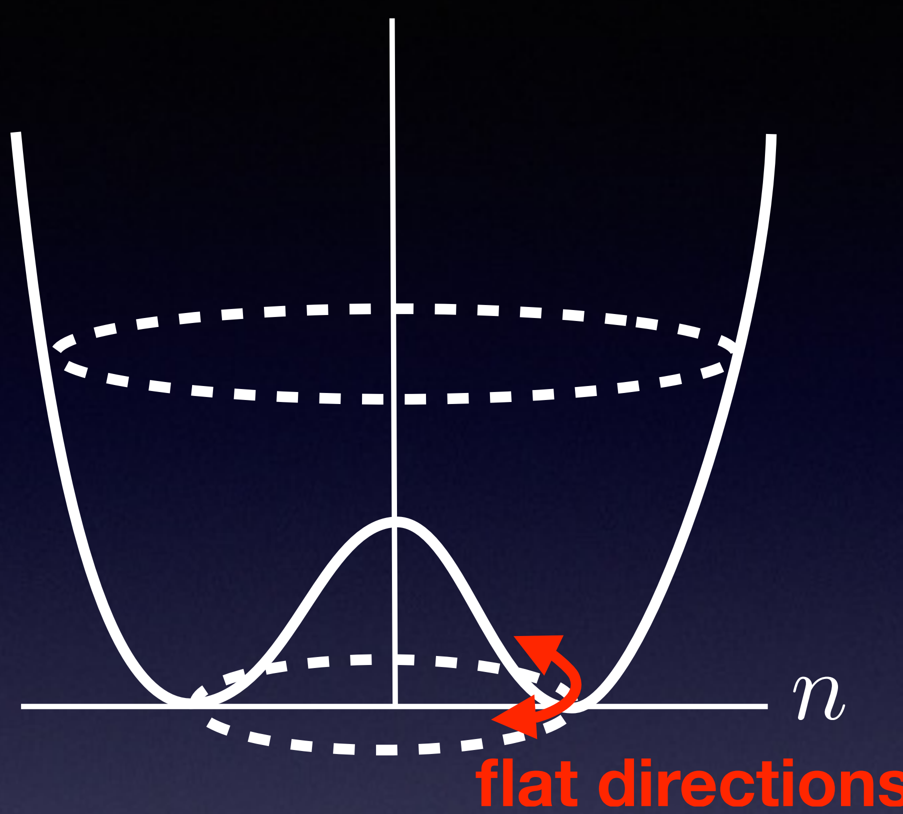
$$\omega = \pm \sqrt{\chi^{-1} \rho} k$$

Type-A = Type-I

Type-B NG mode

$$F = \rho\frac{1}{2}(\nabla n_1)^2 + \rho\frac{1}{2}(\nabla n_2)^2$$

$$\langle [iQ_1, n_2] \rangle = -\langle [iQ_2, n_1] \rangle = c$$



Equations of motion

$$\partial_t n_1 = c\rho\nabla^2 n_2$$
$$\partial_t n_2 = -c\rho\nabla^2 n_1$$

$$\Rightarrow \quad \omega = \pm\rho c k^2$$

Type-B = Type-II

Dissipation effect: Type-A

$$\partial_t A_n = \{A_n, F[A]\}_P - \Gamma_{nm}\frac{\partial F}{\partial A_m} + \xi_n$$

$$F = \frac{\chi_n^{-1}}{2}n^2 + \frac{1}{2}(\nabla\pi)^2 + \cdots$$

Kubo formula

$$\Gamma_{\pi\pi} = \int_0^\infty dt\int_0^\beta d\tau\int d^3x\langle\partial_t\pi(t-i\tau,\boldsymbol{x})\partial_t\pi(0,\boldsymbol{0})\rangle \equiv \gamma$$

$$\Gamma_{nn} = k^2\int_0^\infty dt\int_0^\beta d\tau\int d^3x\langle j^i(t-i\tau,\boldsymbol{x})j^i(0,\boldsymbol{0})\rangle \equiv k^2\sigma$$

Equations of motion

$$\partial_t\pi = \chi_n^{-1}n - \gamma\rho\nabla^2\pi + \xi_\pi$$

$$\partial_t n = \rho\nabla^2\pi + \sigma\chi_n^{-1}\nabla^2 n + \xi_n$$

$$\Rightarrow \quad \omega = \pm\sqrt{\chi^{-1}\rho}k - i\Gamma k^2 \qquad \Gamma = (\sigma\chi_n^{-1}+\gamma\rho)/2$$

Dissipation effect: Type-B

$$\partial_t A_n = \{A_n, F[A]\}_P - \Gamma_{nm}\frac{\partial F}{\partial A_m} + \xi_n$$

$$F = \rho\frac{1}{2}(\nabla n_1)^2 + \rho\frac{1}{2}(\nabla n_2)^2$$

Kubo formula

$$\Gamma_{n_a n_a} = k^2 \int_0^\infty dt \int_0^\beta d\tau \int d^3x \langle j_a^i(t - i\tau, \boldsymbol{x}) j_b^j(0, \mathbf{0})\rangle \equiv k^2\sigma$$

Equations of motion

$$\partial_t n_1 = c\rho\nabla^2 n_2 - \sigma\rho(\nabla^2)^2 n_1 + \xi_1$$

$$\partial_t n_2 = -c\rho\nabla^2 n_1 - \sigma\rho(\nabla^2)^2 n_2 + \xi_2$$

$$\Rightarrow \quad \omega = \pm c\rho k^2 - i\sigma\rho k^4$$

SSB with a small breaking term

$$H = H_0 + \epsilon V$$

small explicit breaking term

Type-A: $\partial_t \phi = \chi_n^{-1} n$

$$\partial_t n = \rho \nabla^2 \phi$$

$$\partial_t n = \epsilon w \phi$$

$$w \equiv \frac{\delta}{\delta \phi} \langle [iQ, V] \rangle$$

Type-B: $\partial_t n_a = \rho_a{}^b \nabla^2 n_b$

$$\partial_t n_a = \epsilon w_a{}^b n_b$$

$$w_a{}^b \equiv \frac{\delta}{\delta n_b} \langle [iQ_a, V] \rangle$$

Gap of Pseudo NG modes

Type-A: $m^2 = \epsilon \chi^{-1} w$

Ex) pions

Type-B: $[m^2]_a{}^b = \epsilon^2 \omega_a{}^c \omega_c{}^b$

Ex) magnon in an external magnetic field

cf.) No higher corrections if the explicit breaking term is proportional to a charge.

Nicolis, Piazza ('12), ('13) Watanabe, Brauner, Murayama ('13)

Summary

For SSB of internal symmetries

- Independent elastic variable=N_{BS}
- $N_{\text{type-B}} = \frac{1}{2}\mathrm{rank}\langle[iQ_a, Q_b]\rangle$
- $N_{\text{type-A}} = N_{\mathrm{BS}} - N_{\text{type-B}}$
- $N_{\text{gapped}} = \frac{1}{2}\left(\mathrm{rank}\langle[iQ_a, \phi_i]\rangle - N_{\text{type-A}}\right)$

The second derivative term in the effective Lagrangian
Karasawa, Gongyo('14)

Type-A (Type-I): $\omega = ak - ibk^2$

Type-B (Type-II): $\omega = a'k^2 - ib'k^4$

Spacetime breaking and dispersion

Ex) Liquid crystal

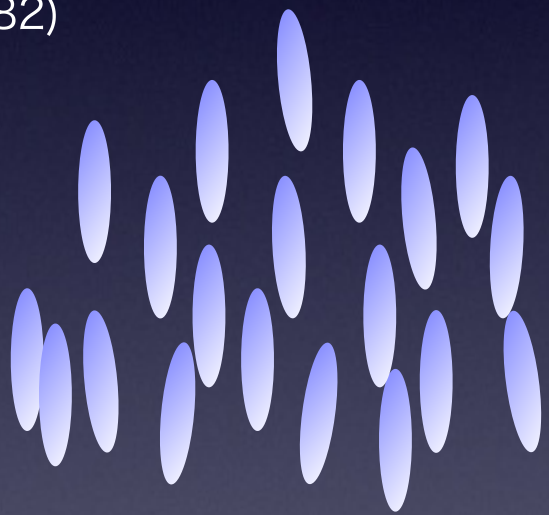
Nematic phase: rotation O(3)→O(2)

$N_{\rm BS} = N_{\rm EV} = 2$ $\quad L_i(x) = \epsilon_{ijk} x^j T^{0k}(x) \quad i = 1, 2$

Dispersion relation: $\omega = ak^2 + ibk^2$ Hosino, Nakano('82)

Real and imaginary parts are the same order (damped oscillation)

In case $a = 0$, (overdamping)



Ex) Capillary wave (ripplon)

cf. Takeuchi, Kasamatsu ('13) Effective Lagrangian: Watanabe, Murayama ('14)

$\frac{1}{V}\langle [P_z, N] \rangle \neq 0$ **Type-B** $\omega \sim k^{3/2}$